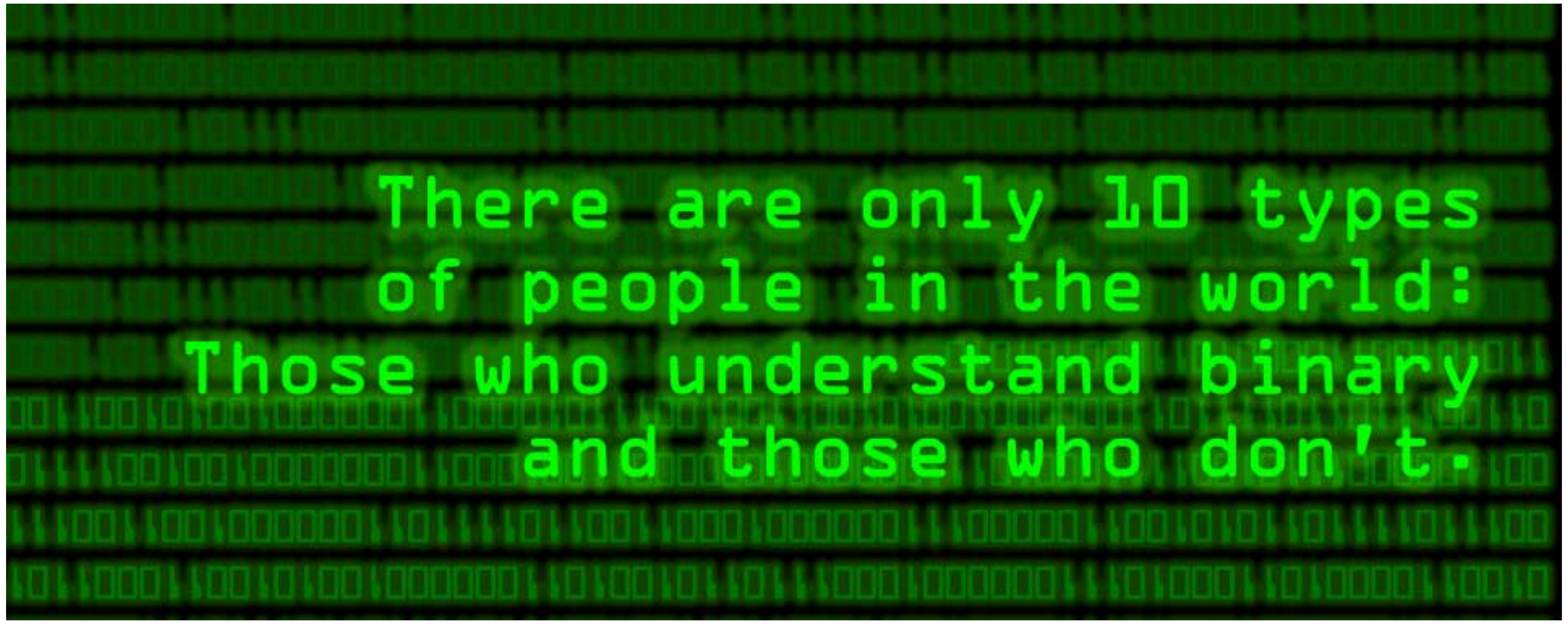
CS 107
Lecture 3: Bits and Bytes

Monday, January 10, 2022



Computer Systems
Winter 2022
Stanford University
Computer Science Department

Reading: Reader: *Number Formats Used in CS 107* and *Bits and Bytes*Textbook: Chapter 2.1

Lecturer: Chris Gregg

Logistics

- Labs start Tuesday -- you will want to watch Monday's lecture before your lab.
- Assign0 Due on Monday at 11:59pm
- Assign1 Released today

Today's Topics

- More on extending the bit representation of numbers
- Truncating numbers
- Data Sizes
- Addressing and Byte Ordering
- Boolean Algebra

Expanding the bit representation of a number

Sometimes we want to convert between two integers having different sizes. E.g., a **`short`** to an **`int`**, or an **`int`** to a **`long`**.

We might not be able to convert from a bigger data type to a smaller data type, but we do want to always be able to convert from a smaller data type to a bigger data type.

