App. divergence theorem

$$W_{tot} = \frac{1}{2}\oint VD \cdot ds - \frac{1}{2}\int D \cdot \nabla V \, dv$$

$$\boxed{VD \propto \frac{1}{r^3}} \quad \boxed{ds \propto r^2}$$

$E = -\nabla V$

As $r \uparrow$, $V.D \to 0$

$$W_{tot} = \frac{1}{2}\underbrace{\cancel{\oint VD \cdot ds}}_{(0)} - \frac{1}{2}\int D \cdot \nabla V dv$$

$$W_{tot} = -\frac{1}{2}\int D \cdot \nabla V \, dv$$

$$= \frac{1}{2}\int D \cdot E \, dv$$

$$= \frac{1}{2}\int \varepsilon E (E) \, dv$$

$$= \frac{1}{2}\int \varepsilon E^2 dv$$

$\frac{\text{Energy}}{\text{Volume}}$ = Energy density

$$\frac{dW_{tot}}{dv} = \frac{1}{2}\varepsilon E^2$$

$$\boxed{W_d = \frac{1}{2}\varepsilon E^2}$$

Energy stored in a Capacitor

$$E = \frac{V}{d}$$

$$W_{tot} = \frac{1}{2}\int \varepsilon \frac{V^2}{d^2} dv$$

Volume

$$\boxed{v = A \times d}$$

$$W_{tot} = \frac{1}{2}\varepsilon \frac{V^2}{d^2}(v)$$

v = volume

$$W_{tot} = \frac{1}{2}\varepsilon \frac{V^2}{d^2} v$$

$$\frac{\varepsilon A}{d} = C$$

$$W_{tot} = \frac{1}{2}\varepsilon \frac{V^2}{d^2} \cdot Ad$$

C - capacitance

V - Voltage applied

$$W_{tot} = \frac{1}{2}\frac{\varepsilon A}{d} V^2$$

$$\boxed{W_{tot} = \frac{1}{2}CV^2}$$

20/04/2021

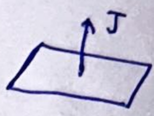
Continuity eqn & KCL

Current density $(J) = \frac{I}{A} = \frac{\text{Current}}{\text{Area}}$

$$\boxed{I = \oint J \cdot ds}$$

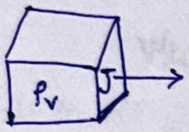
over the surface

(Direction of J will be normal to the surface)



KCL: (Eqn of continuity & KCL)

Based on law of conservation of charge.



($\perp^r$ to surface ds)

Total current through the surface $I = \int_S J \cdot ds$

$$\boxed{I = -\frac{dq}{dt}}$$ (change in charge w.r.t t)

$$I_{out} = \iint J \cdot ds = -\frac{dq}{dt}$$

$$\boxed{q = \rho_v \cdot dv}$$

$$\iint J \cdot ds = \iiint \nabla \cdot J \, dv$$

$$\iint J \cdot ds = \iiint \nabla \cdot J \, dv = -\frac{d}{dt} \iiint \rho_v \cdot dv$$

$$= -\iiint \frac{\partial \rho_v}{\partial t} dv$$

$$\boxed{\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}}$$ → Continuity eqn in differential form

$$\iint J \cdot ds = -\frac{dq}{dt}$$

Divergence of current density is rate of volume charges coming out.

For steady current $\frac{\partial \rho_v}{\partial t} = 0$.

$\boxed{\nabla \cdot J = 0}$ (Solenoidal)

Steady state current is divergenceless (solenoidal)

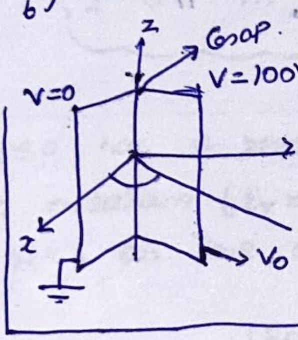
$$\iint J \cdot ds = -\frac{dQ}{dt}$$

$\boxed{\iint J \cdot ds = 0}$ $\boxed{\Sigma I = 0} \rightarrow$ KCL.

Laplace's equation:

semi infinite conducting planes at $\phi = 0$ and $\phi = \frac{\pi}{6}$ are seperated by an infinitesimal insulating gap. If $(V(\phi = 0) = 0)$ & $V(\phi = \frac{\pi}{6}) = 100V$. calc E & V in the region b/w the plates

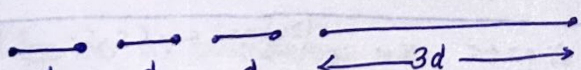
Solu:-



'Boundary → we know'

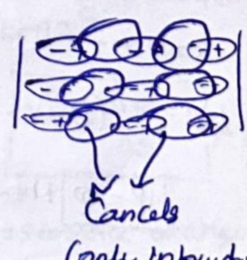
$\rho_v = ?(0)$

As dielectric



$\boxed{\therefore \rho_v = 0 \text{ (Laplace equ)}}$

'cylindrical'



$$\boxed{\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho_v}{\varepsilon}}$$

$\therefore \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} = 0$ $\boxed{\therefore \frac{\partial^2 V}{\partial \phi^2} = 0}$ $\boxed{\text{As } \frac{1}{\rho^2} \neq 0}$

$\frac{\partial^2 V}{\partial \phi^2} = 0$, $\frac{\partial V}{\partial \phi} = \text{Constant } (A) \Rightarrow$ $\boxed{V = A\phi + B}$

Integrate

At $\phi = 0, V = 0$

$0 = 0 + B$

$\boxed{B = 0}$

$V = A\phi$

$\phi = \frac{\pi}{6}, V = 100V$

$100 = A\frac{\pi}{6}$

$\boxed{A = \frac{600}{\pi}}$

$\boxed{V = \frac{600}{\pi}\phi} \longrightarrow (1)$

$\therefore$ From values of $\phi \rightarrow$ we can find V inside the plane.

