Design and Implementation of Sector Rotation Trading Strategy in Hong Kong Stock Market *

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Abstract

We design and implement a sector rotation trading strategy in Hong Kong Stock Market. Strategically, we rotate the focus of our investment from one industry sector to another, according to the fluctuations of the business cycle. The core of our strategy is (1) to detect the stage of the cycle, and (2) to quantify descriptive recommendations such as *strong buy*, *neutral* and *sell*.

Our strategy, which trades quarterly, yields a total return of 297.0% in the investment horizon, which spans from Q1 2009 to Q4 2016, compared to 113.0% for Hang Seng Index (HSI) for the same period. The annualized return is 19.5%, compared to 10.2% for HSI. The annualized volatility is 30.3%, compared to 21.2% for HSI. The maximum drawdown is -31.6%, compared to -23.3% for HSI.

We apply Principal Component Analysis in selecting the stocks, Hidden Markov Model to detect the stage of the business cycle, and employ Monte Carlo methods, with Gibbs sampling ideas, for optimization. We also discuss using statistical techniques, such as skewness, kurtosis, and variance ratio tests, for fine-tuning our strategy.

1 Introduction

A stock market has many sectors. Individual stocks in one sector tend to follow a collective trend that is unique to that sector. At certain times, some sectors are better than others, but not always. These shiny sectors may lose their luster one day, and those that are temporarily out of favor may be in favor again. This turn of fate is predictable to some extent, and closely related to the fluctuations of the business cycle.

These assumptions constitute the premise of *sector rotation*, a trading strategy that dynamically shifts the focus of investment from one sector to another, according to the changing state of the economy.

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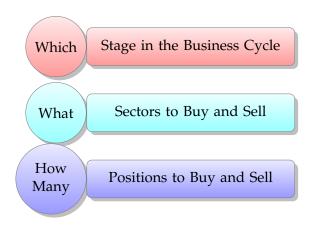
Investment theory has offered clear-cut, buy-or-sell recommendations for a sector given the stage it is in a business cycle. For example, *strong sell* for energy in the early stage of expansion, and *strong buy* for consumer staples in the stage of recession. These recommendations are consensual to a large degree, because they are backed by well-grounded economic theories.

In theory, sector rotation seems like a readily profitable strategy if we can accurately tell and foresee the fluctuations of the business cycle. In practice, however, two fundamental questions have to be answered in order to implement a sector rotation trading strategy:

- 1. How to exactly tell which stage of the business cycle we are in, given observable economic indicators?
- 2. How to quantify the descriptive recommendations for investment, such as *strong buy, neutral,* and *sell*?

It is the core of our trading strategy to these two questions. To put it simply, our corresponding answers are:

- 1. Use Hidden Markov Model to detect the stage of the business cycle;
- 2. Use Monte Carlo methods, with Gibbs sampling ideas, to optimize a multiplier matrix so as to quantify the descriptive recommendations.



A sector rotation trading strategy should properly answer the three questions above. Fortunately, investment theory has provided classic answers to Question 2 — what sectors to buy and sell, given the stage in the cycle. So we focus on answering Questions 1 & 3 — which stage in the business cycle, and how many positions to buy and sell.

In what follows, we first report the results, since for a trading strategy, *all's bad that ends bad*. Next we outline the economic rationale that justifies the strategy. A framework is then set up, under which we discuss the implementation in detail. We conclude our report with a comparison and some final comments.

2 Results

We report major results of our trading strategy below.

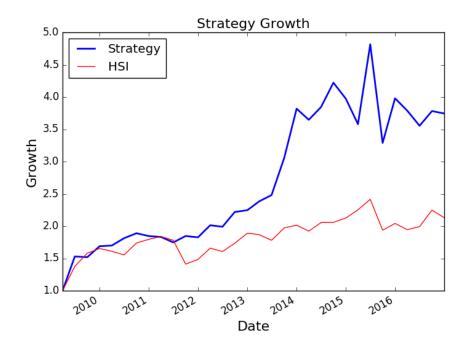
Investment Horizon: Q1 2009 — Q4 2016
 Out-of-Sample Period: Q1 2013 — Q4 2016

• Trading Frequency: Quarterly

• Number of Stocks in Portfolio: 172

Let *S* stand for our trading strategy, *HSI* for the total return of Hang Seng Index, *out* for out-of-sample period, *gross* for gross-of-fees, and *net* for net-of-fees.