This is easy for unsigned values: simply add leading zeros to the representation (called "zero extension").

```
unsigned short s = 4;
// short is a 16-bit format, so                    s = 0000 0000 0000 0100b

unsigned int i = s;
// conversion to 32-bit int, so i = 0000 0000 0000 0000 0000 0000 0000 0100b
```

Expanding the bit representation of a number

For signed values, we want the number to remain the same, just with more bits. In this case, we perform a "sign extension" by repeating the sign of the value for the new digits. E.g.,

```
short s = 4;
// short is a 16-bit format, so                     s = 0000 0000 0000 0100b

int i = s;
// conversion to 32-bit int, so i = 0000 0000 0000 0000 0000 0000 0000 0100b
```

— or —

```
short s = -4;
// short is a 16-bit format, so                     s = 1111 1111 1111 1100b

int i = s;
// conversion to 32-bit int, so i = 1111 1111 1111 1111 1111 1111 1111 1100b
```

Sign-extension Example

```c
// show_bytes() defined on pg. 45, Bryant and O'Halloran
int main() {
    short sx = -12345;         // -12345
    unsigned short usx = sx; // 53191
    int x = sx;               // -12345
    unsigned ux = usx;        // 53191

    printf("sx = %d:\t", sx);
    show_bytes((byte_pointer) &sx, sizeof(short));
    printf("usx = %u:\t", usx);
    show_bytes((byte_pointer) &usx, sizeof(unsigned short));
    printf("x  = %d:\t", x);
    show_bytes((byte_pointer) &x, sizeof(int));
    printf("ux = %u:\t", ux);
    show_bytes((byte_pointer) &ux, sizeof(unsigned));

    return 0;
}
```

```
$ ./sign_extension
sx = -12345:    c7 cf
usx = 53191:    c7 cf
x   = -12345:    c7 cf ff ff
ux = 53191:     c7 cf 00 00
```

(careful: this was printed on the little-endian myth machines!)

Back to right shift: arithmetic -vs- logical

The right-shift (>>) operator behaves differently for unsigned and signed numbers:

- **Unsigned** numbers are **logically**-right shifted (by shifting in 0s, always)
- **Signed** numbers are **arithmetically**-right shifted (by shifting in the sign bit)

```
$ ./right_shift
a = 1048576:        00 00 10 00
a >> 8 = 4096:      00 10 00 00
b = -1048576:       00 00 f0 ff
b >> 8 = -4096:     00 f0 ff ff
```

(run on a little-endian machine)

```
// show_bytes() defined on pg. 45, Bryant and O'Halloran
int main() {
    int a = 1048576;
    int a_rs8 = a >> 8;

    int b = -1048576;
    int b_rs8 = b >> 8;

    printf("a = %d:\t", a);
    show_bytes((byte_pointer) &a, sizeof(int));

    printf("a >> 8 = %d:\t", a_rs8);
    show_bytes((byte_pointer) &a_rs8, sizeof(int));

    printf("b = %d:\t", b);
    show_bytes((byte_pointer) &b, sizeof(int));

    printf("b >> 8 = %d:\t", b_rs8);
    show_bytes((byte_pointer) &b_rs8, sizeof(int));
    return 0;
}
```

Truncating Numbers: Signed

What if we want to reduce the number of bits that a number holds? E.g.

```
int x = 53191;
short sx = (short) x;
int y = sx;
```

What happens here? Let's look at the bits in `x` (a 32-bit `int`), `53191`:

`0000 0000 0000 0000 1100 1111 1100 0111`

When we cast x to a short, it only has 16-bits, and C *truncates* the number:

`1100 1111 1100 0111`

What is this number in decimal? Well, it must be negative (b/c of the initial 1), and it is `-12345`.

Truncating Numbers: Signed

What if we want to reduce the number of bits that a number holds? E.g.

```
int x = 53191;          // 53191
short sx = (short) x;  // -12345
int y = sx;
```

This is a form of *overflow*! We have altered the value of the number. Be careful!

We don't have enough bits to store the int in the short for the value we have in the `int`, so the strange values occur.

What is y above? We are converting a short to an int, so we sign-extend, and we get -12345!

