12k Club (Instituto Militar de Engenharia) ACM-ICPC Team Notebook

Contents

1 Flags + Template + vimrc

1.1 Flags

```
alias comp="g++ -g -Wall -Wextra -std=c++20 -pedantic -O2 -
      Wshadow -Wformat=2 -Wfloat-equal -Wconversion -Wlogical-op
      -Wno-sign-compare -Wno-char-subscripts -Wshift-overflow=2 -
      Wduplicated-cond -Wcast-qual -Wcast-align -D_GLIBCXX_DEBUG
      -D_GLIBCXX_DEBUG_PEDANTIC -D_FORTIFY_SOURCE=2 -fsanitize=
      address,undefined -fno-sanitize-recover -fstack-protector -
      fno-omit-frame-pointer -Wno-unused-result"
```

1.2 Template

```
#define db(x) cerr << #x << " == " << x << endl
#define dbs(x) cerr << x << endl
#define _ << ", " <<
mt19937_64 llrand((int) chrono::steady_clock::now().
    time_since_epoch().count());
int main() {
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  //freopen("in", "r", stdin);
  //freopen("out", "w", stdout);
  return 0;
}
```

1.3 vimrc

```
syntax on
set et ts=2 sw=0 sts=-1 ai nu hls cindent
nnoremap ; :
vnoremap ; :
noremap <c-j> 15gj
noremap <c-k> 15gk
nnoremap <s-k> i<CR><ESC>
inoremap ,. <esc>
vnoremap ,. <esc>
nnoremap ,. <esc>
```

2 Data Structures

2.1 Bit Binary Search

```
// --- Bit Binary Search in o(log(n)) ---
const int M = 20
const int N = 1 << M

int lower_bound(int val){
  int ans = 0, sum = 0;
  for(int i = M - 1; i >= 0; i--){
    int x = ans + (1 << i);
    if(sum + bit[x] < val)
      ans = x, sum += bit[x];
  }

  return ans + 1;
}
```

2.2 Bit

```
// Fenwick Tree / Binary Indexed Tree
ll bit[N];

void add(int p, int v) {
  for (p += 2; p < N; p += p & -p) bit[p] += v;
}

ll query(int p) {
  ll r = 0;
  for (p += 2; p; p -= p & -p) r += bit[p];
  return r;
}
```

2.3 Bit 2D

```
// Thank you for the code tfg!
// O(N(logN)^2)
template<class T = int>
struct Bit2D{
  vector<T> ord;
  vector<vector<T>> fw, coord;

  // pts needs all points that will be used in the upd
  // if range upds remember to build with {x1, y1}, {x1, y2 +
      1}, {x2 + 1, y1}, {x2 + 1, y2 + 1}
  Bit2D(vector<pair<T, T>> pts){
    sort(pts.begin(), pts.end());
    for(auto a : pts)
      if(ord.empty() || a.first != ord.back())
        ord.push_back(a.first);
    fw.resize(ord.size() + 1);
    coord.resize(fw.size());

    for(auto &a : pts)
      swap(a.first, a.second);
    sort(pts.begin(), pts.end());
    for(auto &a : pts){
      swap(a.first, a.second);
      for(int on = std::upper_bound(ord.begin(), ord.end(), a.
          first) - ord.begin(); on < fw.size(); on += on & -on)
        if(coord[on].empty() || coord[on].back() != a.second)
          coord[on].push_back(a.second);
    }

    for(int i = 0; i < fw.size(); i++)
      fw[i].assign(coord[i].size() + 1, 0);
  }

  // point upd
  void upd(T x, T y, T v){
    for(int xx = upper_bound(ord.begin(), ord.end(), x) - ord.
        begin(); xx < fw.size(); xx += xx & -xx)
      for(int yy = upper_bound(coord[xx].begin(), coord[xx].end
          (), y) - coord[xx].begin(); yy < fw[xx].size(); yy +=
           yy & -yy)
        fw[xx][yy] += v;
  }

  // point qry
  T qry(T x, T y){
    T ans = 0;
    for(int xx = upper_bound(ord.begin(), ord.end(), x) - ord.
        begin(); xx > 0; xx -= xx & -xx)
      for(int yy = upper_bound(coord[xx].begin(), coord[xx].end
          (), y) - coord[xx].begin(); yy > 0; yy -= yy & -yy)
        ans += fw[xx][yy];
    return ans;
  }

  // range qry
  T qry(T x1, T y1, T x2, T y2){
    return qry(x2, y2) - qry(x2, y1 - 1) - qry(x1 - 1, y2) + qry
        (x1 - 1, y1 - 1);
  }

  // range upd
  void upd(T x1, T y1, T x2, T y2, T v) {
    upd(x1, y1, v);
    upd(x1, y2 + 1, -v);
    upd(x2 + 1, y1, -v);
    upd(x2 + 1, y2 + 1, v);
  }
};
```

2.4 Centroid Decomposition

```
struct Centroid {
  vector<vector<int>> adj;
  vector<bool> vis;
  vector<int> par, sz;
  int n;
  Centroid(int n_) {
      n = n_;
      adj.resize(n + 1); vis.resize(n + 1);
      par.resize(n + 1); sz.resize(n + 1);
  }
  void add(int a, int b) {
      adj[a].push_back(b);
      adj[b].push_back(a);
  }
  int dfs_sz(int v, int p = -1) {
      if(vis[v]) {
          return 0;
      }
      sz[v] = 1;
      for(auto x : adj[v]) {
          if(x != p) {
              sz[v] += dfs_sz(x, v);
          }
      }
      return sz[v];
  }
  int centroid(int v, int p, int size) {
      for(auto x : adj[v]) {
          if(x != p and !vis[x] and sz[x] > size / 2) {
              return centroid(x, v, size);
          }
      }
      return v;
  }
  void gen_tree(int v = 1, int p = 0) {
      int c = centroid(v, v, dfs_sz(v));
      vis[c] = true;
      par[c] = p;
      for(auto x : adj[c]) {
          if(!vis[x]) {
              gen_tree(x, c);
          }
      }
      vis[c] = false;
  }
  void dfs(vector<int> &path, int i, int p = -1, int d = 0) {
      path.push_back(d);
      for(auto j : adj[i]) {
          if(j != p and !vis[j]) {
              dfs(path, j, i, d + 1);
          }
      }
  }
  //count paths of size k in the tree
  //if you want upto k, just change cnt to be a Fenwick Tree
  long long decomp(int i, int k) {
      int c = centroid(i, i, dfs_sz(i));
      vis[c] = true;
      long long ans = 0;
      vector<int> cnt(sz[i]);
      cnt[0] = 1;
      for(auto j : adj[c]) {
          if(!vis[j]) {
              vector<int> path;
              dfs(path, j);
              for(int d : path) {
                  if(0 <= k - d - 1 and k - d - 1 < sz[i]) {
                      ans += cnt[k - d - 1];
                  }
              }
              for(int d : path) {
                  cnt[d + 1]++;
              }
          }
      }
      for(int j : adj[c]) {
          if(!vis[j]) {
              ans += decomp(j, k);
          }
      }
      vis[c] = false;
      return ans;
  }
};
```

2.5 Heavy-Light Decomposition (Lamarca)

```
#define fr(i,n) for(int i = 0; i<n; i++)
#define all(v) (v).begin(),(v).end()
typedef long long ll;

template<int N> struct Seg{
ll s[4*N], lazy[4*N];
void build(int no = 1, int l = 0, int r = N){
    if(r-l==1){
        s[no] = 0;
        return;
    }
    int mid = (l+r)/2;
    build(2*no,l,mid);
    build(2*no+1,mid,r);
    s[no] = max(s[2*no],s[2*no+1]);
}
Seg(){ //build da HLD tem de ser assim, pq chama sem os
    parametros
  build();
}
void updlazy(int no, int l, int r, ll x){
    s[no] += x;
    lazy[no] += x;
}
void pass(int no, int l, int r){
    int mid = (l+r)/2;
    updlazy(2*no,l,mid,lazy[no]);
    updlazy(2*no+1,mid,r,lazy[no]);
    lazy[no] = 0;
}
void upd(int lup, int rup, ll x, int no = 1, int l = 0, int r =
    N){
    if(rup<=l or r<=lup) return;
    if(lup<=l and r<=rup){
        updlazy(no,l,r,x);
        return;
    }
    pass(no,l,r);
    int mid = (l+r)/2;
    upd(lup,rup,x,2*no,l,mid);
    upd(lup,rup,x,2*no+1,mid,r);
    s[no] = max(s[2*no],s[2*no+1]);
}
ll qry(int lq, int rq, int no = 1, int l = 0, int r = N){
    if(rq<=l or r<=lq) return -LLONG_MAX;
    if(lq<=l and r<=rq){
        return s[no];
    }
    pass(no,l,r);
    int mid = (l+r)/2;
    return max(qry(lq,rq,2*no,l,mid),qry(lq,rq,2*no+1,mid,r));
}
};

template<int N, bool IN_EDGES> struct HLD {
  int t;
  vector<int> g[N];
  int pai[N], sz[N], d[N];
  int root[N], pos[N]; /// vi rpos;
  void ae(int a, int b) { g[a].push_back(b), g[b].push_back(a);
      }
  void dfsSz(int no = 0) {
    if (~pai[no]) g[no].erase(find(all(g[no]),pai[no]));
    sz[no] = 1;
    for(auto &it : g[no]) {
      pai[it] = no; d[it] = d[no]+1;
      dfsSz(it); sz[no] += sz[it];
      if (sz[it] > sz[g[no][0]]) swap(it, g[no][0]);
    }
  }
  void dfsHld(int no = 0) {
    pos[no] = t++; /// rpos.pb(no);
    for(auto &it : g[no]) {
      root[it] = (it == g[no][0] ? root[no] : it);
      dfsHld(it); }
  }
  void init() {
    root[0] = d[0] = t = 0; pai[0] = -1;
    dfsSz(); dfsHld(); }
  Seg<N> tree; //lembrar de ter build da seg sem nada
  template <class Op>
  void processPath(int u, int v, Op op) {
    for (; root[u] != root[v]; v = pai[root[v]]) {
      if (d[root[u]] > d[root[v]]) swap(u, v);
      op(pos[root[v]], pos[v]); }
    if (d[u] > d[v]) swap(u, v);
    op(pos[u]+IN_EDGES, pos[v]);
  }

  /*
  void changeNode(int v, node val){
    tree.upd(pos[v],val);
  }*/
  void modifySubtree(int v, int val) {
    tree.upd(pos[v]+IN_EDGES,pos[v]+sz[v],val);
  }
  ll querySubtree(int v){
    return tree.qry(pos[v]+IN_EDGES,pos[v]+sz[v]);
  }
  void modifyPath(int u, int v, int val) {
    processPath(u,v,[this, &val](int l,int r) {
      tree.upd(l,r+1,val); });
  }
  ll queryPath(int u, int v) { //modificacoes geralmente vem
      aqui (para hld soma)
    ll res = -LLONG_MAX; processPath(u,v,[this,&res](int l,int r
        ) {
```

```
        res = max(tree.qry(l,r+1),res); });
      return res;
    }
};
```

2.6 Lichao Tree (Cosenza)

```
struct lichao {
  struct line {
    long long m, b;
    long long operator()(long long x) const {
      return m * x + b;
    }
    bool cmp(const line &a, long long x) {
      return (*this)(x) < a(x);
    }
  };
  vector<line> seg;
  vector<int> L, R;
  inline void push() {
      seg.push_back({0, -linf});
      L.push_back(-1);
      R.push_back(-1);
  }
  lichao() {
      push();
  }
  void add(line a, int p = 0, long long l = -MAX, long long r =
      MAX) {
    long long mid = (l + r) >> 1;
    if(seg[p].cmp(a, mid)) {
      swap(seg[p], a);
    }
    if(a.b == -linf) {
      return;
    }
    if(seg[p].cmp(a, l) != seg[p].cmp(a, mid)) {
      if(L[p] == -1) {
        L[p] = seg.size();
        push();
      }
      add(a, L[p], l, mid - 1);
    } else if(seg[p].cmp(a, r) != seg[p].cmp(a, mid)) {
      if(R[p] == -1) {
        R[p] = seg.size();
        push();
      }
      add(a, R[p], mid + 1, r);
    }
  }
  long long query(long long x, int p = 0, long long l = -MAX,
    long long r = MAX) {
      if(p < 0) {
        return -linf;
      }
      long long mid = (l + r) >> 1, calc = seg[p](x);
      if(calc == -linf) {
        return calc;
      }
      if(x < mid) {
        return max(calc, query(x, L[p], l, mid - 1));
      } else {
        return max(calc, query(x, R[p], mid + 1, r));
      }
  }
};
```

2.7 Lichao Lazy (UFMG)

```
// Li-Chao Tree - Lazy
//
// Sendo N = MA-MI:
// insert({a, b}) minimiza tudo com ax+b - O(log N)
// insert({a, b}, l, r) minimiza com ax+b no range [l, r] - O(
      log^2 N)
// shift({a, b}) soma ax+b em tudo - O(1)
// shift({a, b}, l, r) soma ax+b no range [l, r] - O(log^2 N)
// query(x) retorna o valor da posicao x - O(log N)
//
// No inicio eh tudo LINF, se inserir {0, 0} fica tudo 0
//
// O(n log N) de memoria ; O(n) de memoria se nao usar as
      operacoes de range

template<int MI = int(-1e9), int MA = int(1e9)> struct lichao {
  struct line {
    ll a, b;
    ll la, lb; // lazy
    array<int, 2> ch;
    line(ll a_ = 0, ll b_ = LINF) :
      a(a_), b(b_), la(0), lb(0), ch({-1, -1}) {}
    ll operator ()(ll x) { return a*x + b; }
  };
  vector<line> ln;

  int ch(int p, int d) {
    if (ln[p].ch[d] == -1) {
      ln[p].ch[d] = ln.size();
      ln.emplace_back();
    }
    return ln[p].ch[d];
  }
  lichao() { ln.emplace_back(); }

  void prop(int p, int l, int r) {
    if (ln[p].la == 0 and ln[p].lb == 0) return;
    ln[p].a += ln[p].la, ln[p].b += ln[p].lb;
    if (l != r) {
      int pl = ch(p, 0), pr = ch(p, 1);
      ln[pl].la += ln[p].la, ln[pl].lb += ln[p].lb;
      ln[pr].la += ln[p].la, ln[pr].lb += ln[p].lb;
    }
    ln[p].la = ln[p].lb = 0;
  }

  ll query(int x, int p=0, int l=MI, int r=MA) {
    prop(p, l, r);
    ll ret = ln[p](x);
    if (ln[p].ch[0] == -1 and ln[p].ch[1] == -1) return ret;
    int m = l + (r-l)/2;
    if (x <= m) return min(ret, query(x, ch(p, 0), l, m));
    return min(ret, query(x, ch(p, 1), m+1, r));
  }

  void push(line s, int p, int l, int r) {
    prop(p, l, r);
    int m = l + (r-l)/2;
    bool L = s(l) < ln[p](l);
    bool M = s(m) < ln[p](m);
    bool R = s(r) < ln[p](r);
    if (M) swap(ln[p].a, s.a), swap(ln[p].b, s.b);
    if (s.b == LINF) return;
    if (L != M) push(s, ch(p, 0), l, m);
    else if (R != M) push(s, ch(p, 1), m+1, r);
  }
  void insert(line s, int a=MI, int b=MA, int p=0, int l=MI, int
      r=MA) {
    prop(p, l, r);
    if (a <= l and r <= b) return push(s, p, l, r);
    if (b < l or r < a) return;
    int m = l + (r-l)/2;
    insert(s, a, b, ch(p, 0), l, m);
    insert(s, a, b, ch(p, 1), m+1, r);
  }

  void shift(line s, int a=MI, int b=MA, int p=0, int l=MI, int
      r=MA) {
    prop(p, l, r);
    int m = l + (r-l)/2;
    if (a <= l and r <= b) {
      ln[p].la += s.a, ln[p].lb += s.b;
      return;
    }
    if (b < l or r < a) return;
    if (ln[p].b != LINF) {
      push(ln[p], ch(p, 0), l, m);
      push(ln[p], ch(p, 1), m+1, r);
      ln[p].a = 0, ln[p].b = LINF;
    }
    shift(s, a, b, ch(p, 0), l, m);
    shift(s, a, b, ch(p, 1), m+1, r);
  }
};
```

2.8 Merge Sort Tree

```
// Mergesort Tree - Time <O(nlogn), O(log^2n)> - Memory O(nlogn)
// Mergesort Tree is a segment tree that stores the sorted
      subarray
// on each node.
vector<int> st[4*N];
void build(int p, int l, int r) {
  if (l == r) { st[p].push_back(s[l]); return; }
  build(2*p, l, (l+r)/2);
  build(2*p+1, (l+r)/2+1, r);
  st[p].resize(r-l+1);
  merge(st[2*p].begin(), st[2*p].end(),
        st[2*p+1].begin(), st[2*p+1].end(),
        st[p].begin());
}

int query(int p, int l, int r, int i, int j, int a, int b) {
  if (j < l or i > r) return 0;
  if (i <= l and j >= r)
    return upper_bound(st[p].begin(), st[p].end(), b) -
           lower_bound(st[p].begin(), st[p].end(), a);
  return query(2*p, l, (l+r)/2, i, j, a, b) +
         query(2*p+1, (l+r)/2+1, r, i, j, a, b);
}
```

2.9 Minimum Queue

```
// O(1) complexity for all operations, except for clear,
// which could be done by creating another deque and using swap
struct MinQueue {
  int plus = 0;
  int sz = 0;
  deque<pair<int, int>> dq;

  bool empty() { return dq.empty(); }
  void clear() { plus = 0; sz = 0; dq.clear(); }
  void add(int x) { plus += x; } // Adds x to every element in
      the queue
  int min() { return dq.front().first + plus; } // Returns the
      minimum element in the queue
  int size() { return sz; }

  void push(int x) {
    x -= plus;
    int amt = 1;
    while (dq.size() and dq.back().first >= x)
      amt += dq.back().second, dq.pop_back();
    dq.push_back({ x, amt });
    sz++;
  }

  void pop() {
    dq.front().second--, sz--;
    if (!dq.front().second) dq.pop_front();
  }
};
```

2.10 Ordered Set

```
#include<bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
using namespace std;
using namespace __gnu_pbds;

typedef tree<int, null_type, less<int>, rb_tree_tag,
      tree_order_statistics_node_update> ordered_set;

ordered_set s;
s.insert(2), s.insert(3), s.insert(7), s.insert(9);

//find_by_order returns an iterator to the element at a given
      position
auto x = s.find_by_order(2);
cout << *x << "\n"; // 7

//order_of_key returns the position of a given element
cout << s.order_of_key(7) << "\n"; // 2

//If the element does not appear in the set, we get the position
      that the element would have in the set
cout << s.order_of_key(6) << "\n"; // 2
cout << s.order_of_key(8) << "\n"; // 3
```

2.11 Dynamic Segment Tree (Lazy)

```
vector<int> e, d, mx, lazy;
//begin creating node 0, then start your segment tree creating
      node 1
int create(){
  mx.push_back(0);
  lazy.push_back(0);
  e.push_back(0);
  d.push_back(0);
  return mx.size() - 1;
}
```

```cpp
void push(int pos, int ini, int fim){
  if(pos == 0) return;
  if (lazy[pos]) {
    mx[pos] += lazy[pos];
    // RMQ (max/min)    -> update: = lazy[p],         incr: +=
        lazy[p]
    // RSQ (sum)        -> update: = (r-l+1)*lazy[p], incr: += (r
        -l+1)*lazy[p]
    // Count lights on -> flip:   = (r-l+1)-st[p];
    if (ini != fim) {
      if(e[pos] == 0){
        int aux = create();
        e[pos] = aux;
      }
      if(d[pos] == 0){
        int aux = create();
        d[pos] = aux;
      }
      lazy[e[pos]] += lazy[pos];
      lazy[d[pos]] += lazy[pos];
      // update:    lazy[2*p] = lazy[p],  lazy[2*p+1] = lazy[p];
      // increment: lazy[2*p] += lazy[p], lazy[2*p+1] += lazy[p
          ];
      // flip:      lazy[2*p] ^= 1,       lazy[2*p+1] ^= 1;
    }
    lazy[pos] = 0;
  }
}

void update(int pos, int ini, int fim, int p, int q, int val){
  if(pos == 0) return;

  push(pos, ini, fim);

  if(q < ini || p > fim) return;

  if(p <= ini and fim <= q){
    lazy[pos] += val;
    // update:    lazy[p] = k;
    // increment: lazy[p] += k;
    // flip:      lazy[p] = 1;
    push(pos, ini, fim);
    return;
  }

  int m = (ini + fim) >> 1;
  if(e[pos] == 0){
    int aux = create();
    e[pos] = aux;
  }
  update(e[pos], ini, m, p, q, val);
  if(d[pos] == 0){
    int aux = create();
    d[pos] = aux;
  }
  update(d[pos], m + 1, fim, p, q, val);
  mx[pos] = max(mx[e[pos]], mx[d[pos]]);
}

int query(int pos, int ini, int fim, int p, int q){
  if(pos == 0) return 0;

  push(pos, ini, fim);

  if(q < ini || p > fim) return 0;

  if(p <= ini and fim <= q) return mx[pos];

  int m = (ini + fim) >> 1;
  return max(query(e[pos], ini, m, p, q) , query(d[pos], m + 1,
      fim, p, q));
}
```

2.12 Iterative Segment Tree

```cpp
int n; // Array size
int st[2*N];

int query(int a, int b) {
  a += n; b += n;
  int s = 0;
  while (a <= b) {
    if (a%2 == 1) s += st[a++];
    if (b%2 == 0) s += st[b--];
    a /= 2; b /= 2;
  }
  return s;
}

void update(int p, int val) {
  p += n;
  st[p] += val;
  for (p /= 2; p >= 1; p /= 2)
    st[p] = st[2*p]+st[2*p+1];
}
```

2.13 Persistent Segment Tree

```cpp
vector<int> e, d, sum;
//begin creating node 0, then start your segment tree creating
    node 1
int create(){
    sum.push_back(0);
    e.push_back(0);
    d.push_back(0);
    return sum.size() - 1;
}

int update(int pos, int ini, int fim, int id, int val){
    int novo = create();

    sum[novo] = sum[pos];
    e[novo] = e[pos];
    d[novo] = d[pos];
    pos = novo;

    if(ini == fim){
        sum[pos] = val;
        return novo;
    }

    int m = (ini + fim) >> 1;
    if(id <= m){
        int aux = update(e[pos], ini, m, id, val);
        e[pos] = aux;
    }
    else{
        int aux = update(d[pos], m + 1, fim, id, val);
        d[pos] = aux;
    }

    sum[pos] = sum[e[pos]] + sum[d[pos]];
    return pos;
}

int query(int pos, int ini, int fim, int p, int q){
    if(q < ini || p > fim) return 0;

    if(pos == 0) return 0;

    if(p <= ini and fim <= q) return sum[pos];

    int m = (ini + fim) >> 1;
    return query(e[pos], ini, m, p, q) + query(d[pos], m + 1,
        fim, p, q);
}
```

2.14 Mod Segment Tree

```cpp
// SegTree with mod
// op1 (l, r) -> sum a[i], i = { l .. r }
// op2 (l, r, x) -> a[i] = a[i] mod x, i = { l .. r }
// op3 (idx, x) -> a[idx] = x;
const int N = 1e5 + 5;

struct segTreeNode { ll sum, mx, mn, lz = -1; };

int n, m;
ll a[N];
segTreeNode st[4 * N];

void push(int p, int l, int r) {
  if (st[p].lz != -1) {

    st[p].mx = st[p].mn = st[p].lz;
    st[p].sum = (r - l + 1) * st[p].lz;

    if (l != r) st[2 * p].lz = st[2 * p + 1].lz = st[p].lz;
    st[p].lz = -1;
  }
}

void merge(int p) {
  st[p].mx = max(st[2 * p].mx, st[2 * p + 1].mx);
  st[p].mn = min(st[2 * p].mn, st[2 * p + 1].mn);
  st[p].sum = st[2 * p].sum + st[2 * p + 1].sum;
}

void build(int p = 1, int l = 1, int r = n) {
  if (l == r) {
    st[p].mn = st[p].mx = st[p].sum = a[l];
    return;
  }

  int mid = (l + r) >> 1;
  build(2 * p, l, mid);
  build(2 * p + 1, mid + 1, r);

  merge(p);
}

ll query(int i, int j, int p = 1, int l = 1, int r = n) {
  push(p, l, r);
  if (r < i or l > j) return 0ll;
  if (i <= l and r <= j) return st[p].sum;
  int mid = (l + r) >> 1;
  return query(i, j, 2 * p, l, mid) + query(i, j, 2 * p + 1, mid
      + 1, r);
}

void module_op(int i, int j, ll x, int p = 1, int l = 1, int r =
     n) {
  push(p, l, r);
  if (r < i or l > j or st[p].mx < x) return;
  if (i <= l and r <= j and st[p].mx == st[p].mn) {
    st[p].lz = st[p].mx % x;
    push(p, l, r);
    return;
  }
  int mid = (l + r) >> 1;
  module_op(i, j, x, 2 * p, l, mid);
  module_op(i, j, x, 2 * p + 1, mid + 1, r);

  merge(p);
}

void set_op(int i, int j, ll x, int p = 1, int l = 1, int r = n)
     {
  push(p, l, r);
  if (r < i or l > j) return;
  if (i <= l and r <= j) {
    st[p].lz = x;
    push(p, l, r);
    return;
  }
  int mid = (l + r) >> 1;
  set_op(i, j, x, 2 * p, l, mid);
  set_op(i, j, x, 2 * p + 1, mid + 1, r);

  merge(p);
}
```

