이진 탐색 트리의 종류 (자동 밸런스)

AVL
2-3
2-3-4
B tree
B+ tree
B** tree
...
RB tree

위키피디아

RB tree 삽입 룰

1

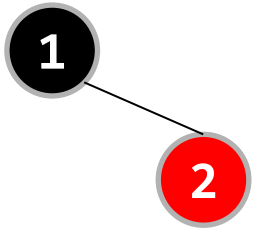
1. 새로운 노드는 빨강이다.

RB tree 삽입 룰

1

1. root 언제나 검정이다.
2. 새로운 노드는 빨강이다.

RB tree 삽입 룰



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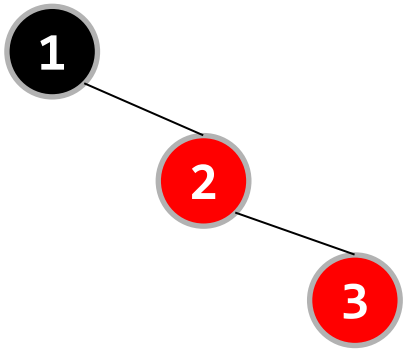

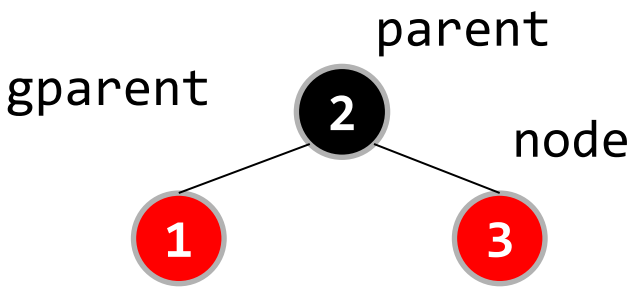

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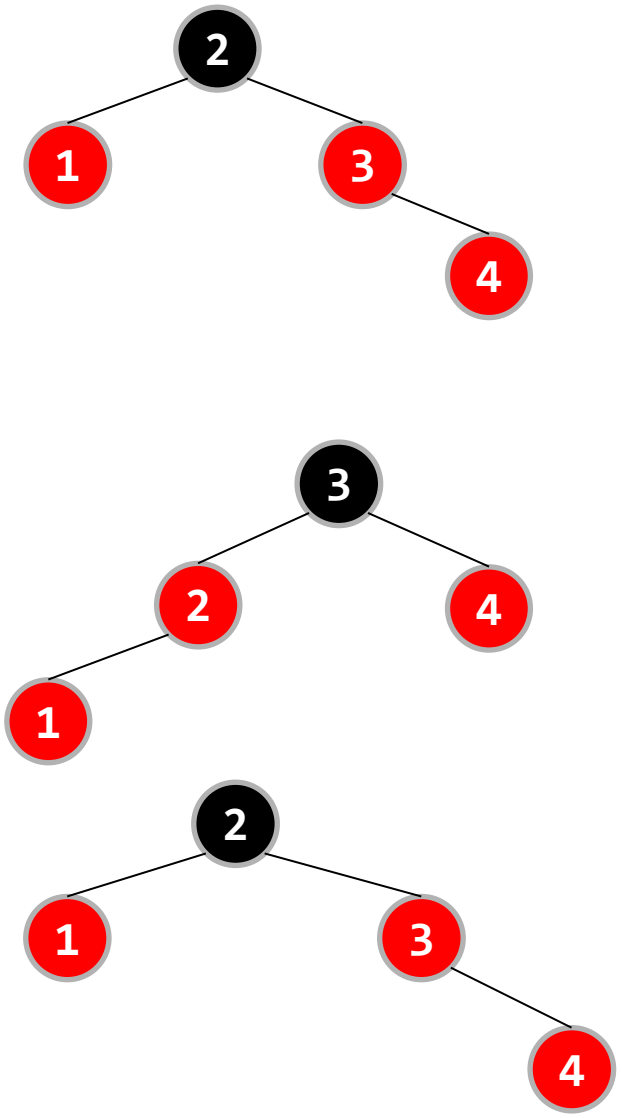
3. 부모가 빨강이면 회전한다.

R-R 경사
color flip
parent => BLACK
gparent => RED
rotate_left(gparent)

RB tree 삽입 룰



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L-L 경사
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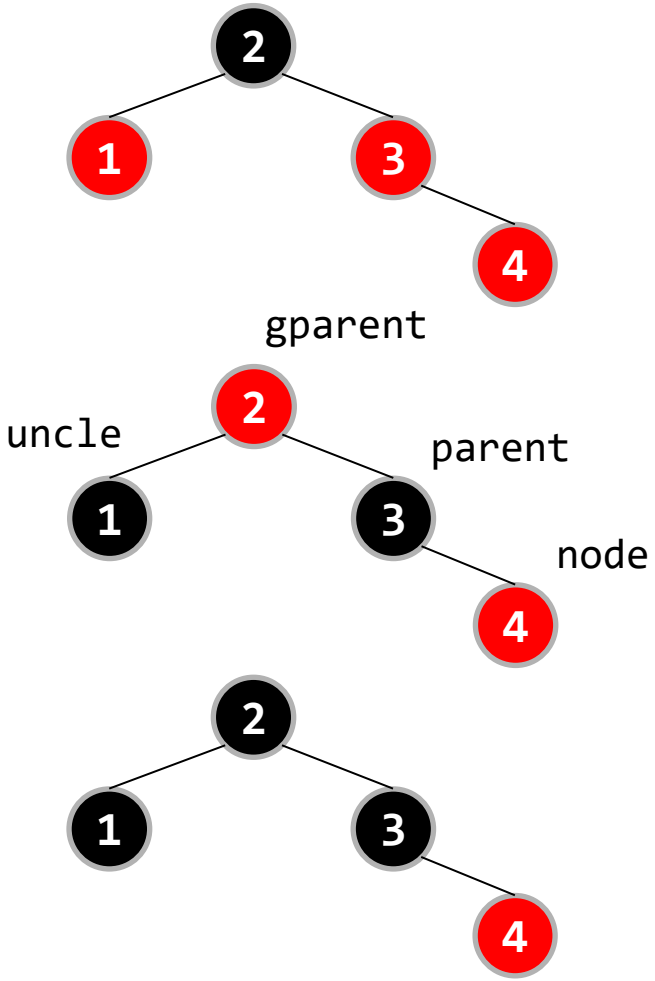
gparent => RED
rotate_right(gparent)

RB tree 삽입 룰



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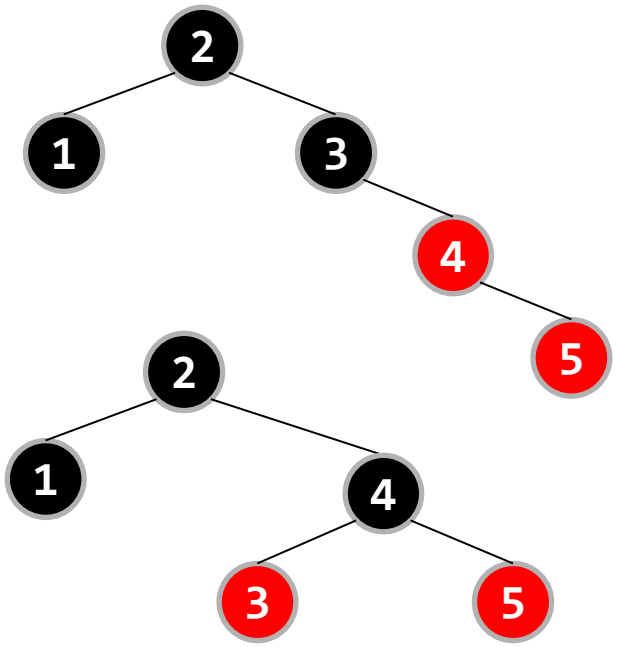

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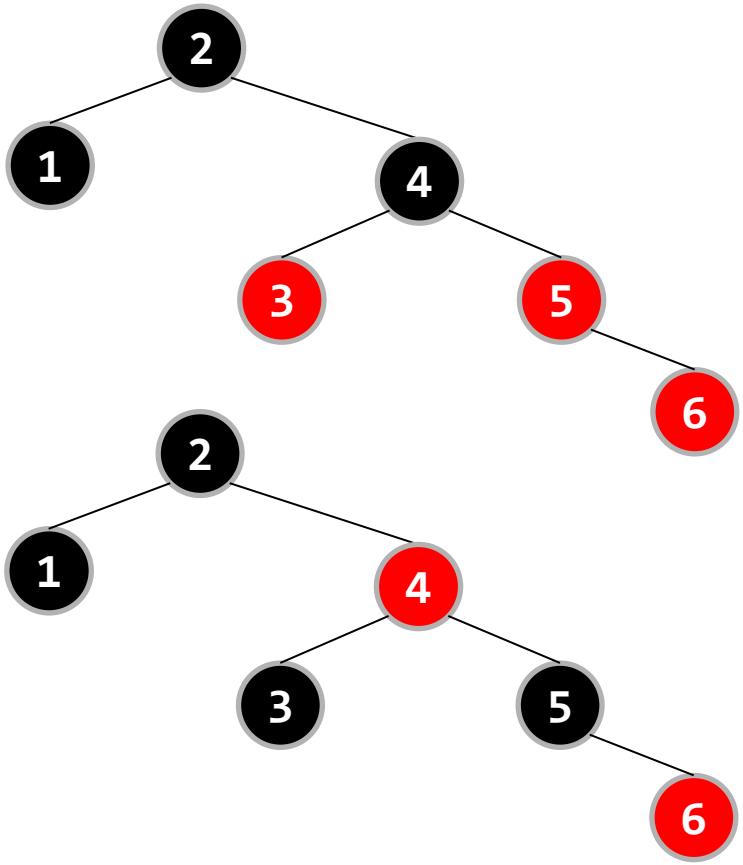

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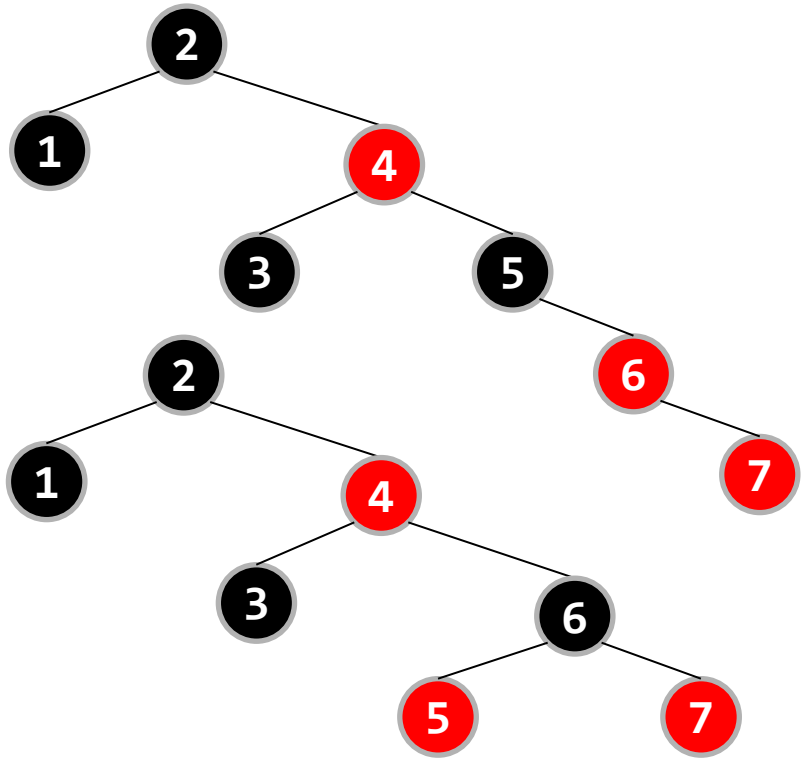

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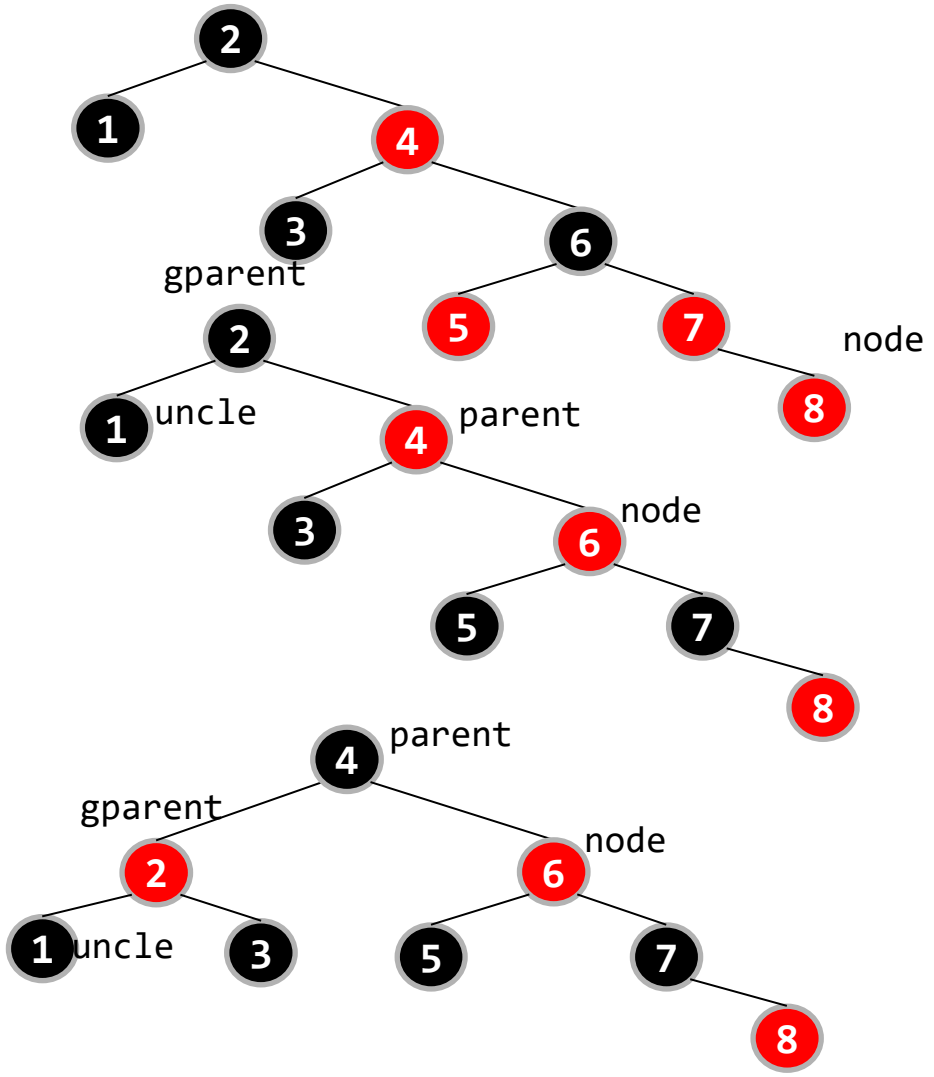

