

IME++ ACM-ICPC Team Notebook

Contents

1	Flags + Template + vimrc	1
1.1	Flags	1
1.2	Template	1
1.3	vimrc	1
2	Data Structures	2
2.1	Bit Binary Search	2
2.2	Bit	2
2.3	Bit 2D	2
2.4	Centroid Decomposition	2
2.5	Heavy-Light Decomposition (Lamarca)	2
2.6	Lichao Tree (ITA)	3
2.7	Merge Sort Tree	3
2.8	Minimum Queue	3
2.9	Ordered Set	3
2.10	Dynamic Segment Tree (Lazy Update)	3
2.11	Dynamic Segment Tree	4
2.12	Iterative Segment Tree	4
2.13	Mod Segment Tree	4
2.14	Persistent Segment Tree (Naum)	5
2.15	Persistent Segment Tree	5
2.16	Struct Segment Tree	5
2.17	Segment Tree 2D	5
2.18	Set Of Intervals	5
2.19	Sparse Table	5
2.20	Sparse Table 2D	6
2.21	KD Tree (Stanford)	6
2.22	Treap	6
2.23	Trie	7
2.24	Union Find	7
3	Dynamic Programming	7
3.1	Convex Hull Trick (emaxx)	7
3.2	Divide and Conquer Optimization	7
3.3	Knuth Optimization	8
3.4	Longest Increasing Subsequence	8
3.5	SOS DP	8
3.6	Steiner tree	8
4	Graphs	8
4.1	2-SAT Kosaraju	8
4.2	Shortest Path (Bellman-Ford)	9
4.3	Block Cut	9
4.4	Articulation points and bridges	9
4.5	Max Flow	9
4.6	Dominator Tree	10
4.7	Erdos Gallai	10
4.8	Eulerian Path	10
4.9	Fast Kuhn	10
4.10	Find Cycle of size 3 and 4	11
4.11	Floyd Warshall	11
4.12	Hungarian Navarro	11
4.13	Strongly Connected Components	11
4.14	MST (Kruskal)	12
4.15	Max Weight on Path (LCA)	12
4.16	Min Cost Max Flow	12
4.17	Shortest Path (SPFA)	13
4.18	Small to Large	13
4.19	Stoer Wagner (Stanford)	13
5	Strings	13

5.1	Aho-Corasick	13
5.2	Aho-Corasick (emaxx)	14
5.3	Booths Algorithm	14
5.4	Knuth-Morris-Pratt (Automaton)	14
5.5	Knuth-Morris-Pratt	14
5.6	Manacher	14
5.7	Manacher 2	14
5.8	Recursive-String Matching	15
5.9	String Hashing	15
5.10	String Multihashing	15
5.11	Suffix Array	16
5.12	Suffix Automaton	16
5.13	Suffix Tree	16
5.14	Z Function	17
6	Mathematics	17
6.1	Basics	17
6.2	Advanced	17
6.3	Discrete Log (Baby-step Giant-step)	18
6.4	Euler Phi	18
6.5	Extended Euclidean and Chinese Remainder	18
6.6	Fast Fourier Transform(Tourist)	18
6.7	Fast Fourier Transform	19
6.8	Fast Walsh-Hadamard Transform	19
6.9	Gaussian Elimination (xor)	20
6.10	Gaussian Elimination (double)	20
6.11	Golden Section Search (Ternary Search)	20
6.12	Josephus	20
6.13	Matrix Exponentiation	20
6.14	Mobius Inversion	21
6.15	Mobius Function	21
6.16	Number Theoretic Transform	21
6.17	Pollard-Rho	21
6.18	Prime Factors	21
6.19	Primitive Root	21
6.20	Sieve of Eratosthenes	22
6.21	Simpson Rule	22
6.22	Simplex (Stanford)	22
7	Geometry	22
7.1	Miscellaneous	22
7.2	Basics (Point)	22
7.3	Radial Sort	23
7.4	Circle	23
7.5	Closest Pair of Points	24
7.6	Half Plane Intersection	24
7.7	Lines	24
7.8	Minkowski Sum	25
7.9	Nearest Neighbour	25
7.10	Polygons	25
7.11	Ternary Search	26
7.12	Delaunay Triangulation	27
7.13	Closest Pair of Points 3D	27
8	Miscellaneous	28
8.1	Bitset	28
8.2	builtin	28
8.3	Date	28
8.4	Parenthesis to Polish (ITA)	28
8.5	Modular Int (Struct)	28
8.6	Parallel Binary Search	29
8.7	prime numbers	29
8.8	Python	29
8.9	Sqrt Decomposition	29
8.10	Latitude Longitude (Stanford)	29
8.11	Week day	29
9	Math Extra	29

9.1	Combinatorial formulas	29
9.2	Number theory identities	30
9.3	Stirling Numbers of the second kind	30
9.4	Burnside's Lemma	30
9.5	Numerical integration	30

1 Flags + Template + vimrc

1.1 Flags

```
g++ -fsanitize=address,undefined -fno-omit-frame-pointer -g -
Wall -Wshadow -std=c++17 -Wno-unused-result -Wno-sign-
compare -Wno-char-subscripts
```

1.2 Template

```
#include <bits/stdc++.h>
using namespace std;

#define st first
#define nd second
#define mp make_pair
#define cl(x, v) memset((x), (v), sizeof(x))
#define gcd(x, y) __gcd((x), (y))

#ifndef ONLINE_JUDGE
#define db(x) cerr << #x << " == " << x << endl
#define dbs(x) cerr << x << endl
#define _ << " " <<
#else
#define db(x) ((void)0)
#define dbs(x) ((void)0)
#endif

typedef long long ll;
typedef long double ld;

typedef pair<int, int> pii;
typedef pair<int, pii> pii;
typedef pair<ll, ll> pll;
typedef pair<ll, pll> pll;

const ld EPS = 1e-9, PI = acos(-1.);
const ll LINF = 0x3f3f3f3f3f3f3f3f;
const int INF = 0x3f3f3f3f, MOD = 1e9+7;;
const int N = 1e5+5;

int main() {
    ios_base::sync_with_stdio(false);
    cin.tie(NULL);
    //freopen("in", "r", stdin);
    //freopen("out", "w", stdout);
    return 0;
}
```

1.3 vimrc

```
syntax on
set et ts=2 sw=0 sts=-1 ai nu hls cindent
nnoremap ; :
vnoremap ; :
noremap <c-j> 15gj
noremap <c-k> 15gk
nnoremap <s-k> i<CR><ESC>
inoremap ,. <esc>
vnoremap ,. <esc>
nnoremap ,. <esc>
```

2 Data Structures

2.1 Bit Binary Search

```
// --- Bit Binary Search in o(log(n)) ---
const int M = 20
const int N = 1 << M

int lower_bound(int val) {
    int ans = 0, sum = 0;
    for(int i = M - 1; i >= 0; i--) {
        int x = ans + (1 << i);
        if(sum + bit[x] < val)
            ans = x, sum += bit[x];
    }

    return ans + 1;
}
```

2.2 Bit

```
// Fenwick Tree / Binary Indexed Tree
ll bit[N];

void add(int p, int v) {
    for (p += 2; p < N; p += p & -p) bit[p] += v;
}

ll query(int p) {
    ll r = 0;
    for (p += 2; p; p -= p & -p) r += bit[p];
    return r;
}
```

2.3 Bit 2D

```
// Thank you for the code tfg!
// O(N(logN)^2)
template<class T = int>
struct Bit2D {
    vector<T> ord;
    vector<vector<T>> fw, coord;

    // pts needs all points that will be used in the upd
    // if range ups remember to build with {x1, y1}, {x1, y2 + 1}, {x2 + 1, y1}, {x2 + 1, y2 + 1}
    Bit2D(vector<pair<T, T>> pts) {
        sort(pts.begin(), pts.end());
        for(auto a : pts)
            if(ord.empty() || a.first != ord.back())
                ord.push_back(a.first);
        fw.resize(ord.size() + 1);
        coord.resize(fw.size());

        for(auto &a : pts)
            swap(a.first, a.second);
        sort(pts.begin(), pts.end());
        for(auto &a : pts) {
            swap(a.first, a.second);
            for(int on = std::upper_bound(ord.begin(), ord.end(), a.first) - ord.begin(); on < fw.size(); on += on & -on)
                if(coord[on].empty() || coord[on].back() != a.second)
                    coord[on].push_back(a.second);
        }

        for(int i = 0; i < fw.size(); i++)
            fw[i].assign(coord[i].size() + 1, 0);
    }

    // point upd
    void upd(T x, T y, T v) {
        for(int xx = upper_bound(ord.begin(), ord.end(), x) - ord.begin(); xx < fw.size(); xx += xx & -xx)
            for(int yy = upper_bound(coord[xx].begin(), coord[xx].end(), y) - coord[xx].begin(); yy < fw[xx].size(); yy += yy & -yy)
                fw[xx][yy] += v;
    }
};
```

```
fw[xx][yy] += v;
}

// point qry
T qry(T x, T y) {
    T ans = 0;
    for(int xx = upper_bound(ord.begin(), ord.end(), x) - ord.begin(); xx > 0; xx -= xx & -xx)
        for(int yy = upper_bound(coord[xx].begin(), coord[xx].end(), y) - coord[xx].begin(); yy > 0; yy -= yy & -yy)
            ans += fw[xx][yy];
    return ans;
}

// range qry
T qry(T x1, T y1, T x2, T y2) {
    return qry(x2, y2) - qry(x2, y1 - 1) - qry(x1 - 1, y2) + qry(x1 - 1, y1 - 1);
}

// range upd
void upd(T x1, T y1, T x2, T y2, T v) {
    upd(x1, y1, v);
    upd(x1, y2 + 1, -v);
    upd(x2 + 1, y1, -v);
    upd(x2 + 1, y2 + 1, v);
}
};
```

2.4 Centroid Decomposition

```
// Centroid decomposition

vector<int> adj[N];
int forb[N], sz[N], par[N];
int n, m;
unordered_map<int, int> dist[N];

void dfs(int u, int p) {
    sz[u] = 1;
    for(int v : adj[u]) {
        if(v != p and !forb[v]) {
            dfs(v, u);
            sz[u] += sz[v];
        }
    }
}

int find_cen(int u, int p, int qt) {
    for(int v : adj[u]) {
        if(v == p or forb[v]) continue;
        if(sz[v] > qt / 2) return find_cen(v, u, qt);
    }
    return u;
}

void getdist(int u, int p, int cen) {
    for(int v : adj[u]) {
        if(v != p and !forb[v]) {
            dist[cen][v] = dist[v][cen] = dist[cen][u] + 1;
            getdist(v, u, cen);
        }
    }
}

void decomp(int u, int p) {
    dfs(u, -1);

    int cen = find_cen(u, -1, sz[u]);
    forb[cen] = 1;
    par[cen] = p;
    dist[cen][cen] = 0;
    getdist(cen, -1, cen);

    for(int v : adj[cen]) if(!forb[v])
        decomp(v, cen);
}

// main
decomp(1, -1);
```

2.5 Heavy-Light (Lamarca)

```
#include <bits/stdc++.h>
using namespace std;

#define fr(i,n) for(int i = 0; i<n; i++)
#define all(v) (v).begin(), (v).end()
typedef long long ll;

template<int N> struct Seg {
    ll s[4*N], lazy[4*N];
    void build(int no = 1, int l = 0, int r = N) {
        if(r-l==1) {
            s[no] = 0;
            return;
        }
        int mid = (l+r)/2;
        build(2*no, l, mid);
        build(2*no+1, mid, r);
        s[no] = max(s[2*no], s[2*no+1]);
    }
    Seg() { //build da HLD tem de ser assim, pq chama sem os parametros
        build();
    }
    void updlazy(int no, int l, int r, ll x) {
        s[no] += x;
        lazy[no] += x;
    }
    void pass(int no, int l, int r) {
        int mid = (l+r)/2;
        updlazy(2*no, l, mid, lazy[no]);
        updlazy(2*no+1, mid, r, lazy[no]);
        lazy[no] = 0;
    }
    void upd(int lup, int rup, ll x, int no = 1, int l = 0, int r = N) {
        if(rup<=l or r<=lup) return;
        if(lup<=l and r<=rup) {
            updlazy(no, l, r, x);
            return;
        }
        pass(no, l, r);
        int mid = (l+r)/2;
        upd(lup, rup, x, 2*no, l, mid);
        upd(lup, rup, x, 2*no+1, mid, r);
        s[no] = max(s[2*no], s[2*no+1]);
    }
    ll qry(int lq, int rq, int no = 1, int l = 0, int r = N) {
        if(rq<=l or r<=lq) return -LLONG_MAX;
        if(lq<=l and r<=rq) {
            return s[no];
        }
        pass(no, l, r);
        int mid = (l+r)/2;
        return max(qry(lq, rq, 2*no, l, mid), qry(lq, rq, 2*no+1, mid, r));
    }
};

template<int N, bool IN_EDGES> struct HLD {
    int t;
    vector<int> g[N];
    int pai[N], sz[N], d[N];
    int root[N], pos[N]; // vi rpos;
    void ae(int a, int b) { g[a].push_back(b), g[b].push_back(a); }
    void dfsSz(int no = 0) {
        if (!pai[no]) g[no].erase(find(all(g[no]), pai[no]));
        sz[no] = 1;
        for(auto &it : g[no]) {
            pai[it] = no; d[it] = d[no]+1;
            dfsSz(it); sz[no] += sz[it];
            if (sz[it] > sz[g[no][0]]) swap(it, g[no][0]);
        }
    }
    void dfsHld(int no = 0) {
        pos[no] = t++; // rpos.pb(no);
        for(auto &it : g[no]) {
            root[it] = (it == g[no][0] ? root[no] : it);
            dfsHld(it);
        }
    }
    void init() {
        root[0] = d[0] = t = 0; pai[0] = -1;
        dfsSz(); dfsHld();
    }
    Seg<N> tree; //lembrar de ter build da seg sem nada
};
```

2.6 Lichao Tree (ITA)

```
#include <cstdio>
#include <vector>
#define INF 0x3f3f3f3f3f3f3f3f
#define MAXN 1009
using namespace std;

typedef long long ll;

/*
 * LiChao Segment Tree
 */

class LiChao {
```

```
template <class Op>
void processPath(int u, int v, Op op) {
    for (; root[u] != root[v]; v = pai[root[v]]) {
        if (d[root[u]] > d[root[v]]) swap(u, v);
        op(pos[root[v]], pos[v]);
        if (d[u] > d[v]) swap(u, v);
        op(pos[u]+IN_EDGES, pos[v]);
    }

    /*
    void changeNode(int v, node val){
        tree.upd(pos[v], val);
    }*/
    void modifySubtree(int v, int val) {
        tree.upd(pos[v]+IN_EDGES, pos[v]+sz[v], val);
    }
    ll querySubtree(int v) {
        return tree.qry(pos[v]+IN_EDGES, pos[v]+sz[v]);
    }
    void modifyPath(int u, int v, int val) {
        processPath(u, v, [this, &val](int l, int r) {
            tree.upd(l, r+1, val); });
    }
    ll queryPath(int u, int v) { //modificacoes geralmente vem
        aqui (para hld soma)
        ll res = -LLONG_MAX; processPath(u, v, [this, &res](int l, int r
        ) {
            res = max(tree.qry(l, r+1), res); });
        return res;
    }
};

//solves https://www.hackerrank.com/challenges/subtrees-and-
//paths/problem
//other problems here: https://blog.anudeep2011.com/heavy-light-
//decomposition/

const int N = 1e5+10;
char str[100];
int main(){
    HLD<N, false> hld;
    int n;
    cin >> n;

    fr(i, n-1){
        int u, v;
        scanf("%d%d", &u, &v);
        u--, v--;
        hld.ae(u, v);
    }
    hld.init();
    int q;
    scanf("%d", &q);
    fr(qg, q){
        scanf("%s", str);
        if(str[0]=='a'){
            int t, val;
            scanf("%d%d", &t, &val);
            t--;
            hld.modifySubtree(t, val);
        } else{
            int u, v;
            scanf("%d%d", &u, &v);
            u--, v--;
            printf("%lld\n", hld.queryPath(u, v));
        }
    }
}
```

2.7 Merge Sort Tree

```
// Mergesort Tree - Time <O(nlogn), O(log^2n)> - Memory O(nlogn)
// Mergesort Tree is a segment tree that stores the sorted
// subarray
// on each node.
vi st[4*N];

void build(int p, int l, int r) {
    if (l == r) { st[p].pb(s[l]); return; }
    build(2*p, l, (l+r)/2);
    build(2*p+1, (l+r)/2+1, r);
    st[p].resize(r-l+1);
    merge(st[2*p].begin(), st[2*p].end(),
          st[2*p+1].begin(), st[2*p+1].end(),
          st[p].begin());
}

int query(int p, int l, int r, int i, int j, int a, int b) {
    if (j < l or i > r) return 0;
    if (i <= l and j >= r)
        return upper_bound(st[p].begin(), st[p].end(), b) -
               lower_bound(st[p].begin(), st[p].end(), a);
    return query(2*p, l, (l+r)/2, i, j, a, b) +
           query(2*p+1, (l+r)/2+1, r, i, j, a, b);
}
```

```
vector<ll> m, b;
int n, sz; ll *x;
#define gx(i) (i < sz ? x[i] : x[sz-1])
void update(int t, int l, int r, ll nm, ll nb) {
    ll xl = nm * gx(l) + nb, xr = nm * gx(r) + nb;
    ll yl = m[t] * gx(l) + b[t], yr = m[t] * gx(r) + b[t];
    if (yl >= xl && yr >= xr) return;
    if (yl <= xl && yr <= xr) {
        m[t] = nm, b[t] = nb; return;
    }
    int mid = (l + r) / 2;
    update(t<<1, l, mid, nm, nb);
    update(1+(t<<1), mid+1, r, nm, nb);
}

public:
LiChao(ll *st, ll *en) : x(st) {
    sz = int(en - st);
    for(n = 1; n < sz; n <= 1);
    m.assign(2*n, 0); b.assign(2*n, -INF);
}

void insert_line(ll nm, ll nb) {
    update(1, 0, n-1, nm, nb);
}

ll query(int i) {
    ll ans = -INF;
    for(int t = i+n; t; t >= 1)
        ans = max(ans, m[t] * x[i] + b[t]);
    return ans;
}
};

/*
 * UVa 12524
 */

ll w[MAXN], x[MAXN], A[MAXN], B[MAXN], dp[MAXN][MAXN];

int main(){
    int N, K;
    while(scanf("%d %d", &N, &K)!=EOF) {
        for(int i=0; i<N; i++){
            scanf("%lld %lld", x+i, w+i);
            A[i] = w[i] + (i>0 ? A[i-1] : 0);
            B[i] = w[i]*x[i] + (i>0 ? B[i-1] : 0);
            dp[i][1] = x[i]*A[i] - B[i];
        }
        for(int k=2; k<=K; k++){
            dp[0][k] = 0;
            LiChao lc(x, x+N);
            for(int i=1; i<N; i++){
                lc.insert_line(A[i-1], -dp[i-1][k-1]-B[i-1]);
                dp[i][k] = x[i]*A[i] - B[i] - lc.query(i);
            }
        }
        printf("%lld\n", dp[N-1][K]);
    }
    return 0;
}
```

2.8 Minimum Queue

```
// O(1) complexity for all operations, except for clear,
// which could be done by creating another deque and using swap

struct MinQueue {
    int plus = 0;
    int sz = 0;
    deque<pair<int, int>> dq;

    bool empty() { return dq.empty(); }
    void clear() { plus = 0; sz = 0; dq.clear(); }
    void add(int x) { plus += x; } // Adds x to every element in
    // the queue
    int min() { return dq.front().first + plus; } // Returns the
    // minimum element in the queue
    int size() { return sz; }

    void push(int x) {
        x -= plus;
        int amt = 1;
        while (dq.size() and dq.back().first >= x)
            amt += dq.back().second, dq.pop_back();
        dq.push_back({ x, amt });
        sz++;
    }

    void pop() {
        dq.front().second--, sz--;
        if (!dq.front().second) dq.pop_front();
    }
};
```

2.9 Ordered Set

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
using namespace std;
using namespace __gnu_pbds;

typedef tree<int, null_type, less<int>, rb_tree_tag,
tree_order_statistics_node_update> ordered_set;

ordered_set s;
s.insert(2), s.insert(3), s.insert(7), s.insert(9);

//find_by_order returns an iterator to the element at a given
// position
auto x = s.find_by_order(2);
cout << *x << "\n"; // 7

//order_of_key returns the position of a given element
cout << s.order_of_key(7) << "\n"; // 2

//If the element does not appear in the set, we get the position
// that the element would have in the set
cout << s.order_of_key(6) << "\n"; // 2
cout << s.order_of_key(8) << "\n"; // 3
```

2.10 Dynamic Segment Tree (Lazy Update)

```
#include <bits/stdc++.h>

/* tested:
https://www.spoj.com/problems/BGSHOOT/
ref:
https://maratona.ic.unicamp.br/MaratonaVerao2022/slides/
AulaSummer-SegmentTree-Aula2.pdf
*/
vector<int> e, d, mx, lazy;
//begin creating node 0, then start your segment tree creating
// node 1
int create() {
    mx.push_back(0);
    lazy.push_back(0);
}
```

```

e.push_back(0);
d.push_back(0);
return mx.size() - 1;
}

void push(int pos, int ini, int fim){
if(pos == 0) return;
if(lazy[pos]) {
mx[pos] += lazy[pos];
// RMQ (max/min) -> update: = lazy[p],      incr: +=
// lazy[p]
// RSQ (sum) -> update: = (r-l+1)*lazy[p], incr: += (r
// -l+1)*lazy[p]
// Count lights on -> flip: = (r-l+1)-st[p];
if(ini != fim) {
if(e[pos] == 0){
int aux = create();
e[pos] = aux;
}
if(d[pos] == 0){
int aux = create();
d[pos] = aux;
}
lazy[e[pos]] += lazy[pos];
lazy[d[pos]] += lazy[pos];
// update: lazy[2*p] = lazy[p], lazy[2*p+1] = lazy[p];
// increment: lazy[2*p] += lazy[p], lazy[2*p+1] += lazy[p]
// flip: lazy[2*p] ^= 1, lazy[2*p+1] ^= 1;
}
lazy[pos] = 0;
}

void update(int pos, int ini, int fim, int p, int q, int val){
if(pos == 0) return;
push(pos, ini, fim);

if(q < ini || p > fim) return;

if(p <= ini and fim <= q){
lazy[pos] += val;
// update: lazy[p] = k;
// increment: lazy[p] += k;
// flip: lazy[p] = 1;
push(pos, ini, fim);
return;
}

int m = (ini + fim) >> 1;
if(e[pos] == 0){
int aux = create();
e[pos] = aux;
}
update(e[pos], ini, m, p, q, val);
if(d[pos] == 0){
int aux = create();
d[pos] = aux;
}
update(d[pos], m + 1, fim, p, q, val);
mx[pos] = max(mx[e[pos]], mx[d[pos]]);
}

int query(int pos, int ini, int fim, int p, int q){
if(pos == 0) return 0;
push(pos, ini, fim);

if(q < ini || p > fim) return 0;

if(p <= ini and fim <= q) return mx[pos];

int m = (ini + fim) >> 1;
return max(query(e[pos], ini, m, p, q), query(d[pos], m + 1,
fim, p, q));
}

```

2.11 Dynamic Segment Tree

```

#include <bits/stdc++.h>

/* tested:
https://www.spoj.com/problems/ORDERSET/

```

```

https://www.eolymp.com/en/contests/8463/problems/72212
https://codeforces.com/contest/474/problem/E
https://codeforces.com/problemset/problem/960/F
ref:
https://maratona.ic.unicamp.br/MaratonaVerao2022/slides/
AulaSummer-SegmentTree-Aula2.pdf
*/

vector<int> e, d, mn;
//begin creating node 0, then start your segment tree creating
node 1
int create(){
mn.push_back(0);
e.push_back(0);
d.push_back(0);
return mn.size() - 1;
}

void update(int pos, int ini, int fim, int id, int val){
if(id < ini || id > fim) return;

if(ini == fim){
mn[pos] = val;
return;
}

int m = (ini + fim) >> 1;
if(id <= m){
if(e[pos] == 0){
int aux = create();
e[pos] = aux;
}
update(e[pos], ini, m, id, val);
}
else{
if(d[pos] == 0){
int aux = create();
d[pos] = aux;
}
update(d[pos], m + 1, fim, id, val);
}

mn[pos] = min(mn[e[pos]], mn[d[pos]]);
}

int query(int pos, int ini, int fim, int p, int q){
if(q < ini || p > fim) return INT_MAX;

if(pos == 0) return 0;

if(p <= ini and fim <= q) return mn[pos];

int m = (ini + fim) >> 1;
return min(query(e[pos], ini, m, p, q), query(d[pos], m + 1,
fim, p, q));
}

```

2.12 Iterative Segment Tree

```

int n; // Array size
int st[2*N];

int query(int a, int b) {
a += n; b += n;
int s = 0;
while (a <= b) {
if (a%2 == 1) s += st[a++];
if (b%2 == 0) s += st[b--];
a /= 2; b /= 2;
}
return s;
}

void update(int p, int val) {
p += n;
st[p] += val;
for (p /= 2; p >= 1; p /= 2)
st[p] = st[2*p] + st[2*p+1];
}

```

2.13 Mod Segment Tree

```

// SegTree with mod
// op1 (l, r) -> sum a[i], i = { 1 .. r }
// op2 (l, r, x) -> a[i] = a[i] mod x, i = { 1 .. r }
// op3 (idx, x) -> a[idx] = x;

const int N = 1e5 + 5;

struct segTreeNode { ll sum, mx, mn, lz = -1; };

int n, m;
ll a[N];
segTreeNode st[4 * N];

void push(int p, int l, int r) {
if(st[p].lz != -1) {
st[p].mx = st[p].mn = st[p].lz;
st[p].sum = (r - l + 1) * st[p].lz;

if(l != r) st[2 * p].lz = st[2 * p + 1].lz = st[p].lz;
st[p].lz = -1;
}
}

void merge(int p) {
st[p].mx = max(st[2 * p].mx, st[2 * p + 1].mx);
st[p].mn = min(st[2 * p].mn, st[2 * p + 1].mn);
st[p].sum = st[2 * p].sum + st[2 * p + 1].sum;
}

void build(int p = 1, int l = 1, int r = n) {
if(l == r) {
st[p].mn = st[p].mx = st[p].sum = a[l];
return;
}

int mid = (l + r) >> 1;
build(2 * p, l, mid);
build(2 * p + 1, mid + 1, r);

merge(p);
}

ll query(int i, int j, int p = 1, int l = 1, int r = n) {
push(p, l, r);
if(r < i or l > j) return 0ll;
if(i <= l and r <= j) return st[p].sum;
int mid = (l + r) >> 1;
return query(i, j, 2 * p, l, mid) + query(i, j, 2 * p + 1, mid
+ 1, r);
}

void module_op(int i, int j, ll x, int p = 1, int l = 1, int r =
n) {
push(p, l, r);
if(r < i or l > j or st[p].mx < x) return;
if(i <= l and r <= j and st[p].mx == st[p].mn) {
st[p].lz = st[p].mx % x;
push(p, l, r);
return;
}

int mid = (l + r) >> 1;
module_op(i, j, x, 2 * p, l, mid);
module_op(i, j, x, 2 * p + 1, mid + 1, r);

merge(p);
}

void set_op(int i, int j, ll x, int p = 1, int l = 1, int r = n)
{
push(p, l, r);
if(r < i or l > j) return;
if(i <= l and r <= j) {
st[p].lz = x;
push(p, l, r);
return;
}

int mid = (l + r) >> 1;
set_op(i, j, x, 2 * p, l, mid);
set_op(i, j, x, 2 * p + 1, mid + 1, r);

merge(p);
}

```

2.14 Persistent Segment Tree (Naum)