$\cancel{\nabla \cdot E = -B}$ $\qquad \boxed{E = -\nabla V}$

$$\nabla F = \frac{\partial F}{\partial \rho}\vec{a_\rho} + \frac{1}{\rho}\frac{\partial F}{\partial \phi}\vec{a_\phi} + \frac{\partial F}{\partial z}\hat{a_z}$$

$$\nabla F = \frac{1}{\rho}\frac{\partial F}{\partial \phi}\vec{a_\phi} \qquad \left[V = \frac{600}{\pi}\phi\right]$$

$$\nabla V = -\frac{1}{\rho}\frac{\partial V}{\partial \phi}\vec{a_\phi}$$

$$\boxed{E = -\frac{1}{\rho}\frac{600}{\pi}\hat{a_\phi}}$$

21/04/21 Three charged cylindrical sheets are present in three spaces with $\sigma = 5$ at $R = 2m$, $\sigma = -2$ at $R = 4m$, $\sigma = -3$ at $R = 5m$. Find the flux density at $R = 4.5m$

Solu: (Gauss law)

$\boxed{\Psi = Q}$ Flux passing through a surface is equal to the charges enclosed

$\boxed{Q = \sigma \times A}$

$\Psi = \oint_S D \cdot d\vec{S}$

$D(2\pi R \cancel{L}) = \sigma(2\pi R \cancel{L})$

$D(2\pi \times 4.5) = Q_1 + Q_2$

$D(2\pi \times 4.5) = \sigma_1(2\pi R) + \sigma_2(2\pi R)$

$\sigma:$ 2 $\quad$ -2 $\quad$ -3

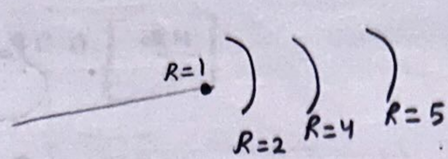
$D(2\pi \times 4.5) = \sigma_1(2\pi \times 2) + \sigma_2(2\pi \times 4)$

$\Rightarrow 4.5D = 2(5) - 2(4)$

$D = \frac{10-8}{4.5} = 0.444$ units.

~~Flux dens~~ $R=1$ (Flux density = ?)

$\therefore$ $R=1$ doesn't enclosed inside the sheet. (So 0)



$V = x^2 + y^2 + z^2 \rightarrow$ satisfies Laplace eqn

$\boxed{\nabla^2 V = 0}$
Laplace eqn

Soln:

$\nabla^2 V = \nabla(2x + 2y + 2z)$

$= (6) \neq 0$

'Doesn't'

$V = r\cos\phi + z$

$\therefore \nabla^2 V$ in cylindrical coordinates

Regn $y \leq 0$ has a perfect conductor while $y \geq 0$ is a dielectric medium ($\varepsilon_r = 2$). If there is a surface charge of $2nC/m^2$) on the conductor, determine E & D at.

(Boundary condition problem: Condt → Dielectric)

A $(3, -2, 2) \rightarrow$ conductor
B $(-4, 1, 5) \rightarrow$ Dielectric.

$E_N = \dfrac{\rho_S}{\varepsilon}$

Dielec ($\varepsilon_r = 2$)

———— $y=0$

Cond

$\boxed{\rho_S = \dfrac{\Delta Q}{\Delta S}}$

D = surface charge density
↓
$\rho_S = D = 2nC/m^2$

A$(3, -2, 2)$

$E = 0$ [conductor]

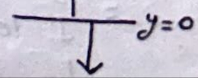
$D = 0$

B$(-4, 1, 5)$

→ conti (next page)

$E_n = \dfrac{\rho_S}{\varepsilon} = \dfrac{2n}{\varepsilon_0 \times \varepsilon_r} = \dfrac{2n}{2 \times 8.854 \times 10^{-12}} = 112.94330025.\hat{a}_y$

Direction → (Normal to surface) → y



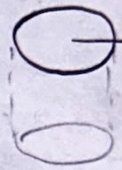
Find the total current in a $\circ^r$ conductor of $r = 4$ mm if

$J = \frac{10^4}{r}$ A/m^2 (varies according to)

(Symmetry: cylindrical)

Solu:

$$I = \oint J \cdot ds$$

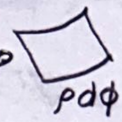


$$I = \oint J \cdot r \cdot d\phi \cdot dr$$

$$= \oint \frac{10^4}{r} \cdot r \cdot d\phi \, dr$$

$ds = \rho \, d\phi \, d\rho$

$$= 10^4 \int_0^4 \int_0^{2\pi} dr \, d\phi$$



$$= 10^4 \times 10^{-3} \int_0^4 \int_0^{2\pi} d\phi \, dr$$

(In metres)

$$= 10 \, (2\pi r) \Big|_0^4$$

constant plane - remove

$$= 10 (2\pi \times 4)$$

$$= 80\pi \text{ (A)}$$

Find the current density $J = 10z \sin^2\phi \; a_r$ A/m^2. Find the current through cylindrical surface $r = 2$, $1 \le z \le 5$ m

Solu:

$$I = \oint J \cdot ds$$

$ds = \rho \, d\phi \, dz \; a_\rho$

$$= \int 10z \sin^2\phi \; a_r \cdot$$

$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$

$$= \int 10z \sin^2\phi \; r \, d\phi \, dz$$

$$= \int_1^5 \int_0^{2\pi} 20z \sin^2\phi \; r \, d\phi \, dz$$

$$I = \int_1^5 \frac{20 r z}{2} \left[\int_0^{2\pi} (1 - \cos 2\theta) \, d\phi \right] dz$$

$$= \int_1^5 10 r z \left[\phi \Big|_0^{2\pi} - \frac{\sin 2\theta}{2} \Big|_0^{2\pi} \right] dz$$

$$= \int_1^5 10 r z \left[2\pi - \frac{\sin 4\pi}{2} \right] dz$$

$$= \int_1^5 10 r z \, (2\pi) \, dz \quad = \int_1^5 20 r z \pi \, dz \quad = 20r \left[\frac{z^2}{2} \pi \right]_1^5$$

$r=2$

$= 20(2)\left[\frac{25}{2}\pi - \frac{\pi}{2}\right]$

$= 20\pi\left[\frac{24}{2}\right]$

$= 20\pi \times 12$

$= 240\pi$

$= 754\ A$

$$\begin{array}{r} 40 \\ 12 \\ \hline 80 \\ 40 \\ \hline 480 \end{array}$$

$\vec{D} = D_N + D_t = D_N = 2n\ \hat{a_y}$

$D_N = \rho_s = 2n\ \frac{c}{m^2}$

$\hat{a_y} = \frac{-4\hat{a_x} + \hat{a_y} + 5\hat{a_z}}{\sqrt{16+1+25}}$

$\boxed{\vec{a_y} = -0.617\ \hat{a_x} + 0.154\ \hat{a_y} + 0.77\ \hat{a_z}}$

$W_{ex} = Q(V_A - V_B)$

$= 50 \times 1.6 \times 10^{-19}\ (7+4)$

$= 50 \times 11 \times 1.6 \times 10^{-19}$

23/04/21

Uniqueness of electrostatic solutions:

Finding answers - using Electrostatic problems through laplace or poisson eqn - unique solution.

