EXAMINING FACTORS PROMOTING CRITICAL THINKING IN THE MATHEMATICS CLASSROOM

An Undergraduate Thesis

Presented to the Faculty of the

Department of Science and Mathematics Education

College of Education

MSU – Iligan Institute of Technology

Iligan City

In Partial Fulfillment of the

Requirements for the Degree

Bachelor of Secondary Education

Major in Mathematics

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May 2025

APPROVAL SHEET

In partial fulfillment of the requirements for the degree Bachelor of Education Major in Mathematics, this undergraduate thesis entitled, **"Examining Factors Promoting Critical Thinking in the Mathematics Classroom"** prepared and submitted by **Champaign Margaux Bongcaron, Jenlyn M. Roda,** and **John P. Urtal,** is hereby recommended for approval.

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ABSTRACT

Bongcaron, Champaign Margaux E., Jenlyn M. Roda and **John P. Urtal:** BSED Mathematics, College of Education, MSU-Iligan Institute of Technology, Tibanga, Iligan City, May 2025, **"Examining Factors Promoting Critical Thinking in The Mathematics Classroom"** Adviser: Joan Rose T. Luib.

Critical thinking is crucial for students' academic understanding and decision-making. However, mathematics instruction in the Philippines often adheres to didactic approaches, hindering its development. This qualitative study aimed to examine factors promoting critical thinking in the mathematics classroom by identifying teacher practices and contributing elements. Employing a multiple case study design, data were gathered through classroom observations and open-ended questionnaires from three purposively sampled secondary mathematics teachers at a local state university. Thematic analysis was then used to interpret the findings. The study revealed that teachers effectively fostered critical thinking through collaborative learning (including group discussions and peer review), consistent use of guiding and probing questions, and timely feedback for reflection. They also incorporated "board works," open-ended problems, student investigations, and multimedia in structured lessons, emphasizing group work to encourage students to explain and justify solutions. These methods promoted critical thinking across dimensions such as skill focus, practice, feedback, active learning, and a supportive environment. The study concludes that fostering critical thinking is vital for developing well-rounded individuals, offering valuable insights for teachers and curriculum planners to enhance instructional practices and design engaging curricula that prioritize this essential skill.

Keywords: Critical Thinking, Providing Practice Opportunities, Providing Feedback, Active Learning Approaches

ACKNOWLEDGEMENT

The researchers wish to express their deepest appreciation to all who have helped and supported in bringing this study to reality. Furthermore, they are grateful to the following:

Ma’am Joan Rose T. Luib, the program and the researchers’ adviser who understands and patiently guides the researchers for this study, especially in giving suggestions and recommendations.

The panel members, whose insights and considerations have helped in the fulfillment of this study, **Dr. Amelia T. Buan**, professor in the Department of Science and Mathematics Education and the school principal of MSU-IIT IDS, and **Ma’am Grace P. Liwanag**, faculty member of MSU-IIT IDS.

Ma’am Mary Joy F. Luga, **Sir Elesar Malicoban,** and **Sir Hassan S. Gandamra** for the valuable comments on the research instruments. And to **Dr. Chrisley Jade C. Saromines**, **Sir Rovic E. Perocho**, and **Ma’am Grace P. Liwanag** for the active cooperation and participation of the study.

Special acknowledgement is also given to researchers’ beloved parents, **Mr. Benjamen C. Urtal & Mrs. Evangeline S. Pabillaran** and **Mr. Francisco S. Roda Jr. & Mrs. Elenita M. Roda**, whose financial support and moral encouragement were vital in completing this study. Special thanks is also given to **Mr. Lachezar Tsanov** and to all the friends for sharing valuable insights and knowledge.

Most of all, to the **Almighty Father**, who is the provider of all things, and the source of all knowledge, wisdom, and strength. It was through His grace that they found the courage to continue and endurance to overcome every challenge along the way.

All glory and honor in His name!

John, Jen & Champ

For the glory of God!

For our supportive parents.

For our teachers, peers, and friends.

And for all pre-service teachers.

This is wholeheartedly dedicated to you!

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CHAPTER I

THE PROBLEM AND ITS SCOPE

Background of the Study

Critical thinking is one of the most essential skills that enhances a student's academic understanding. It empowers students to analyze situations effectively and make appropriate decisions (El Yazidi, 2023). Teaching mathematics requires students to think creatively and critically, solve problems skillfully, make important decisions, collaborate, and communicate in group activities (Wismath et al., 2024). The Department of Education's (DepEd's) mathematics curriculum aims "to enhance students' critical thinking and problem-solving skills" (DepEd, 2012; Alcantara & Bacsa, 2017), which serves as the central focus of learning mathematics.

Marquez (2017) asserts that critical thinking can be cultivated when students are encouraged to raise questions, explore possibilities, and engage in meaningful discussions. The problem, however, is that teaching in the Philippines is still constrained to the didactic approach. As a result, mathematics instruction in many classrooms remains teacher-directed and content-focused, emphasizing the memorization of facts, concepts, and mastery of algorithms. Atteh et al. (2017) argue that this approach does not foster critical thinking or problem-solving skills, as it treats learners as passive recipients of knowledge rather than independent thinkers capable of constructing their own understanding. These concerns led the researchers to examine the factors that promote critical thinking in the mathematics classroom.

Statement of the Problem

This study aimed to examine the factors that promote critical thinking in the mathematics classroom. Specifically, the researchers aimed to answer the following questions:

1. What practices are employed by mathematics teachers that promote critical thinking in the classroom?
2. What factors contribute to promoting critical thinking in the mathematics classroom?

Significance of the Study

The development of critical thinking has been regarded as one of the most significant objectives of education for over a century (Jatmiko et al., 2018; Suarniati et al., 2019). This study highlights how teachers employ various teaching strategies that effectively promote critical thinking in the mathematics classroom. The findings of this study will broaden teachers' perspectives on planning how to enhance their teaching practices in mathematics education to promote critical thinking, potentially leading to improved teaching strategies and better mathematical understanding.

Furthermore, this research will provide valuable insights for curriculum planners, enabling them to design more effective and engaging curricula that foster critical thinking. The study also empowers students by providing evidence-based methods to enhance their learning experiences and outcomes.

In a broader context, this research aims to elevate the quality of mathematics education in the Philippines, contributing to the academic growth and long-term success of its students. The results of this study may also serve as a foundation for future research, encouraging further exploration into innovative teaching methods across different subjects and grade levels.

Scope and Limitations of the Study

This research examined the factors that promote critical thinking in the mathematics classroom, specifically among secondary mathematics teachers. The study is limited to the following: (a) it focuses solely on mathematics teachers within one institution, so the findings may not be generalizable to teachers in other schools or educational settings; (b) the research is based on a random sample of teachers from a single school, limiting the generalizability of the results; (c) the study concentrates on mathematics education, hence the findings may not be directly applicable to other subject areas; (d) the sample size of teachers may limit the ability to generalize the results to a larger population of educators; and (e) the observers conducting the data collection and analysis are pre-service teachers, which may influence the interpretation of classroom interactions due to limited experience.

Conceptual Framework

The conceptual framework of the study is shown in Figure 1.1 on the following page. It shows that the factors that promote critical thinking in the mathematics classroom have five (5) dimensions: focusing on specific critical thinking skills, providing practice opportunities, providing feedback, using active learning approaches, and classroom environment.

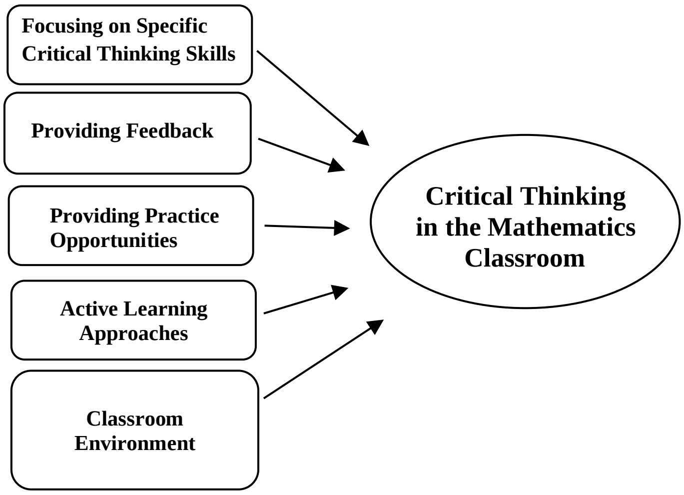

Figure 1.1. Conceptual Framework of Promoting Critical Thinking in the Mathematics Classroom

Note: This conceptual framework is adopted and refined from the study of El Yazidi (2023).

Operational Definition of Terms

To better understand the concepts of significant terms used in this study, their operational definitions are given below.

Classroom Environment— This refers to the psychological, social, and physical conditions within the classroom as experienced by the students, which are established to support the processes of teaching and learning (Lecaros, 2022).

Critical Thinking — In this study, critical thinking refers to the capacity to utilize an intellectually rigorous process of actively and skillfully engaging with information through conceptualization, application, analysis, synthesis, and evaluation (Scriven, 1996) with the specific aim of creating educational experiences that cultivate critical thinking skills in learners, enabling them to become more thoughtful, inquisitive, and actively involved in their education (Rani, 2020).

Focusing on Specific Critical Thinking Skills—In this study, focusing on specific critical thinking skills means that effective instruction should target particular abilities like analyzing arguments, identifying assumptions, and evaluating evidence (El Yazidi, 2023).

Providing Feedback —In this study, providing feedback is defined as the crucial process of offering students information and guidance to facilitate their understanding of how to enhance their critical thinking abilities (El Yazidi, 2023).

Providing Practice Opportunities — Within this study, providing practice opportunities is understood as the deliberate creation of diverse learning experiences, encompassing real-world contexts, that enable students to exercise and thereby enhance their critical thinking abilities (El Yazidi, 2023).

Using Active Learning Approaches — In this study, using active learning approaches is defined as the implementation of instructional methods, such as problem-based and inquiry-based learning, etc., recognized for their effectiveness in fostering critical thinking by actively engaging students in the exploration and questioning of concepts (El Yazidi, 2023).

CHAPTER II

REVIEW OF RELATED LITERATURE AND STUDIES

This chapter discusses the concepts and theories related to this study, which include critical thinking and its application to teaching education. In reviewing the related literature, the researchers systematically searched for articles or journals connected to the themes and used citation chasing to find relevant articles that are related to the study (Haddaway et al., 2022).

Related Literature

Critical thinking is defined as "reflective decision-making and thoughtful problem-solving about what to believe and do" (Facione, 2020). According to Cottrell (2017), critical thinking is a cognitive process used to acquire knowledge. It provides a means to use an organized and systematic method to identify a problem and to develop effective solutions while being able to justify how and why one came to such a conclusion (Gormley, 2017; Wade, 2016).

In essence, critical thinking (Padmanabha, 2018) is a mental activity focused on the interpretation, assessment, evaluation, and explanation of incorrect information (Saputra et al., 2019). The ability to think critically indicates the use of mental processes such as attention, categorization, selection, and judgment (Kumar & James, 2015); concepts such as cognitive and metacognitive skills, practices and abilities, dispositions and character, logic, and reflection (Ennis, 2011); and skills such as analyzing, synthesizing, evaluating, and summarizing (Dwyer et al., 2014). Critical thinking is a higher-order thinking skill that can be considered reflective thinking (Apriliana et al., 2019).

Critical thinking is essential for students to develop in order to succeed academically and navigate complex real-world challenges. According to Sagar and Ranbir (2024), fostering critical thinking in the classroom involves strategies such as inquiry-based learning, Socratic questioning, and problem-solving activities. The study highlights the role of technology and collaborative learning in enhancing students' critical thinking abilities. By integrating these approaches into teaching practices, educators can help students analyze information, evaluate arguments, and make informed decisions, ultimately preparing them for success in school and beyond.

Mathematics provides an effective platform for developing students' critical thinking through reasoning, logic, and metacognition. According to Su et al. (2015), teachers who emphasize reasoning and logical validity help students recognize mathematics as a discipline that enhances critical thinking. The research stated that students develop critical thinking skills by solving mathematical problems, evaluating solutions, and justifying their reasoning. Additionally, metacognition plays a crucial role in helping students understand that mathematical problem-solving is a process of logical reasoning. By applying critical thinking strategies—such as scenario identification, strategy selection, and evaluating conclusions—students improve their problem-solving abilities and become more confident thinkers. Moreover, this attention to critical thinking in mathematics develops in students essential skills applicable to both decision-making processes and practical, real-world situations. Jensen (2011) argues that critical thinking is characterized by a mental process that must be efficient and effective to leverage knowledge that is both appropriate and accurate. Additionally, Mustaji (2012) believes that critical thinking is rooted in making decisions about what to believe or do, which is synonymous with decision-making, strategic planning, and problem-solving (Husnaeni, 2016). It is a method of increasing knowledge and intellect by contrasting several current and potential issues in order to arrive at a conclusion and a solution. Instead of the outright acceptance of various concepts, critical thinking entails deep reasoning and analysis (Fahim & Pezeshki, 2012). This means that people's opinions and suggestions about a phenomenon cannot be believed entirely until they go through a structured and rational process of discovering the facts.

Learning to think critically is essential for applying critical thinking in school settings because people who think critically are able to recognize and correct common logic errors as well as understand and connect logical connections between concepts. Critical thinkers can also construct and test arguments, recognize and correct common logical errors, and solve problems systematically (Chukwuyenum, 2013). Efforts to improve critical thinking in mathematics education have become the focus of global mathematics curriculum development. Critical thinking promotes originality and autonomous thinking by encouraging students to apply critical thinking skills in their regular activities and assignments (Firdaus et al., 2015). According to Bailin, Case, Coombs & Daniels (1999), critical thinking is carried out with the aim of forming beliefs or deciding on actions. They argue that its definition lies in the successful completion of specific intellectual processes, which in turn allows one's thinking to satisfy criteria for being sufficient and accurate. In the context of mathematics education, critical thinking is essential for students to be able to engage with and understand complex concepts and ideas. It enables students to evaluate the credibility of sources, analyze evidence, and construct reasoned arguments. Research has shown that students who develop critical thinking skills are better able to navigate complex academic content, perform better on standardized tests, and are more likely to complete their degrees (Abrami et al., 2008; Ennis, 2011).

Critical thinking is a crucial skill in mathematics education, benefiting both students and teachers in numerous ways. Some of the benefits of critical thinking in mathematics education include the improvement of students’ academic performance. Students who develop critical thinking skills are better equipped to engage with complex academic content, evaluate information, and make informed decisions. Studies indicate that students who utilize critical thinking skills tend to achieve higher levels of academic success (Abrami et al., 2008).

Moreover, critical thinking encourages higher-order thinking skills such as analysis, synthesis, and evaluation. When it comes to problem-solving skills, critical thinking helps students develop problem-solving skills by urging them to analyze information, consider different perspectives, and evaluate evidence. This can be especially beneficial in subjects such as mathematics, science, and technology (Saleh et al. 2021).

The development of critical-thinking dispositions in mathematics can be fostered through problem-solving activities that engage students in meaningful learning experiences. Leader and Middleton (2004) reviewed research on instructional programs that strengthen critical-thinking dispositions by incorporating ill-structured problem-solving tasks. The study highlights that students' inclination to think critically is influenced by attitudinal memory, which can be reinforced through direct experiences, sensory engagement, emotional reactions, freely chosen behaviors, and attitude rehearsal. Programs such as the Jasper series and Decision Making serve as examples of mathematics curricula that promote these dispositions by encouraging students to engage with complex, real-world problems. Based on their review, Leader and Middleton (n.d.) propose a set of design principles that educators can apply to enhance students' critical-thinking skills in mathematics classrooms.

Critical thinking is beneficial in a wide range of subjects, including science and mathematics, where students need to analyze data and evaluate evidence (Bybee, 2014). Critical thinking is not an innate ability; while some students may naturally be inquisitive, they require systematic training to become analytical, fair, and open-minded in their pursuit of knowledge. Equipped with these skills, students can gain confidence in their reasoning and apply critical thinking across various content areas or disciplines (Lundquist, 1999). Scriven and Paul (2007) suggest that critical thinking is akin to the scientific method, representing an organized and structured way of approaching thought. Analogous to the way students learn the stages of scientific inquiry, they also need to grasp the process of critical thinking. Four major challenges often obstruct the inclusion of critical thinking within education: (i) lack of training, (ii) lack of information, (iii) preconceptions, and (iv) time constraints.

Numerous researchers (Landsman and Gorski, 2007; Sandholtz et al., 2004; Sheldon and Biddle, 1998; Wong, 2007) argue that the current trend in education toward standardized curricula and a focus on test scores undermines teachers' ability to foster critical thinking in the classroom. Emphasizing "teaching to the test" diverts the learning process away from student-centered instruction and places the focus on content delivery. If the primary goal is learning, students should be given the freedom (and responsibility) to explore content, analyze resources, and apply information independently.

Yusuf and Adeoye (2012) highlight that critical thinking can help improve communication skills by encouraging individuals to think more deeply about the information they are conveying. It can also help individuals to better understand the perspectives of others and to more effectively evaluate the accuracy of the information they are receiving. By engaging in critical thinking, individuals can become more aware of their own biases and assumptions, as well as help better understand the implications of their words and actions, which can both help them communicate more effectively. As for teachers, critical thinking can also be beneficial by promoting innovation and creativity in the classroom. Teachers who promote critical thinking can create a dynamic learning environment that encourages students to engage with content and explore different perspectives.

Related Studies

El Yazidi (2023) conducted an experimental study on strategies for promoting critical thinking in the classroom. The research was conducted among 160 university students, and the data was analyzed using SPSS statistical software. The study's findings (El Yazidi, 2023) underscored the necessity of promoting critical thinking through collaborative learning for student success, implying that educators need to incorporate this strategy into their classroom practices. Additionally, the study identifies and describes six specific strategies that have been proven effective in fostering critical thinking, namely: socratic questioning, collaborative learning, inquiry-based learning, concept mapping, problem-based learning, and argument mapping.

Mahmud et al. (2021) conducted a qualitative case study with six participants across six Malaysian schools, focusing on how oral questioning in mathematics teaching could enhance students' critical thinking. The findings revealed that mathematics teachers commonly use prompting, reflective, and clarification questions to encourage critical thought, and these question types are effective when students are solving math problems. Additionally, the study found that teachers use reflective questions to help students reconsider their answers. The study suggests that teachers should be cautious when asking oral questions, ensuring that they stimulate students' thinking rather than focusing solely on memorizing mathematical algorithms and procedures.

Dailo & Dailo (2022) carried out a quasi-experimental investigation to examine how differentiated instruction affects the critical thinking abilities of grade 7 mathematics students. The study was implemented over a five-week period across several grade 7 classrooms, where curriculum compacting and flexible grouping strategies were used. The findings of the study revealed that differentiated instruction significantly improved students' critical thinking abilities. The study found that differentiated instructional strategies to meet the needs of individual students helped enhance their ability to think critically in mathematics. This study implies that educators should consider using differentiated instruction techniques to foster the development of critical thinking skills, rather than focusing solely on traditional teaching methods or rote memorization.

Fitriyah & Febrianti (2024) conducted a meta-analysis on the use of the open-ended approach to enhance critical thinking skills in mathematics education. The study analyzed various research articles examining the impact of open-ended tasks on students' critical thinking abilities across different educational settings. The findings revealed that the open-ended approach is effective in promoting critical thinking in mathematics, as it encourages students to explore multiple solutions and engage in deeper problem-solving. The study further highlighted that this approach fosters student autonomy and enhances their ability to think critically when addressing complex mathematical problems. This study suggests that educators should incorporate open-ended tasks into their teaching strategies to stimulate students' critical thinking and move beyond traditional problem-solving methods.

Firdaus and colleagues (2015) undertook a quasi-experimental study focused on cultivating critical thinking skills among students in mathematics education. The research aimed to examine the impact of mathematics learning modules grounded in problem-based learning (PBL) on the critical thinking abilities of secondary school students within the district of Bone. The study's results indicated a positive influence of utilizing PBL-based mathematics learning modules in enhancing students' critical thinking skills across all three dimensions: identifying and interpreting information, analyzing information, and evaluating evidence and arguments.

Chikiwa and Schäfer (2018) conducted a study on how questioning techniques influence critical thinking in grade 11 multilingual mathematics classes. Using a mixed-method approach, the study collected classroom observations and teacher interviews to examine the types of questions used during trigonometry and analytic geometry lessons. The findings revealed that teachers predominantly used lower-order questioning techniques, while higher-order questions—known for promoting critical thinking—were infrequently applied. The study emphasizes that while all levels of Bloom’s taxonomy are important, the higher-order thinking skills necessary for Grade 11 students were not adequately developed through questioning strategies. As a result, the research highlights the need for improved questioning techniques in mathematics teaching to enhance students' critical and analytical reasoning skills.

Ennis (2011) performed a thorough analysis of critical thinking teaching to establish whether the ability to think critically can be effectively imparted and nurtured. This review examined studies investigating diverse strategies for teaching critical thinking, encompassing problem-based learning, inquiry-based learning, and direct teaching. The conclusion of the review was that critical thinking skills can indeed be effectively taught and developed through instruction, application, and feedback. Specifically, the review identified several key factors that are important for effective critical thinking instruction, including:

Focusing on specific critical thinking skills: To effectively teach critical thinking, instruction should focus on specific skills, such as analyzing arguments, identifying assumptions, and evaluating evidence (Yazidi, 2023, p. 30).

Providing practice opportunities: Students need opportunities to practice critical thinking in a variety of contexts, including real-world scenarios, to develop their skills (Yazidi, 2023, p. 30).

Providing feedback: Feedback is essential for helping students understand how to improve their critical thinking (Yazidi, 2023, p. 30).

Using active learning approaches: Active learning approaches, such as problem-based and inquiry-based learning, can be effective for promoting critical thinking by encouraging students to explore and question concepts (Yazidi, 2023, p. 30).

Additionally, Ennis (2011) revealed that "critical thinking can be effectively taught and developed through various instructional approaches." This has important implications for educators, who can use this information to develop effective strategies for teaching critical thinking to their students.

Developing critical thinking skills, defined as the ability to analyze and evaluate information, is significantly influenced by the classroom environment. Studies show that a positive classroom climate directly boosts these skills and indirectly through students' learning goals (Gezer et al., 2018). Teachers can effectively enhance their instruction and integrate critical thinking into the curriculum by fostering a positive learning space (Lecaros, 2022). When students use critical thinking, they engage more deeply with their learning, ask insightful questions, and participate more actively. To prepare students for the demands of the 21st century, educators must prioritize creating learning environments that encourage this kind of thinking (Lecaros, 2022). Research suggests that creating an enabling classroom environment is crucial for fostering critical thinking (CT) skills in students. Faculty practices play a significant role in promoting student engagement and CT development (Cassum & Gul, 2016). A non-oppressive teaching environment encourages students to take responsibility for their learning and curriculum content (Prior, 2000). In e-learning settings, interactive components are perceived to be more effective in promoting CT compared to static resources (Saadé et al., 2012). Expert teachers optimize the learning environment for CT by setting a positive atmosphere, maintaining a supportive climate, and addressing challenges (Jaffe et al., 2019). Key strategies include establishing ground rules, focusing on process rather than answers, building trust, and gently pushing learners while tolerating discomfort. These findings highlight the importance of creating a conducive learning environment to enhance CT skills across various educational contexts, from traditional classrooms to virtual learning environments.

Dalim et al. (2022) conducted a qualitative study to explore teachers' views and challenges in using the Socratic method, a type of inquiry-based learning, in classrooms. They selected five teachers from various subjects using purposive sampling for semi-structured interviews. Thematic analysis of the data revealed that most of the teachers had positive views on the application of the Socratic method, especially to enhance critical thinking skills. Despite that, there were a few challenges faced by the teachers, particularly on their content knowledge, knowledge on questioning methods, class participation, topics and time constraints. Based on the challenges extracted, this study concluded that the teachers require due support particularly in terms of their subject knowledge and teaching skills in carrying out the Socratic method in classrooms effectively. Thus, this study proposes further actions to be taken in helping teachers by providing professional training and developmental programs to effectively implement Socratic questioning in the classrooms.

Orhan (2022) used a non-experimental quantitative approach with 170 high school students to see if gender influenced their critical thinking dispositions, decision-making styles, and perceived problem-solving skills. The study also looked at whether critical thinking dispositions and decision-making styles could predict perceived problem-solving skills. The data was collected using the UF/EMI critical thinking disposition instrument, the problem-solving skills perception scale, and the adolescent decision-making questionnaire. The results showed no significant gender differences in these areas, except for decisional self-esteem.