`1100 1111 1100 0111` becomes

`1111 1111 1111 1111 1100 1111 1100 0111`

Play around here: http://www.convertforfree.com/twos-complement-calculator/

Truncating Numbers: Signed

If the number does fit into the smaller representation in the current form, it will convert just fine.

```
int x = -3;          // -3
short sx = (short) -3; // -3
int y = sx; // -3
```

x: `1111 1111 1111 1111 1111 1111 1111 1101` becomes

sx: `1111 1111 1111 1101`

Play around here: http://www.convertforfree.com/twos-complement-calculator/

Truncating Numbers: Unsigned

We can also lose information with unsigned numbers:

```
unsigned int x = 128000;
unsigned short sx = (short) x;
unsigned int y = sx;
```

Bit representation for x = 128000 (32-bit unsigned int):

`0000 0000 0000 0001 1111 0100 0000 0000`

Truncated unsigned short sx:

`1111 0100 0000 0000`

which equals 62464 decimal.

Converting back to an unsigned int, y = 62464

Overflow in Unsigned Addition

When integer operations overflow in C, the runtime does not produce an error:

```
#include<stdio.h>
#include<stdlib.h>
#include<limits.h> // for UINT_MAX

int main() {
    unsigned int a = UINT_MAX;
    unsigned int b = 1;
    unsigned int c = a + b;

    printf("a = %u\n",a);
    printf("b = %u\n",b);
    printf("a + b = %u\n",c);

    return 0;
}
```

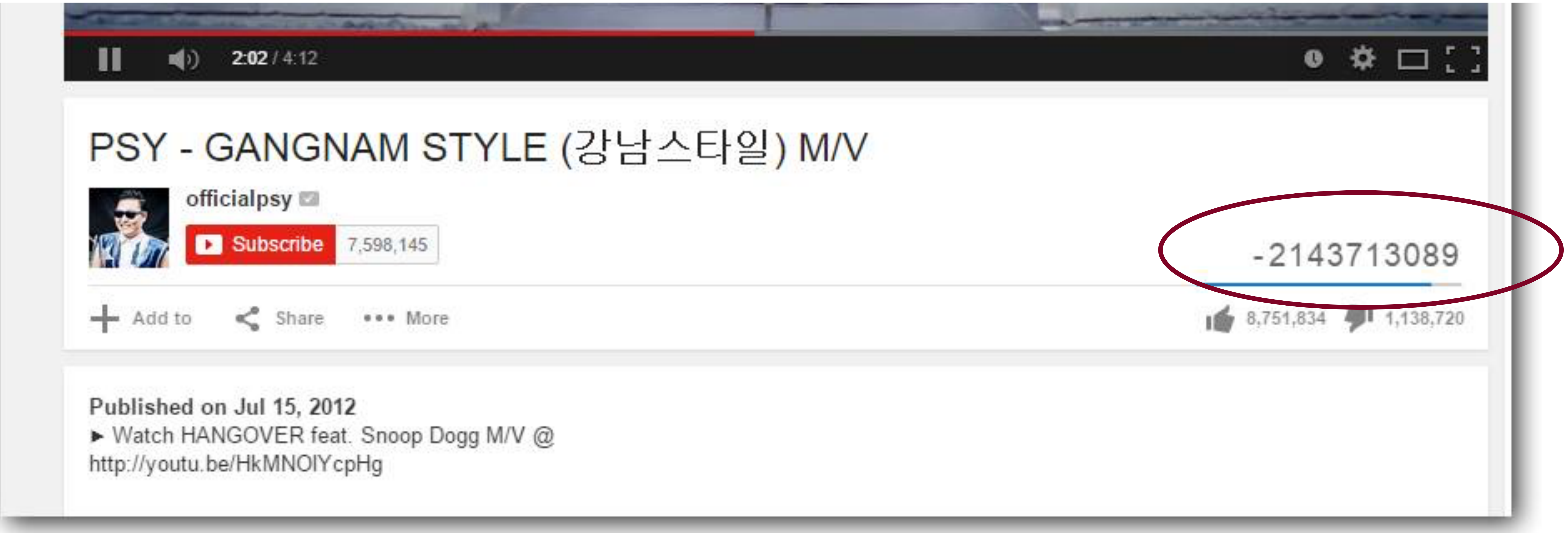
```
$ ./unsigned_overflow
a = 4294967295
b = 1
a + b = 0
```

Technically, unsigned integers in C don't overflow, they just wrap. You need to be aware of the size of your numbers. Here is one way to test if an addition will fail:

```
// for addition
#include <limits.h>
unsigned int a = <something>;
unsigned int x = <something>;
if (a > UINT_MAX - x) /* `a + x` would overflow */;
```

Overflow in Signed Addition

Signed overflow wraps around to the negative numbers:



YouTube fell into this trap — their view counter was a signed, 32-bit int. They fixed it after it was noticed, but for a while, the view count for Gangnam Style (the first video with over `INT_MAX` number of views) was negative.