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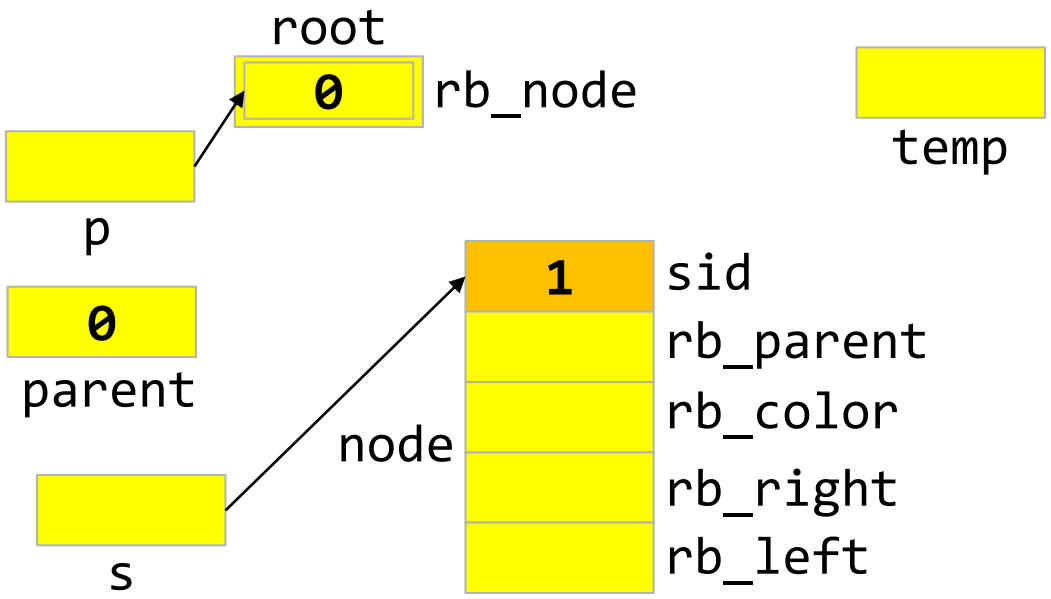
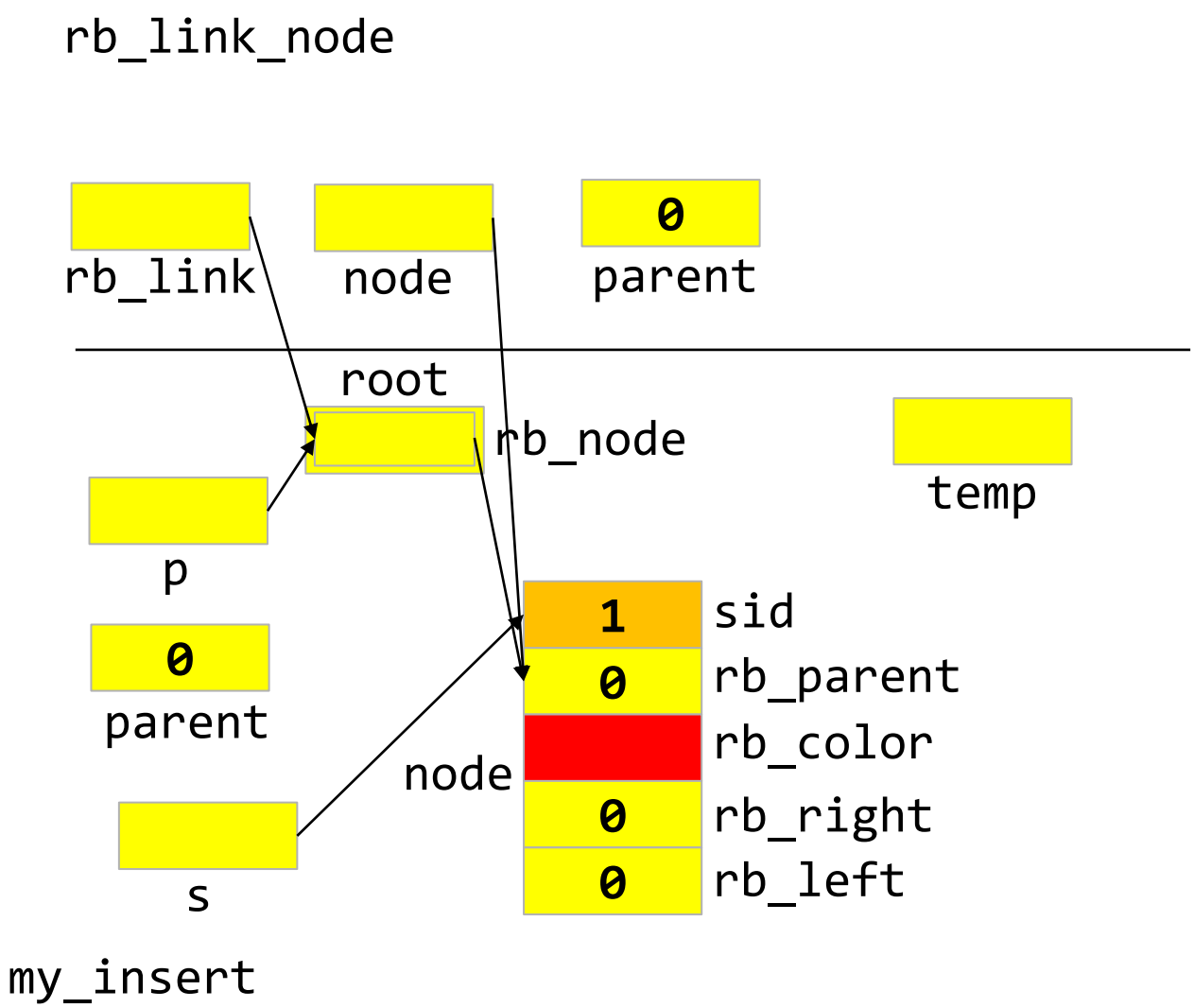
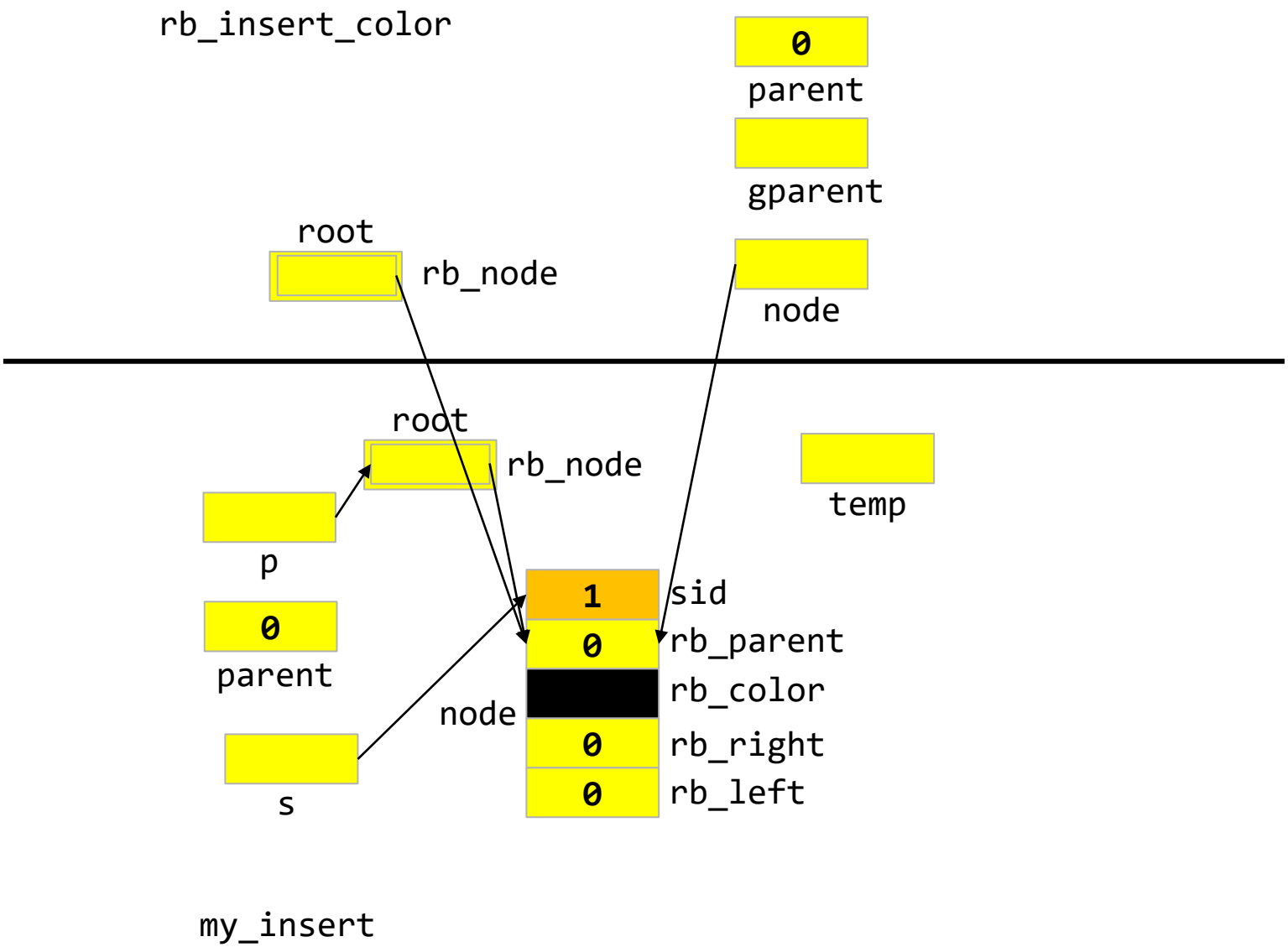
RB tree 삽입 코드 분석



RB tree 삽입 코드 분석



RB tree 삽입 코드 분석



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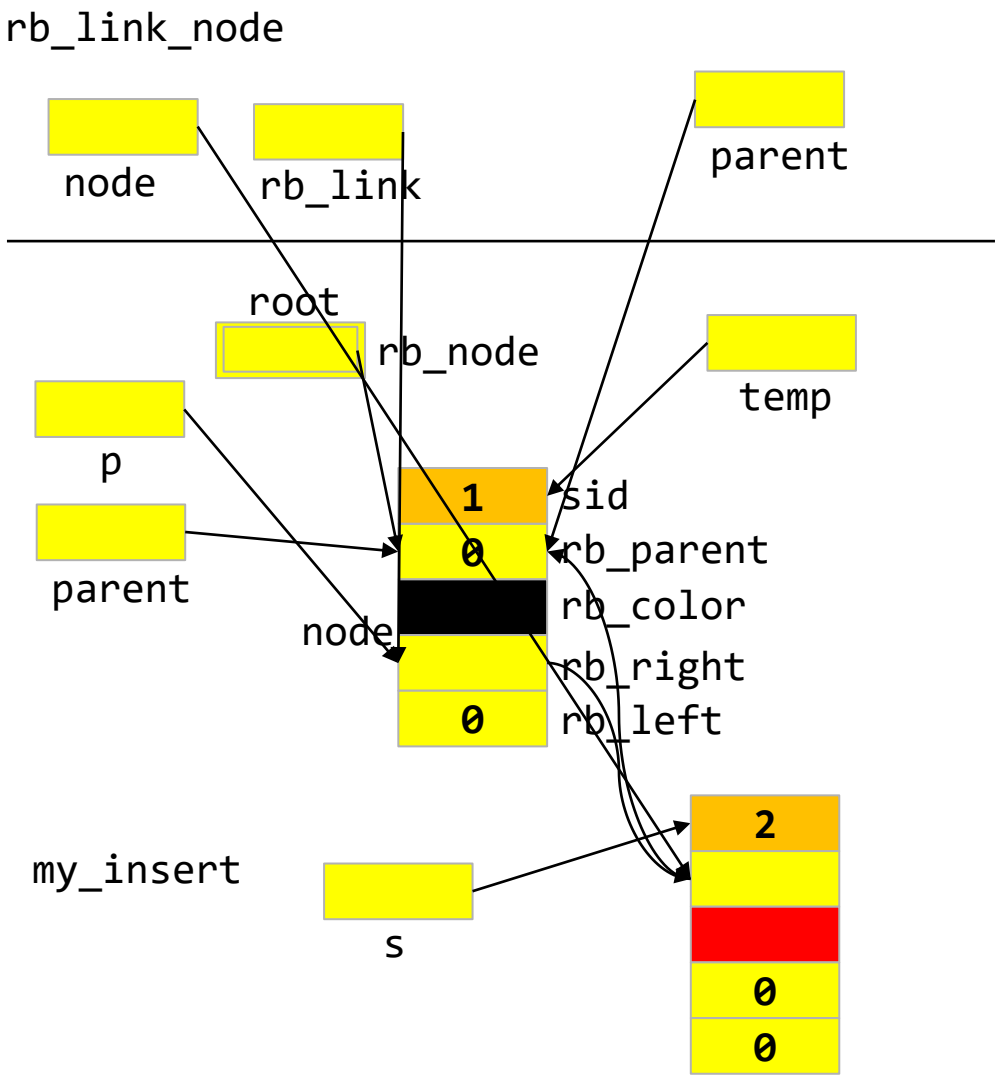
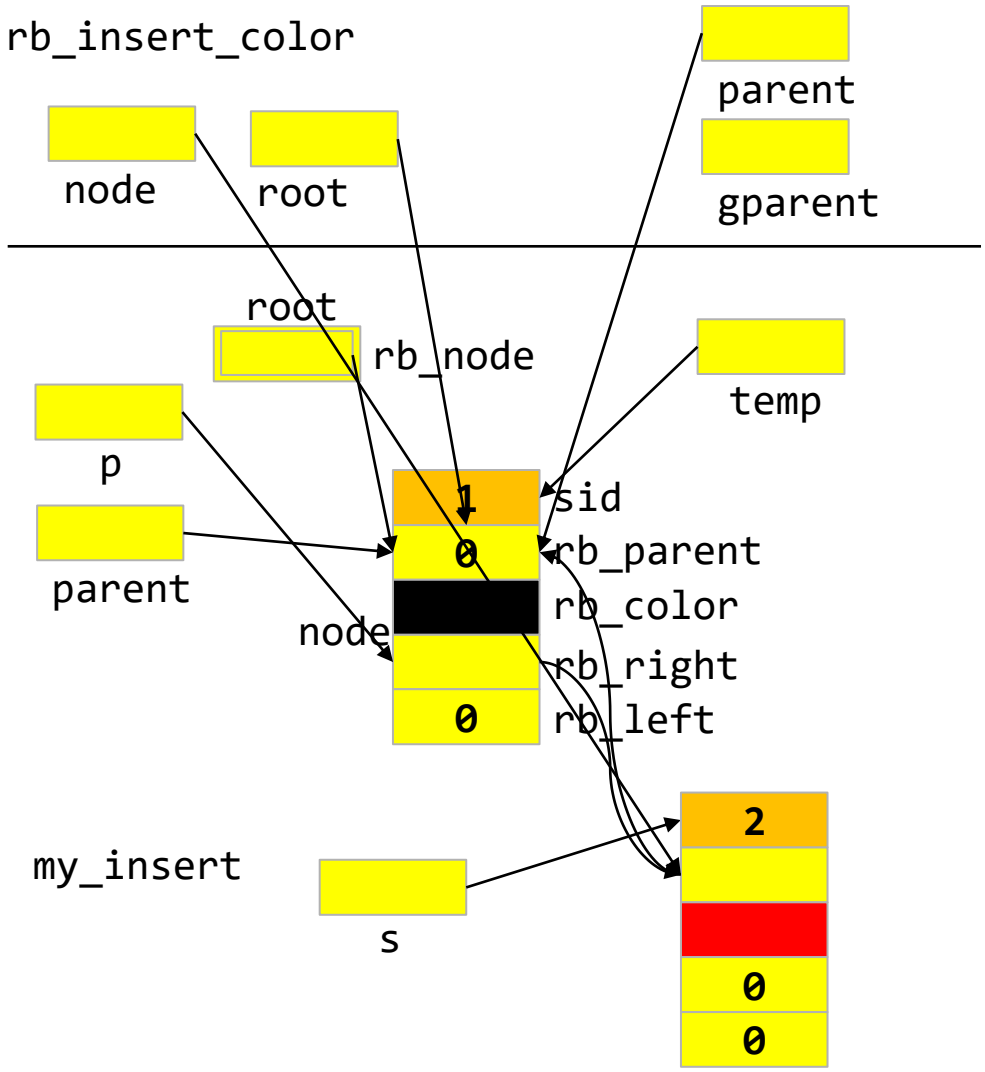