	S (gross)	S (net)	HSI	S (out, gross)	S (out, net)	HSI (out)
Total Return	297.0 %	274.4 %	113.0 %	61.6 %	57.1 %	14.0 %
Annual Return	19.5 %	18.6 %	10.2 %	13.6 %	12.8 %	3.6 %
Annual Vol	30.3 %	30.3 %	21.2 %	34.6 %	34.5 %	16.8 %
Return / Vol	75.2 %	72.3 %	57.8 %	56.1 %	53.8 %	30.5 %
Max Drawdown	-31.6 %	-31.7 %	-23.3 %	-31.6 %	-31.7 %	-19.8~%



Note that the fee structure in the Hong Kong stock market is

1. **Brokers Commission**: 0.08% of trade value;

2. Exchange Fee: 0.005% trade value + HKD 0.50 per trade;

3. Clearing Fee: 0.002% trade value, with

• minimum HKD 2.00 per trade, and

• maximum HKD 100 per trade;

4. Stamp Duty: 0.1% of trade value;

5. **SFC Transaction Levy**: 0.0027% of trade value.

3 Economic Rationale

Over the intermediate term, asset performance is often driven largely by cyclical factors tied to the state of the economy, such as corporate earnings, interest rates, and inflation. The business cycle, which encompasses the cyclical fluctuations in an economy over many months or a few years, can therefore be a critical determinant of equity market returns and the performance of equity sectors. ¹

Business Cycle Every business cycle is unique in its own way, but certain patterns tend to repeat themselves over time. It is a widely accepted practice to divide a business cycle into four stages:

- 1. early expansion;
- 2. mid expansion;
- 3. late expansion;
- 4. recession.

Industry Sectors The GICS ² categorizes all major public companies into 11 sectors — Energy, Materials, Industrials, Consumer Discretionary, Consumer Staples, Health Care, Financials, Information Technology, Telecommunication Services, Utilities, Real Estate.

By and large, the industry sectors can be divided into two categories:

- 1. the economically sensitive, and
- 2. the more defensive.

The former performs well in the early stage of expansion, with information technology and industrials being two examples, whereas the latter enjoys higher returns during a recession — for instance, telecommunication services and utilities.

The links between business cycle and industry sectors are straightforward. For one example, the energy sector performs poorly in the early stage of expansion. With inflation at low level, energy companies cannot charge its clients good prices. For another example, the consumer staples sector (food, beverage, household items, etc.) performs relatively well during a recession, because no matter how badly off consumers are, they have to eat, drink, and take baths.

Approach to Sector Rotation Sector rotation takes advantages of relative sector performance opportunities. As the probability of a shift in stage increases — for instance, from mid expansion to late expansion — such a strategy allows investors to adjust their exposure to sectors that have prominent performance patterns in the next stage of the cycle.

¹ Emsbo-Mattingly et al. The Business Cycle Approach to Equity Sector Investing. Fidelity (2014).

² The Global Industry Classification Standard (GICS) is an industry taxonomy developed by MSCI and Standard & Poor's (S&P). It is widely used by the global financial community.

Sector	Early	Mid	Late	Recess	sion
Energy					
Materials					
Industrials					
Consumer Discretionary					
Consumer Staples					
Health Care					
Financials					
Information Technology					
Telecommunication Services					
Utilities					
Real Estate					

A colored matrix summarizing buy-or-sell recommendations that investment theory gives for a sector given the stage it is in a business cycle. ³ A red cell stands for strong sell, orange for sell, yellow for neutral, light green for buy, and dark green for strong buy.