```
// Persistent Segment Tree
int n;
int rcnt;
int lc[M], rc[M], st[M];

int update(int p, int l, int r, int i, int v) {
    int rt = ++rcnt;
    if (l == r) { st[rt] = v; return rt; }

    int mid = (l+r)/2;
    if (i <= mid) lc[rt] = update(lc[p], l, mid, i, v), rc[rt] = rc[p];
    else rc[rt] = update(rc[p], mid+1, r, i, v), lc[rt] = lc[p];
    st[rt] = st[lc[rt]] + st[rc[rt]];

    return rt;
}

int query(int p, int l, int r, int i, int j) {
    if (l > j or r < i) return 0;
    if (i <= l and r <= j) return st[p];

    return query(lc[p], l, (l+r)/2, i, j)+query(rc[p], (l+r)/2+1, r, i, j);
}

int main() {
    scanf("%d", &n);
    for (int i = 1; i <= n; ++i) {
        int a;
        scanf("%d", &a);
        r[i] = update(r[i-1], 1, n, i, 1);
    }

    return 0;
}
```

2.15 Persistent Segment Tree

```
// Persistent Segtree
// Memory: O(n logn)
// Operations: O(log n)

int li[N], ri[N]; // [li(u), ri(u)] is the interval of node u
int st[N], lc[N], rc[N]; // Value, left son and right son of node u
int stsz; // Size of segment tree

// Returns root of initial tree.
// i and j are the first and last elements of the tree.
int init(int i, int j) {
    int v = ++stsz;
    li[v] = i, ri[v] = j;

    if (i != j) {
        rc[v] = init(i, (i+j)/2);
        lc[v] = init((i+j)/2+1, j);
        st[v] = /* calculate value from rc[v] and lc[v] */;
    } else {
        st[v] = /* insert initial value here */;
    }

    return v;
}

// Gets the sum from i to j from tree with root u
int sum(int u, int i, int j) {
    if (j < li[u] or ri[u] < i) return 0;
    if (i <= li[u] and ri[u] <= j) return st[u];
    return sum(rc[u], i, j) + sum(lc[u], i, j);
}

// Copies node j into node i
void clone(int i, int j) {
    li[i] = li[j], ri[i] = ri[j];
    st[i] = st[j];
    rc[i] = rc[j], lc[i] = lc[j];
}

// Sums v to index i from the tree with root u
```

```
int update(int u, int i, int v) {
    if (i < li[u] or ri[u] < i) return u;

    clone(++stsz, u);
    u = stsz;
    rc[u] = update(rc[u], i, v);
    lc[u] = update(lc[u], i, v);

    if (li[u] == ri[u]) st[u] += v;
    else st[u] = st[rc[u]] + st[lc[u]];

    return u;
}
```

2.16 Struct Segment Tree

```
// Segment Tree (range query and point update)
// Update - O(log n)
// Query - O(log n)
// Memory - O(n)

struct Node {
    ll val;

    Node(ll _val = 0) : val(_val) {}
    Node(const Node& l, const Node& r) : val(l.val + r.val) {}

    friend ostream& operator<<(ostream& os, const Node& a) {
        os << a.val;
        return os;
    }
};

template <class T = Node, class U = int>
struct SimpleSegTree {
    int n;
    vector<T> st;

    SimpleSegTree(int _n) : n(_n), st(4 * n) {}

    SimpleSegTree(vector<U>& v) : n((int)v.size()), st(4 * n) {
        build(v, 1, 0, n - 1);
    }

    void build(vector<U>& v, int p, int l, int r) {
        if (l == r) { st[p] = T(v[l]); return; }
        int mid = (l + r) / 2;
        build(v, 2 * p, l, mid);
        build(v, 2 * p + 1, mid + 1, r);
        st[p] = T(st[2 * p], st[2 * p + 1]);
    }

    T query(int i, int j, int p, int l, int r) {
        if (l >= i and j >= r) return st[p];
        if (l > j or r < i) return T();
        int mid = (l + r) / 2;
        return T(query(i, j, 2 * p, l, mid), query(i, j, 2 * p + 1, mid + 1, r));
    }

    T query(int i, int j) { return query(i, j, 1, 0, n - 1); }

    void update(int idx, U v, int p, int l, int r) {
        if (l == r) { st[p] = T(v); return; }
        int mid = (l + r) / 2;
        if (idx <= mid) update(idx, v, 2 * p, l, mid);
        else update(idx, v, 2 * p + 1, mid + 1, r);
        st[p] = T(st[2 * p], st[2 * p + 1]);
    }

    void update(int idx, U v) { update(idx, v, 1, 0, n - 1); }
};
```

2.17 Segment Tree 2D

```
// Segment Tree 2D - O(nlog(n)log(n)) of Memory and Runtime
const int N = 1e8+5, M = 2e5+5;
int n, k=1, st[N], lc[N], rc[N];

void addx(int x, int l, int r, int u) {
```

```
    if (x < l or r < x) return;

    st[u]++;
    if (l == r) return;

    if (!rc[u]) rc[u] = ++k, lc[u] = ++k;
    addx(x, l, (l+r)/2, lc[u]);
    addx(x, (l+r)/2+1, r, rc[u]);
}

// Adds a point (x, y) to the grid.
void add(int x, int y, int l, int r, int u) {
    if (y < l or r < y) return;

    if (!st[u]) st[u] = ++k;
    addx(x, l, n, st[u]);

    if (l == r) return;

    if (!rc[u]) rc[u] = ++k, lc[u] = ++k;
    add(x, y, l, (l+r)/2, lc[u]);
    add(x, y, (l+r)/2+1, r, rc[u]);
}

int countx(int x, int l, int r, int u) {
    if (!u or x < l) return 0;
    if (r <= x) return st[u];

    return countx(x, l, (l+r)/2, lc[u]) +
           countx(x, (l+r)/2+1, r, rc[u]);
}

// Counts number of points dominated by (x, y)
// Should be called with l = 1, r = n and u = 1
int count(int x, int y, int l, int r, int u) {
    if (!u or y < l) return 0;
    if (r <= y) return countx(x, l, n, st[u]);

    return count(x, y, l, (l+r)/2, lc[u]) +
           count(x, y, (l+r)/2+1, r, rc[u]);
}
```

2.18 Set Of Intervals

```
// Set of Intervals
// Use when you have disjoint intervals
#include <bits/stdc++.h>
using namespace std;

const int N = 2e5 + 5;

typedef pair<int, int> pii;
typedef pair<pii, int> piii;

int n, m, x, t;
set<pii> s;

void in(int l, int r, int i) {
    vector<pii> add, rem;
    auto it = s.lower_bound({l, 0});
    if (it != s.begin()) it--;
    for (; it != s.end(); it++) {
        int ll = it->first.first;
        int rr = it->first.second;
        int idx = it->second;

        if (ll > r) break;
        if (rr < l) continue;
        if (ll < l) add.push_back({ll, l-1, idx});
        if (rr > r) add.push_back({r+1, rr, idx});
        rem.push_back(*it);
    }
    add.push_back({l, r, i});
    for(auto x : rem) s.erase(x);
    for(auto x : add) s.insert(x);
}
```

2.19 Sparse Table

```
const int N;
```

```
const int M; //log2(N)
int sparse[N][M];

void build() {
    for(int i = 0; i < n; i++)
        sparse[i][0] = v[i];

    for(int j = 1; j < M; j++)
        for(int i = 0; i < n; i++)
            sparse[i][j] =
                i + (1 << j - 1) < n
                ? min(sparse[i][j - 1], sparse[i + (1 << j - 1)][j - 1])
                : sparse[i][j - 1];
}

int query(int a, int b) {
    int pot = 32 - __builtin_clz(b - a) - 1;
    return min(sparse[a][pot], sparse[b - (1 << pot) + 1][pot]);
}
```

2.20 Sparse Table 2D

```
// 2D Sparse Table -  $O(n^2 (\log n)^2)$ ,  $O(1)$ 
const int N = 1e3+1, M = 10;
int t[N][N], v[N][N], dp[M][M][N][N], lg[N], n, m;

void build() {
    int k = 0;
    for(int i=1; i<N; ++i) {
        if (1<k == i/2) k++;
        lg[i] = k;
    }

    // Set base cases
    for(int x=0; x<n; ++x) for(int y=0; y<m; ++y) dp[0][0][x][y] =
        v[x][y];
    for(int j=1; j<M; ++j) for(int x=0; x<n; ++x) for(int y=0; y
        + (1<j)<=m; ++y)
        dp[0][j][x][y] = max(dp[0][j-1][x][y], dp[0][j-1][x][y+(1<j<
            -1)]);

    // Calculate sparse table values
    for(int i=1; i<M; ++i) for(int j=0; j<M; ++j)
        for(int x=0; x+(1<i)<=n; ++x) for(int y=0; y+(1<j)<=m; ++y)
            dp[i][j][x][y] = max(dp[i-1][j][x][y], dp[i-1][j][x+(1<i<
                -1)][y]);
}

int query(int x1, int x2, int y1, int y2) {
    int i = lg[x2-x1+1], j = lg[y2-y1+1];
    int m1 = max(dp[i][j][x1][y1], dp[i][j][x2-(1<i)+1][y1]);
    int m2 = max(dp[i][j][x1][y2-(1<j)+1], dp[i][j][x2-(1<i)+1][
        y2-(1<j)+1]);
    return max(m1, m2);
}
```

2.21 KD Tree (Stanford)

```
const int maxn=200005;

struct kdtree
{
    int x1,xr,y1,yr,z1,zr,max,flag; // flag=0:x axis 1:y 2:z
} tree[5000005];

int N,M,lastans,xq,yq;
int a[maxn],pre[maxn],nxt[maxn];
int x[maxn],y[maxn],z[maxn],wei[maxn];
int xc[maxn],yc[maxn],zc[maxn],wc[maxn],hash[maxn],biao[maxn];

bool cmp1(int a,int b)
{
    return x[a]<x[b];
}

bool cmp2(int a,int b)
{
    return y[a]<y[b];
}
```

```
bool cmp3(int a,int b)
{
    return z[a]<z[b];
}

void makekdtree(int node,int l,int r,int flag)
{
    if (l>r)
    {
        tree[node].max=-maxlongint;
        return;
    }
    int xl=maxlongint,xr=-maxlongint;
    int yl=maxlongint,yr=-maxlongint;
    int zl=maxlongint,zr=-maxlongint,maxc=-maxlongint;
    for (int i=l;i<=r;i++)
        xl=min(xl,x[i]),xr=max(xr,x[i]),
        yl=min(yl,y[i]),yr=max(yr,y[i]),
        zl=min(zl,z[i]),zr=max(zr,z[i]),
        maxc=max(maxc,wei[i]),
        xc[i]=x[i],yc[i]=y[i],zc[i]=z[i],wc[i]=wei[i],biao[i]=i;
    tree[node].flag=flag;
    tree[node].xl=xl,tree[node].xr=xr,tree[node].yl=yl;
    tree[node].yr=yr,tree[node].zl=zl,tree[node].zr=zr;
    tree[node].max=maxc;
    if (l==r) return;
    if (flag==0) sort(biao+l,biao+r+1,cmp1);
    if (flag==1) sort(biao+l,biao+r+1,cmp2);
    if (flag==2) sort(biao+l,biao+r+1,cmp3);
    for (int i=l;i<=r;i++)
        x[i]=xc[biao[i]],y[i]=yc[biao[i]],
        z[i]=zc[biao[i]],wei[i]=wc[biao[i]];
    makekdtree(node*2,l,(l+r)/2,(flag+1)%3);
    makekdtree(node*2+1,(l+r)/2+1,r,(flag+1)%3);
}

int getMax(int node,int xl,int xr,int yl,int yr,int zl,int zr)
{
    xl=max(xl,tree[node].xl);
    xr=min(xr,tree[node].xr);
    yl=max(yl,tree[node].yl);
    yr=min(yr,tree[node].yr);
    zl=max(zl,tree[node].zl);
    zr=min(zr,tree[node].zr);
    if (tree[node].max==maxlongint) return 0;
    if ((xr<tree[node].xl)|| (xl>tree[node].xr)) return 0;
    if ((yr<tree[node].yl)|| (yl>tree[node].yr)) return 0;
    if ((zr<tree[node].zl)|| (zl>tree[node].zr)) return 0;
    if ((tree[node].xl==xl)&&(tree[node].xr==xr)&&
        (tree[node].yl==yl)&&(tree[node].yr==yr)&&
        (tree[node].zl==zl)&&(tree[node].zr==zr))
        return tree[node].max;
    else
        return max(getMax(node*2,xl,xr,yl,yr,zl,zr),
            getMax(node*2+1,xl,xr,yl,yr,zl,zr));
}

int main()
{
    // N 3D-rect with weights
    // find the maximum weight containing the given 3D-point
    return 0;
}
```

2.22 Treap

```
// Treap (probabilistic BST)
// O(logn) operations (supports lazy propagation)

mt19937_64 llrand(random_device{} ());

struct node {
    int val;
    int cnt, rev;
    int mn, mx, mindiff; // value-based treap only!
    ll pri;
    node* l;
    node* r;

    node() {}
    node(int x) : val(x), cnt(1), rev(0), mn(x), mx(x), mindiff(
        INF), pri(llrand()), l(0), r(0) {}
};
```

```
struct treap {
    node* root;
    treap() : root(0) {}
    ~treap() { clear(); }

    int cnt(node* t) { return t ? t->cnt : 0; }
    int mn (node* t) { return t ? t->mn : INF; }
    int mx (node* t) { return t ? t->mx : -INF; }
    int mindiff(node* t) { return t ? t->mindiff : INF; }

    void clear() { del(root); }
    void del(node* t) {
        if (!t) return;
        del(t->l); del(t->r);
        delete t;
        t = 0;
    }

    void push(node* t) {
        if (!t or !t->rev) return;
        swap(t->l, t->r);
        if (t->l) t->l->rev ^= 1;
        if (t->r) t->r->rev ^= 1;
        t->rev = 0;
    }

    void update(node& t) {
        if (!t) return;
        t->cnt = cnt(t->l) + cnt(t->r) + 1;
        t->mn = min(t->val, min(mn(t->l), mn(t->r)));
        t->mx = max(t->val, max(mx(t->l), mx(t->r)));
        t->mindiff = min(mn(t->r) - t->val, min(t->val - mx(t->l),
            min(mindiff(t->l), mindiff(t->r))));
    }

    node* merge(node* l, node* r) {
        push(l); push(r);
        node* t;
        if (!l or !r) t = l ? l : r;
        else if (l->pri > r->pri) l->r = merge(l->r, r), t = l;
        else r->l = merge(l, r->l), t = r;
        update(t);
        return t;
    }

    // pos: amount of nodes in the left subtree or
    // the smallest position of the right subtree in a 0-indexed
    // array
    pair<node*, node*> split(node* t, int pos) {
        if (!t) return {0, 0};
        push(t);

        if (cnt(t->l) < pos) {
            auto x = split(t->r, pos-cnt(t->l)-1);
            t->r = x.st;
            update(t);
            return { t, x.nd };
        }

        auto x = split(t->l, pos);
        t->l = x.nd;
        update(t);
        return { x.st, t };
    }

    // Position-based treap
    // used when the values are just additional data
    // the positions are known when it's built, after that you
    // query to get the values at specific positions
    // 0-indexed array!
    /*
    void insert(int pos, int val) {
        push(root);
        node* x = new node(val);
        auto t = split(root, pos);
        root = merge(merge(t.st, x), t.nd);
    }

    void erase(int pos) {
        auto t1 = split(root, pos);
        auto t2 = split(t1.nd, 1);
        delete t2.st;
        root = merge(t1.st, t2.nd);
    }

    int get_val(int pos) { return get_val(root, pos); }
    int get_val(node* t, int pos) {
        push(t);
```

```

if (cnt(t->l) == pos) return t->val;
if (cnt(t->l) < pos) return get_val(t->r, pos-cnt(t->l)-1);
return get_val(t->l, pos);
}
*/
// -----

// Value-based treap
// used when the values needs to be ordered
int order(node* t, int val) {
    if (!t) return 0;
    push(t);
    if (t->val < val) return cnt(t->l) + 1 + order(t->r, val);
    return order(t->l, val);
}

bool has(node* t, int val) {
    if (!t) return 0;
    push(t);
    if (t->val == val) return 1;
    return has((t->val > val ? t->l : t->r), val);
}

void insert(int val) {
    if (has(root, val)) return; // avoid repeated values
    push(root);
    node* x = new node(val);
    auto t = split(root, order(root, val));
    root = merge(merge(t.st, x), t.nd);
}

void erase(int val) {
    if (!has(root, val)) return;

    auto t1 = split(root, order(root, val));
    auto t2 = split(t1.nd, 1);
    delete t2.st;
    root = merge(t1.st, t2.nd);
}

// Get the maximum difference between values
int querymax(int i, int j) {
    if (i == j) return -1;
    auto t1 = split(root, j+1);
    auto t2 = split(t1.st, i);

    int ans = mx(t2.nd) - mn(t2.nd);
    root = merge(merge(t2.st, t2.nd), t1.nd);
    return ans;
}

// Get the minimum difference between values
int querymin(int i, int j) {
    if (i == j) return -1;
    auto t2 = split(root, j+1);
    auto t1 = split(t2.st, i);

    int ans = mindiff(t1.nd);
    root = merge(merge(t1.st, t1.nd), t2.nd);
    return ans;
}

// -----

void reverse(int l, int r) {
    auto t2 = split(root, r+1);
    auto t1 = split(t2.st, l);
    t1.nd->rev = 1;
    root = merge(merge(t1.st, t1.nd), t2.nd);
}

void print() { print(root); printf("\n"); }
void print(node* t) {
    if (!t) return;
    push(t);
    print(t->l);
    printf("%d ", t->val);
    print(t->r);
}
};

```

2.23 Trie

```

// Trie <O(|S|), O(|S|)>
int trie[N][26], trien = 1;

```

```

int add(int u, char c){
    c-'a';
    if (trie[u][c]) return trie[u][c];
    return trie[u][c] = ++trien;
}

//to add a string s in the trie
int u = 1;
for(char c : s) u = add(u, c);

```

2.24 Union Find

```

/*
*****
* DSU (DISJOINT SET UNION / UNION-FIND)
*
* Time complexity: Unite - O(alpha n)
*
* Find - O(alpha n)
*
* Usage: find(node), unite(node1, node2), sz[find(node)]
*
* Notation: par: vector of parents
*
* sz: vector of subsets sizes, i.e. size of the
* subset a node is in
*****
*/

int par[N], sz[N], his[N];
stack <pii> sp, ss;

int find(int a) { return par[a] == a ? a : par[a] = find(par[a]); }

void unite(int a, int b) {
    if ((a = find(a)) == (b = find(b))) return;
    if (sz[a] < sz[b]) swap(a, b);
    par[b] = a; sz[a] += sz[b];
}

//in main
for(int i = 0; i < N; i++) par[i] = i, sz[i] = 1, his[i] = 0;

//Rollback
int find (int a) { return par[a] == a ? a : find(par[a]); }

void unite (int a, int b) {
    if ((a = find(a)) == (b = find(b))) return;
    if (sz[a] < sz[b]) swap(a, b);
    ss.push({a, sz[a]});
    sp.push({b, par[b]});
    sz[a] += sz[b];
    par[b] = a;
}

void rollback() {
    par[sp.top().st] = sp.top().nd; sp.pop();
    sz[ss.top().st] = ss.top().nd; ss.pop();
}

//Partial Persistence
int t, par[N], sz[N]

int find(int a, int t){
    if(par[a] == a) return a;
    if(his[a] > t) return a;
    return find(par[a], t);
}

void unite(int a, int b){
    if(find(a, t) == find(b, t)) return;
    a = find(a, t), b = find(b, t), t++;
    if(sz[a] < sz[b]) swap(a, b);
    sz[a] += sz[b], par[b] = a, his[b] = t;
}

```

3 Dynamic Programming

3.1 Convex Hull Trick (emaxx)

```

struct Point{
    ll x, y;
    Point(ll x = 0, ll y = 0):x(x), y(y) {}
    Point operator-(Point p){ return Point(x - p.x, y - p.y); }
    Point operator+(Point p){ return Point(x + p.x, y + p.y); }
    Point ccw(){ return Point(-y, x); }
    ll operator%(Point p){ return x*p.y - y*p.x; }
    ll operator*(Point p){ return x*p.x + y*p.y; }
    bool operator<(Point p) const { return x == p.x ? y < p.y : x < p.x; }
};

pair<vector<Point>, vector<Point>> ch(Point *v){
    vector<Point> hull, vecs;
    for(int i = 0; i < n; i++){
        if(hull.size() and hull.back().x == v[i].x) continue;

        while(vecs.size() and vecs.back().x*(v[i] - hull.back()) <= 0)
            vecs.pop_back(), hull.pop_back();

        if(hull.size())
            vecs.pb((v[i] - hull.back()).ccw());

        hull.pb(v[i]);
    }
    return {hull, vecs};
}

ll get(ll x) {
    Point query = {x, 1};
    auto it = lower_bound(vecs.begin(), vecs.end(), query, [](
        Point a, Point b) {
            return a%b > 0;
        });
    return query*hull[it - vecs.begin()];
}

```

3.2 Divide and Conquer Optimization

```

/*
*****
* DIVIDE AND CONQUER OPTIMIZATION ( dp[i][k] = min j<k {dp[j][k]
* -1] + C(j,i)} )
* Description: searches for bounds to optimal point using the
* monotocity condition*
* Condition: L[i][k] <= L[i+1][k]
*
* Time Complexity: O(K*N^2) becomes O(K*N*logN)
*
* Notation: dp[i][k]: optimal solution using k positions, until
* position i
* L[i][k]: optimal point, smallest j which minimizes
* dp[i][k]
* C(i,j): cost for splitting range [j,i] to j and i
*****
*/

const int N = 1e3+5;

ll dp[N][N];

//Cost for using i and j
ll C(ll i, ll j);

void compute(ll l, ll r, ll k, ll optl, ll optr){
    // stop condition
    if(l > r) return;

    ll mid = (l+r)/2;
    //best : cost, pos
    pair<ll,ll> best = {LINF,-1};

    //searchs best: lower bound to right, upper bound to left
    for(ll i = optl; i <= min(mid, optr); i++){
        best = min(best, {dp[i][k-1] + C(i,mid), i});
    }
}

```



```

    }
    dp[mid][k] = best.first;
    ll opt = best.second;

    compute(l, mid-1, k, optl, opt);
    compute(mid + 1, r, k, opt, optr);
}

//Iterate over k to calculate
ll solve(){
    //dimensions of dp[N][K]
    int n, k;

    //Initialize DP
    for(ll i = 1; i <= n; i++){
        //dp[i,1] = cost from 0 to i
        dp[i][1] = C(0, i);
    }

    for(ll l = 2; l <= k; l++){
        compute(l, n, l, 1, n);
    }

    /* Iterate over i to get min(dp[i][k]), don't forget cost
       from n to i
       for(ll i=1;i<=n;i++){
           ll rest = ;
           ans = min(ans,dp[i][k] + rest);
       }
    */
}

```

3.3 Knuth Optimization

```

// Knuth DP Optimization - O(n^3) -> O(n^2)
//
// 1) dp[i][j] = min i<k<j { dp[i][k] + dp[k][j] } + C[i][j]
// 2) dp[i][j] = min k<i { dp[k][j-1] + C[k][i] }
//
// Condition: A[i][j-1] <= A[i][j] <= A[i+1][j]
// A[i][j] is the smallest k that gives an optimal answer to dp[
// i][j]
//
// reference (pt-br): https://algorithmmarch.wordpress.com
// /2016/08/12/a-otimizacao-de-pds-e-o-garcom-da-maratona/
//
// 1) dp[i][j] = min i<k<j { dp[i][k] + dp[k][j] } + C[i][j]
int n;
int dp[N][N], a[N][N];

// declare the cost function
int cost(int i, int j) {
    // ...
}

void knuth() {
    // calculate base cases
    memset(dp, 63, sizeof(dp));
    for (int i = 1; i <= n; i++) dp[i][i] = 0;

    // set initial a[i][j]
    for (int i = 1; i <= n; i++) a[i][i] = i;

    for (int j = 2; j <= n; j++)
        for (int i = j; i >= 1; i--) {
            for (int k = a[i][j-1]; k <= a[i+1][j]; ++k) {
                ll v = dp[i][k] + dp[k][j] + cost(i, j);

                // store the minimum answer for d[i][k]
                // in case of maximum, use v > dp[i][k]
                if (v < dp[i][j])
                    a[i][j] = k, dp[i][j] = v;
            }
        }

    /* Iterate over i to get min(dp[i][j]) for each j, don't
       forget cost from n to
    */
}

// 2) dp[i][j] = min k<i { dp[k][j-1] + C[k][i] }
int n, maxj;
int dp[N][J], a[N][J];

```

```

// declare the cost function
int cost(int i, int j) {
    // ...
}

void knuth() {
    // calculate base cases
    memset(dp, 63, sizeof(dp));
    for (int i = 1; i <= n; i++) dp[i][1] = // ...