RB tree 삽입 코드 분석



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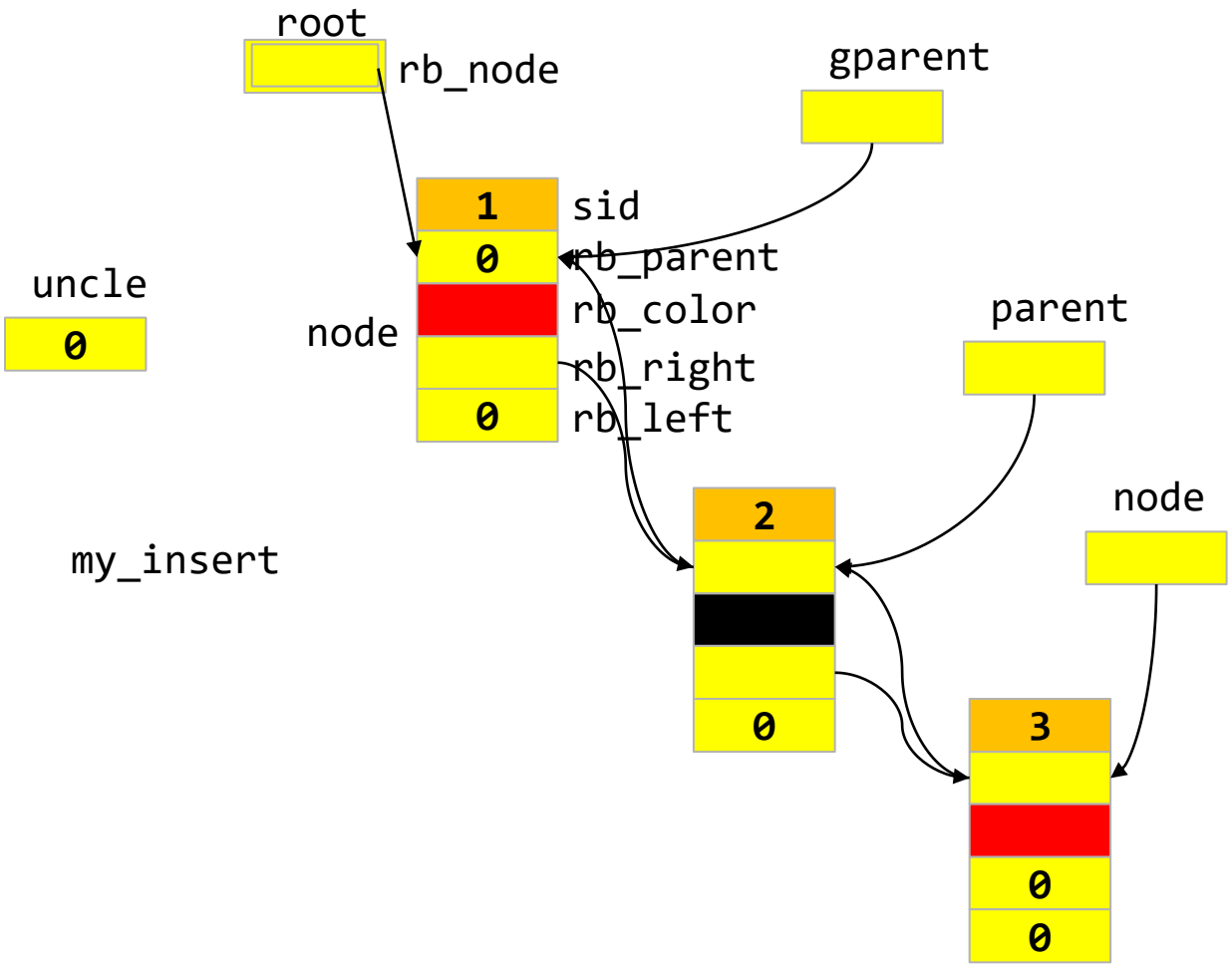
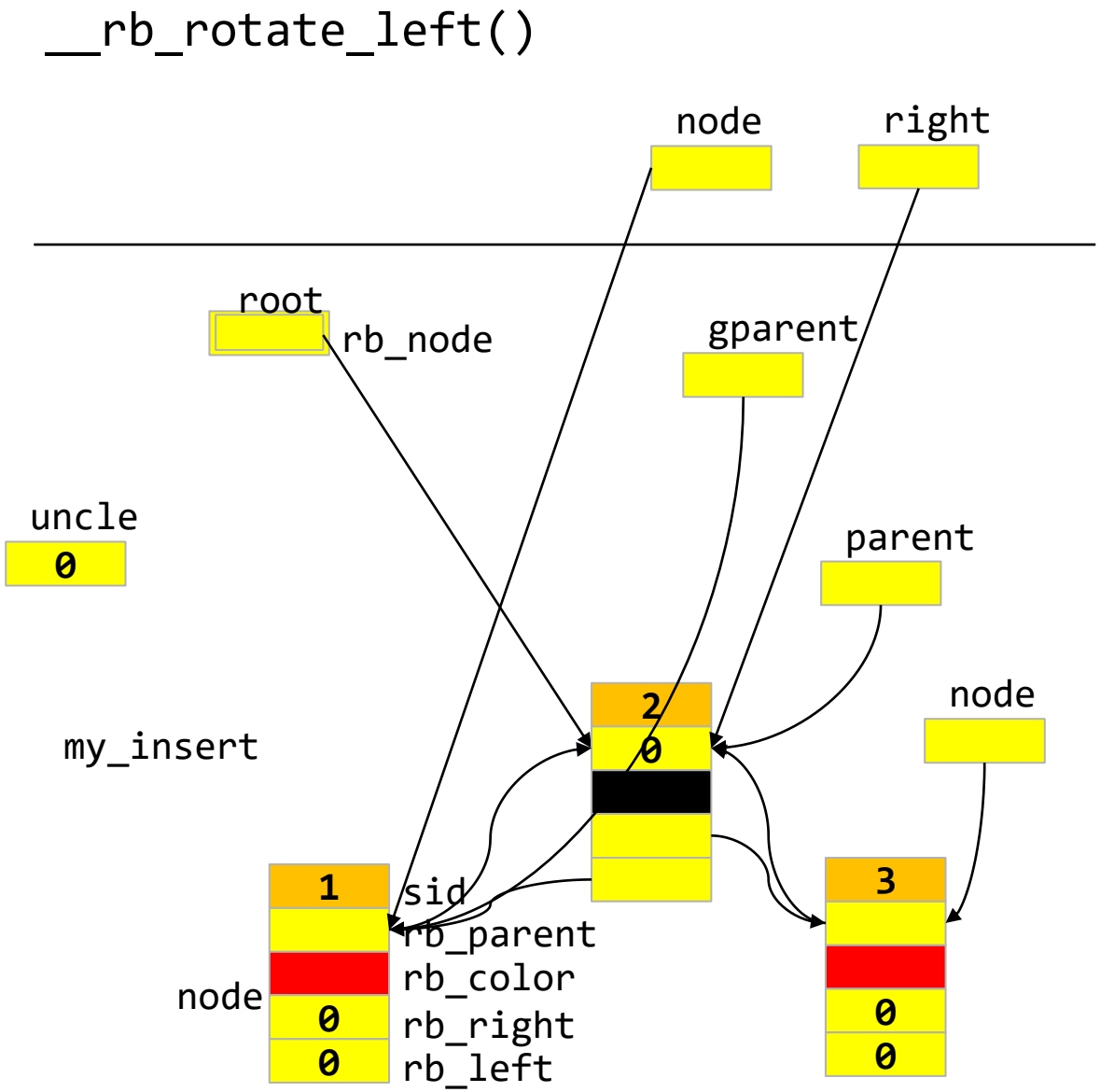
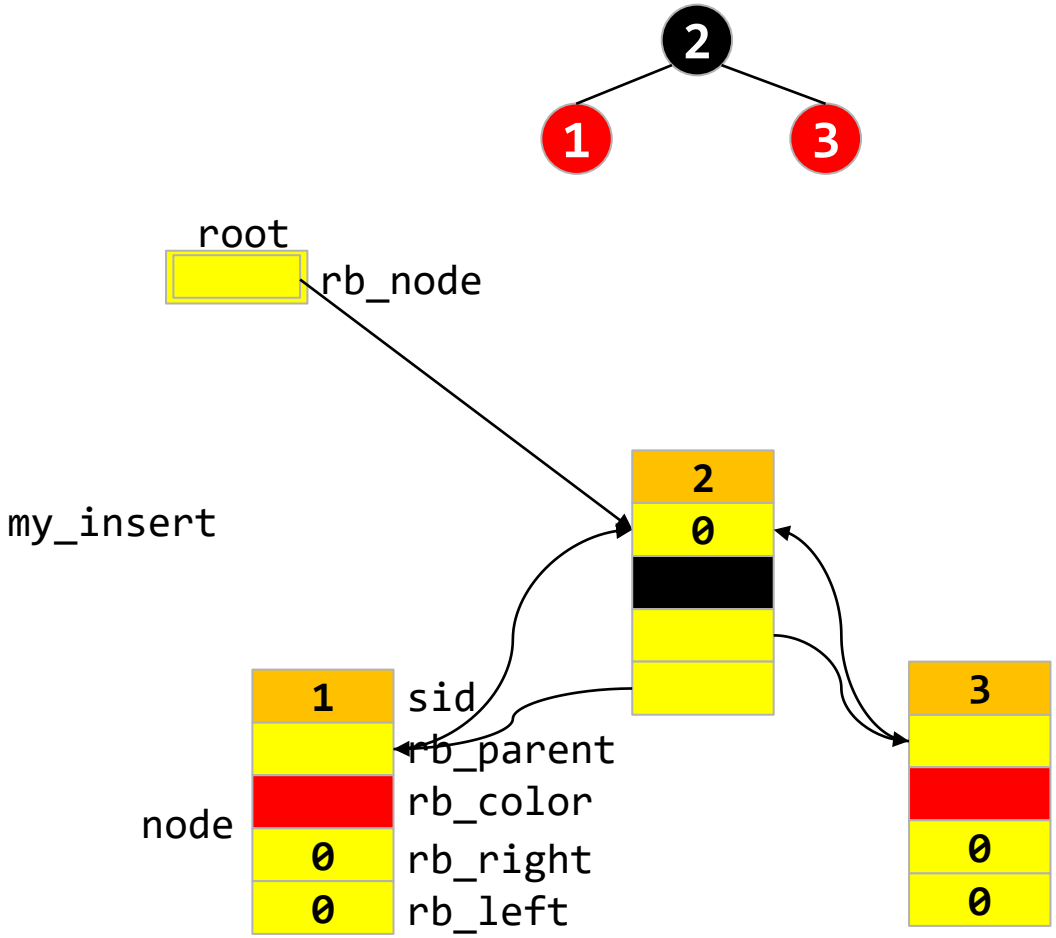