2.15 Segment Tree 2D

```cpp
// Segment Tree 2D - O(nlog(n)log(n)) of Memory and Runtime
const int N = 1e8+5, M = 2e5+5;
int n, k=1, st[N], lc[N], rc[N];

void addx(int x, int l, int r, int u) {
  if (x < l or r < x) return;

  st[u]++;
  if (l == r) return;

  if(!rc[u]) rc[u] = ++k, lc[u] = ++k;
  addx(x, l, (l+r)/2, lc[u]);
  addx(x, (l+r)/2+1, r, rc[u]);
}

// Adds a point (x, y) to the grid.
void add(int x, int y, int l, int r, int u) {
  if (y < l or r < y) return;

  if (!st[u]) st[u] = ++k;
  addx(x, 1, n, st[u]);

  if (l == r) return;

  if(!rc[u]) rc[u] = ++k, lc[u] = ++k;
  add(x, y, l, (l+r)/2, lc[u]);
  add(x, y, (l+r)/2+1, r, rc[u]);
}
```

```
int countx(int x, int l, int r, int u) {
  if (!u or x < l) return 0;
  if (r <= x) return st[u];

  return countx(x, l, (l+r)/2, lc[u]) +
         countx(x, (l+r)/2+1, r, rc[u]);
}

// Counts number of points dominated by (x, y)
// Should be called with l = 1, r = n and u = 1
int count(int x, int y, int l, int r, int u) {
  if (!u or y < l) return 0;
  if (r <= y) return countx(x, 1, n, st[u]);

  return count(x, y, l, (l+r)/2, lc[u]) +
         count(x, y, (l+r)/2+1, r, rc[u]);
}
```

2.16 Set Of Intervals

```
template <class Info = int, class T = int>
struct ColorUpdate {
public:
  struct Range {
    Range(T _l = 0) : l(_l) {}
    Range(T _l, T _r, Info _v) : l(_l), r(_r), v(_v) { }
    T l, r;
    Info v;

    bool operator < (const Range &b) const { return l < b.l; }
  };

  std::vector<Range> erase(T l, T r) {
    std::vector<Range> ans;
    if(l >= r) return ans;
    auto it = ranges.lower_bound(l);
    if(it != ranges.begin()) {
      it--;
      if(it->r > l) {
        auto cur = *it;
        ranges.erase(it);
        ranges.insert(Range(cur.l, l, cur.v));
        ranges.insert(Range(l, cur.r, cur.v));
      }
    }
    it = ranges.lower_bound(r);
    if(it != ranges.begin()) {
      it--;
      if(it->r > r) {
        auto cur = *it;
        ranges.erase(it);
        ranges.insert(Range(cur.l, r, cur.v));
        ranges.insert(Range(r, cur.r, cur.v));
      }
    }
    for(it = ranges.lower_bound(l); it != ranges.end() && it->l
        < r; it++) {
      ans.push_back(*it);
    }
    ranges.erase(ranges.lower_bound(l), ranges.lower_bound(r));
    return ans;
  }

  std::vector<Range> upd(T l, T r, Info v) {
    auto ans = erase(l, r);
    ranges.insert(Range(l, r, v));
    return ans;
  }

  bool exists(T x) {
    auto it = ranges.upper_bound(x);
    if(it == ranges.begin()) return false;
    it--;
    return it->l <= x && x < it->r;
  }
  std::set<Range> ranges;
};

struct CrazySet {
  ColorUpdate<bool, long long> ranges;
  bool inverted = false;
  long long lazy = 0;

  void addLazy(long long x) {
    lazy += x;
  }

  void invert() {
    lazy = -lazy;
    inverted = !inverted;
  }

  void addRange(long long l, long long r) {
    if(!inverted) {
      ranges.upd(l-lazy, r-lazy, true);
    } else {
      ranges.upd(-r+1+lazy, -l+1+lazy, true);
    }
  }

  void removeRange(long long l, long long r) {
    if(!inverted) {
      ranges.erase(l-lazy, r-lazy);
    } else {
      ranges.erase(-r+1+lazy, -l+1+lazy);
    }
  }

  bool exists(long long x) {
    if(!inverted) {
      return ranges.exists(x - lazy);
    } else {
      return ranges.exists(-x + lazy);
    }
  }

  bool empty() { return ranges.ranges.empty(); }
};
```

2.17 Sparse Table

```
const int N;
const int M; //log2(N)
int sparse[N][M];

void build() {
  for(int i = 0; i < n; i++)
    sparse[i][0] = v[i];

  for(int j = 1; j < M; j++)
    for(int i = 0; i < n; i++)
      sparse[i][j] =
        i + (1 << j - 1) < n
        ? min(sparse[i][j - 1], sparse[i + (1 << j - 1)][j - 1])
        : sparse[i][j - 1];
}

int query(int a, int b){
  int pot = 32 - __builtin_clz(b - a) - 1;
  return min(sparse[a][pot], sparse[b - (1 << pot) + 1][pot]);
}
```

2.18 Sparse Table 2D

```
// 2D Sparse Table - <O(n^2 (log n) ^ 2), O(1)>
const int N = 1e3+1, M = 10;
int t[N][N], v[N][N], dp[M][M][N][N], lg[N], n, m;

void build() {
  int k = 0;
  for(int i=1; i<N; ++i) {
    if (1<<k == i/2) k++;
    lg[i] = k;
  }

  // Set base cases
  for(int x=0; x<n; ++x) for(int y=0; y<m; ++y) dp[0][0][x][y] =
      v[x][y];
  for(int j=1; j<M; ++j) for(int x=0; x<n; ++x) for(int y=0; y
     +(1<<j)<=m; ++y)
    dp[0][j][x][y] = max(dp[0][j-1][x][y], dp[0][j-1][x][y+(1<<j
        -1)]);

  // Calculate sparse table values
  for(int i=1; i<M; ++i) for(int j=0; j<M; ++j)
    for(int x=0; x+(1<<i)<=n; ++x) for(int y=0; y+(1<<j)<=m; ++y
       )
      dp[i][j][x][y] = max(dp[i-1][j][x][y], dp[i-1][j][x+(1<<i
          -1)][y]);
}

int query(int x1, int x2, int y1, int y2) {
  int i = lg[x2-x1+1], j = lg[y2-y1+1];
  int m1 = max(dp[i][j][x1][y1], dp[i][j][x2-(1<<i)+1][y1]);
  int m2 = max(dp[i][j][x1][y2-(1<<j)+1], dp[i][j][x2-(1<<i)+1][
      y2-(1<<j)+1]);
  return max(m1, m2);
}
```

2.19 KD Tree (Stanford)

```
const int maxn=200005;

struct kdtree
{
  int xl,xr,yl,yr,zl,zr,max,flag; // flag=0:x axis 1:y 2:z
} tree[5000005];

int N,M,lastans,xq,yq;
int a[maxn],pre[maxn],nxt[maxn];
int x[maxn],y[maxn],z[maxn],wei[maxn];
int xc[maxn],yc[maxn],zc[maxn],wc[maxn],hash[maxn],biao[maxn];

bool cmp1(int a,int b)
{
  return x[a]<x[b];
}

bool cmp2(int a,int b)
{
  return y[a]<y[b];
}

bool cmp3(int a,int b)
{
  return z[a]<z[b];
}

void makekdtree(int node,int l,int r,int flag)
{
  if (l>r)
  {
    tree[node].max=-maxlongint;
    return;
  }
  int xl=maxlongint,xr=-maxlongint;
  int yl=maxlongint,yr=-maxlongint;
  int zl=maxlongint,zr=-maxlongint,maxc=-maxlongint;
  for (int i=l;i<=r;i++)
    xl=min(xl,x[i]),xr=max(xr,x[i]),
    yl=min(yl,y[i]),yr=max(yr,y[i]),
    zl=min(zl,z[i]),zr=max(zr,z[i]),
    maxc=max(maxc,wei[i]),
    xc[i]=x[i],yc[i]=y[i],zc[i]=z[i],wc[i]=wei[i],biao[i]=i;
  tree[node].flag=flag;
  tree[node].xl=xl,tree[node].xr=xr,tree[node].yl=yl;
  tree[node].yr=yr,tree[node].zl=zl,tree[node].zr=zr;
  tree[node].max=maxc;
  if (l==r) return;
  if (flag==0) sort(biao+l,biao+r+1,cmp1);
  if (flag==1) sort(biao+l,biao+r+1,cmp2);
  if (flag==2) sort(biao+l,biao+r+1,cmp3);
  for (int i=l;i<=r;i++)
    x[i]=xc[biao[i]],y[i]=yc[biao[i]],
    z[i]=zc[biao[i]],wei[i]=wc[biao[i]];
  makekdtree(node*2,l,(l+r)/2,(flag+1)%3);
  makekdtree(node*2+1,(l+r)/2+1,r,(flag+1)%3);
}

int getmax(int node,int xl,int xr,int yl,int yr,int zl,int zr)
{
  xl=max(xl,tree[node].xl);
  xr=min(xr,tree[node].xr);
  yl=max(yl,tree[node].yl);
  yr=min(yr,tree[node].yr);
  zl=max(zl,tree[node].zl);
  zr=min(zr,tree[node].zr);
  if (tree[node].max==-maxlongint) return 0;
  if ((xr<tree[node].xl)||(xl>tree[node].xr)) return 0;
  if ((yr<tree[node].yl)||(yl>tree[node].yr)) return 0;
  if ((zr<tree[node].zl)||(zl>tree[node].zr)) return 0;
  if ((tree[node].xl==xl)&&(tree[node].xr==xr)&&
     (tree[node].yl==yl)&&(tree[node].yr==yr)&&
     (tree[node].zl==zl)&&(tree[node].zr==zr))
  return tree[node].max;
  else
  return max(getmax(node*2,xl,xr,yl,yr,zl,zr),
        getmax(node*2+1,xl,xr,yl,yr,zl,zr));
}

int main()
{
  // N 3D-rect with weights
```

```
  // find the maximum weight containing the given 3D-point
  return 0;
}
```

2.20 Treap

```
// Treap (probabilistic BST)
// O(logn) operations (supports lazy propagation)

mt19937_64 llrand(random_device{}());

struct node {
  int val;
  int cnt, rev;
  int mn, mx, mindiff; // value-based treap only!
  ll pri;
  node* l;
  node* r;

  node() {}
  node(int x) : val(x), cnt(1), rev(0), mn(x), mx(x), mindiff(
       INF), pri(llrand()), l(0), r(0) {}
};

struct treap {
  node* root;
  treap() : root(0) {}
  ~treap() { clear(); }

  int cnt(node* t) { return t ? t->cnt : 0; }
  int mn (node* t) { return t ? t->mn  : INF; }
  int mx (node* t) { return t ? t->mx  : -INF; }
  int mindiff(node* t) { return t ? t->mindiff : INF; }

  void clear() { del(root); }
  void del(node* t) {
    if (!t) return;
    del(t->l); del(t->r);
    delete t;
    t = 0;
  }

  void push(node* t) {
    if (!t or !t->rev) return;
    swap(t->l, t->r);
    if (t->l) t->l->rev ^= 1;
    if (t->r) t->r->rev ^= 1;
    t->rev = 0;
  }

  void update(node*& t) {
    if (!t) return;
    t->cnt = cnt(t->l) + cnt(t->r) + 1;
    t->mn = min(t->val, min(mn(t->l), mn(t->r)));
    t->mx = max(t->val, max(mx(t->l), mx(t->r)));
    t->mindiff = min(mn(t->r) - t->val, min(t->val - mx(t->l),
          min(mindiff(t->l), mindiff(t->r))));
  }

  node* merge(node* l, node* r) {
    push(l); push(r);
    node* t;
    if (!l or !r) t = l ? l : r;
    else if (l->pri > r->pri) l->r = merge(l->r, r), t = l;
    else r->l = merge(l, r->l), t = r;
    update(t);
    return t;
  }

  // pos: amount of nodes in the left subtree or
  // the smallest position of the right subtree in a 0-indexed
       array
  pair<node*, node*> split(node* t, int pos) {
    if (!t) return {0, 0};
    push(t);

    if (cnt(t->l) < pos) {
      auto x = split(t->r, pos-cnt(t->l)-1);
      t->r = x.st;
      update(t);
      return { t, x.nd };
    }

    auto x = split(t->l, pos);
    t->l = x.nd;
    update(t);
    return { x.st, t };
  }

  // Position-based treap
  // used when the values are just additional data
  // the positions are known when it's built, after that you
  // query to get the values at specific positions
  // 0-indexed array!
  /*
  void insert(int pos, int val) {
    push(root);
    node* x = new node(val);
    auto t = split(root, pos);
    root = merge(merge(t.st, x), t.nd);
  }

  void erase(int pos) {
    auto t1 = split(root, pos);
    auto t2 = split(t1.nd, 1);
    delete t2.st;
    root = merge(t1.st, t2.nd);
  }

  int get_val(int pos) { return get_val(root, pos); }
  int get_val(node* t, int pos) {
    push(t);
    if (cnt(t->l) == pos) return t->val;
    if (cnt(t->l) < pos) return get_val(t->r, pos-cnt(t->l)-1);
    return get_val(t->l, pos);
  }
  */
  // -------------------

  // Value-based treap
  // used when the values needs to be ordered
  int order(node* t, int val) {
    if (!t) return 0;
    push(t);
    if (t->val < val) return cnt(t->l) + 1 + order(t->r, val);
    return order(t->l, val);
  }

  bool has(node* t, int val) {
    if (!t) return 0;
    push(t);
    if (t->val == val) return 1;
    return has((t->val > val ? t->l : t->r), val);
  }

  void insert(int val) {
    if (has(root, val)) return; // avoid repeated values
    push(root);
    node* x = new node(val);
    auto t = split(root, order(root, val));
    root = merge(merge(t.st, x), t.nd);
  }

  void erase(int val) {
    if (!has(root, val)) return;

    auto t1 = split(root, order(root, val));
    auto t2 = split(t1.nd, 1);
    delete t2.st;
    root = merge(t1.st, t2.nd);
  }

  // Get the maximum difference between values
  int querymax(int i, int j) {
    if (i == j) return -1;
    auto t1 = split(root, j+1);
    auto t2 = split(t1.st, i);

    int ans = mx(t2.nd) - mn(t2.nd);
    root = merge(merge(t2.st, t2.nd), t1.nd);
    return ans;
  }

  // Get the minimum difference between values
  int querymin(int i, int j) {
    if (i == j) return -1;
    auto t2 = split(root, j+1);
    auto t1 = split(t2.st, i);

    int ans = mindiff(t1.nd);
    root = merge(merge(t1.st, t1.nd), t2.nd);
    return ans;
  }
  // ------------------

  void reverse(int l, int r) {
    auto t2 = split(root, r+1);
    auto t1 = split(t2.st, l);
    t1.nd->rev = 1;
    root = merge(merge(t1.st, t1.nd), t2.nd);
  }

  void print() { print(root); printf("\n"); }
  void print(node* t) {
    if (!t) return;
    push(t);
    print(t->l);
    printf("%d ", t->val);
    print(t->r);
  }
};
```

2.21 Trie

```
// Trie <O(|S|), O(|S|)>
int trie[N][26], trien = 1;

int add(int u, char c){
  c-='a';
  if (trie[u][c]) return trie[u][c];
  return trie[u][c] = ++trien;
}

//to add a string s in the trie
int u = 1;
for(char c : s) u = add(u, c);
```

2.22 Union Find

```
// DSU (DISJOINT SET UNION / UNION-FIND)
// Time complexity:  Unite - O(alpha n)
//                   Find - O(alpha n)
// Usage: find(node), unite(node1, node2), sz[find(node)]
// Notation: par: vector of parents
//           sz:  vector of subsets sizes, i.e. size of the
     subset a node is in
int par[N], sz[N], his[N];
stack <pii> sp, ss;

int find(int a) { return par[a] == a ? a : par[a] = find(par[a])
     ; }

void unite(int a, int b) {
  if ((a = find(a)) == (b = find(b))) return;
  if (sz[a] < sz[b]) swap(a, b);
  par[b] = a; sz[a] += sz[b];
}

//in main
for(int i = 0; i < N; i++) par[i] = i, sz[i] = 1, his[i] = 0;

//Rollback
int find (int a) { return par[a] == a ? a : find(par[a]); }

void unite (int a, int b) {
  if ((a = find(a)) == (b = find(b))) return;
  if (sz[a] < sz[b]) swap(a, b);
  ss.push({a, sz[a]});
  sp.push({b, par[b]});
  sz[a] += sz[b];
  par[b] = a;
}

void rollback() {
  par[sp.top().st] = sp.top().nd; sp.pop();
  sz[ss.top().st]  = ss.top().nd; ss.pop();
}

//Partial Persistence
int t, par[N], sz[N]

int find(int a, int t){
  if(par[a] == a) return a;
  if(his[a] > t) return a;
  return find(par[a], t);
}

void unite(int a, int b){
  if(find(a, t) == find(b, t)) return;
  a = find(a, t), b = find(b, t), t++;
  if(sz[a] < sz[b]) swap(a, b);
  sz[a] += sz[b], par[b] = a, his[b] = t;
}
```

3 Dynamic Programming

3.1 Convex Hull Trick (emaxx)

```
struct Point{
  ll x, y;
  Point(ll x = 0, ll y = 0):x(x), y(y) {}
  Point operator-(Point p){ return Point(x - p.x, y - p.y); }
  Point operator+(Point p){ return Point(x + p.x, y + p.y); }
  Point ccw(){ return Point(-y, x); }
  ll operator%(Point p){ return x*p.y - y*p.x; }
  ll operator*(Point p){ return x*p.x + y*p.y; }
  bool operator<(Point p) const { return x == p.x ? y < p.y : x
       < p.x; }};

pair<vector<Point>, vector<Point>> ch(Point *v){
  vector<Point> hull, vecs;
  for(int i = 0; i < n; i++){
    if(hull.size() and hull.back().x == v[i].x) continue;

    while(vecs.size() and vecs.back()*(v[i] - hull.back()) <= 0)
      vecs.pop_back(), hull.pop_back();

    if(hull.size())
      vecs.pb((v[i] - hull.back()).ccw());

    hull.pb(v[i]);
  }
  return {hull, vecs};
}

ll get(ll x) {
    Point query = {x, 1};
    auto it = lower_bound(vecs.begin(), vecs.end(), query, [](
          Point a, Point b) {
        return a%b > 0;
    });
    return query*hull[it - vecs.begin()];
}
```

3.2 Divide and Conquer Optimization

```
// DIVIDE AND CONQUER OPTIMIZATION ( dp[i][k] = min j<k {dp[j][k
     -1] + C(j,i)} )
// Description: searches for bounds to optimal point using the
     monotocity condition
// Condition: L[i][k] <= L[i+1][k]
// Time Complexity: O(KN^2) becomes O(KNlogN)
// Notation: dp[i][k]: optimal solution using k positions, until
      position i
//           L[i][k]: optimal point, smallest j which minimizes
     dp[i][k]
//           C(i,j): cost for splitting range [j,i] to j and i
const int N = 1e3+5;

ll dp[N][N];

//Cost for using i and j
ll C(ll i, ll j);

void compute(ll l, ll r, ll k, ll optl, ll optr){
    // stop condition
    if(l > r) return;

    ll mid = (l+r)/2;
    //best : cost, pos
    pair<ll,ll> best = {LINF,-1};

    //searchs best: lower bound to right, upper bound to left
    for(ll i = optl; i <= min(mid, optr); i++){
        best = min(best, {dp[i][k-1] + C(i,mid), i});
    }
    dp[mid][k] = best.first;
    ll opt = best.second;

    compute(l, mid-1, k, optl, opt);
    compute(mid + 1, r, k, opt, optr);
}

//Iterate over k to calculate
ll solve(){
  //dimensions of dp[N][K]
  int n, k;

  //Initialize DP
  for(ll i = 1; i <= n; i++){
    //dp[i,1] = cost from 0 to i
    dp[i][1] = C(0, i);
  }

  for(ll l = 2; l <= k; l++){
    compute(1, n, l, 1, n);
  }

  /*+ Iterate over i to get min{dp[i][k]}, don't forget cost
       from n to i
    for(ll i=1;i<=n;i++){
        ll rest = ;
        ans = min(ans,dp[i][k] + rest);
    }
  */
}
```

3.3 Knuth Optimization

```
// Knuth DP Optimization - O(n^3) -> O(n^2)
//
// 1) dp[i][j] = min i<k<j { dp[i][k] + dp[k][j] } + C[i][j]
// 2) dp[i][j] = min k<i { dp[k][j-1] + C[k][i] }
//
// Condition: A[i][j-1] <= A[i][j] <= A[i+1][j]
// A[i][j] is the smallest k that gives an optimal answer to dp[
     i][j]
//
// reference (pt-br): https://algorithmmarch.wordpress.com
     /2016/08/12/a-otimizacao-de-pds-e-o-garcom-da-maratona/
//

// 1) dp[i][j] = min i<k<j { dp[i][k] + dp[k][j] } + C[i][j]
int n;
int dp[N][N], a[N][N];

// declare the cost function
int cost(int i, int j) {
  // ...
}

void knuth() {
  // calculate base cases
  memset(dp, 63, sizeof(dp));
  for (int i = 1; i <= n; i++) dp[i][i] = 0;

  // set initial a[i][j]
  for (int i = 1; i <= n; i++) a[i][i] = i;

  for (int j = 2; j <= n; ++j)
    for (int i = j; i >= 1; --i){
      for (int k = a[i][j-1]; k <= a[i+1][j]; ++k) {
        ll v = dp[i][k] + dp[k][j] + cost(i, j);

        // store the minimum answer for d[i][k]
        // in case of maximum, use v > dp[i][k]
        if (v < dp[i][j])
          a[i][j] = k, dp[i][j] = v;
      }
      //+ Iterate over i to get min{dp[i][j]} for each j, don't
           forget cost from n to
    }
}


// 2) dp[i][j] = min k<i { dp[k][j-1] + C[k][i] }
int n, maxj;
int dp[N][J], a[N][J];

// declare the cost function
int cost(int i, int j) {
  // ...
}

void knuth() {
  // calculate base cases
  memset(dp, 63, sizeof(dp));
  for (int i = 1; i <= n; i++) dp[i][1] = // ...

  // set initial a[i][j]
  for (int i = 1; i <= n; i++) a[i][1] = 1, a[n+1][i] = n;

  for (int j = 2; j <= maxj; j++)
    for (int i = n; i >= 1; i--){
      for (int k = a[i][j-1]; k <= a[i+1][j]; k++) {
        ll v = dp[k][j-1] + cost(k, i);

        // store the minimum answer for d[i][k]
        // in case of maximum, use v > dp[i][k]
        if (v < dp[i][j])
          a[i][j] = k, dp[i][j] = v;
      }
      //+ Iterate over i to get min{dp[i][j]} for each j, don't
           forget cost from n to
    }
}
```

3.4 SOS DP

```
// O(N * 2^N)
// A[i] = initial values
// Calculate F[i] = Sum of A[j] for j subset of i
for(int i = 0; i < (1 << N); i++)
  F[i] = A[i];
for(int i = 0; i < N; i++)
  //add to superset
  for(int j = 0; j < (1 << N); j++)
    if(j & (1 << i))
      F[j] += F[j ^ (1 << i)];
  //add to subset
  for(int j = (1 << N) - 1; j >= 0; j--)
    if(j & (1 << i))
      F[j ^ (1 << i)] += F[j];
```

3.5 Steiner tree

```
// Steiner-Tree O(2^t*n^2 + n*3^t + APSP)
// N - number of nodes
// T - number of terminals
// dist[N][N] - Adjacency matrix
// steiner_tree() = min cost to connect first t nodes, 1-indexed
// dp[i][bit_mask] = min cost to connect nodes active in bitmask
     rooting in i
// min{dp[i][bit_mask]}, i <= n if root doesn't matter

int n, t, dp[N][(1 << T)], dist[N][N];

int steiner_tree() {
  for (int k = 1; k <= n; ++k)
    for (int i = 1; i <= n; ++i)
      for (int j = 1; j <= n; ++j)
        dist[i][j] = min(dist[i][j], dist[i][k] + dist[k][j]);

  for(int i = 1; i <= n; i++)
    for(int j = 0; j < (1 << t); j++)
      dp[i][j] = INF;
  for(int i = 1; i <= t; i++) dp[i][1 << (i-1)] = 0;

  for(int msk = 0; msk < (1 << t); msk++) {
    for(int i = 1; i <= n; i++) {
      for(int ss = msk; ss > 0; ss = (ss - 1) & msk)
        dp[i][msk] = min(dp[i][msk], dp[i][ss] + dp[i][msk - ss
            ]);

      if(dp[i][msk] != INF)
        for(int j = 1; j <= n; j++)
          dp[j][msk] = min(dp[j][msk], dp[i][msk] + dist[i][j]);
    }
  }

  int mn = INF;
  for(int i = 1; i <= n; i++) mn = min(mn, dp[i][(1 << t) - 1]);
  return mn;
}
```

