If in a group of people every one loves his friends' friends and also his enemies' enemies and hates his friends' enemies and his enemies' friends, the "signed graph" representing the network of relations will be balanced. The degree of balance of a graph, then, provides a sociometric index which may be of importance in constructing theories of group structure and behavior. There are several ways of defining the degree of balance, and the group has several ways of altering this index, for example by reversing existing relationships, by deleting them, or by expelling members. There may be tendencies other than toward balance operating in a group, and these may be opposed to each other. This paper continues the exploration of the mathematical properties of signed graphs with special reference to specific assumptions about the evolution of human groups.

ON THE MEASUREMENT OF STRUCTURAL BALANCE[1]

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INTRODUCTION

We propose three indices for "structural balance" in a group in which both "positive" and "negative" relationships between pairs of entities (e.g., persons) may occur, and investigate their logical ramifications. The concept of structural balance, as formulated by Cartwright and Harary (1956), serves to generalize the work of Heider (1946). There is more than one way to quantify the "amount of balance" in a group which is not completely balanced. One possible index, proposed in the article by Cartwright and Harary (1956), is based on the relative proportion of positive and negative "cycles." However, in addition to this, there are two other indices, "lines" and "points," which are appropriate for the measurement of structural balance. As developed below, we shall see that these correspond essentially to the number of interpersonal bonds and the number of individuals which must be removed from a group in order to attain balance. In the next three sections, we shall develop some of the theoretical properties of these indices, which are based on cycles, lines, and points of a structure.

These various indices for balance raise some unsolved empirical and theoretical problems concerning group relations. A particular group (with regard to a specified binary relation) may undertake an attempt to achieve structural balance in one way, perhaps by deleting certain interpersonal bonds, while another group may react by the expulsion of one of its members. To formalize the nature of these problems, we propose several reasonable tendencies for group behavior, and discuss their interrelations.

In order to treat these ideas with precision, some definitions from the mathematical theory of graphs are required. Expositions of the theory of graphs, directed graphs, and signed graphs may be found in articles by Harary and Norman (1953), Harary, Norman and Cartwright (in press), and Cartwright and Harary (1956). In a previous article on structural duality (Harary, 1957), the same concepts from the theory of graphs which are required here are utilized. Although an introductory development of the basic concepts of graph theory

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