RB tree 삽입 코드 분석

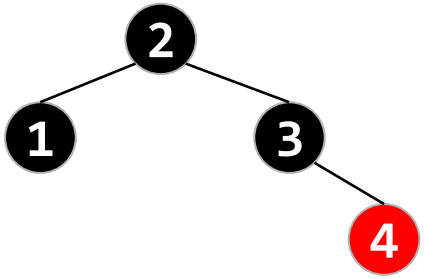


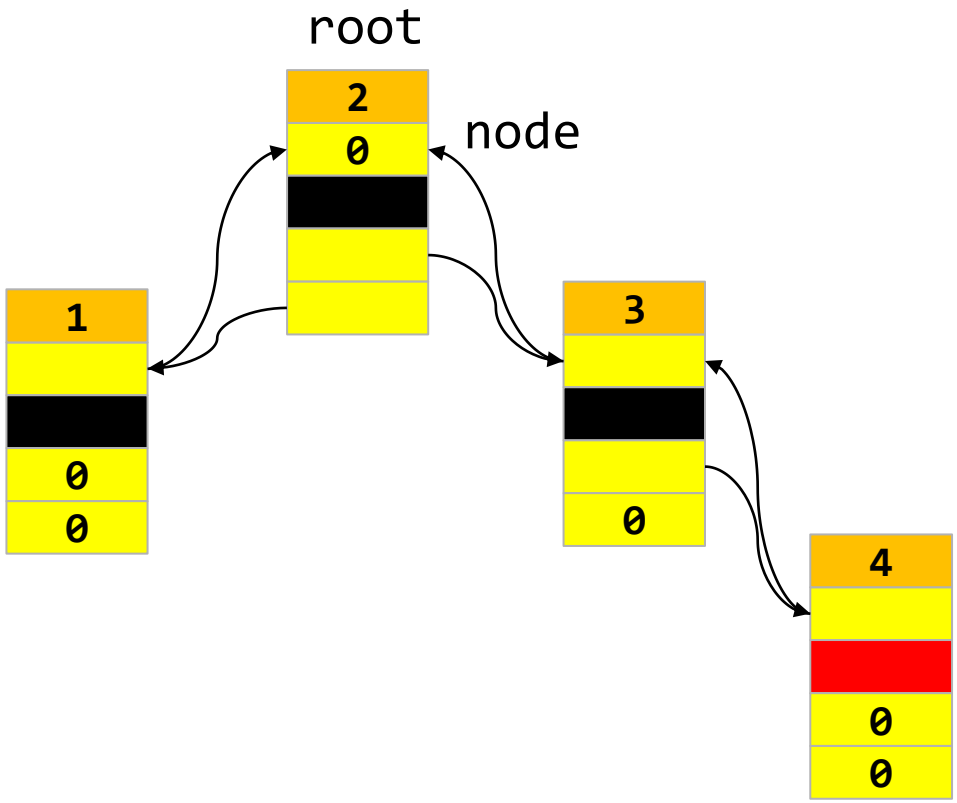
RB tree 삽입 코드 분석

__rb_rotate_left(gparent)

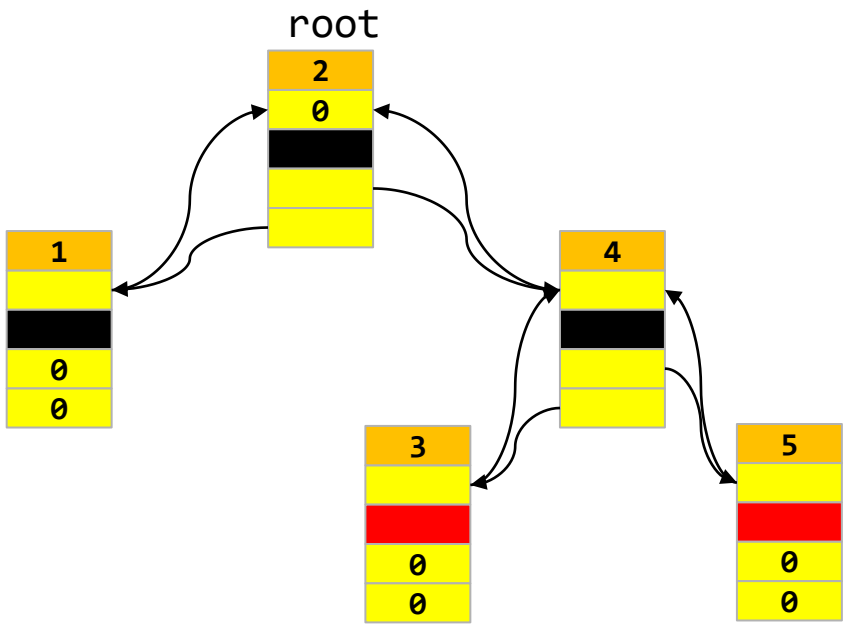


RB tree 삽입 코드 분석

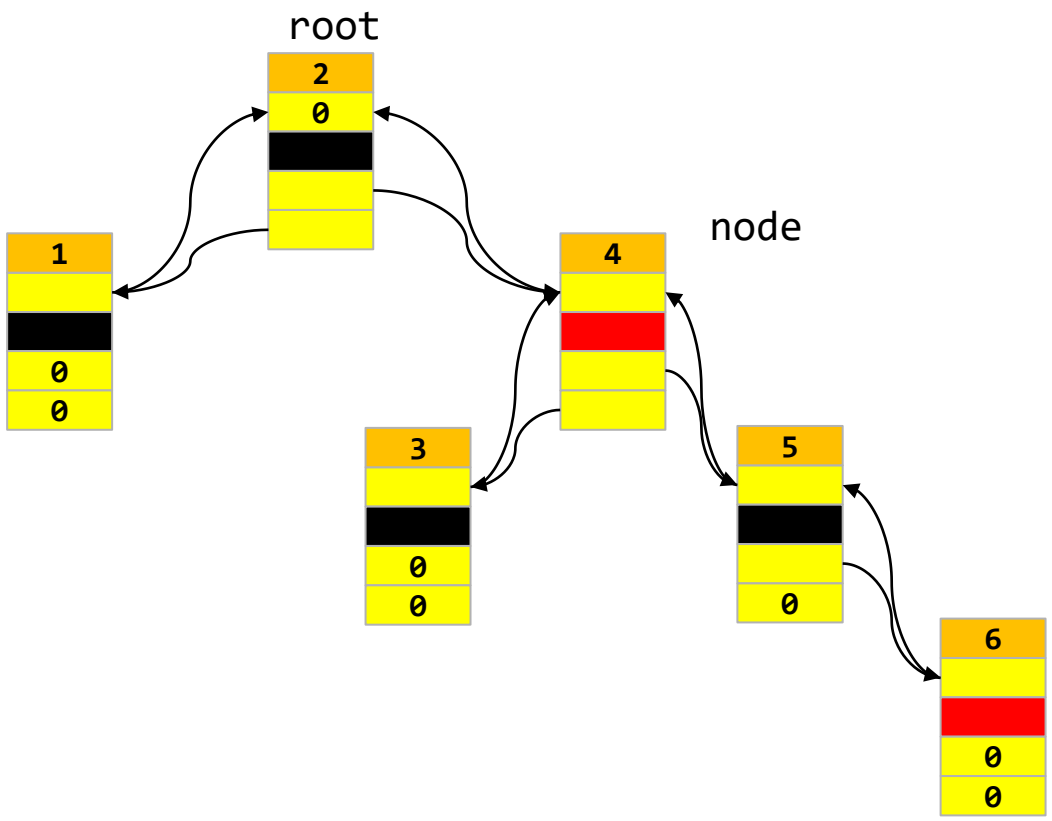




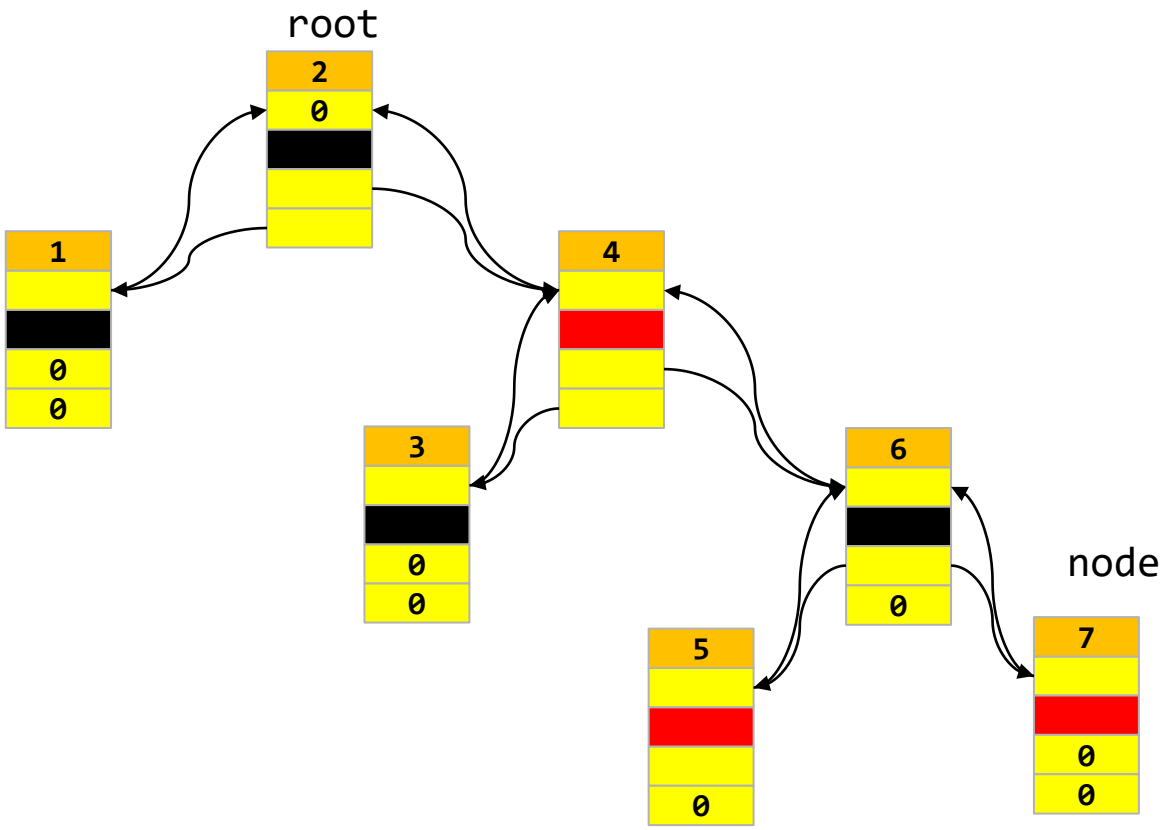
RB tree 삽입 코드 분석



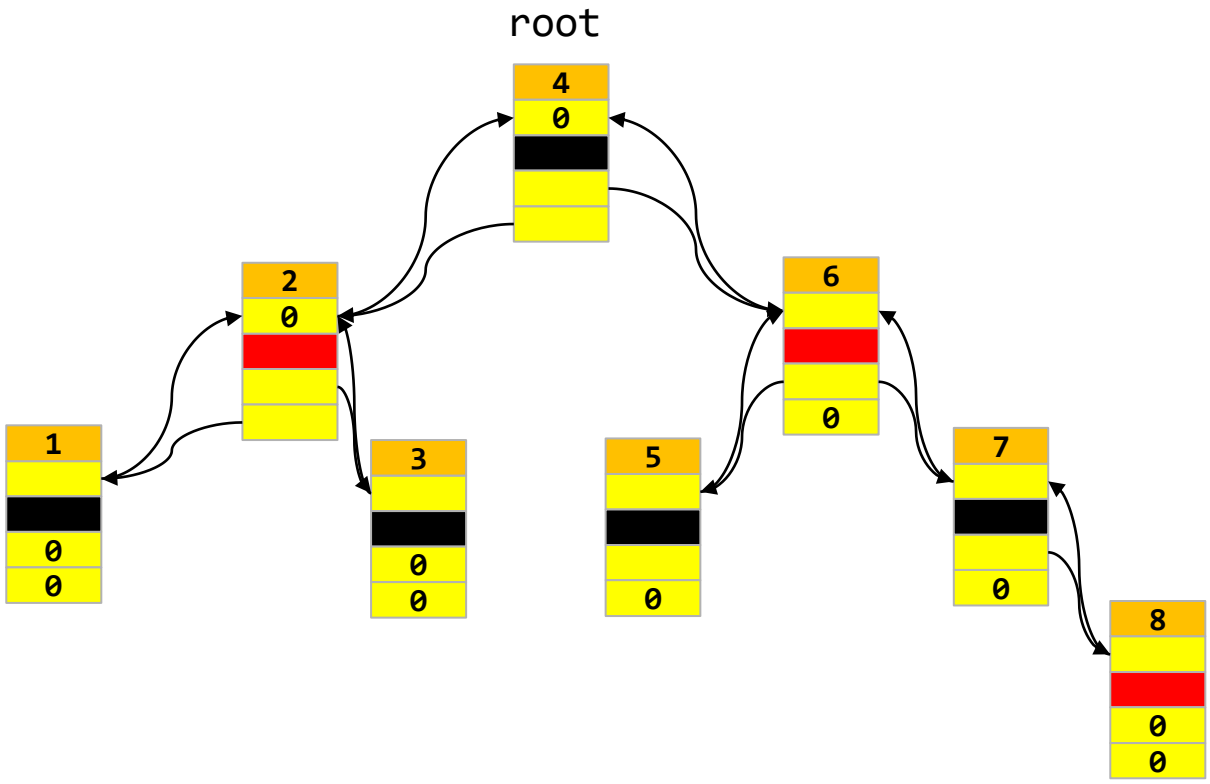
RB tree 삽입 코드 분석



RB tree 삽입 코드 분석

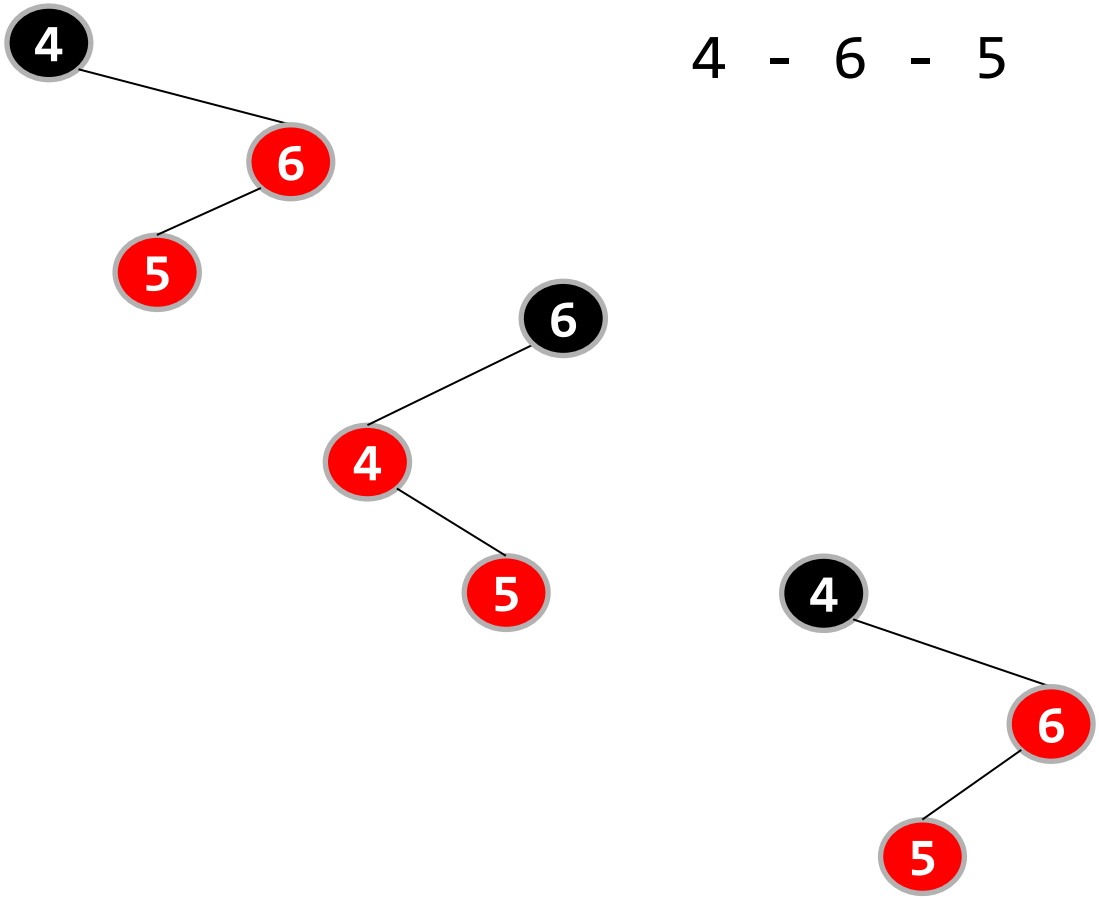


RB tree 삽입 코드 분석



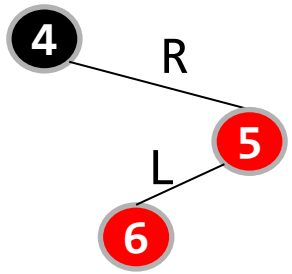
특수한 경사 처리 (R-L 경사)

L-R 경사나 R-L 경사는 한번의 회전으로는 밸런스를 잡을 수 없다.



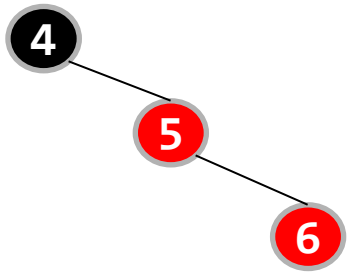
특수한 경사 처리 (R-L 경사)

R-L 경사인 경우는 처음에는 rotate_right(parent)로 들어온 순서를 바꾼후 rotate_left(gparent) 와 color flip으로 밸런스는 잡는다.

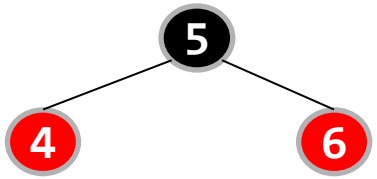


4 - 6 - 5
4 - 5 - 6
5 - 4 - 6

rotate_right(parent);

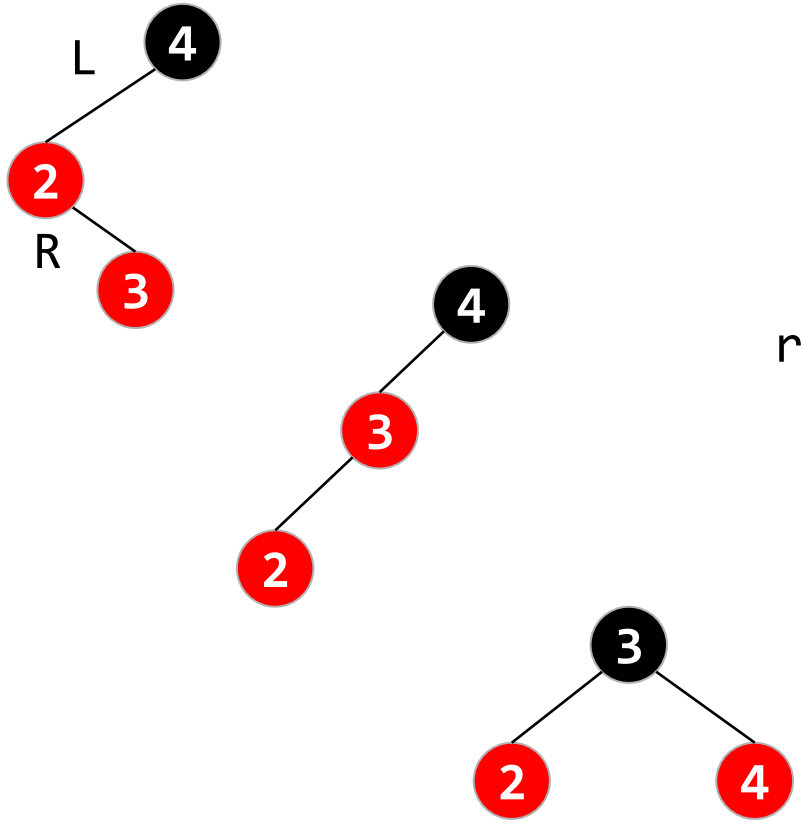


color flip
rotate_left(gparent);



특수한 경사 처리 (L-R 경사)

L-R 경사인 경우는 처음에는 rotate_left(parent)로 들어온 순서를
바꾼후 rotate_right(gparent) 와 color flip으로 밸런스는 잡는다.



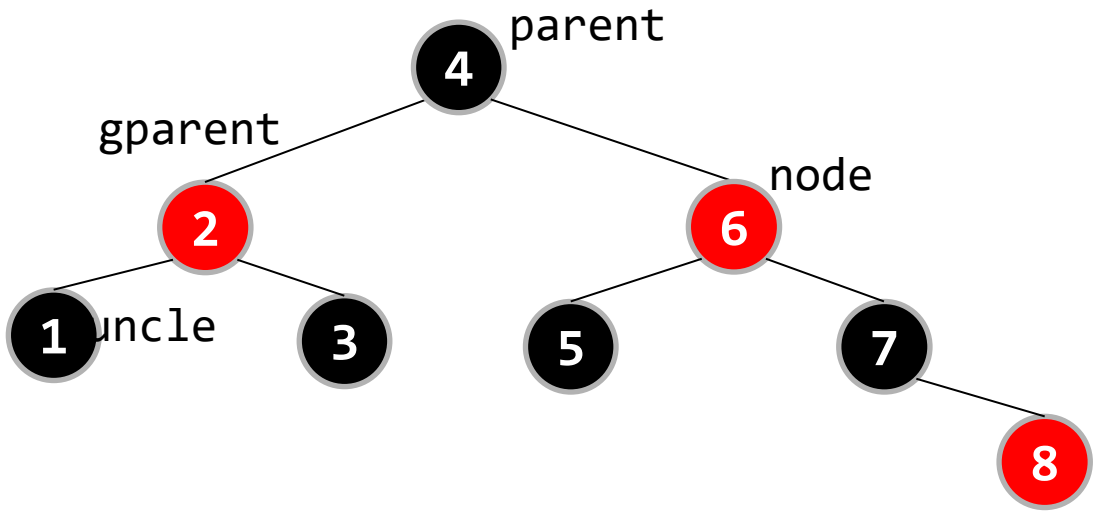
4 - 6 - 5
4 - 5 - 6
5 - 4 - 6

rotate_left(parent);

color flip
rotate_right(gparent);

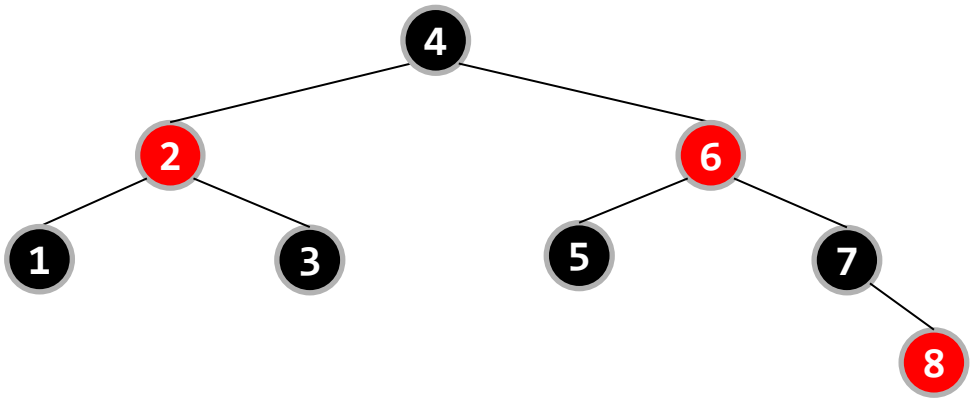
삽입완료 후 결과

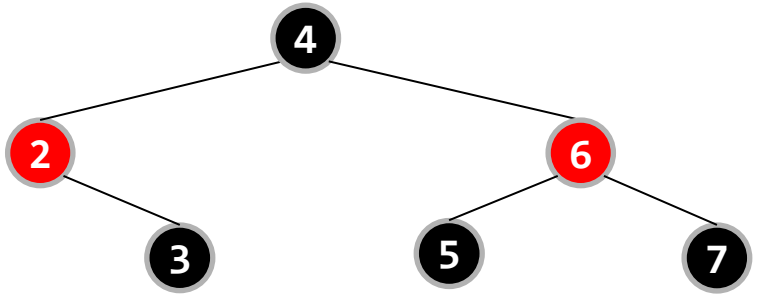
밸런스가 잡힌 후의 트리 모습 과 실제 실행 결과 비교



[4]
< 2>
< 6>
[1]
[3]
[5]
[7]
< 8>

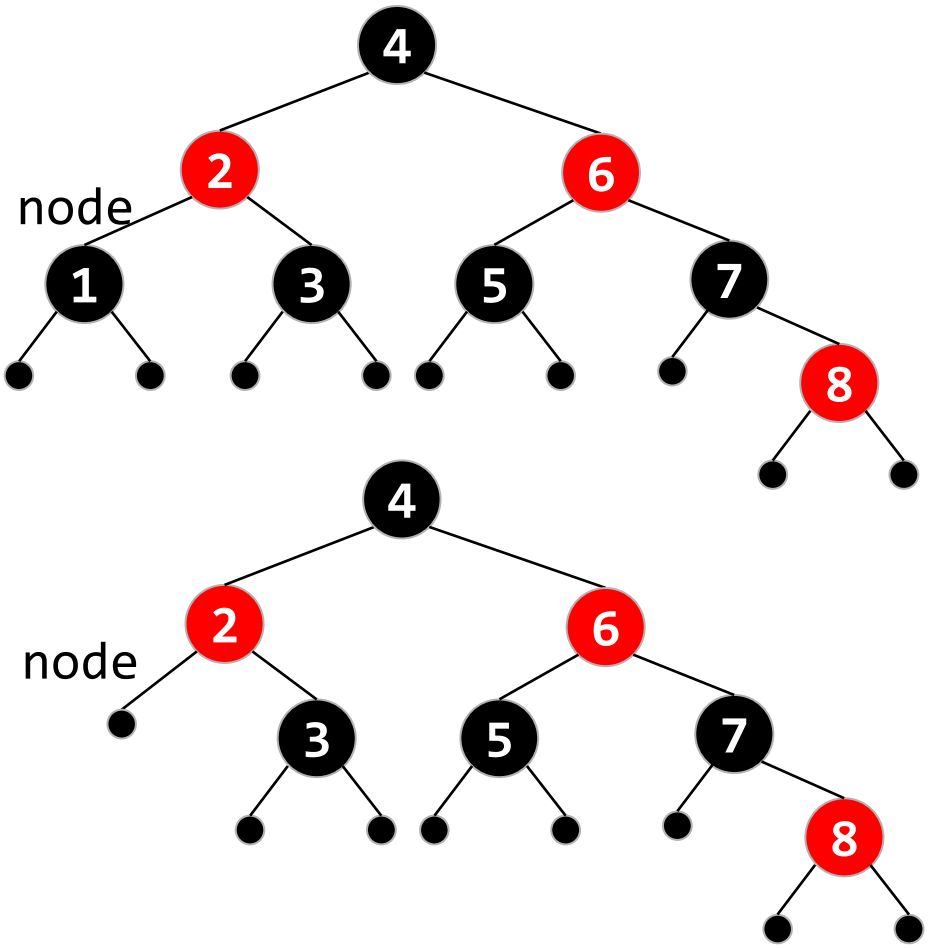
RB tree 의 삭제





만약 1을 지울 경우 ?
밸런스가 깨져 있다.

RB tree 의 삭제

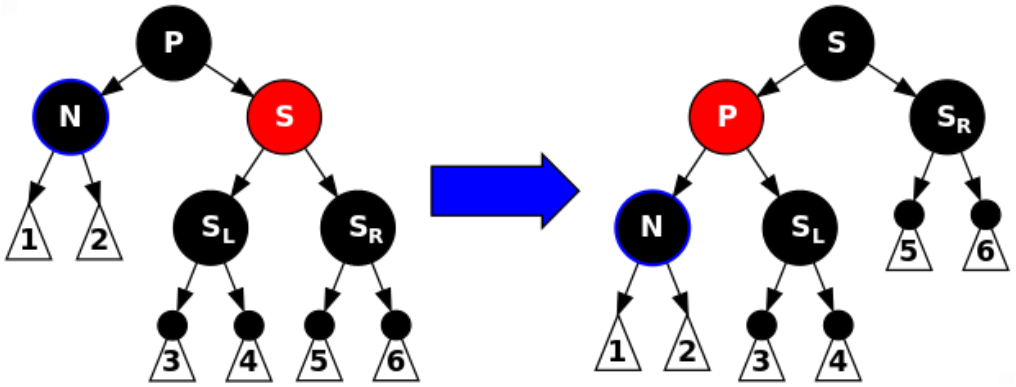


만약 1을 지울 경우 ?
밸런스가 깨져 있다.

NULL은 검정으로 간주 한다.

검정 노드를 지울 경우
밸런스가 깨진다.