Synthesis

This literature review concludes that critical thinking is an invaluable skill with the potential to improve learners' lives both during their education and beyond. Promoting learners’ critical thinking establishes a foundation for analytical and critical thinking to flourish in all areas of their lives. Therefore, if introduced and nurtured strategically, critical thinking can become a beneficial part of learners’ everyday thoughts. In addition, promoting critical thinking from an early age has the potential to develop individuals with the mental abilities to explore new perspectives and foster critical minds. As a result, the future of learners equipped with critical thinking skills may be brighter and more successful in education, employment, finances, relationships, and life as a whole.

In summary, fostering critical thinking plays a crucial role in developing learners who are not only academically proficient but also well-rounded individuals equipped to face the challenges of a rapidly changing world.

CHAPTER III

RESEARCH METHODOLOGY

This chapter presents the methods and procedures followed in this study. It consists of the discussions on the research design, research setting, research subjects, instruments used, data gathering procedure, and data analysis.

Research Design

This study utilized a qualitative research design, employing case studies to explore and understand the factors that promote critical thinking in the mathematics classroom. A qualitative research design is primarily used to generate detailed descriptions and situational interpretations of the phenomenon being studied (Stake & Trumbull, 1982). The present study aimed to capture the nuances of how teachers implement various instructional practices that promote critical thinking and how these practices are perceived and experienced by both teachers and students.

Using a multiple case study approach, the study gathered qualitative data through classroom observations and survey questionnaires. Additionally, thematic analysis was employed to identify recurring patterns and themes that emerged from the collected data, ensuring a systematic and rigorous examination of the factors and practices promoting critical thinking. In this study, three cases were examined to gather relevant information about the practices of teachers that promote critical thinking in the mathematics classroom.

Research Setting

This study was conducted at one of the laboratory schools of a local state university in Iligan city, Philippines. The school caters to both junior and senior high school students and promotes an environment where innovative teaching strategies and approaches, such as teaching through problem solving (TTP) and other instructional frameworks are encouraged and implemented to enhance student learning outcomes. The research focused on observing mathematics classes taught by selected teachers who employed various instructional methods aimed at fostering critical thinking across the different sections they handled.

Research Subject

This study used a purposive sampling method to examine the factors that promote critical thinking in the mathematics classroom. This non-random sampling technique involves selecting participants based on the researchers' judgment and the specific objectives of the study (Supe et al., 2015). The participants included three secondary mathematics teachers from the laboratory school, each bringing diverse academic qualifications and teaching experience. Teacher A (TA), a female educator with a Master of Science in Education (MSciEd), has nearly 25 years of experience teaching various levels of algebra and statistics. Teacher B (TB), a male MSciEd graduate with 3.5 years of teaching experience, teachers Algebra for Grades 7 and 9, as well as Pre-calculus and General Mathematics for higher grades. Teacher C (TC), a female with a Doctor of Philosophy in Mathematics, has been teaching for 2 years and handles Trigonometry for Grade 9 and both General Mathematics and Pre-calculus for

Grade 11. Their combined expertise and perspectives were instrumental in exploring strategies and classroom practices that promote critical thinking in mathematics.

Instruments Used

Open-ended Questionnaire

This study used an open-ended questionnaire to support the observational data. Three questions were adopted and refined from Malepa-Qhobela & Mosimege (2022) and validated through face validity. The researchers used the questions to the three secondary mathematics teachers who were respondents of this study.

Classroom Observation Checklist

This study used a classroom observation checklist to assess teacher practices that promote critical thinking, complementing the data collected from the open-ended questionnaire. The checklist included indicators related to promoting critical thinking in the mathematics classroom, organized under specific categories. These indicators were adapted from El Yazidi's (2023) study, which identified key factors for effective critical thinking instruction based on Ennis's (2011) work and were validated for accuracy. The researchers used this checklist while observing the math classes of the three secondary teachers in the study. It allowed the researchers to systematically record and assess the teaching methods and activities that help foster critical thinking in students.

Data Gathering Procedure

With proper ethical considerations, the researchers developed a classroom observation checklist to assess teacher practices that promote critical thinking, along with open-ended questionnaires. To ensure the validity of the instruments, the researchers sought face validity from local mathematics education experts. The classroom observations were conducted over five days to examine factors promoting critical thinking in mathematics classrooms. During these observations, the researchers focused on the teaching practices of three different teachers across varying grade levels, using a checklist to record key factors such as questioning techniques, classroom activities, and instructional strategies. The checklist was designed to systematically capture these factors, ensuring accurate and consistent data collection through recorded and transcribed notes.

Following the observations, the teachers were invited to complete an open-ended survey questionnaire aimed at gathering insights into their teaching methods, instructional strategies, and approaches to fostering critical thinking in their classrooms. They were informed that their responses would contribute to understanding how their teaching practices influence students' critical thinking skills. After data collection, the information was organized and analyzed to draw accurate conclusions. In line with ethical guidelines, all data was kept confidential and anonymous, with no personal information shared. The researchers adhered to the school’s regulations, and any misconduct would result in the revocation of the agreement. Throughout the process, the researchers maintain transparency regarding the study’s objectives, procedures, and any potential risks.

Data Analysis

To analyze the data gathered from the classroom observation checklist, which assessed teacher practices promoting critical thinking and the open-ended questionnaire, the researchers employed thematic analysis. The purpose of thematic analysis is to identify, classify, analyze, and report patterns (themes) within the data set and to interpret their meaning and importance (Braun, Clark & Weather, 2016). In conducting the thematic analysis, the researcher (i) familiarised the data; (ii) generated initial codes; (iii) searched for themes; (iv) reviewed themes; (v) defined and named themes; and (vi) produced a report (Braun & Clark, 2006).

Coding of Data

To maintain confidentiality, the three teacher participants were assigned pseudonyms. They were labeled as Teacher A (TA), Teacher B (TB), and Teacher C (TC). This approach ensured anonymity while allowing for a meaningful comparison of teaching practices across varying experience levels.

Teacher A

Teacher A is a female educator with 24 years and 9 months of teaching experience, holding a Master of Science in Education (MSciEd). Throughout her career, she has demonstrated versatility and expertise by teaching a range of mathematics subjects, including Elementary Statistics, Elementary Algebra, Intermediate Algebra, and Pre-calculus. Currently, she is handling Elementary Statistics and Advanced Algebra, continuing to apply her deep knowledge and instructional skills in the classroom. Her commitment to professional growth is evident in her active participation in all Project STAR training programs under the Department of Science and Technology – Science Education Institute (DOST-SEI), which aim to enhance the quality of science and mathematics education in the country. Her extensive experience, academic qualifications, and dedication to continuous learning make her a highly competent and impactful mathematics educator.

Teacher B

Teacher B is a male educator who holds a Master of Science in Education (MSciEd) degree and has accumulated 3.5 years of teaching experience. Despite being relatively new in the profession, he has taken on a wide array of teaching responsibilities across different grade levels, showcasing his adaptability and strong foundation in mathematics education. He currently teaches Algebra to students in Grades 7 and 9, Pre-calculus to both Grades 10 and 11, and General Mathematics to Grade 11 students. His role in handling both junior and senior high school classes reflect his competence in delivering complex mathematical concepts to learners at various stages of their academic journey. With a solid academic background and growing teaching experience, Teacher B demonstrates promise as an effective and committed mathematics educator.

Teacher C

Teacher C is a highly qualified female educator who holds a Doctor of Philosophy in Mathematics, specializing in Pure Mathematics. Despite having only two years of teaching experience, she has quickly established herself as a capable and knowledgeable instructor. She currently teaches Trigonometry to Grade 9 students, as well as General Mathematics and Pre-calculus to Grade 11 learners, demonstrating her ability to handle a diverse range of mathematical topics across different levels. Her deep academic background enables her to present complex mathematical concepts with clarity and rigor. In addition to her teaching responsibilities, Teacher C actively engages in mathematics education conferences and research, reflecting her commitment to advancing the field and continuously improving her instructional practice. Her blend of scholarly expertise and emerging classroom experience positions her as a promising contributor to both mathematics education and academic research.

CHAPTER IV

RESULTS AND DISCUSSION

This chapter presents, analyzes, and discusses the data gathered on the mathematics teachers' practices and the factors that promote critical thinking through the open-ended questionnaire and lesson observations.

Mathematics Teachers' Practices in Promoting Critical Thinking

This section presents and discusses the practices of Teacher A, Teacher B, and Teacher C, as observed over five days, that promote critical thinking in the mathematics classroom. The observations were conducted over five days using a structured checklist, which categorized several indicators under key focus areas.

Focusing on Specific Critical Thinking Skills

- ***Analyzing Mathematical Arguments***

  Indicator A.1.3: *The teacher applies the Socratic method to elicit deep thinking and mathematical exploration.*

Teacher A demonstrates the use of the Socratic method to teach adding and subtracting polynomials by guiding the student through a process of inquiry and discovery, rather than directly providing answers. Teacher A uses strategic questioning to guide the students toward discovering the answer themselves. This approach encourages the student to think critically and analyze thoroughly:

> ***TA****: So, look at the figure. What shape is that?*
> ***S****: Rectangle.*
> ***TA****: How did you know that it is a rectangle?*
> ***S:*** *Kay dili man even ang sides ma'am. (Because the sides are not even, ma'am.)*
> ***TA:*** *What do you mean nga dili even ang sides? (What do you mean that the sides are not even?)*

Moreover, Teacher B effectively employs the Socratic method in teaching arithmetic sequences by initiating the lesson with thought-provoking questions that challenge students' initial understanding. Through a series of guided prompts, teacher B encourages critical thinking and deeper exploration, fostering mathematical reasoning and conceptual understanding.

> ***TB:*** *How do we establish 'common difference'?*
> ***S:*** *E-minus nimo ang succeeding terms, let's say "$a_2 - a_1, a_3 - a_2$" . (You just subtract the succeeding terms, let's say "$a_2 - a_1, a_3 - a_2$")*
> ***TB:*** *Difference raman ato gistoryahan , so unsay operation? (We are just talking about difference, so what is the operation?)*
> ***S:*** *Subtraction.*
> ***TB:*** *What operation do you use?*
> ***S:*** *Addition.*
> ***S:*** *Sir connected raman si addition ug subtraction. (Sir, addition and subtraction are connected.)*
> ***TB:*** *Pwede ra gihapon , but again, we have to prove that they have a common difference; wala man siya nag-ingon na common sum. So ning ana pag establish ang common difference. (That's still okay, but again, we have to prove that they have a common difference; it didn't say common sum. So this is how we establish the common difference.)*
> ***TB:*** *"How do we do that? Eh-establish sa nimo ang usa ka difference and we need one more para maka ingon tag naay common. So unsay gamiton nato? " (How do we do that? First, establish one difference, and we need one more so we can say there is a common one. So what do we use?)*
> ***S:*** *"We can subtract a_3 -a_2 ."*

The excerpts from Teachers A and B show how both educators used a Socratic method—asking leading questions instead of giving direct answers—to help the student come to their own understanding of the concept. These findings are supported by the study of Franke et al. (2007), who found that eliciting student thinking through strategic questioning supports conceptual development in mathematics; Chin (2007), who showed how Socratic questioning fosters deep thinking and encourages students to explain and reflect on their reasoning; and Walshaw and Anthony (2008), who emphasized the importance of teacher-led dialogue in promoting mathematical understanding and exploration.

- ***Identifying Assumptions***

Indicator A.2.4: *Higher-order thinking questions (e.g., evaluation, synthesis, and analysis) are regularly incorporated into lessons to challenge students.*

Teacher A presents a problem requiring higher-order thinking skills about permutations, in which students must analyze, evaluate, and justify the correct solution among two presented options. Teacher A challenges the students to study both solutions carefully, determine which one is correct, and explain their reasoning. This type of question moves beyond simple recall and computation, as it demands evaluation of mathematical reasoning and application of combinatorial concepts.

> *"In how many ways can a man distribute gifts consisting of Php 500.00, one skateboard, and one bicycle to 5 boys?"*
>
> - *Solution 1: $3\times3\times3\times3\times3 = 243$ ways*
> - *Solution 2: $5\times5\times5=125$ ways*
>
> *Study each "solution". Describe each why it is what it is. Decide which is correct and explain why.*

Additionally, Teacher B demonstrates a strong emphasis on fostering higher-order thinking skills in teaching geometric sequences by consistently using questioning strategies that encourage students to go beyond recall and engage in analysis, evaluation, and creation.

> ***TB:*** *I think group 7 would like to disagree. Sigi daw can we ask group 7? Sigi naa moy gisulat. Sigi, what do you observe on number 1? Oh, they are saying this is arithmetic. Why are you clustering together no. 1 and no. 5(I think Group 7 would like to disagree. Okay, can we ask Group 7? Okay, you have written*

something. Okay, what do you observe on number 1? Oh, they are saying this is arithmetic. Why are you clustering together no. 1 and no.5?)

***TB:** Can we ask Group 8 nganong gilahi ninyo si number 4? (Can we ask Group 8 why did you separate number 4?)*

***TB:** Giunsa sya pagsulat, gi divide tama? In fractions, is there any other way we can describe that? Unsay pagsulat sa rational number? (How is it written, is the division correct? In fractions, is there any other way we can describe that? How do you write a rational number?)*

***TB:** What do you call this one? Is it an integer or a fraction? (What do you call this one? Is it an integer or a fraction?)*

***TB:** Kinsay gaanser ug dili fraction? Justify nganong inyung answer kay fraction.(Who answered that it is not a fraction? Justify why your answer is a fraction.)*

***TB:** If you want to come up with unsaon sya pag arrive sa formula. Ok inyung himoon tomorrow kay esolve ninyung pattern without using the formula. (If you want to come up with how to arrive at the formula. Okay, you will do it tomorrow by solving the pattern without using the formula.)*

Furthermore, Teacher C incorporates higher-order thinking skills to some extent into the lesson on trigonometric identities by asking reflective and application-based questions, but the approach leans more toward guided practice than independent critical thinking.

***TC:** "But remember, when we take the square root of 5/9, we're solving for sin u. Since sec u < 0… Okay, here it says tan u > 0… What does that mean?"*

***TC:** "So what should be the sign?"*

***TC:** "Now that we have values for cos u, sin u, and sec u, can we find the remaining values?"*

This excerpt shows that Teachers A, B, and C incorporated higher-order thinking by using strategic questioning that promoted evaluation, analysis, and synthesis. Their lessons encouraged students to compare solutions, justify reasoning, and apply concepts across different contexts. This approach aligns with research highlighting the role of guided questioning in developing students' critical thinking and reasoning skills (Mercer & Sams, 2006; Chin, 2007).

- ***Evaluating Evidence***

Indicator A.2.5 *Conceptual understanding is prioritized over procedural fluency, allowing students to build a deeper grasp of the content.*

This classroom interaction from Teacher B illustrates Indicator A.2.5, where conceptual understanding is prioritized over procedural fluency. Instead of simply giving definitions, Teacher B encourages students to explore and reason through the classification of numbers like 100/10. Through guided questions, students reflect on whether such numbers are integers or fractions and justify their answers. The discussion highlights that understanding math involves more than solving—it requires interpreting and explaining concepts. By focusing on reasoning rather than memorization, the teacher helps students build a deeper grasp of the content**.**

> ***TB:*** *Ok, ang tawag sa 2 kay common difference. But this time, unsa may tawag nato sa 10? (Okay, 2 is called the common difference. But this time, what do we call 10?)*
> ***S:*** *Common quotient.*
> ***TB:*** *Common quotient, daw. Technically, yes. (Common quotient, they said. Technically, yes.)*
> ***S:*** *Common product.*
> ***TB:*** *Giunsa sya pagsulat, gi divide tama? In fractions, is there any other way we can describe that? Unsay pagsulat sa rational number? When we are talking with rational numbers, unsamay may naa sa denominator? (How was it written, divided, right? In fractions, is there another way to describe that? How do you write a rational number? When we talk about rational numbers, what's in the denominator?)*
> *S8: Integers. Whole numbers.*
> ***TB:*** *Integers are basically whole numbers. Tama? Question: Asa ninyu na makita na rational numbers? In other words, 100/10 is an integer. (Integers are basically whole numbers, right? Question: where do you see rational numbers? In other words, 100/10 is an integer.)*
> ***S:*** *Yes.*

S: *Fraction.*
TB: *Nganong fraction man sya? (Why is it a fraction?)*
S: *All integers kay fraction man, dapat all fractions are integers. (All integers are fractions, so all fractions should be integers.)*
S: *All integers are fractions man, but not all fractions are integers. (All integers are fractions, but not all fractions are integers.)*
S: *Pwede man na nimo mahimong integer into fraction. (You can turn an integer into a fraction.)*
TB: *If muingon kag common fraction kay mainggon kay common… ratio. Pero, we go back to the definition. What do we call this one? Is it an integer or a fraction? (If you say common fraction, that's like saying common ratio. But let's go back to the definition. What do you call this—an integer or a fraction?)*
S: *No!*
S: *Dili lagi sya pwede ma both. (It can't be both.)*
TB: *Kinsay gaanser ug dili fraction? Justify nganong inyung answer kay fraction. (Who answered that it's not a fraction? Justify why your answer is a fraction.)*
S: *Naay number sa taas, naa poy number sa ubos. (There's a number on top and another below.)*
S: *Sir... ug lantawon nimo siya, sir ba, diba fraction kay naay numerator ug denominator? (Sir… if you look at it, isn't it a fraction because it has a numerator and a denominator?)*
S: *Integer lagi gihapon kay ma 10. (It's still an integer because it becomes 10.)*
TB: *Sigi nganong integer? (Okay, why is it an integer?)*
S: *Actually sir, that is just an unsimplified na fraction. (Actually sir, that's just an unsimplified fraction.)*
S: *Unya, unsa may tawag sa unsimplified na fraction? (So, what do you call an unsimplified fraction?)*
S: *Fraction.*
TB: *Ug tan-awon nimo ang 100/10, unsa sya? (If you look at 100/10, what is it?)*
S: *Fraction.*
S: *Mao na ang goal sa math. Tan-awon nimo. (That's the goal in math. You observe it.)*
S: *It's not something you look at. It's something to solve. (It's not something you just look at. It's something to solve.)*
TB: *Nag overthink rajud mo. Ok, if you are to classify a number gani, you only need to look at its form. Basta ang fraction jud kay ratio of two integers. (You're just overthinking. If you classify a number, just look at its form. A fraction is a ratio of two integers.)*

This conversation from teacher B shows that students evaluated their peers' arguments and countered them with their own logic, weighing definitions and classification techniques. Hence, this indicates that A.3.5: Students are stimulated to question each other's reasoning and consider alternative viewpoints.

Indicator A.3.3: *The teacher encourages students to think meticulously about their methods, test hypotheses, and draw conclusions independently.*

Teacher A challenges the students to carefully study each solution presented on the board. The students are encouraged to describe the reasoning behind each solution and assess whether the calculations and logic are valid based on the given problem. This task not only helps students understand the mathematical process but also promotes critical evaluation of problem-solving approaches. Through this activity, students develop essential skills such as hypothesis testing and logical reasoning

> *"In how many ways can a man distribute gifts consisting of Php 500.00, one skateboard, and one bicycle to 5 boys?"*
>
> - *Solution 1: 3×3×3×3×3 = 243 ways*
> - *Solution 2: 5×5×5=125 ways*
>
> *Study each "solution". Describe each why it is what it is. Decide which is correct and explain why.*

Also, Teacher B effectively facilitates the transition from arithmetic to geometric sequences by presenting a series of ten problems on the board. Throughout the lesson, students are encouraged not only to solve the problems, but also to justify their reasoning, identify patterns, and clarify any misconceptions. This instructional approach promotes a deeper understanding of the concepts by pushing students to evaluate their thought processes critically. Moreover, it fosters a collaborative learning environment where students work together, share strategies, and engage in meaningful mathematical discussions.

TB: *Now narrowing it down, 3, 5, and 9. Sige, pwede masulat sa right side of the board? (Okay, can you write it on board) What do you notice about that kind of sequence?*

S: *Geometric.*

TB: *Sabi nila geometric daw. (They say its geometric.) What do you mean by geometric?*

S: *Iyang common difference, sir kay either gi multiply or gi divide. (Sir, its common difference is either multiplied or divided.)*

TB: *So, what do you call it if it's multiplied?... Unsay pagsulat sa rational number? (How do we write rational number?)*

(Students continue debating whether certain numbers are fractions or integers, leading to classification and justification.)

Moreover, Teacher C encourages students to use their own methods in solving the problem, since the topic is about trigonometric identities, which can be approached in various ways. Teacher C incorporates guided questions to help students think critically and justify their reasoning.

TC: *So, what is your first step?*

S: *Reciprocal secant, ma'am.*

TC: *Correct. Now we have cos u, sec u. What's missing?*

S: *Sine.*

TC: *What identity can we use?*

S: *Pythagorean.*

TC: *Correct. Isolate sin²u. Simplify 1 – 4/9… What's the value? ... Okay, now is sin u negative or positive?*

(Students debate quadrant signs and justify the correct trigonometric values.)

These conversations from Teacher A, B, and C show that the teachers encouraged students to think critically about their problem-solving strategies, test assumptions, and draw conclusions based on analysis. The students were required, not only to solve problems but to explain reasoning, justify answers, and engage in mathematical discourse. The findings are supported by the research of Schoenfeld (1992), which emphasizes the importance of metacognitive engagement and exploratory reasoning for deepening students' understanding. Additionally, the approach aligns with and is supported by Mercer and Sams (2006), who highlight the crucial role of dialogue and reflection in developing logical and independent thought.

Indicator A.3.4: *The teacher encourages students to evaluate their problem-solving strategies and assess the effectiveness of their methods.*

Teacher A is clearly guiding students not just to check whether their answers are correct, but to reflect on their own problem-solving process and revise their work when necessary. In addition, teacher A encourages the student to go through the process again independently to deepen understanding and reinforce learning. Rather than simply accepting the correct answer, the student is required to reconstruct the solution, which is a powerful strategy for assessing the effectiveness of their initial approach. This is evident in the teacher's instruction:

> **TA**: *After checking, show your solution to your groupmates. Pag tan-aw nimo sa imong answer kay wrong, try to answer again. If dili gihapon mao, try to ask your groupmates on how they got their answer. (When you look at your answer and it's wrong, try to solve it again. If it's still wrong, try asking your groupmates how they got their answer.)*
>
> **TA***: Ok, ipa-solve syag utro, Nakita na man ang answer key. Ipa try syag answer na siya ra. (Alright, make him solve it again. The answer key was already shown. Let him try to answer on his own.)*

Moreover, Teacher B effectively demonstrates Indicator A.3.4 by encouraging students to reflect on and justify their problem-solving strategies. The teacher prompts a group to explain why they categorized a particular item differently from others, initiating a thoughtful discussion on their reasoning. The students respond by articulating the basis of their classification, showing that they used structural features of algebraic expressions as a guiding principle. By asking clarifying questions and encouraging peer feedback, the teacher creates an environment where students are not only expected to arrive at correct answers but also to evaluate the validity and effectiveness of their methods.

> ***TB:*** *Group 8, nganong gilahi ninyo si number 4? (Why did you separate number 4?)*
>
> ***S:*** *Ang idea man good namo sir, kay diba ang set A kay contains only binomial factors… SET B kay contains only trinomial factor…(Our idea is good, sir, because set A contains only binomial factors... Set B contains only trinomial factors).*
>
> ***TB:*** *So, number 8 is the same as number 4 and number 5 as well. Can we ask group 8 nganong gilahi ninyo si number 4? (So number 8 is the same as number 4 and number 5 as well. Can we ask group 8 why you divided number 4?).*

(Students explain how they categorized items based on structure and justify their classification.)

Furthermore, Teacher C demonstrates Indicator A.3.4 by guiding students to apply previously found values to discover additional trigonometric functions, prompting them to evaluate and extend their problem-solving strategies. After confirming the students' understanding of the given values, the teacher assigns a task that challenges them to find the remaining trigonometric functions. Rather than providing direct instruction, the teacher allows students to explore different solution paths, promoting independent thinking and deeper engagement. As students present their varied approaches in front of the class, they not only assess the effectiveness of their methods but also learn from each other's reasoning.