4 Graphs

4.1 2-SAT Kosaraju

```
// Time complexity: O(V+E)
int n, vis[2*N], ord[2*N], ordn, cnt, cmp[2*N], val[N];
vector<int> adj[2*N], adjt[2*N];

// for a variable u with idx i
// u is 2*i and !u is 2*i+1
// (a v b) == !a -> b ^ !b -> a

int v(int x) { return 2*x; }
int nv(int x) { return 2*x+1; }

// add clause (a v b)
void add(int a, int b){
  adj[a^1].push_back(b);
```

```
  adj[b^1].push_back(a);
  adjt[b].push_back(a^1);
  adjt[a].push_back(b^1);
}

void dfs(int x){
  vis[x] = 1;
  for(auto v : adj[x]) if(!vis[v]) dfs(v);
  ord[ordn++] = x;
}

void dfst(int x){
  cmp[x] = cnt, vis[x] = 0;
  for(auto v : adjt[x]) if(vis[v]) dfst(v);
}

bool run2sat(){
  for(int i = 1; i <= n; i++) {
    if(!vis[v(i)]) dfs(v(i));
    if(!vis[nv(i)]) dfs(nv(i));
  }
  for(int i = ordn-1; i >= 0; i--)
    if(vis[ord[i]]) cnt++, dfst(ord[i]);
  for(int i = 1; i <= n; i ++){
    if(cmp[v(i)] == cmp[nv(i)]) return false;
    val[i] = cmp[v(i)] > cmp[nv(i)];
  }
  return true;
}

int main () {
    for (int i = 1; i <= n; i++) {
        if (val[i]); // i-th variable is true
        else         // i-th variable is false
}
```

4.2 Shortest Path (Bellman-Ford)

```
//Time complexity: O(VE)
const int N = 1e4+10; // Maximum number of nodes
vector<int> adj[N], adjw[N];
int dist[N], v, w;

memset(dist, 63, sizeof(dist));
dist[0] = 0;
for (int i = 0; i < n-1; ++i)
  for (int u = 0; u < n; ++u)
    for (int j = 0; j < adj[u].size(); ++j)
      v = adj[u][j], w = adjw[u][j],
      dist[v] = min(dist[v], dist[u]+w);
```

4.3 Floyd Warshall

```
int adj[N][N]; // no-edge = INF
for (int k = 0; k < n; ++k)
  for (int i = 0; i < n; ++i) for (int j = 0; j < n; ++j)
    adj[i][j] = min(adj[i][j], adj[i][k]+adj[k][j]);
```

4.4 Block Cut

```
// Tarjan for Block Cut Tree (Node Biconnected Componentes) - O(
     n + m)
const int N = 1e5+5;

// Regular Tarjan stuff
int n, num[N], low[N], cnt, ch[N], art[N];
vector<int> adj[N], st;

int lb[N]; // Last block that node is contained
int bn; // Number of blocks
vector<int> blc[N]; // List of nodes from block

void dfs(int u, int p) {
  num[u] = low[u] = ++cnt;
  ch[u] = adj[u].size();
  st.push_back(u);

  if (adj[u].size() == 1) blc[++bn].push_back(u);

  for(int v : adj[u]) {
    if (!num[v]) {
      dfs(v, u), low[u] = min(low[u], low[v]);
      if (low[v] == num[u]) {
        if (p != -1 or ch[u] > 1) art[u] = 1;
        blc[++bn].push_back(u);
        while(blc[bn].back() != v)
          blc[bn].push_back(st.back()), st.pop_back();
      }
    }
    else if (v != p) low[u] = min(low[u], num[v]), ch[v]--;
  }

  if (low[u] == num[u]) st.pop_back();
}

// Nodes from 1 .. n are blocks
// Nodes from n+1 .. 2*n are articulations
vector<int> bct[2*N]; // Adj list for Block Cut Tree

void build_tree() {
  for(int u=1; u<=n; ++u) for(int v : adj[u]) if (num[u] > num[v
       ]) {
    if (lb[u] == lb[v] or blc[lb[u]][0] == v) /* edge u-v
       belongs to block lb[u] */;
    else { /* edge u-v belongs to block cut tree */;
      int x = (art[u] ? u + n : lb[u]), y = (art[v] ? v + n : lb
           [v]);
      bct[x].push_back(y), bct[y].push_back(x);
    }
  }
}

void tarjan() {
  for(int u=1; u<=n; ++u) if (!num[u]) dfs(u, -1);
  for(int b=1; b<=bn; ++b) for(int u : blc[b]) lb[u] = b;
  build_tree();
}
```

4.5 Articulation points and bridges

```
// Articulation points and Bridges O(V+E)
int par[N], art[N], low[N], num[N], ch[N], cnt;

void articulation(int u) {
  low[u] = num[u] = ++cnt;
  for (int v : adj[u]) {
    if (!num[v]) {
      par[v] = u; ch[u]++;
      articulation(v);
      if (low[v] >= num[u]) art[u] = 1;
      if (low[v] >  num[u]) { /* u-v bridge */ }
      low[u] = min(low[u], low[v]);
    }
    else if (v != par[u]) low[u] = min(low[u], num[v]);
  }
}

for (int i = 0; i < n; ++i) if (!num[i])
  articulation(i), art[i] = ch[i]>1;
```

4.6 Dominator Tree

```
// a node u is said to be dominating node v if, from every path
      from the entry point to v you have to pass through u
// so this code is able to find every dominator from a specific
      entry point (usually 1)
// for directed graphs obviously

const int N = 1e5 + 7;

vector<int> adj[N], radj[N], tree[N], bucket[N];
int sdom[N], par[N], dom[N], dsu[N], label[N], arr[N], rev[N],
     cnt;

void dfs(int u) {
  cnt++;
  arr[u] = cnt;
  rev[cnt] = u;
  label[cnt] = cnt;
  sdom[cnt] = cnt;
  dsu[cnt] = cnt;
  for(auto e : adj[u]) {
    if(!arr[e]) {
      dfs(e);
      par[arr[e]] = arr[u];
    }
    radj[arr[e]].push_back(arr[u]);
  }
}

int find(int u, int x = 0) {
  if(u == dsu[u]) {
    return (x ? -1 : u);
  }
  int v = find(dsu[u], x + 1);
  if(v == -1) {
    return u;
  }
  if(sdom[label[dsu[u]]] < sdom[label[u]]) {
    label[u] = label[dsu[u]];
  }
  dsu[u] = v;
  return (x ? v : label[u]);
}

void unite(int u, int v) {
  dsu[v] = u;
}

// in main

dfs(1);
for(int i = cnt; i >= 1; i--) {
  for(auto e : radj[i]) {
    sdom[i] = min(sdom[i], sdom[find(e)]);
  }
  if(i > 1) {
    bucket[sdom[i]].push_back(i);
  }
  for(auto e : bucket[i]) {
    int v = find(e);
    if(sdom[e] == sdom[v]) {
      dom[e] = sdom[e];
    } else {
      dom[e] = v;
    }
  }
  if(i > 1) {
    unite(par[i], i);
  }
}
for(int i = 2; i <= cnt; i++) {
  if(dom[i] != sdom[i]) {
    dom[i] = dom[dom[i]];
  }
  tree[rev[i]].push_back(rev[dom[i]]);
  tree[rev[dom[i]]].push_back(rev[i]);
}
```

4.7 Erdos Gallai

```
// Erdos-Gallai - O(nlogn)
// check if it's possible to create a simple graph (undirected
     edges) from
// a sequence of vertice's degrees
bool gallai(vector<int> v) {
  vector<ll> sum;
  sum.resize(v.size());

  sort(v.begin(), v.end(), greater<int>());
  sum[0] = v[0];
  for (int i = 1; i < v.size(); i++) sum[i] = sum[i-1] + v[i];
  if (sum.back() % 2) return 0;

  for (int k = 1; k < v.size(); k++) {
    int p = lower_bound(v.begin(), v.end(), k, greater<int>()) -
         v.begin();
    if (p < k) p = k;
    if (sum[k-1] > 1ll*k*(p-1) + sum.back() - sum[p-1]) return
         0;
  }
  return 1;
}
```

4.8 Eulerian Path

```
vector<int> ans, adj[N];
int in[N];

void dfs(int v){
  while(adj[v].size()){
    int x = adj[v].back();
    adj[v].pop_back();
    dfs(x);
  }
  ans.pb(v);
}

// Verify if there is an eulerian path or circuit
vector<int> v;
```

```cpp
for(int i = 0; i < n; i++) if(adj[i].size() != in[i]){
  if(abs((int)adj[i].size() - in[i]) != 1) //-> There is no
        valid eulerian circuit/path
  v.pb(i);
}

if(v.size()){
  if(v.size() != 2) //-> There is no valid eulerian path
  if(in[v[0]] > adj[v[0]].size()) swap(v[0], v[1]);
  if(in[v[0]] > adj[v[0]].size()) //-> There is no valid
        eulerian path
  adj[v[1]].pb(v[0]); // Turn the eulerian path into a eulerian
        circuit
}

dfs(0);
for(int i = 0; i < cnt; i++)
  if(adj[i].size()) //-> There is no valid eulerian circuit/path
        in this case because the graph is not conected

ans.pop_back(); // Since it's a curcuit, the first and the last
        are repeated
reverse(ans.begin(), ans.end());

int bg = 0; // Is used to mark where the eulerian path begins
if(v.size()){
  for(int i = 0; i < ans.size(); i++)
    if(ans[i] == v[1] and ans[(i + 1)%ans.size()] == v[0]){
      bg = i + 1;
      break;
    }
}
```

4.9 Fast Kuhn

```cpp
const int N = 1e5+5;

int x, marcB[N], matchB[N], matchA[N], ans, n, m, p;
vector<int> adj[N];

bool dfs(int v){
  for(int i = 0; i < adj[v].size(); i++){
    int viz = adj[v][i];
    if(marcB[viz] == 1 ) continue;
    marcB[viz] = 1;

    if((matchB[viz] == -1) || dfs(matchB[viz])){
      matchB[viz] = v;
      matchA[v] = viz;
      return true;
    }

  }
  return false;
}

int main(){
  //...
  for(int i = 0; i<=n; i++) matchA[i] = -1;
  for(int j = 0; j<=m; j++) matchB[j] = -1;

  bool aux = true;
  while(aux){
    for(int j=1; j<=m; j++) marcB[j] = 0;
    aux = false;
    for(int i=1; i<=n; i++){
      if(matchA[i] != -1) continue;
      if(dfs(i)){
        ans++;
        aux = true;
      }
    }
  }
  //...
}
```

4.10 Find Cycle of size 3 and 4

```cpp
#define N 330000

int n, m;
vector<int> go[N], lk[N];
int w[N], deg[N], pos[N], id[N];

bool circle3() {
  int ans = 0;
  for(int i = 1; i <= n; i++) w[i] = 0;
  for(int x = 1; x <= n; x++)  {
    for(int y : lk[x]) w[y] = 1;
    for(int y : lk[x]) for(int z:lk[y]) if(w[z]) {
      ans=(ans+go[x].size()+go[y].size()+go[z].size() - 6);
      if(ans) return true;
    }
    for(int y:lk[x]) w[y] = 0;
  }
  return false;
}

bool circle4() {
  for(int i = 1; i <= n; i++) w[i] = 0;
  int ans = 0;
  for(int x = 1; x <= n; x++) {
    for(int y:go[x]) for(int z:lk[y]) if(pos[z] > pos[x]) {
      ans = (ans+w[z]);
      w[z]++;
      if(ans) return true;
    }
    for(int y:go[x]) for(int z : lk[y]) w[z] = 0;
  }
  return false;
}

inline bool cmp(const int &x, const int &y) {
  return deg[x] < deg[y];
}

int main() {
  cin >> n >> m;

  int x, y;
  for(int i = 0; i < n; i++) {
    cin >> x >> y;
  }

  for(int i = 1; i <= n; i++) {
    deg[i] = 0, go[i].clear(), lk[i].clear();
  }
  while (m--){
    int a, b;
    cin >> a >> b;
    deg[a]++, deg[b]++;
    go[a].push_back(b);
    go[b].push_back(a);
  }

  for(int i = 1; i <= n; i++) id[i]= i;
  sort(id+1, id+1+n, cmp);
  for(int i = 1; i<= n; i++) pos[id[i]]=i;
  for(int x = 1; x<= n; x++) {
    for(int y:go[x]) {
      if(pos[y]>pos[x]) lk[x].push_back(y);
    }
  };

  //Check circle3() then circle4()
  return 0;
}
```

4.11 Hungarian Navarro

```cpp
// Hungarian - O(n^2 * m)
template<bool is_max = false, class T = int, bool
      is_zero_indexed = false>
struct Hungarian {
  bool swap_coord = false;
  int lines, cols;
  T ans;

  vector<int> pairV, way;
  vector<bool> used;
  vector<T> pu, pv, minv;
  vector<vector<T>> cost;

  Hungarian(int _n, int _m) {
    if (_n > _m) {
      swap(_n, _m);
      swap_coord = true;
    }

    lines = _n + 1, cols = _m + 1;

    clear();
    cost.resize(lines);
    for (auto& line : cost) line.assign(cols, 0);
  }

  void clear() {
    pairV.assign(cols, 0);
    way.assign(cols, 0);
    pv.assign(cols, 0);
    pu.assign(lines, 0);
  }

  void update(int i, int j, T val) {
    if (is_zero_indexed) i++, j++;
    if (is_max) val = -val;
    if (swap_coord) swap(i, j);

    assert(i < lines);
    assert(j < cols);

    cost[i][j] = val;
  }

  T run() {
    T _INF = numeric_limits<T>::max();
    for (int i = 1, j0 = 0; i < lines; i++) {
      pairV[0] = i;
      minv.assign(cols, _INF);
      used.assign(cols, 0);
      do {
        used[j0] = 1;
        int i0 = pairV[j0], j1;
        T delta = _INF;
        for (int j = 1; j < cols; j++) {
          if (used[j]) continue;
          T cur = cost[i0][j] - pu[i0] - pv[j];
          if (cur < minv[j]) minv[j] = cur, way[j] = j0;
          if (minv[j] < delta) delta = minv[j], j1 = j;
        }

        for (int j = 0; j < cols; j++) {
          if (used[j]) pu[pairV[j]] += delta, pv[j] -= delta;
          else minv[j] -= delta;
        }
        j0 = j1;
      } while (pairV[j0]);

      do {
        int j1 = way[j0];
        pairV[j0] = pairV[j1];
        j0 = j1;
      } while (j0);
    }

    ans = 0;
    for (int j = 1; j < cols; j++) if (pairV[j]) ans += cost[
          pairV[j]][j];

    if (is_max) ans = -ans;
    if (is_zero_indexed) {
      for (int j = 0; j + 1 < cols; j++) pairV[j] = pairV[j +
            1], pairV[j]--;
      pairV[cols - 1] = -1;
    }
    if (swap_coord) {
      vector<int> pairV_sub(lines, 0);
      for (int j = 0; j < cols; j++) if (pairV[j] >= 0)
            pairV_sub[pairV[j]] = j;
      swap(pairV, pairV_sub);
    }

    return ans;
  }
};

template <bool is_max = false, bool is_zero_indexed = false>
struct HungarianMult : public Hungarian<is_max, long double,
      is_zero_indexed> {
  using super = Hungarian<is_max, long double, is_zero_indexed>;

  HungarianMult(int _n, int _m) : super(_n, _m) {}

  void update(int i, int j, long double x) {
    super::update(i, j, log2(x));
  }
};
```

4.12 Strongly Connected Components

```cpp
//Time complexity: O(V+E)
const int N = 2e5 + 5;
```

```
vector<int> adj[N], adjt[N];
int n, ordn, scc_cnt, vis[N], ord[N], scc[N];

//Directed Version
void dfs(int u) {
  vis[u] = 1;
  for (auto v : adj[u]) if (!vis[v]) dfs(v);
  ord[ordn++] = u;
}

void dfst(int u) {
  scc[u] = scc_cnt, vis[u] = 0;
  for (auto v : adjt[u]) if (vis[v]) dfst(v);
}

// add edge: u -> v
void add_edge(int u, int v){
  adj[u].push_back(v);
  adjt[v].push_back(u);
}

//Undirected version:
/*
  int par[N];

  void dfs(int u) {
    vis[u] = 1;
    for (auto v : adj[u]) if(!vis[v]) par[v] = u, dfs(v);
    ord[ordn++] = u;
  }

  void dfst(int u) {
    scc[u] = scc_cnt, vis[u] = 0;
    for (auto v : adj[u]) if(vis[v] and u != par[v]) dfst(v);
  }

  // add edge: u -> v
  void add_edge(int u, int v){
    adj[u].push_back(v);
    adj[v].push_back(u);
  }

*/

// run kosaraju
void kosaraju(){
  for (int i = 1; i <= n; ++i) if (!vis[i]) dfs(i);
  for (int i = ordn - 1; i >= 0; --i) if (vis[ord[i]]) scc_cnt
      ++, dfst(ord[i]);
}
```

4.13 LCA (Max Weight On Path)

```
// Using LCA to find max edge weight between (u, v)
const int N = 1e5+5;  // Max number of vertices
const int K = 20;     // Each 1e3 requires ~ 10 K
const int M = K+5;
int n;                // Number of vertices
vector <pair<int, int>> adj[N];
int vis[N], h[N], anc[N][M], mx[N][M];

void dfs (int u) {
  vis[u] = 1;
  for (auto p : adj[u]) {
    int v = p.st;
    int w = p.nd;
    if (!vis[v]) {
      h[v] = h[u]+1;
      anc[v][0] = u;
      mx[v][0] = w;
      dfs(v);
    }
  }
}

void build () {
  // cl(mn, 63) -- Don't forget to initialize with INF if min
      edge!
  anc[1][0] = 1;
  dfs(1);
  for (int j = 1; j <= K; j++) for (int i = 1; i <= n; i++) {
    anc[i][j] = anc[anc[i][j-1]][j-1];
    mx[i][j] = max(mx[i][j-1], mx[anc[i][j-1]][j-1]);
  }
}

int mxedge (int u, int v) {
  int ans = 0;
  if (h[u] < h[v]) swap(u, v);
  for (int j = K; j >= 0; j--) if (h[anc[u][j]] >= h[v]) {
    ans = max(ans, mx[u][j]);
    u = anc[u][j];
  }
  if (u == v) return ans;
  for (int j = K; j >= 0; j--) if (anc[u][j] != anc[v][j]) {
    ans = max(ans, mx[u][j]);
    ans = max(ans, mx[v][j]);
    u = anc[u][j];
    v = anc[v][j];
  } //LCA: anc[0][u]
  return max({ans, mx[u][0], mx[v][0]});
}
```

4.14 Max Flow

```
// Dinic - O(V^2 * E)
// Bipartite graph or unit flow - O(sqrt(V) * E)
// Small flow - O(F * (V + E))
// USE INF = 1e9!
template <class T = int>
class Dinic {
public:
  struct Edge {
    Edge(int a, T b){to = a;cap = b;}
    int to;
    T cap;
  };

  Dinic(int _n) : n(_n) {
    edges.resize(n);
  }

  T maxFlow(int src, int sink) {
    T ans = 0;
    while(bfs(src, sink)) {
      // maybe random shuffle edges against bad cases?
      T flow;
      pt = std::vector<int>(n, 0);
      while((flow = dfs(src, sink))) {
        ans += flow;
      }
    }
    return ans;
  }

  void addEdge(int from, int to, T cap, T other = 0) {
    edges[from].push_back(list.size());
    list.push_back(Edge(to, cap));
    edges[to].push_back(list.size());
    list.push_back(Edge(from, other));
  }

  bool inCut(int u) const { return h[u] < n; }
  int size() const { return n; }
private:
  int n;
  std::vector<std::vector<int> > edges;
  std::vector<Edge> list;
  std::vector<int> h, pt;

  T dfs(int on, int sink, T flow = 1e9) {
    if(flow == 0) {
      return 0;
    } if(on == sink) {
      return flow;
    }
    for(; pt[on] < (int) edges[on].size(); pt[on]++) {
      int cur = edges[on][pt[on]];
      if(h[on] + 1 != h[list[cur].to]) {
        continue;
      }
      T got = dfs(list[cur].to, sink, std::min(flow, list[cur].
          cap));
      if(got) {
        list[cur].cap -= got;
        list[cur ^ 1].cap += got;
        return got;
      }
    }
    return 0;
  }

  bool bfs(int src, int sink) {
    h = std::vector<int>(n, n);
    h[src] = 0;
    std::queue<int> q;
    q.push(src);
    while(!q.empty()) {
      int on = q.front();
      q.pop();
      for(auto a : edges[on]) {
        if(list[a].cap == 0) {
          continue;
        }
        int to = list[a].to;
        if(h[to] > h[on] + 1) {
          h[to] = h[on] + 1;
          q.push(to);
        }
      }
    }
    return h[sink] < n;
  }
};

// FLOW WITH DEMANDS

// 1 - Finding an arbitrary flow
// Assume a network with [L, R] on edges (some may have L = 0),
    let's call it old network.
// Create a New Source and New Sink (this will be the src and
    snk for Dinic).
// Modelling Network:
// 1) Every edge from the old network will have cost R - L
// 2) Add an edge from New Source to every vertex v with cost:
//   Sum(L) for every (u, v). (sum all L that LEAVES v)
// 3) Add an edge from every vertex v to New Sink with cost:
//   Sum(L) for every (v, w). (sum all L that ARRIVES v)
// 4) Add an edge from Old Source to Old Sink with cost INF (
    circulation problem)
// The Network will be valid if and only if the flow saturates
    the network (max flow == sum(L))

// 2 - Finding Min Flow
// To find min flow that satisfies just do a binary search in
    the (Old Sink -> Old Source) edge
// The cost of this edge represents all the flow from old
    network
// Min flow = Sum(L) that arrives in Old Sink + flow that leaves
     (Old Sink -> Old Source)
```

4.15 Min Cost Max Flow

```
template <class T = int>
class MCMF {
public:
  struct Edge {
    Edge(int a, T b, T c) : to(a), cap(b), cost(c) {}
    int to;
    T cap, cost;
  };

  MCMF(int size) {
    n = size;
    edges.resize(n);
    pot.assign(n, 0);
    dist.resize(n);
    visit.assign(n, false);
  }

  std::pair<T, T> mcmf(int src, int sink) {
    std::pair<T, T> ans(0, 0);
    if(!SPFA(src, sink)) return ans;
    fixPot();
    // can use dijkstra to speed up depending on the graph
    while(SPFA(src, sink)) {
      auto flow = augment(src, sink);
      ans.first += flow.first;
      ans.second += flow.first * flow.second;
      fixPot();
    }
    return ans;
  }

  void addEdge(int from, int to, T cap, T cost) {
    edges[from].push_back(list.size());
    list.push_back(Edge(to, cap, cost));
    edges[to].push_back(list.size());
    list.push_back(Edge(from, 0, -cost));
  }
private:
  int n;
  std::vector<std::vector<int>> edges;
```

```cpp
  std::vector<Edge> list;
  std::vector<int> from;
  std::vector<T> dist, pot;
  std::vector<bool> visit;

  /*bool dij(int src, int sink) {
    T INF = std::numeric_limits<T>::max();
    dist.assign(n, INF);
    from.assign(n, -1);
    visit.assign(n, false);
    dist[src] = 0;
    for(int i = 0; i < n; i++) {
      int best = -1;
      for(int j = 0; j < n; j++) {
        if(visit[j]) continue;
        if(best == -1 || dist[best] > dist[j]) best = j;
      }
      if(dist[best] >= INF) break;
      visit[best] = true;
      for(auto e : edges[best]) {
        auto ed = list[e];
        if(ed.cap == 0) continue;
        T toDist = dist[best] + ed.cost + pot[best] - pot[ed.to
            ];
        assert(toDist >= dist[best]);
        if(toDist < dist[ed.to]) {
          dist[ed.to] = toDist;
          from[ed.to] = e;
        }
      }
    }
    return dist[sink] < INF;
  }*/

  std::pair<T, T> augment(int src, int sink) {
    std::pair<T, T> flow = {list[from[sink]].cap, 0};
    for(int v = sink; v != src; v = list[from[v]^1].to) {
      flow.first = std::min(flow.first, list[from[v]].cap);
      flow.second += list[from[v]].cost;
    }
    for(int v = sink; v != src; v = list[from[v]^1].to) {
      list[from[v]].cap -= flow.first;
      list[from[v]^1].cap += flow.first;
    }
    return flow;
  }

  std::queue<int> q;
  bool SPFA(int src, int sink) {
    T INF = std::numeric_limits<T>::max();
    dist.assign(n, INF);
    from.assign(n, -1);
    q.push(src);
    dist[src] = 0;
    while(!q.empty()) {
      int on = q.front();
      q.pop();
      visit[on] = false;
      for(auto e : edges[on]) {
        auto ed = list[e];
        if(ed.cap == 0) continue;
        T toDist = dist[on] + ed.cost + pot[on] - pot[ed.to];
        if(toDist < dist[ed.to]) {
          dist[ed.to] = toDist;
          from[ed.to] = e;
          if(!visit[ed.to]) {
            visit[ed.to] = true;
            q.push(ed.to);
          }
        }
      }
    }
    return dist[sink] < INF;
  }

  void fixPot() {
    T INF = std::numeric_limits<T>::max();
    for(int i = 0; i < n; i++) {
      if(dist[i] < INF) pot[i] += dist[i];
    }
  }
};
```