RB tree 의 삭제



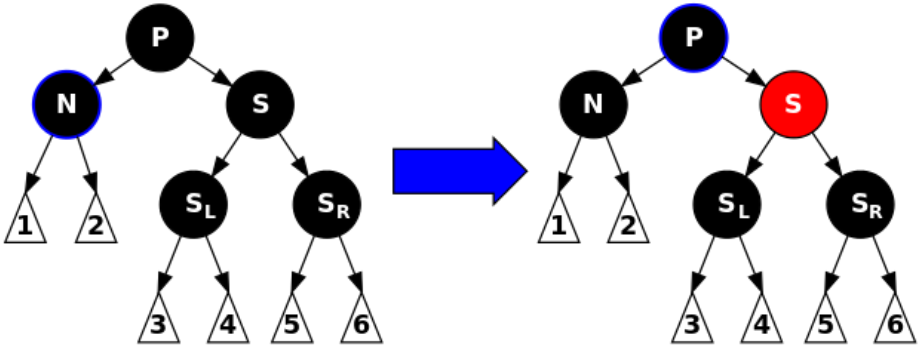
경우 1

형제 노드가 빨강인 경우

형제 노드가 오른쪽

```
color flip
    parent => RED
    형제   => BLACK
rotate_right(parent);
```

RB tree 의 삭제



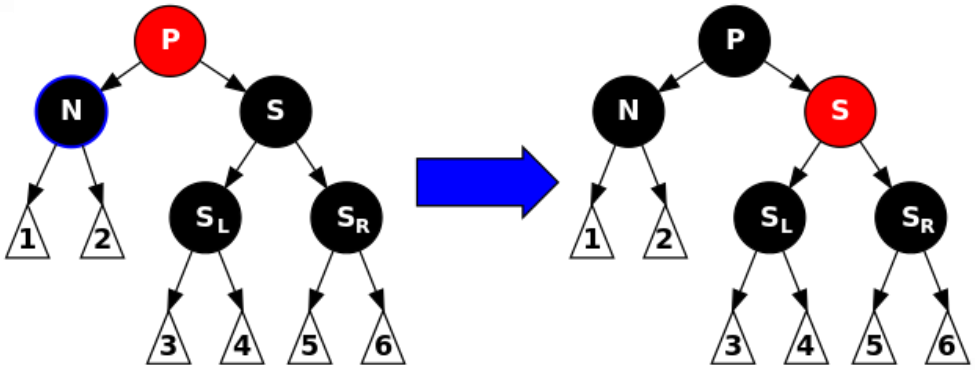
경우 2

부모, 형제, 모든 조카 => 검정

color flip

형제 => RED

RB tree 의 삭제

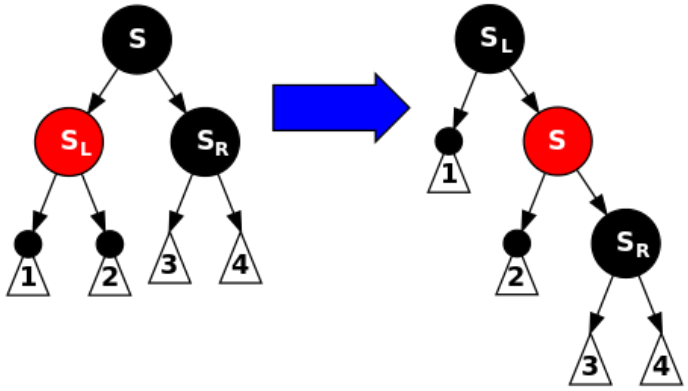


경우 3

모든 조카가 검정이고
부모가 빨강인 경우

color flip
  형제 => RED
  node => RED
  부모 => BLACK

RB tree 의 삭제



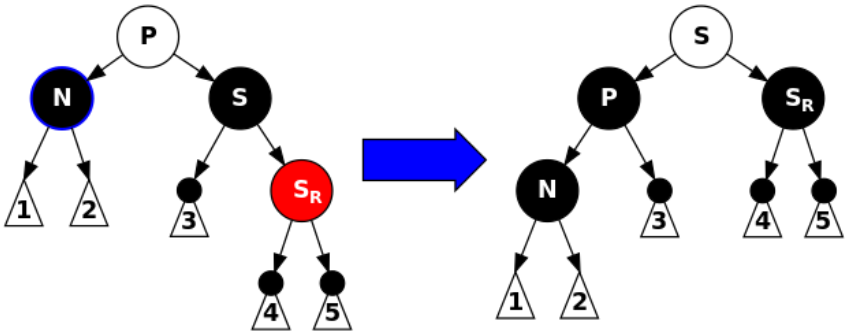
경우 4

가까운 위치의 조카가 빨강인 경우

color flip
    가까운 조카 => BLACK
    형제 => RED

rotate_right(s)

RB tree 의 삭제

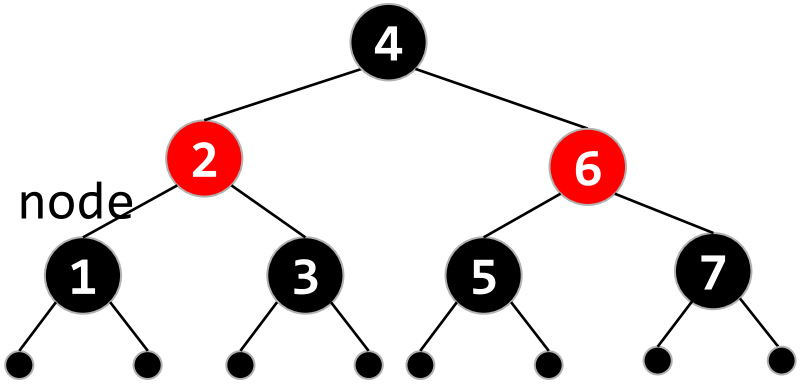


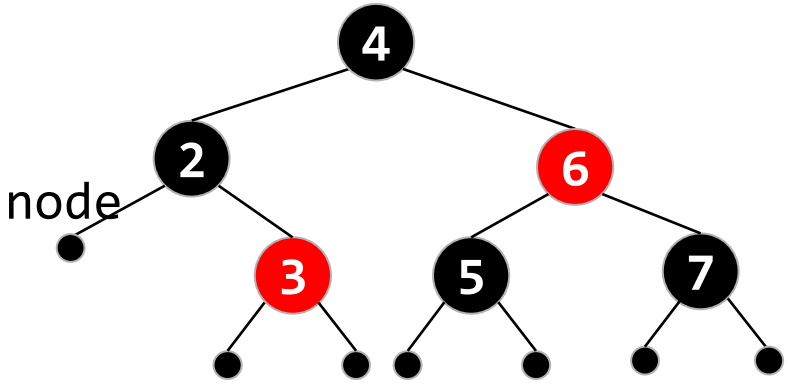
경우 5

위치가 먼 조카가 빨강인 경우

```
color flip
    parent => BLACK
    형제   => parent color
    먼 조카 => BLACK

rotate_left( parent );
```

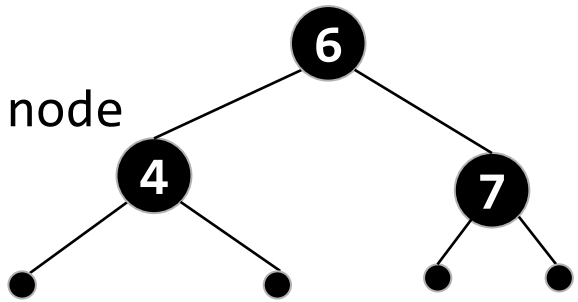
RB tree 의 삭제

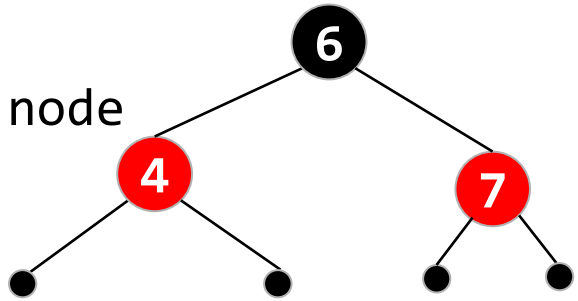


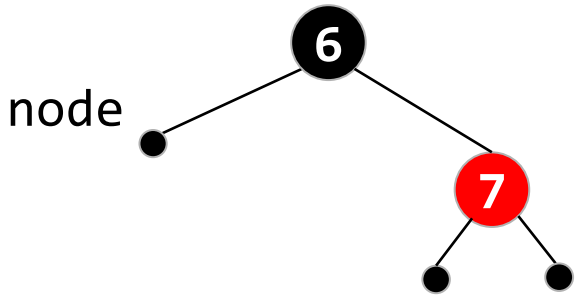


1을 지울 경우

RB tree 의 삭제

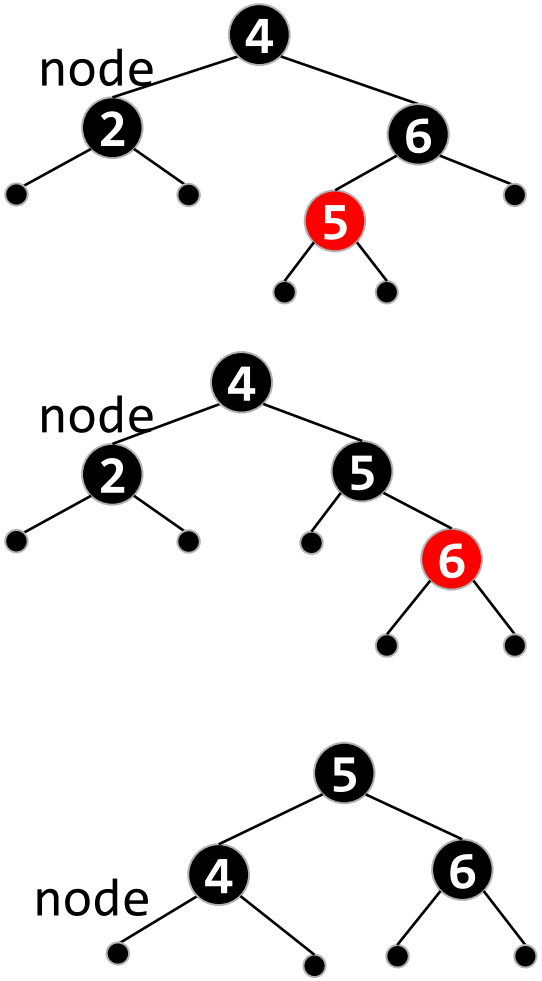






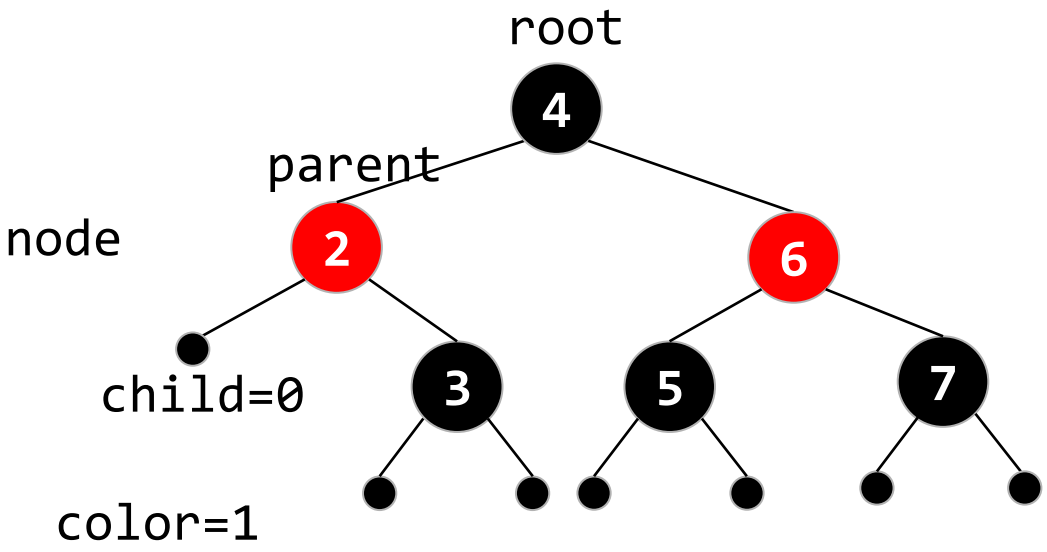
내릴 seed가 없으면
root로 부터 내린다.

RB tree 의 삭제

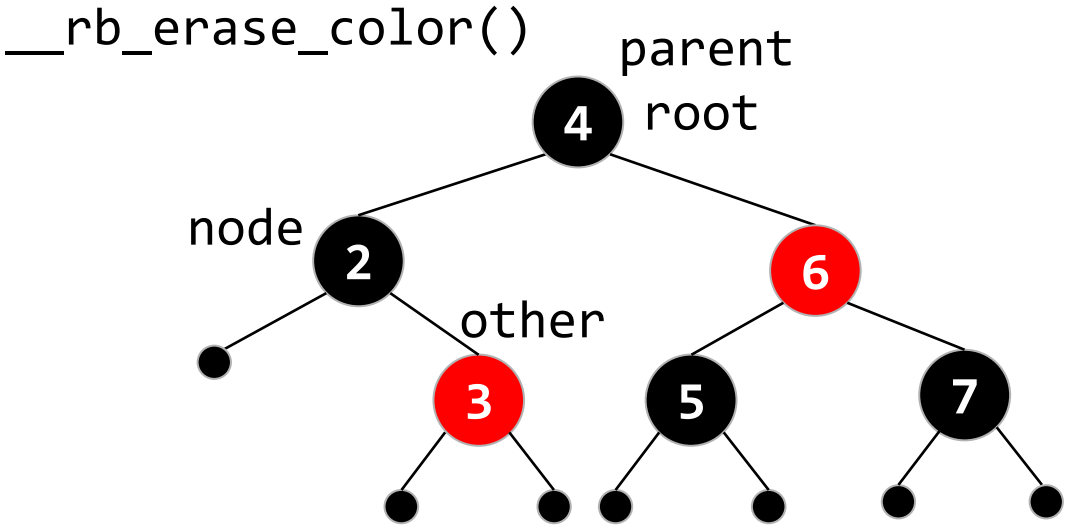


2를 지울 경우 :
가까운 조카가 빨강이므로
먼조카로 바꾼다음
특수한 회전을 통해서
밸런스를 잡는다.