    // set initial a[i][j]
    for (int i = 1; i <= n; i++) a[i][1] = 1, a[n+1][i] = n;

    for (int j = 2; j <= maxj; j++)
        for (int i = n; i >= 1; i--) {
            for (int k = a[i][j-1]; k <= a[i+1][j]; k++) {
                ll v = dp[k][j-1] + cost(k, i);

                // store the minimum answer for d[i][k]
                // in case of maximum, use v > dp[i][k]
                if (v < dp[i][j])
                    a[i][j] = k, dp[i][j] = v;
            }
        }

    /* Iterate over i to get min(dp[i][j]) for each j, don't
       forget cost from n to
    */
}

```

3.4 Longest Increasing Subsequence

```

// Longest Increasing Subsequence - O(nlogn)
//
// dp(i) = max j<i { dp(j) | a[j] < a[i] } + 1
//
// int dp[N], v[N], n, lis;

memset(dp, 63, sizeof dp);
for (int i = 0; i < n; ++i) {
    // increasing: lower_bound
    // non-decreasing: upper_bound
    int j = lower_bound(dp, dp + lis, v[i]) - dp;
    dp[j] = min(dp[j], v[i]);
    lis = max(lis, j + 1);
}

```

3.5 SOS DP

```

// O(N * 2^N)
// A[i] = initial values
// Calculate F[i] = Sum of A[j] for j subset of i
for(int i = 0; i < (1 << N); i++)
    F[i] = A[i];
for(int i = 0; i < N; i++)
    for(int j = 0; j < (1 << N); j++)
        if(j & (1 << i))
            F[j] += F[j ^ (1 << i)];

```

3.6 Steiner tree

```

// Steiner-Tree O(2^t*n^2 + n*3^t + APSP)

// N - number of nodes
// T - number of terminals
// dist[N][N] - Adjacency matrix
// steiner_tree() = min cost to connect first t nodes, 1-indexed
// dp[i][bit_mask] = min cost to connect nodes active in bitmask
// rooting in i
// min(dp[i][bit_mask]), i <= n if root doesn't matter

int n, t, dp[N][(1 << T)], dist[N][N];

int steiner_tree() {
    for (int k = 1; k <= n; ++k)
        for (int i = 1; i <= n; ++i)

```

```

        for (int j = 1; j <= n; ++j)
            dist[i][j] = min(dist[i][j], dist[i][k] + dist[k][j]);

    for(int i = 1; i <= n; i++)
        for(int j = 0; j < (1 << t); j++)
            dp[i][j] = INF;
    for(int i = 1; i <= t; i++) dp[i][1 << (i-1)] = 0;

    for(int msk = 0; msk < (1 << t); msk++) {
        for(int i = 1; i <= n; i++) {
            for(int ss = msk; ss > 0; ss = (ss - 1) & msk)
                dp[i][msk] = min(dp[i][msk], dp[i][ss] + dp[i][msk - ss]);

            if(dp[i][msk] != INF)
                for(int j = 1; j <= n; j++)
                    dp[j][msk] = min(dp[j][msk], dp[i][msk] + dist[i][j]);
        }
    }

    int mn = INF;
    for(int i = 1; i <= n; i++) mn = min(mn, dp[i][(1 << t) - 1]);
    return mn;
}

```

4 Graphs

4.1 2-SAT Kosaraju

```

/*
*****
* 2-SAT (TELL WHETHER A SERIES OF STATEMENTS CAN OR CANNOT BE
* FEASIBLE AT THE SAME TIME) *
*
* Time complexity: O(V+E)
*
* Usage: n          -> number of variables, 1-indexed
*
*         p = v(i)   -> picks the "true" state for variable i
*
*         p = nv(i)  -> picks the "false" state for variable i, i.
*         e.         ^i          *
*         add(p, q) -> add clause (p v q) (which also means ~p =>
*         q, which also means ~q => p) *
*         run2sat() -> true if possible, false if impossible
*
*         val[i]     -> tells if i has to be true or false for
*         that solution *
*****
*/

int n, vis[2*N], ord[2*N], ordn, cnt, cmp[2*N], val[N];
vector<int> adj[2*N], adjt[2*N];

// for a variable u with idx i
// u is 2*i and !u is 2*i+1
// (a v b) == !a -> b ^ !b -> a

int v(int x) { return 2*x; }
int nv(int x) { return 2*x+1; }

// add clause (a v b)
void add(int a, int b){
    adj[a^1].push_back(b);
    adj[b^1].push_back(a);
    adjt[b].push_back(a^1);
    adjt[a].push_back(b^1);
}

void dfs(int x){
    vis[x] = 1;
    for(auto v : adj[x]) if(!vis[v]) dfs(v);
    ord[ordn++] = x;
}

void dfst(int x){
    cmp[x] = cnt, vis[x] = 0;
    for(auto v : adjt[x]) if(vis[v]) dfst(v);
}

```



```

bool run2sat(){
    for(int i = 1; i <= n; i++) {
        if(!vis[v(i)]) dfs(v(i));
        if(!vis[nv(i)]) dfs(nv(i));
    }
    for(int i = ordn-1; i >= 0; i--)
        if(vis[ord[i]]) cnt++, dfst(ord[i]);
    for(int i = 1; i <= n; i++){
        if(cmp[v(i)] == cmp[nv(i)]) return false;
        val[i] = cmp[v(i)] > cmp[nv(i)];
    }
    return true;
}

int main () {
    for (int i = 1; i <= n; i++) {
        if (val[i]); // i-th variable is true
        else // i-th variable is false
    }
}

```

4.2 Shortest Path (Bellman-Ford)

```

/*
*****
* BELLMAN-FORD ALGORITHM (SHORTEST PATH TO A VERTEX - WITH
  NEGATIVE COST) *
* Time complexity: O(VE)
*
* Usage: dist[node]
*
* Notation: m:          number of edges
*
*            n:          number of vertices
*
*            (a, b, w):  edge between a and b with weight w
*
*            s:          starting node
*****
*/
const int N = 1e4+10; // Maximum number of nodes
vector<int> adj[N], adjw[N];
int dist[N], v, w;

memset(dist, 63, sizeof(dist));
dist[0] = 0;
for (int i = 0; i < n-1; ++i)
    for (int u = 0; u < n; ++u)
        for (int j = 0; j < adj[u].size(); ++j)
            v = adj[u][j], w = adjw[u][j],
            dist[v] = min(dist[v], dist[u]+w);

```

4.3 Block Cut

```

// Tarjan for Block Cut Tree (Node Biconnected Componentes) - O(
  n + m)
#define pb push_back
#include <bits/stdc++.h>
using namespace std;

const int N = 1e5+5;

// Regular Tarjan stuff
int n, num[N], low[N], cnt, ch[N], art[N];
vector<int> adj[N], st;

int lb[N]; // Last block that node is contained
int bn; // Number of blocks
vector<int> blc[N]; // List of nodes from block

void dfs(int u, int p) {
    num[u] = low[u] = ++cnt;
    ch[u] = adj[u].size();
    st.pb(u);

    if (adj[u].size() == 1) blc[++bn].pb(u);

```

```

    for(int v : adj[u]) {
        if (!num[v]) {
            dfs(v, u), low[u] = min(low[u], low[v]);
            if (low[v] == num[u]) {
                if (p != -1 or ch[u] > 1) art[u] = 1;
                blc[++bn].pb(u);
                while (blc[bn].back() != v)
                    blc[bn].pb(st.back());
            }
        }
        else if (v != p) low[u] = min(low[u], num[v]), ch[v]--;
    }

    if (low[u] == num[u]) st.pop_back();
}

// Nodes from 1 .. n are blocks
// Nodes from n+1 .. 2*n are articulations
vector<int> bct[2*N]; // Adj list for Block Cut Tree

void build_tree() {
    for(int u=1; u<=n; ++u) for(int v : adj[u]) if (num[u] > num[v]) {
        if (lb[u] == lb[v] or blc[lb[u]][0] == v) /* edge u-v
            belongs to block lb[u] */;
        else /* edge u-v belongs to block cut tree */;
        int x = (art[u] ? u + n : lb[u]), y = (art[v] ? v + n : lb[v]);
        bct[x].pb(y), bct[y].pb(x);
    }
}

void tarjan() {
    for(int u=1; u<=n; ++u) if (!num[u]) dfs(u, -1);
    for(int b=1; b<=bn; ++b) for(int u : blc[b]) lb[u] = b;
    build_tree();
}

```

4.4 Articulation points and bridges

```

// Articulation points and Bridges O(V+E)
int par[N], art[N], low[N], num[N], ch[N], cnt;

void articulation(int u) {
    low[u] = num[u] = ++cnt;
    for (int v : adj[u]) {
        if (!num[v]) {
            par[v] = u; ch[u]++;
            articulation(v);
            if (low[v] >= num[u]) art[u] = 1;
            if (low[v] > num[u]) { /* u-v bridge */
                low[u] = min(low[u], low[v]);
            }
        }
        else if (v != par[u]) low[u] = min(low[u], num[v]);
    }
}

for (int i = 0; i < n; ++i) if (!num[i])
    articulation(i), art[i] = ch[i]>1;

```

4.5 Max Flow

```

// Dinic - O(V^2 * E)
// Bipartite graph or unit flow - O(sqrt(V) * E)
// Small flow - O(F * (V + E))
// USE INF = 1e9!

/*
*****
* DINIC (FIND MAX FLOW / BIPARTITE MATCHING)
*
* Time complexity: O(EV^2)
*
* Usage: dinic()
*
*            add_edge(from, to, capacity)
*
*/

```

```

* Testcase:

*
* add_edge(src, 1, 1);    add_edge(1, snk, 1);    add_edge(2, 3,
  INF);
* add_edge(src, 2, 1);    add_edge(2, snk, 1);    add_edge(3, 4,
  INF);
* add_edge(src, 2, 1);    add_edge(3, snk, 1);

* add_edge(src, 2, 1);    add_edge(4, snk, 1);    => dinic() = 4
*
*****

#include <bits/stdc++.h>
using namespace std;

const int N = 1e5+1, INF = 1e9;
struct edge {int v, c, f;};

int n, src, snk, h[N], ptr[N];
vector<edge> eds;
vector<int> g[N];

void add_edge (int u, int v, int c) {
    int k = eds.size();
    eds.push_back({v, c, 0});
    eds.push_back({u, 0, 0});
    g[u].push_back(k);
    g[v].push_back(k+1);
}

void clear() {
    memset(h, 0, sizeof h);
    memset(ptr, 0, sizeof ptr);
    eds.clear();
    for (int i = 0; i < N; i++) g[i].clear();
    src = 0;
    snk = N-1;
}

bool bfs() {
    memset(h, 0, sizeof h);
    queue<int> q;
    h[src] = 1;
    q.push(src);
    while (!q.empty()) {
        int u = q.front(); q.pop();
        for (int i : g[u]) {
            int v = eds[i].v;
            if (!h[v] and eds[i].f < eds[i].c)
                q.push(v), h[v] = h[u] + 1;
        }
    }
    return h[snk];
}

int dfs (int u, int flow) {
    if (!flow or u == snk) return flow;
    for (int &i = ptr[u]; i < g[u].size(); ++i) {
        edge &dir = eds[g[u][i]], &rev = eds[g[u][i]^1];
        int v = dir.v;
        if (h[v] != h[u] + 1) continue;
        int inc = min(flow, dir.c - dir.f);
        inc = dfs(v, inc);
        if (inc) {
            dir.f += inc, rev.f -= inc;
            return inc;
        }
    }
    return 0;
}

int dinic() {
    int flow = 0;
    while (bfs()) {
        memset(ptr, 0, sizeof ptr);
        while (int inc = dfs(src, INF)) flow += inc;
    }
    return flow;
}

//Recover Dinic
void recover() {
    for (int i = 0; i < eds.size(); i += 2) {
        //edge (u -> v) is being used with flow f
        if (eds[i].f > 0) {
            int v = eds[i].v;
            int u = eds[i^1].v;

```

```

    }
}

/*
*****
* FLOW WITH DEMANDS
*
*
* 1 - Finding an arbitrary flow
*
* Assume a network with [L, R] on edges (some may have L = 0),
* let's call it old network.
* Create a New Source and New Sink (this will be the src and snk
* for Dinic).
* Modelling Network:
*
* 1) Every edge from the old network will have cost R - L
*
* 2) Add an edge from New Source to every vertex v with cost:
*
* Sum(L) for every (u, v). (sum all L that LEAVES v)
*
* 3) Add an edge from every vertex v to New Sink with cost:
*
* Sum(L) for every (v, w). (sum all L that ARRIVES v)
*
* 4) Add an edge from Old Source to Old Sink with cost INF (
* circulation problem)
* The Network will be valid if and only if the flow saturates
* the network (max flow == sum(L)) *
*
*
* 2 - Finding Min Flow
*
* To find min flow that satisfies just do a binary search in the
* (Old Sink -> Old Source) edge *
* The cost of this edge represents all the flow from old network
*
* Min flow = Sum(L) that arrives in Old Sink + flow that leaves
* (Old Sink -> Old Source) *
*****
*/

int main () {
    clear();
    return 0;
}

```

4.6 Dominator Tree

```

// a node u is said to be dominating node v if, from every path
// from the entry point to v you have to pass through u
// so this code is able to find every dominator from a specific
// entry point (usually 1)
// for directed graphs obviously

const int N = 1e5 + 7;

vector<int> adj[N], radj[N], tree[N], bucket[N];
int sdom[N], par[N], dom[N], dsu[N], label[N], arr[N], rev[N],
    cnt;

void dfs(int u) {
    cnt++;
    arr[u] = cnt;
    rev[cnt] = u;
    label[cnt] = cnt;
    sdom[cnt] = cnt;
    dsu[cnt] = cnt;
    for(auto e : adj[u]) {
        if(!arr[e]) {
            dfs(e);
            par[arr[e]] = arr[u];
        }
        radj[arr[e]].push_back(arr[u]);
    }
}

```

```

    }
}

int find(int u, int x = 0) {
    if(u == dsu[u]) {
        return x * 2 * i + u;
    }
    int v = find(dsu[u], x + 1);
    if(v == -1) {
        return u;
    }
    if(sdom[label[dsu[u]]] < sdom[label[u]]) {
        label[u] = label[dsu[u]];
    }
    dsu[u] = v;
    return (x ? v : label[u]);
}

void unite(int u, int v) {
    dsu[v] = u;
}

// in main

dfs(1);
for(int i = cnt; i >= 1; i--) {
    for(auto e : radj[i]) {
        sdom[i] = min(sdom[i], sdom[find(e)]);
    }
    if(i > 1) {
        bucket[sdom[i]].push_back(i);
    }
    for(auto e : bucket[i]) {
        int v = find(e);
        if(sdom[e] == sdom[v]) {
            dom[e] = sdom[e];
        } else {
            dom[e] = v;
        }
    }
    if(i > 1) {
        unite(par[i], i);
    }
}

for(int i = 2; i <= cnt; i++) {
    if(dom[i] != sdom[i]) {
        dom[i] = dom[dom[i]];
    }
    tree[rev[i]].push_back(rev[dom[i]]);
    tree[rev[dom[i]]].push_back(rev[i]);
}

```

4.7 Erdos Gallai

```

// Erdos-Gallai - O(nlogn)
// check if it's possible to create a simple graph (undirected
// edges) from
// a sequence of vertice's degrees
bool gallai(vector<int> v) {
    vector<ll> sum;
    sum.resize(v.size());

    sort(v.begin(), v.end(), greater<int>());
    sum[0] = v[0];
    for (int i = 1; i < v.size(); i++) sum[i] = sum[i-1] + v[i];
    if (sum.back() % 2) return 0;

    for (int k = 1; k < v.size(); k++) {
        int p = lower_bound(v.begin(), v.end(), k, greater<int>()) -
            v.begin();
        if (p < k) p = k;
        if (sum[k-1] > 1ll*k*(p-1) + sum.back() - sum[p-1]) return
            0;
    }
    return 1;
}

```

4.8 Eulerian Path

```

vector<int> ans, adj[N];

```

```

int in[N];

void dfs(int v) {
    while(adj[v].size()) {
        int x = adj[v].back();
        adj[v].pop_back();
        dfs(x);
    }
    ans.pb(v);
}

// Verify if there is an eulerian path or circuit
vector<int> v;
for(int i = 0; i < n; i++) if(adj[i].size() != in[i]) {
    if(abs((int)adj[i].size() - in[i]) != 1) //-> There is no
        valid eulerian circuit/path
    v.pb(i);
}

if(v.size()) {
    if(v.size() != 2) //-> There is no valid eulerian path
    if(in[v[0]] > adj[v[0]].size()) swap(v[0], v[1]);
    if(in[v[0]] > adj[v[0]].size()) //-> There is no valid
        eulerian path
    adj[v[1]].pb(v[0]); // Turn the eulerian path into a eulerian
        circuit
}

dfs(0);
for(int i = 0; i < cnt; i++)
    if(adj[i].size() != 0) //-> There is no valid eulerian circuit/path
        in this case because the graph is not connected

ans.pop_back(); // Since it's a circuit, the first and the last
    are repeated
reverse(ans.begin(), ans.end());

int bg = 0; // Is used to mark where the eulerian path begins
if(v.size()) {
    for(int i = 0; i < ans.size(); i++)
        if(ans[i] == v[1] and ans[(i + 1) % ans.size()] == v[0]) {
            bg = i + 1;
            break;
        }
}

```

4.9 Fast Kuhn

```

const int N = 1e5+5;

int x, marcB[N], matchB[N], matchA[N], ans, n, m, p;
vector<int> adj[N];

bool dfs(int v) {
    for(int i = 0; i < adj[v].size(); i++) {
        int viz = adj[v][i];
        if(marcB[viz] == 1) continue;
        marcB[viz] = 1;

        if((matchB[viz] == -1) || dfs(matchB[viz])) {
            matchB[viz] = v;
            matchA[v] = viz;
            return true;
        }
    }
    return false;
}

int main() {
    //...
    for(int i = 0; i < n; i++) matchA[i] = -1;
    for(int j = 0; j < m; j++) matchB[j] = -1;

    bool aux = true;
    while(aux) {
        for(int j = 1; j <= m; j++) marcB[j] = 0;
        aux = false;
        for(int i = 1; i <= n; i++) {
            if(matchA[i] != -1) continue;
            if(dfs(i)) {
                ans++;
                aux = true;
            }
        }
    }
}

```

```
}
//...
```

4.10 Find Cycle of size 3 and 4

```
#include <bits/stdc++.h>

using lint = int64_t;

constexpr int MOD = int(1e9) + 7;
constexpr int INF = 0x3f3f3f3f;
constexpr int NINF = 0xcfcfcfcf;
constexpr lint LINF = 0x3f3f3f3f3f3f3f3f;

#define endl '\n'

const long double PI = acos(-1.0);

int cmp_double(double a, double b = 0, double eps = 1e-9) {
    return a + eps > b ? b + eps > a ? 0 : 1 : -1;
}

using namespace std;

#define P 1000000007
#define N 330000

int n, m;
vector<int> go[N], lk[N];
int w[N], deg[N], pos[N], id[N];

bool circle3() {
    int ans = 0;
    for(int i = 1; i <= n; i++) w[i] = 0;
    for(int x = 1; x <= n; x++) {
        for(int y : lk[x]) w[y] = 1;
        for(int y : lk[x]) for(int z : lk[y]) if(w[z]) {
            ans = (ans + go[x].size() + go[y].size() + go[z].size() - 6);
            if(ans) return true;
        }
        for(int y : lk[x]) w[y] = 0;
    }
    return false;
}

bool circle4() {
    for(int i = 1; i <= n; i++) w[i] = 0;
    int ans = 0;
    for(int x = 1; x <= n; x++) {
        for(int y : go[x]) for(int z : lk[y]) if(pos[z] > pos[x]) {
            ans = (ans + w[z]);
            w[z]++;
            if(ans) return true;
        }
        for(int y : go[x]) for(int z : lk[y]) w[z] = 0;
    }
    return false;
}

inline bool cmp(const int &x, const int &y) {
    return deg[x] < deg[y];
}

int main() {
    cin.tie(nullptr) -> sync_with_stdio(false);
    cin >> n >> m;

    int x, y;
    for(int i = 0; i < n; i++) {
        cin >> x >> y;

        for(int i = 1; i <= n; i++) {
            deg[i] = 0, go[i].clear(), lk[i].clear();
        }
        while (m--) {
            int a, b;
            cin >> a >> b;
            deg[a]++, deg[b]++;
            go[a].push_back(b);
            go[b].push_back(a);
        }
    }
}
```

```
for(int i = 1; i <= n; i++) id[i] = i;
sort(id+1, id+1+n, cmp);
for(int i = 1; i <= n; i++) pos[id[i]] = i;
for(int x = 1; x <= n; x++) {
    for(int y : go[x]) {
        if(pos[y] > pos[x]) lk[x].push_back(y);
    }
};

if(circle3()) {
    cout << "3" << endl;
    return 0;
};

if(circle4()) {
    cout << "4" << endl;
    return 0;
};

cout << "5" << endl;
return 0;
}
```

4.11 Floyd Warshall

```
/*
*****
* FLOYD-WARSHALL ALGORITHM (SHORTEST PATH TO ANY VERTEX)
* Time complexity: O(V^3)
* Usage: dist[from][to]
*
* Notation: m:          number of edges
*           n:          number of vertices
*           (a, b, w):  edge between a and b with weight w
*****
*/

int adj[N][N]; // no-edge = INF

for (int k = 0; k < n; ++k)
    for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j)
            adj[i][j] = min(adj[i][j], adj[i][k] + adj[k][j]);
```

4.12 Hungarian Navarro

```
// Hungarian - O(n^2 * m)
template<bool is_max = false, class T = int, bool
    is_zero_indexed = false>
struct Hungarian {
    bool swap_coord = false;
    int lines, cols;
    T ans;

    vector<int> pairV, way;
    vector<bool> used;
    vector<T> pu, pv, minv;
    vector<vector<T>> cost;

    Hungarian(int _n, int _m) {
        if (_n > _m) {
            swap(_n, _m);
            swap_coord = true;
        }

        lines = _n + 1, cols = _m + 1;

        clear();
        cost.resize(lines);
        for (auto& line : cost) line.assign(cols, 0);
    }

    void clear() {
        pairV.assign(cols, 0);
    }
}
```

```
way.assign(cols, 0);
pv.assign(cols, 0);
pu.assign(lines, 0);

void update(int i, int j, T val) {
    if (is_zero_indexed) i++, j++;
    if (is_max) val = -val;
    if (swap_coord) swap(i, j);

    assert(i < lines);
    assert(j < cols);

    cost[i][j] = val;
}

T run() {
    T _INF = numeric_limits<T>::max();
    for (int i = 1, j0 = 0; i < lines; i++) {
        pairV[0] = i;
        minv.assign(cols, _INF);
        used.assign(cols, 0);
        do {
            used[j0] = 1;
            int i0 = pairV[j0], j1;
            T delta = _INF;
            for (int j = 1; j < cols; j++) {
                if (used[j]) continue;
                T cur = cost[i0][j] - pu[i0] - pv[j];
                if (cur < minv[j]) minv[j] = cur, way[j] = j0;
                if (minv[j] < delta) delta = minv[j], j1 = j;
            }
            for (int j = 0; j < cols; j++) {
                if (used[j]) pu[pairV[j]] += delta, pv[j] -= delta;
                else minv[j] -= delta;
            }
            j0 = j1;
        } while (pairV[j0]);

        do {
            int j1 = way[j0];
            pairV[j0] = pairV[j1];
            j0 = j1;
        } while (j0);
    }

    ans = 0;
    for (int j = 1; j < cols; j++) if (pairV[j]) ans += cost[
        pairV[j]][j];

    if (is_max) ans = -ans;
    if (is_zero_indexed) {
        for (int j = 0; j + 1 < cols; j++) pairV[j] = pairV[j +
            1], pairV[j]--;
        pairV[cols - 1] = -1;
    }
    if (swap_coord) {
        vector<int> pairV_sub(lines, 0);
        for (int j = 0; j < cols; j++) if (pairV[j] >= 0)
            pairV_sub[pairV[j]] = j;
        swap(pairV, pairV_sub);
    }

    return ans;
}

template <bool is_max = false, bool is_zero_indexed = false>
struct HungarianMult : public Hungarian<is_max, long double,
    is_zero_indexed> {
    using super = Hungarian<is_max, long double, is_zero_indexed>;

    HungarianMult(int _n, int _m) : super(_n, _m) {}

    void update(int i, int j, long double x) {
        super::update(i, j, log2(x));
    }
};
```

4.13 Strongly Connected Components

```
/*
```

```
*****
```

```

* KOSARAJU'S ALGORITHM (GET EVERY STRONGLY CONNECTED COMPONENTS (SCC))
* Description: Given a directed graph, the algorithm generates a list of every strongly connected components. A SCC is a set of points in which you can reach every point regardless of where you start from. For instance, cycles can be a SCC themselves or part of a greater SCC.
* This algorithm starts with a DFS and generates an array called "ord" which stores vertices according to the finish times (i.e. when it reaches "return"). Then, it makes a reversed DFS according to "ord" list. The set of points visited by the reversed DFS defines a new SCC.
* One of the uses of getting all SCC is that you can generate a new DAG (Directed Acyclic Graph), easier to work with, in which each SCC being a "supernode" of the DAG.

* Time complexity: O(V+E)

* Notation: adj[i]: adjacency list for node i
            adjt[i]: reversed adjacency list for node i
            ord: array of vertices according to their finish time
            ordn: ord counter
            scc[i]: supernode assigned to i
            scc_cnt: amount of supernodes in the graph

*****
const int N = 2e5 + 5;
vector<int> adj[N], adjt[N];
int n, ordn, scc_cnt, vis[N], ord[N], scc[N];

//Directed Version
void dfs(int u) {
    vis[u] = 1;
    for (auto v : adj[u]) if (!vis[v]) dfs(v);
    ord[ordn++] = u;
}

void dfst(int u) {
    scc[u] = scc_cnt, vis[u] = 0;
    for (auto v : adjt[u]) if (vis[v]) dfst(v);
}

// add edge: u -> v
void add_edge(int u, int v) {
    adj[u].push_back(v);
    adjt[v].push_back(u);
}

//Undirected version:
/*
int par[N];

void dfs(int u) {
    vis[u] = 1;
    for (auto v : adj[u]) if (!vis[v]) par[v] = u, dfs(v);
    ord[ordn++] = u;
}

void dfst(int u) {
    scc[u] = scc_cnt, vis[u] = 0;
    for (auto v : adj[u]) if (vis[v] and u != par[v]) dfst(v);
}

// add edge: u -> v
void add_edge(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
}

*/

// run kosaraju
void kosaraju() {

```

```

    for (int i = 1; i <= n; ++i) if (!vis[i]) dfs(i);
    for (int i = ordn - 1; i >= 0; --i) if (vis[ord[i]]) scc_cnt++, dfst(ord[i]);
}

4.14 MST (Kruskal)

/*
*****
* KRUSKAL'S ALGORITHM (MINIMAL SPANNING TREE - INCREASING EDGE SIZE)
* Time complexity: O(ElogE)
* Usage: cost, sz[find(node)]
* Notation: cost: sum of all edges which belong to such MST
* sz: vector of subsets sizes, i.e. size of the subset a node is in
*****
// + Union-find

int cost = 0;
vector<pair<int, pair<int, int>>> edges; //mp(dist, mp(node1, node2))

int main () {
    // ...
    sort(edges.begin(), edges.end());
    for (auto e : edges)
        if (find(e.nd.st) != find(e.nd.nd))
            unite(e.nd.st, e.nd.nd), cost += e.st;
    return 0;
}

4.15 Max Weight on Path (LCA)

// Using LCA to find max edge weight between (u, v)
const int N = 1e5+5; // Max number of vertices
const int K = 20; // Each 1e3 requires ~ 10 K
const int M = K+5;
int n; // Number of vertices
vector<pair<int, int>> adj[N];
int vis[N], h[N], anc[N][M], mx[N][M];

void dfs (int u) {
    vis[u] = 1;
    for (auto p : adj[u]) {
        int v = p.st;
        int w = p.nd;
        if (!vis[v]) {
            h[v] = h[u]+1;
            anc[v][0] = u;
            mx[v][0] = w;
            dfs(v);
        }
    }
}

void build () {
    // cl(mn, 63) -- Don't forget to initialize with INF if min edge!
    anc[1][0] = 1;
    dfs(1);
    for (int j = 1; j <= K; j++) for (int i = 1; i <= n; i++) {
        anc[i][j] = anc[anc[i][j-1]][j-1];
        mx[i][j] = max(mx[i][j-1], mx[anc[i][j-1]][j-1]);
    }
}

int mxedge (int u, int v) {
    int ans = 0;

    if (h[u] < h[v]) swap(u, v);
    for (int j = K; j >= 0; j--) if (h[anc[u][j]] >= h[v]) {
        ans = max(ans, mx[u][j]);
    }
}

```

```

    u = anc[u][j];
}
if (u == v) return ans;
for (int j = K; j >= 0; j--) if (anc[u][j] != anc[v][j]) {
    ans = max(ans, mx[u][j]);
    ans = max(ans, mx[v][j]);
    u = anc[u][j];
    v = anc[v][j];
} //LCA: anc[0][u]
return max({ans, mx[u][0], mx[v][0]});
}

```

4.16 Min Cost Max Flow

```

// USE INF = 1e9!