4.16 Small to Large

```cpp
// Imagine you have a tree with colored vertices, and you want
    to do some type of query on every subtree about the colors
    inside
// complexity: O(nlogn)

vector<int> adj[N], vec[N];
int sz[N], color[N], cnt[N];

void dfs_size(int v = 1, int p = 0) {
  sz[v] = 1;
  for (auto u : adj[v]) {
    if (u != p) {
      dfs_size(u, v);
      sz[v] += sz[u];
    }
  }
}

void dfs(int v = 1, int p = 0, bool keep = false) {
  int Max = -1, bigchild = -1;
  for (auto u : adj[v]) {
    if (u != p && Max < sz[u]) {
      Max = sz[u];
      bigchild = u;
    }
  }
  for (auto u : adj[v]) {
    if (u != p && u != bigchild) {
      dfs(u, v, 0);
    }
  }
  if (bigchild != -1) {
    dfs(bigchild, v, 1);
    swap(vec[v], vec[bigchild]);
  }
  vec[v].push_back(v);
  cnt[color[v]]++;
  for (auto u : adj[v]) {
    if (u != p && u != bigchild) {
      for (auto x : vec[u]) {
        cnt[color[x]]++;
        vec[v].push_back(x);
      }
    }
  }
  // now here you can do what the query wants
  // there are cnt[c] vertex in subtree v color with c
  if (keep == 0) {
    for (auto u : vec[v]) {
      cnt[color[u]]--;
    }
  }
}
```

4.17 Stoer Wagner (Stanford)

```cpp
  // a is a N*N matrix storing the graph we use; a[i][j]=a[j][i]
  memset(use,0,sizeof(use));
  ans=maxlongint;
  for (int i=1;i<N;i++)
  {
    memcpy(visit,use,505*sizeof(int));
    memset(reach,0,sizeof(reach));
    memset(last,0,sizeof(last));
    t=0;
    for (int j=1;j<=N;j++)
      if (use[j]==0) {t=j;break;}
    for (int j=1;j<=N;j++)
      if (use[j]==0) reach[j]=a[t][j],last[j]=t;
    visit[t]=1;
    for (int j=1;j<=N-i;j++)
    {
      maxc=maxk=0;
      for (int k=1;k<=N;k++)
        if ((visit[k]==0)&&(reach[k]>maxc)) maxc=reach[k],maxk=k
            ;
      c2=maxk,visit[maxk]=1;
      for (int k=1;k<=N;k++)
        if (visit[k]==0) reach[k]+=a[maxk][k],last[k]=maxk;
    }
    c1=last[c2];
    sum=0;
    for (int j=1;j<=N;j++)
      if (use[j]==0) sum+=a[j][c2];
    ans=min(ans,sum);
    use[c2]=1;
    for (int j=1;j<=N;j++)
      if ((c1!=j)&&(use[j]==0)) {a[j][c1]+=a[j][c2];a[c1][j]=a[j
          ][c1];}
  }
```

4.18 Stable Marriage (Cosenza)

```cpp
std::vector<std::vector<int>> stableMarriage(std::vector<std::
    vector<int>> first, std::vector<std::vector<int>> second,
    std::vector<int> cap) {
  assert(cap.size() == second.size());
  int n = (int) first.size(), m = (int) second.size();
  // init
  // if O(N * M) first in memory, use table
  std::map<std::pair<int, int>, int> prio;
  std::vector<std::set<std::pair<int, int>>> current(m);
  for(int i = 0; i < n; i++) {
    std::reverse(first[i].begin(), first[i].end());
  }
  for(int i = 0; i < m; i++) {
    for(int j = 0; j < (int) second[i].size(); j++) {
      prio[{second[i][j], i}] = j;
    }
  }
  // solve
  for(int i = 0; i < n; i++) {
    int on = i;
    while(!first[on].empty()) {
      int to = first[on].back();
      first[on].pop_back();
      if(cap[to]) {
        cap[to]--;
        assert(prio.count({on, to}));
        current[to].insert({prio[{on, to}], on});
        break;
      }
      assert(!current[to].empty());
      auto it = current[to].end();
      it--;
      if(it->first > prio[{on, to}]) {
        int nxt = it->second;
        current[to].erase(it);
        current[to].insert({prio[{on, to}], on});
        on = nxt;
      }
    }
  }
  // return
  std::vector<std::vector<int>> ans(m);
  for(int i = 0; i < m; i++) {
    for(auto it : current[i]) {
      ans[i].push_back(it.second);
    }
  }
  return ans;
}
```

5 Strings

5.1 Aho-Corasick

```cpp
// Aho-Corasick

// Build: O(sum size of patterns)
// Find total number of matches: O(size of input string)
// Find number of matches for each pattern: O(num of patterns +
    size of input string)

// ids start from 0 by default!

template <int ALPHA_SIZE = 62>
struct Aho {
  struct Node {
    int p, char_p, link = -1, str_idx = -1, nxt[ALPHA_SIZE];
    bool has_end = false;
    Node(int _p = -1, int _char_p = -1) : p(_p), char_p(_char_p)
        {
      fill(nxt, nxt + ALPHA_SIZE, -1);
    }
  };

  vector<Node> nodes = { Node() };
  int ans, cnt = 0;
  bool build_done = false;
  vector<pair<int, int>> rep;
  vector<int> ord, occur, occur_aux;

  // change this if different alphabet
  int remap(char c) {
    if (islower(c)) return c - 'a';
    if (isalpha(c)) return c - 'A' + 26;
    return c - '0' + 52;
```

```
  }

  void add(string &p, int id = -1) {
    int u = 0;
    if (id == -1) id = cnt++;

    for (char ch : p) {
      int c = remap(ch);
      if (nodes[u].nxt[c] == -1) {
        nodes[u].nxt[c] = (int)nodes.size();
        nodes.push_back(Node(u, c));
      }

      u = nodes[u].nxt[c];
    }

    if (nodes[u].str_idx != -1) rep.push_back({ id, nodes[u].
         str_idx });
    else nodes[u].str_idx = id;
    nodes[u].has_end = true;
  }

  void build() {
    build_done = true;
    queue<int> q;

    for (int i = 0; i < ALPHA_SIZE; i++) {
      if (nodes[0].nxt[i] != -1) q.push(nodes[0].nxt[i]);
      else nodes[0].nxt[i] = 0;
    }

    while(q.size()) {
      int u = q.front();
      ord.push_back(u);
      q.pop();

      int j = nodes[nodes[u].p].link;
      if (j == -1) nodes[u].link = 0;
      else nodes[u].link = nodes[j].nxt[nodes[u].char_p];

      nodes[u].has_end |= nodes[nodes[u].link].has_end;

      for (int i = 0; i < ALPHA_SIZE; i++) {
        if (nodes[u].nxt[i] != -1) q.push(nodes[u].nxt[i]);
        else nodes[u].nxt[i] = nodes[nodes[u].link].nxt[i];
      }
    }
  }

  int match(string &s) {
    if (!cnt) return 0;
    if (!build_done) build();

    ans = 0;
    occur = vector<int>(cnt);
    occur_aux = vector<int>(nodes.size());

    int u = 0;
    for (char ch : s) {
      int c = remap(ch);
      u = nodes[u].nxt[c];
      occur_aux[u]++;
    }

    for (int i = (int)ord.size() - 1; i >= 0; i--) {
      int v = ord[i];
      int fv = nodes[v].link;
      occur_aux[fv] += occur_aux[v];
      if (nodes[v].str_idx != -1) {
        occur[nodes[v].str_idx] = occur_aux[v];
        ans += occur_aux[v];
      }
    }

    for (pair<int, int> x : rep) occur[x.first] = occur[x.second
         ];
    return ans;
  }
};
```

5.2 Booths Algorithm

```
// Booth's Algorithm - Find the lexicographically least rotation
     of a string in O(n)

string least_rotation(string s) {
  s += s;
  vector<int> f((int)s.size(), -1);
  int k = 0;
  for (int j = 1; j < (int)s.size(); j++) {
    int i = f[j - k - 1];
    while (i != -1 and s[j] != s[k + i + 1]) {
      if (s[j] < s[k + i + 1]) k = j - i - 1;
      i = f[i];
    }

    if (s[j] != s[k + i + 1]) {
      if (s[j] < s[k]) k = j;
      f[j - k] = -1;
    } else f[j - k] = i + 1;
  }

  return s.substr(k, (int)s.size() / 2);
}
```

5.3 Knuth-Morris-Pratt (Automaton)

```
// KMP Automaton - <O(26*pattern), O(text)>

// max size pattern
const int N = 1e5 + 5;

int cnt, nxt[N+1][26];

void prekmp(string &p) {
  nxt[0][p[0] - 'a'] = 1;
  for(int i = 1, j = 0; i <= p.size(); i++) {
    for(int c = 0; c < 26; c++) nxt[i][c] = nxt[j][c];
    if(i == p.size()) continue;
    nxt[i][p[i] - 'a'] = i+1;
    j = nxt[j][p[i] - 'a'];
  }
}

void kmp(string &s, string &p) {
  for(int i = 0, j = 0; i < s.size(); i++) {
    j = nxt[j][s[i] - 'a'];
    if(j == p.size()) cnt++; //match i - j + 1
  }
}
```

5.4 Knuth-Morris-Pratt

```
// Knuth-Morris-Pratt - String Matching O(n+m)
char s[N], p[N];
int b[N], n, m; // n = strlen(s), m = strlen(p);

void kmppre() {
  b[0] = -1;
  for (int i = 0, j = -1; i < m; b[++i] = ++j)
    while (j >= 0 and p[i] != p[j])
      j = b[j];
}

void kmp() {
  for (int i = 0, j = 0; i < n;) {
    while (j >= 0 and s[i] != p[j]) j=b[j];
    i++, j++;
    if (j == m) {
      // match position i-j
      j = b[j];
    }
  }
}
```

5.5 Manacher

```
// Manacher - O(n)

// d1 -> odd : size = 2*d1[i] - 1, palindrome from i - d1[i] + 1
     to i + d1[i] - 1
// d2 -> even : size = 2*d2[i], palindrome from i - d2[i] to i +
     d2[i] - 1

vector<int> d1, d2;

void manacher(string &s) {
  int n = (int)s.size();
  d1.assign(n, 0);
  d2.assign(n, 0);
  for(int i = 0, l1 = 0, l2 = 0, r1 = -1, r2 = -1; i < n; i++) {
    if(i <= r1) d1[i] = min(d1[r1+l1-i], r1-i+1);
    if(i <= r2) d2[i] = min(d2[r2+l2-i+1], r2-i+1);

    while(i - d1[i] >= 0 and i + d1[i] < n and
        s[i - d1[i]] == s[i + d1[i]]) d1[i]++;

    while(i - d2[i] - 1 >= 0 and i + d2[i] < n and
        s[i - d2[i] - 1] == s[i + d2[i]]) d2[i]++;

    if(i + d1[i] - 1 > r1) l1 = i - d1[i] + 1, r1 = i + d1[i] -
         1;
    if(i + d2[i] - 1 > r2) l2 = i - d2[i], r2 = i + d2[i] - 1;
  }
}
```

5.6 Recursive-String Matching

```
void p_f(char *s, int *pi) {
  int n = strlen(s);
  pi[0]=pi[1]=0;
  for(int i = 2; i <= n; i++) {
    pi[i] = pi[i-1];
    while(pi[i]>0 and s[pi[i]]!=s[i])
      pi[i]=pi[pi[i]];
    if(s[pi[i]]==s[i-1])
      pi[i]++;
  }
}

int main() {
  //...
  //Initialize prefix function
  char p[N]; //Pattern
  int len = strlen(p); //Pattern size
  int pi[N]; //Prefix function
  p_f(p, pi);

  // Create KMP automaton
  int A[N][128]; //A[i][j]: from state i (size of largest suffix
       of text which is prefix of pattern), append character j
       -> new state A[i][j]
  for( char c : ALPHABET )
    A[0][c] = (p[0] == c);
  for( int i = 1; p[i]; i++ ) {
    for( char c : ALPHABET ) {
      if(c==p[i])
        A[i][c]=i+1; //match
      else
        A[i][c]=A[pi[i]][c]; //try second largest suffix
    }
  }

  //Create KMP "string appending" automaton
  // g_n = g_(n-1) + char(n) + g_(n-1)
  // g_0 = "", g_1 = "a", g_2 = "aba", g_3 = "abacaba", ...
  int F[M][N]; //F[i][j]: from state j (size of largest suffix
       of text which is prefix of pattern), append string g_i ->
        new state F[i][j]
  for(int i = 0; i < m; i++) {
    for(int j = 0; j <= len; j++) {
      if(i==0)
        F[i][j] = j; //append empty string
      else {
        int x = F[i-1][j]; //append g_(i-1)
        x = A[x][j]; //append character j
        x = F[i-1][x]; //append g_(i-1)
        F[i][j] = x;
      }
    }
  }

  //Create number of matches matrix
  int K[M][N]; //K[i][j]: from state j (size of largest suffix
       of text which is prefix of pattern), append string g_i ->
        K[i][j] matches
  for(int i = 0; i < m; i++) {
    for(int j = 0; j <= len; j++) {
      if(i==0)
        K[i][j] = (j==len); //append empty string
      else {
        int x = F[i-1][j]; //append g_(i-1)
        x = A[x][j]; //append character j

        K[i][j] = K[i-1][j] /*append g_(i-1)*/ + (x==len) /*
              append character j*/ + K[i-1][x]; /*append g_(i-1)
              */
      }
    }
  }
  //number of matches in g_k
  int answer = K[0][k];
```

```
  //...
}
```

5.7 String Hashing

```
// String Hashing
// Rabin Karp - O(n + m)

// max size txt + 1
const int N = 1e6 + 5;

// lowercase letters p = 31 (remember to do s[i] - 'a' + 1)
// uppercase and lowercase letters p = 53 (remember to do s[i] -
    'a' + 1)
// any character p = 313

const int MOD = 1e9+9;
ull h[N], p[N];
ull pr = 313; //177771

int cnt;


void build(string &s) {
  p[0] = 1, p[1] = pr;
  for(int i = 1; i <= s.size(); i++) {
    h[i] = ((p[1]*h[i-1]) % MOD + s[i-1]) % MOD;
    p[i] = (p[1]*p[i-1]) % MOD;
  }
}

// 1-indexed
ull fhash(int l, int r) {
  return (h[r] - ((h[l-1]*p[r-l+1]) % MOD) + MOD) % MOD;
}

ull shash(string &pt) {
  ull h = 0;
  for(int i = 0; i < pt.size(); i++)
    h = ((h*pr) % MOD + pt[i]) % MOD;
  return h;
}

void rabin_karp(string &s, string &pt) {
  build(s);
  ull hp = shash(pt);
  for(int i = 0, m = pt.size(); i + m <= s.size(); i++) {
    if(fhash(i+1, i+m) == hp) {
      // match at i
      cnt++;
    }
  }
}
```

5.8 String Multihashing

```
// String Hashing
// Rabin Karp - O(n + m)
template <int N = 3>
struct Hash {
  int hs[N];
  static vector<int> mods;

  static int add(int a, int b, int mod) { return a >= mod - b ?
      a + b - mod : a + b; }
  static int sub(int a, int b, int mod) { return a - b < 0 ? a -
       b + mod : a - b; }
  static int mul(int a, int b, int mod) { return 1ll * a * b %
      mod; }

  Hash(int x = 0) { fill(hs, hs + N, x); }

  bool operator<(const Hash& b) const {
    for (int i = 0; i < N; i++) {
      if (hs[i] < b.hs[i]) return true;
      if (hs[i] > b.hs[i]) return false;
    }
    return false;
  }

  Hash operator+(const Hash& b) const {
    Hash ans;
    for (int i = 0; i < N; i++) ans.hs[i] = add(hs[i], b.hs[i],
        mods[i]);
    return ans;
  }

  Hash operator-(const Hash& b) const {
    Hash ans;
    for (int i = 0; i < N; i++) ans.hs[i] = sub(hs[i], b.hs[i],
        mods[i]);
    return ans;
  }

  Hash operator*(const Hash& b) const {
    Hash ans;
    for (int i = 0; i < N; i++) ans.hs[i] = mul(hs[i], b.hs[i],
        mods[i]);
    return ans;
  }

  Hash operator+(int b) const {
    Hash ans;
    for (int i = 0; i < N; i++) ans.hs[i] = add(hs[i], b, mods[i
        ]);
    return ans;
  }

  Hash operator*(int b) const {
    Hash ans;
    for (int i = 0; i < N; i++) ans.hs[i] = mul(hs[i], b, mods[i
        ]);
    return ans;
  }

  friend Hash operator*(int a, const Hash& b) {
    Hash ans;
    for (int i = 0; i < N; i++) ans.hs[i] = mul(b.hs[i], a, b.
        mods[i]);
    return ans;
  }

  friend ostream& operator<<(ostream& os, const Hash& b) {
    for (int i = 0; i < N; i++) os << b.hs[i] << " \n"[i == N -
        1];
    return os;
  }
};

template <int N> vector<int> Hash<N>::mods = { (int) 1e9 + 9, (
     int) 1e9 + 33, (int) 1e9 + 87 };

// In case you need to generate the MODs, uncomment this:
// Obs: you may need this on your template
// mt19937_64 llrand((int) chrono::steady_clock::now().
     time_since_epoch().count());
// In main: gen<>();
/*
template <int N> vector<int> Hash<N>::mods;
template<int N = 3>
void gen() {
  while (Hash<N>::mods.size() < N) {
    int mod;
    bool is_prime;
    do {
      mod = (int) 1e8 + (int) (llrand() % (int) 9e8);
      is_prime = true;
      for (int i = 2; i * i <= mod; i++) {
        if (mod % i == 0) {
          is_prime = false;
          break;
        }
      }
    } while (!is_prime);
    Hash<N>::mods.push_back(mod);
  }
}
*/

template <int N = 3>
struct PolyHash {
  vector<Hash<N>> h, p;

  PolyHash(string& s, int pr = 313) {
    int sz = (int)s.size();
    p.resize(sz + 1);
    h.resize(sz + 1);

    p[0] = 1, h[0] = s[0];
    for (int i = 1; i < sz; i++) {
      h[i] = pr * h[i - 1] + s[i];
      p[i] = pr * p[i - 1];
    }
  }

  Hash<N> fhash(int l, int r) {
    if (!l) return h[r];
    return h[r] - h[l - 1] * p[r - l + 1];
  }

  static Hash<N> shash(string& s, int pr = 313) {
    Hash<N> ans;
    for (int i = 0; i < (int)s.size(); i++) ans = pr * ans + s[i
        ];
    return ans;
  }

  friend int rabin_karp(string& s, string& pt) {
    PolyHash hs = PolyHash(s);
    Hash<N> hp = hs.shash(pt);
    int cnt = 0;
    for (int i = 0, m = (int)pt.size(); i + m <= (int)s.size();
        i++) {
      if (hs.fhash(i, i + m - 1) == hp) {
        // match at i
        cnt++;
      }
    }

    return cnt;
  }
};
```

5.9 Suffix Array

```
// Suffix Array O(nlogn)
// s.push('$');
vector<int> suffix_array(string &s){
  int n = s.size(), alph = 256;
  vector<int> cnt(max(n, alph)), p(n), c(n);

  for(auto c : s) cnt[c]++;
  for(int i = 1; i < alph; i++) cnt[i] += cnt[i - 1];
  for(int i = 0; i < n; i++) p[--cnt[s[i]]] = i;
  for(int i = 1; i < n; i++)
    c[p[i]] = c[p[i - 1]] + (s[p[i]] != s[p[i - 1]]);

  vector<int> c2(n), p2(n);

  for(int k = 0; (1 << k) < n; k++){
    int classes = c[p[n - 1]] + 1;
    fill(cnt.begin(), cnt.begin() + classes, 0);

    for(int i = 0; i < n; i++) p2[i] = (p[i] - (1 << k) + n)%n;
    for(int i = 0; i < n; i++) cnt[c[i]]++;
    for(int i = 1; i < classes; i++) cnt[i] += cnt[i - 1];
    for(int i = n - 1; i >= 0; i--) p[--cnt[c[p2[i]]]] = p2[i];

    c2[p[0]] = 0;
    for(int i = 1; i < n; i++){
      pair<int, int> b1 = {c[p[i]], c[(p[i] + (1 << k))%n]};
      pair<int, int> b2 = {c[p[i - 1]], c[(p[i - 1] + (1 << k))%
          n]};
      c2[p[i]] = c2[p[i - 1]] + (b1 != b2);
    }

    c.swap(c2);
  }
  return p;
}

// Longest Common Prefix with SA O(n)
vector<int> lcp(string &s, vector<int> &p){
  int n = s.size();
  vector<int> ans(n - 1), pi(n);
  for(int i = 0; i < n; i++) pi[p[i]] = i;

  int lst = 0;
  for(int i = 0; i < n - 1; i++){
    if(pi[i] == n - 1) continue;
    while(s[i + lst] == s[p[pi[i] + 1] + lst]) lst++;

    ans[pi[i]] = lst;
    lst = max(0, lst - 1);
  }

  return ans;
}

// Longest Repeated Substring O(n)
int lrs = 0;
for (int i = 0; i < n; ++i) lrs = max(lrs, lcp[i]);

// Longest Common Substring O(n)
```

```
// m = strlen(s);
// strcat(s, "$"); strcat(s, p); strcat(s, "#");
// n = strlen(s);
int lcs = 0;
for (int i = 1; i < n; ++i) if ((sa[i] < m) != (sa[i-1] < m))
  lcs = max(lcs, lcp[i]);

// To calc LCS for multiple texts use a slide window with
     minqueue
// The numver of different substrings of a string is n*(n + 1)/2
      - sum(lcs[i])
```

5.10 Suffix Automaton

```
// Suffix Automaton Construction - O(n)

const int N = 1e6+1, K = 26;
int sl[2*N], len[2*N], sz, last;
ll cnt[2*N];
map<int, int> adj[2*N];

void add(int c) {
  int u = sz++;
  len[u] = len[last] + 1;
  cnt[u] = 1;

  int p = last;
  while(p != -1 and !adj[p][c])
    adj[p][c] = u, p = sl[p];

  if (p == -1) sl[u] = 0;
  else {
    int q = adj[p][c];
    if (len[p] + 1 == len[q]) sl[u] = q;
    else {
      int r = sz++;
      len[r] = len[p] + 1;
      sl[r] = sl[q];
      adj[r] = adj[q];
      while(p != -1 and adj[p][c] == q)
        adj[p][c] = r, p = sl[p];
      sl[q] = sl[u] = r;
    }
  }

  last = u;
}

void clear() {
  for(int i=0; i<=sz; ++i) adj[i].clear();
  last = 0;
  sz = 1;
  sl[0] = -1;
}

void build(char *s) {
  clear();
  for(int i=0; s[i]; ++i) add(s[i]);
}

// Pattern matching - O(|p|)
bool check(char *p) {
  int u = 0, ok = 1;
  for(int i=0; p[i]; ++i) {
    u = adj[u][p[i]];
    if (!u) ok = 0;
  }
  return ok;
}

// Substring count - O(|p|)
ll d[2*N];

void substr_cnt(int u) {
  d[u] = 1;
  for(auto p : adj[u]) {
    int v = p.second;
    if (!d[v]) substr_cnt(v);
    d[u] += d[v];
  }
}

ll substr_cnt() {
  memset(d, 0, sizeof d);
  substr_cnt(0);
  return d[0] - 1;
}

// k-th Substring - O(|s|)
// Just find the k-th path in the automaton.
// Can be done with the value d calculated in previous problem.

// Smallest cyclic shift - O(|s|)
// Build the automaton for string s + s. And adapt previous dp
// to only count paths with size |s|.


// Number of occurences - O(|p|)
vector<int> t[2*N];

void occur_count(int u) {
  for(int v : t[u]) occur_count(v), cnt[u] += cnt[v];
}

void build_tree() {
  for(int i=1; i<=sz; ++i)
    t[sl[i]].push_back(i);
  occur_count(0);
}

ll occur_count(char *p) {
  // Call build tree once per automaton
  int u = 0;
  for(int i=0; p[i]; ++i) {
    u = adj[u][p[i]];
    if (!u) break;
  }
  return !u ? 0 : cnt[u];
}

// First occurence - (|p|)
// Store the first position of occurence fp.
// Add the the code to add function:
// fp[u] = len[u] - 1;
// fp[r] = fp[q];

// To answer a query, just output fp[u] - strlen(p) + 1
// where u is the state corresponding to string p

// All occurences - O(|p| + |ans|)
// All the occurences can reach the first occurence via suffix
     links.
// So every state that contains a occreunce is reacheable by the
// first occurence state in the suffix link tree. Just do a DFS
     in this
// tree, starting from the first occurence.
// OBS: cloned nodes will output same answer twice.


// Smallest substring not contained in the string - O(|s| * K)
// Just do a dynamic programming:
// d[u] = 1 // if d does not have 1 transition
// d[u] = 1 + min d[v] // otherwise


// LCS of 2 Strings - O(|s| + |t|)
// Build automaton of s and traverse the automaton wih string t
// mantaining the current state and the current lenght.
// When we have a transition: update state, increase lenght by
     one.
// If we don't update state by suffix link and the new lenght
     will
// should be reduced (if bigger) to the new state length.
// Answer will be the maximum length of the whole traversal.

// LCS of n Strings - O(n*|s|*K)
// Create a new string S = s_1 + d1 + ... + s_n + d_n,
// where d_i are delimiters that are unique (d_i != d_j).
// For each state use DP + bitmask to calculate if it can
// reach a d_i transition without going through other d_j.
// The answer will be the biggest len[u] that can reach all
// d_i's.
```

