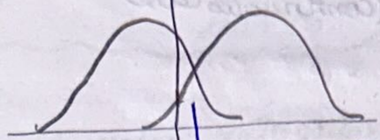
Null = Default = Hypothesis.

'Threshold'

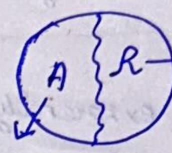


H_0 true if fall in this interval

ξ

Data falls in this H_1 true

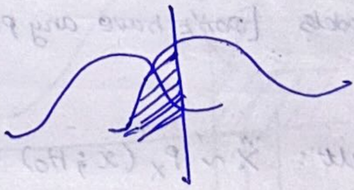
If my data falls here Rejection of H_0 may be error (α)



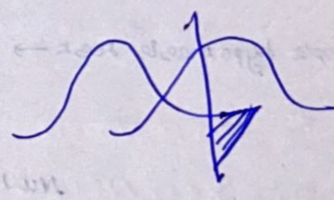
→ Reject H_0 & choose H_1

Accept H_0.

'Reject H_0 even though H_0 is true → true'



β



α

Both α & β must be minimum

(Tradeoff)

Small α, large β → No way.

(Tradeoff is the option)

'where to put the threshold?'

Solution

Choose morelikely one in error prone areas.

$$\lambda(x) = \boxed{\frac{P(X = x \mid H_1)}{P(X = x \mid H_0)} \gtrless \frac{P(H_0)}{P(H_1)}}$$

likelihood ratio test

$$\frac{P(X = x ; H_1)}{P(X = x ; H_0)} > \xi \quad \text{(discrete case)}$$

$$\frac{f_X(x ; H_1)}{f_X(x ; H_0)} > \xi \quad \text{(continuous case)}$$

'How likely to occur if H_1 is true'

'How likely to occur if H_0 is true'

Case: If extremely unlikely to be occured in H_0: this ratio will be big → choose H_1

Threshold chosen by fixing any one of α or β Then choose Threshold.

---

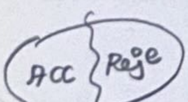
25 → Classical Inference - III

Simple hypothesis test → Two models [Don't have any priors]

Null : Default : $X \sim P_X(x ; H_0)$ (or) $f_X(x ; H_0)$

Alternative hypothesis :

$X \sim P_X(x ; H_1)$ [or $f_X(x ; H_1)$]



Choose a rejection region R

Reject H_0 if data $\in$ R.

* (shape of dividing curve)
* where to put it in space.

Likelihood ratio test: reject H_0 if

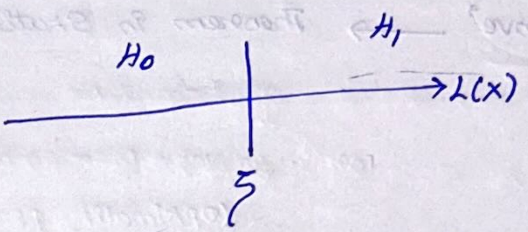
$$\frac{P_X(x ; H_1)}{P_X(x ; H_0)} > \xi \quad \text{(or)} \quad \frac{f_X(x ; H_1)}{f_X(x ; H_0)} > \xi$$

General Shape: Structure of test $\longrightarrow$ chosen by likelihood ratio.

$L(x)$ high means $\rightarrow$ data high chance of occuring in H_1 than H_0.