RB tree 의 삭제 코드 분석

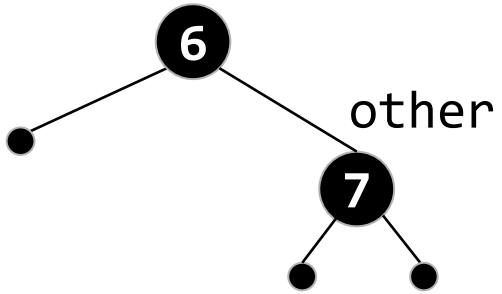


RB tree 의 삭제 코드 분석



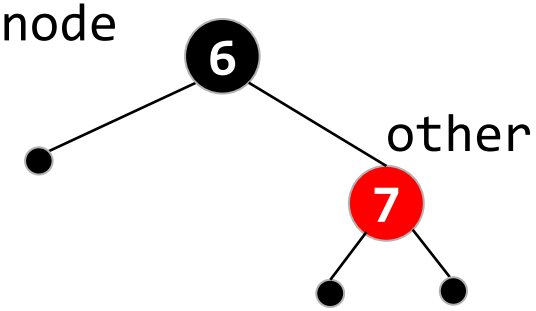
RB tree 의 삭제 코드 분석

color=1

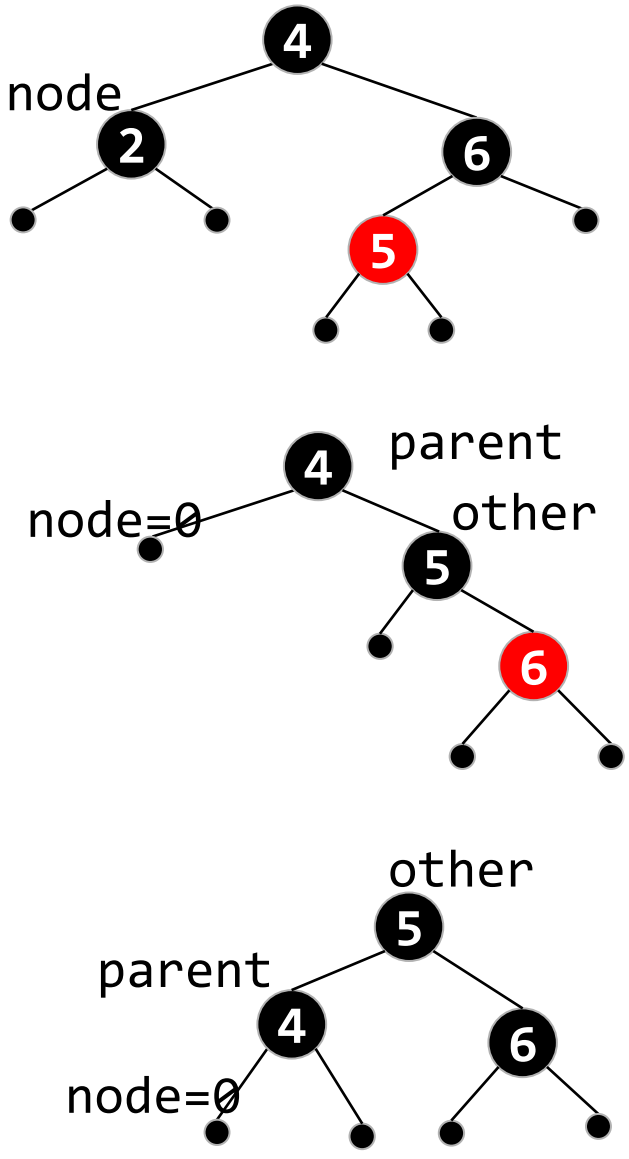


parent=0

node

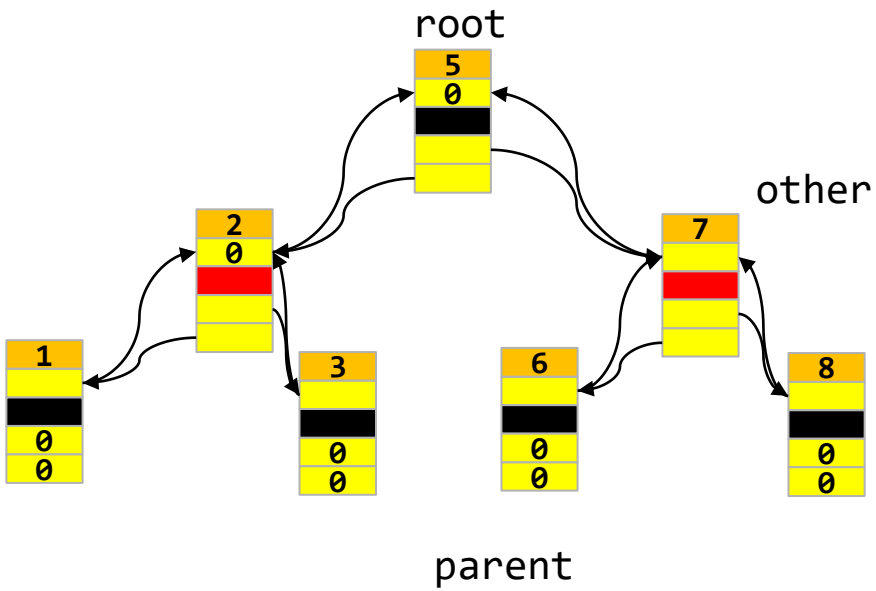


RB tree 의 삭제 코드 분석

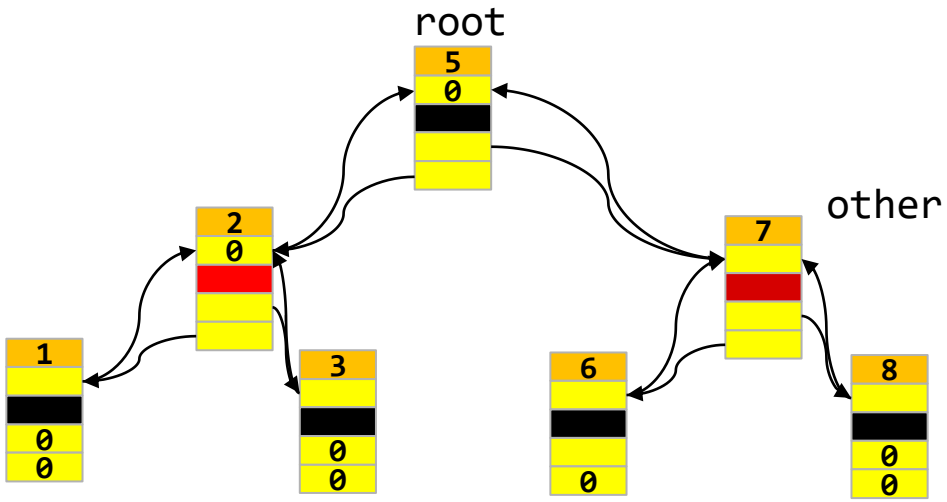


RB tree 의 삭제 코드 분석

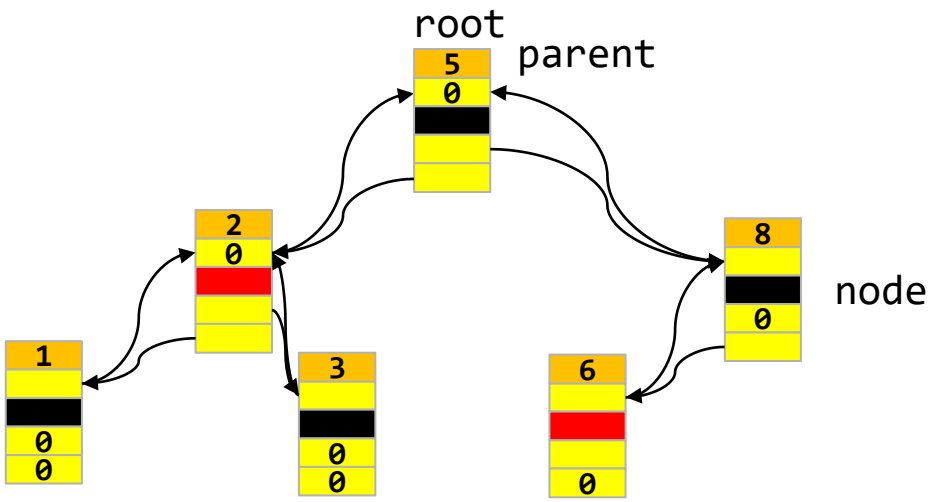
RB tree 에서는 중간 노드를 지울 경우
: 오른쪽 서브 트리중 가장 작은 값을
후보로 올린다.



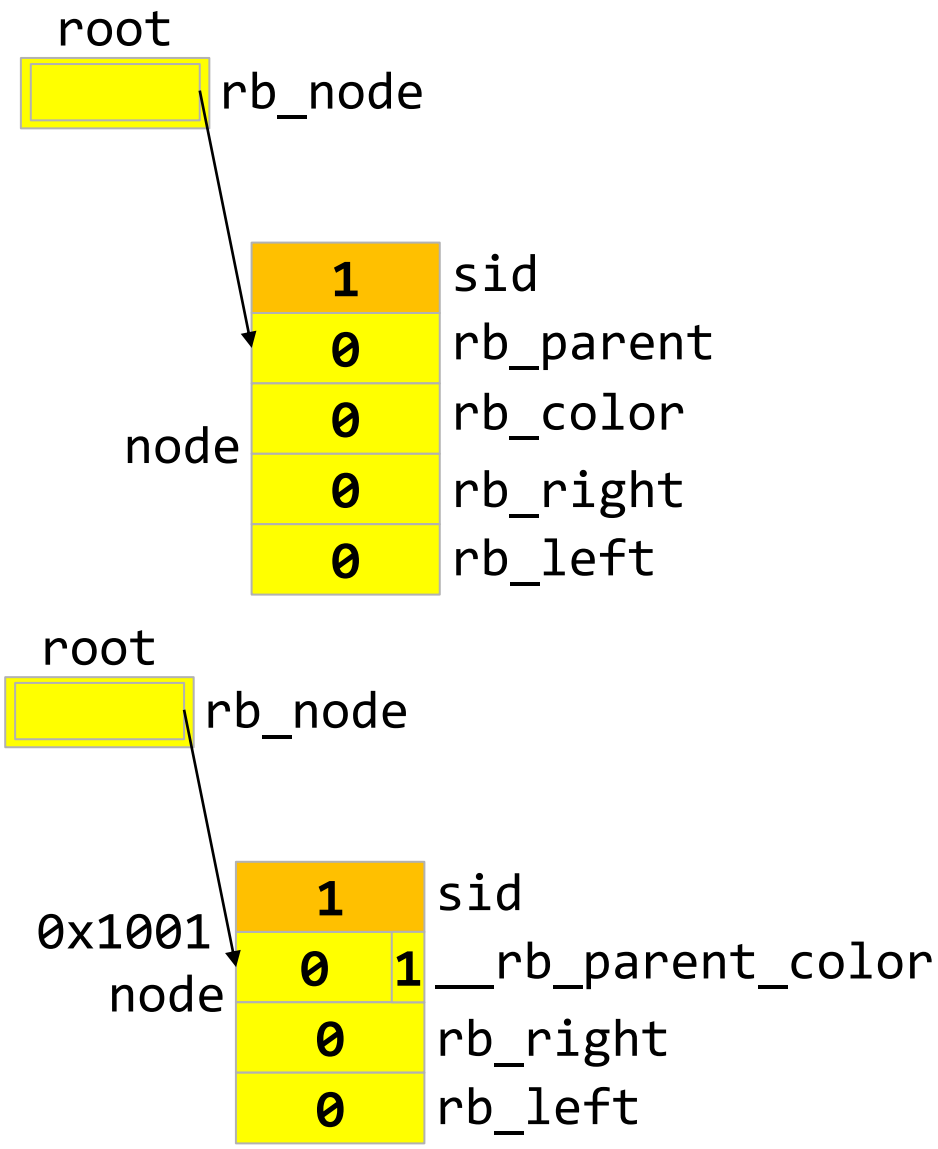
RB tree 의 삭제 코드 분석



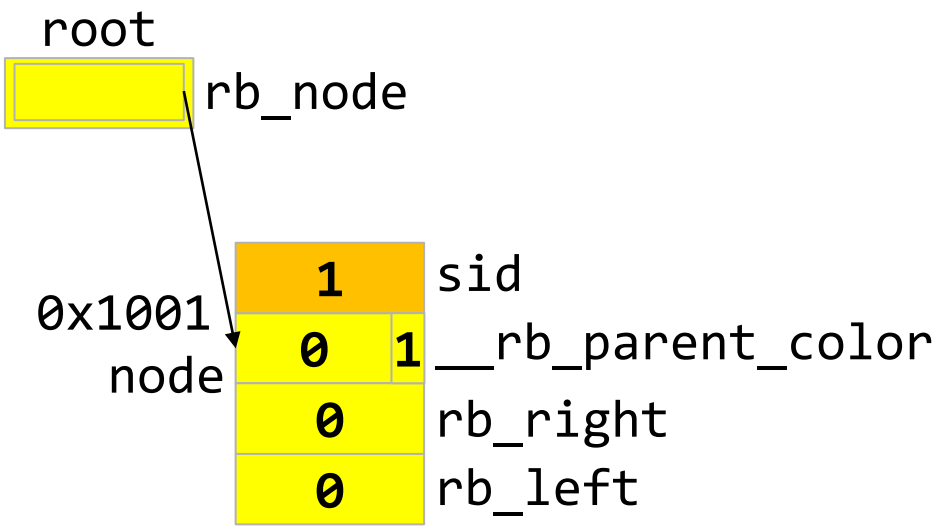
RB tree 의 삭제 코드 분석



RB tree 의 개선

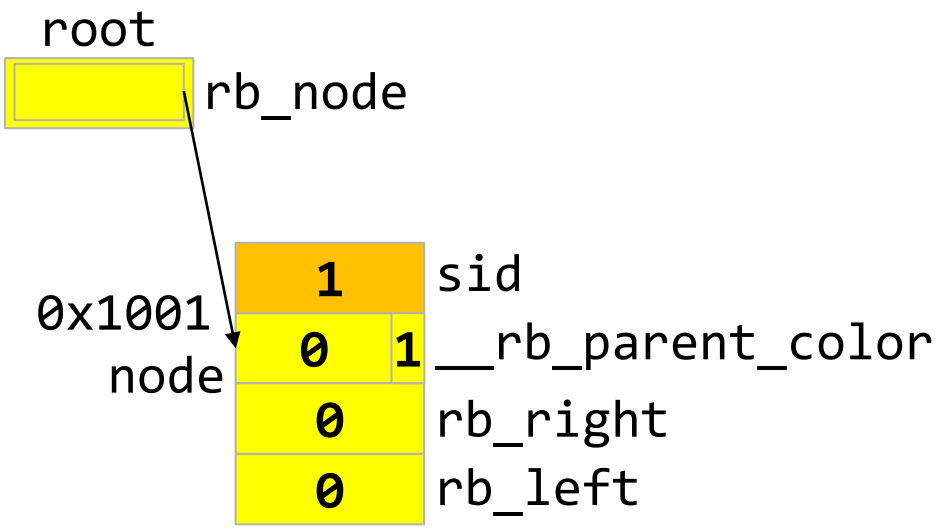

root
rb_node
1
sid
0
rb_parent
0
rb_color
node
0
rb_right
0
rb_left
root
rb_node
1
sid
0x1001
0
1
__rb_parent_color
node
0
rb_right
0
rb_left

RB tree 의 개선




```
rb_parent 매크로
0001000000000001
1111111111111100 &
--------------------
0001000000000000

 0x1000
```

RB tree 의 개선



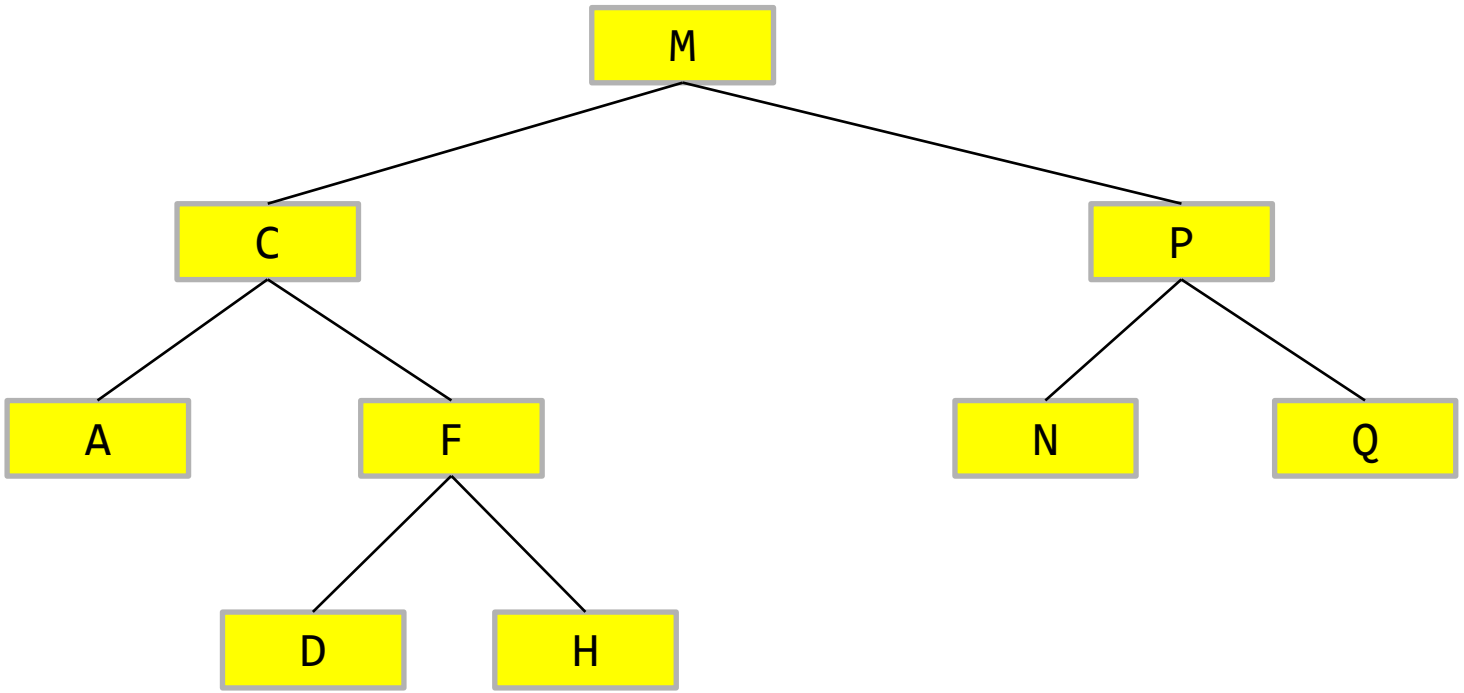

```
rb_color 매크로
0001000000000000
0000000000000001 &
--------------------
0000000000000000
```

증강 트리

알파벳 순서상 5번째 사람 ?

$O(\log_2 N)$

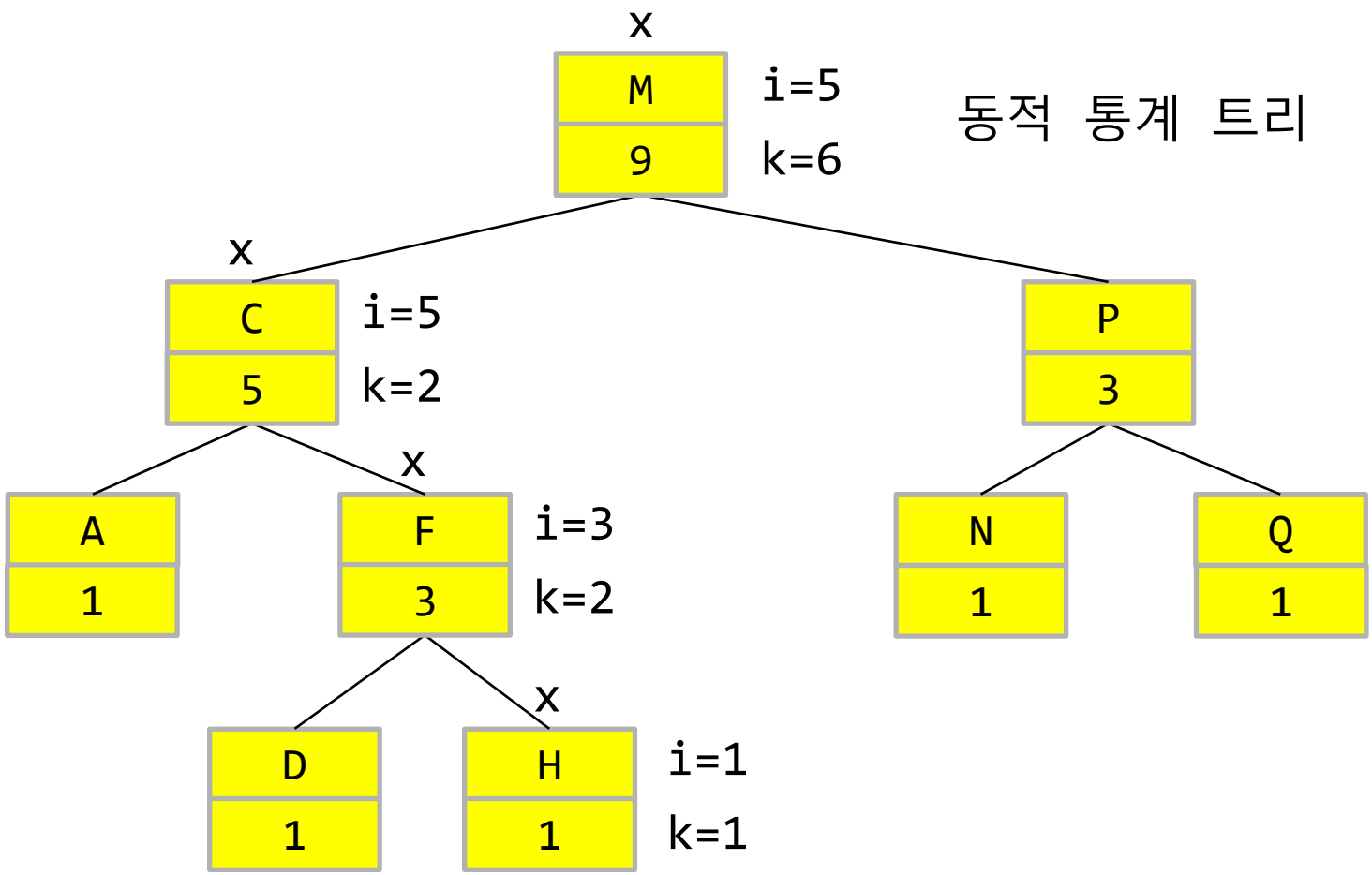
$O(N)$



증강 트리

알파벳 순서상 5번째 사람 ?