> ***Teacher:*** *Now that we have values for cos u, sin u, and sec u, can we find the remaining values?*
>
> ***S:*** *Yes.*
>
> ***Teacher:*** *Okay, that's your task.*
>
> *(The teacher then gives a new problem. Students solve and share different solutions in front of the class.)*

These conversations from Teacher A, B, and C show that the teachers actively prompted students to analyze their solution approaches, justify classifications, compare alternative methods, and reflect on the accuracy and logic of their strategies. The findings regarding this form of metacognitive engagement, which fosters deeper mathematical thinking and the development of independent evaluative skills, are supported by Schoenfeld (1992), emphasizing the importance of structured reflection and reasoning in mathematical problem-solving, and by Mercer and Sams (2006), who noted the benefits of dialogic teaching in promoting critical evaluation.

- ***Encouraging Logical Reasoning***

Indicator A.4.3: *The teacher models metacognitive strategies by verbalizing the thinking process.*

Teacher A demonstrates metacognitive modeling by guiding students to think critically about how they identify and analyze geometric shapes. Rather than simply confirming that the figure is a rectangle, the teacher asks clarifying questions like, "What do you mean nga dili even ang sides?" and models how to measure the sides and calculate the area. This encourages students to reflect on the properties of the shape and connect their observations to mathematical concepts. By verbalizing each step and prompting students to recall and apply the area formula, the teacher shows how to approach problem-solving thoughtfully and systematically. This process helps students understand, not just the answer, but the reasoning behind it.

TA: *So, look at the figure. What shape is that?*
S: *Rectangle.*

TA: *How did you know that it is a rectangle?*

S: *Kay dili man even ang sides, ma'am. (Because the sides are not even, ma'am.)*

TA: *What do you mean nga dili even ang sides? (What do you mean that the sides are not even?)*

(Teacher continues guiding the student by modeling how to measure the sides and calculate the area.)

TA: *Unsa pud ning nasa sulod? (What is this inside?) What figure?*

S: *Rectangle, ma'am.*

TA: *Still a rectangle, tama? (Still a rectangle, right?) What is the area again of a rectangle?*

S: *Length times Width.*

Teacher B models metacognitive strategies by walking students through the process of identifying arithmetic patterns and understanding mathematical forms such as fractions. The teacher prompts students to recall definitions, like arithmetic as repeated addition, and explains how this leads to an arithmetic series. When discussing fractions, the teacher encourages students to think about how fractions are formed, how they can be interpreted, and how to distinguish them from integers. Through continuous questioning and explanation, the teacher verbalizes how to analyze the structure of numbers and reflect on their classification. This not only builds content knowledge, but also supports students in developing habits of thinking critically about the math they encounter.

TB: *Now if you look at numbers 1 and 7, it's arithmetic. (Now if you look at numbers 1 and 7, it's arithmetic.)*

TB: *What's the definition of arithmetic again? Repeated addition. Tama?*

(What's the definition of arithmetic again? Repeated addition, right?)

TB: *Pag i-add ninyo na tanan, then you can come up with arithmetic series. (If you add them all up, then you can come up with an arithmetic series.)*

TB: *Giunsa sya pagsulat, gi-divide tama? In fractions, is there any other way we can describe that? (How is it written, it's divided, right? In fractions, is there any other way we can describe that?)*

(Teacher explains by walking through each step and form of the number, prompting metacognitive reflection.)

Meanwhile, Teacher C uses metacognitive modeling during a trigonometry lesson by guiding students through the process of solving identities step-by-step. The teacher starts with a reciprocal identity and encourages students to substitute values, then carefully walks through the simplification and evaluation of expressions. By asking what is still missing and referring to the quadrant to determine the correct sign, the teacher highlights key decision points in the problem-solving process. This method helps students recognize how and when to apply mathematical rules and strategies. By verbalizing the logic behind each move, the teacher makes thinking visible and teaches students how to monitor and adjust their own reasoning when solving similar problems.

TC: *Okay, we can use the reciprocal identity. So, we have secant u, cos u = 1/sec u. So, can we then?*

S: *Substitute.*

TC: *Correct! Now we substitute. Okay, what's the value of sec u? 1 / -3/2 is equivalent to?*

S: *-2/3.*

TC: *Very good. So naa natay value sa cos u ug sec u. Unsa pa may kulang? (What's still missing?)*

S: *Sine.*

TC: *Correct. So, let's use the Pythagorean identity… simplify 1 - 4/9… square root… now we check which sign applies based on quadrant.*

These conversations from Teacher A, B and C show that the teachers modeled metacognitive strategies by verbalizing their thinking processes—breaking down problems, stating what to do next, and articulating why certain operations or identities are used. The students' ability to internalize a systematic and reflective approach to problems is supported by the research of Mercer & Sams (2006). Their work highlights the importance of structured teacher dialogue in developing students' metacognitive and reasoning abilities through modeled thinking.

Indicator A.4.4: *Time is allocated for students to discuss their thought processes and critique each other's reasoning.*

Teacher A effectively provides students with structured time to reflect on and discuss their thought processes by instructing them to return to their groups and review their mistakes. The teacher encourages peer discussion by allowing the use of whiteboards and explicitly directing students to focus on the incorrect parts of their work. By telling them not to be shy and to engage actively, the teacher fosters a safe and collaborative environment where students are motivated to analyze their reasoning and learn from one another's errors.

TA: *Alright, return the papers. Sit with your groupmates and discuss. You may use the boards to discuss. And please come to me if there's anything you don't understand.*

(The students moved to sit with their groups, grabbed whiteboards, and began discussing.)

TAr: *Tan-awa ninyo ang papel ninyo, tapos katong inyong i-discussan katong mali. (Look at your papers and discuss the parts where you made mistakes.)*

TA: *Walay mag ulaw-ulaw. Pagdali na, katong mali ang i-discuss. (Don't be shy. Hurry up and discuss the incorrect parts.)*

Teacher B promotes critical dialogue among students by inviting groups to respond to and challenge each other's reasoning. For example, when Group 7 disagrees with another group's sequence classification, the teacher encourages them to explain their perspective and asks the original group to consider the critique. This exchange supports students in evaluating the logic behind different answers. The discussion extends to identifying patterns and structures, helping learners refine their understanding while practicing respectful and analytical peer interaction.

TB: *I think group 7 would like to disagree. Can we ask group 7?*

S: *We disagree... A2 and A3, when subtracted, gives 4, murag familiar kaayo sya na pattern.*

TB: *So, can we ask Group 5 if they are part of the same set?*

(Groups continue to share their views and critique the groupings of sequences.)

TB: *Okay, thank you, group 5. Now, narrowing it down, 3, 5, and 9. Sige, pwede masulat sa right side of the board? What do you notice about that kind of sequence?*

Meanwhile, Teacher C facilitates the student-led discussion by allowing learners to present their solutions to trigonometric problems and engage in classroom-wide reflection. After solving identities in groups, students are encouraged to share their approaches and justify their reasoning in front of the class. The teacher creates a space for students to respond to one another, defend their methods, and make adjustments based on feedback. This setup allows for active engagement and peer critique, strengthening both content mastery and collaborative thinking.

(After solving trigonometric identities in groups)

Teacher: *Now that we have cosine u, sine u, and secant u… Can we find the remaining values?*

All: *Yes.*

(Students then share their different solutions in front of the class and discuss.)

(Teacher allowed students to defend and refine their reasoning based on others' input.)

These excerpts from Teacher A, B, and C show that the teachers intentionally created opportunities for peer discussion and critique, allowing students to reflect on their reasoning, share methods, and assess one another's thinking. The deeper conceptual understanding and development of critical thinking fostered by this collaborative analysis are supported by the research of Mercer and Sams (2006), who affirm that such dialogic interaction enhances students' ability to reason logically and engage constructively with diverse perspectives.

Providing Practice Opportunities

- ***Real-World Problem-Solving***

Indicator B.1.4*: Students regularly work on problems that require creativity, fostering self-confidence and independent reasoning.*

Teacher A encourages student creativity and independent thinking during a lesson on permutations by assigning students to design their own problem sets. Instead of relying solely on textbook problems, each group was tasked with creating three original permutation-related problems and then solving them. This allowed learners to apply their conceptual understanding in novel ways and reflect on the logic behind the problems they themselves created. The teacher provided support through group discussions and facilitated peer evaluation of the constructed problems.

Each group is assigned to create a set of problems related to their report. They are given the time to answer.

TA: *Ok, proceed to group discussion.*
(Four groups posted three sample problems each related to the topic. Students were given 10 minutes to answer, followed by group discussion and reflection.)

This task structure demonstrates how Teacher A fosters student ownership of learning. Students were not only solving problems but creating them—a process that involves a higher level of engagement and creativity, leading to deeper mathematical reasoning and confidence in their ability to explore multiple solution paths.

Moreover, Teacher B also promotes independent thinking during a lesson on arithmetic sequences. After reviewing how to establish a common difference, Teacher B assigned an open-ended task that allowed students to explore multiple strategies for identifying sequences and determining patterns. Rather than prescribing a fixed approach, the task invited student interpretation and problem-solving autonomy.

TB: *How do we establish 'common difference'?*
S: *eminus nimo ang succeeding terms…(You subtract the succeeding terms…)*
TB: *Now, if you establish that the common difference is -3 using these different terms, can you put there 2 points?*
(Later, students were given an open task that allowed multiple solution strategies.)
After that, the teacher gives an open task that allows students to explore multiple solutions to the given problem.

This approach supports divergent thinking and encourages learners to take intellectual risks—an important part of developing mathematical resilience. Students were engaged, not only in applying known methods, but in evaluating which methods best suited the context, an indicator of creative mathematical practice.

In addition, Teacher C supported creative reasoning in her trigonometry class by first modeling an example and then offering students an opportunity to solve a related problem using multiple identities. During the problem-solving task, students were free to choose among reciprocal and Pythagorean identities, depending on their interpretation of the given expression. This is seen in the given exchange:

> *(During the activity on trigonometric identities)*
> **Teacher:** *Now here is an example from the book… I'll give you an example first, then I will give you time to answer the next example.*
>
> *(Students worked collaboratively and were encouraged to use different identities to solve problems. They presented their own solutions in front of the class.)*

Teacher C's practice supports open-ended reasoning and mathematical flexibility. By accepting different solution strategies, the teacher sends the message that multiple paths can lead to a valid solution, fostering both creativity and self-confidence in problem-solving.

These conversations from Teacher A, B, and C show that the teachers created open-ended tasks and problem-posing activities that encouraged students to think creatively and independently. By designing their own problems, exploring multiple strategies, and explaining diverse solutions, students developed confidence and flexibility in reasoning. The findings from this method are supported by the research of Zimmerman (2009), which advocates for tasks that nurture creativity and autonomy in mathematical thinking. This approach also aligns with and is supported by Mercer and Sams (2006), who emphasize dialogic and exploratory learning as foundational for building critical thinking.

***Indicator** B.1.5: The teacher provides opportunities for students to work in an open-ended problems that require creativity and critical thinking.*

Teacher A demonstrates this indicator through an activity in which students are presented with a real-world inspired distribution problem related to permutations and combinations. The teacher presents two possible solutions to the problem and asks students to analyze each, identify their assumptions, and evaluate which approach is logically sound. The openness of the task—where more than one interpretation is plausible—requires students to use both critical thinking and creativity in analyzing the distribution scenario.

> *"In how many ways can a man distribute gifts consisting of Php 500.00, one skateboard, and one bicycle to 5 boys?"*
>
> - *Solution 1:* $3\times3\times3\times3\times3 = 243$ *ways*
> - *Solution 2:* $5\times5\times5=125$ *ways*
>
> *Study each "solution". Describe each why it is what it is. Decide which is correct and explain why.*

Here, students are not simply computing but are critically engaging with the logic of multiplication principles, evaluating whether the problem context calls for repetition, selection, or arrangement. This type of activity simulates real-world decision-making, where multiple problem-solving strategies must be compared and validated.

Similarly, Teacher B presents an open-ended task during a lesson on arithmetic sequences. After collaborative problem-solving with specific sequence items, students are tasked to continue solving additional problems in groups. The follow-up task allows for a range of solution strategies, inviting students to compare their reasoning and reflect on alternative approaches.

The class continues in answering the rest of the items such as 12, 14. But this time they answered it by group. Students are given an opportunity to discuss their solutions in front of the class.

After that, the teacher gives an open task that allows students to explore multiple solutions to the given problem.

Teacher B structures lessons to accommodate multiple solution paths, thereby fostering creativity and critical engagement. This approach necessitates that students articulate the underlying structure of arithmetic patterns and defend their reasoning to peers, reinforcing conceptual clarity and cognitive flexibility.

Similarly, Teacher C presents a trigonometry problem that encourages students to apply reciprocal and Pythagorean identities through diverse methods. This task promotes the exploration of identity transformations over algorithmic adherence, with the teacher explicitly sanctioning varied approaches.

TC: *Okay, that's your task.*

(The teacher then gave a new problem where students had to solve it using reciprocal and Pythagorean identities. Multiple solutions were accepted, and students explained their thinking in front of the class.)

Through this structure, Teacher C invites students to make mathematical decisions independently, evaluate identity usage based on the given information, and reflect on their reasoning in a public, constructive manner. The opportunity to explain their thought processes supports not only conceptual understanding but also confidence and metacognitive development.

These excerpts show that the teachers offered open-ended tasks and real-world problem contexts where students had to analyze, compare, and justify multiple solutions. Such tasks naturally require critical thinking and creative reasoning, as students evaluate various methods and articulate their understanding. The findings from this practice are supported by recommendations from educational research by Boaler (1998), which encourages exploratory and open problem-solving to deepen mathematical understanding. Furthermore, this practice aligns with and is supported by Mercer & Sams (2006), who emphasize dialogic and reflective practices in promoting logical and independent thinking.

- ***Collaborative Practice***

Indicator B.3.3: *Students engage in collaborative problem-solving activities, working with peers to discuss and critique each other's reasoning.*

This indicator is demonstrated through a structured group activity on subtracting polynomials. Rather than simply assigning individual seatwork, Teacher A asks students to sit with their groupmates, emphasizing mutual assistance and discussion. Importantly, the teacher guides students not to directly give answers but to support one another's thinking, which fosters shared accountability and critical interaction. Teacher A further encourages them to use tools such as whiteboards and even the windows to visualize and collaboratively solve problems:

> ***TA:*** *Can you please be seated with your groupmates? Para ma alalayan mo pag answer, pero ayaw edritso ug share ang answer. Para dili sila maka feel na alone sila. (So, you can help each other answer, but don't share the answer directly. This way, they won't feel alone.)*
>
> ***TA****: Dili magpa copy sa answer. Make sure na ga-answer sila. Maningkamot mo. (Don't allow copying of answers. Make sure they are answering. Work hard.)*
>
> *(The teacher roams around.)*
>
> ***TA:*** *If you need help, you go to them. Do not let them come to you. Nganong wala man mo gatapok? (Why aren't you gathering?)*
>
> ***TA:*** *Lantawa ang notes. Ipakita sa iyaa ang notes. Ok, mag check na ta sa 4.7. Kay mag-continue tag discuss sa distributive property. (Look at the notes. Show*

your notes to them. Ok, let's check 4.7. We will continue with the discussion of the distributive property.)

This kind of instructional guidance underscores the value of collective reasoning and cooperative problem-solving, creating a classroom culture that encourages students to take ownership of both their own learning and that of their peers.

In another example, Teacher B promotes collaborative classification and critique during a lesson on special products. Students worked in groups to categorize expressions and justify their reasoning using a Venn diagram. Teacher B prompted them to explain their decisions and to engage with contrasting viewpoints from other groups.

TB: *Group 8, nganong gilahi ninyo si number 4? (Why did you separate number 4?)*

S: *Ang idea man good namo sir, kay diba ang set A kay contains only binomial factors… SET B kay contains only trinomial factor. (Our idea, sir, is that Set A contains only binomial factors, while Set B contains only a trinomial factor.)*

(While the representatives solved the problem, the rest of each team's group was also given the opportunity to solve it at their seats.)

In this interaction, Teacher B enables students to articulate and critique mathematical reasoning. He follows up with comparison questions across groups, such as identifying overlaps in classifications and examining the rationale for divergent grouping strategies. This cultivates a space where students are expected to explain, defend, and reconsider their ideas in response to peer perspectives.

Similarly, Teacher C provides a collaborative setting for solving trigonometric identity problems. After working in groups to find values using reciprocal and Pythagorean identities, students present and explain their answers in front of the class.

The teacher facilitates dialogue among students, encouraging them to build on each other's ideas or correct misunderstandings collectively.

***TC**: That's your task. (After solving together)*

(Students solve a new problem using reciprocal identities in groups. Then, they present their answers to the class.)

The emphasis on student-led solution sharing reinforces the value of mathematical communication and the co-construction of knowledge. Students learn to compare multiple methods, justify their solutions publicly, and refine their reasoning based on peer input.

The excerpts above from Teacher A, B, and C show that the teacher created a setting where students not only solved problems but also created and discussed them collaboratively. The structured group discussion afterward helped students reflect on their answers together. The findings are supported by the study of Gillies (2016), who found that structured group discussions enhance student reasoning and understanding; Webb et al. (2014), who emphasized that students develop deeper mathematical understanding through peer interaction and explanation; and National Council of Teachers of Mathematics [NCTM] (2014), which advocates for collaborative learning environments that foster reasoning, critique, and shared problem-solving.

Providing Feedback

- ***Timely and Specific Feedback***

***Indicator** C.1.3 Formative assessments (e.g., quizzes, discussions) are used to monitor understanding and guide instruction.*

Teacher A demonstrates the use of formative assessment during a lesson on subtracting polynomials by administering a quiz and then immediately involving students in peer review and group discussion. Rather than ending the activity with score-checking, the teacher uses the quiz results as a springboard for collaborative reflection. She instructs students to exchange papers, check each other's work, and engage in peer dialogue about their mistakes—an approach that transforms assessment into an active learning opportunity.

The teacher administered a quiz.

TA: *Okay, time's up! Let's check. Exchange papers with your seatmate.*

(Teacher flashed answers on the TV screen for students to check their work.)

TA: *Add up the scores. Over 30. Make sure na naay names written ha? (Make sure names are written, okay?)*

TA: *Return the papers. Sit with your groupmates and discuss. You may use the boards to discuss. And please come to me if there's anything you don't understand.*

(Students discussed their mistakes in groups using whiteboards.)

This process shows that the quiz was not merely evaluative, but formative—used to reveal misunderstandings and redirect instruction. Students were empowered to identify and analyze their own mistakes with teacher support, thus reinforcing metacognitive awareness.

Teacher B incorporates formative feedback through regular board work during a lesson on arithmetic sequences. Homework is checked by calling on volunteers to show their solutions and explain their reasoning to the class. This not only allows the teacher to assess student understanding in real time but also opens opportunities to address misconceptions openly while affirming correct reasoning.

> *(The teacher checks assignments and calls for volunteers to show their solutions on the board and students explain their answers.)*
>
> ***TB:*** *How do we establish common difference?*
>
> *(Students respond and justify their methods; teacher adjusts follow-up based on their answers.)*
>
> *(The class continues answering more items and works in groups, followed by discussion of multiple solution strategies)*

As students explain and reflect on their process, the teacher evaluates their understanding not by simply correcting answers, but by probing their reasoning and adjusting instruction accordingly. This ongoing dialogue supports formative assessment as a tool for adaptive teaching.

Similarly, Teacher C opens the lesson on fundamental trigonometric identities by reviewing homework solutions on the board. The teacher listens to student responses and uses them to guide instructional next steps, tailoring the difficulty or focus of problems based on class performance. Later, students work collaboratively on a new trigonometric problem while the teacher circulates and observes their process, further using informal assessment to guide feedback and support.

> (*The teacher started the class by checking the assignment, calling some student representatives to show their solutions on the board.)*
>
> ***TC:*** *Now we have cos u, sec u… What's missing?*
>
> *(Students respond; teacher uses this to decide which concept to reinforce.)*
>
> *(Teacher C then gives a problem:)*

"Given csc x = -7/6, tan x > 0. Students are given 10 minutes to solve and present."

(Students solve collaboratively, discuss, and present answers to the class.)

These excerpts from Teacher A, B, and C show that the teachers used formative assessments such as quizzes, board work, and group discussions to check students' understanding in real time and adapt instruction accordingly. These practices help identify misconceptions early, provide immediate feedback, and support responsive teaching. The findings from this approach are supported by the research of Black and Wiliam (1998), who highlight formative assessment as a tool to promote learning and guide next instructional steps. This also aligns with and is supported by Mercer and Sams (2006), who emphasize using dialogue and feedback to enhance reasoning and understanding.

***Indicator** C.1.5 Feedback emphasizes effort, process, and progress over correctness.*

Teacher A exemplifies this indicator through her responses to student performance during a lesson on subtracting polynomials. Rather than highlighting errors as failures, she frames mistakes as learning opportunities and shifts attention toward understanding the process. After a quiz, students were grouped to revisit and discuss their errors collaboratively. The teacher made it clear that all ways of taking part, even just asking questions, were important for learning.

***TA:** Alright, return the papers. Sit with your groupmates and discuss. You may use the boards to discuss. And please come to me if there's anything you don't understand.*

***TA:** Tan-awa ninyo ang papel ninyo, tapos katong inyong i-discussan katong mali. (Look at your papers, and discuss the parts where you made mistakes.)*

TA: *Walay mag ulaw-ulaw. Pagdali na, katong mali ang i-discuss. (Don't be shy. Hurry up and discuss the incorrect parts.)*

TA: *Even if your contribution is just asking a question, pwede ra gihapon. (That's still valid.)*

By offering this kind of feedback, the classroom environment transforms from being about getting the right answer to understanding the process. It communicates to students that their learning is valued even if they do not immediately arrive at the correct answer, reinforcing persistence and growth mindset.

Teacher B, in a lesson on special products, provides feedback that invites clarification and deeper reasoning rather than offering direct corrections. When a student group presents an unconventional classification of mathematical expressions, the teacher does not dismiss the response, but instead, encourages justification and open explanation.

TB: *So far, kinsay group nag present ani, 1, 3, and 4? (Which group presented on 1, 3, and 4?)*

TB: *Can we ask group 8 nganong gilahi ninyo si number 4? (Why did you separate number 4?)*

(The teacher accepted various student interpretations and encouraged explanation rather than just confirming a correct answer.)

This method allows students to engage with their thinking more deeply and reflectively. By validating the reasoning process—regardless of whether it leads to the conventionally correct result—Teacher B builds students' confidence and reinforces that understanding evolves through discussion and reflection.

Teacher C similarly demonstrates process-focused feedback during a lesson on parabolas. Instead of simply evaluating answers as right or wrong, she prompts students to explain their reasoning publicly and construct mathematical meaning together. Even when students express initial misconceptions, the teacher guides them patiently toward clarity without judgment.

> ***TC:*** *You can ask questions ha... You can talk with your seatmates.*
> ***TC:*** *(referring to S1) ikaw ready naka? (Referring to S2) Ikaw, ready napud ka? (Are you ready? Are you also ready?) Okay, sige, pag bato bato pick mo... okay go (okay, do rock-paper-scissors… okay go)*
> *(Student 1 and 2 did a rock paper scissors)*
> ***TC:*** *Kinsay daog? (Who won? You?) Ikaw? (Referring to S2)*
> ***S****: Yes, maam.*
> ***TC:*** *Okay, ali. (Come here.)*
> *(While S2 is writing her answer on the board, teacher C asks the whole class.)*
> ***TC****: What's your p?*
> ***S****: Positive 2.*
> ***S****: Negative 2.*
> ***S****: 2*
> ***TC****: Ngano negative 2? Okay, sige daw bi. (Why negative 2? Okay, let's hear it.)*
> ***S****: iyang form maam kay $x^2 = -2y$, so second form sya maam. (Its form is $x^2 = -2y$, so it's in the second standard form, ma'am.)*
> ***TC:*** *So, your p is 2.*
> ***TC:*** *Kinsay naka-draw na ug graph? Tanan? (Who has drawn the graph? Everyone?)*
> *I-practice ninyo ug draw ang graph. (Practice drawing the graph. Try to estimate the distances evenly.)*
> ***TC:*** *Okay let's listen to S2's explanation.*
> ***S:*** *So ang given nga equation kay x^2 equals to -8y—*
> ***TC:*** *"is equal to…"*
> ***S:*** *x^2 is equal to -8y. Gipili nako ang equation sa second form kay ang equation sa second form kay -4py (I chose the equation in the second form because that's the one with -4py.)*
> *Gi-kuan nako sya. Nangita kog number nga if e times sa -4 kay muresult ug -8, which is 2 (I solved it by finding a number that, when multiplied by -4, results in -8, which is 2.). And then ang directrix is y=p, so nakuha nato ganina nga ang p kay 2, so mao nana sya ang directrix (And then the directrix is y = p. Earlier we got that p is 2, so that's the directrix.) And then the axis of symmetry kay x=0 (And then the axis of symmetry is x = 0.) Then opening direction niya kay downward and length of latus rectum kay absolute value of 4p (Then its opening direction is downward and the length of the latus rectum is the absolute value of 4p) so absolute value of -4(2) = -8, and since absolute value of -8 is 8. And then sa graph mao nani sya ang focus. Sakto ba, maam? (And on the graph, this is now the focus. Is that correct, ma'am?)*

> ***TC**: ay, iyang vertex. O dira kasagara ang mistake, dili mo mag draw sa inyong parabola sa focus ha. Dapat sa vertex. (Oh, the vertex. This is where most of the mistakes happen—you don't draw the parabola at the focus, okay? It should be at the vertex.)*

Rather than directly correcting the error, Teacher C uses the moment to reinforce a key concept while affirming the student's effort and partially correct reasoning. The teacher's language encourages the class to view mistakes as steps in the learning journey and clarifies the logic of graphing without embarrassment or judgment.