Overflow in Signed Addition

In the news on January 5, 2022 (!):

ars TECHNICA

BIZ & IT TECH SCIENCE POLICY CARS GAMING & CUL

GOOD THING ANDROID IS GREAT AT ROLLING OUT UPDATES —

Google fixes nightmare Android bug that stopped user from calling 911

An integer overflow/underflow crash lets misbehaving apps lock users out of 911.

RON AMADEO - 1/5/2022, 3:09 PM

https://arstechnica.com/gadgets/2022/01/google-fixes-nightmare-android-bug-that-stopped-user-from-calling-911/

Overflow in Signed Addition

Signed overflow wraps around to the negative numbers.

```
#include<stdio.h>
#include<stdlib.h>
#include<limits.h> // for INT_MAX

int main() {
    int a = INT_MAX;
    int b = 1;
    int c = a + b;

    printf("a = %d\n",a);
    printf("b = %d\n",b);
    printf("a + b = %d\n",c);

    return 0;
}
```

```
$ ./signed_overflow
a = 2147483647
b = 1
a + b = -2147483648
```

Technically, signed integers in C *produce undefined behavior* when they overflow. On two's complement machines (virtually all machines these days), it does overflow predictably. You can test to see if your addition will be correct:

```
// for addition
#include <limits.h>
int a = <something>;
int x = <something>;
if ((x > 0) && (a > INT_MAX - x)) /* `a + x` would overflow */;
if ((x < 0) && (a < INT_MIN - x)) /* `a + x` would underflow */;
```

Data Sizes

Data Sizes

We found out above that on the myth computers, the `int` representation is comprised of 32-bits, or four 8-bit bytes. but the C language does not mandate this. To the right is Figure 2.3 from your textbook:

C declaration		Bytes	
Signed	Unsigned	32-bit	64-bit
[signed] char	unsigned char	1	1
short	unsigned short	2	2
int	unsigned	4	4
long	unsigned long	4	8
int32_t	uint32_t	4	4
int64_t	uint64_t	8	8
char *		4	8
float		4	4
double		8	8

Data Sizes

There are guarantees on the lower-bounds for type sizes, but you should expect that the myth machines will have the numbers in the 64-bit column.

C declaration		Bytes	
Signed	Unsigned	32-bit	64-bit
[signed] char	unsigned char	1	1
short	unsigned short	2	2
int	unsigned	4	4
long	unsigned long	4	8
int32_t	uint32_t	4	4
int64_t	uint64_t	8	8
char *		4	8
float		4	4
double		8	8

Data Sizes

You can be guaranteed the sizes for `int32_t` (4 bytes) and `int64_t` (8 bytes)

C declaration		Bytes	
Signed	Unsigned	32-bit	64-bit
[signed] char	unsigned char	1	1
short	unsigned short	2	2
int	unsigned	4	4
long	unsigned long	4	8
int32_t	uint32_t	4	4
int64_t	uint64_t	8	8
char *		4	8
float		4	4
double		8	8

Data Sizes

We briefly mentioned *unsigned* types on the first day of class. These are integer types that are strictly positive.

By default, integer types are signed.

C declaration		Bytes	
Signed	Unsigned	32-bit	64-bit
[signed] char	unsigned char	1	1
short	unsigned short	2	2
int	unsigned	4	4
long	unsigned long	4	8
int32_t	uint32_t	4	4
int64_t	uint64_t	8	8
char *		4	8
float		4	4
double		8	8

Data Sizes

C allows a variety of ways to order keywords to define a type. The following all have the same meaning:

```
unsigned long
unsigned long int
long unsigned
long unsigned int
```

C declaration		Bytes	
Signed	Unsigned	32-bit	64-bit
[signed] char	unsigned char	1	1
short	unsigned short	2	2
int	unsigned	4	4
long	unsigned long	4	8
int32_t	uint32_t	4	4
int64_t	uint64_t	8	8
char *		4	8
float		4	4
double		8	8

Addressing and Byte Ordering

On the myth machines, pointers are 64-bits long, meaning that a program can "address" up to 2^{64} bytes of memory, because each byte is individually addressable.