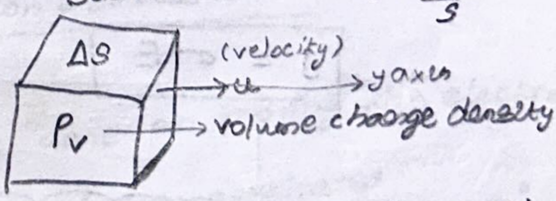
convection current

$I = \frac{Q}{t}$

$J = \frac{I}{S}$

* convection current density

(current)



$\Delta Q = \rho_v \cdot dv \quad \left(\frac{\text{charge}}{\text{unit vol}} = \rho_v\right)$

(Dielectric involved - involved current (convection)
or
vacuum
or
rarified
gases)

$\Delta I = \frac{\Delta Q}{\Delta t} = \frac{\rho_v \cdot dv}{\Delta t}$

$\boxed{\Delta I = \rho_v \cdot \frac{\Delta S \cdot \Delta l}{\Delta t}}$

$\Delta v = \Delta S \cdot \Delta l$
(volume)

$$\Delta I = \rho_v \cdot \Delta S \cdot \frac{\Delta l}{\Delta t}$$

$$\boxed{\Delta I = \rho_v \cdot \Delta S \cdot U_y}$$

$$\frac{\Delta I}{\Delta S} = \rho_v \cdot U_y$$

$$\boxed{J = \rho_v \cdot U_y}$$

$U_y \rightarrow$ velocity along y axes

Conduction current density

Conduction - free e^-s (Electrons - allign in the direction of E)

Drifting - Drift velocity

$e^- \rightarrow e^-$

'collision' - will happen during random movement

$$F = -eE$$

$$E = \frac{F}{q}$$

($q \rightarrow$ charge of e^-)

$$\frac{m \cdot U_d}{\tau} = -eE$$

τ - Average time b/w collisions.

$\therefore F = ma$

$a = \frac{\text{velocity}}{\text{time}}$

$$U_d = -\frac{e\tau}{m} \cdot E$$

$$U_d = -\mu_e E$$

$$\boxed{\mu_e = \frac{e\tau}{m}}$$

mobility of an e^- in E.

$$\boxed{\rho_v = -ne}$$

$$\frac{Q}{V} = \frac{-ne}{1 \text{ (volume unit)}}$$

unit volume

$-ne \rightarrow n$ (numbers) $\times$ electron value charge

$$J = \rho_v \cdot U_d$$

$$J = (-n \cdot e)\, U_d$$

$$J = (-ne)(-\mu_e E)$$

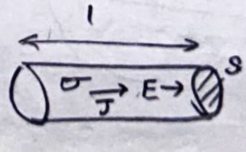
$$\boxed{J = \sigma E}$$

Point form of ohm's law.

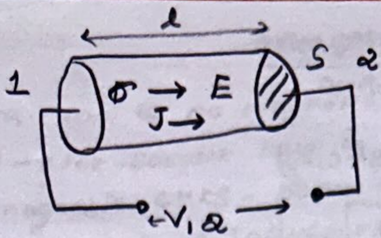
$$\boxed{\sigma = ne\mu_e} \rightarrow \text{conductivity.}$$

Conduction when applied an E

Homogeneous material (E = same) with σ (conductivity) length (l), cross section S



$J \rightarrow$ (flow)



$$V_{12} = E \cdot l$$

$$\boxed{E = \frac{V}{l}}$$

$$\boxed{J = \frac{I}{S}}$$

$$J = \sigma E$$

$$\boxed{\sigma E = \frac{I}{S}}$$

$$\sigma \cdot \frac{V}{l} = \frac{I}{S}$$

$$\frac{V}{I} = R$$

$$\frac{V}{I} = \frac{l}{S} \cdot \frac{1}{\sigma}$$

$\frac{1}{\sigma}$ (Resistivity)

$$R = \rho \frac{l}{S}$$

$$\boxed{\frac{\rho l}{A} = R}$$

$\boxed{\frac{V}{I} = R}$ → Ohm's law.

$\therefore \rho \rightarrow$ Resistivity

Result: conductors will follow Ohm's law.

EMF & KVL

$$E = \frac{V}{l}$$

$$\oint_{l} E \cdot dl = 0$$

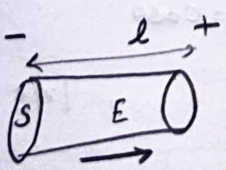
(ckt)

$\oint dl = l$
closed ckt

$$\sum \frac{V}{l} \times l = 0$$

$$\boxed{\sum V = 0}$$

Power dissipation & Joule's law



→ 'Work done on moving the electron'

'Power dissipated in the form of heat'

drifted - collision

e Thermal vibration'

$$\Delta W = F \times \text{displacement}$$
$$= qE \times \Delta l$$

$P = \frac{\text{work}}{\text{time}} = \frac{F \times \text{displacement}}{\text{time}}$

$= \text{Force} \times \text{velocity}$

$\boxed{dP = \rho E \times U} \rightarrow$ unit volume

$\boxed{dP = \rho E\, U \cdot dv}$ (For dv volume)

J=charge × velocity

$dP = J \cdot E \cdot dv$

$\int dP = \int J \cdot E \cdot dv$

$\boxed{P = \int_V J \cdot E \cdot dv} \rightarrow$ Joules law

$P = \int_V J \cdot E \cdot dl \cdot ds$

$= \int_V E \cdot dl \cdot \int J \cdot ds$

$\boxed{P = V \cdot I}$

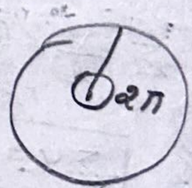
Cont two pages after

Assume free charge density of $-(0.3 \qquad)$ (nC/mm³) in a vacuum tube. For a current density of $-2.4\, a_z$ (A/mm²), find the total current passing through a hemispherical cap specified by R = 5 mm,

