二. 证: $A^2=A \Rightarrow A^2-A=0 \Rightarrow A(A-E)=0$. 又 $A\neq E$. 则 $A=0$. 则主对角元素等于该行其他元素模和

$\Rightarrow A$不是严格对角占优矩阵.

三. 证:

$$B=\begin{pmatrix} |a_{11}| & \cdots & \cdots & |a_{1n}| \\ \vdots & |a_{22}| & & \\ \vdots & & \ddots & \\ |a_{n1}| & \cdots & \cdots & |a_{nn}| \end{pmatrix} \qquad |z-|a_{ii}||\leq \sum_{j\neq i}|a_{ij}|$$

$$D^{-1}BD=\begin{pmatrix} \frac{1}{x_1} & & \\ & \ddots & \\ & & \frac{1}{x_n} \end{pmatrix}\begin{pmatrix} |a_{11}| & \cdots & |a_{nn}| \\ \vdots & & \vdots \\ |a_{n1}| & \cdots & |a_{nn}| \end{pmatrix}\begin{pmatrix} x_1 & & \\ & \ddots & \\ & & x_n \end{pmatrix}$$

$$=\begin{pmatrix} \frac{|a_{11}|}{x_1} & \frac{|a_{12}|}{x_1} & \cdots & \frac{|a_{1n}|}{x_1} \\ \frac{|a_{21}|}{x_2} & \frac{|a_{22}|}{x_2} & & \vdots \\ \vdots & & \ddots & \frac{|a_{nn}|}{x_n} \\ \frac{|a_{n1}|}{x_n} & \cdots & & \end{pmatrix}\begin{pmatrix} x_1 & & \\ & \ddots & \\ & & x_n \end{pmatrix}$$

$$=\begin{pmatrix} |a_{11}| & \frac{x_2}{x_1}|a_{12}| & \cdots & \frac{x_n}{x_1}|a_{1n}| \\ \vdots & & & \vdots \\ \frac{x_1}{x_n}|a_{n1}| & \cdots & \cdots & |a_{nn}| \end{pmatrix} \qquad |z-|a_{ii}||\leq \frac{1}{x_1}\sum_{j\neq i} x_j|a_{ij}|$$

四. 证: ① A为正规矩阵. 则 $U^HAU=\mathrm{diag}(\lambda_1\cdots\lambda_n) \Rightarrow A=U\,\mathrm{diag}(\lambda_1\cdots\lambda_n)U^H$

则 $x^HAx=x^HU\,\mathrm{diag}(\lambda_1\cdots\lambda_n)U^Hx$

$=(U^Hx)^H\,\mathrm{diag}(\lambda_1\cdots\lambda_n)\,U^Hx$

令 $y=U^Hx$. $=y^H\,\mathrm{diag}(\lambda_1\cdots\lambda_n)\,y$

$=\lambda_1|y_1|^2+\cdots+\lambda_n|y_n|^2$

又 $x^HAx\leq 0$. $\Rightarrow \lambda_i\leq 0$. $i=1\cdots n$.

当 $|y_i|^2=1$ 时. 有 $x^HAx=\lambda_1+\cdots+\lambda_n=tr(A)=0$. $\Rightarrow A=0$.

五. 解: $A=BD$

$$A=\begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}\rightarrow\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \end{pmatrix}\rightarrow\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

则 $B=\begin{pmatrix} 1 & 0 \\ -1 & 1 \\ 1 & -1 \end{pmatrix}$ $\quad D=\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

六. 解: $A=\sum_{i=1}^{n}\lambda_iA_i$

$$|\lambda E-A|=\begin{vmatrix} \lambda-4 & -6 & 0 \\ 3 & \lambda+5 & 0 \\ 3 & 6 & \lambda-1 \end{vmatrix}=(\lambda-1)^2(\lambda+2)$$

对于 $\lambda=1$.

$$(E-A)=\begin{pmatrix} -3 & -6 & 0 \\ 3 & 6 & 0 \\ 3 & 6 & 0 \end{pmatrix}\rightarrow\begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

则 $\xi_2=(-2,1,0)$

$\xi_3=(0,0,1)$

对于 $\lambda=-2$.

$$(E-A)=\begin{pmatrix} -6 & -6 & 0 \\ 3 & 3 & 0 \\ 3 & 6 & -3 \end{pmatrix}\rightarrow\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}\rightarrow\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

则 $\xi_1=(-1,1,1)$

则代数重复度等于几何重复度. A为单纯矩阵.