This is a lot of memory! It is 16 *exa*bytes, or 1.84 x 10^{19} bytes. Older, 32-bit machines could only address 2^{32} bytes, or 4 Gigabytes.

64-bit machines can address 4 *billion* times more memory than 32-bit machines...

Machines will not need to address more than 2^{64} bytes of memory for a long, long time.

Addressing and Byte Ordering

We've already talked about the fact that a memory address (pointer) points to a particular byte. But, what if we want to store a data type that has more than one byte?

The `int` type on our machines is 4 bytes long. So, how is a byte stored in memory?

We have choices!

First, let's talk about the ordering of the bytes in a 4-byte hex number. We can represent an `ints` as 8-digit hex numbers:

`0x01234567`

We can separate out the bytes:

`0x   01 23 45 67`

Addressing and Byte Ordering

```
 01          23          45          67

0000 0001   0010 0011   0100 0101   0110 0111
---------   ---------   ---------   ---------
```

most significant → least significant

- Some machines choose to store the bytes ordered from least significant byte to most significant byte, called "little endian" (because the "little end" comes first).

- Other machines choose to store the bytes ordered from most significant byte to least significant byte, called "big endian" (because the "big end" comes first).

Addressing and Byte Ordering

- Our `0x01234567` number would look like this in memory for a little endian computer (which, by the way, is the way the myth computers store ints):

byte:	67	45	23	01
address:	0x100	0x101	0x102	0x103

- A big-endian representation would look like this:

byte:	01	23	45	67
address:	0x100	0x101	0x102	0x103

Many times we don't care how our integers are stored, but in cs107 we will! Let's look at a sample program and dig under the hood to see how little-endian works.

Addressing and Byte Ordering

- Our `0x01234567` number would look like this in memory for a little endian computer (which, by the way, is the way the myth computers store ints):

address:	`0x100`	`0x101`	`0x102`	`0x103`
value:	`67`	`45`	`23`	`01`

- A big-endian representation would look like this:

address:	`0x100`	`0x101`	`0x102`	`0x103`
value:	`01`	`23`	`45`	`67`

Many times we don't care how our integers are stored, but in cs107 we will! Let's look at a sample program and dig under the hood to see how little-endian works.

Addressing and Byte Ordering

```
#include<stdio.h>
#include<stdlib.h>

int main() {
    // a variable
    int a = 0x01234567;

    // print the variable in big endian format
    printf("a's value: 0x%.8x\n",a);
    return 0;
}
```

Addressing and Byte Ordering

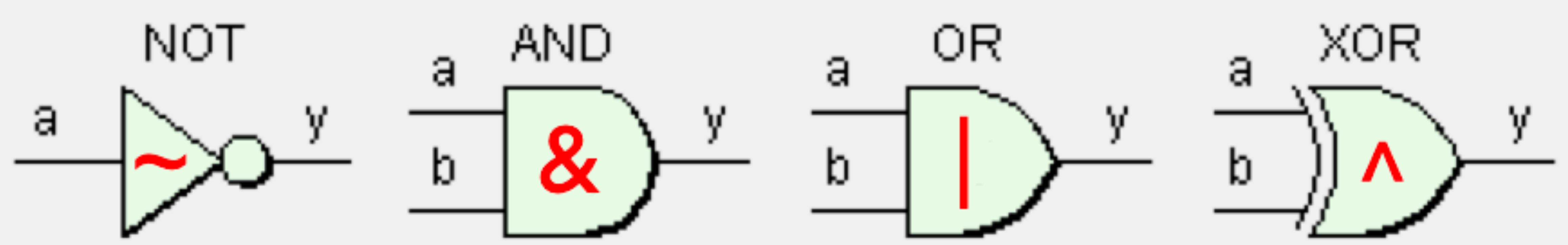
```
$ gcc -g -O0 -std=gnu99 big_endian.c -o big_endian
$ ./big_endian
a's value: 0x01234567

$ gdb big_endian
GNU gdb (Ubuntu 7.7.1-0ubuntu5~14.04.3) 7.7.1
...
(gdb) break main
Breakpoint 1 at 0x400535: file big_endian.c, line 6.
(gdb) run
Starting program: /afs/.ir.stanford.edu/users/c/g/cgregg/107/lectures/lecture2_bits_bytes_continued/big_endian

Breakpoint 1, main () at big_endian.c:6
6	    int a = 0x01234567;
(gdb) n
9	    printf("a's value: 0x%08x\n",a);
(gdb) p/x a
$1 = 0x1234567
(gdb) p &a
$2 = (int *) 0x7fffffffe98c
(gdb) x/16bx &a
0x7fffffffe98c: 0x67    0x45    0x23    0x01    0x00    0x00    0x00    0x00
0x7fffffffe994: 0x00    0x00    0x00    0x00    0x45    0x2f    0xa3    0xf7
(gdb)
```

Note the ordering: `0x01234567` is stored as Little Endian!