$0 \le \theta \le \pi/2,\ 0 \le \psi \le 2\pi$

$\theta \rightarrow 0$ to $\pi/2$

$\phi \rightarrow 0$ to 2π



Solu:

$I = \oint J \cdot ds$

$\boxed{J = -2.4 a_z \ (A/mm^2)}$

$z = r\cos\theta$

$a_z = ?$ $\qquad \boxed{J = -2.4 \cos\theta}$ $\qquad a_z = \cos\theta$

$|r| = 1$

$ds = r\sin\theta\, d\phi\, dr\, \hat{a}_\theta + r^2 \sin\theta\, d\theta\, d\phi\, \hat{a}_r + r\,dr\,d\phi\, \hat{a}_\phi$

[r constant, dr becomes zero, so we can eliminate dr]

$\boxed{ds = r^2 \sin\theta\, d\theta\, d\phi}$ $\qquad I = \oint J \cdot ds$

$I = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} (-2.4\cos\theta)(r^2 \sin\theta\, d\theta\, d\phi) = (-2.4 \times 25) \int_0^{\pi/2} \sin\theta \cos\theta\, d\theta \int^{2\pi} d\phi$

Cont. after 12 pages.

Magnetostatics

Lorentz force eqn – Law of no magnetic monopoles – Ampere's law – vector magnetic potential – Biot-Savart law & app – magnetic field intensity & idea of ϵ_r – magnetic ckts – Behaviour of magnetic materials – Boundary cond – Inductance & Inductors – magnetic Energy, forces and torques

App: magnet in speakers, School bell, motors, Transformers, microphones, compasses, memory storage.

(magnetized when we pass Electric current – Electromagnet.

"Current carrying conductor produces a magnetic field around it"

Path: closed loop (magnetic field)

Thumb – dir of current

(Distance ↑ magnetic field strength ↓)

magnetic field intensity (H)

* Force exp by a unit north pole when placed at that point
* A/m
*

magnetic flux density [B]

* Total magnetic lines of force (flux lines) per unit surface area in the ⊥ direction.

* Wb/m² (Tesla)

$\boxed{\mu_0 = 4\pi \times 10^{-7} \ H/m}$

Relation b/w B & H

μ_0 (Free space) $= 4\pi \times 10^{-7}$ H/m

$B = \mu H$

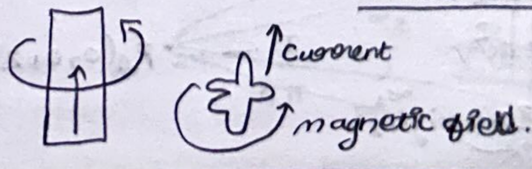
↳ permeability

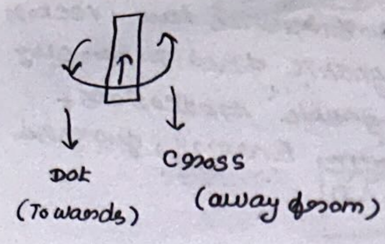
$\boxed{\mu_r = \mu_0 \mu_r}$

← Laws → Ampere's law [special case of Biot savart's law]

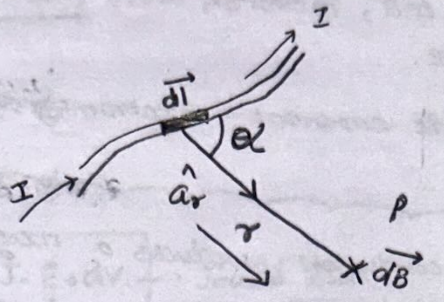
Biot savart's law:

Identification of fields





Biot Savart's law



$I\vec{dl} \rightarrow$ differential current element.

magnetic field intensity (DIA) – at point P.

$$dH \propto \frac{I\,dl\,\sin\alpha}{R^2}$$

$$a_R = \frac{\vec{R}}{|\vec{R}|}$$

$$dH = \frac{K\,I\,dl\,\sin\alpha}{R^2}$$

$$dH = \frac{I\,dl\,\sin\alpha}{4\pi R^2}$$

Sin α – cross product

$$dH = \frac{I\,dl \times |\vec{R}|}{4\pi R^3}$$

$$\boxed{dH = \frac{I\,dl \times \hat{a}_R}{4\pi R^2} \quad \text{(away)}}$$

$\therefore \vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta$

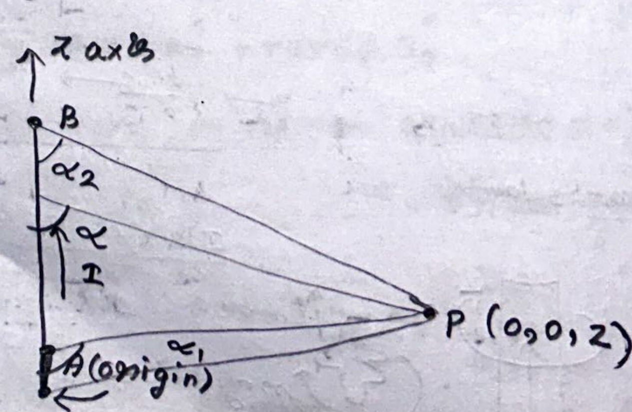
direction of dH : Right hand thumb rule

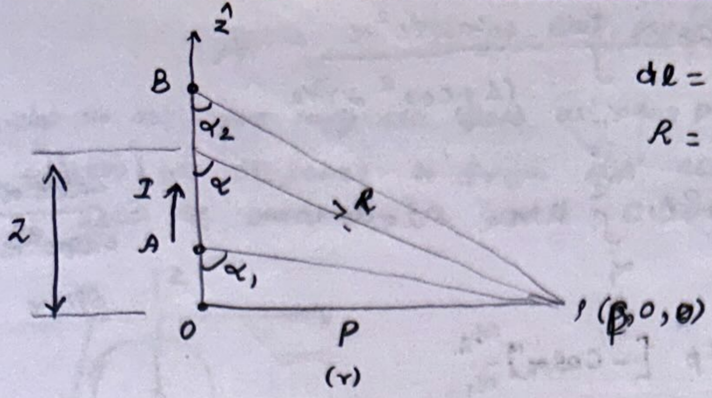
α – angle by $\vec{dl}$ and $\hat{a}_r$

Application

* Estimate H for straight conductor

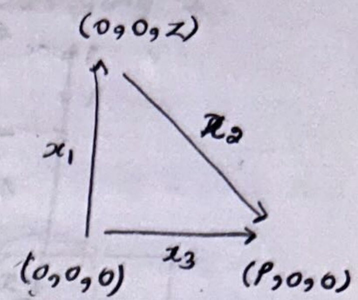
* Description:





$d\ell = dz \cdot a_z$

$R = \rho \cdot a_\rho - z a_z$



$$\boxed{x_2 = R = \rho \cdot \hat{a_\rho} - z \cdot \hat{a_z}}$$

$x_2 = x_3 - x_1$

$(x_1 + x_2 = x_3)$

$x_1 = z \cdot \hat{a_z}$

$x_3 = \rho \cdot a_\rho$

$$dH = \frac{I\, dl \times R}{4\pi R^2} \qquad \boxed{\begin{aligned} dl &= dz \cdot a_z \\ R &= \rho a_\rho - z a_z \end{aligned}}$$

$$dl \times R = dz \cdot a_z \times (\rho \cdot a_\rho - z \cdot a_z)$$

$$= \rho\, dz \cdot a_z \times a_\rho$$

$$= \rho\, dz \cdot a_\phi$$

$\boxed{\text{angle b/w } a_z \text{ \& } a_\rho \text{ is } \phi}$

$$|R| = \sqrt{\rho^2 + z^2} \qquad [R = \rho \cdot a_\rho - z \cdot a_z]$$

$$\left[dH = \frac{I \rho\, dz \cdot \hat{a_\phi}}{4\pi R^2} \right]$$

$$dH = \frac{I\, dl \times R}{4\pi R^3} = \frac{I \rho\, dz \cdot \hat{a_\phi}}{4\pi (\rho^2 + z^2)^{3/2}}$$

$$z = \rho \cot \alpha$$

$$dH = \frac{I \rho\, dz \cdot \hat{a_\phi}}{4\pi (\rho^2 + z^2)^{3/2}}$$

$z = \rho \cot \alpha$

$dz = -\rho \operatorname{cosec}^2 \alpha \cdot d\alpha$

$$dH = \frac{-I\rho\rho \operatorname{cosec}^2 \alpha \cdot \hat{a_\phi}}{4\pi (\rho^2 + (\rho \cot \alpha)^2)^{3/2}}\, d\alpha$$

$$H = \int_{\alpha_1}^{\alpha_2} \frac{-I \rho^2 \operatorname{cosec}^2 \alpha\, a_\phi}{4\pi (\rho^2 + \rho^2 \cot^2 \alpha)^{3/2}}\, d\alpha$$

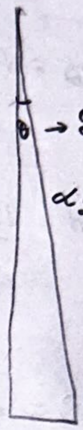
$$H = -\frac{I}{4\pi} \int_{\alpha_1}^{\alpha_2} \frac{\rho^2 \operatorname{cosec}^2 \alpha\, \hat{a_\phi}}{\rho^3 (1 + \cot^2 \alpha)^{3/2}} \cdot d\alpha$$

$$H = -\frac{I}{4\pi\rho}\,\hat{a}_\phi \int_{\alpha_1}^{\alpha_2} \frac{\operatorname{cosec}^2\alpha}{(1+\cot^2\alpha)^{3/2}}\,d\alpha$$

$$= \frac{-I}{4\pi\rho}\,a_\phi \int_{\alpha_1}^{\alpha_2} \sin\alpha\,d\alpha \qquad = \frac{\operatorname{cosec}^2\alpha}{\operatorname{cosec}^3\alpha} = \sin\alpha$$

$$= -\frac{I}{4\pi\rho}\,a_\phi\,\left[-\cos\alpha\right]_{\alpha_1}^{\alpha_2}$$

$$\boxed{H = \frac{+I}{4\pi\rho}\left[\cos\alpha_2 - \cos\alpha_1\right]\cdot\hat{a}_\phi}$$



special cases:

$$H = \frac{+I}{4\pi\rho}\left[\cos\alpha_2 - \cos\alpha_1\right]\hat{a}_\phi$$

i) Semi Infinite conductor

A (0, 0, 0) , B (0, 0, ∞)

$\alpha_1 = 90°,\ \alpha_2 = 0$

$$\boxed{H = \frac{I}{4\pi\rho}\,a_\phi}$$

ii) conductor Infinite

A (0, 0, −∞)

B (0, 0, ∞)