5.11 Suffix Tree

```
// Suffix Tree
// Build: O(|s|)
// Match: O(|p|)

template<int ALPHA_SIZE = 62>
struct SuffixTree {
  struct Node {
    int p, link = -1, l, r, nch = 0;
    vector<int> nxt;

    Node(int _l = 0, int _r = -1, int _p = -1) : p(_p), l(_l), r
         (_r), nxt(ALPHA_SIZE, -1) {}

    int len() { return r - l + 1; }
    int next(char ch) { return nxt[remap(ch)]; }

    // change this if different alphabet
    int remap(char c) {
      if (islower(c)) return c - 'a';
      if (isalpha(c)) return c - 'A' + 26;
      return c - '0' + 52;
    }

    void setEdge(char ch, int nx) {
      int c = remap(ch);
      if (nxt[c] != -1 and nx == -1) nch--;
      else if (nxt[c] == -1 and nx != -1) nch++;
      nxt[c] = nx;
    }
  };

  string s;
  long long num_diff_substr = 0;
  vector<Node> nodes;
  queue<int> leaves;
  pair<int, int> st = { 0, 0 };
  int ls = 0, rs = -1, n;

  int size() { return rs - ls + 1; }

  SuffixTree(string &_s) {
    s = _s;
    // Add this if you want every suffix to be a node
    // s += '$';
    n = (int)s.size();
    nodes.reserve(2 * n + 1);
    nodes.push_back(Node());
    //for (int i = 0; i < n; i++) extend();
  }

  pair<int, int> walk(pair<int, int> _st, int l, int r) {
    int u = _st.first;
    int d = _st.second;

    while (l <= r) {
      if (d == nodes[u].len()) {
        u = nodes[u].next(s[l]), d = 0;
        if (u == -1) return { u, d };
      } else {
        if (s[nodes[u].l + d] != s[l]) return { -1, -1 };
        if (r - l + 1 + d < nodes[u].len()) return { u, r - l +
             1 + d };
        l += nodes[u].len() - d;
        d = nodes[u].len();
      }
    }

    return { u, d };
  }

  int split(pair<int, int> _st) {
    int u = _st.first;
    int d = _st.second;

    if (d == nodes[u].len()) return u;
    if (!d) return nodes[u].p;

    Node& nu = nodes[u];
    int mid = (int)nodes.size();
    nodes.push_back(Node(nu.l, nu.l + d - 1, nu.p));
    nodes[nu.p].setEdge(s[nu.l], mid);
    nodes[mid].setEdge(s[nu.l + d], u);
    nu.p = mid;
    nu.l += d;
    return mid;
  }

  int getLink(int u) {
    if (nodes[u].link != -1) return nodes[u].link;
    if (nodes[u].p == -1) return 0;
    int to = getLink(nodes[u].p);
    pair<int, int> nst = { to, nodes[to].len() };
    return nodes[u].link = split(walk(nst, nodes[u].l + (nodes[u
         ].p == 0), nodes[u].r));
  }

  bool match(string &p) {
    int u = 0, d = 0;
    for (char ch : p) {
```

```cpp
        if (d == min(nodes[u].r, rs) - nodes[u].l + 1) {
          u = nodes[u].next(ch), d = 1;
          if (u == -1) return false;
        } else {
          if (ch != s[nodes[u].l + d]) return false;
          d++;
        }
      }
      return true;
    }

  void extend() {
    int mid;
    assert(rs != n - 1);
    rs++;
    num_diff_substr += (int)leaves.size();
    do {
      pair<int, int> nst = walk(st, rs, rs);
      if (nst.first != -1) { st = nst; return; }
      mid = split(st);
      int leaf = (int)nodes.size();
      num_diff_substr++;
      leaves.push(leaf);
      nodes.push_back(Node(rs, n - 1, mid));
      nodes[mid].setEdge(s[rs], leaf);
      int to = getLink(mid);
      st = { to, nodes[to].len() };
    } while (mid);
  }

  void pop() {
    assert(ls <= rs);
    ls++;
    int leaf = leaves.front();
    leaves.pop();
    Node* nlf = &nodes[leaf];
    while (!nlf->nch) {
      if (st.first != leaf) {
        nodes[nlf->p].setEdge(s[nlf->l], -1);
        num_diff_substr -= min(nlf->r, rs) - nlf->l + 1;
        leaf = nlf->p;
        nlf = &nodes[leaf];
      } else {
        if (st.second != min(nlf->r, rs) - nlf->l + 1) {
          int mid = split(st);
          st.first = mid;
          num_diff_substr -= min(nlf->r, rs) - nlf->l + 1;
          nodes[mid].setEdge(s[nlf->l], -1);
          *nlf = nodes[mid];
          nodes[nlf->p].setEdge(s[nlf->l], leaf);
          nodes.pop_back();
        }
        break;
      }
    }

    if (leaf and !nlf->nch) {
      leaves.push(leaf);
      int to = getLink(nlf->p);
      pair<int, int> nst = { to, nodes[to].len() };
      st = walk(nst, nlf->l + (nlf->p == 0), nlf->r);
      nlf->l = rs - nlf->len() + 1;
      nlf->r = n - 1;
    }
  }
};
```

5.12 Z Function

```cpp
// Z-Function - O(n)

vector<int> zfunction(const string& s){
  vector<int> z (s.size());
  for (int i = 1, l = 0, r = 0, n = s.size(); i < n; i++){
    if (i <= r) z[i] = min(z[i-l], r - i + 1);
    while (i + z[i] < n and s[z[i]] == s[z[i] + i]) z[i]++;
    if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
  }
  return z;
}
```

6 Mathematics

6.1 Basics

```cpp
// Greatest Common Divisor & Lowest Common Multiple
ll gcd(ll a, ll b) { return b ? gcd(b, a%b) : a; }
ll lcm(ll a, ll b) { return a/gcd(a, b)*b; }

// Multiply caring overflow
ll mulmod(ll a, ll b, ll m = MOD) {
  ll r=0;
  for (a %= m; b; b>>=1, a=(a*2)%m) if (b&1) r=(r+a)%m;
  return r;
}

// Another option for mulmod is using long double
ull mulmod(ull a, ull b, ull m = MOD) {
  ull q = (ld) a * (ld) b / (ld) m;
  ull r = a * b - q * m;
  return (r + m) % m;
}

// Fast exponential
ll fexp(ll a, ll b, ll m = MOD) {
  ll r=1;
  for (a %= m; b; b>>=1, a=(a*a)%m) if (b&1) r=(r*a)%m;
  return r;
}

//cfloor
ll cfloor(ll a, ll b) {
  ll c = abs(a);
  ll d = abs(b);
  if (a * b > 0) return c/d;
  return -(c + d - 1)/d;
}
```

6.2 Advanced

```cpp
/* Line integral = integral(sqrt(1 + (dy/dx)^2)) dx */

/* Multiplicative Inverse over MOD for all 1..N - 1 < MOD in O(N
    )
 Only works for prime MOD. If all 1..MOD - 1 needed, use N = MOD
    */
ll inv[N];
inv[1] = 1;
for(int i = 2; i < N; ++i)
  inv[i] = MOD - (MOD / i) * inv[MOD % i] % MOD;

/* Catalan
 f(n) = sum(f(i) * f(n - i - 1)), i in [0, n - 1] = (2n)! / ((n
    +1)! * n!) = ...
 If you have any function f(n) (there are many) that follows
    this sequence (0-indexed):
 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012,
    742900, 2674440
 than it's the Catalan function */
ll cat[N];
cat[0] = 1;
for(int i = 1; i + 1 < N; i++) // needs inv[i + 1] till inv[N -
    1]
  cat[i] = 2ll * (2ll * i - 1) * inv[i + 1] % MOD * cat[i - 1] %
      MOD;

/* Floor(n / i), i = [1, n], has <= 2 * sqrt(n) diff values.
 Proof: i = [1, sqrt(n)] has sqrt(n) diff values.
 For i = [sqrt(n), n] we have that 1 <= n / i <= sqrt(n)
 and thus has <= sqrt(n) diff values.
*/
/* l = first number that has floor(N / l) = x
 r = last number that has floor(N / r) = x
 N / r >= floor(N / l)
 r <= N / floor(N / l)*/
for(int l = 1, r; l <= n; l = r + 1){
  r = n / (n / l);
  // floor(n / i) has the same value for l <= i <= r
}

/* Recurrence using matriz
 h[i + 2] = a1 * h[i + 1] + a0 * h[i]
 [h[i] h[i-1]] = [h[1] h[0]] * [a1 1] ^ (i - 1)
                               [a0 0]      */

/* Fibonacci in O(log(N)) with memoization
 f(0) = f(1) = 1
 f(2*k) = f(k)^2 + f(k - 1)^2
 f(2*k + 1) = f(k)*[f(k) + 2*f(k - 1)] */

/* Wilson's Theorem Extension
 B = b1 * b2 * ... * bm (mod n) = +-1, all bi <= n such that gcd
    (bi, n) = 1
 if(n <= 4 or n = (odd prime)^k or n = 2 * (odd prime)^k) B =
    -1; for any k
 else B = 1; */

/* Stirling numbers of the second kind
 S(n, k) = Number of ways to split n numbers into k non-empty
    sets
 S(n, 1) = S(n, n) = 1
 S(n, k) = k * S(n - 1, k) + S(n - 1, k - 1)
 Sr(n, k) = S(n, k) with at least r numbers in each set
 Sr(n, k) = k * Sr(n - 1, k) + (n - 1) * Sr(n - r, k - 1)
            (r - 1)
 S(n - d + 1, k - d + 1) = S(n, k) where if indexes i, j belong
    to the same set, then |i - j| >= d */

/* Burnside's Lemma
 |Classes| = 1 / |G| * sum(K ^ C(g)) for each g in G
 G = Different permutations possible
 C(g) = Number of cycles on the permutation g
 K = Number of states for each element

 Different ways to paint a necklace with N beads and K colors:
 G = {(1, 2, ... N), (2, 3, ... N, 1), ... (N, 1, ... N - 1)}
 gi = (i, i + 1, ... i + N), (taking mod N to get it right) i =
    1 ... N
 i -> 2i -> 3i ..., Cycles in gi all have size n / gcd(i, n), so
    C(gi) = gcd(i, n)
 Ans = 1 / N * sum(K ^ gcd(i, n)), i = 1 ... N
 (For the brave, you can get to Ans = 1 / N * sum(euler_phi(N /
    d) * K ^ d), d | N) */

/* Mobius Inversion
 Sum of gcd(i, j), 1 <= i, j <= N?
 sum(k->N) k * sum(i->N) sum(j->N) [gcd(i, j) == k], i = a * k,
    j = b * k
 = sum(k->N) k * sum(a->N/k) sum(b->N/k) [gcd(a, b) == 1]
 = sum(k->N) k * sum(a->N/k) sum(b->N/k) sum(d->N/k) [d | a] * [
    d | b] * mi(d)
 = sum(k->N) k * sum(d->N/k) mi(d) * floor(N / kd)^2, l = kd, l
    <= N, k | l, d = l / k
 = sum(l->N) floor(N / l)^2 * sum(k|l) k * mi(l / k)
 If f(n) = sum(x|n)(g(x) * h(x)) with g(x) and h(x)
    multiplicative, than f(n) is multiplicative
 Hence, g(l) = sum(k|l) k * mi(l / k) is multiplicative
 = sum(l->N) floor(N / l)^2 * g(l) */

/* Frobenius / Chicken McNugget
n, m given, gcd(n, m) = 1, we want to know if it's possible to
    create N = a * n + b * m
N, a, b >= 0
The greatest number NOT possible is n * m - n - m
We can NOT create (n - 1) * (m - 1) / 2 numbers */
```

6.3 Discrete Log (Baby-step Giant-step)

```cpp
// O(sqrt(m))
// Solve c * a^x = b mod(m) for integer x >= 0.
// Return the smallest x possible, or -1 if there is no solution
// If all solutions needed, solve c * a^x = b mod(m) and (a*b) *
    a^y = b mod(m)
// x + k * (y + 1) for k >= 0 are all solutions
// Works for any integer values of c, a, b and positive m

// Corner Cases:
// 0^x = 1 mod(m) returns x = 0, so you may want to change it to
    -1
// You also may want to change for 0^x = 0 mod(1) to return x =
    1 instead
// We leave it like it is because you might be actually checking
    for m^x = 0^x mod(m)
// which would have x = 0 as the actual solution.
ll discrete_log(ll c, ll a, ll b, ll m){
  c = ((c % m) + m) % m, a = ((a % m) + m) % m, b = ((b % m) + m
      ) % m;
  if(c == b)
    return 0;

  ll g = __gcd(a, m);
  if(b % g) return -1;

  if(g > 1){
    ll r = discrete_log(c * a / g, a, b / g, m / g);
    return r + (r >= 0);
  }

  unordered_map<ll, ll> babystep;
  ll n = 1, an = a % m;
```

```
  // set n to the ceil of sqrt(m):
  while(n * n < m) n++, an = (an * a) % m;

  // babysteps:
  ll bstep = b;
  for(ll i = 0; i <= n; i++){
    babystep[bstep] = i;
    bstep = (bstep * a) % m;
  }

  // giantsteps:
  ll gstep = c * an % m;
  for(ll i = 1; i <= n; i++){
    if(babystep.find(gstep) != babystep.end())
      return n * i - babystep[gstep];
    gstep = (gstep * an) % m;
  }
  return -1;
}
```

6.4 Euler Phi

```
// Euler phi (totient)
int ind = 0, pf = primes[0], ans = n;
while (1ll*pf*pf <= n) {
  if (n%pf==0) ans -= ans/pf;
  while (n%pf==0) n /= pf;
  pf = primes[++ind];
}
if (n != 1) ans -= ans/n;

// IME2014
int phi[N];
void totient() {
  for (int i = 1; i < N; ++i)  phi[i]=i;
  for (int i = 2; i < N; i+=2) phi[i]>>=1;
  for (int j = 3; j < N; j+=2) if (phi[j]==j) {
    phi[j]--;
    for (int i = 2*j; i < N; i+=j) phi[i]=phi[i]/j*(j-1);
  }
}
```

6.5 Extended Euclidean and Chinese Remainder

```
// Extended Euclid:
void euclid(ll a, ll b, ll &x, ll &y) {
  if (b) euclid(b, a%b, y, x), y -= x*(a/b);
  else x = 1, y = 0;
}

// find (x, y) such that a*x + b*y = c or return false if it's
     not possible
// [x + k*b/gcd(a, b), y - k*a/gcd(a, b)] are also solutions
bool diof(ll a, ll b, ll c, ll &x, ll &y){
  euclid(abs(a), abs(b), x, y);
  ll g = abs(__gcd(a, b));
  if(c % g) return false;
  x *= c / g;
  y *= c / g;
  if(a < 0) x = -x;
  if(b < 0) y = -y;
  return true;
}

// auxiliar to find_all_solutions
void shift_solution (ll &x, ll &y, ll a, ll b, ll cnt) {
  x += cnt * b;
  y -= cnt * a;
}

// Find the amount of solutions of
// ax + by = c
// in given intervals for x and y
ll find_all_solutions (ll a, ll b, ll c, ll minx, ll maxx, ll
    miny, ll maxy) {
  ll x, y, g = __gcd(a, b);
  if(!diof(a, b, c, x, y)) return 0;
  a /= g; b /= g;

  int sign_a = a>0 ? +1 : -1;
  int sign_b = b>0 ? +1 : -1;

  shift_solution (x, y, a, b, (minx - x) / b);
  if (x < minx)
    shift_solution (x, y, a, b, sign_b);
  if (x > maxx)
    return 0;
  int lx1 = x;

  shift_solution (x, y, a, b, (maxx - x) / b);
  if (x > maxx)
    shift_solution (x, y, a, b, -sign_b);
  int rx1 = x;

  shift_solution (x, y, a, b, - (miny - y) / a);
  if (y < miny)
    shift_solution (x, y, a, b, -sign_a);
  if (y > maxy)
    return 0;
  int lx2 = x;

  shift_solution (x, y, a, b, - (maxy - y) / a);
  if (y > maxy)
    shift_solution (x, y, a, b, sign_a);
  int rx2 = x;

  if (lx2 > rx2)
    swap (lx2, rx2);
  int lx = max (lx1, lx2);
  int rx = min (rx1, rx2);

  if (lx > rx) return 0;
  return (rx - lx) / abs(b) + 1;
}

bool crt_auxiliar(ll a, ll b, ll m1, ll m2, ll &ans){
  ll x, y;
  if(!diof(m1, m2, b - a, x, y)) return false;
  ll lcm = m1 / __gcd(m1, m2) * m2;
  ans = ((a + x % (lcm / m1) * m1) % lcm + lcm) % lcm;
  return true;
}

// find ans such that ans = a[i] mod b[i] for all 0 <= i < n or
     return false if not possible
// ans + k * lcm(b[i]) are also solutions
bool crt(int n, ll a[], ll b[], ll &ans){
  if(!b[0]) return false;
  ans = a[0] % b[0];
  ll l = b[0];
  for(int i = 1; i < n; i++){
   if(!b[i]) return false;
    if(!crt_auxiliar(ans, a[i] % b[i], l, b[i], ans)) return
        false;
    l *= (b[i] / __gcd(b[i], l));
  }
  return true;
}
```

6.6 Fast Fourier Transform(Tourist)

```
//  FFT made by tourist. It if faster and more supportive,
     although it requires more lines of code.
//  Also, it allows operations with MOD, which is usually an
     issue in FFT problems.
namespace fft {
  typedef double dbl;

  struct num {
    dbl x, y;
    num() { x = y = 0; }
    num(dbl x, dbl y) : x(x), y(y) {}
  };

  inline num operator+ (num a, num b) { return num(a.x + b.x, a.
      y + b.y); }
  inline num operator- (num a, num b) { return num(a.x - b.x, a.
      y - b.y); }
  inline num operator* (num a, num b) { return num(a.x * b.x - a
      .y * b.y, a.x * b.y + a.y * b.x); }
  inline num conj(num a) { return num(a.x, -a.y); }

  int base = 1;
  vector<num> roots = {{0, 0}, {1, 0}};
  vector<int> rev = {0, 1};

  const dbl PI = acosl(-1.0);

  void ensure_base(int nbase) {
    if(nbase <= base) return;

    rev.resize(1 << nbase);
    for(int i=0; i < (1 << nbase); i++) {
      rev[i] = (rev[i >> 1] >> 1) + ((i & 1) << (nbase - 1));
    }
    roots.resize(1 << nbase);

    while(base < nbase) {
      dbl angle = 2*PI / (1 << (base + 1));
      for(int i = 1 << (base - 1); i < (1 << base); i++) {
        roots[i << 1] = roots[i];
        dbl angle_i = angle * (2 * i + 1 - (1 << base));
        roots[(i << 1) + 1] = num(cos(angle_i), sin(angle_i));
      }
      base++;
    }
  }

  void fft(vector<num> &a, int n = -1) {
    if(n == -1) {
      n = a.size();
    }
    assert((n & (n-1)) == 0);
    int zeros = __builtin_ctz(n);
    ensure_base(zeros);
    int shift = base - zeros;
    for(int i = 0; i < n; i++) {
      if(i < (rev[i] >> shift)) {
        swap(a[i], a[rev[i] >> shift]);
      }
    }
    for(int k = 1; k < n; k <<= 1) {
      for(int i = 0; i < n; i += 2 * k) {
        for(int j = 0; j < k; j++) {
          num z = a[i+j+k] * roots[j+k];
          a[i+j+k] = a[i+j] - z;
          a[i+j] = a[i+j] + z;
        }
      }
    }
  }

  vector<num> fa, fb;
  vector<int> multiply(vector<int> &a, vector<int> &b) {
    int need = a.size() + b.size() - 1;
    int nbase = 0;
    while((1 << nbase) < need) nbase++;
    ensure_base(nbase);
    int sz = 1 << nbase;
    if(sz > (int) fa.size()) {
      fa.resize(sz);
    }
    for(int i = 0; i < sz; i++) {
      int x = (i < (int) a.size() ? a[i] : 0);
      int y = (i < (int) b.size() ? b[i] : 0);
      fa[i] = num(x, y);
    }
    fft(fa, sz);
    num r(0, -0.25 / sz);
    for(int i = 0; i <= (sz >> 1); i++) {
      int j = (sz - i) & (sz - 1);
      num z = (fa[j] * fa[j] - conj(fa[i] * fa[i])) * r;
      if(i != j) {
        fa[j] = (fa[i] * fa[i] - conj(fa[j] * fa[j])) * r;
      }
      fa[i] = z;
    }
    fft(fa, sz);
    vector<int> res(need);
    for(int i = 0; i < need; i++) {
      res[i] = fa[i].x + 0.5;
    }
    return res;
  }

  vector<int> multiply_mod(vector<int> &a, vector<int> &b, int m
      , int eq = 0) {
    int need = a.size() + b.size() - 1;
    int nbase = 0;
    while ((1 << nbase) < need) nbase++;
    ensure_base(nbase);
    int sz = 1 << nbase;
    if (sz > (int) fa.size()) {
      fa.resize(sz);
    }
    for (int i = 0; i < (int) a.size(); i++) {
      int x = (a[i] % m + m) % m;
      fa[i] = num(x & ((1 << 15) - 1), x >> 15);
    }
    fill(fa.begin() + a.size(), fa.begin() + sz, num {0, 0});
    fft(fa, sz);
    if (sz > (int) fb.size()) {
      fb.resize(sz);
    }
```

```
    if (eq) {
      copy(fa.begin(), fa.begin() + sz, fb.begin());
    } else {
      for (int i = 0; i < (int) b.size(); i++) {
        int x = (b[i] % m + m) % m;
        fb[i] = num(x & ((1 << 15) - 1), x >> 15);
      }
      fill(fb.begin() + b.size(), fb.begin() + sz, num {0, 0});
      fft(fb, sz);
    }
    dbl ratio = 0.25 / sz;
    num r2(0, -1);
    num r3(ratio, 0);
    num r4(0, -ratio);
    num r5(0, 1);
    for (int i = 0; i <= (sz >> 1); i++) {
      int j = (sz - i) & (sz - 1);
      num a1 = (fa[i] + conj(fa[j]));
      num a2 = (fa[i] - conj(fa[j])) * r2;
      num b1 = (fb[i] + conj(fb[j])) * r3;
      num b2 = (fb[i] - conj(fb[j])) * r4;
      if (i != j) {
        num c1 = (fa[j] + conj(fa[i]));
        num c2 = (fa[j] - conj(fa[i])) * r2;
        num d1 = (fb[j] + conj(fb[i])) * r3;
        num d2 = (fb[j] - conj(fb[i])) * r4;
        fa[i] = c1 * d1 + c2 * d2 * r5;
        fb[i] = c1 * d2 + c2 * d1;
      }
      fa[j] = a1 * b1 + a2 * b2 * r5;
      fb[j] = a1 * b2 + a2 * b1;
    }
    fft(fa, sz);
    fft(fb, sz);
    vector<int> res(need);
    for (int i = 0; i < need; i++) {
      long long aa = fa[i].x + 0.5;
      long long bb = fb[i].x + 0.5;
      long long cc = fa[i].y + 0.5;
      res[i] = (aa + ((bb % m) << 15) + ((cc % m) << 30)) % m;
    }
    return res;
  }

  vector<int> square_mod(vector<int> &a, int m) {
    return multiply_mod(a, a, m, 1);
  }
}
```