有 $P=\begin{pmatrix} -1 & -2 & 0 \\ 1 & 1 & 0 \end{pmatrix}$ 使 $P^{-1}AP=\mathrm{diag}(-2.1.1)$

$(1 \quad 2 \quad 0$

$$\begin{pmatrix} 1 & 0 & 1 \end{pmatrix}$$

即 $A = P\,\mathrm{diag}(-2,1,1)P^{-1}$ $P^{-1} = \begin{pmatrix} -1 & -1 & 0 \\ -1 & -2 & 1 \end{pmatrix}$

$$= \sum_{i=1}^{3} \lambda_i A_i$$

$$= -2\begin{pmatrix} -1 & -2 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 0 \end{pmatrix} + \begin{pmatrix} 2 & 2 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & -2 & 1 \end{pmatrix}$$

又 $A = P\,\mathrm{diag}(-2,1,1)P^{-1}$ 则 $A^{10} = P(\mathrm{diag}(-2,1,1))^{10}P^{-1}$ 没有必要这样计算

$$= \begin{pmatrix} -1 & -2 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2^{10} & & \\ & 1 & \\ & & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 0 \\ -1 & -1 & 0 \\ -1 & -2 & 1 \end{pmatrix}$$

$A^{10} = (-2)^{10}A_1 + (1)^{10}A_2 + (1)^{10}A_3.$

$$= \begin{pmatrix} -2^{10} & -2 & 0 \\ 2^{10} & 1 & 0 \\ 2^{10} & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 0 \\ -1 & -1 & 0 \\ -1 & -2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2-2^{10} & 2-2^{11} & 0 \\ 2^{10}-1 & 2^{11}-1 & 0 \\ 2^{10}-1 & 2^{11}-2 & 1 \end{pmatrix}$$

七、(1) 证: $\|B\|_2^2 = r(B^HB)$

则 $B^HB = \begin{pmatrix} 0 & A \\ A^H & 0 \end{pmatrix}\begin{pmatrix} 0 & A \\ A^H & 0 \end{pmatrix} = \begin{pmatrix} AA^H & 0 \\ 0 & A^HA \end{pmatrix}$

设 $A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}$ 则 $AA^H = A^HA = \begin{pmatrix} \sum_{j=1}^{n}|a_{1j}\overline{a_{1j}}| & & * \\ & \ddots & \\ * & & \sum_{j=1}^{n}|a_{nj}\overline{a_{nj}}| \end{pmatrix}$

$\Rightarrow \lambda(B^HB) = \lambda(A^HA)$

$\Rightarrow r(B^HB) = r(A^HA)$ $\Rightarrow \|B\|_2^2 = \|A\|_2^2$ $\Rightarrow \|B\|_2 = \|A\|_2$

(2) 证: $A \cdot A^{-1} = E$ $B = A-(A-B) = A[E-A^{-1}(A-B)]$ $\Rightarrow$ $E-A^{-1}(A-B)$ 不可逆

$1 = \|A\cdot A^{-1}\| \le \|A\|\,\|A^{-1}\|$ $\Rightarrow$ $1 \le r[A^{-1}(A-B)] \le \max_i |\lambda_i| \le \|A^{-1}(A-B)\|$

$\le \|A^{-1}\|\,\|A-B\|$

$\frac{1}{\|A^{-1}\|} \le \|A\|$

$\Rightarrow \|A-B\| \ge \|A^{-1}\|^{-1}$

一、1. $\|Ux\|_2^2 = [(Ux)^HUx] = (x^HU^HUx) = (x^Hx) = \|x\|_2^2$

2. Schur不等式.

3. $\|\lambda x\| = |\lambda x_1|^2 = \lambda^2\|x\|$. 不!满足齐次性.

4. $\max_i |x_i| \le \sum_{i=1}^{n}|x_i| \le n \max_i |x_i|$

5. ✓

6. $A_L^{-1} = (A^HA)^{-1}A^H$

7. $AA^{-1} = E \Rightarrow$ $1 = \|AA^{-1}\| \le \|A\|\,\|A^{-1}\|$. $\Rightarrow$ $\|A^{-1}\| \ge \|A\|^{-1}$

8. $\begin{pmatrix} \vdots & \vdots \end{pmatrix}$ 非正规 $AA^H = A^HA$

9. $r(A) = \max_i |\lambda_i|$ $\|A\|_{m\infty} = \max_{i,j}|a_{ij}|$ 无关系

10. $\|Ax\|_a \le \|A\|_m\|x\|_a$

$\Rightarrow \|Ax\| = \|Axa^H\|_m \le \|A\|_m\|xa^H\|_m = \|A\|_m\|x\|$ 相容

5. $AA^H = E$ $A^+ = A^H(AA^H)^+$

$A\cdot A^+ = AA^H(AA^H)^+ = E$

$A^+A = (A^HA)^+A^HA = E$