$\alpha_1 = 180°,\ \alpha_2 = 0°$

$$\boxed{H = \frac{+I}{2\pi\rho}\,a_\phi}$$

---

continuation

$u = \sin\theta$

$du = \cos\theta\,d\theta$

θ	$\pi/2$	0
$u = \sin\theta$	1	0

$$= -60 \times 2\pi \int_0^1 u.du$$

$$= -376.99\left[\frac{u^2}{2}\right]_0^1$$

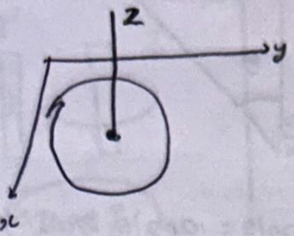
$$= -376.99\left[\frac{1}{2}\right]$$

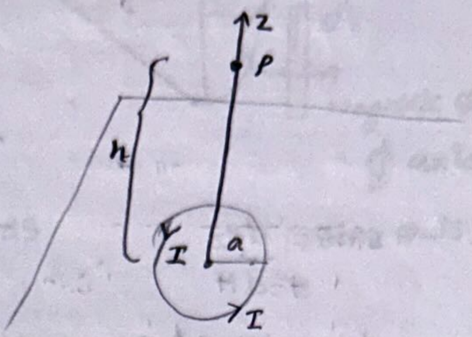
$I = -188.496\ A$

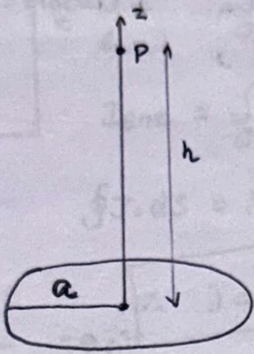
$I = -0.188\ KA$

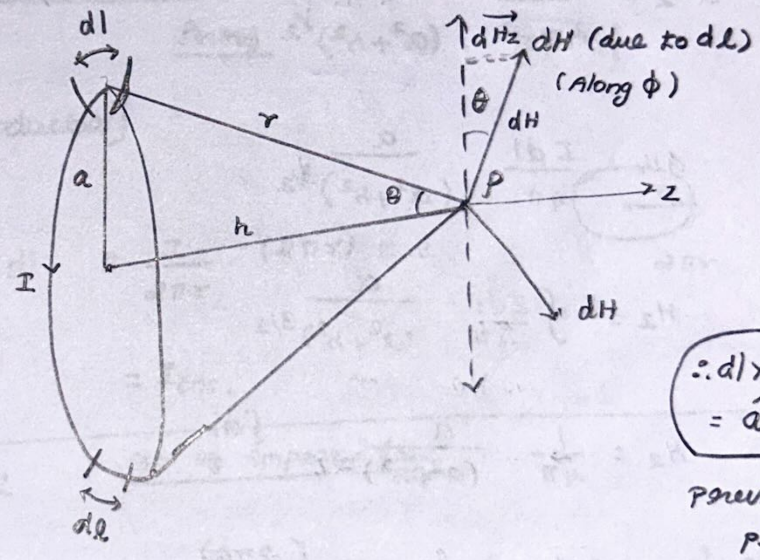
App. of Biot-savart's law

Develop an exp for magnetic field at any point on the line through the center at distance h from the center & $\perp^r$ to the plane of 0^r loop of radius a and current carrying 'I'.



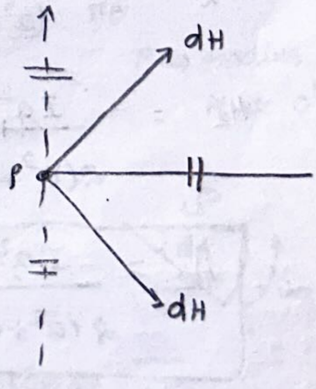






$\therefore dl \times R = \hat{a}_\phi (dl)$

Previous prob



$\therefore$ As vertical component gones Along horizontal alone.

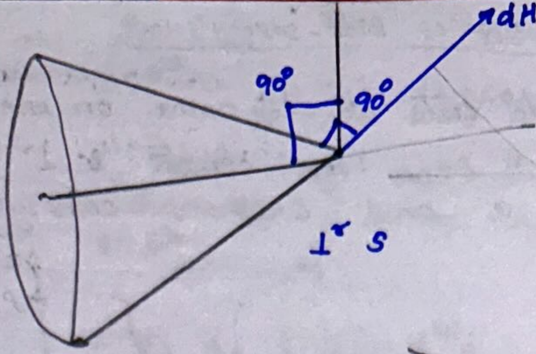
'Resolving two components'

$$dH = \frac{I\, dl \times a_R}{4\pi R^2}$$

$$|dH| = \frac{I\, dl}{4\pi R^2}$$

$\therefore$ y cancels each other.

(Angle b/w 2 lines = Angle b/w their $\perp^r$s)



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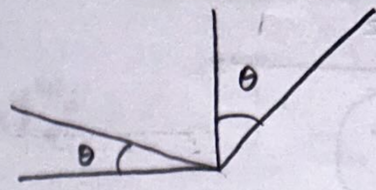
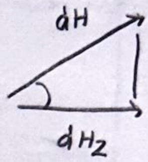


$\sin\theta = \frac{dH_z}{dH}$

"⊥r angle = angle b/w two lines"

$dH_z = dH \sin\theta$



$|dH| = \frac{I\,dl}{4\pi r^2}$

$|dH_z| = \frac{I\,dl}{4\pi r^2}\sin\theta$

$\sin\theta = \frac{a}{(a^2+h^2)^{1/2}} = \frac{a}{r}$

$dH_z = \frac{I\,dl}{4\pi r^2}\,\frac{a}{(a^2+h^2)^{1/2}}$

$\boxed{r = \sqrt{a^2+h^2}}$

$dH_z = \frac{I\,dl}{4\pi}\,\frac{a}{(a^2+h^2)^{3/2}}$

$H_z = \int \frac{I\,dl}{4\pi}\,\frac{a}{(a^2+h^2)^{3/2}}$

$H_z = \frac{1}{4\pi}\,\frac{a}{(a^2+h^2)^{3/2}}\int dl$

$l = 2\pi r$

$H_z = \frac{1}{4\pi}\,\frac{a}{(a^2+h^2)^{3/2}}\,(2\pi a)$

$H_z = \frac{I a^2}{2(a^2+h^2)^{3/2}}$

$$\boxed{H_z = \frac{I a^2}{2(a^2+h^2)^{3/2}}\,a_z}$$

If $h=0$ (At centre):

$H_z = \frac{I a^2}{(2a^2)^{3/2}}\,a_z = \frac{I}{2a}\,\hat{a}_z$

Ampere's law:

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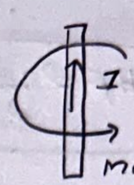
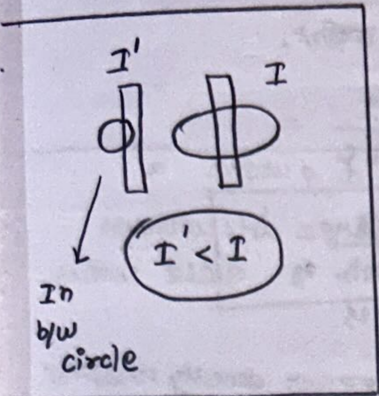
$\oint H \cdot dl = I_{enc}$ States that the line $\int$ of H around a closed path is equal to the net current enclosed by the path.





Stokes theorem:

$$\oint_{l} H \cdot dl = \int_{S} \nabla \times \vec{H} \; ds$$

$$I_{enc} = \int_{S} \nabla \times \vec{H} \; ds$$

$$\oint J \cdot ds = \oint \nabla \times \vec{H} \; ds$$

$$\boxed{\therefore \; J = \nabla \times \vec{H}}$$

[Point form of Gauss law $\nabla \cdot D = \rho_v$]

Differential form of Ampere's circuital law

Proof

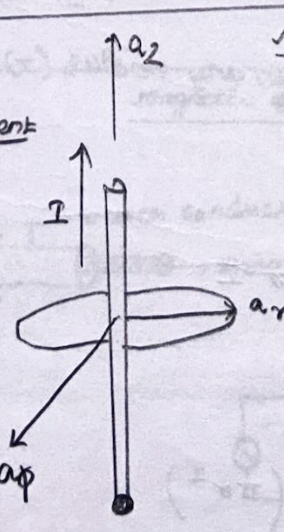
$H = \dfrac{I}{2\pi r}$ [∞ conductors]

$$\oint H \cdot dl = \int \frac{I}{2\pi r} \cdot dl = \frac{I}{2\pi r}(2\pi r)$$

$$= I_{enc}.$$

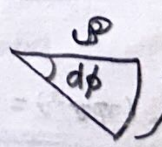
App of Amperes law

∞ line current



'Amperial path' : circle.