6.7 Fast Walsh-Hadamard Transform

```
template<const char ch, class T>
std::vector<T> FWHT(std::vector<T> a, const bool inv = false) {
  int n = (int) a.size();
  for(int len = 1; len < n; len += len) {
    for(int i = 0; i < n; i += 2 * len) {
      for(int j = 0; j < len; j++) {
        auto u = a[i + j], v = a[i + j + len];
        if(ch == '^') {
          a[i + j] = u + v;
          a[i + j + len] = u - v;
        }
        if(ch == '|') {
          if(!inv) {
            a[i + j + len] += a[i + j];
          } else {
            a[i + j + len] -= a[i + j];
          }
        }
        if(ch == '&') {
          if(!inv) {
            a[i + j] += a[i + j + len];
          } else {
            a[i + j] -= a[i + j + len];
          }
        }
      }
    }
  }
  if(ch == '^' && inv) {
    for(int i = 0; i < n; i++) {
      a[i] = a[i] / n;
    }
  }
  return a;
}
```

6.8 Gaussian Elimination (xor)

```
// Gauss Elimination for xor boolean operations
// Return false if not possible to solve
// Use boolean matrixes 0-indexed
// n equations, m variables, O(n * m * m)
// eq[i][j] = coefficient of j-th element in i-th equation
// r[i] = result of i-th equation
// Return ans[j] = xj that gives the lexicographically greatest
     solution (if possible)
// (Can be changed to lexicographically least, follow the
     comments in the code)
// WARNING!! The arrays get changed during de algorithm

bool eq[N][M], r[N], ans[M];

bool gauss_xor(int n, int m){
  for(int i = 0; i < m; i++)
    ans[i] = true;
  int lid[N] = {0}; // id + 1 of last element present in i-th
      line of final matrix
  int l = 0;
  for(int i = m - 1; i >= 0; i--){
    for(int j = l; j < n; j++)
      if(eq[j][i]){ // pivot
        swap(eq[l], eq[j]);
        swap(r[l], r[j]);
      }
    if(l == n || !eq[l][i])
      continue;
    lid[l] = i + 1;
    for(int j = l + 1; j < n; j++){ // eliminate column
      if(!eq[j][i])
        continue;
      for(int k = 0; k <= i; k++)
        eq[j][k] ^= eq[l][k];
      r[j] ^= r[l];
    }
    l++;
  }
  for(int i = n - 1; i >= 0; i--){ // solve triangular matrix
    for(int j = 0; j < lid[i + 1]; j++)
      r[i] ^= (eq[i][j] && ans[j]);
    // for lexicographically least just delete the for bellow
    for(int j = lid[i + 1]; j + 1 < lid[i]; j++){
      ans[j] = true;
      r[i] ^= eq[i][j];
    }
    if(lid[i])
      ans[lid[i] - 1] = r[i];
    else if(r[i])
      return false;
  }
  return true;
}
```

6.9 Gaussian Elimination (double)

```
//Gaussian Elimination
//double A[N][M+1], X[M]

// if n < m, there's no solution
// column m holds the right side of the equation
// X holds the solutions

for(int j=0; j<m; j++) { //collumn to eliminate
  int l = j;
  for(int i=j+1; i<n; i++) //find largest pivot
    if(abs(A[i][j])>abs(A[l][j]))
      l=i;
  if(abs(A[i][j]) < EPS) continue;
  for(int k = 0; k < m+1; k++) { //Swap lines
    swap(A[l][k],A[j][k]);
  }
  for(int i = j+1; i < n; i++) { //eliminate column
    double t=A[i][j]/A[j][j];
    for(int k = j; k < m+1; k++)
      A[i][k]-=t*A[j][k];
  }
}

for(int i = m-1; i >= 0; i--) { //solve triangular system
  for(int j = m-1; j > i; j--)
    A[i][m] -= A[i][j]*X[j];
  X[i]=A[i][m]/A[i][i];
}
```

6.10 Ternary Search

```
//Ternary Search - O(log(n))
//Max version, for minimum version just change signals
//Faster version - 300 iteratons up to 1e-6 precision
//For integers do (r - l > 3) and beware of boundaries
double ternary_search(double l, double r, int No = 300){
  // for(int i = 0; i < No; i++){
  while(r - l > EPS){
    double m1 = l + (r - l) / 3;
    double m2 = r - (r - l) / 3;
    // if (f(m1) > f(m2))
    if (f(m1) < f(m2))
      l = m1;
    else
      r = m2;
  }
  return f(l);
}
```

6.11 Golden Section Search (Ternary Search)

```
double gss(double l, double r) {
  double m1 = r-(r-l)/gr, m2 = l+(r-l)/gr;
  double f1 = f(m1), f2 = f(m2);
  while(fabs(l-r)>EPS) {
    if(f1>f2) l=m1, f1=f2, m1=m2, m2=l+(r-l)/gr, f2=f(m2);
    else r=m2, f2=f1, m2=m1, m1=r-(r-l)/gr, f1=f(m1);
  }
  return l;
}
```

6.12 Josephus

```
// UFMG
/* Josephus Problem - It returns the position to be, in order to
      not die. O(n)*/
/* With k=2, for instance, the game begins with 2 being killed
     and then n+2, n+4, ... */
ll josephus(ll n, ll k) {
  if(n==1) return 1;
  else return (josephus(n-1, k)+k-1)%n+1;
}

/* Another Way to compute the last position to be killed - O(d *
      log n) */
ll josephus(ll n, ll d) {
  ll K = 1;
  while (K <= (d - 1)*n) K = (d * K + d - 2) / (d - 1);
  return d * n + 1 - K;
}
```

6.13 Mobius Inversion

```
// multiplicative function calculator
// euler_phi and mobius are multiplicative
// if another f[N] needed just remove comments
// O(N)

bool p[N];
vector<ll> primes;
ll g[N];
// ll f[N];

void mfc(){
  // if g(1) != 1 than it's not multiplicative
  g[1] = 1;
  // f[1] = 1;
  primes.clear();
  primes.reserve(N / 10);
  for(ll i = 2; i < N; i++){
    if(!p[i]){
      primes.push_back(i);
      for(ll j = i; j < N; j *= i){
        g[j] = // g(p^k) you found
        // f[j] = f(p^k) you found
        p[j] = (j != i);
      }
    }
    for(ll j : primes){
      if(i * j >= N || i % j == 0)
        break;
      for(ll k = j; i * k < N; k *= j){
        g[i * k] = g[i] * g[k];
```

```
        // f[i * k] = f[i] * f[k];
        p[i * k] = true;
      }
    }
  }
}
```

6.14 Mobius Function

```
// 1 if n == 1
// 0 if exists x | n%(x^2) == 0
// else (-1)^k, k = #(p) | p is prime and n%p == 0

//Calculate Mobius for all integers using sieve
//O(n*log(log(n)))
void mobius() {
  for(int i = 1; i < N; i++) mob[i] = 1;

  for(ll i = 2; i < N; i++) if(!sieve[i]){
    for(ll j = i; j < N; j += i) sieve[j] = i, mob[j] *= -1;
    for(ll j = i*i; j < N; j += i*i) mob[j] = 0;
  }
}

/*
//Calculate Mobius for 1 integer
//O(sqrt(n))
int mobius(int n){
  if(n == 1) return 1;
  int p = 0;
  for(int i = 2; i*i <= n; i++)
    if(n%i == 0){
      n /= i;
      p++;
      if(n%i == 0) return 0;
    }
  if(n > 1) p++;
  return p&1 ? -1 : 1;
}
*/
```

6.15 Number Theoretic Transform

```
namespace ntt {
    long long w[N], k, nrev, fact[N], ifact[N];
    void f(int n) {
        fact[0] = 1;
        for(int i = 1; i <= n; i++) {
            fact[i] = (fact[i - 1] * i) % mod;
        }
        ifact[n] = binexp(fact[n], mod - 2);
        for(int i = n - 1; i >= 0; i--) {
            ifact[i] = (ifact[i + 1] * (i + 1)) % mod;
        }
    }
    void init(int n, int root) {
        w[0] = 1;
        k = binexp(root, (mod - 1) / n);
        nrev = binexp(n, mod - 2);
        for(int i = 1; i <= n; i++) {
            w[i] = (w[i - 1] * k) % mod;
        }
    }
    inline void ntt(vector<long long> &a, int n, bool inv =
         false) {
        a.resize(n);
        for(int i = 0, j = 0; i < n; i++) {
            if(i > j) swap(a[i], a[j]);
            for(int l = n / 2; (j ^= l) < l; l >>= 1);
        }
        for(int i = 2; i <= n; i <<= 1) {
            for(int j = 0; j < n; j += i) {
                for(int l = 0; l < i / 2; l++) {
                    int x = j + l, y = j + l + (i / 2), z = (n /
                        i) * l;
                    long long tmp = (a[y] * w[(inv ? (n - z) : z
                        )]) % mod;
                    a[y] = (a[x] - tmp + mod) % mod;
                    a[x] = (a[j + l] + tmp) % mod;
                }
            }
        }
        if(inv) {
            for(int i = 0; i < n; i++) {
                a[i] = (a[i] * nrev) % mod;
            }
        }
     }
     // use search() from PrimitiveRoot.cpp if MOD isn't
         998244353
    vector<long long> multiply(vector<long long>& a, vector<long
         long>& b, int root = 3) {
        int n = a.size() + b.size() - 1;
        while(n & (n - 1)) n++;
        a.resize(n);
        b.resize(n);
        init(n, root);
        ntt(a, n);
        ntt(b, n);
        vector<long long> ans(n);
        for(int i = 0; i < n; i++) {
            ans[i] = (a[i] * b[i]) % mod;
        }
        ntt(ans, n, true);
        return ans;
    }
    vector<long long> poly_shift(vector<long long>& a, int shift
        ) {
        int n = a.size() - 1;
        f(n);
        vector<long long> x(n + 1), y(n + 1);
        long long cur = 1;
        for(int i = 0; i <= n; i++) {
            x[i] = cur * ifact[i] % mod;
            cur = (cur * shift) % mod;
            y[i] = a[n - i] * fact[n - i] % mod;
        }
        vector<long long> tmp = multiply(x, y), res(n + 1);
        for(int i = 0; i <= n; i++) {
            res[i] = tmp[n - i] * ifact[i] % mod;
        }
        return res;
    }
}
```

6.16 Pollard-Rho

```
// factor(N, v) to get N factorized in vector v
// O(N ^ (1 / 4)) on average
// Miller-Rabin - Primarily Test O(|base|*(logn)^2)
ll addmod(ll a, ll b, ll m){
  if(a >= m - b) return a + b - m;
  return a + b;
}

ll mulmod(ll a, ll b, ll m){
  ll ans = 0;
  while(b){
    if(b & 1) ans = addmod(ans, a, m);
    a = addmod(a, a, m);
    b >>= 1;
  }
  return ans;
}

ll fexp(ll a, ll b, ll n){
  ll r = 1;
  while(b){
    if(b & 1) r = mulmod(r, a, n);
    a = mulmod(a, a, n);
    b >>= 1;
  }
  return r;
}

bool miller(ll a, ll n){
  if (a >= n) return true;
  ll s = 0, d = n - 1;
  while(d % 2 == 0) d >>= 1, s++;
  ll x = fexp(a, d, n);
  if (x == 1 || x == n - 1) return true;
  for (int r = 0; r < s; r++, x = mulmod(x,x,n)){
    if (x == 1) return false;
    if (x == n - 1) return true;
  }
  return false;
}

bool isprime(ll n){
  if(n == 1) return false;
  int base[] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
  for (int i = 0; i < 12; ++i) if (!miller(base[i], n)) return
       false;
  return true;
}

ll pollard(ll n){
  ll x, y, d, c = 1;
  if (n % 2 == 0) return 2;
  while(true){
    y = x = 2;
    while(true){
      x = addmod(mulmod(x,x,n), c, n);
      y = addmod(mulmod(y,y,n), c, n);
      y = addmod(mulmod(y,y,n), c, n);
      if (x == y) break;
      d = __gcd(abs(x-y), n);
      if (d > 1) return d;
    }
    c++;
  }
}

vector<ll> factor(ll n){
  if (n == 1 || isprime(n)) return {n};
  ll f = pollard(n);
  vector<ll> l = factor(f), r = factor(n / f);
  l.insert(l.end(), r.begin(), r.end());
  sort(l.begin(), l.end());
  return l;
}

//n < 2,047 base = {2};
//n < 9,080,191 base = {31, 73};
//n < 2,152,302,898,747 base = {2, 3, 5, 7, 11};
//n < 318,665,857,834,031,151,167,461 base = {2, 3, 5, 7, 11,
     13, 17, 19, 23, 29, 31, 37};
//n < 3,317,044,064,679,887,385,961,981 base = {2, 3, 5, 7, 11,
     13, 17, 19, 23, 29, 31, 37, 41};
```

6.17 Primitive Root

```
// Finds a primitive root modulo p
// To make it works for any value of p, we must add calculation
     of phi(p)
// n is 1, 2, 4 or p^k or 2*p^k (p odd in both cases)

//is n primitive root of p ?
bool test(long long x, long long p) {
  long long m = p - 1;
  for(int i = 2; i * i <= m; ++i) if(!(m % i)) {
    if(fexp(x, i, p) == 1) return false;
    if(fexp(x, m / i, p) == 1) return false;
  }
  return true;
}
//find the smallest primitive root for p
int search(int p) {
  for(int i = 2; i < p; i++) if(test(i, p)) return i;
  return -1;
}
```

6.18 Sieve of Eratosthenes

```
// Sieve of Erasthotenes
int p[N]; vi primes;

for (ll i = 2; i < N; ++i) if (!p[i]) {
  for (ll j = i*i; j < N; j+=i) p[j]=1;
  primes.pb(i);
}
```

6.19 Simpson Rule

```
// Simpson Integration Rule
// define the function f
double f(double x) {
  // ...
}

double simpson(double a, double b, int n = 1e6) {
    double h = (b - a) / n;
    double s = f(a) + f(b);
    for (int i = 1; i < n; i += 2) s += 4 * f(a + h*i);
    for (int i = 2; i < n; i += 2) s += 2 * f(a + h*i);
    return s*h/3;
}
```

6.20 Simplex (Stanford)

```cpp
// Two-phase simplex algorithm for solving linear programs of
//   the form
//
//     maximize     c^T x
//     subject to   Ax <= b
//                  x >= 0
//
// INPUT: A -- an m x n matrix
//        b -- an m-dimensional vector
//        c -- an n-dimensional vector
//        x -- a vector where the optimal solution will be
//   stored
//
// OUTPUT: value of the optimal solution (infinity if unbounded
//         above, nan if infeasible)
//
// To use this code, create an LPSolver object with A, b, and c
//   as
// arguments.  Then, call Solve(x).
typedef long double DOUBLE;
typedef vector<DOUBLE> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;
const DOUBLE EPS = 1e-9;

struct LPSolver {
  int m, n;
  VI B, N;
  VVD D;

  LPSolver(const VVD &A, const VD &b, const VD &c) :
    m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2, VD(n + 2)
        ) {
    for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) D[i
        ][j] = A[i][j];
    for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n] = -1; D[
        i][n + 1] = b[i]; }
    for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
    N[n] = -1; D[m + 1][n] = 1;
  }

  void Pivot(int r, int s) {
    for (int i = 0; i < m + 2; i++) if (i != r)
      for (int j = 0; j < n + 2; j++) if (j != s)
        D[i][j] -= D[r][j] * D[i][s] / D[r][s];
    for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] /= D[r][
        s];
    for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] /= -D[r
        ][s];
    D[r][s] = 1.0 / D[r][s];
    swap(B[r], N[s]);
  }

  bool Simplex(int phase) {
    int x = phase == 1 ? m + 1 : m;
    while (true) {
      int s = -1;
      for (int j = 0; j <= n; j++) {
        if (phase == 2 && N[j] == -1) continue;
        if (s == -1 || D[x][j] < D[x][s] || D[x][j] == D[x][s]
            && N[j] < N[s]) s = j;
      }
      if (D[x][s] > -EPS) return true;
      int r = -1;
      for (int i = 0; i < m; i++) {
        if (D[i][s] < EPS) continue;
        if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r
            ][s] ||
          (D[i][n + 1] / D[i][s]) == (D[r][n + 1] / D[r][s]) &&
              B[i] < B[r]) r = i;
      }
      if (r == -1) return false;
      Pivot(r, s);
    }
  }

  DOUBLE Solve(VD &x) {
    int r = 0;
    for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1]) r
        = i;
    if (D[r][n + 1] < -EPS) {
      Pivot(r, n);
      if (!Simplex(1) || D[m + 1][n + 1] < -EPS) return -
          numeric_limits<DOUBLE>::infinity();
      for (int i = 0; i < m; i++) if (B[i] == -1) {
        int s = -1;
        for (int j = 0; j <= n; j++)
          if (s == -1 || D[i][j] < D[i][s] || D[i][j] == D[i][s]
              && N[j] < N[s]) s = j;
        Pivot(i, s);
      }
    }
    if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
    x = VD(n);
    for (int i = 0; i < m; i++) if (B[i] < n) x[B[i]] = D[i][n +
        1];
    return D[m][n + 1];
  }
};

int main() {

  const int m = 4;
  const int n = 3;
  DOUBLE _A[m][n] = {
    { 6, -1, 0 },
    { -1, -5, 0 },
    { 1, 5, 1 },
    { -1, -5, -1 }
  };
  DOUBLE _b[m] = { 10, -4, 5, -5 };
  DOUBLE _c[n] = { 1, -1, 0 };

  VVD A(m);
  VD b(_b, _b + m);
  VD c(_c, _c + n);
  for (int i = 0; i < m; i++) A[i] = VD(_A[i], _A[i] + n);

  LPSolver solver(A, b, c);
  VD x;
  DOUBLE value = solver.Solve(x);

  cerr << "VALUE: " << value << endl; // VALUE: 1.29032
  cerr << "SOLUTION:"; // SOLUTION: 1.74194 0.451613 1
  for (size_t i = 0; i < x.size(); i++) cerr << " " << x[i];
  cerr << endl;
  return 0;
}
```

7 Geometry

7.1 Miscellaneous

```cpp
/*
1)  Square (n = 4) is the only regular polygon with integer
      coordinates

2)  Pick's theorem:  A = i + b/2 - 1
  A: area of the polygon
  i: number of interior points
  b: number of points on the border

3) Conic Rotations
  Given elipse: Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0
  Convert it to: Ax^2 + Bxy + Cy^2 + Dx + Ey = 1 (this formula
        suits better for elipse, before doing this verify F = 0)
  Final conversion: A(x + D/2A)^2 + C(y + E/2C)^2 = 1 + D^2/4A +
        E^2/4C
    B != 0 (Rotate):
      theta = atan2(b, c-a)/2.0;
      A' = (a + c + b/sin(2.0*theta))/2.0; // A
      C' = (a + c - b/sin(2.0*theta))/2.0; // C
      D' = d*sin(theta) + e*cos(theta); // D
      E' = d*cos(theta) - e*sin(theta); // E
    If you do any point calculation, for example finding elipses
        focus, remember to rotate the points by theta after!
*/
```

7.2 Basics (Point)

```cpp
const long double EPS = 1e-9;

typedef long double type;
//for big coordinates change to long long

bool ge(type x, type y) { return x + EPS > y; }
bool le(type x, type y) { return x - EPS < y; }
bool eq(type x, type y) { return ge(x, y) and le(x, y); }
int sign(type x) { return ge(x, 0) - le(x, 0); }

struct point {
  type x, y;

  point() : x(0), y(0) {}
  point(type _x, type _y) : x(_x), y(_y) {}

  point operator -() { return point(-x, -y); }
  point operator +(point p) { return point(x + p.x, y + p.y); }
  point operator -(point p) { return point(x - p.x, y - p.y); }
  point operator *(type k) { return point(x*k, y*k); }
  point operator /(type k) { return point(x/k, y/k); }
  type operator *(point p) { return x*p.x + y*p.y; }
  type operator %(point p) { return x*p.y - y*p.x; }
  bool operator ==(const point &p) const{ return x == p.x and y
      == p.y; }
  bool operator !=(const point &p) const{ return x != p.x  or y
      != p.y; }
  bool operator <(const point &p) const { return (x < p.x) or (x
        == p.x and y < p.y); }
  // 0 => same direction
  // 1 => p is on the left
  //-1 => p is on the right
  int dir(point o, point p) {
    type x = (*this - o) % (p - o);
    return ge(x,0) - le(x,0);
  }

  bool on_seg(point p, point q) {
    if (this->dir(p, q)) return 0;
    return ge(x, min(p.x, q.x)) and le(x, max(p.x, q.x)) and ge(
        y, min(p.y, q.y)) and le(y, max(p.y, q.y));
  }
  //rotation: cos * x - sin * y, sin * x + cos * y
};
int direction(point o, point p, point q) { return p.dir(o, q); }
point rotate_ccw90(point p)   { return point(-p.y,p.x); }
point rotate_cw90(point p)    { return point(p.y,-p.x); }

//double area
type area_2(point a, point b, point c) { return cross(a,b) +
     cross(b,c) + cross(c,a); }

//angle between (a1 and b1) vs angle between (a2 and b2)
//1  : bigger
//-1 : smaller
//0  : equal
int angle_less(const point& a1, const point& b1, const point& a2
      , const point& b2) {
  point p1(dot(   a1, b1), abs(cross(   a1, b1)));
  point p2(dot(   a2, b2), abs(cross(   a2, b2)));
  if(cross(p1, p2) < 0) return 1;
  if(cross(p1, p2) > 0) return -1;
  return 0;
}

ostream &operator<<(ostream &os, const point &p) {
  os << "(" << p.x << "," << p.y << ")";
  return os;
}
```

7.3 Radial Sort

```cpp
point origin;
//  below < above
//  order: [pi, 2 * pi)


int above(point p){
  if(p.y == origin.y) return p.x > origin.x;
  return p.y > origin.y;
}

bool cmp(point p, point q){
  int tmp = above(q) - above(p);
  if(tmp) return tmp > 0;
  return p.dir(origin,q) > 0;
  //Be Careful: p.dir(origin,q) == 0
}
```

7.4 Lines

```cpp
//Suggestion: for line intersections check
     line_line_intersection and then use
     compute_line_intersection
//Distance(point - segment): Project point and calculate
     distance
//Segments Distance: brute distance (point - segment) for all
     border points

point project_point_line(point c, point a, point b) {
  ld r = dot(b - a,b - a);
```

```
  if (fabs(r) < EPS) return a;
  return a + (b - a)*dot(c - a, b - a)/dot(b - a, b - a);
}

point project_point_ray(point c, point a, point b) {
  ld r = dot(b - a, b - a);
  if (fabs(r) < EPS) return a;
  r = dot(c - a, b - a) / r;
  if (le(r, 0)) return a;
  return a + (b - a)*r;
}

point project_point_segment(point c, point a, point b) {
  ld r = dot(b - a, b - a);
  if (fabs(r) < EPS) return a;
  r = dot(c - a, b - a)/r;
  if (le(r, 0)) return a;
  if (ge(r, 1)) return b;
  return a + (b - a)*r;
}

ld distance_point_plane(ld x, ld y, ld z,
            ld a, ld b, ld c, ld d)
{
  return fabs(a*x + b*y + c*z - d)/sqrt(a*a + b*b + c*c);
}

bool lines_parallel(point a, point b, point c, point d) {
  return fabs(cross(b - a, d - c)) < EPS;
}

bool lines_collinear(point a, point b, point c, point d) {
  return lines_parallel(a, b, c, d)
    && fabs(cross(a-b, a-c)) < EPS
    && fabs(cross(c-d, c-a)) < EPS;
}

point lines_intersect(point p, point q, point a, point b) {
  point r = q - p, s = b - a, c(p%q, a%b);
  if (eq(r%s,0)) return point(LINF, LINF);
  return point(point(r.x, s.x) % c, point(r.y, s.y) % c) / (r%s)
      ;
}

//be careful: test line_line_intersection before using this
    function
point compute_line_intersection(point a, point b, point c, point
    d) {
  b = b - a; d = c - d; c = c - a;
  assert(dot(b, b) > EPS && dot(d, d) > EPS);
  return a + b*cross(c, d)/cross(b, d);
}

bool line_line_intersect(point a, point b, point c, point d) {
  if(!lines_parallel(a, b, c, d)) return true;
  if(lines_collinear(a, b, c, d)) return true;
  return false;
}

//rays in direction a -> b, c -> d
bool ray_ray_intersect(point a, point b, point c, point d){
  if (a.dist2(c) < EPS || a.dist2(d) < EPS ||
    b.dist2(c) < EPS || b.dist2(d) < EPS) return true;
  if (lines_collinear(a, b, c, d)) {
    if(ge(dot(b - a, d - c), 0)) return true;
    if(ge(dot(a - c, d - c), 0)) return true;
    return false;
  }
  if(!line_line_intersect(a, b, c, d)) return false;
  point inters = lines_intersect(a, b, c, d);
  if(ge(dot(inters - c, d - c), 0) && ge(dot(inters - a, b - a),
      0)) return true;
  return false;
}

bool segment_segment_intersect(point a, point b, point c, point
    d) {
  if (a.dist2(c) < EPS || a.dist2(d) < EPS ||
    b.dist2(c) < EPS || b.dist2(d) < EPS) return true;
  int d1, d2, d3, d4;
  d1 = direction(a, b, c);
  d2 = direction(a, b, d);
  d3 = direction(c, d, a);
  d4 = direction(c, d, b);
  if (d1*d2 < 0 and d3*d4 < 0) return 1;
  return a.on_seg(c, d) or b.on_seg(c, d) or
      c.on_seg(a, b) or d.on_seg(a, b);
}

bool segment_line_intersect(point a, point b, point c, point d){
  if(!line_line_intersect(a, b, c, d)) return false;
  point inters = lines_intersect(a, b, c, d);
  if(inters.on_seg(a, b)) return true;
  return false;
}

//ray in direction c -> d
bool segment_ray_intersect(point a, point b, point c, point d){
  if (a.dist2(c) < EPS || a.dist2(d) < EPS ||
    b.dist2(c) < EPS || b.dist2(d) < EPS) return true;
  if (lines_collinear(a, b, c, d)) {
    if(c.on_seg(a, b)) return true;
    if(ge(dot(d - c, a - c), 0)) return true;
    return false;
  }
  if(!line_line_intersect(a, b, c, d)) return false;
  point inters = lines_intersect(a, b, c, d);
  if(!inters.on_seg(a, b)) return false;
  if(ge(dot(inters - c, d - c), 0)) return true;
  return false;
}

//ray in direction a -> b
bool ray_line_intersect(point a, point b, point c, point d){
  if (a.dist2(c) < EPS || a.dist2(d) < EPS ||
    b.dist2(c) < EPS || b.dist2(d) < EPS) return true;
  if (!line_line_intersect(a, b, c, d)) return false;
  point inters = lines_intersect(a, b, c, d);
  if(!line_line_intersect(a, b, c, d)) return false;
  if(ge(dot(inters - a, b - a), 0)) return true;
  return false;
}
```