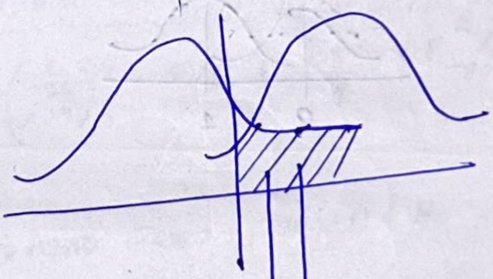
* Fix false rejection probability α (eg: $\alpha = 0.05$)

* choose ξ, so that $P(\text{reject } H_0 ; H_0) = \alpha$



choose threshold

(Look at $L(x)$ distribution)



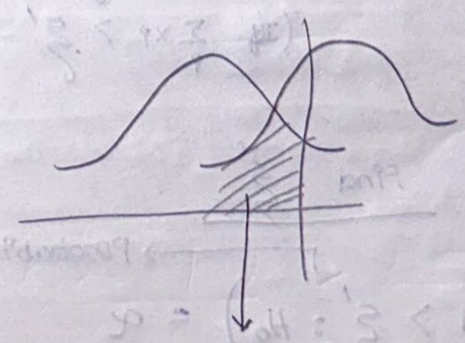
Incorrect decision (force rejection of H_0)

$\alpha = 5\%$ (choose ξ accordingly)

If $H_0 \rightarrow$ True [Prob will be less than 5%] $\rightarrow$ more than 5%

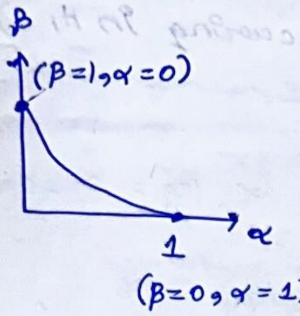
reject H_0

Other kind



Other kind β

"Trade off b/w α and β"

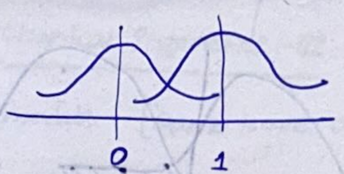


$\longrightarrow$ "Trade off curve"

"when we use likelihood ratio test - we get best possible trade off curve" $\longrightarrow$ Theorem in Statistics.

For a gn value of $\alpha \leftrightarrow$ minimize the prob of error
(optimality prop of likelihood ratio test.

Example (normal mean)



$H_0 : X_i \sim N(0,1)$

$H_1 : X_i \sim N(1,1)$

$$\frac{\left(\frac{1}{\sqrt{2\pi}}\right)^n \exp\left(-\sum_i \frac{(x_i - 1)^2}{2}\right)}{\left(\frac{1}{\sqrt{2\pi}}\right)^n \exp\left(-\sum_i x_i^2/2\right)} > \xi$$

$\longrightarrow$ after log & derivative.

$$\boxed{\text{Reject } H_0 \text{ if : } \sum_i x_i > \xi' \text{ (Threshold)}}$$

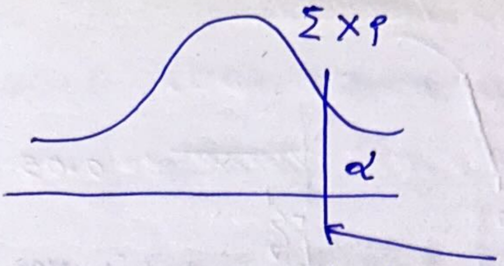
Summary: Statistic

(If $\sum_i x_i > \xi' \longrightarrow$ Reject)

Find ξ'

$\longrightarrow$ Probability of incorrect decision (False rejection)

$$P\left(\sum_{i=1}^{n} x_i > \xi' : H_0\right) = \alpha$$



ξ' (using normal table)

Go to normal table

Example

(Same mean — difference variance).

n data points, i.i.d:

H_0: $x_i \sim N(0, 1)$

H_1: $x_i \sim N(0, 4)$

Likelihood ratio

$$\frac{\left(\frac{1}{2\sqrt{2\pi}}\right)^n \exp\left(-\sum_i x_i^2 / (2 \cdot \underset{\text{variance}}{4})\right)}{\left(\frac{1}{\sqrt{2\pi}}\right)^n \exp\left(-\sum_i x_i^2 / 2\right)} > \xi$$

Take log then derivative

Idea: $x \to$ near zero choose H_0

Big on either side $\to$ choose H_1

Reject H_0 if $\boxed{\sum_i x_i^2 > \xi'}$ $\to$ fix the shape of the rejection region.