4 Framework

matrix of multipliers To begin with, we convert the descriptive recommendations dictated by investment theory to a matrix T as follows.

	early exp.	mid exp.	late exp.	recession
Energy	$ hilde{ heta}_{SS}$	$ heta_N$	$ heta_{SB}$	$ heta_N$
Materials	θ_N	$ heta_{SS}$	$ heta_{SB}$	θ_S
Industrials	$ heta_{SB}$	$ heta_B$	$ heta_N$	$ heta_{SS}$
Consumer Discretionary	$ heta_{SB}$	$ heta_N$	$ heta_{SS}$	θ_N
Consumer Staples	$ heta_S$	$ heta_N$	$ heta_B$	$ heta_{SB}$
Health Care	θ_B	$ heta_N$	$ heta_{SB}$	$ heta_{SB}$
Financials	$ heta_B$	$ heta_N$	$ heta_N$	θ_S
Information Technology	$ heta_B$	$ heta_B$	$ heta_{SS}$	$ heta_{SS}$
Telecomm. Services	$ heta_{SS}$	$ heta_N$	$ heta_N$	$ heta_{SB}$
Utilities	$ heta_{SS}$	$ heta_S$	$ heta_B$	$ heta_{SB}$
Real Estate	θ_{SB}	$ heta_N$	$ heta_B$	$ heta_{SS}$

The elements of matrix $T - \theta_{SS}$, θ_S , θ_S , θ_B , θ_{SB} are multipliers corresponding to *strong sell*, *sell*, *neutral*, *buy*, and *strong buy* respectively.

To illustrate, suppose a *strong sell* recommendation at $\theta_{SS} = 1/3$. It means to sell 2/3 and retain only 1/3 of the current position.

At this moment, these θ s are parameters to be optimized. To stay in line with the investment theory, we subject them to the constraints

$$0 < \theta_{SS} < \theta_S < \theta_N \equiv 1 < \theta_B < \theta_{SB} < +\infty$$
 .

³ extracted from multiple sources from Fidelity

stage of the cycle We use a vector α_t to express the stage of the business cycle at time t.

$$\alpha_t = \begin{bmatrix} \text{early exp.} & \alpha_{1,t} \\ \text{mid exp.} & \alpha_{2,t} \\ \text{late exp.} & \alpha_{3,t} \\ \text{recession} & \alpha_{4,t} \end{bmatrix}$$

In a more general setting, we only require $\alpha_{i,t} \geq 0$ and that $\sum_i \alpha_{i,t} = 1$, which represents a probabilistic view on the current economic stage. For example, $\alpha_t = (0.8, 0.2, 0, 0)^{T}$ indicates 80% chance of early expansion and 20% chance of mid expansion.

In our simplified setting, we demand that α_t take only 0-or-1 values corresponding to a definite stage. That is, for early expansion, $\alpha_t = \epsilon_1 \equiv (1, 0, 0, 0)^{\mathsf{T}}$, for mid expansion, $\alpha_t = \epsilon_2 \equiv (0, 1, 0, 0)^{\mathsf{T}}$, and so on.

multipliers for the stage Given stage α_t at time t and the multiplier matrix T, we may work out the multipliers θ_t for N sectors at the stage. That is,

$$m{T}m{lpha}_t = m{ heta}_t \equiv egin{bmatrix} m{ heta}_t^{(1)} \ dots \ m{ heta}_t^{(N)} \end{bmatrix}$$

market capitalization Let $\kappa_{i,t}^{(j)}$, $S_{i,t}^{(j)}$, $F_{i,t}^{(j)}$ respectively stand for the market capitalization, stock price, and number of free floats for the *i*th stock in the *j*th sector at time *t*. We have

$$\kappa_{i,t}^{(j)} = S_{i,t}^{(j)} F_{i,t}^{(j)}$$

weights Let $w_{i,t}^{(j)}$ stand for the weight for the *i*th stock in the *j*th sector at time *t*. If an index/trading strategy is purely cap-weighted, we should observe

$$w_{i,t}^{(j)} \propto \kappa_{i,t}^{(j)}$$

Our trading strategy uses market cap $\kappa_{i,t}^{(j)}$ as the starting point for weighting a stock, and adjusts the weight $w_{i,t}^{(j)}$ by multiplying a multiplier $\theta_t^{(j)}$, which is jointly determined by the economic stage and the industry sector that the stock is in.