7.5 Circle

```
struct circle {
  point c;
  ld r;
  circle() { c = point(); r = 0; }
  circle(point _c, ld _r) : c(_c), r(_r) {}
  ld area() { return acos(-1.0)*r*r; }
  ld chord(ld rad) { return  2*r*sin(rad/2.0); }
  ld sector(ld rad) { return 0.5*rad*area()/acos(-1.0); }
  bool intersects(circle other) {
    return le(c.dist(other.c), r + other.r);
  }
  bool contains(point p) { return le(c.dist(p), r); }
  pair<point, point> getTangentPoint(point p) {
    ld d1 = c.dist(p), theta = asin(r/d1);
    point p1 = (c - p).rotate(-theta);
    point p2 = (c - p).rotate(theta);
    p1 = p1*(sqrt(d1*d1 - r*r)/d1) + p;
    p2 = p2*(sqrt(d1*d1 - r*r)/d1) + p;
    return make_pair(p1,p2);
  }
};

circle circumcircle(point a, point b, point c) {
  circle ans;
  point u = point((b - a).y, -(b - a).x);
  point v = point((c - a).y, -(c - a).x);
  point n = (c - b)*0.5;
  ld t = cross(u,n)/cross(v,u);
  ans.c = ((a + c)*0.5) + (v*t);
  ans.r = ans.c.dist(a);
  return ans;
}

point compute_circle_center(point a, point b, point c) {
  //circumcenter
  b = (a + b)/2;
  c = (a + c)/2;
  return compute_line_intersection(b, b + rotate_cw90(a - b), c,
       c + rotate_cw90(a - c));
}

int inside_circle(point p, circle c) {
  if (fabs(p.dist(c.c) - c.r)<EPS) return 1;
  else if (p.dist(c.c) < c.r) return 0;
  else return 2;
} //0 = inside/1 = border/2 = outside

circle incircle( point p1, point p2, point p3 ) {
  ld m1 = p2.dist(p3);
  ld m2 = p1.dist(p3);
  ld m3 = p1.dist(p2);
  point c = (p1*m1 + p2*m2 + p3*m3)*(1/(m1 + m2 + m3));
  ld s = 0.5*(m1 + m2 + m3);
  ld r = sqrt(s*(s - m1)*(s - m2)*(s - m3))/s;
  return circle(c, r);
}

circle minimum_circle(vector<point> p) {
  random_shuffle(p.begin(), p.end());
  circle C = circle(p[0], 0.0);
  for(int i = 0; i < (int)p.size(); i++) {
    if (C.contains(p[i])) continue;
    C = circle(p[i], 0.0);
    for(int j = 0; j < i; j++) {
      if (C.contains(p[j])) continue;
      C = circle((p[j] + p[i])*0.5, 0.5*p[j].dist(p[i]));
      for(int k = 0; k < j; k++) {
        if (C.contains(p[k])) continue;
        C = circumcircle(p[j], p[i], p[k]);
      }
    }
  }
  return C;
}

// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<point> circle_line_intersection(point a, point b, point c
    , ld r) {
  vector<point> ret;
  b = b - a;
  a = a - c;
  ld A = dot(b, b);
  ld B = dot(a, b);
  ld C = dot(a, a) - r*r;
  ld D = B*B - A*C;
  if (D < -EPS) return ret;
  ret.push_back(c + a + b*(sqrt(D + EPS) - B)/A);
  if (D > EPS)
    ret.push_back(c + a + b*(-B - sqrt(D))/A);
  return ret;
}

vector<point> circle_circle_intersection(point a, point b, ld r,
     ld R) {
  vector<point> ret;
  ld d = sqrt(a.dist2(b));
  if (d > r + R || d + min(r, R) < max(r, R)) return ret;
  ld x = (d*d - R*R + r*r)/(2*d);
  ld y = sqrt(r*r - x*x);
  point v = (b - a)/d;
  ret.push_back(a + v*x + rotate_ccw90(v)*y);
  if (y > 0)
    ret.push_back(a + v*x - rotate_ccw90(v)*y);
  return ret;
}

//GREAT CIRCLE
double gcTheta(double pLat, double pLong, double qLat, double
     qLong) {
  pLat *= acos(-1.0) / 180.0; pLong *= acos(-1.0) / 180.0; //
      convert degree to radian
  qLat *= acos(-1.0) / 180.0; qLong *= acos(-1.0) / 180.0;
  return acos(cos(pLat)*cos(pLong)*cos(qLat)*cos(qLong) +
    cos(pLat)*sin(pLong)*cos(qLat)*sin(qLong) +
    sin(pLat)*sin(qLat));
}

double gcDistance(double pLat, double pLong, double qLat, double
     qLong, double radius) {
  return radius*gcTheta(pLat, pLong, qLat, qLong);
}
```

7.6 Polygons

```
//Monotone chain O(nlog(n))
#define REMOVE_REDUNDANT
#ifdef REMOVE_REDUNDANT
bool between(const point &a, const point &b, const point &c) {
  return (fabs(area_2(a,b,c)) < EPS && (a.x-b.x)*(c.x-b.x) <= 0
      && (a.y-b.y)*(c.y-b.y) <= 0);
}
#endif

//new change: <= 0 / >= 0 became < 0 / > 0 (yet to be tested)

void convex_hull(vector<point> &pts) {
  sort(pts.begin(), pts.end());
```

```cpp
  pts.erase(unique(pts.begin(), pts.end()), pts.end());
  vector<point> up, dn;
  for (int i = 0; i < pts.size(); i++) {
    while (up.size() > 1 && area_2(up[up.size()-2], up.back(),
          pts[i]) > 0) up.pop_back();
    while (dn.size() > 1 && area_2(dn[dn.size()-2], dn.back(),
          pts[i]) < 0) dn.pop_back();
    up.push_back(pts[i]);
    dn.push_back(pts[i]);
  }
  pts = dn;
  for (int i = (int) up.size() - 2; i >= 1; i--) pts.push_back(
      up[i]);

  #ifdef REMOVE_REDUNDANT
  if (pts.size() <= 2) return;
  dn.clear();
  dn.push_back(pts[0]);
  dn.push_back(pts[1]);
  for (int i = 2; i < pts.size(); i++) {
    if (between(dn[dn.size()-2], dn[dn.size()-1], pts[i])) dn.
        pop_back();
    dn.push_back(pts[i]);
  }
  if (dn.size() >= 3 && between(dn.back(), dn[0], dn[1])) {
    dn[0] = dn.back();
    dn.pop_back();
  }
  pts = dn;
  #endif
}

//avoid using long double for comparisons, change type and add
     division by 2
type compute_signed_area(const vector<point> &p) {
  type area = 0;
  for(int i = 0; i < p.size(); i++) {
    int j = (i+1) % p.size();
    area += p[i].x*p[j].y - p[j].x*p[i].y;
  }
  return area;
}

point compute_centroid(vector<point> &p) {
  point c(0,0);
  ld scale = 3.0 * compute_signed_area(p);
  for (int i = 0; i < p.size(); i++){
    int j = (i+1) % p.size();
    c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
  }
  return c / scale;
}

bool is_ccw(vector<point> &p) {
  type area = 0;
  for(int i = 2; i < p.size(); i++) {
    area += cross(p[i] - p[0], p[i - 1] - p[0]);
  }
  return area > 0;
}

bool point_in_triangle(point a, point b, point c, point cur){
  ll s1 = abs(cross(b - a, c - a));
  ll s2 = abs(cross(a - cur, b - cur)) + abs(cross(b - cur, c -
      cur)) + abs(cross(c - cur, a - cur));
  return s1 == s2;
}

void sort_lex_hull(vector<point> &hull){
  if(compute_signed_area(hull) < 0) reverse(hull.begin(), hull.
      end());
  int n = hull.size();

  //Sort hull by x
  int pos = 0;
  for(int i = 1; i < n; i++) if(hull[i] < hull[pos]) pos = i;
  rotate(hull.begin(), hull.begin() + pos, hull.end());
}

//determine if point is inside or on the boundary of a polygon (
     O(logn))
bool point_in_convex_polygon(vector<point> &hull, point cur){
  int n = hull.size();
  //Corner cases: point outside most left and most right wedges
  if(cur.dir(hull[0], hull[1]) != 0 && cur.dir(hull[0], hull[1])
        != hull[n - 1].dir(hull[0], hull[1]))
    return false;
  if(cur.dir(hull[0], hull[n - 1]) != 0 && cur.dir(hull[0], hull
        [n - 1]) != hull[1].dir(hull[0], hull[n - 1]))
    return false;

  //Binary search to find which wedges it is between
  int l = 1, r = n - 1;
  while(r - l > 1){
    int mid = (l + r)/2;
    if(cur.dir(hull[0], hull[mid]) <= 0)l = mid;
    else r = mid;
  }
  return point_in_triangle(hull[l], hull[l + 1], hull[0], cur);
}

//Simple Polygons

// this code assumes that there are no 3 colinear points
int maximize_scalar_product(vector<point> &hull, point vec /*,
     int dir_flag*/) {
  /*
    For Minimize change: >= becomes <= and > becomes <
    For finding tangents, use same code passing direction flag
    dir_flag = -1 for right tangent
    dir_flag =  1 for left tangent
    >= or > becomes: == dir_flag
    < or <= becomes != dir_flag
    commentaries below for better clarification
  */
  int ans = 0;
  int n = hull.size();
  if(n < 20) {
    for(int i = 0; i < n; i++) {
      if(hull[i] * vec > hull[ans] * vec) {
        //hull[ans].dir(vec, hull[i]) == dir_flag
        ans = i;
      }
    }
  } else {
    if(hull[1] * vec > hull[ans] * vec) {
      //hull[ans].dir(vec, hull[1]) == dir_flag
      ans = 1;
    }
    for(int rep = 0; rep < 2; rep++) {
      int l = 2, r = n - 1;
      while(l != r) {
        int mid = (l + r + 1) / 2;
        bool flag = hull[mid] * vec >= hull[mid-1] * vec;
        //(hull[ans].dir(vec, hull[l]) == dir_flag
        if(rep == 0) { flag = flag && hull[mid] * vec >= hull[0]
              * vec; }
        //(hull[ans].dir(vec, hull[l]) == dir_flag
        else { flag = flag || hull[mid-1] * vec < hull[0] * vec;
              }
        //(hull[ans].dir(vec, hull[l]) != dir_flag
        if(flag) {
          l = mid;
        } else {
          r = mid - 1;
        }
      }
      if(hull[l] * vec > hull[ans] * vec) {
        //(hull[ans].dir(vec, hull[l]) == dir_flag
        ans = l;
      }
    }
  }
  return ans;
}
```

7.7 Shamos Hoey

```cpp
//Shamos - Hoey for test polygon simple in O(nlog(n))
inline bool adj(int a, int b, int n) {return (b == (a + 1)%n or
     a == (b + 1)%n);}

struct edge{
  point ini, fim;
  edge(point ini = point(0,0), point fim = point(0,0)) : ini(ini
       ), fim(fim) {}
};

struct lower_hull{
    point ini, fim;
    int id_ini, id_fim;

    lower_hull(point ini = point(LINF, LINF), point fim = point
         (-LINF, -LINF)) : ini(ini), fim(fim) {
        id_ini = id_fim = -1;
    }
};

//< here means the edge on the top will be at the begin
bool operator < (const edge& a, const edge& b) {
  if (a.ini == b.ini) return direction(a.ini, a.fim, b.fim) < 0;
  if (a.ini.x < b.ini.x) return direction(a.ini, a.fim, b.ini) <
        0;
  return direction(a.ini, b.fim, b.ini) < 0;
}

int n, k[N], p[N], a[N], root;
vector<int> par_upd[N], adj[N];
set<int> paired;
vector <point> hull[N];
pair<point, int> end_hull[N];
lower_hull low[N];

bool is_simple_polygon(const vector<point> &pts){
  //remember to change events style if it is a bunch of convex
       polygons
  vector <pair<point, pii>> eve;
  vector <pair<edge, int>> edgs;
  set <pair<edge, int>> sweep;
  int n = (int)pts.size();
  for(int i = 0; i < n; i++){
    point l = min(pts[i], pts[(i + 1)%n]);
    point r = max(pts[i], pts[(i + 1)%n]);
    eve.pb({l, {0, i}});
    eve.pb({r, {1, i}});
    edgs.pb(make_pair(edge(l, r), i));
  }
  //{point, {initial/final endpoint, edge index in vector}}
  sort(eve.begin(), eve.end());
  for(auto e : eve){
    if(!e.nd.st){
      auto cur = sweep.lower_bound(edgs[e.nd.nd]);
      pair<edge, int> above, below;
      if(cur != sweep.end()){
        below = *cur;
        if(!adj(below.nd, e.nd.nd, n) and
              segment_segment_intersect(pts[below.nd], pts[(below
              .nd + 1)%n], pts[e.nd.nd], pts[(e.nd.nd + 1)%n]))
          return false;
      }
      if(cur != sweep.begin()){
        above = *(--cur);
        if(!adj(above.nd, e.nd.nd, n) and
              segment_segment_intersect(pts[above.nd], pts[(above
              .nd + 1)%n], pts[e.nd.nd], pts[(e.nd.nd + 1)%n]))
          return false;
      }
      sweep.insert(edgs[e.nd.nd]);
    }
    else{
      auto below = sweep.upper_bound(edgs[e.nd.nd]);
      auto cur = below, above = --cur;
      if(below != sweep.end() and above != sweep.begin()){
        --above;
        if(!adj(below->nd, above->nd, n) and
              segment_segment_intersect(pts[below->nd], pts[(
              below->nd + 1)%n], pts[above->nd], pts[(above->nd +
               1)%n]))
          return false;
      }

      //For convex polygons:
      //Process things if the point is the endpoint of this
            convex hull
      //event: {point, {initial/final endpoint, {hull index,
            edge index in hull}}}
      if(above != sweep.begin() and end_hull[e.nd.nd.st].nd == e
            .nd.nd.nd){
        --above;
        //if below lower hull then its father is the father from
              the polygon with edge above
        if(above->nd.nd < low[above->nd.st].id_fim){
            a[e.nd.nd.st] = above->nd.st;
            par_upd[above->nd.st].pb(e.nd.nd.st);
        }
        //if below upper hull then it is inside the polygon with
              edge above
        else{
            p[e.nd.nd.st] = above->nd.st;
            paired.insert(e.nd.nd.st);
        }
      }

      sweep.erase(cur);
    }
  }
```

```
  return true;
}


//calculate lower hull
//sort lex hull to make most left point to have index 0
sort_lex_hull(hull[i]);

low[i].ini = hull[i][0];
low[i].id_ini = 0;
end_hull[i] = {point(-LINF, -LINF), -1};
//search for point that ends lower hull
//end_hull[i] = point that will mark the end of the hull so we
    can process the polygon in the sweep line
for(int j = 0; j < k[i]; j++){
    point u = hull[i][j];
    if((u.x > low[i].fim.x) or (u.x == low[i].fim.x and u.y <
        low[i].fim.y)) low[i].fim = u, low[i].id_fim = j;
    end_hull[i] = max(end_hull[i], {u, j});
}

//calculate simple polygon to generate graph of convex hulls
//for all nodes with parent add parent to the nodes that depend
    on them
while(!paired.empty()){
    auto cur = paired.begin();
    for(auto v : par_upd[*cur]){
        p[v] = p[*cur];
        paired.insert(v);
    }
    par_upd[*cur].clear();
    paired.erase(cur);
}

//generate graph
//n = virtual node;
for(int i = 0; i < n; i++){
    if(p[i] != -1){
        adj[p[i]].pb(i);
    }
    else{
        adj[n].pb(i);
    }
}
```

7.8 Winding Number

```
bool upward_edge(point a, point b, point c, point d){
    //Line: a - b
    //Edge: c - d
    //Edge who comes from bottom to top (or from right to left),
        but does not consider the final endpoint
    return (direction(a, b, c) < 1 and direction(a, b, d) == 1);
}

bool downward_edge(point a, point b, point c, point d){
    //Line: a - b
    //Edge: c - d
    //Edge who comes from top to bottom (or from left to right),
        but does not consider the initial endpoint
    return (direction(a, b, c) == 1 and direction(a, b, d) < 1);
}

//Crossing Number
//Check ray intersection if point aiming to the infinite hits a
    bound of the polygon
//Direction: Ray_a -> Ray_b
//upward and downward disconsider parallel edges to the ray
if(upward_edge(ray_a, ray_b, pts[j], pts[(j + 1)%n]) ||
    downward_edge(ray_a, ray_b, pts[j], pts[(j + 1)%n]))
    if(segment_ray_intersect(ray_a, ray_b, pts[j], pts[(j + 1)%n
        ]))
     crossing_number++;

//Winding Number
//Check ray intersection if point aiming to the infinite hits a
    bound of the polygon
//upward and downward disconsider parallel edges to the ray
if(upward_edge(ray_a, ray_b, pts[j], pts[(j + 1)%n]))
  if(segment_ray_intersect(ray_a, ray_b, pts[j], pts[(j + 1)%n])
      )
    winding_number++;
if(downward_edge(ray_a, ray_b, pts[j], pts[(j + 1)%n]))
  if(segment_ray_intersect(ray_a, ray_b, pts[j], pts[(j + 1)%n])
      )
    winding_number--;
```

```
//Check if edge is inside simple polygon (assure it is ccw using
    geometry/polygons.cpp)
bool check_edge(int i, int j, vector<point>& pts){
    int n = pts.size();
    int wn = 0;
    for(int k = 0; k < n; k++){
        if(k % n == i or (k + 1)%n == i) continue;
        if(upward_edge(pts[i], pts[j], pts[k % n], pts[(k + 1) %
            n]) or downward_edge(pts[i], pts[j], pts[k % n],
            pts[(k + 1) % n])){
            if(segment_ray_intersect(pts[k % n], pts[(k + 1) % n
                ], pts[i], pts[j])){
                wn++;
            }
        }
    }
    return wn % 2;
}
```

7.9 Closes Point Approach

```
//Closest Point Approach
ld CPA(point p, point u, point q, point v){
    point w = p - q;
    if(fabs(dot(u - v, u - v)) < EPS) return LINF;
    return -dot(w, u - v)/dot(u - v, u - v);
}

pair <bool, ld> time_intersects(point p, point a, point b, point
    v, point u){
    ld num = (p.x - a.x)*(b.y - a.y) - (p.y - a.y)*(b.x - a.x);
    ld den = (v.x - u.x)*(b.y - a.y) - (v.y - u.y)*(b.x - a.x);
    if(eq(abs(num), 0.0) and eq(abs(u%v), 0.0)){
      if(!ge((b - a)*(u), 0)) swap(b, a);
      if(!le((p - a)*(b - a), 0)){
        if(le(u * v,  0) or !le(v.abs2(), u.abs2()))
          return{true, p.dist(b)/(u - v).abs()};
        else
          return {false, LINF};
      }
      else{
        if(ge(u * v, 0) and !le(u.abs2(), v.abs2()))
          return{true, p.dist(a)/(u - v).abs()};
        else
          return {false, LINF};
      }
    }
    if(eq(abs(den), 0)) return {false, LINF};
    ld ans = -num/den;
    if(ge(ans, 0)) return {true, ans};
    return {false, LINF};
}
//check intersection
if((p[i][j] + v[i]*t.nd).on_seg((p[i^1][0] + v[i^1]*t.nd), (p[i
    ^1][1] + v[i^1]*t.nd))) ans = min(ans, t.nd), ok = true;
```

7.10 Rotating Calipers

```
vector<pair<point, point>> edges;

//add id to point struct, mark the point with an Id, better if
    long long coordinates
sort(pts.begin(), pts.end());
for(int i = 0; i < pts.size(); i++){
    point p = pts[i];
    id[p] = i;
}

//create edges and sort perpendicular radially
for(int i = 0; i < n; i++){
    for(int j = i + 1; j < n; j++){
        edges.pb({pts[i], pts[j]});
    }
}
//geometry/radial_sort.cpp
sort(edges.begin(), edges.end(), cmp);

//smaller triangle
for(auto e : edges){
    int tmp = INF;
    int l = id[e.st], r = id[e.nd];
    //do stuff
    //1- remember points will for sure be adjacents.
    //2- if you are not sure about adjacency, the points in
        beetween will be collinear
    //3- point [r + 1, ..., n] are ordered by distance to r (r +
        1: closest, n: furthest)
    //4- point [0, ..., l - 1] are ordered by distance to l (l -
        1: closest, 0: furthest)
    //5- you can find max, min triangles and do binary search /
        two points using this information.
    swap(pts[l], pts[r]);
    swap(id[e.nd], id[e.st]);
}
```

7.11 Closest Pair of Points

```
//DIVIDE AND CONQUER METHOD
//Warning: include variable id into the struct point

struct cmp_y {
  bool operator()(const point & a, const point & b) const {
    return a.y < b.y;
  }
};

ld min_dist = LINF;
pair<int, int> best_pair;
vector<point> pts, stripe;
int n;

void upd_ans(const point & a, const point & b) {
  ld dist = sqrt((a.x - b.x)*(a.x - b.x) + (a.y - b.y)*(a.y - b.
      y));
  if (dist < min_dist) {
    min_dist = dist;
    // best_pair = {a.id, b.id};
  }
}

void closest_pair(int l, int r) {
  if (r - l <= 3) {
    for (int i = l; i < r; ++i) {
      for (int j = i + 1; j < r; ++j) {
        upd_ans(pts[i], pts[j]);
      }
    }
    sort(pts.begin() + l, pts.begin() + r, cmp_y());
    return;
  }

  int m = (l + r) >> 1;
  type midx = pts[m].x;
  closest_pair(l, m);
  closest_pair(m, r);

  merge(pts.begin() + l, pts.begin() + m, pts.begin() + m, pts.
      begin() + r, stripe.begin(), cmp_y());
  copy(stripe.begin(), stripe.begin() + r - l, pts.begin() + l);

  int stripe_sz = 0;
  for (int i = l; i < r; ++i) {
    if (abs(pts[i].x - midx) < min_dist) {
      for (int j = stripe_sz - 1; j >= 0 && pts[i].y - stripe[j
          ].y < min_dist; --j)
        upd_ans(pts[i], stripe[j]);
      stripe[stripe_sz++] = pts[i];
    }
  }

  //3D (sort points by Z before starting)(cfloor in math/basics)
  //map opposite side
  map<pll, vector<int>> f;
  for(int i = m; i < r; i++){
    f[{cfloor(pts[i].x, min_dist), cfloor(pts[i].y, min_dist)}].
        push_back(i);
  }
  //find
  for(int i = l; i < m; i++){
    if((midz - pts[i].z) * (midz - pts[i].z) >= min_dist)
      continue;

    pll cur = {cfloor(pts[i].x, min_dist), cfloor(pts[i].y,
        min_dist)};
    for(int dx = -1; dx <= 1; dx++)
      for(int dy = -1; dy <= 1; dy++)
        for(auto p : f[{cur.st + dx, cur.nd + dy}])
          min_dist = min(min_dist, pts[i].dist2(pts[p]));
  }
}

int main(){
  //read and save in vector pts
  min_dist = LINF;
  stripe.resize(n);
```

```
  sort(pts.begin(), pts.end());
  closest_pair(0, n);
}
```