/*
*****
* MIN COST MAX FLOW (MINIMUM COST TO ACHIEVE MAXIMUM FLOW)
* Description: Given a graph which represents a flow network where every edge has a capacity and a cost per unit, find the minimum cost to establish the maximum possible flow from s to t.
* Note: When adding edge (a, b), it is a directed edge!
* Usage: min_cost_max_flow()
* add_edge(from, to, cost, capacity)
* Notation: flw: max flow
* cst: min cost to achieve flw
* Testcase:
* add_edge(src, 1, 0, 1); add_edge(1, snk, 0, 1); add_edge(2, 3, 1, INF);
* add_edge(src, 2, 0, 1); add_edge(2, snk, 0, 1); add_edge(3, 4, 1, INF);
* add_edge(src, 2, 0, 1); add_edge(3, snk, 0, 1);
* add_edge(src, 2, 0, 1); add_edge(4, snk, 0, 1); => flw = 4, cst = 3
*****
// w: weight or cost, c : capacity
struct edge {int v, f, w, c;};

int n, flw_lmt=INF, src, snk, flw, cst, p[N], d[N], et[N];
vector<edge> e;
vector<int> g[N];

void add_edge(int u, int v, int w, int c) {
    int k = e.size();
    g[u].push_back(k);
    g[v].push_back(k+1);
    e.push_back({v, 0, w, c});
    e.push_back({u, 0, -w, 0});
}

void clear() {
    flw_lmt = INF;
    for(int i=0; i<=n; ++i) g[i].clear();
    e.clear();
}

void min_cost_max_flow() {
    flw = 0, cst = 0;
    while (flw < flw_lmt) {
        memset(et, 0, (n+1) * sizeof(int));
        memset(d, 63, (n+1) * sizeof(int));
        deque<int> q;
        q.push_back(src), d[src] = 0;

        while (!q.empty()) {
            int u = q.front(); q.pop_front();
            et[u] = 2;

            for(int i : g[u]) {

```

```

    edge &dir = e[i];
    int v = dir.v;
    if (dir.f < dir.c and d[u] + dir.w < d[v]) {
        d[v] = d[u] + dir.w;
        if (et[v] == 0) q.push_back(v);
        else if (et[v] == 2) q.push_front(v);
        et[v] = 1;
        p[v] = i;
    }
}

if (d[snk] > INF) break;

int inc = flw_lmt - flw;
for (int u=snk; u != src; u = e[p[u]^1].v) {
    edge &dir = e[p[u]];
    inc = min(inc, dir.c - dir.f);
}

for (int u=snk; u != src; u = e[p[u]^1].v) {
    edge &dir = e[p[u]], &rev = e[p[u]^1];
    dir.f += inc;
    rev.f -= inc;
    cst += inc * dir.w;
}

if (!inc) break;
flw += inc;
}
}

```

4.17 Shortest Path (SPFA)

```

// Shortest Path Faster Algorithm O(VE)
int dist[N], inq[N];

cl(dist,63);
queue<int> q;
q.push(0); dist[0] = 0; inq[0] = 1;

while (!q.empty()) {
    int u = q.front(); q.pop(); inq[u]=0;
    for (int i = 0; i < adj[u].size(); ++i) {
        int v = adj[u][i], w = adjw[u][i];
        if (dist[v] > dist[u] + w) {
            dist[v] = dist[u] + w;
            if (!inq[v]) q.push(v), inq[v] = 1;
        }
    }
}

```

4.18 Small to Large

```

// Imagine you have a tree with colored vertices, and you want
// to do some type of query on every subtree about the colors
// inside
// complexity: O(nlogn)

vector<int> adj[N], vec[N];
int sz[N], color[N], cnt[N];

void dfs_size(int v = 1, int p = 0) {
    sz[v] = 1;
    for (auto u : adj[v]) {
        if (u != p) {
            dfs_size(u, v);
            sz[v] += sz[u];
        }
    }
}

void dfs(int v = 1, int p = 0, bool keep = false) {
    int Max = -1, bigchild = -1;
    for (auto u : adj[v]) {
        if (u != p && Max < sz[u]) {
            Max = sz[u];
            bigchild = u;
        }
    }
}

```

```

for (auto u : adj[v]) {
    if (u != p && u != bigchild) {
        dfs(u, v, 0);
    }
}

if (bigchild != -1) {
    dfs(bigchild, v, 1);
    swap(vec[v], vec[bigchild]);
}

vec[v].push_back(v);
cnt[color[v]]++;
for (auto u : adj[v]) {
    if (u != p && u != bigchild) {
        for (auto x : vec[u]) {
            cnt[color[x]]++;
            vec[v].push_back(x);
        }
    }
}

// now here you can do what the query wants
// there are cnt[c] vertex in subtree v color with c
if (keep == 0) {
    for (auto u : vec[v]) {
        cnt[color[u]]--;
    }
}
}

```

4.19 Stoer Wagner (Stanford)

```

// a is a N*N matrix storing the graph we use; a[i][j]=a[j][i]
memset(use,0,sizeof(use));
ans=maxlongint;
for (int i=1;i<N;i++)
{
    memcpy(visit,use,505*sizeof(int));
    memset(reach,0,sizeof(reach));
    memset(last,0,sizeof(last));
    t=0;
    for (int j=1;j<=N;j++)
        if (use[j]==0) {t=j;break;}
    for (int j=1;j<=N;j++)
        if (use[j]==0) reach[j]=a[t][j],last[j]=t;
    visit[t]=1;
    for (int j=1;j<=N-i;j++)
    {
        maxc=maxk=0;
        for (int k=1;k<=N;k++)
            if ((visit[k]==0)&&(reach[k]>maxc)) maxc=reach[k],maxk=k;
        c2=maxk,visit[maxk]=1;
        for (int k=1;k<=N;k++)
            if (visit[k]==0) reach[k]+=a[maxk][k],last[k]=maxk;
    }
    c1=last[c2];
    sum=0;
    for (int j=1;j<=N;j++)
        if (use[j]==0) sum+=a[j][c2];
    ans=min(ans,sum);
    use[c2]=1;
    for (int j=1;j<=N;j++)
        if ((c1!=j)&&(use[j]==0)) {a[j][c1]+=a[j][c2];a[c1][j]=a[j][c1];}
}
}

```

5 Strings

5.1 Aho-Corasick

```

// Aho-Corasick
// Build: O(sum size of patterns)
// Find total number of matches: O(size of input string)
// Find number of matches for each pattern: O(num of patterns +
// size of input string)

// ids start from 0 by default!

```

```

template <int ALPHA_SIZE = 62>
struct Aho {
    struct Node {
        int p, char_p, link = -1, str_idx = -1, nxt[ALPHA_SIZE];
        bool has_end = false;
        Node(int _p = -1, int _char_p = -1) : p(_p), char_p(_char_p)
        {
            fill(nxt, nxt + ALPHA_SIZE, -1);
        }
    };

    vector<Node> nodes = { Node() };
    int ans, cnt = 0;
    bool build_done = false;
    vector<pair<int, int>> rep;
    vector<int> ord, occur, occur_aux;

    // change this if different alphabet
    int remap(char c) {
        if (islower(c)) return c - 'a';
        if (isalpha(c)) return c - 'A' + 26;
        return c - '0' + 52;
    }

    void add(string &p, int id = -1) {
        int u = 0;
        if (id == -1) id = cnt++;

        for (char ch : p) {
            int c = remap(ch);
            if (nodes[u].nxt[c] == -1) {
                nodes[u].nxt[c] = (int)nodes.size();
                nodes.push_back(Node(u, c));
            }

            u = nodes[u].nxt[c];
        }

        if (nodes[u].str_idx != -1) rep.push_back({ id, nodes[u].str_idx });
        else nodes[u].str_idx = id;
        nodes[u].has_end = true;
    }

    void build() {
        build_done = true;
        queue<int> q;

        for (int i = 0; i < ALPHA_SIZE; i++) {
            if (nodes[0].nxt[i] != -1) q.push(nodes[0].nxt[i]);
            else nodes[0].nxt[i] = 0;
        }

        while(q.size()) {
            int u = q.front();
            ord.push_back(u);
            q.pop();

            int j = nodes[nodes[u].p].link;
            if (j == -1) nodes[u].link = 0;
            else nodes[u].link = nodes[j].nxt[nodes[u].char_p];

            nodes[u].has_end |= nodes[nodes[u].link].has_end;

            for (int i = 0; i < ALPHA_SIZE; i++) {
                if (nodes[u].nxt[i] != -1) q.push(nodes[u].nxt[i]);
                else nodes[u].nxt[i] = nodes[nodes[u].link].nxt[i];
            }
        }
    }

    int match(string &s) {
        if (!cnt) return 0;
        if (!build_done) build();

        ans = 0;
        occur = vector<int>(cnt);
        occur_aux = vector<int>(nodes.size());

        int u = 0;
        for (char ch : s) {
            int c = remap(ch);
            u = nodes[u].nxt[c];
            occur_aux[u]++;
        }

        for (int i = (int)ord.size() - 1; i >= 0; i--) {
            int v = ord[i];

```

```

    int fv = nodes[v].link;
    occur_aux[fv] += occur_aux[v];
    if (nodes[v].str_idx != -1) {
        occur[nodes[v].str_idx] = occur_aux[v];
        ans += occur_aux[v];
    }
}

for (pair<int, int> x : rep) occur[x.first] = occur[x.second];
return ans;
}
};

```

5.2 Aho-Corasick (emaxx)

```

// Aho Corasick - <O(sum(m)), O(n + #matches)>
// Multiple string matching

#include <bits/stdc++.h>
using namespace std;

int remap(char c) {
    if (islower(c)) return c - 'a';
    return c - 'A' + 26;
}

const int K = 52;

struct Aho {
    struct Node {
        int nxt[K];
        int par = -1;
        int link = -1;
        int go[K];
        bitset<1005> ids;
        char pch;
    };

    Node(int p = -1, char ch = '$') : par { p }, pch { ch } {
        fill(begin(nxt), end(nxt), -1);
        fill(begin(go), end(go), -1);
    };

    vector<Node> nodes;

    Aho() : nodes (1) {}

    void add_string(const string& s, int id) {
        int u = 0;
        for (char ch : s) {
            int c = remap(ch);
            if (nodes[u].nxt[c] == -1) {
                nodes[u].nxt[c] = nodes.size();
                nodes.emplace_back(u, ch);
            }

            u = nodes[u].nxt[c];
        }

        nodes[u].ids.set(id);
    }

    int get_link(int u) {
        if (nodes[u].link == -1) {
            if (u == 0 or nodes[u].par == 0) nodes[u].link = 0;
            else nodes[u].link = go(get_link(nodes[u].par), nodes[u].pch);
        }
        return nodes[u].link;
    }

    int go(int u, char ch) {
        int c = remap(ch);
        if (nodes[u].go[c] == -1) {
            if (nodes[u].nxt[c] != -1) nodes[u].go[c] = nodes[u].nxt[c];
            else nodes[u].go[c] = (u == 0) ? 0 : go(get_link(u), ch);
            nodes[u].ids |= nodes[nodes[u].go[c]].ids;
        }
        return nodes[u].go[c];
    }

    bitset<1005> run(const string& s) {

```

```

        bitset<1005> bs;
        int u = 0;
        for (char ch : s) {
            int c = remap(ch);
            if (go(u, ch) == -1) assert(0);
            bs |= nodes[u].ids;
            u = nodes[u].nxt[c];
            if (u == -1) u = 0;
        }
        bs |= nodes[u].ids;
        return bs;
    }
};

```

5.3 Booths Algorithm

```

// Booth's Algorithm - Find the lexicographically least rotation
// of a string in O(n)

string least_rotation(string s) {
    s += s;
    vector<int> f((int)s.size(), -1);
    int k = 0;
    for (int j = 1; j < (int)s.size(); j++) {
        int i = f[j - k - 1];
        while (i != -1 and s[j] != s[k + i + 1]) {
            if (s[j] < s[k + i + 1]) k = j - i - 1;
            i = f[i];
        }

        if (s[j] != s[k + i + 1]) {
            if (s[j] < s[k]) k = j;
            f[j - k] = -1;
        } else f[j - k] = i + 1;
    }

    return s.substr(k, (int)s.size() / 2);
}

```

5.4 Knuth-Morris-Pratt (Automaton)

```

// KMP Automaton - <O(26*pattern), O(text)>

// max size pattern
const int N = 1e5 + 5;

int cnt, nxt[N+1][26];

void prekmp(string &p) {
    nxt[0][p[0] - 'a'] = 1;
    for (int i = 1, j = 0; i <= p.size(); i++) {
        for (int c = 0; c < 26; c++) nxt[i][c] = nxt[j][c];
        if (i == p.size()) continue;
        nxt[i][p[i] - 'a'] = i+1;
        j = nxt[j][p[i] - 'a'];
    }
}

void kmp(string &s, string &p) {
    for (int i = 0, j = 0; i < s.size(); i++) {
        j = nxt[j][s[i] - 'a'];
        if (j == p.size()) cnt++; //match i - j + 1
    }
}

```

5.5 Knuth-Morris-Pratt

```

// Knuth-Morris-Pratt - String Matching O(n+m)
char s[N], p[N];
int b[N], n, m; // n = strlen(s), m = strlen(p);

void kmppre() {
    b[0] = -1;
    for (int i = 0, j = -1; i < m; b[++i] = ++j)
        while (j >= 0 and p[i] != p[j])

```

```

        j = b[j];
    }

    void kmp() {
        for (int i = 0, j = 0; i < n; i++) {
            while (j >= 0 and s[i] != p[j]) j = b[j];
            i++, j++;
            if (j == m) {
                // match position i-j
                j = b[j];
            }
        }
    }
}

```

5.6 Manacher

```

// Manacher (Longest Palindromic String) - O(n)
int lps[2*N+5];
char s[N];

int manacher() {
    int n = strlen(s);

    string p (2*n+3, '#');
    p[0] = '^';
    for (int i = 0; i < n; i++) p[2*(i+1)] = s[i];
    p[2*n+2] = '$';

    int k = 0, r = 0, m = 0;
    int l = p.length();
    for (int i = 1; i < l; i++) {
        int o = 2*k - i;
        lps[i] = (r > i) ? min(r-i, lps[o]) : 0;
        while (p[i + 1 + lps[i]] == p[i - 1 - lps[i]]) lps[i]++;
        if (i + lps[i] > r) k = i, r = i + lps[i];
        m = max(m, lps[i]);
    }
    return m;
}

```

5.7 Manacher 2

```

// Manacher O(n)

vector<int> d1, d2;

// d1 -> odd : size = 2 * d1[i] - 1, palindrome from i - d1[i] + 1 to i + d1[i] - 1
// d2 -> even : size = 2 * d2[i], palindrome from i - d2[i] to i + d2[i] - 1

void manacher(string &s) {
    int n = s.size();
    d1.resize(n), d2.resize(n);
    for (int i = 0, l1 = 0, l2 = 0, r1 = -1, r2 = -1; i < n; i++) {
        if (i <= r1) {
            d1[i] = min(d1[r1 + l1 - i], r1 - i + 1);
        }
        if (i <= r2) {
            d2[i] = min(d2[r2 + l2 - i + 1], r2 - i + 1);
        }
        while (i - d1[i] >= 0 and i + d1[i] < n and s[i - d1[i]] == s[i + d1[i]]) {
            d1[i]++;
        }
        while (i - d2[i] - 1 >= 0 and i + d2[i] < n and s[i - d2[i] - 1] == s[i + d2[i]]) {
            d2[i]++;
        }
        if (i + d1[i] - 1 > r1) {
            l1 = i - d1[i] + 1;
            r1 = i + d1[i] - 1;
        }
        if (i + d2[i] - 1 > r2) {
            l2 = i - d2[i];
            r2 = i + d2[i] - 1;
        }
    }
}

```

5.8 Recursive-String Matching

```

void p_f(char *s, int *pi) {
    int n = strlen(s);
    pi[0]=pi[1]=0;
    for(int i = 2; i <= n; i++) {
        pi[i] = pi[i-1];
        while(pi[i]>0 and s[pi[i]]!=s[i])
            pi[i]=pi[pi[i]];
        if(s[pi[i]]==s[i-1])
            pi[i]++;
    }
}

int main() {
    //....
    //Initialize prefix function
    char p[N]; //Pattern
    int len = strlen(p); //Pattern size
    int pi[N]; //Prefix function
    p_f(p, pi);

    // Create KMP automaton
    int A[N][128]; //A[i][j]: from state i (size of largest suffix
    // of text which is prefix of pattern), append character j
    // -> new state A[i][j]
    for( char c : ALPHABET )
        A[0][c] = (p[0] == c);
    for( int i = 1; p[i]; i++ ) {
        for( char c : ALPHABET ) {
            if(c==p[i])
                A[i][c]=i+1; //match
            else
                A[i][c]=A[pi[i]][c]; //try second largest suffix
        }
    }

    //Create KMP "string appending" automaton
    // g_n = g_(n-1) + char(n) + g_(n-1)
    // g_0 = "", g_1 = "a", g_2 = "aba", g_3 = "abacaba", ...
    int F[M][N]; //F[i][j]: from state j (size of largest suffix
    // of text which is prefix of pattern), append string g_i ->
    // new state F[i][j]
    for(int i = 0; i < m; i++) {
        for(int j = 0; j <= len; j++) {
            if(i==0)
                F[i][j] = j; //append empty string
            else {
                int x = F[i-1][j]; //append g_(i-1)
                x = A[x][j]; //append character j
                x = F[i-1][x]; //append g_(i-1)
                F[i][j] = x;
            }
        }
    }

    //Create number of matches matrix
    int K[M][N]; //K[i][j]: from state j (size of largest suffix
    // of text which is prefix of pattern), append string g_i ->
    // K[i][j] matches
    for(int i = 0; i < m; i++) {
        for(int j = 0; j <= len; j++) {
            if(i==0)
                K[i][j] = (j==len); //append empty string
            else {
                int x = F[i-1][j]; //append g_(i-1)
                x = A[x][j]; //append character j
                K[i][j] = K[i-1][j] /*append g_(i-1)*/ + (x==len) /*
                append character j*/ + K[i-1][x]; /*append g_(i-1)
                */
            }
        }
    }

    //number of matches in g_k
    int answer = K[0][k];
    //....
}

```

5.9 String Hashing

```

// String Hashing
// Rabin Karp - O(n + m)

// max size txt + 1
const int N = 1e6 + 5;

// lowercase letters p = 31 (remember to do s[i] - 'a' + 1)
// uppercase and lowercase letters p = 53 (remember to do s[i] -
// 'a' + 1)
// any character p = 313

const int MOD = 1e9+9;
ull h[N], p[N];
ull pr = 313;

int cnt;

void build(string &s) {
    p[0] = 1, p[1] = pr;
    for(int i = 1; i <= s.size(); i++) {
        h[i] = ((p[1]*h[i-1]) % MOD + s[i-1]) % MOD;
        p[i] = (p[1]*p[i-1]) % MOD;
    }
}

// 1-indexed
ull fhash(int l, int r) {
    return (h[r] - ((h[l-1]*p[r-l+1]) % MOD) + MOD) % MOD;
}

ull shash(string &pt) {
    ull h = 0;
    for(int i = 0; i < pt.size(); i++)
        h = ((h*pr) % MOD + pt[i]) % MOD;
    return h;
}

void rabin_karp(string &s, string &pt) {
    build(s);
    ull hp = shash(pt);
    for(int i = 0, m = pt.size(); i + m <= s.size(); i++) {
        if(fhash(i+1, i+m) == hp) {
            // match at i
            cnt++;
        }
    }
}

```

5.10 String Multihashing

```

// String Hashing
// Rabin Karp - O(n + m)
template <int N = 3>
struct Hash {
    int hs[N];
    static vector<int> mods;

    static int add(int a, int b, int mod) { return a >= mod - b ?
        a + b - mod : a + b; }
    static int sub(int a, int b, int mod) { return a - b < 0 ? a -
        b + mod : a - b; }
    static int mul(int a, int b, int mod) { return 1ll * a * b %
        mod; }

    Hash(int x = 0) { fill(hs, hs + N, x); }

    bool operator<(const Hash& b) const {
        for (int i = 0; i < N; i++) {
            if (hs[i] < b.hs[i]) return true;
            if (hs[i] > b.hs[i]) return false;
        }
        return false;
    }

    Hash operator+(const Hash& b) const {
        Hash ans;
        for (int i = 0; i < N; i++) ans.hs[i] = add(hs[i], b.hs[i],
            mods[i]);
        return ans;
    }

    Hash operator-(const Hash& b) const {
        Hash ans;

```

```

        for (int i = 0; i < N; i++) ans.hs[i] = sub(hs[i], b.hs[i],
            mods[i]);
        return ans;
    }

    Hash operator*(const Hash& b) const {
        Hash ans;
        for (int i = 0; i < N; i++) ans.hs[i] = mul(hs[i], b.hs[i],
            mods[i]);
        return ans;
    }

    Hash operator+(int b) const {
        Hash ans;
        for (int i = 0; i < N; i++) ans.hs[i] = add(hs[i], b, mods[i]
            );
        return ans;
    }

    Hash operator*(int b) const {
        Hash ans;
        for (int i = 0; i < N; i++) ans.hs[i] = mul(hs[i], b, mods[i]
            );
        return ans;
    }

    friend Hash operator*(int a, const Hash& b) {
        Hash ans;
        for (int i = 0; i < N; i++) ans.hs[i] = mul(b.hs[i], a, b.
            mods[i]);
        return ans;
    }

    friend ostream& operator<<(ostream& os, const Hash& b) {
        for (int i = 0; i < N; i++) os << b.hs[i] << " \n"[i == N -
            1];
        return os;
    }
};

template <int N> vector<int> Hash<N>::mods = { (int) 1e9 + 9, (
    int) 1e9 + 33, (int) 1e9 + 87 };

// In case you need to generate the MODs, uncomment this:
// Obs: you may need this on your template
// mt19937_64 llrand((int) chrono::steady_clock::now().
//     time_since_epoch().count());
// In main: gen<>();
/*
template <int N> vector<int> Hash<N>::mods;
template<int N = 3>
void gen() {
    while (Hash<N>::mods.size() < N) {
        int mod;
        bool is_prime;
        do {
            mod = (int) 1e8 + (int) (llrand() % (int) 9e8);
            is_prime = true;
            for (int i = 2; i * i <= mod; i++) {
                if (mod % i == 0) {
                    is_prime = false;
                    break;
                }
            }
        } while (!is_prime);
        Hash<N>::mods.push_back(mod);
    }
}
*/

template <int N = 3>
struct PolyHash {
    vector<Hash<N>> h, p;

    PolyHash(string& s, int pr = 313) {
        int sz = (int)s.size();
        p.resize(sz + 1);
        h.resize(sz + 1);

        p[0] = 1, h[0] = s[0];
        for (int i = 1; i < sz; i++) {
            h[i] = pr * h[i - 1] + s[i];
            p[i] = pr * p[i - 1];
        }
    }

    Hash<N> fhash(int l, int r) {
        if (!l) return h[r];
        return h[r] - h[l - 1] * p[r - l + 1];
    }
}

```



```

}

static Hash<N> shash(string& s, int pr = 313) {
    Hash<N> ans;
    for (int i = 0; i < (int)s.size(); i++) ans = pr * ans + s[i];
    return ans;
}

friend int rabin_karp(string& s, string& pt) {
    PolyHash hs = PolyHash(s);
    Hash<N> hp = hs.shash(pt);
    int cnt = 0;
    for (int i = 0, m = (int)pt.size(); i + m <= (int)s.size(); i++) {
        if (hs.fhash(i, i + m - 1) == hp) {
            // match at i
            cnt++;
        }
    }
    return cnt;
};

```

5.11 Suffix Array

```

// Suffix Array O(nlogn)
// s.push('$');
vector<int> suffix_array(string &s) {
    int n = s.size(), alph = 256;
    vector<int> cnt(max(n, alph), p(n), c(n);

    for(auto c : s) cnt[c]++;
    for(int i = 1; i < alph; i++) cnt[i] += cnt[i - 1];
    for(int i = 0; i < n; i++) p[--cnt[s[i]]] = i;
    for(int i = 1; i < n; i++)
        c[p[i]] = c[p[i - 1]] + (s[p[i]] != s[p[i - 1]]);

    vector<int> c2(n), p2(n);

    for(int k = 0; (1 << k) < n; k++) {
        int classes = c[p[n - 1]] + 1;
        fill(cnt.begin(), cnt.begin() + classes, 0);

        for(int i = 0; i < n; i++) p2[i] = (p[i] - (1 << k) + n) % n;
        for(int i = 0; i < n; i++) cnt[c[i]]++;
        for(int i = 1; i < classes; i++) cnt[i] += cnt[i - 1];
        for(int i = n - 1; i >= 0; i--) p[--cnt[c[p2[i]]]] = p2[i];

        c2[p[0]] = 0;
        for(int i = 1; i < n; i++) {
            pair<int, int> b1 = {c[p[i]], c[(p[i] + (1 << k)) % n]};
            pair<int, int> b2 = {c[p[i - 1]], c[(p[i - 1] + (1 << k)) % n]};
            c2[p[i]] = c2[p[i - 1]] + (b1 != b2);
        }
        c.swap(c2);
    }
    return p;
}

// Longest Common Prefix with SA O(n)
vector<int> lcp(string &s, vector<int> &p) {
    int n = s.size();
    vector<int> ans(n - 1, pi(n);
    for(int i = 0; i < n; i++) pi[p[i]] = i;

    int lst = 0;
    for(int i = 0; i < n - 1; i++) {
        if(pi[i] == n - 1) continue;
        while(s[i + lst] == s[p[pi[i] + 1] + lst]) lst++;

        ans[pi[i]] = lst;
        lst = max(0, lst - 1);
    }

    return ans;
}

// Longest Repeated Substring O(n)
int lrs = 0;
for (int i = 0; i < n; ++i) lrs = max(lrs, lcp[i]);

```

```

// Longest Common Substring O(n)
// m = strlen(s);
// strcat(s, "$"); strcat(s, p); strcat(s, "#");
// n = strlen(s);
int lcs = 0;
for (int i = 1; i < n; ++i) if ((sa[i] < m) != (sa[i-1] < m))
    lcs = max(lcs, lcp[i]);

// To calc LCS for multiple texts use a slide window with
// minqueue
// The number of different substrings of a string is n*(n + 1)/2
// - sum(lcs[i])

```

5.12 Suffix Automaton

```

// Suffix Automaton Construction - O(n)

const int N = 1e6+1, K = 26;
int sl[2*N], len[2*N], sz, last;
ll cnt[2*N];
map<int, int> adj[2*N];

void add(int c) {
    int u = sz++;
    len[u] = len[last] + 1;
    cnt[u] = 1;

    int p = last;
    while(p != -1 and !adj[p][c])
        adj[p][c] = u, p = sl[p];

    if (p == -1) sl[u] = 0;
    else {
        int q = adj[p][c];
        if (len[p] + 1 == len[q]) sl[u] = q;
        else {
            int r = sz++;
            len[r] = len[p] + 1;
            sl[r] = sl[q];
            adj[r] = adj[q];
            while(p != -1 and adj[p][c] == q)
                adj[p][c] = r, p = sl[p];
            sl[q] = sl[u] = r;
        }
    }

    last = u;
}

void clear() {
    for(int i=0; i<sz; ++i) adj[i].clear();
    last = 0;
    sz = 1;
    sl[0] = -1;
}

void build(char *s) {
    clear();
    for(int i=0; s[i]; ++i) add(s[i]);
}

// Pattern matching - O(|p|)
bool check(char *p) {
    int u = 0, ok = 1;
    for(int i=0; p[i]; ++i) {
        u = adj[u][p[i]];
        if (!u) ok = 0;
    }
    return ok;
}

// Substring count - O(|p|)
ll d[2*N];

void substr_cnt(int u) {
    d[u] = 1;
    for(auto p : adj[u]) {
        int v = p.second;
        if (!d[v]) substr_cnt(v);
        d[u] += d[v];
    }
}

```

```

ll substr_cnt() {
    memset(d, 0, sizeof d);
    substr_cnt(0);
    return d[0] - 1;
}

// k-th Substring - O(|s|)
// Just find the k-th path in the automaton.
// Can be done with the value d calculated in previous problem.