$$\boxed{w_{i,t}^{(j)} \propto \theta_t^{(j)} \kappa_{i,t}^{(j)}}$$

To write explicitly, we have

$$w_{i,t}^{(j)} = rac{ heta_t^{(j)} \kappa_{i,t}^{(j)}}{\sum\limits_{j=1}^{N} \sum\limits_{i=1}^{m(j)} heta_t^{(j)} \kappa_{i,t}^{(j)}}$$

positions and fees Once we obtain the weight for each stock, it would be straightforward, though complicated ⁴, to work out the positions for each stock in the portfolio, as well as the transaction fees incurred when changing the positions.

We omit the details, and represent the procedures symbolically as functions ϕ and ψ , which map weights to, respectively, numbers of stock

$$n_{i,t}^{(j)} = \phi\left(w_{i,t}^{(j)}\right)$$

and transaction fees

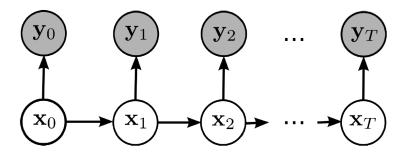
$$f_{i,t}^{(j)} = \psi\left(w_{i,t}^{(j)}, w_{i,t-1}^{(j)}\right)$$

portfolio value Given the numbers and prices of stocks in our portfolio, we may readily obtain its value at time *t*

$$V_t = \sum_{i=1}^{N} \sum_{i} n_{i,t}^{(j)} S_{i,t}^{(j)}$$

5 Stage Detection with Hidden Markov Model

Hidden Markov Model Hidden Markov Model (HMM) is ideal in recovering a series of hidden states from a series of observations. In our case, the stage of the business cycle is the hidden state which we would like to recover. That is, given observable economic indicators at that time t, we would like to produce the stage vector α_t at that time.

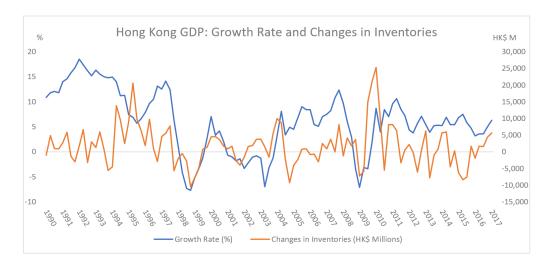


With Hidden Markov Model, we try to recover a series of **hidden states** $\{y_t\}_0^T$ from a series of **observations** $\{x_t\}_0^T$.

leading & lagging indicators It is ultimately important to pick the right economic indicators as observations. Since we may tell much about the economy from GDP, any indicator that lags GDP is meaningless — for instance, the unemployment rate.

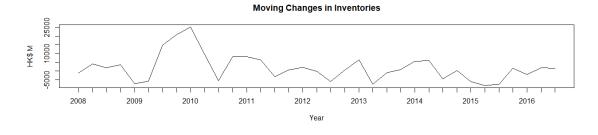
⁴ For example, while weights can be any real numbers between 0 and 1, the numbers of stocks in our portfolio can be whole numbers only — multiples of 100, to be exact.

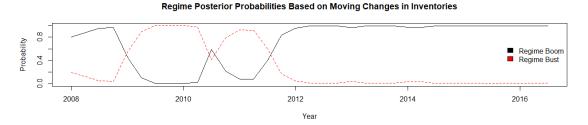
In our case, we choose to observe Hong Kong's changes in inventories ⁵, which are widely accepted as a leading indicator for the economy.