These excerpts from Teacher A, B, and C show that they gave feedback focused on student effort, self-correction, and the thinking process, rather than only correctness. Students were encouraged to analyze their own mistakes, justify reasoning, and feel safe making errors to grow. The findings from this approach are supported by the research of Hattie & Timperley (2007), which highlights the value of process-oriented feedback for fostering resilience, motivation, and critical thinking. This is also consistent with and supported by Mercer & Sams (2006), who emphasize reflective dialogue as key to developing student reasoning.

- ***Opportunities for Revision***

Indicator C.4.1: *The teacher allows students to revise their work based on feedback to strengthen their critical thinking and problem-solving approaches.*

Teacher A exemplifies this indicator by encouraging students to revise their work after receiving feedback, thus positioning errors not as endpoints but as opportunities for intellectual growth. During a class on subtracting polynomials, after students checked their answers, the teacher instructed them to revisit and correct their mistakes either independently or with peer assistance. This allowed students to reflect on the feedback they received and apply it to refine their understanding and solution methods.

> ***TA:*** *After checking, show your solution to your groupmates. Pag tan-aw nimo sa imong answer kay wrong, try to answer again. If dili gihapon mao, try to ask your groupmates on how they got their answer. (When you look at your answer and it's wrong, try to solve it again. If it's still wrong, try asking your groupmates how they got their answer.)*
>
> ***TA:*** *You can use your whiteboards. Pwede pod sa window mag write. (You can also write on the window.)*
>
> *(The students can discuss their answer to their groupmates.)*
>
> ***TA:*** *Class, kanang ga ask ug question ba, point na sya na nag-participate. (Class, those who are asking questions are already participating.) Show your solutions. Dili kay maglomba, tan-awa inyung mga mali. When someone is solving, tan-awa giunsa niya. (Don't compete, look at your mistakes. When someone is solving, see how they do it.)*
>
> ***TA:*** *Ok, ipa-solve syag utro, Nakita na man ang answer key. Ipa try syag answer na siya ra. (Alright, make him solve it again. The answer key was already shown. Let him try to answer on his own.) …*

This approach allows students to revisit their initial misconceptions and make meaningful corrections, thereby promoting deeper conceptual understanding and problem-solving resilience. Rather than focusing on a single chance at correctness, the emphasis is on learning through iteration and reflection.

Teacher B applies a similar strategy in an arithmetic sequence lesson. Instead of limiting the learning moment to the completion of a solution, Teacher B invites students to expand or improve on their previous work, even after sharing their answers. This shows a commitment to valuing mathematical reasoning as an ongoing process.

> ***S2:*** *Sir, pwede nako dagdagan ang solution, sir? (Sir, can I add to my solution?)*

TB: *Aw, ikaw gud... Ikaw bahala. (Oh, of course... It's up to you.)*

S3*: Excuse me, sir, pwede ko mag sulat sa number 2? (Excuse me, sir, can I write for number 2?)*

By encouraging additional contributions, Teacher B empowers students to refine and elaborate their thinking**.**

These excerpts from Teacher A and B show that the teacher encouraged the student to revise their work and try solving again independently even after seeing the correct answer, focusing on learning through correction. The findings are supported by the study of Hattie and Timperley (2007), who emphasized that effective feedback leads to student improvement when it is followed by opportunities to revise and re-engage with the task. They are also consistent with Black and Wiliam (2009), who stressed that formative feedback helps students reflect and adjust their strategies, and Butler and Winne (2005), who highlighted that revision based on feedback is essential for self-regulated learning and the development of critical thinking.

Using Active Learning Approaches

- ***Inquiry-Based Learning***

Indicator *D.2.4 Scaffolding techniques (e.g., guided questions or hints) help students engage with complex tasks.*

Teacher A utilizes scaffolding during a lesson on identifying shapes by guiding students through a series of prompts to analyze a figure. Rather than directly telling the answer, the teacher encourages students to observe and reason through the process. As the student initially identifies the shape as a rectangle, the teacher probes further by asking what the student means by “uneven sides,” prompting them to think critically. TA then draws attention to specific measurements and supports students in exploring the properties of the shape.

TA: *So, look at the figure. What shape is that?*
S: *Rectangle.*
TA: *How did you know that it is a rectangle?*
S: *Kay dili man even ang sides ma'am. (Because the sides are not even, ma'am.)*
TA: *What do you mean nga dili even ang sides? (What do you mean that the sides are not even?)*
S: *(analyzing)*
TA: *What is the measure of this side? (referring to the width of the big rectangle)*
Student: 4x.

This interaction shows teacher A scaffolding understanding through questioning rather than direct instruction. The student's reasoning is supported through step-by-step prompts, helping them engage more deeply with the task.

Additionally, teacher B demonstrates scaffolding strategies during a lesson on geometric sequences by prompting students to articulate their observations and define mathematical terms based on patterns. Rather than providing definitions outright, the teacher builds understanding through a sequence of guided questions that link observation with terminology.

TB: *What can you observe? What did you do to reach the next term?*
S*: You need to multiply the number.*

TB*: Okay, so you need to multiply the number to reach the next term. What do we call the constant number when we multiply a certain value?*
S: *Common ratio. Teacher: So, again, what is a geometric sequence?*
S: *It is a sequence where each term after the first is found by multiplying the previous one by a fixed, non-zero number.*

The teacher scaffolds learning by guiding students from concrete examples to abstract definitions. Through strategic questioning, students arrive at key concepts independently, demonstrating deeper engagement with the content.

These conversations from Teacher A, B, and C show that both teachers encouraged students to revise their work and try solving again independently even after seeing the correct answer, focusing on learning through correction. The findings are supported by the study of Hattie and Timperley (2007), who emphasized that effective feedback leads to student improvement when it is followed by opportunities to revise and re-engage with the task. The findings also align with Black and Wiliam (2009), who stressed that formative feedback helps students reflect and adjust their strategies, and Butler and Winne (2005), who highlighted that revision based on feedback is essential for self-regulated learning and the development of critical thinking.

- ***Collaborative Learning and Discussion***

Indicator D.4.5: *Students engage in collaborative problem-solving activities.*

Teacher A facilitates collaborative problem-solving by encouraging students to work in groups to analyze and correct their quiz errors. Rather than simply returning the papers for individual review, the teacher instructs students to sit with their group mates and use whiteboards to focus discussion on incorrect answers. The environment is made supportive by encouraging open discussion and emphasizing that mistakes are learning opportunities.

> ***TA:*** *Alright, return the papers. Sit with your groupmates and discuss. You may use the boards to discuss. And please come to me if there's anything you don't understand.*
> ***TA:*** *Tan-awa ninyo ang papel ninyo, tapos katong inyong i-discussan katong mali. (Look at your papers, and discuss the parts where you made mistakes.)*
> ***TA:*** *Walay mag ulaw-ulaw. Pagdali na, katong mali ang i-discuss. (Don't be shy. Hurry up and discuss the incorrect parts.)*

This setup enables students to collaboratively reflect on their errors and deepen their understanding through peer discussion and group reasoning.

Similarly, Teacher B fosters collaborative problem-solving by engaging students in a group classification task, where they justify the placement of mathematical expressions into different sets. The teacher facilitates inter-group dialogue, prompting students to explain their decisions and encouraging critical thinking across groups.

(While groups work on sorting expressions.)

TB: *Si group 8 kay gilahi nila si number 4. (Group 8 separated number 4.)*
TB: *Can we ask group 8 nganong gilahi ninyo si number 4? (Can we ask group 8 why they separated number 4?)*
S: *Ang idea man good namo sir, kay diba ang set A kay contains only binomial factors (1, 2, 3, 6, 7, 9, 10, 12, 13, 15) SET B kay contains only trinomial factor which is (4 5 8 14) ang 4 kay naa sa B kay ang 4 man only contain trinomial. (Our idea, sir, is that set A contains only binomial factors (1, 2, 3, 6, 7, 9, 10, 12, 13, 15), while SET B contains only trinomial factors (4, 5, 8, 14), and number 4 is in set B because it only contains a trinomial.)*
S*: Along with 5 and 8*

Students reason through their categorization and defend their logic with peers. This dialogue illustrates collaborative engagement, where students construct meaning together and clarify their thinking.

Meanwhile, Teacher C initiates a collaborative activity by organizing students into groups, assigning leaders, and preparing a shared task presented via PowerPoint. After giving instructions, the teacher ensures that each group works together on a problem based on a model example.

(Before the group task begins.)

TC: *Did you bring your whiteboard pen?*
TC: *Are you okay with group activity?*
S: *Yes*
(The teacher called some names that will serve as a leader in a group activity.)
TC: *Okay, count.*
S: *1 , 2…. 7.*

(The teacher assigned designated areas where students work collaboratively.)
TC: *Okay, ready na? (Okay, are you ready?)*
(The teacher presented a PPT and instructed students.)
TC: *Okay, that will be your task.*
(The teacher presented a problem as an example and read it from the PPT and each group to the task after the example presented)

This structured collaboration gives students a clear framework for solving problems together, promoting active engagement and shared responsibility in learning.

These excerpts from Teacher A , B and C show that both teachers encouraged students to revise their work and try solving again independently even after seeing the correct answer, focusing on learning through correction. The findings are supported by the study of Hattie and Timperley (2007), who emphasized that effective feedback leads to student improvement when it is followed by opportunities to revise and re-engage with the task; Black and Wiliam (2009), who stressed that formative feedback helps students reflect and adjust their strategies; and Butler and Winne (2005), who highlighted that revision based on feedback is essential for self-regulated learning and the development of critical thinking.

- ***Independent Thinking and Creativity***

Indicator D.5.3: *The teacher provides tasks that require independent critical thinking and creativity.*

Teacher A promotes independent critical thinking and creativity by assigning a task that requires students to construct and solve their own problems related to the topic. This approach encourages learners to apply conceptual understanding in novel ways while also fostering ownership of learning. After completing the task, students engage in group discussions to compare solutions and reflect on their reasoning. This not only deepens comprehension but also provides a space for diverse problem-solving strategies to emerge.

(*Teacher A assigned students to create and answer their own problems related to the topic. After solving, they had group discussions to reflect on their answers.)*

Teacher B challenges students' critical thinking through an open-ended group task. Each group is tasked with deriving the explicit formula of a sequence whose total sum is 784. The activity requires students to work backward from the given condition to determine the formula—an exercise in both analytical reasoning and creative problem construction. By leaving the solution path open, the task encourages exploration and application of multiple strategies, promoting flexible thinking.

(Teacher B presented an open-ended task in which each group was instructed to find the explicit formula of a sequence whose sum is 784.)

These excerpts from Teacher A and B show that both teachers encouraged students to revise their work and try solving again independently even after seeing the correct answer, focusing on learning through correction. The findings are supported by the study of Hattie and Timperley (2007), who emphasized that effective feedback leads to student improvement when it is followed by opportunities to revise and re-engage with the task; Black and Wiliam (2009), who stressed that formative feedback helps students reflect and adjust their strategies; and Butler and Winne (2005), who highlighted that revision based on feedback is essential for self-regulated learning and the development of critical thinking.

Classroom Environment

- ***Psychological Safety***

Indicator E.1.1: *The teacher fosters a psychologically safe classroom where mistakes are part of the learning process.*

Teacher A fosters a psychologically safe classroom by encouraging students to reflect on their thinking without fear of being wrong. She probes students' responses with clarifying questions instead of dismissing errors and allows the use of notes and phones to support learning. She also promotes peer support by encouraging students to sit with their group mates and discuss their answers together, emphasizing that it's okay to make mistakes and try again.

> ***TA****: You can open your notes while answering.*
>
> ***S****: Pwede maka-use ug phone, ma'am? (Can we use phones, ma'am?)*
>
> ***TA:*** *Sa phone? Oh, sige pwede ra. Pag connect mog internet. (On the phone? Oh, yes, that's fine. Connect to the internet.)*
>
> ***TA:*** *Can you please be seated with your groupmates? Para ma alalayan mo pag answer, pero ayaw edritso ug share ang answer. Para dili sila maka feel na alone sila. (So, you can help each other answer, but don't share the answer directly. This way, they won't feel alone.)*
>
> ***TA:*** *After checking, show your solution to your groupmates. Pag tan-aw nimo sa imong answer kay wrong, try to answer again. If dili gihapon mao, try to ask your groupmates how they got their answer. (When you look at your answer and it's wrong, try to solve it again. If it's still wrong, try asking your groupmates how they got their answer.)*

Teacher B creates a safe environment by accepting students' incorrect terms, like "common quotient," without directly correcting them. Instead, she builds on their responses with guiding questions that lead them to the correct concept, helping them refine their understanding without embarrassment.

(*Students openly made mistakes (e.g., calling common ratio "common quotient") and the teacher responded with follow-up questions rather than corrections.*

TB: *Ok, ang tawag sa 2 kay common difference. But this time, unsa may tawag nato sa 10? (Okay, 2 is called the common difference. But this time, what do we call 10?)*

S7: *Common quotient.*

TB: *Common quotient, daw. Technically, yes. (Common quotient, they said. Technically, yes.)*

S7: *Common product.*

TB: *Giunsa sya pagsulat, gi divide tama? In fractions, is there any other way we can describe that? Unsay pagsulat sa rational number? When we are talking with rational numbers, unsa may may naa sa denominator? (How was it written, divided, right? In fractions, is there another way to describe that? How do you write a rational number? When we talk about rational numbers, what's in the denominator?)*

Teacher C also maintains a supportive atmosphere by acknowledging errors calmly and moving forward with clarification rather than criticism. When a student corrects their own mistake, the teacher responds without judgment, reinforcing that mistakes are part of the learning process.

TC*: Yes, correct. Isolate sin* u^2 *then simplify 1- 4/9. What is 1 - 4/9?*

S1*: 5/9*

S2: *- 5/9*

TC: *Okay, 5/9.*

S2: *ayy 5/9 diay (Oh, it's 5/9.)*

(No shaming, continues with clarification)

The aforementioned excerpts from Teacher A, B, and C show that the teacher welcomed the student's misconception and responded with guiding questions instead of criticism, creating a safe space for learning through mistakes. Hence, this indicates that E.1.1: the teacher fosters a psychologically safe classroom where mistakes are part of the learning process. The findings are supported by the study of Hattie and Timperley

(2007), who found that feedback and questioning that promote understanding, rather than criticism, contribute to a safe environment where mistakes are seen as part of the learning process; Dweck (2006), who emphasized that fostering a growth mindset through encouragement of mistakes supports student learning and resilience; and Boaler (2016), who highlighted that mistakes are essential to the learning process in mathematics and should be embraced for deep understanding.

- ***Physical Space and Collaboration***

Indicator E.2*: Physical space is organized for collaborative learning with flexible seating and group work areas.*

Teacher A organizes the physical space to support collaborative learning by having students sit with their group mates to discuss their work. She encourages them to use available resources like the boards for group discussions and invites them to approach her with questions, creating an open and interactive learning environment

> ***TA:*** *Alright, return the papers. Sit with your groupmates and discuss. You may use the boards to discuss. And please come to me if there's anything you don't understand.*

Teacher B also promotes collaboration by engaging students in a Venn diagram activity where they work together to categorize problems. Through this group task, students share ideas, explain their reasoning, and learn from each other in a setup that supports flexible and cooperative learning.

> *(During the Venn diagram activity, students worked collaboratively to share ideas and make conclusions about categorizing problems, with Teacher B prompting them to explain their reasoning.)*

These excerpts from Teacher A and B show that both teachers encouraged students to revise their work and try solving again independently even after seeing the correct answer, focusing on learning through correction. The findings are supported by the study of Hattie and Timperley (2007), who emphasized that effective feedback leads to student improvement when it is followed by opportunities to revise and re-engage with the task; Black and Wiliam (2009), who stressed that formative feedback helps students reflect and adjust their strategies; and Butler and Winne (2005), who highlighted that revision based on feedback is essential for self-regulated learning and the development of critical thinking.

- ***Valuing Diverse Approaches***

Indicator E.3.1: *The classroom culture values diverse mathematical approaches and strategies.*

Teacher A fosters a classroom culture that values diverse mathematical approaches by presenting a problem with two different solutions and encouraging students to analyze, compare, and reflect on each method. This invites students to see that problems can be approached in different ways and promotes discussion about the reasoning behind each strategy.

> *(Teacher A presented a problem to the class with two different solutions already provided. She then asked the students to study both solutions, describe what each approach meant, determine which one was correct, and finally, share their thoughts on the different methods.)*

Teacher B reinforces this culture by posing an open-ended question—"How do we establish common difference?"—and then offering a task that allows for multiple solution paths. By doing so, she encourages students to explore various methods and strategies, recognizing that there is not always just one correct way to solve a problem.

> *(Teacher B asked, "How do we establish common difference?" Following this, she provided an open task that allowed students to explore multiple solutions to the problem.)*

These excerpts from Teacher A and B show that both teachers encouraged students to revise their work and try solving again independently even after seeing the correct answer, focusing on learning through correction. The findings are supported by the study of Hattie and Timperley (2007), who emphasized that effective feedback leads to student improvement when it is followed by opportunities to revise and re-engage with the task. They are also consistent with Black and Wiliam (2009), who stressed that formative feedback helps students reflect and adjust their strategies, and Butler and Winne (2005), who highlighted that revision based on feedback is essential for self-regulated learning and the development of critical thinking.

- ***Communication and Respect***

Indicator E.4.1: *Clear expectations are established for respectful communication and active listening.*

Teacher A creates a supportive environment by encouraging students to openly discuss their mistakes without hesitation. By inviting them to explain the incorrect parts of their work, she shifts the focus from getting the right answer to understanding the process. She also affirms that even asking a question counts as a meaningful contribution, which helps reduce fear of participation and promotes a classroom culture where reasoning, inquiry, and learning from errors are valued.

> ***TA:*** *Walay mag ulaw-ulaw. Pagdali na, katong mali ang i-discuss. (Don't be*

shy. Hurry up and discuss the incorrect parts.)

TA: *Okay, listen... This will be 1 pre-assessment. And even if your contribution is just asking a question, pwede ra gihapon (that's still valid).*

Teacher B invites a group of students to share their differing opinion on the sequence, recognizing their written work and encouraging them to explain their reasoning in front of the class. By asking probing questions like why they identified the sequence as arithmetic and why they grouped certain terms together, he creates an opportunity for students to justify their thinking and reflect on their observations. This strategy fosters classroom discussion, values student input, and emphasizes the importance of reasoning in understanding mathematical patterns:

TB: *I think group 7 would like to disagree. Sigi daw can we ask group 7? Sigi naa moy gisulat. Sigi, what do you observe on number 1? Oh, they are saying this is arithmetic. Why are you clustering together number 1 and number 5? (I think group 7 would like to disagree. Okay, can we ask group 7? You've written something. What do you observe about number 1? Oh, they are saying it's arithmetic. Why are you grouping together numbers 1 and 5?)*

S: *Sa pagbantay man gud nimo sa sequence, if e minus nimo si $a1$ ug $a2$ kay ang difference is 2. Whereas si $a2$ ug $a3$ ug e minus kay 4, murag familiar kaayo sya na pattern. (When you observe the sequence, if you subtract A1 from A2, the difference is 2. But A2 and A3, when subtracted, gives 4. It seems like a familiar pattern.)*

These excerpts from Teacher A and B show that both teachers encouraged students to revise their work and try solving again independently even after seeing the correct answer, focusing on learning through correction. The findings are supported by the study of Hattie and Timperley (2007), who emphasized that effective feedback leads to student improvement when it is followed by opportunities to revise and re-engage with the task. They are also consistent with Black and Wiliam (2009), who stressed that formative feedback helps students reflect and adjust their strategies, and Butler and Winne (2005), who highlighted that revision based on feedback is essential for self-regulated learning and the development of critical thinking.

- ***Communication and Respect***

Indicator E.4.2: *The teacher encourages students to share and justify their ideas in front of the class, emphasizing the importance of reasoning.*

Teacher A presents a problem with two different solutions and asks students to examine both approaches carefully. She then encourages them to analyze what each solution represents, determine which one is correct, and explain their reasoning. By having students share their thoughts with the class, Teacher A promotes an environment where critical thinking and mathematical reasoning are valued. This strategy helps students develop confidence in expressing their ideas publicly while also learning to justify their answers clearly and thoughtfully.

> *(Teacher A presented a problem with two different solutions. She asked students to study both solutions, describe what each meant, determine which one was correct, and then share their thoughts with the class.)*

Teacher B encourages students to explain their problem-solving process by asking how they approached the sequence and prompting them to identify patterns. He engages the class by inviting a specific group—Group 7—to share a differing viewpoint, highlighting their written observations and asking them to explain their reasoning. By questioning why the students categorized certain numbers together and prompting them to justify their thinking, Teacher B creates a space where multiple ideas are welcomed and students are encouraged to defend their reasoning in front of the class. This approach supports meaningful mathematical discourse and active participation.

TB: *Giunsa ninyu sya pag solve? Diba naa silay common, something. All you need to do is pangitaon ninyu iyahang center. You have 10 minutes to answer. (How did you solve it? They have something in common, right? All you need to do is find its center. You have 10 minutes to answer.)*

TB: *I think group 7 would like to disagree. Sigi daw can we ask group 7? Sigi naa moy gisulat. Sigi, what do you observe on number 1? Oh, they are saying this is arithmetic. Why are you clustering together number 1 and number 5? (I think group 7 would like to disagree. Okay, can we ask group 7? You've written something. What do you observe about number 1? Oh, they are saying it's arithmetic. Why are you grouping together numbers 1 and 5?)*

S*: Sa pagbantay man gud nimo sa sequence, if e minus nimo si* $a1$ *ug* $a2$ *kay ang difference is 2. Whereas si* $a2$ *ug* $a3$ *ug e minus kay 4, murag familiar kaayo sya na pattern. (When you observe the sequence, if you subtract A1 from A2, the difference is 2. But A2 and A3, when subtracted, gives 4. It seems like a familiar pattern.)*

These excerpts from Teacher A and B show that both teachers encouraged students to revise their work and try solving again independently even after seeing the correct answer, focusing on learning through correction. The findings are supported by the study of Hattie and Timperley (2007), who emphasized that effective feedback leads to student improvement when it is followed by opportunities to revise and re-engage with the task. The findings are also consistent with Black and Wiliam (2009), who stressed that formative feedback helps students reflect and adjust their strategies, and Butler and Winne (2005), who highlighted that revision based on feedback is essential for self-regulated learning and the development of critical thinking.

Factors Promoting Critical Thinking in the Mathematics Classroom

This section presents and discusses classroom observation data on the instructional practices of Teacher A, Teacher B, and Teacher C, as well as the factors that promote critical thinking in the mathematics classroom from the survey questionnaire answers.

Teacher A

Over five days of observation, Teacher A strategically employed different practices to promote critical thinking in mathematics. These included collaborative learning, questioning techniques, and feedback. During the classroom observations, collaborative activities were evident in Teacher A's class, wherein students worked together in small groups or pairs to solve problems, complete tasks, or discuss concepts. This observation supports Teacher A’s response in the survey administered by the researchers, in which she indicated that she regularly employs collaborative activities to foster students’ critical thinking.