Find ξ'

$\to$ Distribution of $\sum_i x_i^2$ is known as

(derived distribution problem)

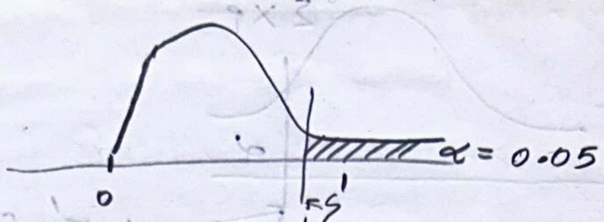
$\to$ $P\left(\sum_{i=1}^{n} x_i^2 > \xi' ; H_0\right) = \alpha$

$\hookrightarrow$ false rejection.

$\xi' = ?$

$\sum x_i^2$

(Non negative



95% of distribution.
Normal table.

$\sum x_i^2$ Knowing its distribution, we know ξ'

<u>x_i ? distribution:</u>

* we know x_i's distribution
* Derive for x_i^2 $\longrightarrow$ Tables available for it

$\because$ x_i's are independent, x_i^2's are too

[convolution formula]

95% of distribution: ξ'

$\downarrow$

Common problem - chi-square problem

$\xi <$ Coin - Fair coin

$\downarrow$

Is it fair

Null: Fair

Alternative hypothesis: Not Fair $\rightarrow$ 0.6, 0.7, 0.51, 0.48

Lot of cases.
(family)

* Got some data [472 heads]

$H_0 : p = \frac{1}{2}$ — Is this true

If it looks like extreme outliers → Remove

within acceptable range → Accept.

'we don't care about : sequence'

Summary of data: No. of head → Does it like a outliers
→ OK?

Pick shape of rejection ratio

eg: $|S - n/2| > \xi$ → rejection ratio.

pick a statistic — No. of heads.

Boom

$\boxed{\alpha = 0.05}$ → significance level

5% → False rejection.

Pick ξ (ch9)

Use a critical value

CLT – Central limit theorem.

ξ ; $P(\text{reject } H_0 : H_0) = \alpha$

CLT :

$P(|S - 500| \le 31 ; H_0) \approx 0.95$, $\xi = 31$

↓

Threshold.

95% → prob will be within 31 heads from the mean. [Normal table]

our example: 472 heads → 28 away from mean.

$\boxed{\therefore \text{Not rejected}}$ → H_0 Stands.

Terminology

* H_0 is not rejected (at the 5% level)

'Still alive'

Complicated ways:

'Independent or Not Independent?'

No amount of data: can prove your theory

↓

experiments may strengthen it.

---

may be bias 0.500001 or anything

A die is fair

Null hypothesis: $1/6$ each. [Statistically independent]

Alternate hypothesis: No

observed occurrence i (specific result): N_i

$$\text{reject } H_0 \text{ if } T = \sum_i \frac{(N_i - n p_i)^2}{n p_i} > \xi$$

p_i → $1/6$ (our example)

np_i → mean (Tailor series approximation)

'How far away allowed: ?'

* Too far: Die not fair → reject null hypothesis.

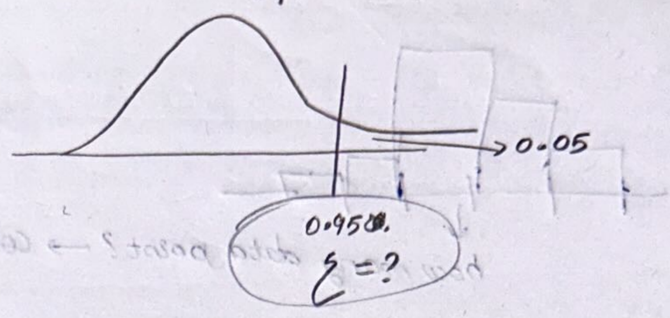
Choose ξ

$P(\text{rejection } H_0 : H_0) = 0.05$ (False rejection)

$$\boxed{P(T > \xi : H_0) = 0.05}$$

Need the distribution of T

$f_T(t)$ under H_0



T → Some messy derived distribution problem.