3 Minute Break!

Boolean Algebra

Boolean Algebra



a	y
0	1
1	0

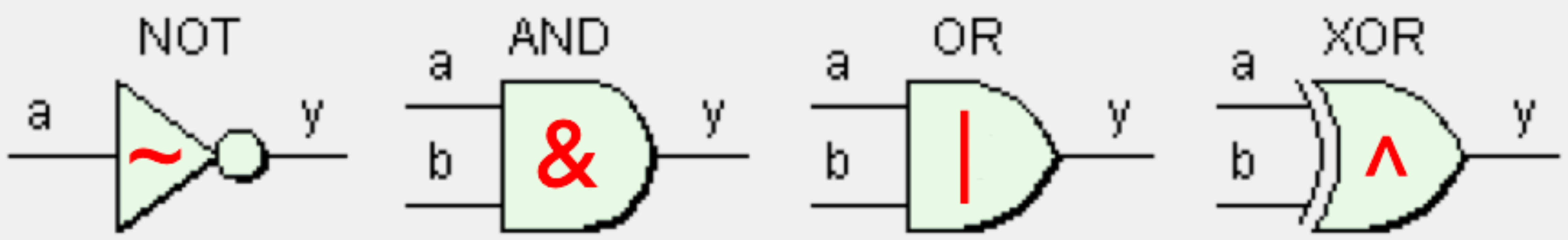
a	b	y
0	0	0
0	1	0
1	0	0
1	1	1

a	b	y
0	0	0
0	1	1
1	0	1
1	1	1

a	b	y
0	0	0
0	1	1
1	0	1
1	1	0

- Because computers store values in binary, we need to learn about boolean algebra. Most of you have already studied this in some form in math classes before, but we are going to quantify it and discuss it in the context of computing and programming.
- We can define Boolean algebra over a 2-element set, 0 and 1, where 0 represents `false` and 1 represents `true`.
- The symbols are: ~ for NOT, `&` for AND, | for OR, and ^ for "exclusive or," which means that if one and only one of the values is true, the expression is true.