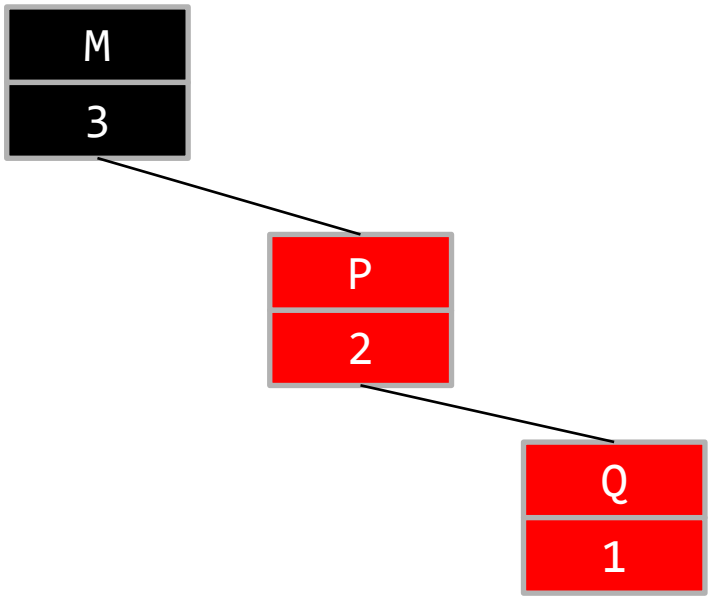
$O(\log_2 N)$

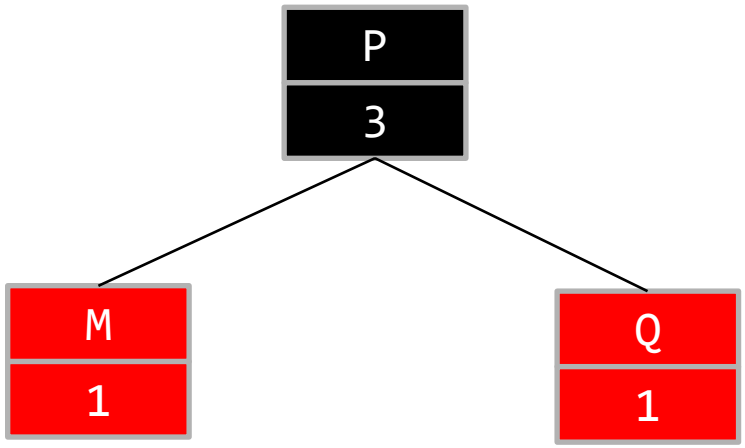


증강 트리

```
SAWON *my_select( struct rb_node *node, int i )
{
   int k=1;
   if(node == 0 )
      return 0;

   if( node->rb_left )
      k = rb_entry(node->rb_left,SAWON,node)->augmented+1;

   if( i==k )
      return rb_entry( node, SAWON, node );

   if( i<k )
     my_select( node->rb_left, i );
   else
     my_select( node->rb_right,i-k);
}
```

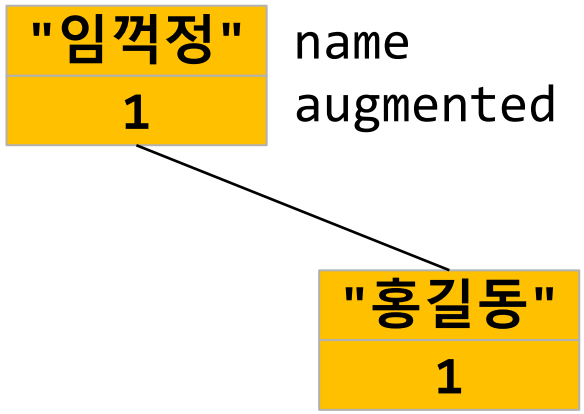
증강 트리



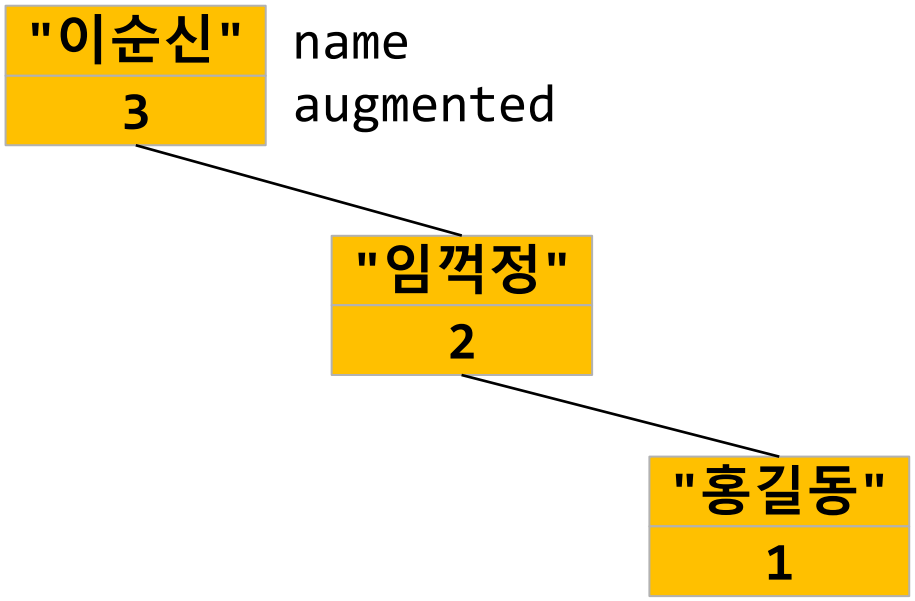



```
new->augmented = old->augmented;
old->augmented = recompute( old );
```

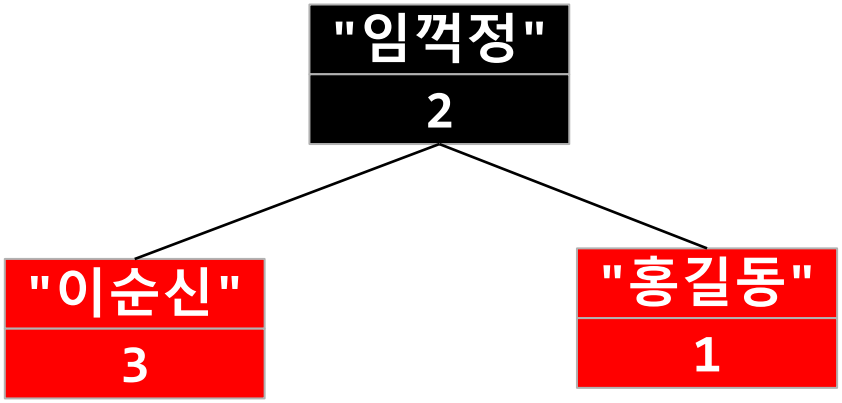
증강 트리



증강 트리



증강 트리

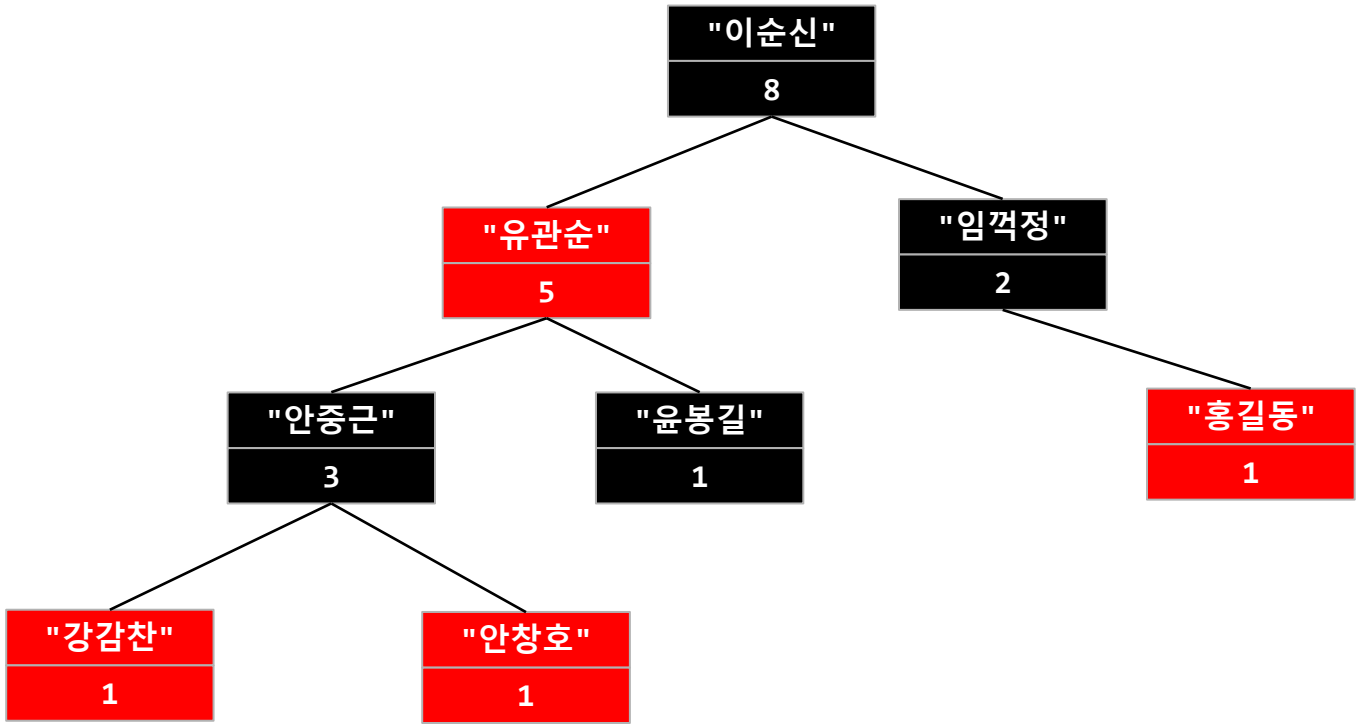


증강 트리

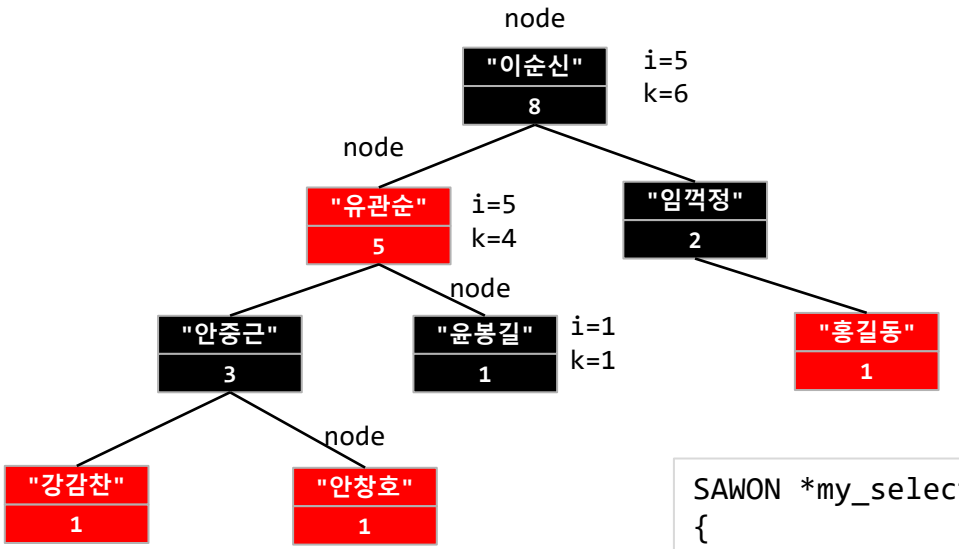
```
static void
augment_callbacks_rotate(struct rb_node *rb_old, struct rb_node *rb_new)
{
    SAWON *old = rb_entry(rb_old, SAWON, node);
    SAWON *new = rb_entry(rb_new, SAWON, node);
    new->augmented = old->augmented;
    old->augmented = my_compute(old);
}

RB_DECLARE_CALLBACKS( static, augment_callbacks, SAWON, node, int, augmented,
                      my_compute);
```

증강 트리

[이순신,8]
<유관순,5> [임꺽정,2]
[안중근,3] [윤봉길,1] <홍길동,1>
<강감찬,1> <안창호,1>

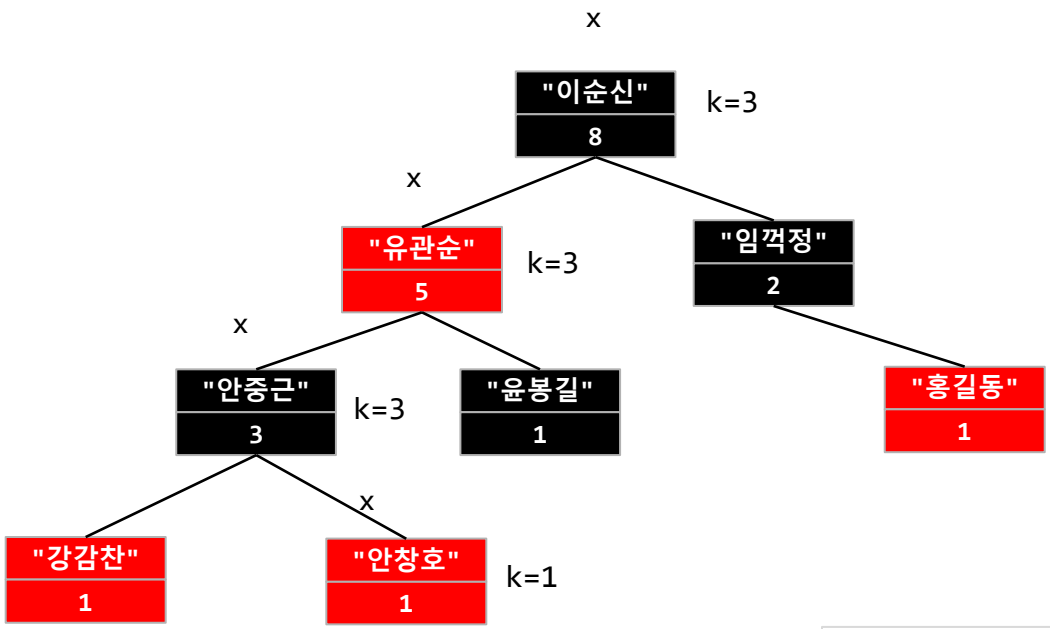


증강 트리



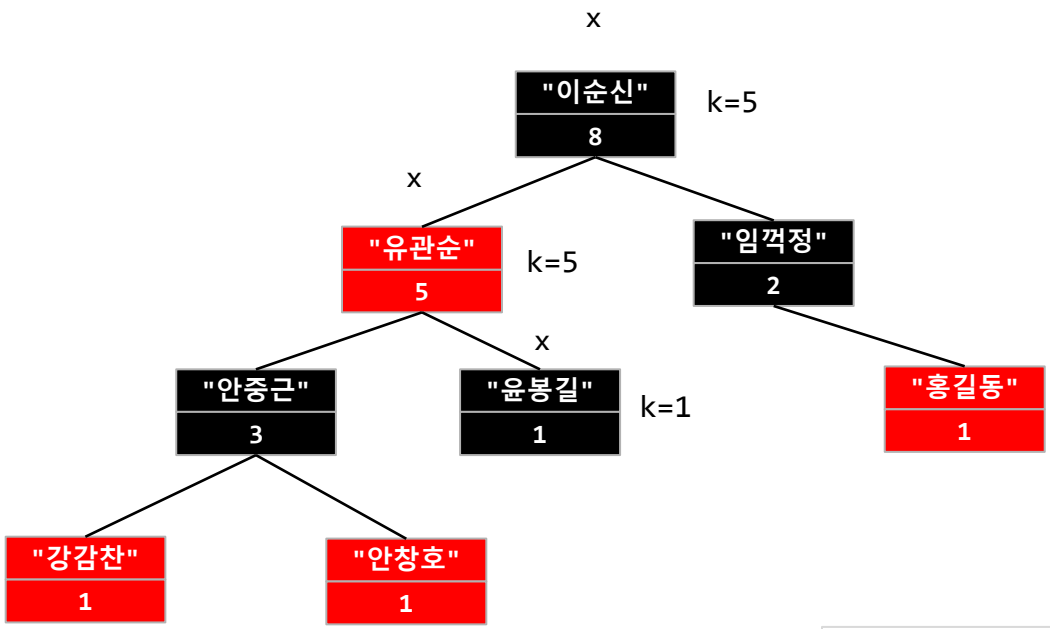

```
SAWON *my_select( struct rb_node *node, int i )
{
   int k=1;
   if(node == 0 )
      return 0;

   if( node->rb_left )
      k = rb_entry(node->rb_left,SAWON,node)->augmented+1;

   if( i==k )
      return rb_entry( node, SAWON, node );

   if( i<k )
     my_select( node->rb_left, i );
   else
     my_select( node->rb_right,i-k);
}
```

증강 트리



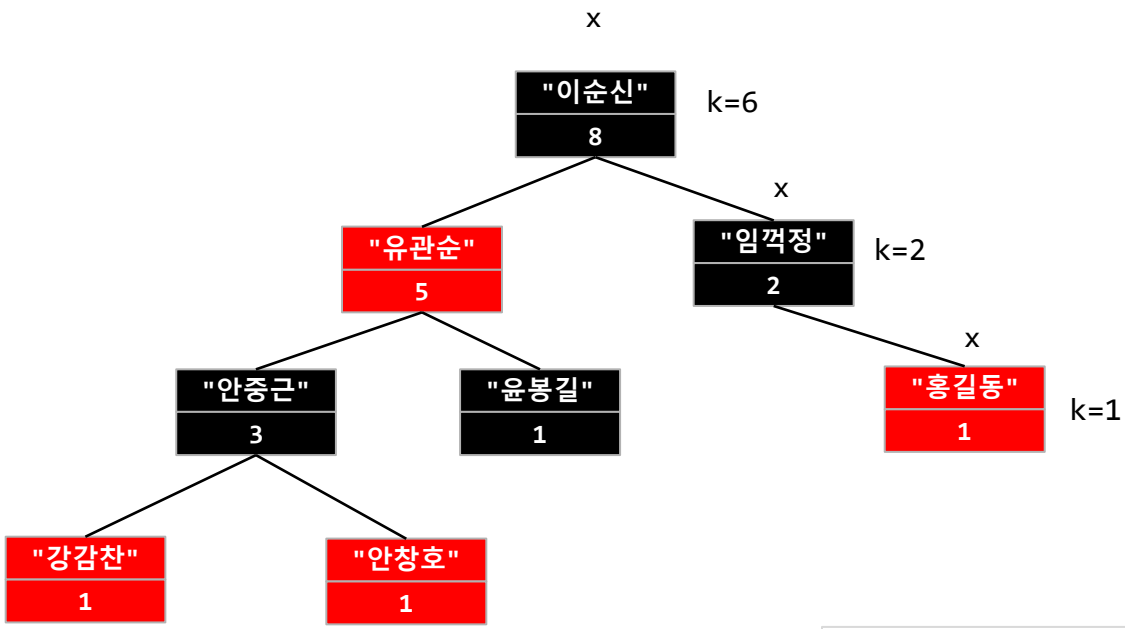

```
int my_rank( x )
{
   k <- size[left[x]]+1;

   while(x != 0)
   {
      if( x->parent->right == x )
        k+=size[x->parent->left]+1;
      x=x->parent;
   }
   return k;
}
```