// Smallest cyclic shift - O(|s|)
// Build the automaton for string s + s. And adapt previous dp
// to only count paths with size |s|.

// Number of occurrences - O(|p|)
vector<int> t[2*N];

void occur_count(int u) {
    for(int v : t[u]) occur_count(v), cnt[u] += cnt[v];
}

void build_tree() {
    for(int i=1; i<=sz; ++i)
        t[sl[i]].push_back(i);
    occur_count(0);
}

ll occur_count(char *p) {
    // Call build tree once per automaton
    int u = 0;
    for(int i=0; p[i]; ++i) {
        u = adj[u][p[i]];
        if (!u) break;
    }
    return !u ? 0 : cnt[u];
}

// First occurrence - (|p|)
// Store the first position of occurrence fp.
// Add the the code to add function:
// fp[u] = len[u] - 1;
// fp[r] = fp[q];

// To answer a query, just output fp[u] - strlen(p) + 1
// where u is the state corresponding to string p

// All occurrences - O(|p| + |ans|)
// All the occurrences can reach the first occurrence via suffix
// links.
// So every state that contains a occurrence is reachable by the
// first occurrence state in the suffix link tree. Just do a DFS
// in this
// tree, starting from the first occurrence.
// OBS: cloned nodes will output same answer twice.

// Smallest substring not contained in the string - O(|s| * K)
// Just do a dynamic programming:
// d[u] = 1 // if d does not have 1 transition
// d[u] = 1 + min d[v] // otherwise

// LCS of 2 Strings - O(|s| + |t|)
// Build automaton of s and traverse the automaton with string t
// maintaining the current state and the current length.
// When we have a transition: update state, increase length by
// one.
// If we don't update state by suffix link and the new length
// will
// should be reduced (if bigger) to the new state length.
// Answer will be the maximum length of the whole traversal.

// LCS of n Strings - O(n*|s|*K)
// Create a new string S = s_1 + d_1 + ... + s_n + d_n,
// where d_i are delimiters that are unique (d_i != d_j).
// For each state use DP + bitmask to calculate if it can
// reach a d_i transition without going through other d_j.
// The answer will be the biggest len[u] that can reach all
// d_i's.

```

5.13 Suffix Tree

```
// Suffix Tree
// Build: O(|s|)
// Match: O(|p|)

template<int ALPHA_SIZE = 62>
struct SuffixTree {
    struct Node {
        int p, link = -1, l, r, nch = 0;
        vector<int> nxt;
        Node(int _l = 0, int _r = -1, int _p = -1) : p(_p), l(_l), r(_r), nxt(ALPHA_SIZE, -1) {}

        int len() { return r - l + 1; }
        int next(char ch) { return nxt[remap(ch)]; }

        // change this if different alphabet
        int remap(char c) {
            if (islower(c)) return c - 'a';
            if (isalpha(c)) return c - 'A' + 26;
            return c - '0' + 52;
        }

        void setEdge(char ch, int nx) {
            int c = remap(ch);
            if (nxt[c] != -1 and nx == -1) nch--;
            else if (nxt[c] == -1 and nx != -1) nch++;
            nxt[c] = nx;
        }
    };

    string s;
    long long num_diff_substr = 0;
    vector<Node> nodes;
    queue<int> leaves;
    pair<int, int> st = { 0, 0 };
    int ls = 0, rs = -1, n;

    int size() { return rs - ls + 1; }

    SuffixTree(string &s) {
        s = _s;
        // Add this if you want every suffix to be a node
        // s += '$';
        n = (int)s.size();
        nodes.reserve(2 * n + 1);
        nodes.push_back(Node());
        //for (int i = 0; i < n; i++) extend();
    }

    pair<int, int> walk(pair<int, int> _st, int l, int r) {
        int u = _st.first;
        int d = _st.second;

        while (l <= r) {
            if (d == nodes[u].len()) {
                u = nodes[u].next(s[l]); d = 0;
                if (u == -1) return { u, d };
            } else {
                if (s[nodes[u].l + d] != s[l]) return { -1, -1 };
                if (r - l + 1 + d < nodes[u].len()) return { u, r - l + 1 + d };
                l += nodes[u].len() - d;
                d = nodes[u].len();
            }
        }

        return { u, d };
    }

    int split(pair<int, int> _st) {
        int u = _st.first;
        int d = _st.second;

        if (d == nodes[u].len()) return u;
        if (!d) return nodes[u].p;

        Node& nu = nodes[u];
        int mid = (int)nodes.size();
        nodes.push_back(Node(nu.l, nu.l + d - 1, nu.p));
        nodes[nu.p].setEdge(s[nu.l], mid);
        nodes[mid].setEdge(s[nu.l + d], u);
        nu.p = mid;
        nu.l += d;
        return mid;
    }

    int getLink(int u) {
        if (nodes[u].link != -1) return nodes[u].link;

```

```
        if (nodes[u].p == -1) return 0;
        int to = getLink(nodes[u].p);
        pair<int, int> nst = { to, nodes[to].len() };
        return nodes[u].link = split(walk(nst, nodes[u].l + (nodes[u].p == 0), nodes[u].r));
    }

    bool match(string &p) {
        int u = 0, d = 0;
        for (char ch : p) {
            if (d == min(nodes[u].r, rs) - nodes[u].l + 1) {
                u = nodes[u].next(ch); d = 1;
                if (u == -1) return false;
            } else {
                if (ch != s[nodes[u].l + d]) return false;
                d++;
            }
        }
        return true;
    }

    void extend() {
        int mid;
        assert(rs != n - 1);
        rs++;
        num_diff_substr += (int)leaves.size();
        do {
            pair<int, int> nst = walk(st, rs, rs);
            if (nst.first != -1) { st = nst; return; }
            mid = split(st);
            int leaf = (int)nodes.size();
            num_diff_substr++;
            leaves.push(leaf);
            nodes.push_back(Node(rs, n - 1, mid));
            nodes[mid].setEdge(s[rs], leaf);
            int to = getLink(mid);
            st = { to, nodes[to].len() };
        } while (mid);

        void pop() {
            assert(ls <= rs);
            ls++;
            int leaf = leaves.front();
            leaves.pop();
            Node* nlf = &nodes[leaf];
            while (!nlf->nch) {
                if (st.first != leaf) {
                    nodes[nlf->p].setEdge(s[nlf->l], -1);
                    num_diff_substr -= min(nlf->r, rs) - nlf->l + 1;
                    leaf = nlf->p;
                    nlf = &nodes[leaf];
                } else {
                    if (st.second != min(nlf->r, rs) - nlf->l + 1) {
                        int mid = split(st);
                        st.first = mid;
                        num_diff_substr -= min(nlf->r, rs) - nlf->l + 1;
                        nodes[mid].setEdge(s[nlf->l], -1);
                        *nlf = nodes[mid];
                        nodes[nlf->p].setEdge(s[nlf->l], leaf);
                        nodes.pop_back();
                    }
                    break;
                }
            }

            if (leaf and !nlf->nch) {
                leaves.push(leaf);
                int to = getLink(nlf->p);
                pair<int, int> nst = { to, nodes[to].len() };
                st = walk(nst, nlf->l + (nlf->p == 0), nlf->r);
                nlf->l = rs - nlf->len() + 1;
                nlf->r = n - 1;
            }
        }
    }
};
```

5.14 Z Function

```
// Z-Function - O(n)

vector<int> zfunction(const string& s) {
    vector<int> z(s.size());
    for (int i = 1, l = 0, r = 0, n = s.size(); i < n; i++) {
        if (i <= r) z[i] = min(z[i-l], r - i + 1);
    }
}
```

```
        while (i + z[i] < n and s[z[i]] == s[z[i] + i]) z[i]++;
        if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
    }
    return z;
}
```

6 Mathematics

6.1 Basics

```
// Greatest Common Divisor & Lowest Common Multiple
ll gcd(ll a, ll b) { return b ? gcd(b, a%b) : a; }
ll lcm(ll a, ll b) { return a/gcd(a, b)*b; }

// Multiply caring overflow
ll mulmod(ll a, ll b, ll m = MOD) {
    ll r=0;
    for (a %= m; b;>=1, a=(a*2)%m; if (b&1) r=(r+a)%m;
    return r;
}

// Another option for mulmod is using long double
ull mulmod(ull a, ull b, ull m = MOD) {
    ull q = (ld) a * (ld) b / (ld) m;
    ull r = a * b - q * m;
    return (r + m) % m;
}

// Fast exponential
ll fexp(ll a, ll b, ll m = MOD) {
    ll r=1;
    for (a %= m; b;>=1, a=(a*a)%m; if (b&1) r=(r*a)%m;
    return r;
}
```

6.2 Advanced

```
/* Line integral = integral(sqrt(1 + (dy/dx)^2)) dx */

/* Multiplicative Inverse over MOD for all 1..N - 1 < MOD in O(N)
   Only works for prime MOD. If all 1..MOD - 1 needed, use N = MOD
   */
ll inv[N];
inv[1] = 1;
for(int i = 2; i < N; ++i)
    inv[i] = MOD - (MOD / i) * inv[MOD % i] % MOD;

/* Catalan
   f(n) = sum(f(i) * f(n - i - 1)), i in [0, n - 1] = (2n)! / ((n + 1)! * n!) = ...
   If you have any function f(n) (there are many) that follows
   this sequence (0-indexed):
   1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012,
   742900, 2674440
   then it's the Catalan function */
ll cat[N];
cat[0] = 1;
for(int i = 1; i + 1 < N; i++) // needs inv[i + 1] till inv[N - 1]
    cat[i] = 211 * (211 * i - 1) * inv[i + 1] % MOD * cat[i - 1] % MOD;

/* Floor(n / i), i = [1, n], has <= 2 * sqrt(n) diff values.
   Proof: i = [1, sqrt(n)] has sqrt(n) diff values.
   For i = [sqrt(n), n] we have that 1 <= n / i <= sqrt(n)
   and thus has <= sqrt(n) diff values.
   */
/* l = first number that has floor(N / l) = x
   r = last number that has floor(N / r) = x
   N / r >= floor(N / l)
   r <= N / floor(N / l) */
for(int l = 1, r; l <= n; l = r + 1) {
    r = n / (n / l);
    // floor(n / i) has the same value for l <= i <= r
}

/* Recurrence using matrix
```

```

h[i + 2] = a1 * h[i + 1] + a0 * h[i]
[h[i] h[i-1]] = [h[1] h[0]] * [a1 1] ^ (i - 1)
                             [a0 0]    */

/* Fibonacci in O(log(N)) with memoization
f(0) = f(1) = 1
f(2*k) = f(k)^2 + f(k - 1)^2
f(2*k + 1) = f(k)*f(k) + 2*f(k - 1) */

/* Wilson's Theorem Extension
B = b1 * b2 * ... * bm (mod n) = +-1, all bi <= n such that gcd
    (bi, n) = 1
if(n <= 4 or n = (odd prime)^k or n = 2 * (odd prime)^k) B =
    -1; for any k
else B = 1; */

/* Stirling numbers of the second kind
S(n, k) = Number of ways to split n numbers into k non-empty
    sets
S(n, 1) = S(n, n) = 1
S(n, k) = k * S(n - 1, k) + S(n - 1, k - 1)
Sr(n, k) = S(n, k) with at least r numbers in each set
Sr(n, k) = k * Sr(n - 1, k) + (n - 1) * Sr(n - r, k - 1)
    (r - 1)
S(n - d + 1, k - d + 1) = S(n, k) where if indexes i, j belong
    to the same set, then |i - j| >= d */

/* Burnside's Lemma
|Classes| = 1 / |G| * sum(K ^ C(g)) for each g in G
G = Different permutations possible
C(g) = Number of cycles on the permutation g
K = Number of states for each element

Different ways to paint a necklace with N beads and K colors:
G = {(1, 2, ... N), (2, 3, ... N, 1), ... (N, 1, ... N - 1)}
gi = (i, i + 1, ... i + N), (taking mod N to get it right) i =
    1 ... N
i -> 2i -> 3i ..., Cycles in gi all have size n / gcd(i, n), so
    C(gi) = gcd(i, n)
Ans = 1 / N * sum(K ^ gcd(i, n)), i = 1 ... N
(For the brave, you can get to Ans = 1 / N * sum(euler_phi(N /
    d) * K ^ d), d | N) */

/* Mobius Inversion
Sum of gcd(i, j), 1 <= i, j <= N?
sum(k->N) k * sum(i->N) sum(j->N) [gcd(i, j) == k], i = a * k,
    j = b * k
= sum(k->N) k * sum(a->N/k) sum(b->N/k) [gcd(a, b) == 1]
= sum(k->N) k * sum(a->N/k) sum(b->N/k) sum(d->N/k) [d | a] * [
    d | b] * mi(d)
= sum(k->N) k * sum(d->N/k) mi(d) * floor(N / kd)^2, 1 = kd, 1
    <= N, k | 1, d = 1 / k
= sum(1->N) floor(N / l)^2 * sum(k|l) k * mi(l / k)
If f(n) = sum(x|n) (g(x) * h(x)) with g(x) and h(x)
    multiplicative, than f(n) is multiplicative
Hence, g(1) = sum(k|1) k * mi(l / k) is multiplicative
= sum(1->N) floor(N / l)^2 * g(1) */

/* Frobenius / Chicken McNugget
n, m given, gcd(n, m) = 1, we want to know if it's possible to
    create N = a * n + b * m
N, a, b >= 0
The greatest number NOT possible is n * m - n - m
We can NOT create (n - 1) * (m - 1) / 2 numbers */

```

6.3 Discrete Log (Baby-step Giant-step)

```

// O(sqrt(m))
// Solve c * a^x = b mod(m) for integer x >= 0.
// Return the smallest x possible, or -1 if there is no solution
// If all solutions needed, solve c * a^x = b mod(m) and (a*b) *
    a^y = b mod(m)
// x + k * (y + 1) for k >= 0 are all solutions
// Works for any integer values of c, a, b and positive m

// Corner Cases:
// 0^x = 1 mod(m) returns x = 0, so you may want to change it to
    -1
// You also may want to change for 0^x = 0 mod(1) to return x =
    1 instead
// We leave it like it is because you might be actually checking
    for m^x = 0^x mod(m)

```

```

// which would have x = 0 as the actual solution.
ll discrete_log(ll c, ll a, ll b, ll m) {
    c = ((c % m) + m) % m, a = ((a % m) + m) % m, b = ((b % m) + m)
        % m;
    if(c == b)
        return 0;

    ll g = __gcd(a, m);
    if(b % g) return -1;

    if(g > 1) {
        ll r = discrete_log(c * a / g, a, b / g, m / g);
        return r + (r >= 0);
    }

    unordered_map<ll, ll> babystep;
    ll n = 1, an = a % m;

    // set n to the ceil of sqrt(m):
    while(n * n < m) n++, an = (an * a) % m;

    // babysteps:
    ll bstep = b;
    for(ll i = 0; i <= n; i++) {
        babystep[bstep] = i;
        bstep = (bstep * a) % m;
    }

    // giantsteps:
    ll gstep = c * an % m;
    for(ll i = 1; i <= n; i++) {
        if(babystep.find(gstep) != babystep.end())
            return n * i - babystep[gstep];
        gstep = (gstep * an) % m;
    }
    return -1;
}

```

6.4 Euler Phi

```

// Euler phi (totient)
int ind = 0, pf = primes[0], ans = n;
while (1ll*pf*pf <= n) {
    if (n%pf==0) ans -= ans/pf;
    while (n%pf==0) n /= pf;
    pf = primes[++ind];
}
if (n != 1) ans -= ans/n;

// IME2014
int phi[N];
void totient() {
    for (int i = 1; i < N; ++i) phi[i]=i;
    for (int i = 2; i < N; i+=2) phi[i]>=1;
    for (int j = 3; j < N; j+=2) if (phi[j]==j) {
        phi[j]--;
        for (int i = 2*j; i < N; i+=j) phi[i]=phi[i]/j*(j-1);
    }
}

```

6.5 Extended Euclidean and Chinese Remainder

```

// Extended Euclid:
void euclid(ll a, ll b, ll &x, ll &y) {
    if (b) euclid(b, a%b, y, x), y -= x*(a/b);
    else x = 1, y = 0;
}

// find (x, y) such that a*x + b*y = c or return false if it's
    not possible
// [x + k*b/gcd(a, b), y - k*a/gcd(a, b)] are also solutions
bool diof(ll a, ll b, ll c, ll &x, ll &y) {
    euclid(abs(a), abs(b), x, y);
    ll g = abs(__gcd(a, b));
    if(c % g) return false;
    x *= c / g;
    y *= c / g;
    if(a < 0) x = -x;
}

```

```

if(b < 0) y = -y;
return true;
}

// auxiliar to find_all_solutions
void shift_solution (ll &x, ll &y, ll a, ll b, ll cnt) {
    x += cnt * b;
    y -= cnt * a;
}

// Find the amount of solutions of
// ax + by = c
// in given intervals for x and y
ll find_all_solutions (ll a, ll b, ll c, ll minx, ll maxx, ll
    miny, ll maxy) {
    ll x, y, g = __gcd(a, b);
    if(!diof(a, b, c, x, y)) return 0;
    a /= g; b /= g;

    int sign_a = a>0 ? +1 : -1;
    int sign_b = b>0 ? +1 : -1;

    shift_solution (x, y, a, b, (minx - x) / b);
    if (x < minx)
        shift_solution (x, y, a, b, sign_b);
    if (x > maxx)
        return 0;
    int lx1 = x;

    shift_solution (x, y, a, b, (maxx - x) / b);
    if (x > maxx)
        shift_solution (x, y, a, b, -sign_b);
    int rx1 = x;

    shift_solution (x, y, a, b, - (miny - y) / a);
    if (y < miny)
        shift_solution (x, y, a, b, -sign_a);
    if (y > maxy)
        return 0;
    int lx2 = x;

    shift_solution (x, y, a, b, - (maxy - y) / a);
    if (y > maxy)
        shift_solution (x, y, a, b, sign_a);
    int rx2 = x;

    if (lx2 > rx2)
        swap (lx2, rx2);
    int lx = max (lx1, lx2);
    int rx = min (rx1, rx2);

    if (lx > rx) return 0;
    return (rx - lx) / abs(b) + 1;
}

bool crt_auxiliar(ll a, ll b, ll m1, ll m2, ll &ans) {
    ll x, y;
    if(!diof(m1, m2, b - a, x, y)) return false;
    ll lcm = m1 / __gcd(m1, m2) * m2;
    ans = ((a + x % (lcm / m1) * m1) % lcm + lcm) % lcm;
    return true;
}

// find ans such that ans = a[i] mod b[i] for all 0 <= i < n or
    return false if not possible
// ans + k * lcm(b[i]) are also solutions
bool crt(int n, ll a[], ll b[], ll &ans) {
    if(!b[0]) return false;
    ans = a[0] % b[0];
    ll l = b[0];
    for(int i = 1; i < n; i++) {
        if(!b[i]) return false;
        if(!crt_auxiliar(ans, a[i] % b[i], l, b[i], ans)) return
            false;
        l *= (b[i] / __gcd(b[i], l));
    }
    return true;
}

```

6.6 Fast Fourier Transform(Tourist)

```

//
// FFT made by tourist. It is faster and more supportive,
    although it requires more lines of code.

```

```
// Also, it allows operations with MOD, which is usually an
// issue in FFT problems.
namespace fft {
    typedef double dbl;

    struct num {
        dbl x, y;
        num() { x = y = 0; }
        num(dbl x, dbl y) : x(x), y(y) {}
    };

    inline num operator+ (num a, num b) { return num(a.x + b.x, a.y + b.y); }
    inline num operator- (num a, num b) { return num(a.x - b.x, a.y - b.y); }
    inline num operator* (num a, num b) { return num(a.x * b.x - a.y * b.y, a.x * b.y + a.y * b.x); }
    inline num conj(num a) { return num(a.x, -a.y); }

    int base = 1;
    vector<num> roots = {{0, 0}, {1, 0}};
    vector<int> rev = {0, 1};

    const dbl PI = acos(-1.0);

    void ensure_base(int nbase) {
        if(nbase <= base) return;
        rev.resize(1 << nbase);
        for(int i=0; i < (1 << nbase); i++) {
            rev[i] = (rev[i] >> 1) >> 1 + ((i & 1) << (nbase - 1));
        }
        roots.resize(1 << nbase);
        while(base < nbase) {
            dbl angle = 2*PI / (1 << (base + 1));
            for(int i = 1 << (base - 1); i < (1 << base); i++) {
                roots[i << 1] = roots[i];
                dbl angle_i = angle * (2 * i + 1 - (1 << base));
                roots[(i << 1) + 1] = num(cos(angle_i), sin(angle_i));
            }
            base++;
        }
    }

    void fft(vector<num> &a, int n = -1) {
        if(n == -1) {
            n = a.size();
        }
        assert((n & (n-1)) == 0);
        int zeros = __builtin_ctz(n);
        ensure_base(zeros);
        int shift = base - zeros;
        for(int i = 0; i < n; i++) {
            if(i < (rev[i] >> shift)) {
                swap(a[i], a[rev[i] >> shift]);
            }
        }
        for(int k = 1; k < n; k <= 1) {
            for(int i = 0; i < n; i += 2 * k) {
                for(int j = 0; j < k; j++) {
                    num z = a[i+j+k] * roots[j+k];
                    a[i+j+k] = a[i+j] - z;
                    a[i+j] = a[i+j] + z;
                }
            }
        }
    }

    vector<num> fa, fb;
    vector<int> multiply_mod(vector<int> &a, vector<int> &b) {
        int need = a.size() + b.size() - 1;
        int nbase = 0;
        while((1 << nbase) < need) nbase++;
        ensure_base(nbase);
        int sz = 1 << nbase;
        if(sz > (int) fa.size()) {
            fa.resize(sz);
        }
        for(int i = 0; i < sz; i++) {
            int x = (i < (int) a.size() ? a[i] : 0);
            int y = (i < (int) b.size() ? b[i] : 0);
            fa[i] = num(x, y);
        }
        fft(fa, sz);
        num r(0, -0.25 / sz);
        for(int i = 0; i <= (sz >> 1); i++) {
            int j = (sz - i) & (sz - 1);
```

```
        num z = (fa[j] * fa[j] - conj(fa[i] * fa[i])) * r;
        if(i != j) {
            fa[j] = (fa[i] * fa[i] - conj(fa[j] * fa[j])) * r;
        }
        fa[i] = z;
    }
    fft(fa, sz);
    vector<int> res(need);
    for(int i = 0; i < need; i++) {
        res[i] = fa[i].x + 0.5;
    }
    return res;
}

vector<int> multiply_mod(vector<int> &a, vector<int> &b, int m, int eq = 0) {
    int need = a.size() + b.size() - 1;
    int nbase = 0;
    while((1 << nbase) < need) nbase++;
    ensure_base(nbase);
    int sz = 1 << nbase;
    if(sz > (int) fa.size()) {
        fa.resize(sz);
    }
    for(int i = 0; i < (int) a.size(); i++) {
        int x = (a[i] % m + m) % m;
        fa[i] = num(x & ((1 << 15) - 1), x >> 15);
    }
    fill(fa.begin() + a.size(), fa.begin() + sz, num{0, 0});
    fft(fa, sz);
    if(sz > (int) fb.size()) {
        fb.resize(sz);
    }
    if(eq) {
        copy(fa.begin(), fa.begin() + sz, fb.begin());
    }
    else {
        for(int i = 0; i < (int) b.size(); i++) {
            int x = (b[i] % m + m) % m;
            fb[i] = num(x & ((1 << 15) - 1), x >> 15);
        }
        fill(fb.begin() + b.size(), fb.begin() + sz, num{0, 0});
        fft(fb, sz);
    }
    dbl ratio = 0.25 / sz;
    num r2(0, -1);
    num r3(ratio, 0);
    num r4(0, -ratio);
    num r5(0, 1);
    for(int i = 0; i <= (sz >> 1); i++) {
        int j = (sz - i) & (sz - 1);
        num a1 = (fa[i] + conj(fa[j]));
        num a2 = (fa[i] - conj(fa[j])) * r2;
        num b1 = (fb[i] + conj(fb[j])) * r3;
        num b2 = (fb[i] - conj(fb[j])) * r4;
        if(i != j) {
            num c1 = (fa[j] + conj(fa[i]));
            num c2 = (fa[j] - conj(fa[i])) * r2;
            num d1 = (fb[j] + conj(fb[i])) * r3;
            num d2 = (fb[j] - conj(fb[i])) * r4;
            fa[i] = c1 * d1 + c2 * d2 * r5;
            fb[i] = c1 * d2 + c2 * d1;
        }
        fa[j] = a1 * b1 + a2 * b2 * r5;
        fb[j] = a1 * b2 + a2 * b1;
    }
    fft(fa, sz);
    fft(fb, sz);
    vector<int> res(need);
    for(int i = 0; i < need; i++) {
        long long aa = fa[i].x + 0.5;
        long long bb = fb[i].x + 0.5;
        long long cc = fa[i].y + 0.5;
        res[i] = (aa + ((bb % m) << 15) + ((cc % m) << 30)) % m;
    }
    return res;
}

vector<int> square_mod(vector<int> &a, int m) {
    return multiply_mod(a, a, m, 1);
}
```

6.7 Fast Fourier Transform

```
// Fast Fourier Transform - O(nlogn)
```

```
/*
// Use struct instead. Performance will be way better!
typedef complex<ld> T;
T a[N], b[N];
*/

struct T {
    ld x, y;
    T() : x(0), y(0) {}
    T(ld a, ld b) : x(a), y(b) {}

    T operator/=(ld k) { x/=k; y/=k; return (*this); }
    T operator*(T a) const { return T(x*a.x - y*a.y, x*a.y + y*a.x); }
    T operator+(T a) const { return T(x+a.x, y+a.y); }
    T operator-(T a) const { return T(x-a.x, y-a.y); }
} a[N], b[N];

// a: vector containing polynomial
// n: power of two greater or equal product size
/*
// Use iterative version!
void fft_recursive(T* a, int n, int s) {
    if(n == 1) return;
    T tmp[n];
    for(int i = 0; i < n/2; ++i)
        tmp[i] = a[2*i], tmp[i+n/2] = a[2*i+1];

    fft_recursive(&tmp[0], n/2, s);
    fft_recursive(&tmp[n/2], n/2, s);