Growth rate of GDP (**blue**) vs. changes in inventories (**orange**) in Hong Kong (1990 – 2017). It is evident that changes in inventories leads growth rate of GDP after the 1997 Asian financial crisis.

hindsight bias To avoid hindsight bias, we assume one quarter's lag before we can know the data for the period concerned. This is sufficient in Hong Kong. Quarterly data on GDP and changes in inventories are normally available after two months. However, we dismiss revisions to the figures.





A series of moving-window guesses: at the end of each quarter, we obtain a pair of posterior probabilities corresponding to regime boom (**black solid**) and regime bust (**red dashed**), based on the best available data at that time. Information is updated as time advances.

 $^{^{\}rm 5}$ data obtained from censtatd.gov.hk, Census and Statistics Department, Hong Kong Government

number of hidden states We presets *two* hidden states (regimes) in the application of HMM. The model then produces a pair of posterior probabilities corresponding the two states (regimes).

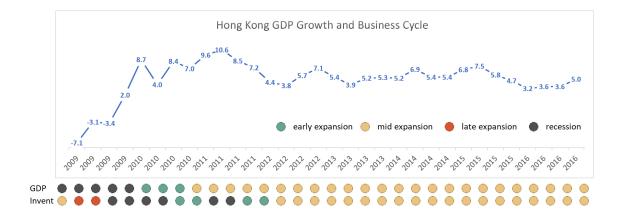
Why don't we preset four hidden states, since there are four stages in the cycle? Because we cannot control the relationship between the four hidden states. Note that the four stages of the business cycle are interrelated — An economy cannot go suddenly from recession to mid expansion. It must go through an early expansion.

Thus, a judicious choice would be presetting only two states. The two states, which should be as distinct as possible, are very likely to model the boom and bust. With the posterior probabilities for the two states, we further determines the four stages.

from two states to four stages Let p_t stand for the period-t posterior probability for one of the regimes, which, against the backdrop of GDP growth, we may call regime boom. We convert p_t to the four stages using the following rule:

- 1. if $p_t < 0.1$, it is a recession;
- 2. else if $p_t > 0.9$, it is a mid expansion;
- 3. else if $p_t > \frac{1}{4} \sum_{i=1}^{4} p_{t-i}$, it is an early expansion;
- 4. else, it is a late expansion.

In one word, if it is very close to regime bust, it is a recession; if it is very close to regime boom, it is a mid expansion; if it is in the middle of regimes boom and bust, we will see whether it comes from a recession or a mid expansion, to determine whether it is an early or late expansion.



Hong Kong GDP growth rate, over the stages of business cycle recovered using the GDP growth itself (first row) and changes in inventories (second row). A green dot stands for early expansion, golden for mid expansion, red for late expansion, and black for recession.

6 Data Preprocessing with PCA

We obtain stock data from Webb-site Reports ⁶, Wind Info, and Bloomberg. The key information includes the history of adjusted close prices, market values, and suspension states of individual stocks.

There are around 2,000 stocks in the Hong Kong market. After removing stocks that miss key information, we still have over 1,000 stocks in our pool.

Many stocks are homogeneous to a large degree. We would like to bring the total number of stocks below 200, using Principal Component Analysis (PCA).

challenges There are two challenges when applying PCA to stock selection:

- PCA essentially is a rotation, which makes the first several principal components (PCs) most representative of the entirety. However, since PCs are linear combinations of original variables (stocks), PCA does not naturally delete stocks;
- The total numbers of stocks in different sectors are very uneven. For example, there are 360 stocks in the consumer discretionary sector, but only 14 stocks in the telecommunication services sector ⁷

Our corresponding solutions are:

- 1. Apply an algorithm proposed by *Yang*, *Rea* & *Rea* ⁸, which removes the stocks that are most representative of the less representative;
- 2. Revise *Yang, Rea & Rea*'s algorithm with regard to uneven sectors delete *aggressively* if there are many stocks in one sector, and delete *conservatively*, or cease to delete, if there are only few stocks remaining.