$\rho \rightarrow$ radius of that o^{le}



$H_\phi \cdot a_\phi \rightarrow$ vector along ϕ axis

(using Right hand thumb)

$$I = \oint H \cdot dl$$

$dl = \rho \, d\phi$ (Small part)

$$I = \oint H_\phi \cdot a_\phi \cdot dl$$

$$= \oint H_\phi \cdot a_\phi \cdot \rho d\phi \cdot a_\phi$$

$$= \oint H_\phi \, \rho \; d\phi$$

$$I = H_\phi \rho \int_0^{2\pi} d\phi$$

$$I = 2\pi \rho H_\phi$$

ρ - distance b/w conductor & point.

$$\boxed{H_\phi = \frac{I}{2\pi\rho} \cdot \hat{a}_\phi}$$

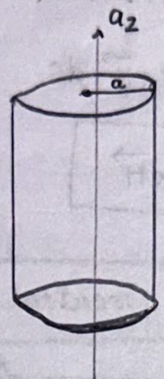
Der an exp for mag field intensity both inside & outside a solid cylindrical conductor of radius a carrying a current I with uniform density & sketch the variation of field intensity as a function of distance from the cond axis.

*current density uniform

solu:

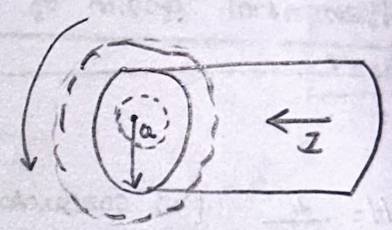
★ H = ?

★ sketch the variation of field intensity.



$\boxed{H = ?}$ $\quad H = \frac{I}{2\pi\rho} \cdot \hat{a}_\phi$

$\rho > a$



(Inside & outside the conductor)

Inside the conductor:

$J \rightarrow$ uniform current density $\quad r < a$

$I \rightarrow$ Total current

$I' \rightarrow$ current enclosed within any radius (r)

$$J = \frac{I}{A}$$

$$\boxed{J = \frac{I}{\pi a^2}} \longrightarrow \text{For } I$$

$$J \cdot (\pi r^2) = I'$$

$$\boxed{I' = \frac{J}{\pi r^2}}$$

$$I' = \frac{I}{\pi a^2}(\pi r^2)$$

$$\boxed{I' = I\left(\frac{r}{a}\right)^2}$$

$$H' = \frac{I'}{2\pi r}.$$

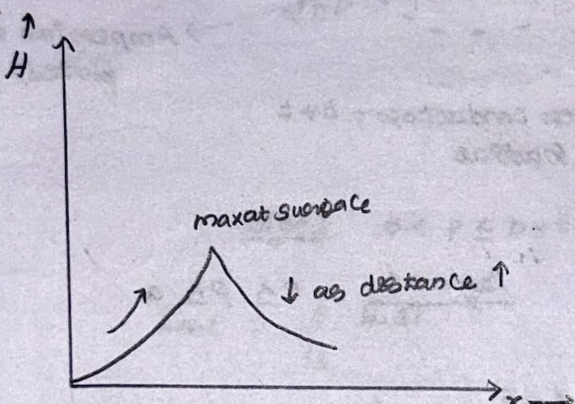
$$= \frac{I\left(\frac{r^2}{a^2}\right)}{2\pi r}$$

$$= \frac{I}{2\pi}\frac{r}{a^2}$$

$$\boxed{H' = \frac{Ir}{2\pi a^2}}$$

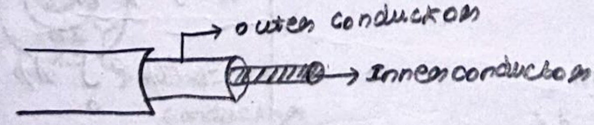
At the surface $r = a$

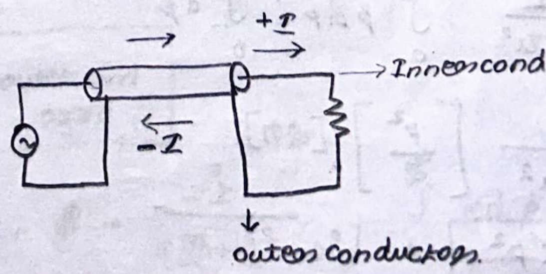
$$\boxed{H' = \frac{I}{2\pi a}}$$

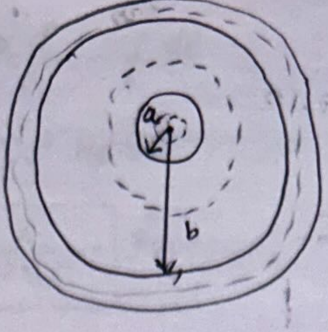


$H = \frac{I}{2\pi \rho}$ (out), $H = \frac{Ir}{2\pi a^2}$ ins, $H' = \frac{I}{2\pi a}$ (boundary)

Magnetic field intensity of a inf long coaxial transmission line







$0 \le \rho \le a$

$a < \rho \le b$

$b < \rho \le b + t$

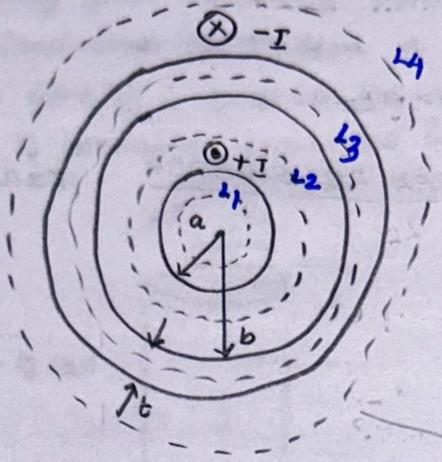
$\rho > b + t$

Inner conductor

Insulator

outer cond

(Three amperical paths in each layers)