    T wn = T(cos(s*2*PI/n), sin(s*2*PI/n)), w(1,0);
    for(int i = 0; i < n/2; i++, w=w*wn)
        a[i] = tmp[i] + w*tmp[i+n/2],
        a[i+n/2] = tmp[i] - w*tmp[i+n/2];
}
*/

void fft(T* a, int n, int s) {
    for(int i=0, j=0; i<n; i++) {
        if(i>j) swap(a[i], a[j]);
        for(int l=n/2; (j^=1) < 1; l>=1);
    }

    for(int i = 1; (1<<i) <= n; i++){
        int M = 1 << i;
        int K = M >> 1;
        T wn = T(cos(s*2*PI/M), sin(s*2*PI/M));
        for(int j = 0; j < n; j += M) {
            T w = T(1, 0);
            for(int l = j; l < K + j; ++l){
                T t = w*a[l + K];
                a[l + K] = a[l] - t;
                a[l] = a[l] + t;
                w = wn*w;
            }
        }
    }

    // assert n is a power of two greater of equal product size
    // n = na + nb; while (n&(n-1)) n++;
    void multiply(T* a, T* b, int n) {
        fft(a, n, 1);
        fft(b, n, 1);
        for(int i = 0; i < n; i++) a[i] = a[i]*b[i];
        fft(a, n, -1);
        for(int i = 0; i < n; i++) a[i] /= n;
    }

    // Convert to integers after multiplying:
    // (int)(a[i].x + 0.5);
```

6.8 Fast Walsh-Hadamard Transform

```
// Fast Walsh-Hadamard Transform - O(nlogn)
//
// Multiply two polynomials, but instead of x^a * x^b = x^(a+b)
// we have x^a * x^b = x^(a XOR b).
//
// WARNING: assert n is a power of two!
void fwht(ll* a, int n, bool inv) {
    for(int l=1; 2*l <= n; l<=1) {
        for(int i=0; i < n; i+=2*l) {
```

```

for(int j=0; j<l; j++) {
    ll u = a[i+j], v = a[i+l+j];

    a[i+j] = (u+v) % MOD;
    a[i+l+j] = (u-v+MOD) % MOD;
    // % is kinda slow, you can use add() macro instead
    // #define add(x,y) (x+y >= MOD ? x+y-MOD : x+y)
}
}

if(inv) {
    for(int i=0; i<n; i++) {
        a[i] = a[i] / n;
    }
}

/* FWHT AND
Matrix : Inverse
0 1      -1 1
1 1      1 0
*/
void fwht_and(vi &a, bool inv) {
    vi ret = a;
    ll u, v;
    int tam = a.size() / 2;
    for(int len = 1; 2 * len <= tam; len <= 1) {
        for(int i = 0; i < tam; i += 2 * len) {
            for(int j = 0; j < len; j++) {
                u = ret[i + j];
                v = ret[i + len + j];
                if(!inv) {
                    ret[i + j] = v;
                    ret[i + len + j] = u + v;
                }
                else {
                    ret[i + j] = -u + v;
                    ret[i + len + j] = u;
                }
            }
        }
    }
    a = ret;
}

/* FWHT OR
Matrix : Inverse
1 1      0 1
1 0      1 -1
*/
void fft_or(vi &a, bool inv) {
    vi ret = a;
    ll u, v;
    int tam = a.size() / 2;
    for(int len = 1; 2 * len <= tam; len <= 1) {
        for(int i = 0; i < tam; i += 2 * len) {
            for(int j = 0; j < len; j++) {
                u = ret[i + j];
                v = ret[i + len + j];
                if(!inv) {
                    ret[i + j] = u + v;
                    ret[i + len + j] = u;
                }
                else {
                    ret[i + j] = v;
                    ret[i + len + j] = u - v;
                }
            }
        }
    }
    a = ret;
}

```

6.9 Gaussian Elimination (xor)

```

// Gauss Elimination for xor boolean operations
// Return false if not possible to solve
// Use boolean matrixes 0-indexed
// n equations, m variables, O(n * m * m)
// eq[i][j] = coefficient of j-th element in i-th equation
// r[i] = result of i-th equation

```

```

// Return ans[j] = xj that gives the lexicographically greatest
// solution (if possible)
// (Can be changed to lexicographically least, follow the
// comments in the code)
// WARNING!! The arrays get changed during de algorithm

bool eq[N][M], r[N], ans[M];

bool gauss_xor(int n, int m) {
    for(int i = 0; i < m; i++)
        ans[i] = true;
    int lid[N] = {0}; // id + 1 of last element present in i-th
    // line of final matrix
    int l = 0;
    for(int i = m - 1; i >= 0; i--) {
        for(int j = 1; j < n; j++)
            if(eq[j][i]) { // pivot
                swap(eq[l], eq[j]);
                swap(r[l], r[j]);
            }
        if(l == n || !eq[l][i])
            continue;
        lid[l] = i + 1;
        for(int j = l + 1; j < n; j++) { // eliminate column
            if(!eq[j][i])
                continue;
            for(int k = 0; k <= i; k++)
                eq[j][k] ^= eq[l][k];
            r[j] ^= r[l];
        }
        l++;
    }
    for(int i = n - 1; i >= 0; i--) { // solve triangular matrix
        for(int j = 0; j < lid[i + 1]; j++)
            r[i] ^= (eq[i][j] && ans[j]);
        // for lexicographically least just delete the for bellow
        for(int j = lid[i + 1]; j + 1 < lid[i]; j++) {
            ans[j] = true;
            r[i] ^= eq[i][j];
        }
        if(lid[i])
            ans[lid[i] - 1] = r[i];
        else if(r[i])
            return false;
    }
    return true;
}

```

6.10 Gaussian Elimination (double)

```

//Gaussian Elimination
//double A[N][M+1], X[M]

// if n < m, there's no solution
// column m holds the right side of the equation
// X holds the solutions

for(int j=0; j<m; j++) { //column to eliminate
    int l = j;
    for(int i=j+1; i<n; i++) //find largest pivot
        if(abs(A[i][j])>abs(A[l][j]))
            l=i;
    if(abs(A[l][j]) < EPS) continue;
    for(int k = 0; k < m+1; k++) { //Swap lines
        swap(A[l][k], A[j][k]);
    }
    for(int i = j+1; i < n; i++) { //eliminate column
        double t=A[i][j]/A[j][j];
        for(int k = j; k < m+1; k++)
            A[i][k]-=t*A[j][k];
    }
}

for(int i = m-1; i >= 0; i--) { //solve triangular system
    for(int j = m-1; j > i; j--)
        A[i][m] -= A[i][j]*X[j];
    X[i]=A[i][m]/A[i][i];
}

```

6.11 Golden Section Search (Ternary Search)

```

double gss(double l, double r) {
    double m1 = r-(r-l)/gr, m2 = l+(r-l)/gr;
    double f1 = f(m1), f2 = f(m2);
    while(fabs(l-r)>EPS) {
        if(f1>f2) l=m1, f1=f2, m1=m2, m2=l+(r-l)/gr, f2=f(m2);
        else r=m2, f2=f1, m2=m1, m1=r-(r-l)/gr, f1=f(m1);
    }
    return l;
}

```

6.12 Josephus

```

// UFMG
/* Josephus Problem - It returns the position to be, in order to
   not die. O(n) */
/* With k=2, for instance, the game begins with 2 being killed
   and then n+2, n+4, ... */
ll josephus(ll n, ll k) {
    if(n==1) return 1;
    else return (josephus(n-1, k)+k-1)%n+1;
}

/* Another Way to compute the last position to be killed - O(d *
   log n) */
ll josephus(ll n, ll d) {
    ll K = 1;
    while (K <= (d - 1)*n) K = (d * K + d - 2) / (d - 1);
    return d * n + 1 - K;
}

```

6.13 Matrix Exponentiation

```

/*
   This code assumes you are multiplying two matrices that can be
   multiplied: (A nxp * B pxm)
   Matrix fexp assumes square matrices
*/

const int MOD = 1e9 + 7;
typedef long long ll;
typedef long long type;

struct matrix {
    //matrix n x m
    vector<vector<type>>> a;
    int n, m;
    matrix() = default;

    matrix(int _n, int _m) : n(_n), m(_m) {
        a.resize(n, vector<type>(m));
    }

    matrix operator *(matrix other) {
        matrix result(this->n, other.m);
        for(int i = 0; i < result.n; i++) {
            for(int j = 0; j < result.m; j++) {
                for(int k = 0; k < this->m; k++) {
                    result.a[i][j] = (result.a[i][j] + a[i][k] * other.a[k][j]);
                    //result.a[i][j] = (result.a[i][j] + (a[i][k] * other.a[k][j]) % MOD) % MOD;
                }
            }
        }
        return result;
    };
};

matrix identity(int n) {
    matrix id(n, n);
    for(int i = 0; i < n; i++) id.a[i][i] = 1;
    return id;
}

```

```

matrix fexp(matrix b, ll e){
    matrix ans = identity(b.n);
    while(e){
        if(e & 1) ans = (ans * b);
        b = b * b;
        e >>= 1;
    }
    return ans;
}

```

6.14 Mobius Inversion

```

// multiplicative function calculator
// euler_phi and mobius are multiplicative
// if another f[N] needed just remove comments
// O(N)

bool p[N];
vector<ll> primes;
ll g[N];
// ll f[N];

void mfc(){
    // if g(1) != 1 than it's not multiplicative
    g[1] = 1;
    // f[1] = 1;
    primes.clear();
    primes.reserve(N / 10);
    for(ll i = 2; i < N; i++){
        if(!p[i]){
            primes.push_back(i);
            for(ll j = i; j < N; j *= i){
                g[j] = // g(p^k) you found
                // f[j] = f(p^k) you found
                p[j] = (j != i);
            }
        }
        for(ll j : primes){
            if(i * j >= N || i % j == 0)
                break;
            for(ll k = j; i * k < N; k *= j){
                g[i * k] = g[i] * g[k];
                // f[i * k] = f[i] * f[k];
                p[i * k] = true;
            }
        }
    }
}

```

6.15 Mobius Function

```

// 1 if n == 1
// 0 if exists x | n%(x^2) == 0
// else (-1)^k, k = #(p) | p is prime and n%p == 0

//Calculate Mobius for all integers using sieve
//O(n*log(log(n)))
void mobius() {
    for(int i = 1; i < N; i++) mob[i] = 1;

    for(ll i = 2; i < N; i++) if(!sieve[i]){
        for(ll j = i; j < N; j += i) sieve[j] = i, mob[j] *= -1;
        for(ll j = i*i; j < N; j += i*i) mob[j] = 0;
    }
}

/*
//Calculate Mobius for 1 integer
//O(sqrt(n))
int mobius(int n){
    if(n == 1) return 1;
    int p = 0;
    for(int i = 2; i*i <= n; i++)
        if(n%i == 0){
            n /= i;
            p++;
            if(n%i == 0) return 0;
        }
    if(n > 1) p++;
    return p%1 ? -1 : 1;
}

```

```

}
*/

```

6.16 Number Theoretic Transform

```

// Number Theoretic Transform - O(nlogn)

// if long long is not necessary, use int instead to improve
// performance
const int mod = 20*(1<<23)+1;
const int root = 3;

ll w[N];

// a: vector containing polynomial
// n: power of two greater or equal product size
void ntt(ll* a, int n, bool inv) {
    for (int i=0, j=0; i<n; i++) {
        if (i>j) swap(a[i], a[j]);
        for (int l=n/2; (j^=l) < 1; l>>=1);
    }

    // TODO: Rewrite this loop using FFT version
    ll k, t, nrev;
    w[0] = 1;
    k = exp(root, (mod-1) / n, mod);
    for (int i=1; i<=n; i++) w[i] = w[i-1] * k % mod;
    for(int i=2; i<=n; i<=1) for(int j=0; j<n; j+=i) for(int l=0;
        l<(i/2); l++) {
        int x = j+l, y = j+l+(i/2), z = (n/i)*l;
        t = a[y] * w[inv ? (n-z) : z] % mod;
        a[y] = (a[x] - t + mod) % mod;
        a[x] = (a[j+l] + t) % mod;
    }

    nrev = exp(n, mod-2, mod);
    if (inv) for(int i=0; i<n; ++i) a[i] = a[i] * nrev % mod;
}

// assert n is a power of two greater of equal product size
// n = na + nb; while (n&(n-1)) n++;
void multiply(ll* a, ll* b, int n) {
    ntt(a, n, 0);
    ntt(b, n, 0);
    for (int i = 0; i < n; i++) a[i] = a[i]*b[i] % mod;
    ntt(a, n, 1);
}

```

6.17 Pollard-Rho

```

// factor(N, v) to get N factorized in vector v
// O(N ^ (1 / 4)) on average
// Miller-Rabin - Primarily Test O(|base|*(logn)^2)
ll addmod(ll a, ll b, ll m){
    if(a >= m - b) return a + b - m;
    return a + b;
}

ll mulmod(ll a, ll b, ll m){
    ll ans = 0;
    while(b){
        if(b & 1) ans = addmod(ans, a, m);
        a = addmod(a, a, m);
        b >>= 1;
    }
    return ans;
}

ll fexp(ll a, ll b, ll n){
    ll r = 1;
    while(b){
        if(b & 1) r = mulmod(r, a, n);
        a = mulmod(a, a, n);
        b >>= 1;
    }
    return r;
}

bool miller(ll a, ll n){
    if (a >= n) return true;
}

```

```

ll s = 0, d = n - 1;
while(d % 2 == 0) d >>= 1, s++;
ll x = fexp(a, d, n);
if (x == 1 || x == n - 1) return true;
for (int r = 0; r < s; r++, x = mulmod(x,x,n)){
    if (x == 1) return false;
    if (x == n - 1) return true;
}
return false;
}

bool isprime(ll n){
    if(n == 1) return false;
    int base[] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
    for (int i = 0; i < 12; ++i) if (!miller(base[i], n)) return
        false;
    return true;
}

ll pollard(ll n){
    ll x, y, d, c = 1;
    if (n % 2 == 0) return 2;
    while(true){
        y = x = 2;
        while(true){
            x = addmod(mulmod(x,x,n), c, n);
            y = addmod(mulmod(y,y,n), c, n);
            y = addmod(mulmod(y,y,n), c, n);
            if (x == y) break;
            d = __gcd(abs(x-y), n);
            if (d > 1) return d;
        }
        c++;
    }
}

vector<ll> factor(ll n){
    if (n == 1 || isprime(n)) return {n};
    ll f = pollard(n);
    vector<ll> l = factor(f), r = factor(n / f);
    l.insert(l.end(), r.begin(), r.end());
    sort(l.begin(), l.end());
    return l;
}

//n < 2,047 base = {2};
//n < 9,080,191 base = {31, 73};
//n < 2,152,302,898,747 base = {2, 3, 5, 7, 11};
//n < 318,665,857,834,031,151,167,461 base = {2, 3, 5, 7, 11,
13, 17, 19, 23, 29, 31, 37};
//n < 3,317,044,064,679,887,385,961,981 base = {2, 3, 5, 7, 11,
13, 17, 19, 23, 29, 31, 37, 41};

```

6.18 Prime Factors

```

// Prime factors (up to 9*10^13. For greater see Pollard Rho)
vi factors;
int ind=0, pf = primes[0];
while (pf*pf <= n) {
    while (n%pf == 0) n /= pf, factors.pb(pf);
    pf = primes[++ind];
}
if (n != 1) factors.pb(n);

```

6.19 Primitive Root

```

// Finds a primitive root modulo p
// To make it works for any value of p, we must add calculation
// of phi(p)
// n is i, 2, 4 or p^k or 2*p^k (p odd in both cases)
ll root(ll p) {
    ll n = p-1;
    vector<ll> fact;

    for (int i=2; i*i<=n; ++i) if (n % i == 0) {
        fact.push_back (i);
        while (n % i == 0) n /= i;
    }

    if (n > 1) fact.push_back (n);
}

```

```

for (int res=2; res<=p; ++res) {
    bool ok = true;
    for (size_t i=0; i<fact.size() && ok; ++i)
        ok &= exp(res, (p-1) / fact[i], p) != 1;
    if (ok) return res;
}

return -1;
}

```

6.20 Sieve of Eratosthenes

```

// Sieve of Eratosthenes
int p[N]; vi primes;

for (ll i = 2; i < N; ++i) if (!p[i]) {
    for (ll j = i*i; j < N; j+=i) p[j]=1;
    primes.pb(i);
}

```

6.21 Simpson Rule

```

// Simpson Integration Rule
// define the function f
double f(double x) {
    // ...
}

double simpson(double a, double b, int n = 1e6) {
    double h = (b - a) / n;
    double s = f(a) + f(b);
    for (int i = 1; i < n; i += 2) s += 4 * f(a + h*i);
    for (int i = 2; i < n; i += 2) s += 2 * f(a + h*i);
    return s*h/3;
}

```

6.22 Simplex (Stanford)

```

// Two-phase simplex algorithm for solving linear programs of
// the form
//
//      maximize      c^T x
//      subject to    Ax <= b
//                  x >= 0
//
// INPUT: A -- an m x n matrix
//        b -- an m-dimensional vector
//        c -- an n-dimensional vector
//        x -- a vector where the optimal solution will be
//             stored
//
// OUTPUT: value of the optimal solution (infinity if unbounded
//        above, nan if infeasible)
//
// To use this code, create an LPSolver object with A, b, and c
// as
// arguments. Then, call Solve(x).

#include <iostream>
#include <iomanip>
#include <vector>
#include <cmath>
#include <limits>

using namespace std;
typedef long double DOUBLE;
typedef vector<DOUBLE> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;
const DOUBLE EPS = 1e-9;

struct LPSolver {
    int m, n;
    VI B, N;

```

```

VVD D;

LPSolver(const VVD &A, const VD &b, const VD &c) :
    m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2, VD(n + 2)) {
    for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) D[i][j] = A[i][j];
    for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n] = -1; D[i][n + 1] = b[i]; }
    for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
    N[n] = -1; D[m + 1][n] = 1;
}

void Pivot(int r, int s) {
    for (int i = 0; i < m + 2; i++) if (i != r)
        for (int j = 0; j < n + 2; j++) if (j != s)
            D[i][j] -= D[r][j] * D[i][s] / D[r][s];
    for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] /= D[r][s];
    for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] /= -D[r][s];
    D[r][s] = 1.0 / D[r][s];
    swap(B[r], N[s]);
}

bool Simplex(int phase) {
    int x = phase == 1 ? m + 1 : m;
    while (true) {
        int s = -1;
        for (int j = 0; j <= n; j++) {
            if (phase == 2 && N[j] == -1) continue;
            if (s == -1 || D[x][j] < D[x][s] || D[x][j] == D[x][s] && N[j] < N[s]) s = j;
        }
        if (D[x][s] > -EPS) return true;
        int r = -1;
        for (int i = 0; i < m; i++) {
            if (D[i][s] < EPS) continue;
            if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s] ||
                (D[i][n + 1] / D[i][s] == D[r][n + 1] / D[r][s] && B[i] < B[r]) r = i;
        }
        if (r == -1) return false;
        Pivot(r, s);
    }
}

DOUBLE Solve(VD &x) {
    int r = 0;
    for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1]) r = i;
    if (D[r][n + 1] < -EPS) {
        Pivot(r, n);
        if (!Simplex(1) || D[m + 1][n + 1] < -EPS) return -
            numeric_limits<DOUBLE>::infinity();
        for (int i = 0; i < m; i++) if (B[i] == -1) {
            int s = -1;
            for (int j = 0; j <= n; j++)
                if (s == -1 || D[i][j] < D[i][s] || D[i][j] == D[i][s] && N[j] < N[s]) s = j;
            Pivot(i, s);
        }
    }
    if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
    x = VD(n);
    for (int i = 0; i < m; i++) if (B[i] < n) x[B[i]] = D[i][n + 1];
    return D[m][n + 1];
}

int main() {
    const int m = 4;
    const int n = 3;
    DOUBLE _A[m][n] = {
        { 6, -1, 0 },
        { -1, -5, 0 },
        { 1, 5, 1 },
        { -1, -5, -1 }
    };
    DOUBLE _b[m] = { 10, -4, 5, -5 };
    DOUBLE _c[n] = { 1, -1, 0 };

    VVD A(m);
    VD b(_b, _b + m);
    VD c(_c, _c + n);

```

```

for (int i = 0; i < m; i++) A[i] = VD(_A[i], _A[i] + n);

LPSolver solver(A, b, c);
VD x;
DOUBLE value = solver.Solve(x);

cerr << "VALUE: " << value << endl; // VALUE: 1.29032
cerr << "SOLUTION:"; // SOLUTION: 1.74194 0.451613 1
for (size_t i = 0; i < x.size(); i++) cerr << " " << x[i];
cerr << endl;
return 0;
}

```

7 Geometry

7.1 Miscellaneous

```

/*
1) Square (n = 4) is the only regular polygon with integer
   coordinates

2) Pick's theorem: A = i + b/2 - 1
   A: area of the polygon
   i: number of interior points
   b: number of points on the border

3) Conic Rotations
   Given ellipse: Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0
   Convert it to: Ax^2 + Bxy + Cy^2 + Dx + Ey = 1 (this formula
   suits better for ellipse, before doing this verify F = 0)
   Final conversion: A(x + D/2A)^2 + C(y + E/2C)^2 = 1 + D^2/4A +
   E^2/4C
   B != 0 (Rotate):
   theta = atan2(b, c-a)/2.0;
   A' = (a + c + b/sin(2.0*theta))/2.0; // A
   C' = (a + c - b/sin(2.0*theta))/2.0; // C
   D' = d*sin(theta) + e*cos(theta); // D
   E' = d*cos(theta) - e*sin(theta); // E
   If you do any point calculation, for example finding ellipses
   focus, remember to rotate the points by theta after!
*/

```

7.2 Basics (Point)

```

#include <bits/stdc++.h>

using namespace std;

#define st first
#define nd second
#define pb push_back
#define cl(x,v) memset((x), (v), sizeof(x))
#define db(x) cerr << #x << " == " << x << endl
#define dbs(x) cerr << x << endl
#define _ << ", " <<

typedef long long ll;
typedef long double ld;
typedef pair<int,int> pii;
typedef pair<int,pii> piii;
typedef pair<ll,ll> pll;
typedef pair<ll,pll> pll1;
typedef vector<int> vi;
typedef vector<vi> vii;

const ld EPS = 1e-9, PI = acos(-1.);
const ll LINF = 0x3f3f3f3f3f3f3f3f;
const int INF = 0x3f3f3f3f, MOD = 1e9+7;
const int N = 1e5+5;

typedef long double type;
//for big coordinates change to long long

bool ge(type x, type y) { return x + EPS > y; }
bool le(type x, type y) { return x - EPS < y; }
bool eq(type x, type y) { return ge(x, y) and le(x, y); }
int sign(type x) { return ge(x, 0) - le(x, 0); }

```



```

struct point {
    type x, y;

    point() : x(0), y(0) {}
    point(type _x, type _y) : x(_x), y(_y) {}

    point operator -() { return point(-x, -y); }
    point operator +(point p) { return point(x + p.x, y + p.y); }
    point operator -(point p) { return point(x - p.x, y - p.y); }

    point operator *(type k) { return point(x*k, y*k); }
    point operator /(type k) { return point(x/k, y/k); }

    //inner product
    type operator *(point p) { return x*p.x + y*p.y; }
    //cross product
    type operator %(point p) { return x*p.y - y*p.x; }

    bool operator ==(const point &p) const{ return x == p.x and y
    == p.y; }
    bool operator !=(const point &p) const{ return x != p.x or y
    != p.y; }
    bool operator <(const point &p) const { return (x < p.x) or (x
    == p.x and y < p.y); }

    // 0 => same direction
    // 1 => p is on the left
    //-1 => p is on the right
    int dir(point o, point p) {
        type x = (*this - o) % (p - o);
        return ge(x,0) - le(x,0);
    }

    bool on_seg(point p, point q) {
        if (this->dir(p, q)) return 0;
        return ge(x, min(p.x, q.x)) and le(x, max(p.x, q.x)) and ge(
        y, min(p.y, q.y)) and le(y, max(p.y, q.y));
    }

    ld abs() { return sqrt(x*x + y*y); }
    type abs2() { return x*x + y*y; }
    ld dist(point q) { return (*this - q).abs(); }
    type dist2(point q) { return (*this - q).abs2(); }

    ld arg() { return atan2l(y, x); }

    // Project point on vector y
    point project(point y) { return y * ((*this * y) / (y * y)); }

    // Project point on line generated by points x and y
    point project(point x, point y) { return x + (*this - x).
        project(y-x); }

    ld dist_line(point x, point y) { return dist(project(x, y)); }

    ld dist_seg(point x, point y) {
        return project(x, y).on_seg(x, y) ? dist_line(x, y) : min(
        dist(x, dist(y));
    }

    point rotate(ld sin, ld cos) { return point(cos*x - sin*y, sin
        *x + cos*y); }
    point rotate(ld a) { return rotate(sin(a), cos(a)); }

    // rotate around the argument of vector p
    point rotate(point p) { return rotate(p.y / p.abs(), p.x / p.
        abs()); }
};

int direction(point o, point p, point q) { return p.dir(o, q); }

point rotate_ccw90(point p) { return point(-p.y,p.x); }
point rotate_cw90(point p) { return point(p.y,-p.x); }

//for reading purposes avoid using * and % operators, use the
functions below:
type dot(point p, point q) { return p.x*q.x + p.y*q.y; }
type cross(point p, point q) { return p.x*q.y - p.y*q.x; }

//double area
type area_2(point a, point b, point c) { return cross(a,b) +
    cross(b,c) + cross(c,a); }

//angle between (a1 and b1) vs angle between (a2 and b2)
//1 : bigger
//-1 : smaller
//0 : equal

```

```

int angle_less(const point& a1, const point& b1, const point& a2
    , const point& b2) {
    point p1(dot( a1, b1), abs(cross( a1, b1)));
    point p2(dot( a2, b2), abs(cross( a2, b2)));
    if(cross(p1, p2) < 0) return 1;
    if(cross(p1, p2) > 0) return -1;
    return 0;
}

ostream &operator<<(ostream &os, const point &p) {
    os << "(" << p.x << ", " << p.y << ")";
    return os;
}

```

7.3 Radial Sort

```

#include "basics.cpp"
point origin;

/*
    below < above
    order: [pi, 2 * pi)
*/

int above(point p){
    if(p.y == origin.y) return p.x > origin.x;
    return p.y > origin.y;
}

bool cmp(point p, point q){
    int tmp = above(q) - above(p);
    if(tmp) return tmp > 0;
    return p.dir(origin,q) > 0;
} //Be Careful: p.dir(origin,q) == 0

```

7.4 Circle

```

#include "basics.cpp"
#include "lines.cpp"

struct circle {
    point c;
    ld r;
    circle() { c = point(); r = 0; }
    circle(point _c, ld _r) : c(_c), r(_r) {}
    ld area() { return acos(-1.0)*r*r; }
    ld chord(ld rad) { return 2*r*sin(rad/2.0); }
    ld sector(ld rad) { return 0.5*rad*area()/acos(-1.0); }
    bool intersects(circle other) {
        return le(c.dist(other.c), r + other.r);
    }
    bool contains(point p) { return le(c.dist(p), r); }
    pair<point, point> getTangentPoint(point p) {
        ld dl = c.dist(p), theta = asin(r/dl);
        point p1 = (c - p).rotate(-theta);
        point p2 = (c - p).rotate(theta);
        p1 = p1*(sqrt(dl*dl - r*r)/dl) + p;
        p2 = p2*(sqrt(dl*dl - r*r)/dl) + p;
        return make_pair(p1,p2);
    }
};

circle circumcircle(point a, point b, point c) {
    circle ans;
    point u = point((b - a).y, -(b - a).x);
    point v = point((c - a).y, -(c - a).x);
    point n = (c - b)*0.5;
    ld t = cross(u,n)/cross(v,u);
    ans.c = ((a + c)*0.5) + (v*t);
    ans.r = ans.c.dist(a);
    return ans;
}

point compute_circle_center(point a, point b, point c) {
    //circumcenter
    b = (a + b)/2;
    c = (a + c)/2;
    return compute_line_intersection(b, b + rotate_cw90(a - b), c,
        c + rotate_cw90(a - c));
}

```