essence of algorithm We apply PCA in stock selection, with constant λ_D , λ_S :

- Step 1. apply PCA to the correlation matrix of stock prices;
- Step 2. for each principal component $PC^{(k)} = \sum_j c_j s_j^{(k)}$ whose eigenvalue $\lambda_k < \lambda_D$, find the stock $s_m^{(k)}$ with the largest coefficient in absolute value $|c_m|$, and then delete it;
- Step 3. repeat steps 1 and 2 until
 - (a) the number of stocks is below a minimum, or
 - (b) the lowest eigenvalues are above λ_S .

7 Optimization with Monte Carlo methods

Definition Let θ_{SS} , θ_{S} , θ_{B} , and θ_{SB} be the multipliers to be optimized. They respectively correspond to *strong sell*, *sell*, *buy*, and *strong buy*. The multiplier for *neutral* is fixed at 1. That is, $\theta_{N} \equiv 1$. Let X be the available data.

⁶ webb-site.com

⁷ As a side note, telecommunication services and utilities sectors fit the concept of *natural monopoly* in microeconomics. There must be very few companies in these sectors.

⁸ Yang, Rea & Rea. Stock Selection with Principal Component Analysis. Working paper (2015).

Objective Find θ_{SS} , θ_{S} , θ_{B} , and θ_{SB} such that the portfolio value $V(\theta_{SS}, \theta_{S}, \theta_{B}, \theta_{SB})$ is maximized in the in-sample period.

Constraints $0 < \theta_{SS} < \theta_{S} < \theta_{N} \equiv 1 < \theta_{B} < \theta_{SB} < +\infty$

Method with Gibbs sampling ideas

- 1. initialize $\theta_{SS}^{(0)} = 0.25$, $\theta_{S}^{(0)} = 0.5$, $\theta_{B}^{(0)} = 2$, $\theta_{SB}^{(0)} = 4$;
- 2. for k = 0, 1, ..., iteratively draw one θ while fixing three others, using the most updated draws
 - (a) draw a random sample $\theta_{SS}^{(k+1)}$ from $f_{SS}(\theta_{SS} \mid \theta_S^{(k)}, \theta_B^{(k)}, \theta_{SB}^{(k)}; \boldsymbol{X})$, s.t. $\theta_{SS}^{(k+1)} < \theta_S^{(k)}$,
 - (b) draw a random sample $\theta_S^{(k+1)}$ from $f_S(\theta_S \mid \theta_{SS}^{(k+1)}, \theta_B^{(k)}, \theta_{SB}^{(k)}; \boldsymbol{X})$, s.t. $\theta_{SS}^{(k+1)} < \theta_S^{(k+1)} < \theta_N \equiv 1$,
 - (c) draw a random sample $\theta_B^{(k+1)}$ from $f_B(\theta_B \mid \theta_{SS}^{(k+1)}, \theta_S^{(k+1)}, \theta_{SB}^{(k)}; \boldsymbol{X})$, s.t. $\theta_N \equiv 1 < \theta_B^{(k+1)} < \theta_{SB}^{(k)}$,
 - (d) draw a random sample $\theta_{SB}^{(k+1)}$ from $f_{SB}(\theta_{SB} \mid \theta_{SS}^{(k+1)}, \theta_{S}^{(k+1)}, \theta_{B}^{(k+1)}; \boldsymbol{X})$, s.t. $\theta_{B}^{(k+1)} < \theta_{SB}^{(k+1)}$;
- 3. repeat *m* times to obtain a sequence of random draws

$$\left(\theta_{SS}^{(1)},\theta_{S}^{(1)},\theta_{B}^{(1)},\theta_{SB}^{(1)}\right),\,\cdots,\left(\theta_{SS}^{(m)},\theta_{S}^{(m)},\theta_{B}^{(m)},\theta_{SB}^{(m)}\right)\;;$$

4. for each set of random draws $(\theta_{SS}^{(k)}, \theta_S^{(k)}, \theta_B^{(k)}, \theta_{SB}^{(k)})$, calculate the portfolio value $V(\theta_{SS}^{(k)}, \theta_S^{(k)}, \theta_B^{(k)}, \theta_{SB}^{(k)})$, and set $\theta_{SS}, \theta_S, \theta_B, \theta_{SB}$ to be the set of random draws that maximizes V.