증강 트리




```
int my_rank( x )
{
   k <- size[left[x]]+1;

   while(x != 0)
   {
      if( x->parent->right == x )
        k+=size[x->parent->left]+1;
      x=x->parent;
   }
   return k;
}
```

증강 트리

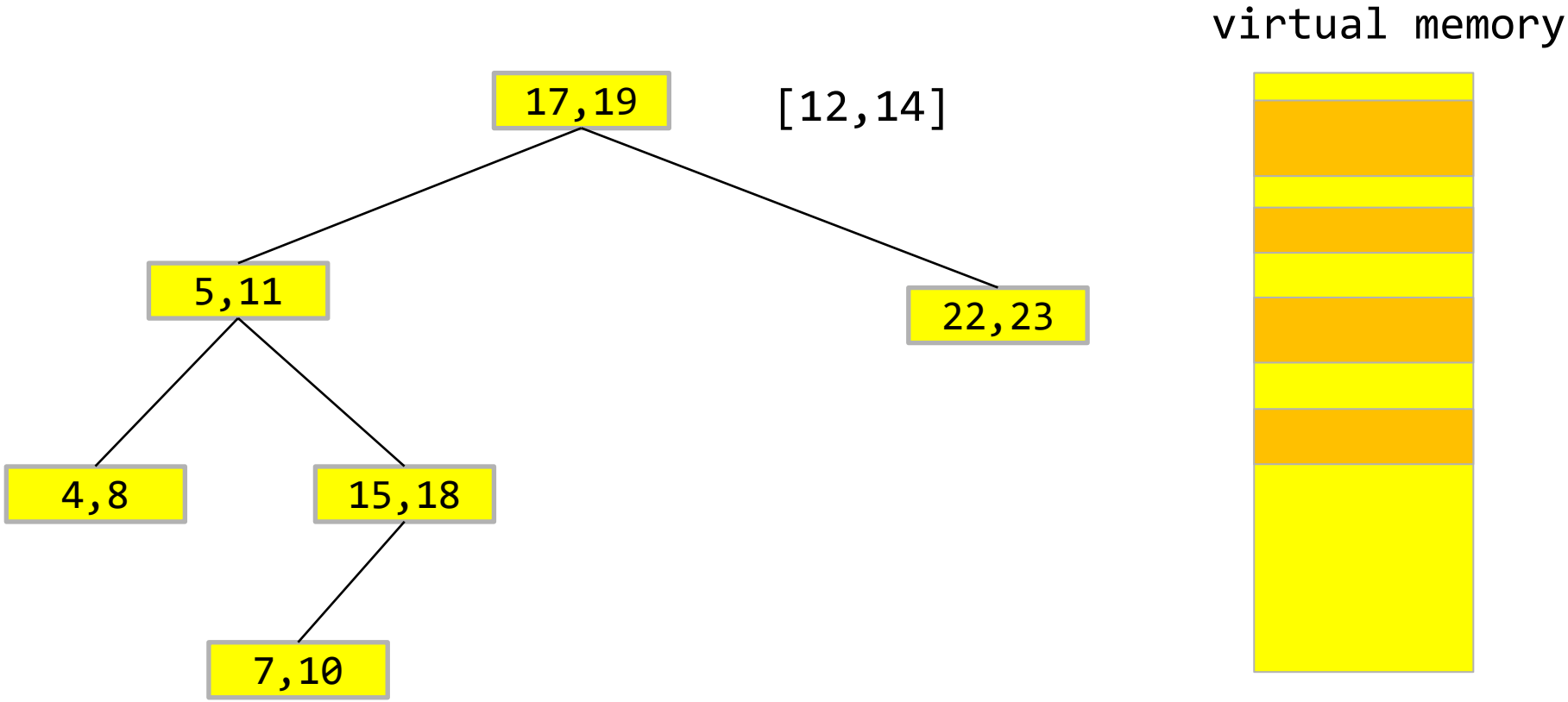



```
int my_rank( x )
{
   k <- size[left[x]]+1;

   while(x != 0)
   {
      if( x->parent->right == x )
        k+=size[x->parent->left]+1;
      x=x->parent;
   }
   return k;
}
```

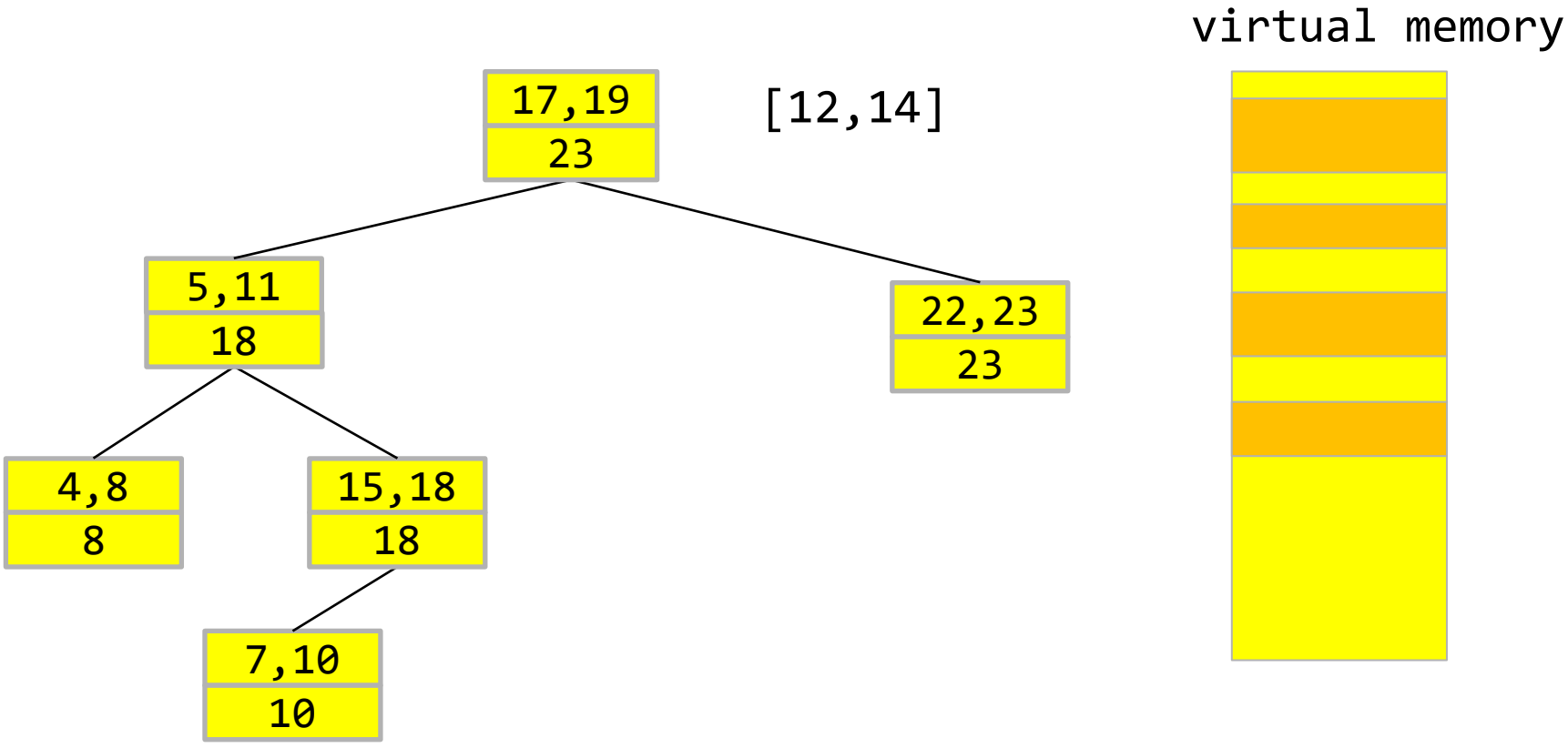
증강 트리

```
int my_rank( struct rb_node *node )
{
   int k=1;

   if( node->rb_left )
      k = rb_entry(node->rb_left, SAWON, node)->augmented+1;

   while(rb_parent(node) != 0)
   {
      if( rb_parent(node)->rb_right == node )
      {
        if(rb_parent(node)->rb_left )
          k+=rb_entry(rb_parent(node)->rb_left,SAWON,node)->augmented+1;
        else
          k++;
      }
      node=rb_parent(node);
   }
   return k;
}
```

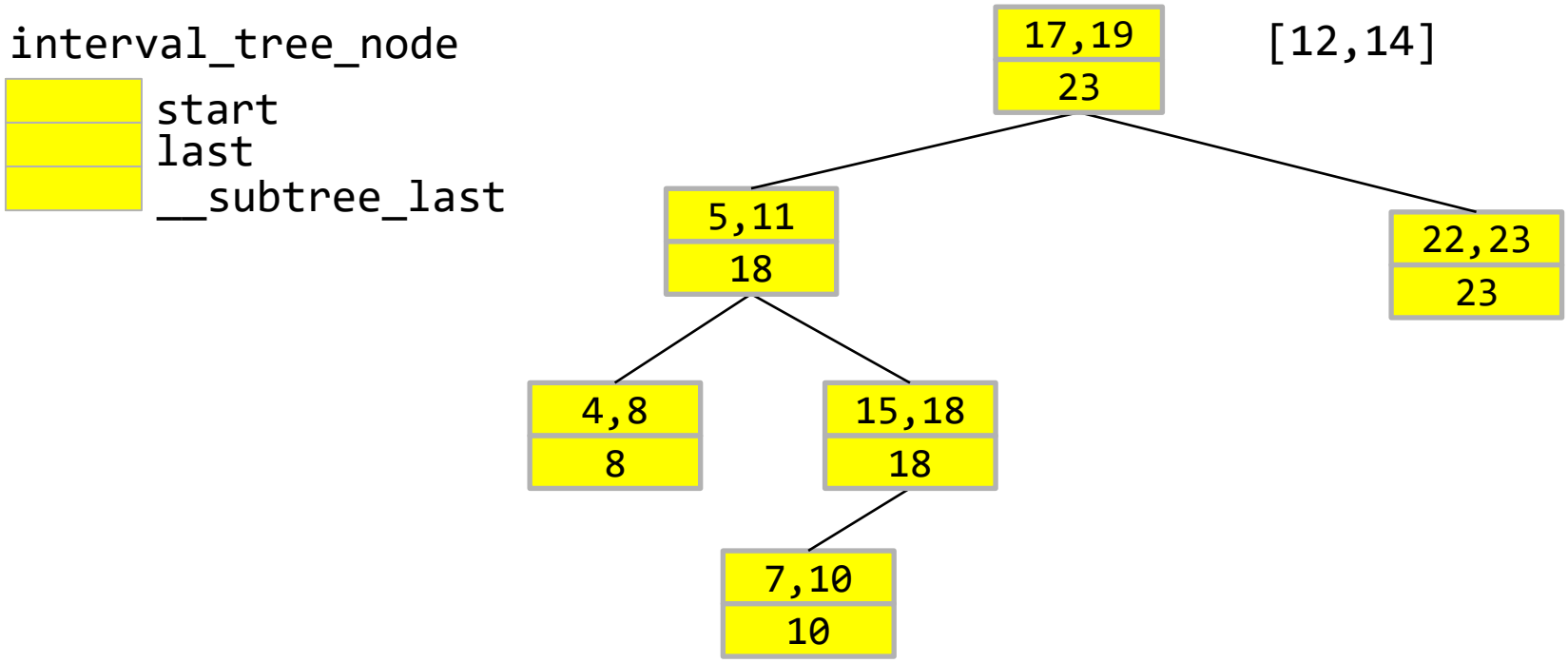
Interval 트리



Interval 트리



Interval 트리


interval_tree_node
start
last
__subtree_last
17,19
23
[12,14]
5,11
18
22,23
23
4,8
8
15,18
18
7,10
10

Interval 트리

```
INTERVAL_TREE_DEFINE(ITSTRUCT, ITRB, ITTYPE,
                     ITSUBTREE, ITSTART, ITLAST,
                     ITSTATIC, ITPREFIX)
INTERVAL_TREE_DEFINE(struct interval_tree_node,
                rb, unsigned long, __subtree_last,
                node->start, node->last, ,
                interval_tree)

unsigned long
interval_tree_compute_subtree_last
(struct interval_tree_node *node)
{
    unsigned long max = node->last;
    unsigned long subtree_last;
    if( node->rb.rb_left )
    {
      subtree_last = rb_left->__subtree_last;
      if( max < subtree_last )
        max = subtree_last;
    }
}
```

Interval 트리

```
unsigned long
interval_tree_compute_subtree_last
(struct interval_tree_node *node)
{
    unsigned long max = node->last;
    unsigned long subtree_last;
    if( node->rb.rb_left )
    {
      subtree_last = rb_left->__subtree_last;
      if( max < subtree_last )
        max = subtree_last;
    }

    if( node->rb.rb_right )
    {
      subtree_last = rb_right->__subtree_last;
      if( max < subtree_last )
        max = subtree_last;
    }
    return max;
}
```

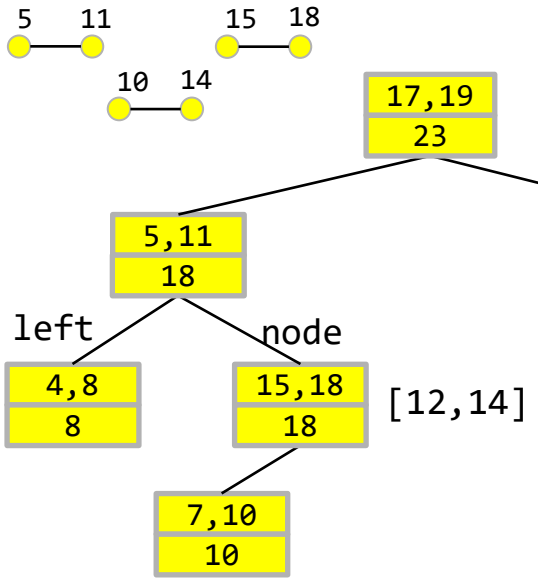
Interval 트리

```
void interval_tree_insert(struct interval_tree_node *node,
                          struct rb_root *root)
{
    struct rb_node **link = &root->rb_node, *rb_parent = NULL;
    unsigned long start = node->start, last = node->last;
    struct interval_tree_node *parent;

    while (*link) {
        rb_parent = *link;
        parent = rb_entry(rb_parent, struct interval_tree_node
                          , rb);
        if (parent->__subtree_last < last)
            parent->__subtree_last  = last;
        if (start < parent->start)
            link = &parent->rb.rb_left;
        else
            link = &parent->rb.rb_right;
    }
    rb_link_node(&node->rb, rb_parent, link);
    rb_insert_augmented(&node->rb, root,
                 &interval_tree_augment);
}
```

Interval 트리

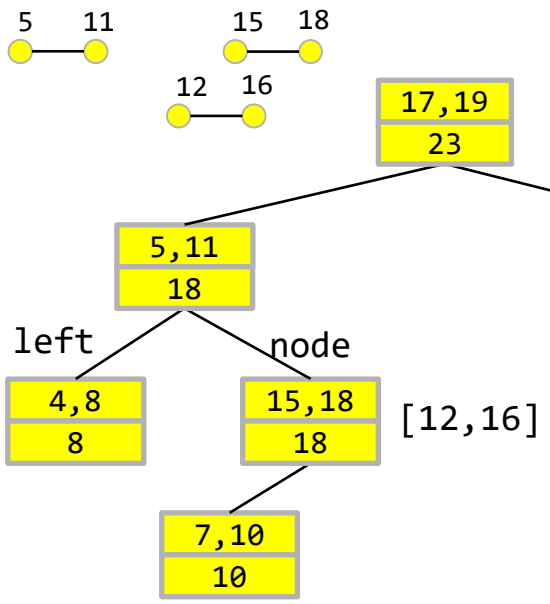
```
struct interval_tree_node *
interval_tree_subtree_search(
struct interval_tree_node *node,
unsigned long start,
unsigned long last)
{
    while (true) {
        if (node->rb.rb_left) {
             struct interval_tree_node *left = node->rb.rb_left;
             if (start <= left->__subtree_last) {
                node = left;
                continue;
             }
        }
        if (node->start <= last) {
            if (start <= node->last )
                return node;
            if (node->rb.rb_right) {
                node = rb_entry(node->ITRB.rb_right, ITSTRUCT, ITRB);
                if (start <= node->__sub_tree_last)
                    continue;
            }
        }
        return NULL;
    }
}
```

Interval 트리




```
struct interval_tree_node *
interval_tree_subtree_search(struct interval_tree_node *node,
unsigned long start,
unsigned long last)
{
    while (true) {
        if (node->rb.rb_left) {
             struct interval_tree_node *left = node->rb.rb_left;
             if (start <= left->__subtree_last) {
                node = left;
                continue;
             }
        }
        if (node->start <= last) {
            if (start <= node->last )
                return node;
            if (node->rb.rb_right) {
                node = rb_entry(node->ITRB.rb_right, ITSTRUCT, ITRB);
                if (start <= node->__sub_tree_last)
                    continue;
            }
        }
        return NULL;
    }
}
```

Interval 트리




```
struct interval_tree_node *
interval_tree_subtree_search(struct interval_tree_node *node,
unsigned long start,
unsigned long last)
{
    while (true) {
        if (node->rb.rb_left) {
             struct interval_tree_node *left = node->rb.rb_left;
             if (start <= left->__subtree_last) {
                node = left;
                continue;
             }
        }
        if (node->start <= last) {
            if (start <= node->last )
                return node;
            if (node->rb.rb_right) {
                node = rb_entry(node->ITRB.rb_right, ITSTRUCT, ITRB);
                if (start <= node->__sub_tree_last)
                    continue;
            }
        }
        return NULL;
    }
}
```