```

}

int inside_circle(point p, circle c) {
    if (fabs(p.dist(c.c) - c.r)<EPS) return 1;
    else if (p.dist(c.c) < c.r) return 0;
    else return 2;
} //0 = inside/1 = border/2 = outside

circle incircle( point p1, point p2, point p3 ) {
    ld m1 = p2.dist(p3);
    ld m2 = p1.dist(p3);
    ld m3 = p1.dist(p2);
    point c = (p1*m1 + p2*m2 + p3*m3)*(1/(m1 + m2 + m3));
    ld s = 0.5*(m1 + m2 + m3);
    ld r = sqrt(s*(s - m1)*(s - m2)*(s - m3))/s;
    return circle(c, r);
}

circle minimum_circle(vector<point> p) {
    random_shuffle(p.begin(), p.end());
    circle C = circle(p[0], 0.0);
    for(int i = 0; i < (int)p.size(); i++) {
        if (C.contains(p[i])) continue;
        C = circle(p[i], 0.0);
        for(int j = 0; j < i; j++) {
            if (C.contains(p[j])) continue;
            C = circle((p[j] + p[i])*0.5, 0.5*p[j].dist(p[i]));
            for(int k = 0; k < j; k++) {
                if (C.contains(p[k])) continue;
                C = circumcircle(p[j], p[i], p[k]);
            }
        }
    }
    return C;
}

// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<point> circle_line_intersection(point a, point b, point c
    , ld r) {
    vector<point> ret;
    b = b - a;
    a = a - c;
    ld A = dot(b, b);
    ld B = dot(a, b);
    ld C = dot(a, a) - r*r;
    ld D = B*B - A*C;
    if (D < -EPS) return ret;
    ret.push_back(c + a + b*(sqrt(D + EPS) - B)/A);
    if (D > EPS)
        ret.push_back(c + a + b*(-B - sqrt(D))/A);
    return ret;
}

vector<point> circle_circle_intersection(point a, point b, ld r,
    ld R) {
    vector<point> ret;
    ld d = sqrt(a.dist2(b));
    if (d > r + R || d + min(r, R) < max(r, R)) return ret;
    ld x = (d*d - R*R + r*r)/(2*d);
    ld y = sqrt(r*r - x*x);
    point v = (b - a)/d;
    ret.push_back(a + v*x + rotate_ccw90(v)*y);
    if (y > 0)
        ret.push_back(a + v*x - rotate_ccw90(v)*y);
    return ret;
}

//GREAT CIRCLE

double gcTheta(double pLat, double pLong, double qLat, double
    qLong) {
    plat *= acos(-1.0) / 180.0; pLong *= acos(-1.0) / 180.0; //
    convert degree to radian
    qlat *= acos(-1.0) / 180.0; qLong *= acos(-1.0) / 180.0;
    return acos(cos(pLat)*cos(pLong)*cos(qLat)*cos(qLong) +
        cos(pLat)*sin(pLong)*cos(qLat)*sin(qLong) +
        sin(pLat)*sin(qLat));
}

double gcDistance(double pLat, double pLong, double qLat, double
    qLong, double radius) {
    return radius*gcTheta(pLat, pLong, qLat, qLong);
}

```

7.5 Closest Pair of Points

```
#include "basics.cpp"
//DIVIDE AND CONQUER METHOD
//Warning: include variable id into the struct point

struct cmp_y {
    bool operator()(const point & a, const point & b) const {
        return a.y < b.y;
    }
};

ld min_dist = LINF;
pair<int, int> best_pair;
vector<point> pts, stripe;
int n;

void upd_ans(const point & a, const point & b) {
    ld dist = sqrt((a.x - b.x)*(a.x - b.x) + (a.y - b.y)*(a.y - b.y));
    if (dist < min_dist) {
        min_dist = dist;
        // best_pair = {a.id, b.id};
    }
}

void closest_pair(int l, int r) {
    if (r - l <= 3) {
        for (int i = l; i < r; ++i) {
            for (int j = i + 1; j < r; ++j) {
                upd_ans(pts[i], pts[j]);
            }
        }
        sort(pts.begin() + l, pts.begin() + r, cmp_y());
        return;
    }

    int m = (l + r) >> 1;
    type midx = pts[m].x;
    closest_pair(l, m);
    closest_pair(m, r);

    merge(pts.begin() + l, pts.begin() + m, pts.begin() + m, pts.begin() + r, stripe.begin(), cmp_y());
    copy(stripe.begin(), stripe.begin() + r - l, pts.begin() + l);

    int stripe_sz = 0;
    for (int i = l; i < r; ++i) {
        if (abs(pts[i].x - midx) < min_dist) {
            for (int j = stripe_sz - 1; j >= 0 && pts[i].y - stripe[j].y < min_dist; --j) {
                upd_ans(pts[i], stripe[j]);
                stripe[stripe_sz++] = pts[i];
            }
        }
    }

    int main() {
        //read and save in vector pts
        min_dist = LINF;
        stripe.resize(n);
        sort(pts.begin(), pts.end());
        closest_pair(0, n);
    }
}
```

7.6 Half Plane Intersection

```
// Intersection of halfplanes - O(nlogn)
// Points are given in counterclockwise order
//
// by Agnez

typedef vector<point> polygon;

int cmp(ld x, ld y = 0, ld tol = EPS) {
    return (x <= y + tol) ? (x + tol < y) ? -1 : 0 : 1;
}

bool cmp(point a, point b) {
    if ((cmp(a.x) > 0 || (cmp(a.x) == 0 && cmp(a.y) > 0)) && (cmp(b.x) < 0 || (cmp(b.x) == 0 && cmp(b.y) < 0)))
        return 1;
    if ((cmp(b.x) > 0 || (cmp(b.x) == 0 && cmp(b.y) > 0)) && (cmp(a.x) < 0 || (cmp(a.x) == 0 && cmp(a.y) < 0)))
        return -1;
    return 0;
}
```

```
return 0;
ll R = a%b;
if(R) return R > 0;
return false;
}

namespace halfplane{
    struct L{
        point p,v;
        L(){}
        L(point P, point V):p(P),v(V){}
        bool operator<(const L &b)const{ return comp(v, b.v); }
    };
    vector<L> line;
    void addL(point a, point b){line.pb(L(a,b-a));}
    bool left(point &p, L &l){ return cmp(l.v % (p-l.p))>0; }
    bool left_equal(point &p, L &l){ return cmp(l.v % (p-l.p))>=0; }
    void init(){ line.clear(); }

    point pos(L &a, L &b){
        point x=a.p-b.p;
        ld t = (b.v % x)/(a.v % b.v);
        return a.p+a.v*t;
    }

    polygon intersect(){
        sort(line.begin(), line.end());
        deque<L> q; //linhas da intersecao
        deque<point> p; //pontos de intersecao entre elas
        q.push_back(line[0]);
        for(int i=1; i < (int) line.size(); i++){
            while(q.size()>1 && !left(p.back(), line[i]))
                q.pop_back(), p.pop_back();
            while(q.size()>1 && !left(p.front(), line[i]))
                q.pop_front(), p.pop_front();
            if(!cmp(q.back().v % line[i].v) && !left(q.back().p, line[i].p))
                q.back() = line[i];
            else if(cmp(q.back().v % line[i].v))
                q.push_back(line[i]), p.push_back(point());
            if(q.size()>1)
                p.back()=pos(q.back(),q[q.size()-2]);
        }
        while(q.size()>1 && !left(p.back(),q.front()))
            q.pop_back(), p.pop_back();
        if(q.size() <= 2) return polygon(); //Nao forma poligono (
            pode nao ter intersecao)
        if(!cmp(q.back().v % q.front().v) return polygon(); //Lados
            paralelos -> area infinita
        point ult = pos(q.back(),q.front());

        bool ok = 1;
        for(int i=0; i < (int) line.size(); i++){
            if(!left_equal(ult,line[i])){ ok=0; break; }

            if(ok) p.push_back(ult); //Se formar um poligono fechado
        }
        polygon ret;
        for(int i=0; i < (int) p.size(); i++){
            ret.pb(p[i]);
        }
        return ret;
    }
}
```

7.7 Lines

```
#include "basics.cpp"
//functions tested at: https://codeforces.com/group/3qadGzUdR4/contest/101706/problem/B

//WARNING: all distance functions are not realizing sqrt operation
//Suggestion: for line intersections check line_line_intersection and then use compute_line_intersection

point project_point_line(point c, point a, point b) {
    ld r = dot(b - a, b - a);
    if (fabs(r) < EPS) return a;
    return a + (b - a)*dot(c - a, b - a)/dot(b - a, b - a);
}

point project_point_ray(point c, point a, point b) {
    ld r = dot(b - a, b - a);
    if (fabs(r) < EPS) return a;
    return a + (b - a)*dot(c - a, b - a)/dot(b - a, b - a);
}

r = dot(c - a, b - a) / r;
if (le(r, 0)) return a;
return a + (b - a)*r;

point project_point_segment(point c, point a, point b) {
    ld r = dot(b - a, b - a);
    if (fabs(r) < EPS) return a;
    r = dot(c - a, b - a)/r;
    if (le(r, 0)) return a;
    if (ge(r, 1)) return b;
    return a + (b - a)*r;
}

ld distance_point_line(point c, point a, point b) {
    return c.dist2(project_point_line(c, a, b));
}

ld distance_point_ray(point c, point a, point b) {
    return c.dist2(project_point_ray(c, a, b));
}

ld distance_point_segment(point c, point a, point b) {
    return c.dist2(project_point_segment(c, a, b));
}

//not tested
ld distance_point_plane(ld x, ld y, ld z, ld a, ld b, ld c, ld d) {
    return fabs(a*x + b*y + c*z - d)/sqrt(a*a + b*b + c*c);
}

bool lines_parallel(point a, point b, point c, point d) {
    return fabs(cross(b - a, d - c)) < EPS;
}

bool lines_collinear(point a, point b, point c, point d) {
    return lines_parallel(a, b, c, d) && fabs(cross(a-b, a-c)) < EPS && fabs(cross(c-d, c-a)) < EPS;
}

point lines_intersect(point p, point q, point a, point b) {
    point r = q - p, s = b - a, c(p%q, a%b);
    if (eq(r%s,0)) return point(LINF, LINF);
    return point((point(r.x, s.x) % c, point(r.y, s.y) % c) / (r%s));
}

//be careful: test line_line_intersection before using this function
point compute_line_intersection(point a, point b, point c, point d) {
    b = b - a; d = c - d; c = c - a;
    assert(dot(b, b) > EPS && dot(d, d) > EPS);
    return a + b*cross(c, d)/cross(b, d);
}

bool line_line_intersect(point a, point b, point c, point d) {
    if(!lines_parallel(a, b, c, d)) return true;
    if(lines_collinear(a, b, c, d)) return true;
    return false;
}

//rays in direction a -> b, c -> d
bool ray_ray_intersect(point a, point b, point c, point d) {
    if (a.dist2(c) < EPS || a.dist2(d) < EPS || b.dist2(c) < EPS || b.dist2(d) < EPS) return true;
    if (lines_collinear(a, b, c, d)) {
        if(ge(dot(b - a, d - c), 0)) return true;
        if(ge(dot(a - c, d - c), 0)) return true;
        return false;
    }
    if(!line_line_intersect(a, b, c, d)) return false;
    point inters = lines_intersect(a, b, c, d);
    if(ge(dot(inters - c, d - c), 0) && ge(dot(inters - a, b - a), 0)) return true;
    return false;
}

bool segment_segment_intersect(point a, point b, point c, point d) {
    if (a.dist2(c) < EPS || a.dist2(d) < EPS || b.dist2(c) < EPS || b.dist2(d) < EPS) return true;
    int d1, d2, d3, d4;
    d1 = direction(a, b, c);
    d2 = direction(a, b, d);
    d3 = direction(c, d, a);
    d4 = direction(c, d, b);
    return (d1 < 0 && d2 > 0 && d3 < 0 && d4 > 0) || (d1 > 0 && d2 < 0 && d3 > 0 && d4 < 0);
}
```

```

d3 = direction(c, d, a);
d4 = direction(c, d, b);
if (d1*d2 < 0 and d3*d4 < 0) return 1;
return a.on_seg(c, d) or b.on_seg(c, d) or
c.on_seg(a, b) or d.on_seg(a, b);
}

bool segment_line_intersect(point a, point b, point c, point d) {
    if(!line_line_intersect(a, b, c, d)) return false;
    point inters = lines_intersect(a, b, c, d);
    if(inters.on_seg(a, b)) return true;
    return false;
}

//ray in direction c -> d
bool segment_ray_intersect(point a, point b, point c, point d) {
    if (a.dist2(c) < EPS || a.dist2(d) < EPS ||
        b.dist2(c) < EPS || b.dist2(d) < EPS) return true;
    if (lines_collinear(a, b, c, d)) {
        if(c.on_seg(a, b)) return true;
        if(ge(dot(d - c, a - c), 0)) return true;
        return false;
    }
    if(!line_line_intersect(a, b, c, d)) return false;
    point inters = lines_intersect(a, b, c, d);
    if(!inters.on_seg(a, b)) return false;
    if(ge(dot(inters - c, d - c), 0)) return true;
    return false;
}

//ray in direction a -> b
bool ray_line_intersect(point a, point b, point c, point d) {
    if (a.dist2(c) < EPS || a.dist2(d) < EPS ||
        b.dist2(c) < EPS || b.dist2(d) < EPS) return true;
    if (!line_line_intersect(a, b, c, d)) return false;
    point inters = lines_intersect(a, b, c, d);
    if(!line_line_intersect(a, b, c, d)) return false;
    if(ge(dot(inters - a, b - a), 0)) return true;
    return false;
}

ld distance_segment_line(point a, point b, point c, point d) {
    if(segment_line_intersect(a, b, c, d)) return 0;
    return min(distance_point_line(a, c, d), distance_point_line(b, c, d));
}

ld distance_segment_ray(point a, point b, point c, point d) {
    if(segment_ray_intersect(a, b, c, d)) return 0;
    ld min1 = distance_point_segment(c, a, b);
    ld min2 = min(distance_point_ray(a, c, d), distance_point_ray(b, c, d));
    return min(min1, min2);
}

ld distance_segment_segment(point a, point b, point c, point d) {
    if(segment_segment_intersect(a, b, c, d)) return 0;
    ld min1 = min(distance_point_segment(c, a, b), distance_point_segment(d, a, b));
    ld min2 = min(distance_point_segment(a, c, d), distance_point_segment(b, c, d));
    return min(min1, min2);
}

ld distance_ray_line(point a, point b, point c, point d) {
    if(ray_line_intersect(a, b, c, d)) return 0;
    ld min1 = distance_point_line(a, c, d);
    return min1;
}

ld distance_ray_ray(point a, point b, point c, point d) {
    if(ray_ray_intersect(a, b, c, d)) return 0;
    ld min1 = min(distance_point_ray(c, a, b), distance_point_ray(a, c, d));
    return min1;
}

ld distance_line_line(point a, point b, point c, point d) {
    if(line_line_intersect(a, b, c, d)) return 0;
    return distance_point_line(a, c, d);
}

```

```

#include "basics.cpp"
#include "polygons.cpp"

//ITA MINKOWSKI

typedef vector<point> polygon;

/*
 * Minkowski sum
 * Distance between two polygons P and Q:
 * Do Minkowski(P, Q)
 * Ans = min(ans, dist((0, 0), edge))
 */

polygon minkowski(polygon &A, polygon &B) {
    polygon P; point v1, v2;
    sort_lex_hull(A), sort_lex_hull(B);
    int n1 = A.size(), n2 = B.size();
    P.push_back(A[0] + B[0]);
    for(int i = 0, j = 0; i < n1 || j < n2; ) {
        v1 = A[(i + 1)%n1] - A[i%n1];
        v2 = B[(j + 1)%n2] - B[j%n2];
        if (j == n2 || cross(v1, v2) > EPS) {
            P.push_back(P.back() + v1); i++;
        }
        else if (i == n1 || cross(v1, v2) < -EPS) {
            P.push_back(P.back() + v2); j++;
        }
        else {
            P.push_back(P.back() + (v1 + v2));
            i++; j++;
        }
    }
    P.pop_back();
    sort_lex_hull(P);
    return P;
}

```

7.9 Nearest Neighbour

```

// Closest Neighbor - O(n * log(n))
const ll N = 1e6+3, INF = 1e18;
ll n, cn[N], x[N], y[N]; // number of points, closes neighbor, x
                           // coordinates, y coordinates

ll sqr(ll i) { return i*i; }
ll dist(int i, int j) { return sqr(x[i]-x[j]) + sqr(y[i]-y[j]); }
ll dist(int i) { return i == cn[i] ? INF : dist(i, cn[i]); }

bool cpx(int i, int j) { return x[i] < x[j] or (x[i] == x[j] and
y[i] < y[j]); }
bool cpy(int i, int j) { return y[i] < y[j] or (y[i] == y[j] and
x[i] < x[j]); }

ll calc(int i, ll x0) {
    ll dlt = dist(i) - sqr(x[i]-x0);
    return dlt >= 0 ? ceil(sqrt(dlt)) : -1;
}

void updt(int i, int j, ll x0, ll &dlt) {
    if (dist(i) > dist(i, j)) cn[i] = j, dlt = calc(i, x0);
}

void cmp(vi &u, vi &v, ll x0) {
    for(int a=0, b=0; a<u.size(); ++a) {
        ll i = u[a], dlt = calc(i, x0);
        while(b < v.size() and y[i] > y[v[b]]) b++;
        for(int j = b-1; j >= 0 and y[i] - dlt <= y[v[j]]; j--)
            updt(i, v[j], x0, dlt);
        for(int j = b; j < v.size() and y[i] + dlt >= y[v[j]]; j++)
            updt(i, v[j], x0, dlt);
    }
}

void slv(vi &ix, vi &iy) {
    int n = ix.size();
    if (n == 1) { cn[ix[0]] = ix[0]; return; }

    int m = ix[n/2];

    vi ix1, ix2, iy1, iy2;
    for(int i=0; i<n; ++i) {
        if (cpx(ix[i], m)) ix1.push_back(ix[i]);
    }
}

```

```

else ix2.push_back(ix[i]);

if (cpx(iy[i], m)) iy1.push_back(iy[i]);
else iy2.push_back(iy[i]);
}

slv(ix1, iy1);
slv(ix2, iy2);

cmp(iy1, iy2, x[m]);
cmp(iy2, iy1, x[m]);
}

void slv(int n) {
    vi ix, iy;
    ix.resize(n);
    iy.resize(n);
    for(int i=0; i<n; ++i) ix[i] = iy[i] = i;
    sort(ix.begin(), ix.end(), cpx);
    sort(iy.begin(), iy.end(), cpy);
    slv(ix, iy);
}

//new change: <= 0 / >= 0 became < 0 / > 0 (yet to be tested)

```

7.10 Polygons

```

#include "basics.cpp"
#include "lines.cpp"

//Monotone chain O(nlog(n))
#define REMOVE_REDUNDANT
#define REMOVE_REDUNDANT
bool between(const point &a, const point &b, const point &c) {
    return (fabs(area_2(a,b,c)) < EPS && (a.x-b.x)*(c.x-b.x) <= 0 &&
        (a.y-b.y)*(c.y-b.y) <= 0);
}
#endif

void convex_hull(vector<point> &pts) {
    sort(pts.begin(), pts.end());
    pts.erase(unique(pts.begin(), pts.end(), pts.end()));
    vector<point> up, dn;
    for (int i = 0; i < pts.size(); i++) {
        while (up.size() > 1 && area_2(up[up.size()-2], up.back(),
pts[i]) > 0) up.pop_back();
        while (dn.size() > 1 && area_2(dn[dn.size()-2], dn.back(),
pts[i]) < 0) dn.pop_back();
        up.push_back(pts[i]);
        dn.push_back(pts[i]);
    }
    pts = dn;
    for (int i = (int) up.size() - 2; i >= 1; i--) pts.push_back(
up[i]);

#ifdef REMOVE_REDUNDANT
    if (pts.size() <= 2) return;
    dn.clear();
    dn.push_back(pts[0]);
    dn.push_back(pts[1]);
    for (int i = 2; i < pts.size(); i++) {
        if (between(dn[dn.size()-2], dn[dn.size()-1], pts[i])) dn.
pop_back();
        dn.push_back(pts[i]);
    }
    if (dn.size() >= 3 && between(dn.back(), dn[0], dn[1])) {
        dn[0] = dn.back();
        dn.pop_back();
    }
    pts = dn;
#endif

//avoid using long double for comparisons, change type and
remove division by 2
type compute_signed_area(const vector<point> &p) {
    type area = 0;
    for(int i = 0; i < p.size(); i++) {
        int j = (i+1) % p.size();
        area += p[i].x*p[j].y - p[j].x*p[i].y;
    }
    return area;
}

```

7.8 Minkowski Sum

```
ld compute_area(const vector<point> &p) {
    return fabs(compute_signed_area(p) / 2.0);
}

ld compute_perimeter(vector<point> &p) {
    ld per = 0;
    for(int i = 0; i < p.size(); i++) {
        int j = (i+1) % p.size();
        per += p[i].dist(p[j]);
    }
    return per;
}

//not tested
// TODO: test this code. This code has not been tested, please
// do it before proper use.
// http://codeforces.com/problemset/problem/975/E is a good
// problem for testing.
point compute_centroid(vector<point> &p) {
    point c(0,0);
    ld scale = 6.0 * compute_signed_area(p);
    for (int i = 0; i < p.size(); i++) {
        int j = (i+1) % p.size();
        c = c + (p[i].x*p[j].y - p[j].x*p[i].y);
    }
    return c / scale;
}

// TODO: test this code. This code has not been tested, please
// do it before proper use.
// http://codeforces.com/problemset/problem/975/E is a good
// problem for testing.
point centroid(vector<point> &v) {
    int n = v.size();
    type da = 0;
    point m, c;

    for(point p : v) m = m + p;
    m = m / n;

    for(int i=0; i<n; ++i) {
        point p = v[i] - m, q = v[(i+1)%n] - m;
        type x = p.x * q.y;
        c = c + (p + q) * x;
        da += x;
    }

    return c / (3 * da);
}

//O(n^2)
bool is_simple(const vector<point> &p) {
    for (int i = 0; i < p.size(); i++) {
        for (int k = i+1; k < p.size(); k++) {
            int j = (i+1) % p.size();
            int l = (k+1) % p.size();
            if (i == l || j == k) continue;
            if (segment_segment_intersect(p[i], p[j], p[k], p[l]))
                return false;
        }
    }
    return true;
}

bool point_in_triangle(point a, point b, point c, point cur){
    ll s1 = abs(cross(b - a, c - a));
    ll s2 = abs(cross(a - cur, b - cur)) + abs(cross(b - cur, c - cur)) + abs(cross(c - cur, a - cur));
    return s1 == s2;
}

void sort_lex_hull(vector<point> &hull){
    if(compute_signed_area(hull) < 0) reverse(hull.begin(), hull.end());
    int n = hull.size();

    //Sort hull by x
    int pos = 0;
    for(int i = 1; i < n; i++) if(hull[i].x < hull[pos].x) pos = i;
    rotate(hull.begin(), hull.begin() + pos, hull.end());
}

//determine if point is inside or on the boundary of a polygon (O(logn))
bool point_in_convex_polygon(vector<point> &hull, point cur){
    int n = hull.size();
    //Corner cases: point outside most left and most right wedges
    if(cur.dir(hull[0], hull[1]) != 0 && cur.dir(hull[0], hull[1])
```

```
!= hull[n - 1].dir(hull[0], hull[1]))
    return false;
if(cur.dir(hull[0], hull[n - 1]) != 0 && cur.dir(hull[0], hull[n - 1]) != hull[1].dir(hull[0], hull[n - 1]))
    return false;

//Binary search to find which wedges it is between
int l = 1, r = n - 1;
while(r - l > 1) {
    int mid = (l + r) / 2;
    if(cur.dir(hull[0], hull[mid]) <= 0) l = mid;
    else r = mid;
}
return point_in_triangle(hull[l], hull[l + 1], hull[0], cur);
}

// determine if point is on the boundary of a polygon (O(N))
bool point_on_polygon(vector<point> &p, point q) {
    for (int i = 0; i < p.size(); i++)
        if (q.dist2(project_point_segment(p[i], p[(i+1)%p.size()], q)) < EPS) return true;
    return false;
}

//Shamos - Hoey for test polygon simple in O(nlog(n))
inline bool adj(int a, int b, int n) {return (b == (a + 1)%n or a == (b + 1)%n);}

struct edge{
    point ini, fim;
    edge(point ini = point(0,0), point fim = point(0,0)) : ini(ini), fim(fim) {}
};

//< here means the edge on the top will be at the begin
bool operator < (const edge& a, const edge& b) {
    if (a.ini == b.ini) return direction(a.ini, a.fim, b.fim) < 0;
    if (a.ini.x < b.ini.x) return direction(a.ini, a.fim, b.ini) < 0;
    return direction(a.ini, b.fim, b.ini) < 0;
}

bool is_simple_polygon(const vector<point> &pts){
    vector <pair<point, pii>> eve;
    vector <pair<edge, int>> edgs;
    set <pair<edge, int>> sweep;
    int n = (int)pts.size();
    for(int i = 0; i < n; i++){
        point l = min(pts[i], pts[(i + 1)%n]);
        point r = max(pts[i], pts[(i + 1)%n]);
        eve.pb({l, {0, i}});
        eve.pb({r, {1, i}});
        edgs.pb(make_pair(edge(l, r), i));
    }
    sort(eve.begin(), eve.end());
    for(auto e : eve){
        if(!e.nd.st){
            auto cur = sweep.lower_bound(edgs[e.nd.nd]);
            pair<edge, int> above, below;
            if(cur != sweep.end()){
                below = *cur;
                if(!adj(below.nd, e.nd.nd, n) and segment_segment_intersect(pts[below.nd], pts[(below.nd + 1)%n], pts[e.nd.nd], pts[(e.nd.nd + 1)%n]))
                    return false;
            }
            if(cur != sweep.begin()){
                above = *(--cur);
                if(!adj(above.nd, e.nd.nd, n) and segment_segment_intersect(pts[above.nd], pts[(above.nd + 1)%n], pts[e.nd.nd], pts[(e.nd.nd + 1)%n]))
                    return false;
            }
            sweep.insert(edgs[e.nd.nd]);
        }
        else{
            auto below = sweep.upper_bound(edgs[e.nd.nd]);
            auto cur = below, above = --cur;
            if(below != sweep.end() and above != sweep.begin()){
                --above;
                if(!adj(below->nd, above->nd, n) and segment_segment_intersect(pts[below->nd], pts[(below->nd + 1)%n], pts[above->nd], pts[(above->nd + 1)%n]))
                    return false;
            }
            sweep.erase(cur);
        }
    }
}
```

```
return true;
}

// this code assumes that there are no 3 colinear points
int maximize_scalar_product(vector<point> &hull, point vec /*, int dir_flag*/) {
    /*
    For Minimize change: >= becomes <= and > becomes <
    For finding tangents, use same code passing direction flag
    dir_flag = -1 for right tangent
    dir_flag = 1 for left tangent
    >= or > becomes: == dir_flag
    < or <= becomes != dir_flag
    commentaries below for better clarification
    */
    int ans = 0;
    int n = hull.size();
    if(n < 20) {
        for(int i = 0; i < n; i++) {
            if(hull[i] * vec > hull[ans] * vec) {
                //hull[ans].dir(vec, hull[i]) == dir_flag
                ans = i;
            }
        }
    }
    else {
        if(hull[1] * vec > hull[ans] * vec) {
            //hull[ans].dir(vec, hull[i]) == dir_flag
            ans = 1;
        }
        for(int rep = 0; rep < 2; rep++) {
            int l = 2, r = n - 1;
            while(l != r) {
                int mid = (l + r + 1) / 2;
                bool flag = hull[mid] * vec >= hull[mid-1] * vec;
                //hull[ans].dir(vec, hull[l]) == dir_flag
                if(rep == 0) { flag = flag && hull[mid] * vec >= hull[0] * vec; }
                //hull[ans].dir(vec, hull[l]) == dir_flag
                else { flag = flag || hull[mid-1] * vec < hull[0] * vec; }
                //hull[ans].dir(vec, hull[l]) != dir_flag
                if(flag) {
                    l = mid;
                }
                else {
                    r = mid - 1;
                }
            }
            if(hull[l] * vec > hull[ans] * vec) {
                //hull[ans].dir(vec, hull[l]) == dir_flag
                ans = l;
            }
        }
    }
    return ans;
}
```

7.11 Ternary Search

```
//Ternary Search - O(log(n))
//Max version, for minimum version just change signals

ll ternary_search(ll l, ll r){
    while(r - l > 3) {
        ll m1 = (l+r)/2;
        ll m2 = (l+r)/2 + 1;
        ll f1 = f(m1), f2 = f(m2);
        //if(f1 > f2) l = m1;
        if (f1 < f2) l = m1;
        else r = m2;
    }
    ll ans = 0;
    for(int i = l; i <= r; i++){
        ll tmp = f(i);
        //ans = min(ans, tmp);
        ans = max(ans, tmp);
    }
    return ans;
}

//Faster version - 300 iterations up to 1e-6 precision
double ternary_search(double l, double r, int No = 300){
    // for(int i = 0; i < No; i++){
    while(r - l > EPS){
        double m1 = l + (r - l) / 3;
```

```
double m2 = r - (r - 1) / 3;
// if (f(m1) > f(m2))
if (f(m1) < f(m2))
    l = m1;
else
    r = m2;
}
return f(l);
}
```

7.12 Delaunay Triangulation

```
/*
Complexity: O(nlogn)
Code by Bruno Maletta (UFMG): https://github.com/brunomaletta/
Biblioteca

The definition of the Voronoi diagram immediately shows signs of
applications.
* Given a set S of n points and m query points p1,...,pm, we
can answer for each query point, its nearest neighbor in S.
This can be done in O((n+q)log(n+q)) offline by sweeping the
Voronoi diagram and query points.
Or it can be done online with persistent data structures.