Conditional Distributions The conditional distributions for θ_{SS} , θ_{S} , θ_{B} , and θ_{SB} should be set in accordance with the constraints $0 < \theta_{SS} < \theta_{S} < \theta_{N} \equiv 1 < \theta_{B} < \theta_{SB} < +\infty$ and the sampling method.

At the bottom line, the random samples should be drawn from a closed interval whose endpoints may change for each iteration. Thus Gaussian distributions are not suitable for this case.

We assume uniform distributions \mathcal{U} for θ_{SS} and θ_{S} , and inverse uniform distributions \mathcal{I} for θ_{B} and θ_{SB} . In particular,

$$\begin{split} \theta_{SS} \mid \theta_S^{(k)}, & \theta_B^{(k)}, \theta_{SB}^{(k)} \sim \mathcal{U}\left(0, \, \theta_S^{(k)}\right) \;; \\ \theta_{S} \mid \theta_{SS}^{(k+1)}, & \theta_B^{(k)}, \theta_{SB}^{(k)} \sim \mathcal{U}\left(\theta_{SS}^{(k+1)}, \, 1\right) \;; \\ \theta_{B} \mid \theta_{SS}^{(k+1)}, & \theta_S^{(k+1)}, \theta_{SB}^{(k)} \sim \mathcal{I}\left(1, \, \theta_{SB}^{(k)}\right) \;; \\ \theta_{SB} \mid \theta_{SS}^{(k+1)}, & \theta_S^{(k+1)}, \theta_B^{(k+1)} \sim \mathcal{I}\left(\theta_B^{(k+1)}, \, +\infty\right) \;. \end{split}$$

Note that

$$X \sim \mathcal{I}(a, b) \Longleftrightarrow X^{-1} \sim \mathcal{U}\left(b^{-1}, a^{-1}\right)$$
 , where

$$0 < a < b \le +\infty$$
, and $+\infty^{-1} \stackrel{\text{def}}{=\!\!=\!\!=} 0$.

Alternatively, we may preset a sufficiently large upper bound M, and instead assume

$$\begin{split} \theta_B \mid \theta_{SS}^{(k+1)}, & \theta_S^{(k+1)}, \theta_{SB}^{(k)} \sim \mathcal{U}\left(1, \; \theta_{SB}^{(k)}\right) \;; \\ \theta_{SB} \mid \theta_{SS}^{(k+1)}, & \theta_S^{(k+1)}, \theta_B^{(k+1)} \sim \mathcal{U}\left(\theta_B^{(k+1)}, \; M\right) \;. \end{split}$$

8 Fine-Tuning

Up to this point we have not shown much interest in the paths of individual stocks. However, if the history does bear a relation to the future, we may consider using statistical techniques to tactically fine-tune our portfolio, making use of the historical data of individual stocks.

skewness, kurtosis, and variance ratio test For example, stocks with negative mean and negative skewness in returns are considered dangerous. A fat tail on the negative side says its price must have plunged for several times. On the contrary, positive mean / positive skewness is a desirable combination to which we should give more weight.

	Negative Mean	Positive Mean
Positive Skewness	No Man's Land	Holy Grail of Investing
Negative Skewness	Dead on Arrival	Most Cases

Kurtosis can be made use of, too. We favor small kurtosis, for thinner tails indicate more robust performance.

Variance ratio tests tell if a stock follows a random walk. If the movement is completely random, we can do nothing to add any value. The gain is simply fortuitous like tossing a coin. However, most stocks are not completely random, according to our tests.