“I incorporate collaborative activities where students work together to accomplish challenging tasks. These are in small groups, 3–5 members. Listening to group discussions, some members kay maka-input thru questions especially kadtong na-confuse sila. Thru these questions, ma-lead to different realizations” **- [Teacher A]**

In the excerpt, it appears that Teacher A intentionally incorporates collaborative activities into her instruction by organizing students into small groups of 3 to 5 members to solve challenging tasks. This practice is indeed effective in promoting critical thinking, as it provides students with opportunities to engage in active learning, exchange ideas, and challenge one another’s perspectives on how to solve problems or provide solutions (El Yazidi, 2023). These findings are supported by the study of Warsah et al. (2021) and Rokhaniyah (2016), who stated that collaborative learning activities improved critical thinking and academic achievement among students in various learning subjects. Similarly, Strickroth (2023) and Smith et al. (2021) suggest that peer discussions improve critical thinking and performance by helping students to clarify, explain, and enhance their understanding cooperatively.

This implies that collaborative learning can be an effective practice for teachers to promote critical thinking in the mathematics classroom rather than individual learning (El Yazidi, 2023) , particularly when it involves well-crafted activities or tasks and clear expectations for participation and communication.

The use of questioning techniques were evident in Teacher A's practice of promoting critical thinking in the mathematics classroom. Questioning is a vital teaching strategy that involves probing students' reasoning and prompting them to justify their answers. Shanmugavelu et al. (2020) emphasize that questioning is one of the most effective ways to engage students in the learning process. By asking questions, teachers not only encourage students to think critically about the lesson content but also gather valuable feedback that reflects the effectiveness of their teaching.

This was clearly observed in Teacher A's classroom, particularly during the question-and-answer sessions, where a variety of questioning techniques were employed to motivate students and stimulate their thinking. Open-ended and follow-up questions were used effectively to prompt students to reflect on their reasoning and deepen their understanding. This classroom observation is consistent with Teacher A's survey response, where she emphasized her intentional use of questioning techniques to foster critical thinking among her students.

" *…usually, I just give them a problem to start the lesson, then employ the TTP approach. If the instructional materials available to me work, I use them too. I try to ask them questions too.*" - [**Teacher A]**

The aforementioned excerpt indicates that Teacher A clearly shows that questioning is part of her lesson. These findings are supported by the study of Shanmugavelu et al. (2020), who stated that questioning techniques stimulate learning, enhance students' thinking abilities, clarify ideas, spark creativity, and motivate action. This implies that teachers should focus on using questioning techniques in the teaching and learning process to nurture students' critical thinking and sustain their interest in learning.

Furthermore, Teacher A consistently used feedback strategies that promote critical thinking among students. These included probing questions that guided students to justify their reasoning, opportunities for peer assessment, and activities that encouraged reflection and evaluation of solutions. For instance, the teacher asked students to analyze and compare different answers, reflect on their mistakes, and discuss with peers before arriving at conclusions. This was also evident to her answer on the survey emphasizing the use of feedback promoting critical thinking.

"... I believe that if a teacher knows how to respond to students' answers or how to give appropriate feedback, it can also foster critical thinking. For me, appropriate feedback promotes a safe space—and a safe space promotes critical thinking." **[Teacher A]**

The excerpt above implies that Teacher A values the role of appropriate feedback in promoting critical thinking among students. It suggests that the teacher understands that how feedback is given can significantly influence students' willingness to participate, reflect, and think deeply. By providing constructive responses, the teacher creates a safe and supportive learning environment where students feel encouraged to share their ideas, make mistakes, and learn from them—an essential condition for developing critical thinking skills. Such practices align with studies by Hattie and Timperley (2007) and Black and Wiliam (2009), which emphasize that effective feedback enhances learners' understanding by encouraging them to engage deeply with the content, reflect on their thought processes, and improve their reasoning skills. Teacher A's feedback approach fostered an active learning environment where students were not passive recipients of information but were empowered to think critically, ask questions, and collaboratively construct knowledge. Teacher A refers to critical thinking as ***"pagpanimbang"***, emphasizing that it is not developed instantly. TA allows students to make mistakes, process their thoughts, and facilitates responses to guide them toward the desired understanding of critical thinking.

Ultimately, Teacher A thoughtfully implemented a range of strategies aimed at nurturing critical thinking in the mathematics classroom. By incorporating collaborative learning, employing varied questioning techniques, providing meaningful feedback, and creating opportunities for reflection and peer interaction, TA fostered a classroom environment where students actively engaged in reasoning, shared diverse ideas, and deepened their understanding. These practices align with research-based approaches that support the development of critical thinking in mathematics education.

Teacher B

Teacher B used a variety of teaching strategies that helped students think more deeply about math. Over five observed lessons, TB consistently used activities that encouraged students to think critically, work with others, and understand math concepts better. These practices often included board work, collaborative learning, questioning techniques, open tasks, and investigation.

A common feature in TB's lessons was the use of *"boardworks"*—interactive activities that helped review key math concepts. These not only made the lessons engaging but also created a sense of challenge and accountability among students. Such gamified approaches are known to support active learning and critical thinking (Lai & Hwang, 2016).

Teacher B balanced independent work with group activities. Students were often asked to solve problems on their own, then discuss their solutions with peers. This setup gave them space to think on their own and also learn from others. TB also asked follow-up questions that pushed students to explain their thinking, justify their answers, and use correct math terms—especially when learning about arithmetic and geometric sequences. This kind of guided questioning helps students build deeper understanding (King, 1994).

Teacher B made sure students could tell the difference between related concepts, such as *common difference* and *common ratio*, and encouraged them to look for patterns and categorize expressions. More importantly, the classroom was a place where students felt comfortable sharing their thoughts, questioning ideas, and discussing whether solutions made sense. This kind of environment helps students become confident thinkers and problem solvers. In addition, Teacher B encouraged students to listen to each other's reasoning, explain their own ideas, and participate in discussions. This reflects research that says students improve their thinking when they talk about their ideas with others (Zohar & Dori, 2003).

What is also important is how TB's teaching practices match their beliefs. In the survey, Teacher B emphasized the value of giving students *"time to investigate and solve problems"* and using "*open tasks*" with more than one possible solution. These beliefs were clearly seen in practice. The *"boardworks"* and problem-solving tasks gave students chances to explore, try different approaches, and learn from mistakes.

Teacher B also described their role as a *"facilitator"* who brings out students' ideas and uses them to build class discussions. This was seen in how TB guided conversations—asking questions that helped students move from basic answers to deeper thinking.

Overall, Teacher B demonstrated effective use of instructional strategies that promote critical thinking in mathematics. By incorporating open-ended tasks, encouraging peer collaboration or collaborative learning, boardworks, questioning techniques, investigation, and facilitating classroom discussions, TB created a learning environment where students could explore ideas, justify their reasoning, and engage in thoughtful problem-solving. These practices reflect well-established approaches that support the development of critical thinking skills in mathematics education.

Teacher C

Teacher C used a combination of structured teaching and collaborative activities to help students think more critically about math. Across the observed sessions, she began her lessons by reviewing previous content or playing short, topic-related videos. These videos helped introduce or reinforce key ideas and kept students engaged. After the introduction, she used PowerPoint presentations to explain the lesson clearly and step-by-step. She also used sample problems to model how to solve different types of questions.

A big part of Teacher C's teaching involved group work and student presentations. Students worked in teams to solve problems and were often asked to explain their answers in front of the class. Many groups found different ways to solve the same problem, especially in lessons about trigonometric identities. This showed that students were learning to explore different strategies and think flexibly. Teacher C encouraged this by asking guiding questions like, "*Why do you think that works?"* or *"Is there another way to do this? ,"* helping students reflect more deeply on their work.

The classroom environment also supported learning. Students had materials like identity cards to help them during problem-solving, and they were comfortable working together and sharing ideas. This setup supports active learning and helps build students' confidence in explaining their thinking.

Teacher C's survey responses match what was observed in class. She shared that she is still learning as a new teacher but is already using strategies like real-life problems, open-ended questions, and group discussions to help students move beyond memorizing formulas. As she explained, *"I'm learning to cultivate critical thinking in my students by engaging them in meaningful problem-solving tasks."* Her goal is to become a facilitator who guides students to think independently, explore different methods, and learn from their mistakes. These beliefs were clearly shown in her teaching practices.

According to Lai and Hwang (2016), using multimedia tools and group activities in math helps students stay engaged and think critically. Zohar and Dori (2003) also found that students—regardless of skill level—can develop higher-order thinking when given open tasks and meaningful problems. Teacher C's use of videos, group work, and reflective questioning reflects these proven strategies.

Teachers A, B, and C employed a range of instructional strategies that promote critical thinking in the mathematics classroom. All three emphasized collaborative learning, where students worked in groups to solve problems, discuss answers, and reflect on their thinking, an approach that fosters peer interaction and deeper understanding. They also consistently used questioning techniques, particularly open-ended and follow-up questions, to challenge students' reasoning and encourage them to justify their answers. Teacher A focused on guided group discussions, reflective error analysis, and feedback that created a safe space for learning from mistakes. Teacher B integrated open-ended tasks, investigative activities, and "board works" to stimulate exploration and critical engagement. Teacher C, on the other hand, used structured teaching tools such as videos and PowerPoints while still promoting student-led presentations and flexible thinking through group work and real-life problem contexts. Across all three classrooms, the teachers created supportive learning environments, provided opportunities for reflection, and encouraged multiple solution strategies, demonstrating a shared commitment to nurturing critical thinking in their mathematics instruction.

CHAPTER V

SUMMARY, CONCLUSION AND RECOMMENDATIONS

This chapter provides a summary of the findings, conclusion, and recommendations derived from the overall results of this study.

5.1 Summary of Findings

This study aimed to explore the factors that promote critical thinking in the mathematics classroom by observing the teaching practices of three secondary mathematics teachers at one of the laboratory schools of a state university in the Philippines. Through a combination of classroom observations and open-ended questionnaires, the study identified various teaching practices and behaviors that facilitate the development of students' critical thinking skills.

Over five days of classroom observation, each teacher demonstrated specific practices aligned with the dimensions of critical thinking identified in the study:

- **Teacher A** utilized collaborative learning strategies such as group discussions, peer review, and team-based tasks. She also employed guiding and probing questions to stimulate student reasoning and provided timely feedback to support reflection and revision.
- **Teacher B** incorporated "board works", collaborative tasks, open-ended problems, and student investigations. He made consistent use of questioning techniques to prompt deeper understanding and encourage student exploration.

- **Teacher C** engaged students through structured lessons supported by multimedia tools. Her teaching emphasized collaborative group work and guiding questions that prompted students to explain and justify their solutions using various methods.

These instructional practices were found to promote critical thinking across the five main factors identified in the study: (1) focusing on specific critical thinking skills, (2) providing practice opportunities, (3) providing feedback, (4) using active learning approaches, and (5) creating a supportive classroom environment.

Further analysis revealed specific subfactors and indicators under these dimensions:

Focusing on Specific Critical Thinking Skills

Most of the teachers demonstrated strategies aligned with analyzing mathematical arguments. This included prompting students to explore the validity of solutions, using the Socratic method to guide reasoning, and encouraging justification of answers and exploration of multiple problem-solving approaches.

- ***Practice Opportunities***

  Collaborative practice was most evident in this dimension. Students were regularly encouraged to work together on problem-solving tasks, share perspectives, and critique one another's reasoning. These opportunities fostered meaningful dialogue and supported deeper engagement with mathematical content.

- ***Providing Feedback***

  Teachers created opportunities for students to revise their work based on both peer and teacher feedback. This feedback process helped students refine their solutions, correct misunderstandings, and strengthen their critical thinking and problem-solving strategies.

- **Using Active Learning Approaches**

  Lessons were designed to be interactive and student-centered, incorporating activities such as problem-based learning, open-ended tasks, peer presentations, and real-world scenarios that promoted inquiry and active participation.

- **Classroom Environment**

A respectful and supportive atmosphere was consistently observed. Teachers cultivated safe spaces where students felt comfortable asking questions, discussing mistakes, and engaging in collaborative problem solving—key elements that allowed critical thinking to thrive.

Overall, the findings demonstrate that critical thinking in mathematics can be effectively promoted when instructional strategies are intentionally aligned with key dimensions of cognitive engagement and when the classroom environment supports open dialogue, exploration, and reflection.

5.2 Conclusion

This study concludes that promoting critical thinking in the mathematics classroom is both achievable and essential for developing independent, reflective, and competent learners. The instructional practices observed indicate that when teachers design their lessons with intentional strategies—such as collaborative learning, inquiry-based tasks, strategic questioning, and timely feedback—students become better equipped to analyze, evaluate, and solve complex mathematical problems.

The integration of active learning approaches proved particularly effective in transforming traditional mathematics instruction into an engaging and intellectually stimulating experience. By shifting from teacher-centered to student-centered methods, learners were not only encouraged to participate actively but were also empowered to take ownership of their learning. This autonomy fosters the development of higher-order thinking skills, which are critical for success in both academic and real-world contexts.

Moreover, the classroom environment emerged as a foundational element in supporting students' cognitive development. A safe, respectful, and open atmosphere nurtures curiosity, creativity, and confidence—qualities that are indispensable for critical thinking to flourish.

In essence, this study affirms that promoting critical thinking is not achieved through a single method or tool; rather, it is the result of cultivating a teaching mindset that prioritizes exploration, reasoning, and reflective learning. It calls on educators to move beyond traditional, procedural instruction and embrace strategies that challenge students to think deeply and independently.

5.3 Recommendations

Based on the study's findings, the following recommendations are proposed to enhance the promotion of critical thinking in mathematics education:

Incorporate Collaborative and Inquiry-Based Learning. Teachers should design group tasks and inquiry activities that allow students to explore, discuss, and reason out solutions together. These promote active engagement and deeper understanding of mathematical concepts.

Use Strategic Questioning and the Socratic Method. Effective questioning, especially open-ended and probing types—should be part of every lesson. It encourages students to justify their thinking, examine ideas critically, and engage in meaningful dialogue.

Provide Open-Ended Problems with Multiple Solution Paths: Teachers are encouraged to use real-world or complex problems that allow for different solving methods. This not only stimulates creativity but also fosters analytical skills and flexible thinking.

Give Timely and Constructive Feedback: Feedback should go beyond checking answers. Teachers must focus on guiding students' reasoning and providing opportunities for self-correction and peer feedback, which are vital for reflection and improvement.

Foster a Safe and Respectful Classroom Environment: A supportive atmosphere where mistakes are seen as part of learning helps students become more confident and willing to think critically. Teachers should create a space that values all contributions and encourages respectful discussion.

Promote Ongoing Teacher Development: To sustain these practices, schools and teacher education programs should offer regular training focused on critical thinking strategies, classroom discourse, and reflective teaching.

To further improve this study, future researchers should extend the observation period to two to four weeks to allow for a more comprehensive analysis of teaching practices. Additionally, administering face-to-face surveys and interviews would be more effective than using online platforms, as it ensures real-time responses and allows for more in-depth interactions with participants. It is recommended to observe together in one class with one teacher, so observers can agree on whether a certain indicator is present. This helps reduce personal bias and makes the results more reliable. Lastly, during open classroom observations, researchers should ensure that detailed notes are taken of all relevant conversations between teachers and students. Obtaining permission to use audio and/or video recording devices would also enhance the accuracy and reliability of the collected data.

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APPENDIX A

Permission to conduct a study from the School Principal of IDS

COLLEGE OF EDUCATION
Center of Excellence in Teacher Education

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December 9, 2024

Ms. Amelia T. Buan, PhD
School Principal
MSU-IIT IDS

Dear Ma'am:

We are the students of MSU-IIT from College of Education, Department of Science and Mathematics Education. We would like to ask your permission to allow us to conduct a class observation among the ~~students among~~ junior high school students at Mindanao State University-Iligan Institute of Technology Integrated Developmental School (MSU-IIT IDS) to support our thesis, "Examining Factors that Promote Critical Thinking in the Mathematics Classroom."

The said observation would last about 1-2 weeks depending on the range of 1 unit coverage and would be scheduled at a time that was convenient for both students and teachers during their mathematics class. There are no known or anticipated risks to participating in this study. All given information will be kept strictly confidential and used only for academic reasons. The names of the respondents will not be included in any dissertations or publications that result from this study. You will receive a copy of the executive summary once the data has been analyzed. If you are interested in more information, you can request an electronic copy (e.g., PDF) of the whole thesis.

We would like to sincerely and humbly ask for your kind consideration to permit us to conduct this study/observation at your school. Your approval to conduct this study will be greatly appreciated. Thank you in advance for your interest and assistance with this research.

Sincerely yours,

Researchers

[redacted] CHAMPAIGN MARGAUX E. BONGCARON

[redacted] JENLYN M. RODA

[redacted] JOHN P. URTAL

Noted by:

[redacted]
Prof. Joan Rose T. Luib, MSciEd
Thesis Adviser

Approved by:

[redacted]
Ms. Amelia T. Buan, PhD
School Principal

Be sure to observe ethical procedure in conducting research

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ILIGAN ADDRESS
Andres Bonifacio Avenue, Tibanga, Iligan City
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MANILA ADDRESS
MSU-IIT MANILA INFORMATION & LIAISON OFFICE
12th Floor G.E. Antonino Building
T.M. Kalaw Avenue Cor. J. Bocobo St. Ermita, Manila
Tel. 527-5595

APPENDIX B

Permission to conduct a study from the Math, IT& ABM Department Chairperson

COLLEGE OF
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January 20, 2025

ALEXIS MICHAEL B. OLEDAN
Math, IT, & ABM Department Chairperson,
Mindanao State University- Iligan Institute of Technology
Iligan City, 9200

Thru: **ROVIC E. PEROCHO**
Faculty, Math Department

GRACE P. LIWANAG
Faculty, Math Department

CHRISLEY JADE C. SAROMINES
Faculty, Math Department

Greetings of Peace!

We, the undersigned researchers from the College of Education, Department of Science and Mathematics Education at MSU-IIT, are currently conducting our thesis study to examine factors promoting critical thinking in the mathematics classroom. To support our data collection, we respectfully seek your permission to conduct a class observation in your mathematics class.

We would like to inform you that our request has already been reviewed and officially approved by our thesis adviser, Ma'am Joan Rose T. Luib, MSciEd, and IDS School Principal, Ms. Amelia T. Buan, PhD., as indicated in the formal letter previously submitted and signed.

The class observation will be scheduled at a time most convenient for you and your students. All information gathered will be treated with strict confidentiality and used solely for academic purposes. The identities of participants will remain anonymous, and you will receive a summary of the findings upon completion of data analysis.

We sincerely hope for your kind consideration and approval of this request. Thank you very much for your support and assistance in our academic endeavor.

Sincerely,

JOHN P. URTAL
Researcher

JENLYN M. RODA
Researcher

CHAMPAIGN MARGAUX E. BONGCARON
Researcher

Approved by:

ALEXIS MICHAEL B. OLEDAN
Math, IT, & ABM Department Chairperson

APPENDIX C

Request for Validation of Checklist & Survey Questionnaires

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January 28, 2025

Hassan S. Gandamra
Associate Professor, DSME
College of Education
MSU- Iligan Institute of Technology

Dear Sir,

We are students from MSU-IIT, College of Education, Department of Science and Mathematics Education, currently conducting a research study titled **"Examining Factors that Promote Critical Thinking in the Mathematics Classroom"** as part of our academic requirements. In line with this, we have developed a set of indicators relevant to our research objectives.

Understanding the importance of ensuring the validity and reliability of these indicators, we respectfully seek your expertise in evaluating their relevance, clarity, and accuracy. Your valuable feedback will significantly contribute to improving the quality and credibility of our research.

Your insights and expertise are vital to the success of our study, and we sincerely appreciate your time and assistance.

Thank you very much for considering our request. We look forward to your positive response.

Sincerely,

JOHN P. URTAL
Researcher

JENLYN M. RODA
Researcher

CHAMPAIGN MARGAUX E. BONGCARON
Researcher

Approved by:

Hassan S. Gandamra, MSciEd
Associate Professor, DSME

College of
Education

www.msuiit.edu.ph

March 31, 2025

ALEASAR V. MALICOBAN
Faculty, Department of Science and Mathematics,
Mindanao State University-Iligan Institute of Technology
Iligan City, 9200

Dear Sir,

We are students from MSU-IIT, College of Education, Department of Science and Mathematics Education, currently conducting a research study titled **"Examining Factors that Promote Critical Thinking in the Mathematics Classroom"** as part of our academic requirements. In line with this, we have developed a set of indicators relevant to our research objectives.

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Your insights and expertise are vital to the success of our study, and we sincerely appreciate your time and assistance.

Thank you very much for considering our request. We look forward to your positive response.

Sincerely,

[redacted]
JOHN P. CURTAL
Researcher

[redacted]
JENIVYN M. RODA
Researcher

[redacted]
CHAMPAIGN MARGAUX E. BONGCARON
Researcher

Approved by:

[redacted]
ALEASAR V. MALICOBAN, MSciEd
Faculty, Department of Science and Mathematics

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April 2, 2025

MARY JOY F. LUGA
Faculty, Math Department
MSU-IIT IDS

Dear Ma'am,

We are students from MSU-IIT, College of Education, Department of Science and Mathematics Education, currently conducting a research study titled **"Examining Factors that Promote Critical Thinking in the Mathematics Classroom"** as part of our academic requirements. In line with this, we have developed a set of indicators relevant to our research objectives.

Understanding the importance of ensuring the validity and reliability of these indicators, we respectfully seek your expertise in evaluating their relevance, clarity, and accuracy. Your valuable feedback will significantly contribute to improving the quality and credibility of our research.

Your insights and expertise are vital to the success of our study, and we sincerely appreciate your time and assistance.

Thank you very much for considering our request. We look forward to your positive response.

Sincerely,

[redacted]
JOHN P. [illegible]URTAL
Researcher

[redacted]
JENLYN M. RODA
Researcher

[redacted]
CHAMPAIGN MARGAUX E. BONGCARON
Researcher

Approved by:

[redacted]
MARY JOY F. LUGA
Faculty, Math Department

APPENDIX D

Participant Consent Form

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January 20, 2025

Grace P. Liwanag
Faculty, Math Department
MSU-IIT IDS

Dear Ma'am,

We, the undersigned researchers from the College of Education, Department of Science and Mathematics Education at MSU-IIT, are currently conducting our thesis study to examine factors promoting critical thinking in the mathematics classroom. To support our data collection, we respectfully seek your permission to conduct a class observation in your mathematics class.

We would like to inform you that our request has already been reviewed and officially approved by our thesis adviser, Ma'am Joan Rose T. Luib, MSciEd, and IDS School Principal, Ms. Amelia T. Buan, PhD., as indicated in the formal letter previously submitted and signed.

The class observation will be scheduled at a time most convenient for you and your students. All information gathered will be treated with strict confidentiality and used solely for academic purposes. The identities of participants will remain anonymous, and you will receive a summary of the findings upon completion of data analysis.

We sincerely hope for your kind consideration and approval of this request. Thank you very much for your support and assistance in our academic endeavor.

Sincerely,

[redacted]
JOHN P. URTAL
Researcher

[redacted]
JENLYN M. RODA
Researcher

[redacted]
CHAMPAIGN MARGAUX E. BONGCARON
Researcher

Approved by:

[redacted]
GRACE P. LIWANAG
Faculty, Math Department

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January 20, 2025

Rovic E. Perocho
Faculty, Math Department
MSU-IIT IDS

Dear Sir,

We, the undersigned researchers from the College of Education, Department of Science and Mathematics Education at MSU-IIT, are currently conducting our thesis study to examine factors promoting critical thinking in the mathematics classroom. To support our data collection, we respectfully seek your permission to conduct a class observation in your mathematics class.

We would like to inform you that our request has already been reviewed and officially approved by our thesis adviser, Ma'am Joan Rose T. Luib, MSciEd, and IDS School Principal, Ms. Amelia T. Buan, PhD., as indicated in the formal letter previously submitted and signed.

The class observation will be scheduled at a time most convenient for you and your students. All information gathered will be treated with strict confidentiality and used solely for academic purposes. The identities of participants will remain anonymous, and you will receive a summary of the findings upon completion of data analysis.

We sincerely hope for your kind consideration and approval of this request. Thank you very much for your support and assistance in our academic endeavor.

Sincerely,

[redacted]
JOHN P. URTAL
Researcher

[redacted]
JENLYN M. RODA
Researcher

[redacted]
CHAMPAIGN MARGAUX E. BONGCARON
Researcher

Approved by:

[redacted]
ROVIC E. PEROCHO
Faculty, Math Department

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January 20, 2025

Chrisley Jade C. Saromines
Faculty, Math Department
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Dear Ma'am,

We, the undersigned researchers from the College of Education, Department of Science and Mathematics Education at MSU-IIT, are currently conducting our thesis study to examine factors promoting critical thinking in the mathematics classroom. To support our data collection, we respectfully seek your permission to conduct a class observation in your mathematics class.