* For each Delaunay triangle, its circumcircle does not
strictly contain any points in S. (In fact, you can also
consider this the defining property of Delaunay
triangulation)

* The number of Delaunay edges is at most 3n - 6, so there is
hope for an efficient construction.

* Each point p belongs to S is adjacent to its nearest
neighbor with a Delaunay edge.

* The Delaunay triangulation maximizes the minimum angle in
the triangles among all possible triangulations.

* The Euclidean minimum spanning tree is a subset of Delaunay
edges.
*/

#include "basics.cpp"

bool ccw(point a, point b, point c) { return area_2(a, b, c) > 0; }

typedef struct QuadEdge* Q;
struct QuadEdge {
    int id;
    point o;
    Q rot, nxt;
    bool used;

    QuadEdge(int id_ = -1, point o_ = point(INF, INF)) :
        id(id_), o(o_), rot(nullptr), nxt(nullptr), used(false) {}

    Q rev() const { return rot->rot; }
    Q next() const { return nxt; }
    Q prev() const { return rot->next()->rot; }
    point dest() const { return rev()->o; }
};

Q edge(point from, point to, int id_from, int id_to) {
    Q e1 = new QuadEdge(id_from, from);
    Q e2 = new QuadEdge(id_to, to);
    Q e3 = new QuadEdge;
    Q e4 = new QuadEdge;
    tie(e1->rot, e2->rot, e3->rot, e4->rot) = {e3, e4, e2, e1};
    tie(e1->nxt, e2->nxt, e3->nxt, e4->nxt) = {e1, e2, e4, e3};
    return e1;
}

void splice(Q a, Q b) {
    swap(a->nxt->rot->nxt, b->nxt->rot->nxt);
    swap(a->nxt, b->nxt);
}

void del_edge(Q& e, Q ne) { // delete e and assign e <- ne
    splice(e, e->prev());
    splice(e->rev(), e->rev()->prev());
    delete e->rev()->rot, delete e->rev();
}
```

```
delete e->rot; delete e;
e = ne;
}

Q conn(Q a, Q b) {
    Q e = edge(a->dest(), b->o, a->rev()->id, b->id);
    splice(e, a->rev()->prev());
    splice(e->rev(), b);
    return e;
}

bool in_c(point a, point b, point c, point p) { // p ta na
    circumf. (a, b, c) ?
    type p2 = p*p, A = a*a - p2, B = b*b - p2, C = c*c - p2;
    return area_2(p, a, b) * C + area_2(p, b, c) * A + area_2(p, c
        , a) * B > 0;
}

pair<Q, Q> build_tr(vector<point>& p, int l, int r) {
    if (r-l+1 <= 3) {
        Q a = edge(p[l], p[l+1], l, l+1), b = edge(p[l+1], p[r], l
            +1, r);
        if (r-l+1 == 2) return {a, a->rev()};
        splice(a->rev(), b);
        type ar = area_2(p[l], p[l+1], p[r]);
        Q c = ar ? conn(b, a) : 0;
        if (ar >= 0) return {a, b->rev()};
        return {c->rev(), c};
    }
    int m = (l+r)/2;
    auto [la, ra] = build_tr(p, l, m);
    auto [lb, rb] = build_tr(p, m+1, r);
    while (true) {
        if (ccw(lb->o, ra->o, ra->dest())) ra = ra->rev()->prev();
        else if (ccw(lb->o, ra->o, lb->dest())) lb = lb->rev()->next
            ();
        else break;
    }
    Q b = conn(lb->rev(), ra);
    auto valid = [&](Q e) { return ccw(e->dest(), b->dest(), b->o)
        ; };
    if (ra->o == la->o) la = b->rev();
    if (lb->o == rb->o) rb = b;
    while (true) {
        Q L = b->rev()->next();
        if (valid(L)) while (in_c(b->dest(), b->o, L->dest(), L->
            next()->dest()))
            del_edge(L, L->next());
        Q R = b->prev();
        if (valid(R)) while (in_c(b->dest(), b->o, R->dest(), R->
            prev()->dest()))
            del_edge(R, R->prev());
        if (!valid(L) and !valid(R)) break;
        if (!valid(L) or (valid(R) and in_c(L->dest(), L->o, R->o, R->
            dest()))
            b = conn(R, b->rev());
        else b = conn(b->rev(), L->rev());
    }
    return {la, rb};
}

//NOTE: Before calculating Delaunay add a bound triangle: (-INF,
// -INF), (INF, INF), (0, INF)
vector<vector<int>> delaunay(vector<point> v) {
    int n = v.size();
    auto tmp = v;
    vector<int> idx(n);
    iota(idx.begin(), idx.end(), 0);
    sort(idx.begin(), idx.end(), [&](int l, int r) { return v[l] <
        v[r]; });
    for (int i = 0; i < n; i++) v[i] = tmp[idx[i]];
    assert(unique(v.begin(), v.end()) == v.end());
    vector<vector<int>> g(n);
    bool col = true;
    for (int i = 2; i < n; i++) if (area_2(v[i], v[i-1], v[i-2]))
        col = false;
    if (col) {
        for (int i = 1; i < n; i++)
            g[idx[i-1]].push_back(idx[i]), g[idx[i]].push_back(idx[i
                -1]);
        return g;
    }
    Q e = build_tr(v, 0, n-1).first;
    vector<Q> edg = {e};
    for (int i = 0; i < edg.size(); i = edg[i++]) {
        for (Q at = e; !at->used; at = at->next()) {
            at->used = true;
            g[idx[at->id]].push_back(idx[at->rev()->id]);
            edg.push_back(at->prev());
        }
    }
}
```

```

    }
    return g;
}

vector<vector<point>> voronoi(const vector<point>& points, const
    vector<point>& delaunay) {
    int n = delaunay.size();
    vector<vector<point>> voronoi(n, vector<point>());
    for (int i = 0; i < n; i++) {
        for (int d = 0; d < delaunay[i].size(); d++) {
            int j = delaunay[i][d], k = delaunay[i][(d + 1) %
                delaunay[i].size()];
            circle c = circumcircle(points[i], points[j], points[k
                ]);
            voronoi[i].push_back(c.c);
            voronoi[j].push_back(c.c);
            voronoi[k].push_back(c.c);
        }
    }
}
```

7.13 Closest Pair of Points 3D

```
#include <bits/stdc++.h>

using namespace std;

#define st first
#define nd second

typedef long long ll;
typedef long double ld;
typedef pair<ll,ll> pll;

const ld EPS = 1e-9, PI = acos(-1.);
const ll LINF = 0x3f3f3f3f3f3f3f3f;
const int N = 1e5+5;

typedef long long type;

struct point {
    type x, y, z;

    point() : x(0), y(0), z(0) {}
    point(type _x, type _y, type _z) : x(_x), y(_y), z(_z) {}

    point operator -() const { return point(-x, -y, -z); }
    point operator +(point p) const { return point(x + p.x, y + p.y, z +
        p.z); }
    point operator -(point p) const { return point(x - p.x, y - p.y, z -
        p.z); }

    point operator *(type k) const { return point(x*k, y*k, z*k); }
    point operator /(type k) const { return point(x/k, y/k, z/k); }

    bool operator ==(const point &p) const { return x == p.x and y
        == p.y and z == p.z; }
    bool operator !=(const point &p) const { return x != p.x or y
        != p.y or z != p.z; }
    bool operator <(const point &p) const { return (z < p.z) or (z
        == p.z and y < p.y) or (z == p.z and y == p.y and x < p.
        x); }

    type abs2() const { return x*x + y*y + z*z; }
    type dist2(point q) const { return (*this - q).abs2(); }
};

ll cfloor(ll a, ll b) {
    ll c = abs(a);
    ll d = abs(b);
    if (a * b > 0) return c/d;
    return -(c + d - 1)/d;
}

ll min_dist = LINF;
pair<int, int> best_pair;
vector<point> pts;
int n;

//Warning: include variable id into the struct point
void upd_ans(const point &a, const point &b) {
    ll dist = (a.x - b.x)*(a.x - b.x) + (a.y - b.y)*(a.y - b.y) +
        (a.z - b.z)*(a.z - b.z);
}
```

```

    if (dist < min_dist) {
        min_dist = dist;
        // best_pair = {a.id, b.id};
    }
}

void closest_pair(int l, int r) {
    if (r - l <= 3) {
        for (int i = l; i < r; ++i) {
            for (int j = i + 1; j < r; ++j) {
                upd_ans(pts[i], pts[j]);
            }
        }
        return;
    }

    int m = (l + r) >> 1;
    type midz = pts[m].z;
    closest_pair(l, m);
    closest_pair(m, r);

    //map opposite side
    map<pll, vector<int>>> f;
    for(int i = m; i < r; i++){
        f[{cfloor(pts[i].x, min_dist), cfloor(pts[i].y, min_dist)}].
            push_back(i);
    }
    //find
    for(int i = l; i < m; i++){
        if((midz - pts[i].z) * (midz - pts[i].z) >= min_dist)
            continue;

        pll cur = {cfloor(pts[i].x, min_dist), cfloor(pts[i].y,
            min_dist)};
        for(int dx = -1; dx <= 1; dx++){
            for(int dy = -1; dy <= 1; dy++){
                for(auto p : f[{cur.st + dx, cur.nd + dy}])
                    min_dist = min(min_dist, pts[i].dist2(pts[p]));
            }
        }
    }

    int main(){
        ios_base::sync_with_stdio(false);
        cin.tie(NULL);
        cin >> n;
        pts.resize(n);
        for(int i = 0; i < n; i++) cin >> pts[i].x >> pts[i].y >> pts[i].z;
        sort(pts.begin(), pts.end());
        closest_pair(0, n);
        cout << setprecision(15) << fixed << sqrt((ld)min_dist) << "\n";
        return 0;
    }
}

```

8 Miscellaneous

8.1 Bitset

```

//Goes through the subsets of a set x :
int b = 0;
do {
    // process subset b
} while (b=(b-x)&x);

```

8.2 builtin

```

__builtin_ctz(x) // trailing zeroes
__builtin_clz(x) // leading zeroes
__builtin_popcount(x) // # bits set
__builtin_ffs(x) // index(LSB) + 1 [0 if x==0]

// Add ll to the end for long long [__builtin_clzll(x)]

```

8.3 Date

```

struct Date {
    int d, m, y;
    static int mnt[], mntsum[];

    Date() : d(1), m(1), y(1) {}
    Date(int d, int m, int y) : d(d), m(m), y(y) {}
    Date(int days) : d(1), m(1), y(1) { advance(days); }

    bool bissext() { return (y%4 == 0 and y%100) or (y%400 == 0); }

    int mdays() { return mnt[m] + (m == 2)*bissext(); }
    int ydays() { return 365+bissext(); }

    int msum() { return mntsum[m-1] + (m > 2)*bissext(); }
    int ysum() { return 365*(y-1) + (y-1)/4 - (y-1)/100 + (y-1)/400; }

    int count() { return (d-1) + msum() + ysum(); }

    int day() {
        int x = y - (m<3);
        return (x + x/4 - x/100 + x/400 + mntsum[m-1] + d + 6)%7;
    }

    void advance(int days) {
        days += count();
        d = m = 1, y = 1 + days/366;
        days -= count();
        while(days >= ydays()) days -= ydays(), y++;
        while(days >= mdays()) days -= mdays(), m++;
        d += days;
    }
};

int Date::mnt[13] = {0, 31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30, 31};
int Date::mntsum[13] = {};
for(int i=1; i<13; ++i) Date::mntsum[i] = Date::mntsum[i-1] + Date::mnt[i];

```

8.4 Parenthesis to Polish (ITA)

```

#include <cstdio>
#include <map>
#include <stack>
using namespace std;

/*
 * Parenthetic to polish expression conversion
 */

inline bool isOp(char c) {
    return c=='+' || c=='-' || c=='*' || c=='/' || c=='^';
}

inline bool isCarac(char c) {
    return (c>='a' && c<='z') || (c>='A' && c<='Z') || (c>='0' && c<='9');
}

int paren2polish(char* paren, char* polish) {
    map<char, int> prec;
    prec['('] = 0;
    prec['+'] = prec['-'] = 1;
    prec['*'] = prec['/'] = 2;
    prec['^'] = 3;
    int len = 0;
    stack<char> op;
    for (int i = 0; paren[i]; i++) {
        if (isOp(paren[i])) {
            while (!op.empty() && prec[op.top()] >= prec[paren[i]]) {
                polish[len++] = op.top(); op.pop();
            }
            op.push(paren[i]);
        }
        else if (paren[i]=='(') op.push('(');
        else if (paren[i]==')') {
            for (; op.top()!='('; op.pop())
                polish[len++] = op.top();
            op.pop();
        }
    }
}

```

```

    }
    else if (isCarac(paren[i]))
        polish[len++] = paren[i];
    }
    for(; !op.empty(); op.pop())
        polish[len++] = op.top();
    polish[len] = 0;
    return len;
}

/*
 * TEST MATRIX
 */

int main() {
    int N, len;
    char polish[400], paren[400];
    scanf("%d", &N);
    for (int j=0; j<N; j++) {
        scanf("%s", paren);
        paren2polish(paren, polish);
        printf("%s\n", polish);
    }
    return 0;
}

```

8.5 Modular Int (Struct)

```

// Struct to do basic modular arithmetic

template <int MOD>
struct Modular {
    int v;

    static int minv(int a, int m) {
        a %= m;
        assert(a);
        return a == 1 ? 1 : int(m - 1l(minv(m, a)) * 1l(m) / a);
    }

    Modular(1l _v = 0) : v(int(_v % MOD)) {
        if (v < 0) v += MOD;
    }

    bool operator==(const Modular& b) const { return v == b.v; }
    bool operator!=(const Modular& b) const { return v != b.v; }

    friend Modular inv(const Modular& b) { return Modular(minv(b.v, MOD)); }

    friend ostream& operator<<(ostream& os, const Modular& b) {
        return os << b.v; }
    friend istream& operator>>(istream& is, Modular& b) {
        1l _v;
        is >> _v;
        b = Modular(_v);
        return is;
    }

    Modular operator+(const Modular& b) const {
        Modular ans;
        ans.v = v >= MOD - b.v ? v + b.v - MOD : v + b.v;
        return ans;
    }

    Modular operator-(const Modular& b) const {
        Modular ans;
        ans.v = v < b.v ? v - b.v + MOD : v - b.v;
        return ans;
    }

    Modular operator*(const Modular& b) const {
        Modular ans;
        ans.v = int(1l(v) * 1l(b.v) % MOD);
        return ans;
    }

    Modular operator/(const Modular& b) const {
        return (*this) * inv(b);
    }

    Modular& operator+=(const Modular& b) { return *this = *this + b; }
    Modular& operator-=(const Modular& b) { return *this = *this - b; }
}

```



```

Modular& operator*=(const Modular& b) { return *this = *this *
    b; }
Modular& operator/=(const Modular& b) { return *this = *this /
    b; }
};

using Mint = Modular<MOD>;

```

8.6 Parallel Binary Search

```

// Parallel Binary Search - O(nlog n * cost to update data
// structure + qlog n * cost for binary search condition)

struct Query { int l, ans; /*+ query related info*/ };
vector<Query> req;

void pbs(vector<Query>& qs, int l /*= min value*/, int r /*=
    max value*/) {
    if (qs.empty()) return;

    if (l == r) {
        for (auto& q : qs) req[q.l].ans = l;
        return;
    }

    int mid = (l + r) / 2;
    // mid = (l + r + 1) / 2 if different from simple upper/lower
    // bound

    for (int i = l; i <= mid; i++) {
        // add value to data structure
    }

    vector<Query> vl, vr;
    for (auto& q : qs) {
        if (/* cond */) vl.push_back(q);
        else vr.push_back(q);
    }

    pbs(vr, mid + 1, r);

    for (int i = l; i <= mid; i++) {
        // remove value from data structure
    }

    pbs(vl, l, mid);
}

```

8.7 prime numbers

	2	3	5	7	11	13	17	19	23	29
127	131	137	139	149	151	157	163	167	173	
179	181	191	193	197	199	211	223	227	229	
233	239	241	251	257	263	269	271	277	281	
283	293	307	311	313	317	331	337	347	349	
353	359	367	373	379	383	389	397	401	409	
419	421	431	433	439	443	449	457	461	463	
467	479	487	491	499	503	509	521	523	541	
547	557	563	569	571	577	587	593	599	601	
607	613	617	619	631	641	643	647	653	659	
661	673	677	683	691	701	709	719	727	733	
739	743	751	757	761	769	773	787	797	809	
811	821	823	827	829	839	853	857	859	863	
877	881	883	887	907	911	919	929	937	941	
947	953	967	971	977	983	991	997	1009	1013	
1019	1021	1031	1033	1039	1049	1051	1061	1063	1069	
1087	1091	1093	1097	1103	1109	1117	1123	1129	1151	
1153	1163	1171	1181	1187	1193	1201	1213	1217	1223	
1229	1231	1237	1249	1259	1277	1279	1283	1289	1291	
1297	1301	1303	1307	1319	1321	1327	1361	1367	1373	
1381	1399	1409	1423	1427	1429	1433	1439	1447	1451	
1453	1459	1471	1481	1483	1487	1489	1493	1499	1511	
1523	1531	1543	1549	1553	1559	1567	1571	1579	1583	
1597	1601	1607	1609	1613	1619	1621	1627	1637	1657	
1663	1667	1669	1693	1697	1699	1709	1721	1723	1733	
1741	1747	1753	1759	1777	1783	1787	1789	1801	1811	
1823	1831	1847	1861	1867	1871	1873	1877	1879	1889	
1901	1907	1913	1931	1933	1949	1951	1973	1979	1987	

```

970'997      971'483      921'281'269      999'279'733
1'000'000'009 1'000'000'021 1'000'000'409 1'005'012'527

```

8.8 Python

```

# reopen
import sys
sys.stdout = open('out', 'w')
sys.stdin = open('in', 'r')

//Dummy example
R = lambda: map(int, input().split())
n, k = R(),
v, t = [], [0]*n
for p, c, i in sorted(zip(R(), R(), range(n))):
    t[i] = sum(v)+c
    v += [c]
    v = sorted(v)[:n-1]
    if len(v) > k:
        v.pop()
print(' '.join(map(str, t)))

```

8.9 Sqrt Decomposition

```

// Square Root Decomposition (Mo's Algorithm) - O(n^(3/2))
const int N = 1e5+1, SQ = 500;
int n, m, v[N];

void add(int p) { /* add value to aggregated data structure */ }
void rem(int p) { /* remove value from aggregated data structure
    */ }

struct query { int l, r, ans; } qs[N];

bool c1(query a, query b) {
    if (a.l/SQ != b.l/SQ) return a.l < b.l;
    return a.l/SQ & 1 ? a.r > b.r : a.r < b.r;
}

bool c2(query a, query b) { return a.i < b.i; }

/* inside main */
int l = 0, r = -1;
sort(qs, qs+m, c1);
for (int i = 0; i < m; ++i) {
    query &q = qs[i];
    while (r < q.r) add(v[++r]);
    while (r > q.r) rem(v[r--]);
    while (l < q.l) rem(v[l--]);
    while (l > q.l) add(v[--l]);

    q.ans = /* calculate answer */;
}

sort(qs, qs+m, c2); // sort to original order

```

8.10 Latitude Longitude (Stanford)

```

/*
Converts from rectangular coordinates to latitude/longitude and
vice versa. Uses degrees (not radians).
*/

#include <iostream>
#include <cmath>

using namespace std;

struct ll
{
    double r, lat, lon;
};

```

```

struct rect
{
    double x, y, z;
};

ll convert(rect& P)
{
    ll Q;
    Q.r = sqrt(P.x*P.x+P.y*P.y+P.z*P.z);
    Q.lat = 180/M_PI*asin(P.z/Q.r);
    Q.lon = 180/M_PI*acos(P.x/sqrt(P.x*P.x+P.y*P.y));

    return Q;
}

rect convert(ll& Q)
{
    rect P;
    P.x = Q.r*cos(Q.lon*M_PI/180)*cos(Q.lat*M_PI/180);
    P.y = Q.r*sin(Q.lon*M_PI/180)*cos(Q.lat*M_PI/180);
    P.z = Q.r*sin(Q.lat*M_PI/180);

    return P;
}

int main()
{
    rect A;
    ll B;

    A.x = -1.0; A.y = 2.0; A.z = -3.0;

    B = convert(A);
    cout << B.r << " " << B.lat << " " << B.lon << endl;

    A = convert(B);
    cout << A.x << " " << A.y << " " << A.z << endl;
}

```

8.11 Week day

```

int v[] = { 0, 3, 2, 5, 0, 3, 5, 1, 4, 6, 2, 4 };
int day(int d, int m, int y) {
    y -= m<3;
    return (y + y/4 - y/100 + y/400 + v[m-1] + d)%7;
}

```

9 Math Extra

9.1 Combinatorial formulas

$$\begin{aligned}
 \sum_{k=0}^n k^2 &= n(n+1)(2n+1)/6 \\
 \sum_{k=0}^n k^3 &= n^2(n+1)^2/4 \\
 \sum_{k=0}^n k^4 &= (6n^5 + 15n^4 + 10n^3 - n)/30 \\
 \sum_{k=0}^n k^5 &= (2n^6 + 6n^5 + 5n^4 - n^2)/12 \\
 \sum_{k=0}^n x^k &= (x^{n+1} - 1)/(x - 1) \\
 \sum_{k=0}^n kx^k &= (x - (n+1)x^{n+1} + nx^{n+2})/(x - 1)^2 \\
 \binom{n}{k} &= \frac{n!}{(n-k)!k!} \\
 \binom{n}{k} &= \binom{n-1}{k} + \binom{n-1}{k-1} \\
 \binom{n}{k} &= \frac{n}{n-k} \binom{n-1}{k} \\
 \binom{n}{k} &= \frac{n-k+1}{k} \binom{n}{k-1} \\
 \binom{n+1}{k} &= \frac{n+1}{n-k+1} \binom{n}{k} \\
 \binom{n}{k+1} &= \frac{n-k}{k+1} \binom{n}{k}
 \end{aligned}$$

$$\begin{aligned}\sum_{k=1}^n k \binom{n}{k} &= n2^{n-1} \\ \sum_{k=1}^n k^2 \binom{n}{k} &= (n + n^2)2^{n-2} \\ \binom{m+n}{r} &= \sum_{k=0}^r \binom{m}{k} \binom{n}{r-k} \\ \binom{n}{k} &= \prod_{i=1}^k \frac{n-k+i}{i}\end{aligned}$$

9.2 Number theory identities

Lucas' Theorem: For non-negative integers m and n and a prime p ,

$$\binom{m}{n} \equiv \prod_{i=0}^k \binom{m_i}{n_i} \pmod{p},$$

where

$$m = m_k p^k + m_{k-1} p^{k-1} + \cdots + m_1 p + m_0$$

is the base p representation of m , and similarly for n .

9.3 Stirling Numbers of the second kind

Number of ways to partition a set of n numbers into k non-empty subsets.

$$\left\{ \begin{matrix} n \\ k \end{matrix} \right\} = \frac{1}{k!} \sum_{j=0}^k (-1)^{(k-j)} \binom{k}{j} j^n$$

Recurrence relation:

$$\begin{aligned}\left\{ \begin{matrix} 0 \\ 0 \end{matrix} \right\} &= 1 \\ \left\{ \begin{matrix} n \\ 0 \end{matrix} \right\} &= \left\{ \begin{matrix} 0 \\ n \end{matrix} \right\} = 1 \\ \left\{ \begin{matrix} n+1 \\ k \end{matrix} \right\} &= k \left\{ \begin{matrix} n \\ k \end{matrix} \right\} + \left\{ \begin{matrix} n \\ k-1 \end{matrix} \right\}\end{aligned}$$

9.4 Burnside's Lemma

Let G be a finite group that acts on a set X . For each g in G let X^g denote the set of elements in X that are

fixed by g , which means $X^g = \{x \in X | g(x) = x\}$. Burnside's lemma asserts the following formula for the number of orbits, denoted $|X/G|$:

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

9.5 Numerical integration

RK4: to integrate $\dot{y} = f(t, y)$ with $y_0 = y(t_0)$, compute

$$\begin{aligned}k_1 &= f(t_n, y_n) \\ k_2 &= f(t_n + \frac{h}{2}, y_n + \frac{h}{2} k_1) \\ k_3 &= f(t_n + \frac{h}{2}, y_n + \frac{h}{2} k_2) \\ k_4 &= f(t_n + h, y_n + h k_3) \\ y_{n+1} &= y_n + \frac{h}{6} (k_1 + 2k_2 + 2k_3 + k_4)\end{aligned}$$