Boolean Algebra



a	y
0	1
1	0

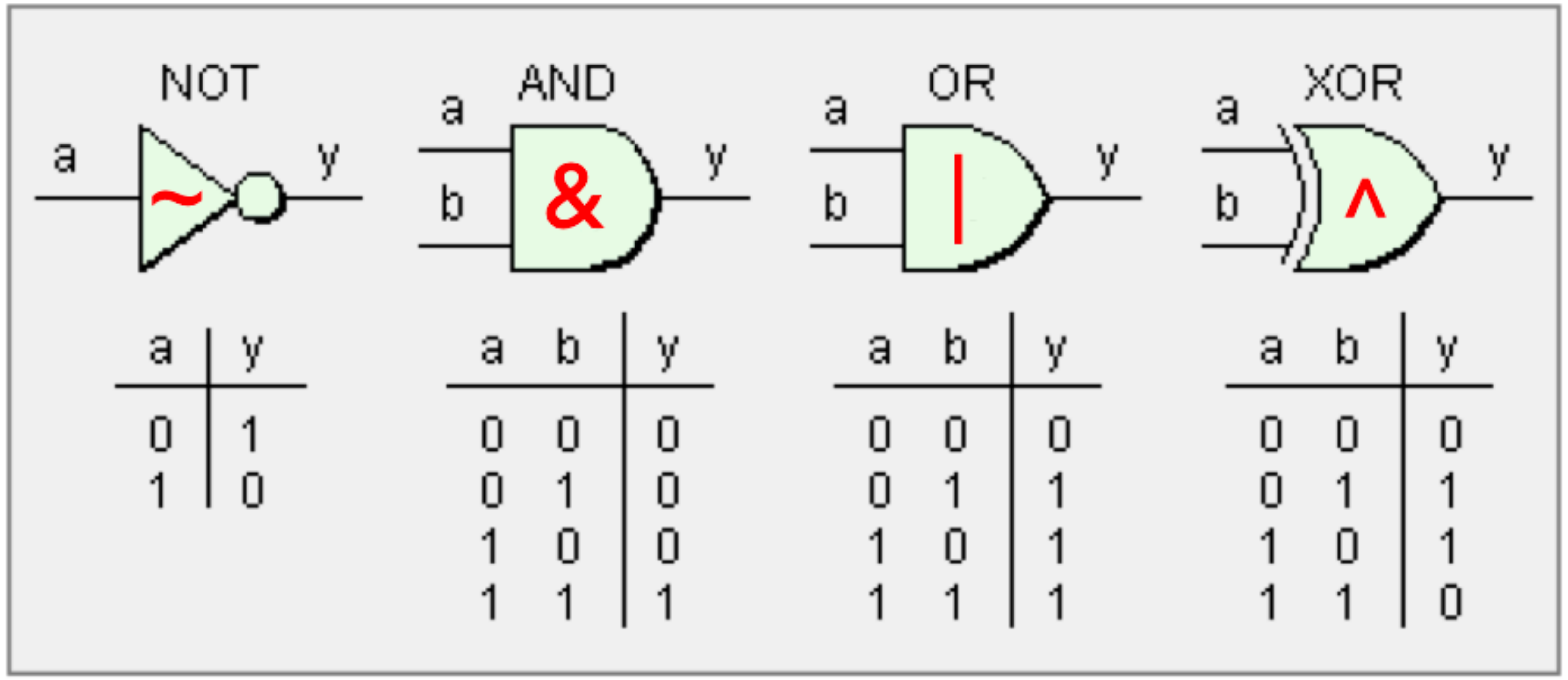
a	b	y
0	0	0
0	1	0
1	0	0
1	1	1

a	b	y
0	0	0
0	1	1
1	0	1
1	1	1

a	b	y
0	0	0
0	1	1
1	0	1
1	1	0

- Be careful! There are *logical* analogs to some of these that you have used in C++ and other programming languages: `!` (logical NOT), `&&` (logical AND), and `||` (logical OR), but we are now talking about *bit* operations that result in 0 or 1 for each bit in a number.
- The bitwise operators use single character representations for AND and OR, not double-characters.

Boolean Algebra



- When a boolean operator is applied to two numbers (or, in the case of ~, a single number), the operator is applied to the corresponding bits in each number. For example:

```
  0110
& 1100
  ----
  0100
```

```
  0110
| 1100
  ----
  1110
```

```
  0110
^ 1100
  ----
  1010
```

```
~ 1100
  ----
  0011
```

Boolean Algebra: Mystery Function

- Let's look at a mystery function!

```
// mystery1.c
#include<stdlib.h>
#include<stdio.h>

void mystery(int *x, int *y) {
    if (x != y) {
        *y = *x ^ *y;
        *x = *x ^ *y;
        *y = *x ^ *y;
    }
}

int main(int argc, char *argv[]) {
    int x = atoi(argv[1]);
    int y = atoi(argv[2]);

    printf("x:%d, y:%d\n",x,y);

    mystery(&x,&y);

    printf("x:%d, y:%d\n",x,y);
    return 0;
}
```

```
$ ./mystery 4 5
```

Boolean Algebra: Mystery Function

- Let's look at a mystery function!

```c
// mystery1.c
#include<stdlib.h>
#include<stdio.h>

void mystery(int *x, int *y) {
    if (x != y) {
        *y = *x ^ *y;
        *x = *x ^ *y;
        *y = *x ^ *y;
    }
}

int main(int argc, char *argv[]) {
    int x = atoi(argv[1]);
    int y = atoi(argv[2]);

    printf("x:%d, y:%d\n",x,y);

    mystery(&x,&y);

    printf("x:%d, y:%d\n",x,y);
    return 0;
}
```

```
$ ./mystery 4 5
x:4, y:5
x:5, y:4
```

https://en.wikipedia.org/wiki/XOR_swap_algorithm

This relies on the fact that `x^x == 0`, and the associativity and commutativity of the exclusive or function.

Incidentally, if you XOR a number with all 1s, you get the complement!