We would like to inform you that our request has already been reviewed and officially approved by our thesis adviser, Ma'am Joan Rose T. Luib, MSciEd, and IDS School Principal, Ms. Amelia T. Buan, PhD., as indicated in the formal letter previously submitted and signed.

The class observation will be scheduled at a time most convenient for you and your students. All information gathered will be treated with strict confidentiality and used solely for academic purposes. The identities of participants will remain anonymous, and you will receive a summary of the findings upon completion of data analysis.

We sincerely hope for your kind consideration and approval of this request. Thank you very much for your support and assistance in our academic endeavor.

Sincerely,

[redacted]
JOHN R. URTAL
Researcher

[redacted]
JENLYN M. RODA
Researcher

[redacted]
CHAMPAIGN MARGAUX E. BONGCARON
Researcher

Approved by:

[redacted]
Chrisley Jade C. Saromines
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APPENDIX E

Checklist of Practice of Teachers Promoting Critical Thinking

"EXAMINING FACTORS PROMOTING CRITICAL THINKING IN THE MATHEMATICS CLASSROOM"

Directions for the Evaluator:

1. Evaluate each indicator to determine if it correctly belongs to its respective category. Place a checkmark (✓) in the appropriate box: Accept, Revise, or Reject.
2. Provide remarks in the comment section to explain your evaluation decision.

Key:

- *Accept:* The indicator is correctly categorized and valid for use.
- *Revise:* The indicator is valid but may need slight revisions or adjustments before being reused.
- *Reject:* The indicator does not belong in this category and should be discarded or revised

Category	Indicator	Accept	Revise	Reject	Remarks
A. Focusing on Specific Critical Thinking Skills	**1. Analyzing Mathematical Arguments:**				
	The teacher guides students in identifying and evaluating the structure of arguments presented in the lesson.		✓		The teacher gives prompts to guide the students into analyzing math'l. arguments.
	Students are asked to differentiate between valid and invalid arguments within the context of the subject matter.		✓		Students determine and are able to ~~[illegible]~~ reason out ~~[illegible]~~ [illegible] regarding argument validity.
	The teacher uses Socratic questioning to guide students in deeper exploration of mathematical ideas (e.g., "What is the reasoning behind your answer?").	~~✓~~	✓		The teacher applies Socratic ~~questioning~~ method to ask to elicit deeper thinking and mathematical exploration.

	Teachers use "thinking aloud" techniques, demonstrating how to approach problems critically and reasoning through each step.	✓			
	The teacher encourages students to ask their own questions and engage with mathematical problems from multiple angles.		✓		The teacher [illegible] the learners to go/delve deeper into their own queries.
	Students are encouraged to justify their answers and explore multiple ways to approach a problem.			✓	Similar [illegible] in ② already.
	2. Identifying Assumptions:				
	The teacher encourages students to identify hidden assumptions in problems, scenarios, or arguments.	✓			
	Students are challenged to raise questions about what is supposed in the material being discussed.	✓			
	The teacher emphasizes inquiry-based learning, where students explore mathematical concepts through guided discovery.	✓			
	Higher-order thinking questions (e.g., evaluation, synthesis, and analysis) are regularly incorporated into lessons to challenge students.	✓			

	Conceptual understanding is prioritized over procedural fluency, allowing students to build a deeper grasp of the content.	✓				
	3. Evaluating Evidence:					This focus [illegible] on?
	The teacher instructs students on how to assess the reliability and validity of evidence used in arguments or problem-solving tasks.?		✓			The students are taught about evaluating reliability and validity of arguments.
	Students are taught to weigh different types of evidence and evaluate the strength of their conclusions.		✓			weigh? [illegible]?
	The teacher encourages students to think critically about their methods, test hypotheses, and draw conclusions independently.		✓			Cannot see the term critically. ([illegible] I think will be better)
	The teacher encourages students to reflect on their problem-solving strategies and assess the effectiveness of their methods.		✓			I don't think reflect is the word you're really trying to imply here.
	Students are encouraged to question each other's reasoning and consider alternative viewpoints.		✓			Students [illegible] peers' reasonings ...
	4. Encouraging Logical Reasoning:					
	The teacher highlights the importance of logical consistency in students' reasoning.	✓				

	Students are prompted to detect and correct logical fallacies in their own or others' reasoning.	✓			
	The teacher models metacognitive strategies by verbalizing the thinking process.	✓			
	Time is allocated for students to discuss their thought processes and critique each other's reasoning.	✓			
	Students use metacognitive strategies to monitor their understanding and approach to solve problems.	✓			
B. Providing Practice Opportunities	**1. Real-World Problem-Solving:**				
	Students are regularly given opportunities to apply critical thinking to solve real-world problems.	✓			
	The teacher incorporates practical scenarios or case studies that require students to critically analyze and solve problems.	✓			
	The teacher provides opportunities for project-based learning that requires students to solve real-world problems.	?	? ✓		[illegible] ?

	Students regularly work on problems that require creativity, fostering self-confidence and independent reasoning.	✓				
	The teacher provides opportunities for students to work in an open-ended problems that require creativity and critical thinking.	✓				
	2. Variety of Contexts:					
	Students engage in critical thinking tasks across different subjects or disciplines.		✓			please clarify. interdisciplinary?
	Students are exposed to interdisciplinary tasks that allow them to apply critical thinking skills in diverse contexts.		✓			redundancy
	Active learning techniques, such as flipped classrooms and problem-based learning, are used to engage students.			✓		this refers to varied methods, not contexts.
	Students are encouraged to tackle complex tasks requiring independent thinking and multiple solution strategies.	✓				
	Instruction is tailored to meet the needs of diverse learners, providing differentiated tasks based on students' skill levels.	✓				
	3. Collaborative Practice:					

	Students are encouraged to work together on problem-solving tasks, allowing for diverse perspectives and critical exchange of ideas.	✓			
	The teacher fosters group discussions where students can collaboratively analyze and critique each other's reasoning.	✓			
	Students regularly engage in collaborative problem-solving activities, working with peers to discuss and critique each other's reasoning.		✓		regularly?
	Student participation is encouraged through interactive activities like math games, group problem-solving tasks, and mathematical discussions.		✓		replace the "encouraged" term with "practiced" or "promoted"
	4. Reflection on Learning:				
	Students are given opportunities to reflect on their thought processes and problem-solving strategies.	✓			
	The teacher prompts students to think analyze what could be improved and worked on after solving a problem.		✓		analyze → think
	Students' reflection on their thinking process (e.g., metacognition) is integrated into regular classroom practices.	✓			

	Students receive time to share strategies, discuss solutions, and critique their own work.	✓			
C. Providing Feedback	**1. Timely and Specific Feedback:**				
	The teacher provides feedback that is given promptly after the task or assignment, focusing on the reasoning and thought processes rather than just correct answers.	✓			
	The feedback is specific and constructive, guiding students on how to improve their critical thinking.	✓			
	Formative assessments (e.g., quizzes, discussions) are used to monitor understanding and guide instruction.	✓			
	Rubrics are utilized to evaluate the depth of mathematical reasoning, not just final answers.	✓			
	Feedback emphasizes effort, process, and progress over correctness.	✓			
	2. Self-Assessment:				
	The teacher prompts students to assess their own critical thinking skills and identify areas for improvement.	✓			

	Students are encouraged to reflect on their own strengths and weaknesses in reasoning.	✓			
	Self-assessment practices (e.g., reflective journaling) help students evaluate their own problem-solving and thinking.	✓			
	Students are encouraged to assess their own reasoning and identify areas for growth.	✓			
	3. Peer Feedback:				
	The teacher facilitates opportunities for students to give and receive feedback from their peers on their reasoning and arguments.	✓	✓		
	Students are taught how to provide constructive criticism and are encouraged to question the reasoning and supporting data of their peers.	✓			
	Peer-assessment is encouraged, allowing students to evaluate and give feedback on each other's problem-solving processes.	✓			
	Teachers use digital resources for real-time feedback on student progress.	✓			
	4. Opportunities for Revision:				

	The teacher allows students to revise their work based on feedback to strengthen their critical thinking and problem-solving approaches.	✓			
	Students are encouraged to revisit and refine their arguments and evidence after receiving feedback.	✓			
	Assessment data informs instruction and provides personalized feedback for growth.	✓			
	Reflection tools such as journals or portfolios help track cognitive development.	✓			
D. Using Active Learning Approaches	1. **Problem-Based Learning (PBL):**				
	The teacher implements problem-based learning activities where students must actively explore, question, and solve complex problems.	✓			
	Students are given the opportunity to take ownership of their learning by investigating real-world problems that require critical thinking.	✓			
	The teacher provides minimal support during independent tasks to promote problem-solving and critical reflection.	✓			

	Tasks are open-ended, allowing students to take ownership of their learning and determine the best approach to problem-solving.	✓			
	Students work on complex tasks requiring independent thinking and multiple solution strategies.	✓			
	2. Inquiry-Based Learning				
	The teacher encourages students to ask questions, explore various answers, and engage in deep inquiry into the subject matter.		✓		The teacher [illegible] inquiries on students [illegible]
	Students are guided to investigate problems through hypothesis testing, experimentation, and data collection.	✓			
	~~The teacher emphasizes inquiry-based learning where~~ students explore mathematical concepts through guided discovery.		✓		Students explore math. concepts through guided discovery
	Scaffolding techniques (e.g., guided questions or hints) help students engage with complex tasks.	✓			
	The teacher adjusts tasks to provide varying levels of challenge and encourages all students to tackle problems at their own pace.	✓			

	3. Student-Centered Learning:				
	The teacher creates opportunities for students to explore concepts, formulate their own questions, and construct their understanding.	✓			
	Students are encouraged to be active participants in their learning process, rather than passive recipients of information.	✓			
	Teacher only facilitates the learning.	✓			
	4. Collaborative Learning and Discussion:				
	Students are ~~encouraged~~ motivated to engage in group discussions where they must defend their reasoning and challenge the ideas of others.		✓		
	The teacher promotes dialogue that encourages students to question and refine each other's arguments.	✓			
	Metacognitive strategies, such as self-monitoring and explaining thought processes, are integrated into lessons.	✓			
	Time is allocated for students to share strategies, discuss solutions, and critique each other's work.	✓			

	Students ~~regularly~~ engage in collaborative problem-solving activities.		✓		
	5. Independent Thinking and Creativity:				
	The teacher creates an environment that encourages students to explore multiple solutions or perspectives to a problem.	✓			
	Students are encouraged to experiment with different approaches, fostering creativity and critical thinking.	✓			
	The teacher encourages students to explore multiple solution strategies independently.	✓			
	The teacher provides tasks that require independent critical thinking and creativity.	✓			
E. Classroom Environment	**1. Psychological Safety**				
	The teacher fosters a psychologically safe classroom where mistakes are part of the learning process.	✓			
	The teacher celebrates mistakes as a natural part of learning and encourages students to view errors as stepping stones to mastery.	✓			

	2. Physical Space and Collaboration				
	Physical space is organized for collaborative learning with flexible seating and group work areas.	✓			
	3. Valuing Diverse Approaches				
	The classroom culture values diverse mathematical approaches and strategies.	✓			
	The teacher reinforces the belief that mathematical skills can improve with effort and perseverance.	✓			
	4. Communication and Respect				
	Clear expectations are established for respectful communication and active listening.	✓			
	The teacher encourages students to share and justify their ideas in front of the class, emphasizing the importance of reasoning.	✓			

APPENDIX F

SURVEY QUESTIONNAIRE

This survey is strictly confidential. Your responses will be used solely for research purposes and will not be shared with any third party. Your identity will remain anonymous. We appreciate your time in completing this questionnaire. Your responses will provide valuable insights into teaching strategies that promote critical thinking in mathematics. Please answer the questions honestly and to the best of your ability.

I. Background and Experience:
1. How many years have you been teaching Mathematics?
** Please specify the number of years..*

2. What specific areas of mathematics or grade levels have you taught, and are you currently teaching?
* Please *list the subjects and grade levels you have experience teaching.*

3. Could you describe your educational background and any relevant professional development you've participated in related to mathematics education?
** Include degree(s) earned, certifications, training, and workshops attended*

II. Teaching Strategies and Practices:
(Adopted and refined from the study of Malepa-Qhobela & Mosimege, 2022.)
1. What are the specific ways that foster critical thinking among learners in a mathematics classroom discussion?

2. What methods and instructional practices do you normally employ, and what classroom activities do you engage your learners in to help them develop critical thinking? Why?

3. How would you describe your role as a teacher in helping/fostering your learners' development of critical thinking?
** Discuss how you facilitate, guide, and encourage critical thinking in your classroom.*

APPENDIX G

RAW SURVEY DATA

Teacher Responses to Questionnaire

Teacher A

I. Background and Experience:

1. How many years have you been teaching Mathematics?

** Please specify the number of years.*

Answer: *24 years and 9 months*

2. What specific areas of mathematics or grade levels have you taught, and are you currently teaching?

* Please *list the subjects and grade levels you have experience teaching.*

Answer: *Currently: Elementary Statistics and Advanced Algebra*

Past: Elementary Statistics, Elementary Algebra, Intermediate Algebra, and Precalculus (1 Year)

3. Could you describe your educational background and any relevant professional development you've participated in related to mathematics education?

** Include degree(s) earned, certifications, training, and workshops attended.*

Answer: *I have attended all Project STAR (DOST-SEI) training.*

II. Teaching Strategies and Practices:

(Adopted and refined from the study of Malepa-Qhobela & Mosimege, 2022.)

1. What are the specific ways that foster critical thinking among learners in a mathematics classroom discussion?

Answer: *"In my practice, a well-crafted activity or task given regularly provides opportunities to promote students' ability to think critically. I believe that if a teacher knows how to respond to students' answers or how to give appropriate feedback, it can also foster critical thinking. For me, appropriate feedback promotes a safe space—and a safe space promotes critical thinking*.

2. What methods and instructional practices do you normally employ, and what classroom activities do you engage your learners in to help them develop critical thinking? Why?

Answer: *"I am a traditional teacher. Usually, I just give them a problem to start the lesson, then employ the TTP approach. If the instructional materials available to me work, I use them too. I try to ask them questions too. I incorporate collaborative activities where students work together to accomplish challenging tasks.*

-These are in small groups, 3–5 members.

-Some students do not benefit well from these activities though.

-Listening to group discussions, some members kay maka-input thru questions especially kadtong na-confuse sila. Thru these questions, ma-lead to different realizations.

-I call critical thinking as pagpanimbang.

-Critical thinking kay dili outright nga ma-develop.

3. How would you describe your role as a teacher in helping/fostering your learners' development of critical thinking?

** Discuss how you facilitate, guide, and encourage critical thinking in your classroom.*

Answer: *Allowing students to make mistakes, process, and facilitating their responses to get to the desired answers. In my classes, I allow them to see what it is and how it is to be critical.*

Teacher B

I. Background and Experience:

1. How many years have you been teaching Mathematics?

** Please specify the number of years*

Answer: 3 and a half years.

2. What specific areas of mathematics or grade levels have you taught, and are you currently teaching?

* Please *list the subjects and grade levels you have experience teaching.*

Answer: *Grade 7&9 (Algebra), Grade 10&11 (Pre-calculus), and Grade 11 (GenMath)*

3. Could you describe your educational background and any relevant professional development you've participated in related to mathematics education?

** Include degree(s) earned, certifications, training, and workshops attended.*

Answer: *Master's Degree*

II. Teaching Strategies and Practices:

(Adopted and refined from the study of Malepa-Qhobela & Mosimege, 2022.)

1. ***What are the specific ways that foster critical thinking among learners in a mathematics classroom discussion?***

Answer: *When learners are given time to investigate and solve problems. Open tasks also helps.*

2. ***What methods and instructional practices do you normally employ, and what classroom activities do you engage your learners in to help them develop critical thinking? Why?***

Answer: Open tasks, investigation, problem-solving with lots of possible solution.

3. How would you describe your role as a teacher in helping/fostering your learners' development of critical thinking?

** Discuss how you facilitate, guide, and encourage critical thinking in your classroom.*

Answer: *facilitator. I elicit what's their ideas and use that to build the classroom discussion and bring them to a level of my expected level of critical thinking.*

Teacher C

I. Background and Experience:

1. How many years have you been teaching Mathematics?

** Please specify the number of years.*

Answer: 2 years

2. What specific areas of mathematics or grade levels have you taught, and are you currently teaching?

* Please *list the subjects and grade levels you have experience teaching.*

Answer: *Grade 9 (Trigonometry) and Grade 11 (GenMath and PreCal)*

3. Could you describe your educational background and any relevant professional development you've participated in related to mathematics education?

** Include degree(s) earned, certifications, training, and workshops attended.*

Answer: *I am a graduate of Doctor of Philosophy in Mathematics(Pure Math) and but currently active in participating conferences and writing papers related to MathEd(Mathematics Education).*

II. Teaching Strategies and Practices:

(Adopted and refined from the study of Malepa-Qhobela & Mosimege, 2022.)

1. ***What are the specific ways that foster critical thinking among learners in a mathematics classroom discussion?***

Answer: *As a new mathematics teacher, I am currently exploring ways to go beyond simply teaching formulas and procedures. I'm learning to cultivate critical thinking in my students by engaging them in meaningful problem-solving tasks. Though I'm still working on refining my approach, I've started using open-ended problems and real-life scenarios to encourage students to think flexibly, try different methods, and explain their reasoning.*

2. ***What methods and instructional practices do you normally employ, and what classroom activities do you engage your learners in to help them develop critical thinking? Why?***

Answer: *In the classroom, I try to create opportunities for group discussions where students can verbalize their ideas and learn from each other. I'm practicing the use of guiding and probing questions—like "Why do you think that works?" or "Is there another way to approach this?"—to help students reflect more deeply. While I know I still have a lot to improve, I believe these small steps are helping me grow as an educator and support my students in becoming more independent and critical thinkers.*

3. How would you describe your role as a teacher in helping/fostering your learners' development of critical thinking?

** Discuss how you facilitate, guide, and encourage critical thinking in your classroom.*

Answer: *As a new mathematics teacher, I see my role in fostering critical thinking as more than just delivering content—it's about guiding my students to think independently, question concepts, and make sense of problems on their own. I try to* *create a learning environment where it's okay to explore, make mistakes, and learn from them. I see myself as a facilitator who encourages students to ask "why" and "what if" rather than just focus on the "how." Even though I'm still in the process of learning and improving my teaching strategies, I do my best to provide tasks and questions that require reasoning, not just memorization. My goal is to help students develop the confidence to explain their thinking, consider other perspectives, and approach problems creatively. Little by little, I'm learning how to support them in becoming not just better math students, but better thinkers.*

APPENDIX H

NARRATIVE OF 5 DAYS OBSERVATION

Class Observations of Teacher A from Day 1 to Day 5

Day 1	Teacher A began the lesson by posting problems on the board related to adding and subtracting polynomials and finding the area of shaded regions, giving students enogh time to answer them individually. As students answered, she read the questions aloud and emphasized key terms to guide their understanding. She asked probing questions to help students identify shapes, justify their reasoning, and recall relevant formulas. After the individual work, Teacher A encouraged students to come to the board and present their solutions. Multiple students volunteered to solve different problems, and their answers were written and explained step-by-step in front of the class. As students worked through their solutions, Teacher A facilitated peer learning by allowing students to discuss and review each other's work.
Day 2	Teacher A administered a quiz, setting a 30-second time limit for each item. After a few minutes, the answers were checked, and the class transitioned into a collaborative activity. During this phase, the teacher encouraged students to focus on correcting the questions they got wrong rather than merely reviewing the correct ones. She urged them to identify and explain the errors in their solutions to their group mates, fostering peer learning and deeper understanding. Teacher A intentionally provided an incorrect answer for item no. 26. During the collaborative session, a group of students approached her to present their corrected solution (*referring to item no. 26*). After reviewing their work, the teacher approved their answer, using it to reinforce the importance of careful analysis and learning from mistakes. As the activity progressed, Teacher A reminded the class to observe others' solutions and prioritize understanding over speed, carefully. While moving around the room, she facilitated discussions, encouraged reflection, guided students in correcting their mistakes, and promoted peer assistance. The class concluded with meaningful collaborative learning, highlighting a supportive and reflective classroom environment.

Day 3	Teacher A facilitated a quiz session with a dynamic and interactive approach that fostered accountability, resourcefulness, and collaborative learning. At the start, students were instructed to show their solutions and allowed students to open their notes and online resources, encouraging students to be resourceful in finding answers. After the quiz, the answer key was projected on the screen, and students exchanged papers for peer checking. Following the peer checking, a collaborative activity was conducted. Students sat with their group mates to share and discuss answers, with a particular focus on understanding incorrect responses rather than merely celebrating correct ones. This approach encouraged students to reflect on their mistakes, learn from one another, and offer peer support without resorting to direct copying. Teacher A actively roamed the classroom, facilitating discussions, prompting reflection, and encouraging students to ask questions. She emphasized that asking questions is a form of participation and a sign of engagement. During the collaboration, students were to use whiteboards and windows to write and present their solutions, supporting active learning.
Day 4	The teacher facilitated a collaborative and student-centered activity where each group was tasked with creating a set of problems related to their report topic. Students were given time to answer the problems individually, followed by a prompt from the teacher to begin group discussion: *"Ok, proceed to group discussion."* After discussing their answers, the teacher instructed, *"Submit your papers to the reporters."* Each of the four groups posted three sample problems, which were then answered by classmates within a 10-minute period. This was followed by a reflective group discussion, allowing students to analyze and compare their solutions.
Day 5	Teacher A started the class with a collaborative activity. The students were grouped and assigned different topics such as permutations, combinations, and probability of mutually exclusive events. Teacher A encouraged the students to ask questions to their peers and group mates to gain a better understanding of the topic assigned to them. After the collaborative activity, the teacher presented a problem and asked the students to solve it individually for about 20 minutes. The problem was: *"In how many ways can a man distribute gifts consisting of Php 500.00, one skateboard, and one bicycle to 5 boys?"* Two sample solutions were given: ***Solution 1:*** $3\times3\times3\times3\times3 = 243$ *ways* ***Solution 2:*** $5\times5\times5=125$ *ways*

	The students were asked to study both solutions, describe what each solution means, and determine which one is correct. After the students answered, Teacher A asked the students to share their thoughts on which solution was correct, and then the class was dismissed

Class Observations of Teacher B from Day 1 to Day 5

Day 1	Teacher B began his class with an activity called boardworks, which was designed to assess mastery of the lesson or topic in multiplying polynomials. The students were divided into two teams, Team A and Team B, each of which had its own respective group members. Each team selected one representative to solve a given problem. The team whose representative solved the problem first earned points based on the result of a dice roll. After a team solved the problem, the representative was required to explain their solution to the class. Meanwhile, the rest of the team members had the opportunity to solve the problem at their seats. If the representatives were unable to solve the problem correctly, Teacher B would call on a group that had found the correct answer. The boardworks activity lasted for about 40 minutes. After the boardworks activity, Teacher B posted another problem for the students to solve individually. However, they were allowed to share their solutions with their peers or seatmates once they arrived at the correct answer. The class ended after this individual activity, which encouraged collaboration among students while also promoting independent problem-solving. The sequence of activities, from team-based problem-solving to individual work with peer collaboration, was designed to engage students in both cooperative and independent learning. The use of competition through the dice roll added an element of excitement, while the individual challenge reinforced personal understanding of the lesson.
Day 2	Teacher B began the class by checking assignments 1, 3, 5, and 11, calling for volunteers to show their solutions on the board and explain their answers. Upon the students explaining their answer, Teacher B asks several guiding and probing questions that help students to think critically and explain their solutions comprehensively. Teacher B started by asking, "*What is your common difference?"* , "*When can we say that there is something common? Follow* up with, "*What makes a sequence arithmetic? ",* prompting the students to consider the concept of a common difference. Teacher B then asked what the result would be if two terms were subtracted, with a student answering *"difference."* Teacher B emphasized that for a sequence to be arithmetic, there must be a common difference and prompted the students to

	explain how to establish this difference. After a student gave a low-volume answer, Teacher B asked another student to clarify. The student explained that the common difference could be found by subtracting successive terms. Teacher B followed up with questions to reinforce the subtraction operation and clarified the difference between addition and subtraction in this context. Teacher B highlighted the need to prove that there is a common difference and emphasized that the common difference must be established using subtraction, not addition. The class discussion concluded with Teacher B prompting the students to use subtraction (a_3 - a_2) to determine the common difference, which was identified as -3. Also, Teacher B conducted a "boardworks" activity using the same mechanics as mentioned on Day 1. After that, Teacher B posted an open task problem, which offers multiple answers and solutions. The students engaged in a collaborative activity to solve the open task problem.
Day 3	The teacher guided the class through an analytical discussion on classifying algebraic expressions, using strategic questioning to prompt student reasoning. As students compared different terms, the teacher asked why certain expressions were enclosed in parentheses, encouraging them to articulate their understanding. When a student responded that it was to avoid shortening terms, the teacher probed further, clarifying concepts such as the distinction between binomials and trinomials. Through continuous questioning, the teacher helped students identify that items 1 and 3 were binomials while item 4 was a trinomial, leading to a deeper classification of expressions. The discussion then shifted to comparing item 8 with others, where students recognized its similarity to item 4, reinforcing the pattern. The teacher encouraged group accountability by referencing specific group presentations and inviting clarifications. Group 8 was asked to explain their classification logic, and they clearly articulated the basis of their sorting—dividing Set A and Set B by binomial and trinomial factors respectively. The teacher's facilitation highlighted concept development, peer explanation, and critical thinking in a collaborative setting.
Day 4	Teacher B started the class by asking students to exchange notebooks and check each other's work. He then chose a few numbers from the activity and asked for volunteers to show and explain their answers on the board. While students explained, TB asked follow-up questions to help everyone understand better. When a student asked about a_0, TB used the question to explain that it refers to a term before the first term in a sequence. He also gave an example of a sequence (1, 2, 5) that is not arithmetic to show that not all sequences have a common difference. TB reminded the class that arithmetic sequences are formed by repeatedly adding or subtracting the same number. Also, Teacher B conducted a "Boardworks" activity using the same mechanics as mentioned on Day 1, this is all about the arithmetic sequence. Overall, he

	guided the class with clear questions and let students take part in the discussion, helping them understand the topic better.
Day 5	Teacher B handled the class through an engaging and highly interactive discussion focused on identifying and analyzing patterns in a geometric sequence. The lesson began with a posted problem from 1 to 10, prompting students to analyze the problem and work collaboratively to determine the center or key structure of the sequence. Through strategic questioning, the teacher guided students to distinguish between arithmetic and geometric sequences, challenging their initial assumptions and encouraging group comparisons and justifications. Groups shared their observations, such as noticing differences or multipliers between terms, while the teacher facilitated deeper exploration by connecting these patterns to mathematical definitions like common difference, common ratio, and the nature of rational numbers. A spirited exchange unfolded as students debated whether certain terms were fractions or integers, with the teacher pushing for precise reasoning and mathematical justification. By the end of the discussion, students clarified that the term "common ratio" best described the relationship in geometric sequences. The session concluded with a reflective question and an assignment, tasking students to find the 100th term of a pattern without using the formula, reinforcing conceptual understanding over memorization. The teacher's approach promoted critical thinking, peer collaboration, and the development of strong mathematical arguments.