∴ N_i → binomial R.V

(No. of 1 got in n rolls of my die)

yes / no

↓

$n \to \infty$, Normal

$$\text{Normal} - \text{constant} \longrightarrow \frac{(\text{Normal})^2}{\text{Scale}} \longrightarrow \text{Normal.}$$

(CLT + derived distribution problem)

For large data n,

T has appr chi-square distribution.

Null hypothesis: Is my pmf correct?

$$p_i = \frac{1}{6} \quad (i = 1, \ldots, 6)$$

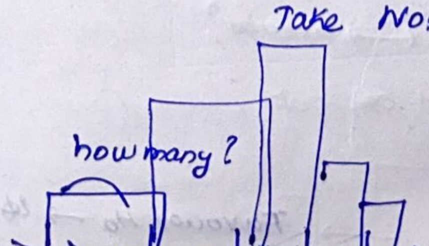
<u>Is my pdf correct: continuous data</u>

↓

Take Normal result → Discretize.

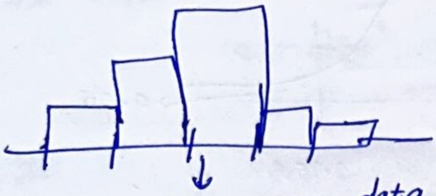
↓

solve for discrete problem



→ histograms.

Does it look like normal?



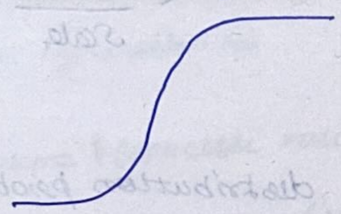
how many data point? → compare it with actual

How to choose bin size

narrow (empty bins) | large (losing info)

Kolmogorov-Smirnov test

Plotting a PMF (appro pdf) → work with cumulative distr- function.

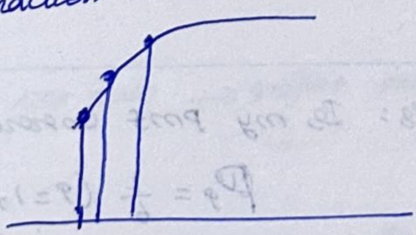


$H_0 =$ $X \sim N(0,1)$

CDF = Continuous looking curve

Through data : Draw empirical CDF.

∴ what fraction below each number



'like — how much'

If my hypothesis be true

↓ 'CDF → data based ⟷ actual'

closed or not.

Difference

$$\boxed{D_n = \max_{x} \left| F_X(x) - \hat{F}X(x) \right|}$$

→ Favours H_0 → if small distance b/w two CDFs (Data & actual).

$P(\sqrt{n} D_n \geq 1.36) \approx 0.05.$

↓

5% False rejection.

$$\boxed{P\left(D_n \geq \frac{1.36}{\sqrt{n}}\right) \approx 0.05}$$

<u>Pdf of data</u>

↓

Histogram → smooth curve

* (Bin size ?) → Depends on how many data

↓

Smooth

* Signals: Fast in real time (need with accuracy)

* Can I pick x or x^2 to estimate y

<u>why most published research findings are False</u>

* wrong statistics

* (when you see something: prior) → Not possible.

Null: drug doesn't work

By accident 5% → seems to be working (Bias)

↓

without foundation

* may be an outlier.

* Some tests may favour by accident.

(collection: making an error lot more each: 5%.)

pick hypothesis get data Do test	→ correct

wrong

1) Get data
2) Test
3) Choose hypothesis (Abnormal)