9 Comparison

The project requires to imitate, improve, and innovate with the new ideas that *Arnott et al* have done in their paper entitled *Fundamental Indexation*, in which stocks are weighted according to their fundamental figures such as revenues and employment.

Our trading strategy, which involves macro-economy and industry sectors, is indifferent to fundamental figures of individual firms.

As a comparison, we also implement a trading strategy which is related to fundamental figures, including assets, sales and income.

fundamental data Like *Arnott et al*, this trading strategy, proposed by *Haugen & Bakerb*, also makes uses of fundamental data, but in a more adroit way — it embeds the fundamental figures in the framework of mean-variance efficient frontier.

To put it simply, rather than using stock returns directly obtained from historical data, the trading strategy forecasts them — using fundamental data. With predicted returns, the trading strategy then constructs the efficient portfolio.

The factors utilized to predict returns of the stocks include

- (1) past returns;
- (2) changing rate of trading volume;
- (3) assets / sales;
- (4) income / sales;
- (5) trading volume / market value.

With notation

- $r_{j, t}$ return of jth stock at time t,
- R_t return of portfolio at time t,
- $\widehat{P}_{i,t}$ predicted value of *i*th factor at time *t*,
- $F_{j, i, t-1}$ factor loading of jth stock's ith factor at time t-1,
- $u_{j,t}$ error term corresponding to jth stock at time t, and
- U_t error term corresponding to portfolio at time t,
- $w_{j,t}$ weight of jth stock at time t,

we sketch the trading strategy as follows.

Step 1. Conduct cross-sectional multiple regression

$$r_{j,t} = \sum_{i} \widehat{P}_{i,t} F_{j,i,t-1} + u_{j,t}$$

Step 2. Calculate expected returns and covariance matrix

$$\mathbb{E}(r_{j,t}) = \sum_{i} \mathbb{E}(\widehat{P}_{i,t}) F_{j,i,t-1}$$

$$\mathbb{V}(R_t) = \mathbf{F}_{t-1} \mathbb{V}(\widehat{\mathbf{P}}_t) \mathbf{F}_{t-1}^{\mathsf{T}} + \operatorname{diag}(\mathbb{V}(U_t))$$

Step 3. Solve optimization problem

$$\max_{w_t} \quad \frac{\mathbb{E}(R_t)}{\mathbb{V}(R_t)} \quad s.t. \quad 0 \le w_{j, t} \le 0.15 \quad \text{where}$$

$$\mathbb{E}(R_t) = \mathbb{E}\left(\sum_{j} \left(w_{j,\,t} \, r_{j,\,t}\right)\right) = \sum_{j} w_{j,t} \mathbb{E}\left(r_{j,\,t}\right)$$

Step 4. Find weights w_t for stocks in the portfolio.

results for comparison For the investment period spanning from 2010 to 2016, this trading strategy achieves an annual return of 10.88%, with annual volatility of 12.75%, and maximum drawdown of -26%. Compared with our sector rotation trading strategy, this one has lower annual return, but lower volatility.

10 Final Comments

We make three final comments about our trading strategy, which also represent possible improvements we may make in future.

- 1. Our trading strategy heavily relies on the fluctuations of long-term cycles. This carries two implications:
 - (a) We profit (or at least beat the benchmark) only if the stage of the cycle changes, which takes months or even years to happen;
 - (b) We need long-term data for prediction / optimization / calibration.
- Shifting from one sector to another may cause volatility and transaction fees to rise. Thus sector rotation is meaningful only when marginal benefit outweighs marginal cost.
- 3. In evaluating a trading strategy, one should give proper treatment to stocks in long-term suspension. For example, Hanergy Thin Film (0566.HK) has been suspended since 20 May, 2015. Thus its adjusted close price is frozen at HK\$3.91. However, in all likelihood, the stock is unworthy of the price, taking the scandal into consideration. While we cannot sell it since it is in suspension, we should properly evaluate the true value of our portfolio.