Class Observations of Teacher C from Day 1 to Day 4

Day 1	Teacher C began by asking students about their previous lesson and homework. She introduced the new topic with a 6-minute YouTube video, then reiterated the properties of a parabola and referred students to their books. She explained further, gave a sample problem for the class to solve, then showed another 10-minute video related to Pre-calculus. Afterward, she gave two more examples—each answered and discussed by different students at the board. The lesson ended with a seatwork activity as an assessment.
Day 2	Teacher C began her class by calling on different students to serve as leaders. She then projected her PowerPoint presentation with the new topic and instructed the students on what they were going to do. Teacher C provided a step-by-step process for solving the problem. Afterward, she facilitated a collaborative activity where students worked in groups to solve the given problem. Each group arrived at the same answer but used different methods, demonstrating that the problem had multiple solutions due to various trigonometric identities. Teacher C called particular groups to explain their

	answer in the class. The students were able to solve the problem quickly because they had index cards containing all the trigonometric identities. The class ended with collaborative activity.
Day 3	Teacher C began her class by playing a video about the Super Hexagon related to trigonometric identities. Afterward, she called on different students to serve as group leaders. She then presented her PowerPoint presentation, which was similar to what she used on Day 2. Each group solved the given problem using different methods but arrived at the same final answer. Teacher C asked each group to present their output in front of the class and explain their solutions. With the help of guiding and probing questions from Teacher C, the students were able to explain their answers more thoroughly.
Day 4	Teacher C began the class by reviewing the past quiz on trigonometric identities. She demonstrated how to answer the questions on the board, prompting students to fill in the missing steps by asking guiding questions. Afterward, she distributed the students' scores. TC then grouped the students and assigned each group a problem to solve. Each group went to their designated board and started discussing their answers. She called on one group to present their solution, and after reviewing their answers, the class celebrated as most of the groups got the correct answer

APPENDIX I

OBSERVATION NOTES

Classroom Observations from Teacher A to Day 1 to Day 5

Topic: Subtracting Polynomials
Teacher: (writing the problems on the board)
Problems:
1. How do you add and subtract polynomials?
2. Find the difference wh
3. $8x - 4$ is subtracted from $10x - 3$.
4. From $5x + 3$, subtract the sum of $-3x + 7$ and $4x + 8$.
5. Find the area of the shaded region:
Teacher: Okay, go. You have 15 minutes.
Student: (start answering)
Teacher: (reads the questions and emphasizing some key words)
Teacher: Are you listening?
Student: Yes ma'am.
Teacher: So, look at the figure. What shape is that?
Student: Rectangle.
Teacher: How did you know that it is a rectangle?
Student: Kay dili man even ang sides ma'am. (Because the sides are not even, ma'am.)
Teacher: What do you mean nga dili even ang sides? (What do you mean that the sides are not even?)
Student: (analyzing)
Teacher: What is the measure of this side? (referring to the width of the big rectangle)
Student: 4x.
Teacher: This side? (referring to the length of the big rectangle)
Student: 3x.
Teacher: If this has been a square? (referring to the sides of the big rectangle)
Student: Parihas. (same)
Teacher: Parihas dapat. But then, unsa? Dili man?
- So, this must be a? (It should be the same. But then what? Isn't it?)
Student: Rectangle.
Teacher: Rectangle.
- Okay, how do you find the r = area of the rectangle?
- Ha? (What) (referring to S10)
Student: Length times Width.
Teacher: Pangitaon man nimo diha ang squares. Nga 1x1 nga squares, then ihapon nimo pila ka squares. (referring to the problem no.4) (You need to find the squares there. The 1x1 squares, then count how many squares there are.)
Student: (listening)
Teacher: How are you going to do that?
Students: (listening)

Teacher: Pila ka squares diari? (How many squares are here?) (pointing at the width side). Pila pud ka squares diari? (How many squares are here too?) (pointing at the length side). Dayon, pila ka squares tanan? (Then, how many squares in total?) (referring to the area of the rectangle).
- So, how will you find that one?
Student: Multiply.
Teacher: Multiply ra, kung pila ni diri diba? (referring to the figure) (Just multiply, if this is how many here, right?)
Students: (continue answering)
Teacher: Unsa pud ning nasa sulod? (What is this inside?) (referring to the small shape that is inside in the big rectangle).
- What figure?
Students: (analyzing)
- Rectangle, ma'am.
Teacher: Still a rectangle. Tama? (Still a rectangle, right?)
Student: Yes, ma'am.
Teacher: What is the area again of a rectangle?
Student: Length times Width.
Teacher: Okay.
Student: (continues answering).
Teacher: Go to the board, whoever is ready na mo answer diha. (Go to the board, whoever is ready to answer, go ahead.)
Student: (Students are going to the board.)
Student 8: Ako sa number 2. Ikaw sa number 3. (I'll do number 2. You do number 3.)
Student 9: Sege ako sa number 3. (Okay, I'll do number 3.)
Student 10: Ako sa number 4. (I'll do number 4.)
Student 8: (solution)
2.) $10x - 3$
(s) $8x - 4$

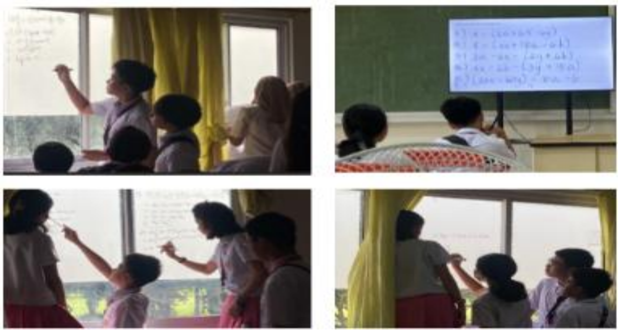
$10x - 3$
(A) $8x - 4$

$2x + 1$
Student 9: (solution)
3.) $5x + 3 - (-3x + 7 + 4x + 8)$
$= 5x + 3 - (x + 15)$
$= 5x + 3 + (-x - 15)$
$= 4x - 12.$
Student 10: (solution)
4.)
Big Rectangle = $LxW = (4x)(3x) = 12x2$
Small Rectangle = $LxW = (2)(x) = 2x$
Shaded region:
$Area(BR) - Area(SM)$
$= 12x2 - 2x.$

Day 2: Quiz Day

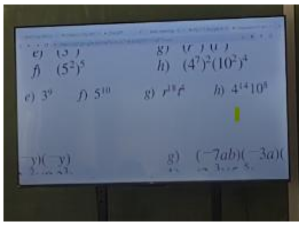
(The teacher presented a portion of the problems that the students needed to solve.)

T*: Cover your papers… Are you done?*
S: *Wala pa ma'am. (Not yet, ma'am.)*
T: *Okay, 1 minute. Paspas (Hurry up.)*
(After 1 minute)
T: *Okay, time's up! Let's check. Exchange papers with your seatmate.*
(The teacher flashed the answers beside each problem on the TV screen.)
T: *You just have to remove the grouping symbols.*
T: *Okay, write the score. Over 18. Okay, next..*
S: Wait sa ma'am (Wait a moment, ma'am.)
T: You have to learn to work fast.
T: Okay, use the back of the paper now. Ready? Okay, next.
(The teacher displayed a new set of problems for the students to solve. Once finished, the students exchanged papers again, and the teacher flashed the answers.)T: You have to learn to work fast.(After checking) T: Add up the scores. Over 30. Make sure na naay names written ha? (Make sure there are names written, okay?)
S: i-check gihapon kung wala siya naka-simplify? (Should we still mark it check if it's not simplified?)
T: Like what? Like how?
S: Like... 39/54.
T: So, what's the GCF of that?
S: 3.
T: dili pwede.. dapat kay reduced form (That's not acceptable… it should be in reduced form.)
T: Alright, return the papers. Sit with your groupmates and discuss. You may use the boards to discuss. And please come to me if there's anything you don't understand.
(The students moved to sit with their groups, grabbed whiteboards, and began discussing.)
T: Tan-awa ninyo ang papel ninyo, tapos katong inyong i-discussan katong mali. (Look at your papers, and discuss the parts where you made mistakes.)
T: Walay mag ulaw-ulaw. Pagdali na, katong mali ang i-discuss. (Don't be shy. Hurry up and discuss the incorrect parts.)
T: Okay, listen… This will be 1 pre-assessment. And even if your contribution is just asking a question, pwede ra gihapon (that's still valid)



Day 3: Grade 7 - Gamma
Topic: Subtracting Polynomials

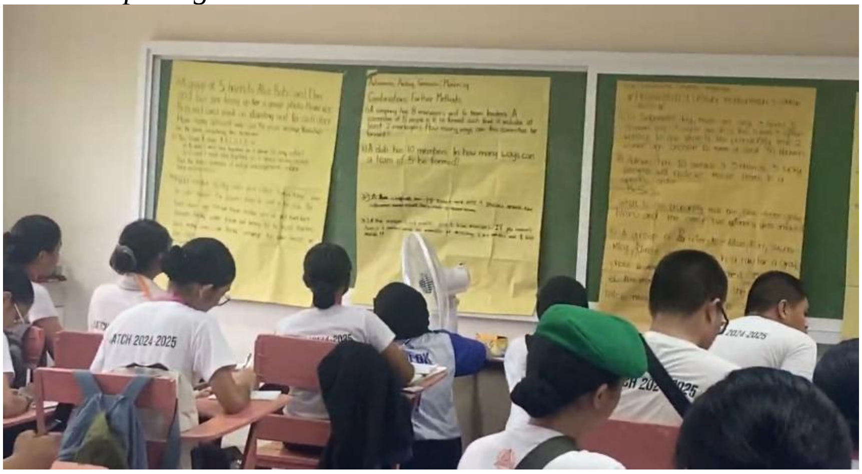
The teacher administered a quiz and a collaborative activity.
TA: Show your solution. Ngano na? Gi open ninyu inyung notes?
(Why is that? Did you open your notes?)
TA: Exchange your papers.
- Answers are posted on the screen.



TA: Are you done? For you to successfully answer this one, you need to have the notes. Kinsay nag-open sa notes? (Who opened the notes?) Raise your right hand. Ok, very good.
-CONTINUE ON CHECKING-
TA: Alright, you answer now for item 4.27. I'll give you 3 minutes to do that.
The window was checked/opened to improve ventilation.
TA: Eopen ning bintana para makasulod ang hangin. (Open the window so that the air can come in.)
TA: You can open your notes while answering.
S: Pwede maka-use ug phone, ma'am? (Can we use phones, ma'am?)
TA: Sa phone? Oh, sigi pwede ra. Pag connect mog internet. (On the phone? Oh, yes, that's fine. Connect to the internet.)
TA: Can you please be seated with your groupmates? Para ma alalayan mo pag answer, pero ayaw edritso ug share ang answer. Para dili sila maka feel na alone sila. (So you can help each other answer, but don't share the answer directly. This way, they won't feel alone.)
TA: Dili magpa copy sa answer. Make sure na ga-answer sila. Maningkamot mo. (Don't allow copying of answers. Make sure they are answering. Work hard.)
-COLLABORATIVE LEARNING (The teacher roams around.)
TA: If you need help, you go to them. Do not let them come to you. Nganong wala man mo gatapok? (Why aren't you gathering?)
TA: Lantawa ang notes. Ipakita sa iyaa ang notes. Ok, mag check na ta sa 4.7. Kay mag-continue tag discuss sa distributive property. (Look at the notes. Show your notes to them. ok, let's check 4.7. We will continue with the discussion of the distributive property.)
-CHECKING OF ANSWERS-
TA: After checking, show your solution to your groupmates. Pag tan-aw nimo sa imong answer kay wrong, try to answer again. If dili gihapon mao, try to ask your groupmates on how they got their answer. (When you look at your answer and it's wrong, try to solve it again. If it's still wrong, try asking your groupmates how they got their answer.)
TA: You can use your whiteboards. Pwede pod sa window mag write. (You can also write on the window.)
-The students can discuss their answer to their groupmates.-
TA: Class, kanang ga ask ug question ba, point na sya na nag-participate. (Class, those who are asking questions are already participating.) Show your solutions. Dili kay maglomba, tan-awa inyung mga mali. When someone is solving, tan-awa giunsa niya. (Don't compete, look at your mistakes. When someone is solving, see how they do it.)

TA: Ok, ipa-solve syag utro, Nakita na man ang answer key. Ipa try syag answer na siya ra.
(Alright, make him solve it again. The answer key was already shown. Let him try to answer on his own.)

Day 4:
Topic: Permutations
Problem-posing



Each group is assigned to create a set of problems related to their report. They are given the time to answer.
After that they are given time for group discussion.
TA: Ok, proceed to group discussion.
TA: Submit your papers to the reporters.
-GROUP DISCUSSION-
There are four groups posting 3 sample problems related to the topic. The students were given 10 minutes to answer. After that is group discussion. (to reflect on their answers). The reporters collected the papers.

Day 5
Topic: *Permutations, combinations, and probability of mutually exclusive events.*

Teacher A started the class with a collaborative activity. The students were grouped and assigned different topics such as permutations, combinations, and probability of mutually exclusive events. Teacher A encouraged the students to ask questions to their peers to gain a better understanding of the topic assigned to them.
After the collaborative activity, the teacher presented a problem and asked the students to solve it individually for about 20 minutes. The problem was:

"In how many ways can a man distribute gifts consisting of Php 500.00, one skateboard, and one bicycle to 5 boys?"
Two sample solutions were given:
Solution 1: $3\times3\times3\times3\times3 = 243$ *ways*
Solution 2: $5\times5\times5=125$ *ways*

The students were asked to study both solutions, describe what each solution means, and determine which one is correct. After the students answered, Teacher A asked the students to share their thoughts on which solution was correct, and then the class was dismissed.

Classroom Observations of Teacher B from Day 1 to Day 5

Day 1

Topic: Multiplying Polynomials

Grade 7 Board work

The class began with an activity called "Boardworks" to assess mastery of the lesson/topic.

The class was divided into two groups: Team A vs. Team B (each team had its own respective subgroups). Each team selected one representative to solve a given problem.

The team whose representative solved the problem first earned points based on the result of a dice roll. The team that solved the problem first explained their answers to the class.

While the representatives solved the problem, the rest of each team's group was also given the opportunity to solve it at their seats. If the representatives did not get the correct answer, the teacher called on a group that had the correct answer.

The board work activity lasted for 40 minutes.

Afterward, Teacher A posted a problem for the students to solve individually, but they could share their solutions with their peers or seatmates once they got the correct answer.

The class ended after that.

DAY 2

Topic: Arithmetic Sequence

The teacher checks assignments and calls for volunteers to show their solutions on the board and students explain their answers.

TB: What's your difference?

- *You mentioned their this common difference*
- *"What makes it common?"*
- *"When can you say there is something common?"*

S: Responds but confused to her answer.

TB: "Question: What makes it arithmetic? "

TB: "Unsay result if mag subtract ta ug duha ka terms?" ***(What is the result if we subtract two terms?)***

S: "Difference."

TB: "Going back to the statement that it has to be …"

TB: "Dapat naa siyay common difference." ***(It should have a common difference.)***

TB: "How do we establish common difference?

S: (The student answer the question but the voice is low)

TB: "I think dili kana ang pasabot ni S5" (I think that's not what S5 meant.)

TB: "E-try daw ug explain ang gi ingon nimo ganiha S5" ***(Try to explain what you said earlier S5.)***

S: (The student explain in front.)

TB: "How do we establish 'common difference ? ".

*S: "Eh minus nimo ang succeeding terms, let's say "*a2-a1,a3-a2" . ***(You just subtract the succeeding terms, let's say*** "a2-a1,a3-a2"***)***

TB: "Difference raman ato gistoryahan , so unsay operation? ***(We are just talking about difference, so what is the operation?)***

S: "Subtraction."

TB: "What operation do you use? "

S: "Addition."

S: "Sir connected raman si addition ug subtraction." (Sir, addition and subtraction are connected.)

TB: Pwede ra gihapon , but again, we have to prove that they have a common difference; wala man siya nag-ingon na common sum. So ning ana pag establish ang common difference. ***(That's still okay, but again, we have to prove that they have a common difference; it didn't say common sum. So this is how we establish the common difference.)***

TB: "How do we do that? Eh-establish sa nimo ang usa ka difference and we need one more para maka ingon tag naay common. So unsay gamiton nato? " ***(How do we do that? First, establish one difference, and we need one more so we can say there is a common one. So what do we use?)***

S: "We can subtract a3-a2 *."*

TB: "Now, if you establish that the common difference is -3 using these different terms, can you put there 2 points?"

The class continues in answering the rest of the items such as 12, 14. But this time they answered it by group. Students are given an opportunity to discuss their solutions in front of the class. After that, the teacher gives an open task that allows students to explore multiple solutions to the given problem.

Day 3

Grade 7 - Special Product

- *The class began by posting the problem (1-15)*
- *The collaborative activity was conducted where students analyzed the 15 sets of problems by putting it into a Venn diagram.*
- *Students work collaboratively by sharing their ideas and making conclusions where what number/item be put on Set A, B, and C with an intersection showing the relationship of its item.*
- *The teacher called some groups to discuss their output in front of the class.*

T: Because again, you observed that they have three terms compared to 1 and 2.

*T: Okay, ngano gani gi parenthesis ni ninyo? (**Okay, why did you put this in parentheses?**)*

*S: Kay para dili ma-short. (**So it won't be shortened.**)*

*T: Nganong di mashort?(**Why won't it be shortened?**)*

*S: Kay lahi man ang kuan sir 11.(**Because it's different, sir, 11.**)*

*T: Lahi ang unsa? (**Different from what?**)*

*T: 1 and 3 are binomial , Tama? (**Right?**) But number 4 is ?*

S: Trinomial

*T: Okay, trinomial daw number 4. (**So number 4 is trinomial.**)*

*T: Wala na aside from 2,7 and 12? (**Anything else besides 2, 7, and 12?**)*

S: 8

*T: Yes, how about 8? Unsay naa sa 8? (**What's in 8?**)*

*S: Parehas ra siya sa kuan 4 sir. (**It's the same as number 4, sir.**)*

*T: Number 8 daw same sa number 4 and number 5 as well. **Number 8 is the same as number 4 and number 5.**)*

*T: So far, kinsay group nag present ani, 1, 3 and 4? (**So far, which group presented on 1, 3, and 4?**)*

*TB: Si group 8 kay gilahi nila si number 4. (**Group 8 separated number 4.**)*

***TB**: Can we ask group 8 nganong gilahi ninyo si number 4? (**Can we ask group 8 why they separated number 4?**)*

***S**: Ang idea man good namo sir, kay diba ang set A kay contains only binomial factors (1, 2, 3, 6, 7, 9, 10, 12, 13, 15) SET B kay contains only trinomial factor which is (4 5 8 14) ang 4 kay naa sa B kay ang 4 man only contain trinomial. (**Our idea, sir, is that set A contains only binomial factors (1, 2, 3, 6, 7, 9, 10, 12, 13, 15), while SET B contains only trinomial factors (4, 5, 8, 14), and number 4 is in set B because it only contains a trinomial.**)*

***S2:** Along with 5 and 8*

DAY 4:

Topic: Arithmetic Sequence

S: *Sir, dili lang ta mag-dice, mag-check rata. sir naningkamot baya mig answer (**Sir, let's not use dice, let's just check. Sir, we really worked hard on our answers.**)*

T: *Recap, sa ta **(Let's do a recap).** Exchange notebooks.*

S*: Sir, we negotiate.*

S2*: Sir, wala pami nahuman (**Sir, we're not done yet.**)*
T: *Ngano wa paman na nahuman? (**Why isn't that finished yet?**)*

T: *Okay, number 1. Anyone who would like to show their answers. Okay, volunteer for number 1? How about number 2?*
*S: Sir mag explain pa sir? (**Sir, do we need to explain it too?**)*

*T: Kabalo naman mo unsay protocol ana. (**You already know the protocol for that.**)*

T*: Okay, next number 3? Okay, how about number 5? Kana lang numbers 1, 3, and 5, and siguro, can we have number 7? (**Just those numbers—1, 3, and 5—and maybe number 7?**)*

S*: Sir, magdice ta unya, sir? (**Sir, are we going to use the dice later?**)*

T*: Depende sa ilang answer. (**It depends on their answer.**) Okay, number 7? (A student raises his hand.) Okay, go.*

S2*: Sir, pwede nako dagdagan ang solution, sir? (**Sir, can I add to my solution?**)*

T*: Aw, ikaw gud... Ikaw bahala. (**Oh, of course... It's up to you.**)*

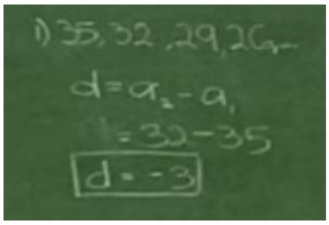
S3*: Excuse me, sir, pwede ko mag sulat sa number 2? (**Excuse me, sir, can I write for number 2?**)*

T: *Walay, number 2, later na. (**There's no number 2 for now, we'll do that later.**)*

T: *And please, make sure to write "corrected by" sa ibabaw (**on top**).*

(Checking of answers on the board)T: *Ok, let's start with number 1.*
S*: Sir, self-explanatory naman ni... (**Sir, this is already self-explanatory...**)*

S*: So ang gipangita man ani kay ang common difference, and we all know nga mao ni ang formula so ang and 2nd term i-minus sa 1st term. So naa naman tay given terms, ato nalang sya i-substitute ang variable and makuha na dayon nato ang* answer. *(**So what's being asked here is the common difference, and we all know this is the formula: the 2nd term minus the 1st term. Since we already have the given terms, we just substitute the variable and we'll immediately get the answer.**)*



T: *Sakto? (**Correct?**)*

S*: Yes, sir*

T: *Okay, questions?*

S4*: Dili possible ang a_0 sir, no? (**a_0 isn't possible, sir, right?**)*

*T: Possible ba daw ang a_o? (**Is a_0 possible?**) Okay, sige, unsay pasabot ni a_0? (**Alright, what does a_0 mean?**)*

S4: *The term before the first term*
T*: Ok, the term before the first term. Ok, question: What's the term before the first term?*
S4: *38*
T: *Ok, 38, unsay point sa a_o? (**What's the point of a_0?) So** naa pud tay term nga a_{-1}? pwede? (**so we also have a term a_{-1}? is that possible?**)*

S: *Ohhhh, pwede, sir, pwede. (**Ohhhh, it's possible, sir.**)*

*T: Ok, sige, we have the common difference, -3. Sakto o dili? (**Alright, is it correct or not?**)*

S: Correct.
*T: Ok, sakto **(correct)**. Kindly give 2 points. Wala gi-identify kung arithmetic ba sya o dili (**It wasn't identified whether it's arithmetic or not.**)*

S: Yes, arithmetic, yes, sir.
*T: Okay. This is arithmetic. You have to identify kay ang question man gud didto: "Determine whether the sequence is arithmetic" (**because the question there says, "Determine whether the sequence is arithmetic").***

*1 point for common difference and 1 point if gibutngan niyag arithmetic. (**1 point for the common difference and 1 point if it was labeled as arithmetic.**)*

S4: Sir, when you find the common difference, we already know its arithmetic, sir.