7.12 Nearest Neighbour

```
// Closest Neighbor - O(n * log(n))
const ll N = 1e6+3, INF = 1e18;
ll n, cn[N], x[N], y[N]; // number of points, closes neighbor, x coordinates, y coordinates

ll sqr(ll i) { return i*i; }
ll dist(int i, int j) { return sqr(x[i]-x[j]) + sqr(y[i]-y[j]); }
ll dist(int i) { return i == cn[i] ? INF : dist(i, cn[i]); }

bool cpx(int i, int j) { return x[i] < x[j] or (x[i] == x[j] and y[i] < y[j]); }
bool cpy(int i, int j) { return y[i] < y[j] or (y[i] == y[j] and x[i] < x[j]); }

ll calc(int i, ll x0) {
  ll dlt = dist(i) - sqr(x[i]-x0);
  return dlt >= 0 ? ceil(sqrt(dlt)) : -1;
}

void updt(int i, int j, ll x0, ll &dlt) {
  if (dist(i) > dist(i, j)) cn[i] = j, dlt = calc(i, x0);
}

void cmp(vi &u, vi &v, ll x0) {
  for(int a=0, b=0; a<u.size(); ++a) {
    ll i = u[a], dlt = calc(i, x0);
    while(b < v.size() and y[i] > y[v[b]]) b++;
    for(int j = b-1; j >= 0     and y[i] - dlt <= y[v[j]]; j--)
      updt(i, v[j], x0, dlt);
    for(int j = b; j < v.size() and y[i] + dlt >= y[v[j]]; j++)
      updt(i, v[j], x0, dlt);
  }
}

void slv(vi &ix, vi &iy) {
  int n = ix.size();
  if (n == 1) { cn[ix[0]] = ix[0]; return; }

  int m = ix[n/2];

  vi ix1, ix2, iy1, iy2;
  for(int i=0; i<n; ++i) {
    if (cpx(ix[i], m)) ix1.push_back(ix[i]);
    else ix2.push_back(ix[i]);

    if (cpx(iy[i], m)) iy1.push_back(iy[i]);
    else iy2.push_back(iy[i]);
  }

  slv(ix1, iy1);
  slv(ix2, iy2);

  cmp(iy1, iy2, x[m]);
  cmp(iy2, iy1, x[m]);
}

void slv(int n) {
  vi ix, iy;
  ix.resize(n);
  iy.resize(n);
  for(int i=0; i<n; ++i) ix[i] = iy[i] = i;
  sort(ix.begin(), ix.end(), cpx);
  sort(iy.begin(), iy.end(), cpy);
  slv(ix, iy);
}
```

7.13 Minkowski Sum

```
//ITA MINKOWSKI
typedef vector<point> polygon;
/*
 * Minkowski sum
   Distance between two polygons P and Q:
    Do Minkowski(P, Q)
    Ans = min(ans, dist((0, 0), edge))
 */

polygon minkowski(polygon & A, polygon & B) {
  polygon P; point v1, v2;
  sort_lex_hull(A), sort_lex_hull(B);
  int n1 = A.size(), n2 = B.size();
  P.push_back(A[0] + B[0]);
  for(int i = 0, j = 0; i < n1 || j < n2;) {
    v1 = A[(i + 1)%n1] - A[i%n1];
    v2 = B[(j + 1)%n2] - B[j%n2];
    if (j == n2 || cross(v1, v2) > EPS) {
      P.push_back(P.back() + v1); i++;
    }
    else if (i == n1 || cross(v1, v2) < -EPS) {
      P.push_back(P.back() + v2); j++;
    }
    else {
      P.push_back(P.back() + (v1 + v2));
      i++; j++;
    }
  }
  P.pop_back();
  sort_lex_hull(P);
  return P;
}

/*
Computing the Minkowski sum of multiple polygons:
the resulting polygon will have the number of sides equal to the number of vectors in all sequences for given polygons, if we count all parallel vectors as one.
Now we can solve the problem in such a way: construct the sequences of vectors for the given polygons and divide these vectors into equivalence classes
in such a way that vectors belong to the same class if and only if they are parallel.
The answer to each query is equal to the number of equivalence classes such that at least one vector belonging to this class is contained in at least one sequence on the segment of polygons;
this can be modeled as the query "count the number of distinct values on the given segment of the given array".
*/
//cmp from radial sort
//build equivalence classes from here with resizing unique and giving id to edges
struct edge{
    point l, r;
    edge(point _l = point(), point _r = point()) : l(_l), r(_r) {}

    bool operator <(const edge& p) const{
        point u = p.r, v = r;
        return cmp(v - l, u - p.l);
    }
    //actually this operator is checking >= not ==
    bool operator ==(const edge& p) const{
        point u = p.r, v = r;
        return cmp(v - l, u - p.l) == 0;
    }
};
```

7.14 Half Plane Intersection

```
// Intersection of halfplanes - O(nlogn)
// Points are given in counterclockwise order
//
// by Agnez

typedef vector<point> polygon;

int cmp(ld x, ld y = 0, ld tol = EPS) {
    return (x <= y + tol) ? (x + tol < y) ? -1 : 0 : 1; }

bool comp(point a, point b){
    if((cmp(a.x) > 0 || (cmp(a.x) == 0 && cmp(a.y) > 0) ) && (
        cmp(b.x) < 0 || (cmp(b.x) == 0 && cmp(b.y) < 0)))
        return 1;
    if((cmp(b.x) > 0 || (cmp(b.x) == 0 && cmp(b.y) > 0) ) && (
        cmp(a.x) < 0 || (cmp(a.x) == 0 && cmp(a.y) < 0)))
        return 0;
    ll R = a%b;
    if(R) return R > 0;
    return false;
}

namespace halfplane{
  struct L{
    point p,v;
    L(){}
    L(point P, point V):p(P),v(V){}
    bool operator<(const L &b)const{ return comp(v, b.v); }
  };
  vector<L> line;
  void addL(point a, point b){line.pb(L(a,b-a));}
  bool left(point &p, L &l){ return cmp(l.v % (p-l.p))>0; }
  bool left_equal(point &p, L &l){ return cmp(l.v % (p-l.p))>=0; }
  void init(){ line.clear(); }

  point pos(L &a, L &b){
    point x=a.p-b.p;
    ld t = (b.v % x)/(a.v % b.v);
    return a.p+a.v*t;
  }

  polygon intersect(){
    sort(line.begin(), line.end());
    deque<L> q; //linhas da intersecao
    deque<point> p; //pontos de intersecao entre elas
    q.push_back(line[0]);
    for(int i=1; i < (int) line.size(); i++){
      while(q.size()>1 && !left(p.back(), line[i]))
        q.pop_back(), p.pop_back();
      while(q.size()>1 && !left(p.front(), line[i]))
        q.pop_front(), p.pop_front();
      if(!cmp(q.back().v % line[i].v) && !left(q.back().p,line[i]))
        q.back() = line[i];
      else if(cmp(q.back().v % line[i].v))
        q.push_back(line[i]), p.push_back(point());
      if(q.size()>1)
        p.back()=pos(q.back(),q[q.size()-2]);
    }
    while(q.size()>1 && !left(p.back(),q.front()))
      q.pop_back(), p.pop_back();
    if(q.size() <= 2) return polygon(); //Nao forma poligono (pode nao ter intersecao)
    if(!cmp(q.back().v % q.front().v)) return polygon(); //Lados paralelos -> area infinita
    point ult = pos(q.back(),q.front());

    bool ok = 1;
    for(int i=0; i < (int) line.size(); i++)
      if(!left_equal(ult,line[i])){ ok=0; break; }

    if(ok) p.push_back(ult); //Se formar um poligono fechado
    polygon ret;
    for(int i=0; i < (int) p.size(); i++)
      ret.pb(p[i]);
    return ret;
  }
};
```

7.15 Delaunay Triangulation

```
/*
Complexity: O(nlogn)
Code by Bruno Maletta (UFMG): https://github.com/brunomaletta/Biblioteca

The definition of the Voronoi diagram immediately shows signs of applications.
*  Given a set S of n points and m query points p1,...,pm, we can answer for each query point, its nearest neighbor in S.
  This can be done in O((n+q)log(n+q)) offline by sweeping the Voronoi diagram and query points.
  Or it can be done online with persistent data structures.

*  For each Delaunay triangle, its circumcircle does not strictly contain any points in S. (In fact, you can also consider this the defining property of Delaunay triangulation)

*  The number of Delaunay edges is at most 3n - 6, so there is hope for an efficient construction.

*  Each point p belongs to S is adjacent to its nearest neighbor with a Delaunay edge.

*  The Delaunay triangulation maximizes the minimum angle in the triangles among all possible triangulations.

*  The Euclidean minimum spanning tree is a subset of Delaunay edges.

*/

bool ccw(point a, point b, point c){ return area_2(a, b, c) > 0; }
```

```
typedef struct QuadEdge* Q;
struct QuadEdge {
  int id;
  point o;
  Q rot, nxt;
  bool used;

  QuadEdge(int id_ = -1, point o_ = point(INF, INF)) :
    id(id_), o(o_), rot(nullptr), nxt(nullptr), used(false) {}

  Q rev() const { return rot->rot; }
  Q next() const { return nxt; }
  Q prev() const { return rot->next()->rot; }
  point dest() const { return rev()->o; }
};

Q edge(point from, point to, int id_from, int id_to) {
  Q e1 = new QuadEdge(id_from, from);
  Q e2 = new QuadEdge(id_to, to);
  Q e3 = new QuadEdge;
  Q e4 = new QuadEdge;
  tie(e1->rot, e2->rot, e3->rot, e4->rot) = {e3, e4, e2, e1};
  tie(e1->nxt, e2->nxt, e3->nxt, e4->nxt) = {e1, e2, e4, e3};
  return e1;
}

void splice(Q a, Q b) {
  swap(a->nxt->rot->nxt, b->nxt->rot->nxt);
  swap(a->nxt, b->nxt);
}

void del_edge(Q& e, Q ne) { // delete e and assign e <- ne
  splice(e, e->prev());
  splice(e->rev(), e->rev()->prev());
  delete e->rev()->rot, delete e->rev();
  delete e->rot; delete e;
  e = ne;
}

Q conn(Q a, Q b) {
  Q e = edge(a->dest(), b->o, a->rev()->id, b->id);
  splice(e, a->rev()->prev());
  splice(e->rev(), b);
  return e;
}

bool in_c(point a, point b, point c, point p) { // p ta na
    circunf. (a, b, c) ?
  type p2 = p*p, A = a*a - p2, B = b*b - p2, C = c*c - p2;
  return area_2(p, a, b) * C + area_2(p, b, c) * A + area_2(p, c
      , a) * B > 0;
}

pair<Q, Q> build_tr(vector<point>& p, int l, int r) {
  if (r-l+1 <= 3) {
    Q a = edge(p[l], p[l+1], l, l+1), b = edge(p[l+1], p[r], l
        +1, r);
    if (r-l+1 == 2) return {a, a->rev()};
    splice(a->rev(), b);
    type ar = area_2(p[l], p[l+1], p[r]);
    Q c = ar ? conn(b, a) : 0;
    if (ar >= 0) return {a, b->rev()};
    return {c->rev(), c};
  }
  int m = (l+r)/2;
  auto [la, ra] = build_tr(p, l, m);
  auto [lb, rb] = build_tr(p, m+1, r);
  while (true) {
    if (ccw(lb->o, ra->o, ra->dest())) ra = ra->rev()->prev();
    else if (ccw(lb->o, ra->o, lb->dest())) lb = lb->rev()->next
        ();
    else break;
  }
  Q b = conn(lb->rev(), ra);
  auto valid = [&](Q e) { return ccw(e->dest(), b->dest(), b->o)
      ; };
  if (ra->o == la->o) la = b->rev();
  if (lb->o == rb->o) rb = b;
  while (true) {
    Q L = b->rev()->next();
    if (valid(L)) while (in_c(b->dest(), b->o, L->dest(), L->
        next()->dest()))
      del_edge(L, L->next());
    Q R = b->prev();
    if (valid(R)) while (in_c(b->dest(), b->o, R->dest(), R->
        prev()->dest()))
      del_edge(R, R->prev());
    if (!valid(L) and !valid(R)) break;
    if (!valid(L) or (valid(R) and in_c(L->dest(), L->o, R->o, R
        ->dest())))
      b = conn(R, b->rev());
    else b = conn(b->rev(), L->rev());
  }
  return {la, rb};
}

//NOTE: Before calculating Delaunay add a bound triangle: (-INF,
    -INF), (INF, INF), (0, INF)
vector<vector<int>> delaunay(vector<point> v) {
  int n = v.size();
  auto tmp = v;
  vector<int> idx(n);
  iota(idx.begin(), idx.end(), 0);
  sort(idx.begin(), idx.end(), [&](int l, int r) { return v[l] <
      v[r]; });
  for (int i = 0; i < n; i++) v[i] = tmp[idx[i]];
  assert(unique(v.begin(), v.end()) == v.end());
  vector<vector<int>> g(n);
  bool col = true;
  for (int i = 2; i < n; i++) if (area_2(v[i], v[i-1], v[i-2]))
      col = false;
  if (col) {
    for (int i = 1; i < n; i++)
      g[idx[i-1]].push_back(idx[i]), g[idx[i]].push_back(idx[i
          -1]);
    return g;
  }
  Q e = build_tr(v, 0, n-1).first;
  vector<Q> edg = {e};
  for (int i = 0; i < edg.size(); e = edg[i++]) {
    for (Q at = e; !at->used; at = at->next()) {
      at->used = true;
      g[idx[at->id]].push_back(idx[at->rev()->id]);
      edg.push_back(at->rev());
    }
  }
  return g;
}

vector<vector<point>> voronoi(const vector<point>& points, const
    vector<point>& delaunay){
  int n = delaunay.size();
  vector<vector<point>> voronoi(n, vector<point>());
  for(int i = 0; i < n; i++){
    for(int d = 0; d < delaunay[i].size(); d++){
      int j = delaunay[i][d], k = delaunay[i][(d + 1) % delaunay
          [i].size()];
      circle c = circumcircle(points[i], points[j], points[k]);
      voronoi[i].push_back(c.c);
      voronoi[j].push_back(c.c);
      voronoi[k].push_back(c.c);
    }
  }
}
```

8 Miscellaneous

8.1 Bitset

```
//Goes through the subsets of a set x :
int b = 0;
do {
// process subset b
} while (b=(b-x)&x);
```

8.2 builtin

```
__builtin_ctz(x) // trailing zeroes
__builtin_clz(x) // leading zeroes
__builtin_popcount(x) // # bits set
__builtin_ffs(x) // index(LSB) + 1 [0 if x==0]

// Add ll to the end for long long [__builtin_clzll(x)]
```

8.3 Date

```
struct Date {
  int d, m, y;
  static int mnt[], mntsum[];

  Date() : d(1), m(1), y(1) {}
  Date(int d, int m, int y) : d(d), m(m), y(y) {}
  Date(int days) : d(1), m(1), y(1) { advance(days); }

  bool bissexto() { return (y%4 == 0 and y%100) or (y%400 == 0);
      }

  int mdays() { return mnt[m] + (m == 2)*bissexto(); }
  int ydays() { return 365+bissexto(); }

  int msum()  { return mntsum[m-1] + (m > 2)*bissexto(); }
  int ysum()  { return 365*(y-1) + (y-1)/4 - (y-1)/100 + (y-1)
      /400; }

  int count() { return (d-1) + msum() + ysum(); }

  int day() {
    int x = y - (m<3);
    return (x + x/4 - x/100 + x/400 + mntsum[m-1] + d + 6)%7;
  }

  void advance(int days) {
    days += count();
    d = m = 1, y = 1 + days/366;
    days -= count();
    while(days >= ydays()) days -= ydays(), y++;
    while(days >= mdays()) days -= mdays(), m++;
    d += days;
  }
};

int Date::mnt[13] = {0, 31, 28, 31, 30, 31, 30, 31, 31, 30, 31,
    30, 31};
int Date::mntsum[13] = {};
for(int i=1; i<13; ++i) Date::mntsum[i] = Date::mntsum[i-1] +
    Date::mnt[i];
```

8.4 Parentesis to Polish (ITA)

```
//Parenthetic to polish expression conversion

inline bool isOp(char c) {
  return c=='+' || c=='-' || c=='*' || c=='/' || c=='^';
}

inline bool isCarac(char c) {
  return (c>='a' && c<='z') || (c>='A' && c<='Z') || (c>='0' &&
      c<='9');
}

int paren2polish(char* paren, char* polish) {
  map<char, int> prec;
  prec['('] = 0;
  prec['+'] = prec['-'] = 1;
  prec['*'] = prec['/'] = 2;
  prec['^'] = 3;
  int len = 0;
  stack<char> op;
  for (int i = 0; paren[i]; i++) {
    if (isOp(paren[i])) {
      while (!op.empty() && prec[op.top()] >= prec[paren[i]]) {
        polish[len++] = op.top(); op.pop();
      }
      op.push(paren[i]);
    }
    else if (paren[i]=='(') op.push('(');
    else if (paren[i]==')') {
      for (; op.top()!='('; op.pop())
        polish[len++] = op.top();
      op.pop();
    }
    else if (isCarac(paren[i]))
      polish[len++] = paren[i];
  }
  for(; !op.empty(); op.pop())
    polish[len++] = op.top();
  polish[len] = 0;
  return len;
}

/*
 * TEST MATRIX
 */

int main() {
  int N, len;
  char polish[400], paren[400];
  scanf("%d", &N);
  for (int j=0; j<N; j++) {
    scanf(" %s", paren);
    paren2polish(paren, polish);
```

```
    printf("%s\n", polish);
  }
  return 0;
}
```

8.5 Parallel Binary Search

```
// Parallel Binary Search - O(nlog n * cost to update data
    structure + qlog n * cost for binary search condition)

struct Query { int i, ans; /*+ query related info*/ };
vector<Query> req;

void pbs(vector<Query>& qs, int l /* = min value*/, int r /* =
      max value*/) {
  if (qs.empty()) return;

  if (l == r) {
    for (auto& q : qs) req[q.i].ans = l;
    return;
  }

  int mid = (l + r) / 2;
  // mid = (l + r + 1) / 2 if different from simple upper/lower
      bound

  for (int i = l; i <= mid; i++) {
    // add value to data structure
  }

  vector<Query> vl, vr;
  for (auto& q : qs) {
    if (/* cond */) vl.push_back(q);
    else vr.push_back(q);
  }

  pbs(vr, mid + 1, r);

  for (int i = l; i <= mid; i++) {
    // remove value from data structure
  }

  pbs(vl, l, mid);
}
```

8.6 Python

```python
# reopen
import sys
sys.stdout = open('out','w')
sys.stdin  = open('in' ,'r')

//Dummy example
R = lambda: map(int, input().split())
n, k = R(),
v, t = [], [0]*n
for p, c, i in sorted(zip(R(), R(), range(n))):
        t[i] = sum(v)+c
        v += [c]
        v = sorted(v)[::-1]
        if len(v) > k:
                v.pop()
print(' '.join(map(str, t)))
```

8.7 Sqrt Decomposition

```
// Square Root Decomposition (Mo's Algorithm) - O(n^(3/2))
const int N = 1e5+1, SQ = 500;
int n, m, v[N];

void add(int p) { /* add value to aggregated data structure */ }
void rem(int p) { /* remove value from aggregated data structure
     */ }

struct query { int i, l, r, ans; } qs[N];

bool c1(query a, query b) {
  if(a.l/SQ != b.l/SQ) return a.l < b.l;
  return a.l/SQ&1 ? a.r > b.r : a.r < b.r;
}

bool c2(query a, query b) { return a.i < b.i; }

/* inside main */
int l = 0, r = -1;
sort(qs, qs+m, c1);
for (int i = 0; i < m; ++i) {
  query &q = qs[i];
  while (r < q.r) add(v[++r]);
  while (r > q.r) rem(v[r--]);
  while (l < q.l) rem(v[l++]);
  while (l > q.l) add(v[--l]);

  q.ans = /* calculate answer */;
}

sort(qs, qs+m, c2); // sort to original order
```

8.8 Latitude Longitude (Stanford)

```
// Converts from rectangular coordinates to latitude/longitude
    and vice
// versa. Uses degrees (not radians).
struct ll
{
  double r, lat, lon;
};

struct rect
{
  double x, y, z;
};

ll convert(rect& P)
{
  ll Q;
  Q.r = sqrt(P.x*P.x+P.y*P.y+P.z*P.z);
  Q.lat = 180/M_PI*asin(P.z/Q.r);
  Q.lon = 180/M_PI*acos(P.x/sqrt(P.x*P.x+P.y*P.y));

  return Q;
}

rect convert(ll& Q)
{
  rect P;
  P.x = Q.r*cos(Q.lon*M_PI/180)*cos(Q.lat*M_PI/180);
  P.y = Q.r*sin(Q.lon*M_PI/180)*cos(Q.lat*M_PI/180);
  P.z = Q.r*sin(Q.lat*M_PI/180);

  return P;
}

int main()
{
  rect A;
  ll B;

  A.x = -1.0; A.y = 2.0; A.z = -3.0;

  B = convert(A);
  cout << B.r << " " << B.lat << " " << B.lon << endl;

  A = convert(B);
  cout << A.x << " " << A.y << " " << A.z << endl;
}
```

8.9 Week day

```
int v[] = { 0, 3, 2, 5, 0, 3, 5, 1, 4, 6, 2, 4 };
int day(int d, int m, int y) {
  y -= m<3;
  return (y + y/4 - y/100 + y/400 + v[m-1] + d)%7;
}
```

9 Math Extra

9.1 Combinatorial formulas

$\sum_{k=0}^{n} k^2 = n(n+1)(2n+1)/6$
$\sum_{k=0}^{n} k^3 = n^2(n+1)^2/4$
$\sum_{k=0}^{n} k^4 = (6n^5 + 15n^4 + 10n^3 - n)/30$
$\sum_{k=0}^{n} k^5 = (2n^6 + 6n^5 + 5n^4 - n^2)/12$
$\sum_{k=0}^{n} x^k = (x^{n+1} - 1)/(x - 1)$
$\sum_{k=0}^{n} kx^k = (x - (n+1)x^{n+1} + nx^{n+2})/(x - 1)^2$
$\binom{n}{k} = \frac{n!}{(n-k)!k!}$
$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$
$\binom{n}{k} = \frac{n}{n-k}\binom{n-1}{k}$
$\binom{n}{k} = \frac{n-k+1}{k}\binom{n}{k-1}$
$\binom{n+1}{k} = \frac{n+1}{n-k+1}\binom{n}{k}$
$\binom{n}{k+1} = \frac{n-k}{k+1}\binom{n}{k}$
$\sum_{k=1}^{n} k\binom{n}{k} = n2^{n-1}$
$\sum_{k=1}^{n} k^2\binom{n}{k} = (n+n^2)2^{n-2}$
$\binom{m+n}{r} = \sum_{k=0}^{r}\binom{m}{k}\binom{n}{r-k}$
$\binom{n}{k} = \prod_{i=1}^{k}\frac{n-k+i}{i}$

9.2 Number theory identities

Lucas' Theorem: For non-negative integers m and n and a prime p,

$$\binom{m}{n} \equiv \prod_{i=0}^{k}\binom{m_i}{n_i} \pmod p,$$

where

$$m = m_k p^k + m_{k-1}p^{k-1} + \cdots + m_1 p + m_0$$

is the base p representation of m, and similarly for n.

9.3 Stirling Numbers of the second kind

Number of ways to partition a set of n numbers into k non-empty subsets.

$$\left\{ {n \atop k} \right\} = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{(k-j)}\binom{k}{j}j^n$$

Recurrence relation: $\left\{ {0 \atop 0} \right\} = 1$

$$\left\{ {n \atop 0} \right\} = \left\{ {0 \atop n} \right\} = 1$$

$$\left\{ {n+1 \atop k} \right\} = k\left\{ {n \atop k} \right\} + \left\{ {n \atop k-1} \right\}$$

9.4 Numerical integration

RK4: to integrate $\dot{y} = f(t, y)$ with $y_0 = y(t_0)$, compute

$$k_1 = f(t_n, y_n)$$
$$k_2 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1)$$
$$k_3 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2)$$
$$k_4 = f(t_n + h, y_n + hk_3)$$
$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$