Boolean Algebra: Operations on bit flags

We can represent finite sets with bit vectors, where we can perform set functions such as union, intersection, and complement. For example:

bit vector a = [01101001] encodes the set A = {0,3,5,6} (reading the 1 positions from *right to left*, with #0 being the right-most, #7 being the left-most)

bit vector b = [01010101] encodes the set B = {0,2,4,6}

The | operator produces a set union:
`a | b` → [01111101], or $A \cup B = \{0,2,3,4,5,6\}$

The `&` operator produces a set intersection:
`a & b` → [01000001], or $A \cap B = \{0,6\}$

Boolean Algebra: Bit Masking

A common use of bit-level operations is to implement *masking* operations, where a mask is a bit pattern that will be used to choose a selected set of bits in a word. For example, the mask of `0xFF` means the lowest byte in an integer. To get the low-order byte out of an integer, we simply use the bitwise AND operator with the mask:

```
int j = 0x89ABCDEF;
int k = j & 0xFF; // k now holds the value 0xEF,
                  // which is the low-order byte of j
```

A useful expression is `~0`, which makes an integer with all 1s, regardless of the size of the integer.

Boolean Algebra: Bit Masking

Challenge 1: write an expression that sets the least significant byte to all ones, and all other bytes of the number (assume it is the variable `j`) left unchanged E.g.

`0x87654321` → `0x876543FF`

Possible answer: `j | 0xFF`

Challenge 2: write an expression that complements all but the least significant byte of `j`, with the least significant byte unchanged. E.g.

`0x87654321` → `0x789ABC21`

Possible answer: `j ^ ~0xFF`

Boolean Algebra: Shift Operations

C provides operations to shift bit patterns to the left and to the right.

The `<<` operator moves the bits to the left, replacing the lower order bits with zeros and dropping any values that would be bigger than the type can hold:

`x << k` will shift **`x`** to the left by **`k`** number of bits.

Examples for an 8-bit binary number:

`00110111 << 2` returns `110111`*`00`*
`01100011 << 4` returns `0011`*`0000`*
`10010101 << 4` returns `0101`*`0000`*

Boolean Algebra: Shift Operations

There are actually two flavors of *right* shift, which work differently depending on the value and type of the number you are shifting.

A *logical* right shift moves the values to the right, replacing the upper bits with 0s.

An *arithmetic* right shift moves the values to the right, replacing the upper bits with a copy of the most significant bit. This may seem weird! But, we will see why this is useful soon!

Examples for an 8-bit binary number:
Logical right shift:

`00110111 >> 2` returns *`00001101`*
`10110111 >> 2` returns *`00101101`*
`01100011 >> 4` returns *`00000110`*
`10010101 >> 4` returns *`00001001`*

Examples for an 8-bit binary number:
Artithmetic right shift:

`00110111 >> 2` returns *`00001101`*
`10110111 >> 2` returns *`11101101`*
`01100011 >> 4` returns *`00000110`*
`10010101 >> 4` returns *`11111001`*

Shift Operation Pitfalls

There are two important things you need to consider when using the shift operators:

1. The C standard does not precisely define whether a right shift for signed integers is logical or arithmetic. *Almost all* compilers / machines use arithmetic shifts for signed integers, and you can most likely assume this. Don't be surprised if some Internet pedant yells at you about it some day. :) All *unsigned* integers will always use a logical right shift (more on this later!)

2. Operator precedence can be tricky! Example:

   `1<<2 + 3<<4` means this: `1 << (2 + 3) << 4`, because *addition and subtraction have a higher precedence than shifts!*

   Always parenthesize to be sure:
   `(1<<2) + (3<<4)`

Practice!

Let's take a look at lots of examples:

If you want to try the examples out yourself. On myth:

```
$ cd CS107
$ cp -r /afs/ir/class/cs107/lecture-code/lect3 .
cd lect3
make
ls # to see the files
```

References and Advanced Reading

- **References:**
  - argc and argv: http://crasseux.com/books/ctutorial/argc-and-argv.html
  - The C Language: https://en.wikipedia.org/wiki/C_(programming_language)
  - Kernighan and Ritchie (K&R) C: https://www.youtube.com/watch?v=de2Hsvxaf8M
  - C Standard Library: http://www.cplusplus.com/reference/clibrary/
  - https://en.wikipedia.org/wiki/Bitwise_operations_in_C
  - http://en.cppreference.com/w/c/language/operator_precedence

- **Advanced Reading:**
  - After All These Years, the World is Still Powered by C Programming
  - Is C Still Relevant in the 21st Century?
  - Why Every Programmer Should Learn C