*T: Tanan ba naay common difference? (**Do all sequences have a common difference?**)*

*S: Dili, sir. **(No, sir.)***
T: Ok, what if your sequence is 1, 2, and 5? What's the common difference?

*S4: Wala, sir. (**None, sir.**)*

*T: Wala, so is it arithmetic? (**None, so is it arithmetic?**)*

S4: Sir, bag-o ni nga question. so imong gihatag nga example ganina sir is 1,2,5 dili sya arithmetic because wala siyay common difference. It's just a sequence, pero wala gi-specify if *arithmetic ba sya. (**Sir, this is a new question. So your earlier example of 1, 2, 5—it's not arithmetic because it has no common difference. It's just a sequence, but it wasn't specified if it's arithmetic.**)*

*T: So based on your readings last time, the question is if arithmetic ba daw tawag ana? (**Is that called arithmetic?**) So wala nato gi-specify unsa sya nga klase nga sequence. (**We didn't specify what type of sequence is that?**) Unsa nga klase nga sequence si 1, 2, 5? (**What kind of sequence is 1, 2, 5?**)*

S4: 1 to 5, sir? Or 1, 2, 5?

*T: 1 comma 2 comma 5. Unsa na sya nga klase nga sequence? (**What kind of sequence is that?**)*

*S4: Unsa na sya, sir? (**What kind is that, sir?**)*

T: We just know that it is a sequence, but how did we define arithmetic this time?

*S5: Naay, d sir. (**There is a common difference, sir.**) (d = common difference)*

*T: So unsaon nato pag expand ana? (**How do we expand that?**)*

*S4: Mag-involve ug adding a negative or positive value. (**It involves adding a negative or positive value.**)*

*T: Adding a positive or negative value but definitely sa adding (**with addition**). So it's a repeated addition mao na ginatawag na sya ug arithmetic (**that's why it's called arithmetic**). Pag i-add ninyo na tanan, then you can come up with arithmetic series. (**When you add all of it, then you can form an arithmetic series.**)*

DAY 5

***Topic:* Geometric Sequence**

(Posting a Problem)

TB: *Giunsa ninyu sya pag solve? Diba naa silay common, something. All you need to do is pangitaon ninyu iyahang center. You have 10 minutes to answer. (**How did you solve it? They have something in common, right? All you need to do is find its center. You have 10 minutes to answer.**)*

TB: *e-figure out sa na. Before nimo e write totally. Unsa na mga items ang similar? Stop writing, lantawon sa nato sakto ba ilang Anthony. Unsa daw ang question? Numbers 1 and 5. (**Figure it out first before you completely write your answers. Which items are similar? Stop writing, let's first check if Anthony's answer is correct. What's the question again? Numbers 1 and 5.**)*

S1: *The difference of 3, 5, and 7 is 2…*

TB: *You mean to say kay numbers 1 and 5. (**You mean to say it's numbers 1 and 5.**)*

TB: *I think group 7 would like to disagree. Sigi daw can we ask group 7? Sigi naa moy gisulat. Sigi, what do you observe on number 1? Oh, they are saying this is arithmetic. Why are you clustering together #1 and #5? (**I think group 7 would like to disagree. Okay, can we ask group 7? You've written something. What do you observe about number 1? Oh, they are saying it's arithmetic. Why are you grouping together numbers 1 and 5?**)*

TB: *Nganong si 1 and 5? (**Why numbers 1 and 5?**)*

S2: *2 ang difference. (**The difference is 2.**)*

TB: *Okey 2 daw ang difference. Do you agree? Mo agree daw ang group 4. Sigi daw G4? So unsay pasabot ninyu na naay difference of 2? (**Okay, the difference is 2, they said. Do you agree? Group 4 agrees. Group 4, what do you mean by a difference of 2?**)*

S3: *We disagree.*

TB: *Sigi, why do you disagree? (**Okay, why do you disagree?**)*

S3: *Sa pagbantay man gud nimo sa sequence, if e minus nimo si* a1 *ug* a2 *kay ang difference is 2. Whereas si* a2 *ug* a3 *ug e minus kay 4, murag familiar kaayo sya na pattern. (**When you observe the sequence, if you subtract A1 from A2, the difference is 2. But A2 and A3, when subtracted, gives 4. It seems like a familiar pattern.**)*

TB: *Okay. Naay gi multiply instead of gi add. Tama? But what you know about arithmetic is that...unsa man imong gibuhat? Kaning sa number 5 gi multiply. So, can we ask Group 5 if they are part of the same set? (**Okay. It was multiplied instead of added, right? But what do you do in arithmetic? Number 5 was multiplied. So can we ask Group 5 if they are in the same set?**)*

S2: *From what I have said earlier, 5, 7, and 9 are similar.*

TB: *Lahi napod na item ilang gi cluster. Do you agree? (**They grouped different items again. Do you agree?**)*

TB: *Mo agree raba ang group 2, na si 1 ug si 5 kay pareha sa 7? (**Group 2 agrees that 1 and 5 are the same as 7?**)*

S4: *Dili. Unahon sa natong 5, sir. So, ang partner sa number 5 kay si 5 ug 3. Naa pa nay lain ha. I would say na 9. You know why? Kay gi times 1 raman na. (**No. Let's start with number 5, sir. The pair of 5 is 5 and 3. There's another one — I'd say 9. You know why? Because it's just multiplied by 1.**)*

TB: *How about number 5, unsay gi multiply nimo? How about number 3? (**How about number 5, what did you multiply it with? And number 3?**)*

S4: *Number 3, kay gi multiply by 10. (**Number 3 was multiplied by 10.**)*

TB: *Unsay by 10? Gi add by 10? Ay e times UG 10, and its different from the discussions nato last time sa arithmetic. Unsay definition sa arithmetic diay? Repeated addition. Tama? So now if you look at numbers 1 and 7, it's arithmetic. (**By 10? Or added by 10? Oh, it was multiplied by 10, and that's different from what we discussed last time in arithmetic. What's the***

definition of arithmetic again? Repeated addition. Right? So if you look at numbers 1 and 7, that's arithmetic.*)* *Okay, thank you, group 5. Now, narrowing it down, 3,5,and 9. Sigi, pwede masulat sa right side of the board? What do you notice about that kind of sequence?(****Okay, thank you, Group 5. Now narrowing it down to 3, 5, and 9. Can you write it on the right side of the board? What do you notice about that sequence?****)*

S5: *Geometric.*

TB: *Sabi nila geometric daw. What do you mean by geometric? (****They said it's geometric. What do you mean by geometric?****)*

S6: *Iyang common difference, sir kay either gi multiply or gi divide. (***Its common difference, sir, is either multiplication or division.***)*

TB: *Common difference, daw. Ngano man? Unsaman diay tawag kon gi multiply na? (****Common difference, they said. Why? What do you call it if it's multiplied?****)*

S6: *Naay multiplier. (****There's a multiplier.****)*

TB: *Aha diay gikan nang difference diay? (***Where does that difference come from?***)*

S6: a0-a1.

TB: *What is in common? parehas ra sa* A3-A2. *So, 5-3, 7-5…Question, gi unsa ninyu pag arrive na? Ok, let's say na number 3 ang example. Sigi, unsa inyung gi-divide? (****What is in common? It's the same as A3 - A2. So, 5 - 3, 7 - 5… Question, how did you arrive at that? Let's use number 3 as an example. What did you divide?****)*

TB: *When we start with 3, how do you get the next term? Diba, ga-add mog 2? Unsay tawag ninyu sa 2? (***When you start with 3, how do you get the next term? You add 2, right? What do you call the 2?***)*

S7: *Common difference*

TB: *Ok, ang tawag sa 2 kay common difference. But this time, unsa may tawag nato sa 10? (***Okay, 2 is called the common difference. But this time, what do we call 10?***)*

S7: *Common quotient.*

TB: *Common quotient, daw. Technically, yes. (***Common quotient, they said. Technically, yes.***)*

S7: *Common product.*

TB: *Giunsa sya pagsulat, gi divide tama? In fractions, is there any other way we can describe that? Unsay pagsulat sa rational number? When we are talking with rational numbers, unsamay may naa sa denominator? (***How was it written, divided, right? In fractions, is there another way to describe that? How do you write a rational number? When we talk about rational numbers, what's in the denominator?***)*

S8: *Integers. Whole numbers.*

TB: *Integers are basically whole numbers. Tama? Question: Asa ninyu na makita na rational numbers? In other words, 100/10 is an integer.* (***Integers are basically whole numbers, right? Question: where do you see rational numbers? In other words, 100/10 is an integer.***)

S8: Yes.

S9: *Fraction.*

TB: *Nganong fraction man sya?* (***Why is it a fraction?***)

S9: *All integers kay fraction man, dapat all fractions are integers.* (***All integers are fractions, so all fractions should be integers.***)

S8: *All integers are fractions man, but not all fractions are integers.* (***All integers are fractions, but not all fractions are integers.***)

S9: Pwede man na nimo mahimong integer into fraction. (***You can turn an integer into a fraction.***)

TB: *If muingon kag common fraction kay mainggon kay common… ratio. Pero, we go back to the definition. What do we call this one? Is it an integer or a fraction?* (***If you say common fraction, that's like saying common ratio. But let's go back to the definition. What do you call this—an integer or a fraction?***)

S5: *No!*

S9: *Dili lagi sya pwede ma both..* (***It can't be both.***)

TB: *Kinsay gaanser ug dili fraction? Justify nganong inyung answer kay fraction.* (***Who answered that it's not a fraction? Justify why your answer is a fraction.***)

S8: *Naay number sa taas, naa poy number sa ubos.* (***There's a number on top and another below.***)

S10: *Sir... ug lantawon nimo siya, sir ba, diba fraction kay naay numerator ug denominator?* (***Sir… if you look at it, isn't it a fraction because it has a numerator and a denominator?***)

S9: *Integer lagi gihapon kay ma 10.* (***It's still an integer because it becomes 10.***)

TB: *Sigi nganong integer?* (***Okay, why is it an integer?***)

S9: *Actually sir, that is just an unsimplified na fraction.* (***Actually sir, that's just an unsimplified fraction.***)

S8: *Unya, unsa may tawag sa unsimplified na fraction?* (***So what do you call an unsimplified fraction?***)

S9: *Fraction.*

(MATHEMATICAL ARGUMENT)

TB: *Ug tan-awon nimo ang 100/10, unsa sya?* (***If you look at 100/10, what is it?***)

ALL: *Fraction.*

S8: *Mao na ang goal sa math. Tan-awon nimo. (**That's the goal in math. You observe it.**)*

S9: *It's not something you look at. It's something to solve. (**It's not something you just look at. It's something to solve.**)*

TB: *Nag overthink rajud mo. Ok, if you are to classify a number gani, you only need to look at its form. Basta ang fraction jud kay ratio of two integers. (**You're just overthinking. If you classify a number, just look at its form. A fraction is a ratio of two integers.**)*

END OF MATHEMATICAL ARGUMENT

TB: *Speaking of Ratio, dili na sya common fraction ha. Dili na sya common quotient but common ratio. Question? Ok, for number 5, unsay common ratio?*

S: *2*

S9: *If you want to come up with unsaon sya pag arrive sa formula. ok inyung himoon tommorrow kay e solve ninyung pattern without using the formula. (**Speaking of ratio, it's not called a common fraction. It's not common quotient, but common ratio. Question? For number 5, what's the common ratio?**)*

Assignment: How would you find the 100th term of item number 5 without using the formula ?

Classroom Observations of Teacher B from Day 1 to Day 5

Day 1 - G11- CAELUM

(After watching the video discussion about parabola)

TC*: Our lesson for today is all about what?*

S: Parabola

TC*: And you see there on the video that parabola is (meaning)*

TC*: Much better nga e-open? (referring to the windows) (**Would it be better to open [the window]** igang? (**Is it humid?**)*

TC*: These are the parts: we have the focus, vertex, directrix, and axis, axis of symmetry and naa pay kulang...(**there's still something missing...**) Naa sainyong book, unsay kulang? (**It's in your book, what's missing?**) Naa pud tay gitawag nga length of the latus rectum. (**We also have what we call the length of the latus rectum.**) Aha mana sya? (**Where is that located?**) The endpoints of our directrix.*

TC: *So this morning, we will learn about the properties of the different forms of parabola.*

*Sa circle class, unsa inyong maremember? (**In the topic on circles, class, what do you remember?**) Sa standard form sa circle, diba naa man to silay duha ka klase? Naa silay vertex nga? (**It has a vertex that is...**) Ang vertex is naa sa origin, which is (0,0), and then ang sa another form is asa man ang vertex? (**in the other form, where is the vertex?**)*

TC: *So in the parabola class, we also have 0,0 and vertex nga not in the origin (***that is not at the origin.***) So this is the equation of the parabola in general form found in your book on page 29. This is the standard form of the equation of the parabola:*

$$x^2 = 4py \quad or \qquad y^2 = 4px$$

TC: *So let's start with the vertex at (0,0), this is the standard form:*

Okay, pag x^2 means upward or downward. Pag y^2, left or right. So what are the properties? (when it's x^2 it means it opens upward or downward. When it's y^2, it opens left or right.)

From the parabola, from the vertex here, dapat daw same ang distance ni kinsa? (it should have the same distance from whom?)

S: *Distance sa focus ma'am.(Distance to the focus, ma'am.)*

TC*: From the vertex to the focus and from the vertex to the?*

S*: Directrix*

TC: *Let's have our first example:*

This is the parabola, y^2 . diba basta y^2 opens to left or right ***(Right? When it's y^2, it opens to the left or right.)****. We have our vertex, p, and the directrix. Unsaon nato pagkuha?****(How do we find it?)*** *Determine first unsaon pagkuha ang p* ***(how to get p)****, if $p > 0$ the graph opens to the right and if $p < 0$ opens to the left.*

Here, for x^2, if $p > 0$ the graph opens upward and if $p < 0$ the graph opens downward

Take note, ngano man nga -p ni sha? ***(Why is this -p?)***

S: *Kay naa na sa negative iyang directrix.(Because its directrix is on the negative side.)*

TC: *Kay naa na sya didto and negative na iyang value. Okay good*. (*Because it's already there and its value is negative.*)

Instructions: *Find the focus and directrix of the parabola.*

TC: *Find the focus and directrix of the parabola given by $y^2 = 12x$.*

Sige daw bi (Let's see), what is p?

S: *3*

TC: *Why 3?*

S: *Kay 12x man, maam.(Because it's 12x, ma'am.)*

TC: *Okay so nagkuha ramog factors no..(so you just got the factors, right?) 12x is 4p as a whole. So naa na tay p (so we have p). Which is 3. So makuha na nato ang focus because p is* *positive thats our hint. Asa man sya padulong? (So we can now get the focus because p is positive — that's our hint. (Where does it go?)*

S: *Right.*

TC: *Asa man? (Where?)*

S: *And opening naa sa right.(And the opening is on the right.)*

TC: *The graph opens to the right. Since p is positive. Are we clear?*

S: *Yes, maam.*

TC*: Ang focus is (p,0) since we have*

$y^2 = 12x$

$p=3$

$F(p,0) = (3,0)$

$V\ (0,0)$

directrix: $x = -p$

$x = -3$

Makagraph na ba ta? Kani, (0,0) tapos atong focus kay opens to the right, mag ihap ko ug pila ka unit? (Can we now graph it? This is (0,0), then our focus opens to the right, let me count how many units?)

S: 3

TC: *Ok 1, 2, and 3. Asa man nako i-draw ang parabola? Ana ba? Sure ba mo? (Where should I draw the parabola? Like this? Are you sure?)*

S: *yes, yes.*

T: *This is the focus; this is the vertex. So, our directrix is?*

S: *-3*

TC: *So, ihap tag 1,2,3 then line. mao pani atong na-drawing pero naa pa tay e-add nag property*

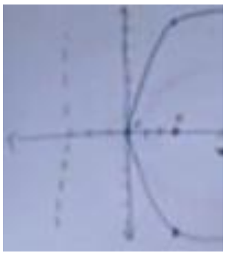
Which is the length of the latus rectum. Para kabalo ta sa buka bitaw (we count 1, 2, 3 then draw the line. This is what we have drawn so far but we still have one more property to add,)

TC: So we substitute x into 3.

$y^2 = 12\ (3)$

$y^2 = 36$

$y = 6$



We have 6 units up here and down. This is the length of our latus rectum.

TC: *Find the focus, the directrix, and other properties of a parabola $x^2 = -8y$.*

If naa namoy answer, ayaw sa e-shout (If you already have an answer, don't shout it yet.) Let others have the chance to answer

S: *Tanan properties, maam? (All properties, ma'am?)*

TC: *Oo, tanan properties. First, you determine asa man sya ma-fall (which direction it falls or opens to.). What's your p? Hatagan nakog points ang makaanswer sa front. If gusto mo mu-answer sa front, raise your hand.(I will give points to those who answer in front. (If you want to answer in front, raise your hand.)*

TC: *You can ask questions ha... You can talk with your seatmates.*

TC: *(referring to S1) ikaw ready naka? (referring to S2) Ikaw, ready napud ka? (Are you ready? Are you also ready?) Okay, sige, pag bato bato pick mo... okay go (okay, do rock-paper-scissors… okay go)*

(student 1 and 2 did a rock paper scissors)

TC: *Kinsay daog? (Who won? You?) Ikaw? (referring to S2)*

S: *Yes, maam.*

TC: *Okay, ali. (come here.)*

(while S2 is writing her answer on the board, teacher C asks the whole class.)
TC: *What's your p?*

S1: *Positive 2.*
S3: *Negative 2.*
S1*: 2*

TC: Ngano negative 2? Okay, sige daw bi. (*Why negative 2? Okay, let's hear it.*)

S1: iyang form maam kay $x^2 = -2y$, so second form sya maam. (*Its form is $x^2 = -2y$, so it's in the second standard form, ma'am.*)

TC: *So your p is 2.*

TC: *Kinsay naka-draw na ug graph? Tanan? (Who has drawn the graph? Everyone?)*

I-practice ninyo ug draw ang graph. (Practice drawing the graph. Try to estimate the distances evenly.)

TC: *Okay let's listen to S2's explanation.*

S2: So ang given nga equation kay x^2 equals to -8y—
TC*: "is equal to…"*

S2: x^2 *is equal to -8y. Gipili nako ang equation sa second form kay ang equation sa second form kay -4py (I chose the equation in the second form because that's the one with -4py.)*

Gi-kuan nako sya. Nangita kog number nga if e times sa -4 kay muresult ug -8, which is 2 (I solved it by finding a number that, when multiplied by -4, results in -8, which is 2.)

. And then ang directrix is y=p, so nakuha nato ganina nga ang p kay 2, so mao nana sya ang directrix (And then the directrix is y = p. Earlier we got that p is 2, so that's the directrix.) And then the axis of symmetry kay x=0 (And then the axis of symmetry is x = 0.) Then opening direction niya kay downward *and length of latus rectum kay absolute value of 4p (Then its opening direction is downward and the length of the latus rectum is the absolute value of 4p) so absolute value of -4(2) = -8, and since absolute value of -8 is 8. And then sa graph mao nani sya ang focus. Sakto ba, maam? (And on the graph, this is now the focus. Is that correct, ma'am?)*

TC: *ay, iyang vertex. O dira kasagara ang mistake, dili mo mag draw sa inyong parabola sa focus ha. Dapat sa vertex. (Oh, the vertex. This is where most of the mistakes happen—you don't draw the parabola at the focus, okay? It should be at the vertex.)*
(S2 proceeds on explaining the graph.)

Day 2:

Topic: Fundamental Trigonometric Identities

(The teacher started the class by checking the assignment, calling some student representatives to show their solutions on the board.)

T: Did you bring your whiteboard pen?

T: Are you okay with group activity?

S: Yes

(The classroom is noisy due to the returning the papers to the rightful owner after checking their assignment).

(The teacher called some names that will serve as a leader in a group activity.)

T: Okay, count.

S: 1, 2…. 7.

(The teacher assigned designated areas where students work collaboratively.)

T: hala wait asa inyuha index card? inyuha book? *(Wait, where are your index cards? Your books?)*

T: Okay, ready na? *(Okay, are you ready?)*

(The teacher presented a PPT and instructed students.)

T: Okay, that will be your task.

(The teacher presented a problem as an example and read it from the PPT.)

T: Okay we're done with simplifying trigonometric expressions, now we will use those identities. Now here is an example from the book..

T: Paminaw sa mo, I'll give you an example huh, Then I will give you time to answer the next example. *(Listen up, I'll give you an example first, then you'll have time to try the next one.)*

T: So in solving this, what is your first step?

S: Reciprocal sa secant ma'am. *(Use the reciprocal of secant, ma'am.)*

T: Okay, we can use the reciprocal identity. so we have secant u, cos u = 1/sec u , so we can then?

S: substitute

T: Substitute, correct! the value of sec u. Okay, what's the value of sec u?

S: -3/2

T: so 1 / -3/2 is equivalent to?

S: - ⅔.

T: Very good, so naa natay value sa cos u ug sec u , unsa pa may kulang? *(So we now have values for cos u and sec u. What's missing?)*

S: Sine

T: Unsaon pagkuha sa Sine? *(How do we find sine?)* What identity can we use?

S: ah , katong pythagorean ma'am. *(Oh, that Pythagorean one, ma'am.)*

T: Correct, using a Pythagorean identity , you have sin^2 u = 1 - cos^2 u. So naa man tay cos u , atong eh? *(Since we have cos u, we'll?)*

S: e-substitute

T: Yes, correct. Isolate sin u^2 then simplify 1- 4/9. What is 1 - 4/9?

S1: 5/9

S2: - 5/9

T: Okay, 5/9.

S2: ayy 5/9 diay *(Oh, it's 5/9.)*

T: But, because diba if square root nato ni 5/9. $\sin^2$ u na siya ha. atong goal is sin u because sec u < 0, meaning negative. Okay, here is the tangent u > 0. Tangent u is equivalent to what? sin u/ cos u. If si tangent u > 0 , unsay meaning ana? (*But remember, when we take the square root* *of 5/9, we're solving for sin u. Since sec u < 0, meaning secant is negative… Okay, here it says tan u > 0. Tangent u is equivalent to what?)*

S: Greater ang value, ay positive ang ni sin u. (*The value is greater… oh, sine u is positive.)*

T: Very good, positive si sin u and positive si cos u. What other possibilities?

S: Negative silang duha. *(Both are negative.)*

T: Negative silang duha? *(Both are negative?)* okay, correct! now unsay man ang nakuha nato na value ni cos u? (*Now what did we get as the value of cos u?)*

S: negative si cosine u. (Cosine *u is negative.)*

T: so unsa man atong sign dapat? *(So what should be the sign?)*

S: negative.

T: okay, negative. okay atong square both sides. it follows u lies in the third quadrant , right? if the sin u is negative , we can choose the negative root. So we have sin u -5/3. so naa natay cosine u , sine u, and secant u. makuha naba nato ang remaining na mga values? *(Now that we have values for cos u, sin u, and sec u, Can we find the remaining values?)*

All: Yes.

T: Using reciprocal identities. Okay, that's your task.

(Teacher then gives a problem, given: csc x = -7/6, tan x > 0. 10 minutes to answer)

(The students solve the problem collaboratively with different solutions using reciprocal identities. The students then discuss and share their solutions in front of the class

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