4 layers

$+I \rightarrow$ towards us

$-I \rightarrow$ Away

$\rightarrow$ Amperian paths (dotted)

outer conductor = b+t radius

<u>H at 4 layers:</u>

<u>Region 1</u> $\quad 0 \le \rho \le a$

$$I_{enc} = \oint H \cdot dl = \oint J \cdot ds$$

$$ds = \rho \, d\phi \, d\rho \, a_z \quad [\text{Along z direction}]$$

$$\boxed{J = \frac{I}{A}}$$

$$I_{enc} = \oint J \cdot ds$$

$$I_{enc} = \oint \frac{I}{\pi a^2} a_z \quad \rho \, d\phi \, d\rho \, a_z$$

$$= \oint \frac{I}{\pi a^2} \rho \, d\phi \, d\rho$$

$$= \frac{I}{\pi a^2} \int_0^{\rho} \rho \, d\rho \int_0^{2\pi} d\phi$$

$$I_{enc} = \frac{I}{\pi a^2} \left[\frac{\rho^2}{2}\right] [2\pi]$$

$$\boxed{I_{enc} = \frac{I \rho^2}{a^2}}$$

$$I = \oint H_\phi \, a_\phi \, \rho \, d\phi \, a_\phi$$

$$= H_\phi \, \rho \int_0^{2\pi} d\phi$$

$$= H_\phi (2\pi \rho)$$

$$I_{enc} = \oint_{L_1} H \cdot dl \qquad \int dl = 2\pi\rho$$

$$I_{enc} = H_\phi (2\pi \rho)$$

$I_{enc} = H_\phi (2\pi\rho)$ $\qquad$ $I_{enc} = \frac{I\rho^2}{a^2}$

$$\frac{I\rho^2}{a^2} = H_\phi (2\pi\rho)$$

$$H_\phi = \frac{I\rho^2}{(2\pi\rho)a^2}$$

$$\boxed{H_\phi = \frac{I\rho}{2\pi a^2}}$$

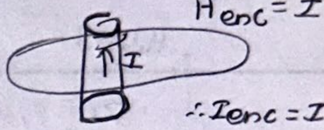
<u>Reign 2</u> $\quad a < \rho \leq b$

$$I_{enc} = \oint_{L_2} H \cdot dl$$

$$I_{enc} = H_\phi (2\pi\rho)$$

$$H = \frac{I}{2\pi\rho}$$

$\boxed{I_{enc} = I}$



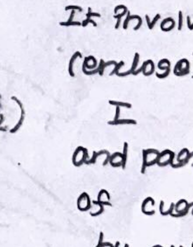
$\therefore I_{enc} = I$

Innercond

<u>Reign 3</u> $\quad b < \rho \leq b+t$

$$I_{enc} = \oint_{L_3} H \cdot dl$$

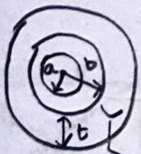
$$I_{enc} = I + \oint J \cdot ds = I' \longrightarrow$$



It involves (enclose) I and part of current by outer conductor.

$$\boxed{J = -\frac{I}{\pi[(b+t)^2 - b^2]} a_z}$$

$$I_{enc} = I + \oint -\frac{I}{\pi[(b+t)^2 - b^2]} a_z (\rho d\phi \, d\rho \, a_z)$$



alone outer conductor

$\boxed{\therefore \text{outer } \odot^{le} - \text{Inner } \odot^{le} = \text{outer cond area}}$

$$I_{enc} = I + \oint - \frac{I}{\pi[(b+t)^2 - b^2]} \int_b^\rho \rho \, d\rho \int_0^{2\pi} d\phi$$

$$= I - \frac{I}{\pi[(b+t)^2 - b^2)} \left[\frac{\rho^2 - b^2}{2}\right] [2\pi]$$

$$= I - \frac{I(\rho^2-b^2)}{[(b+t)^2-b^2]}$$

$\therefore -I$ in outer conductor

$$I_{enc} = H_\phi (2\pi\rho)$$

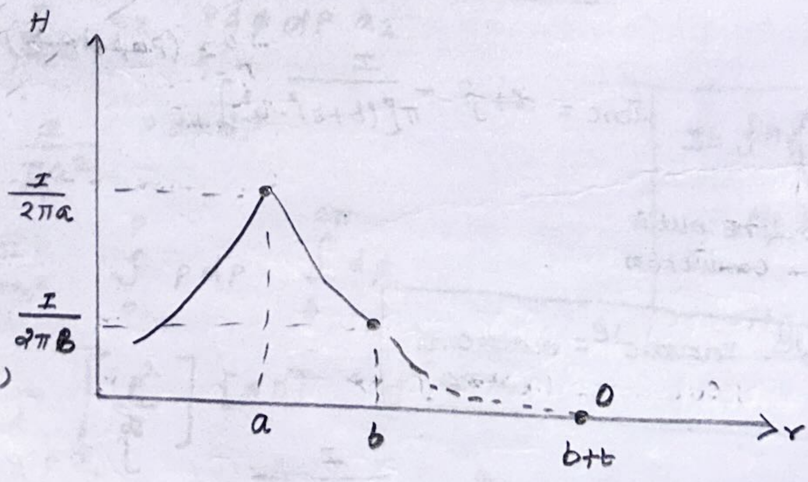
$$H_\phi (2\pi\rho) = I\left[1 - \frac{(\rho^2-b^2)}{[(b+t)^2-b^2]}\right]$$

$$H_\phi = \frac{I}{2\pi\rho}\left[1 - \frac{(\rho^2-b^2)}{[(b+t)^2-b^2]}\right]$$

Region 4

$H_\phi = 0$ [$\therefore$ Inner conductor $= +I$
outer conductor $= -I$]

$$H = \begin{cases} \dfrac{I\rho}{2\pi a^2} & 0 \le \rho \le a \\[2ex] \dfrac{I}{2\pi\rho} & a < \rho \le b \\[2ex] \dfrac{I}{2\pi\rho}\left[1 - \dfrac{(\rho^2-b^2)}{[(b+t)^2-b^2]}\right] & b < \rho \le b+t \\[2ex] 0 & \rho > b+t \end{cases}$$



$\rho \to a$ (Increase)

$\rho = a$ (maximum)

Law of no magnetic poles (Gauss theorem)

$\int D \cdot ds = Q$. [Divergence]

Source Sink.

$\therefore$ divergence = 0 [In magnetism] → flux lines starts from N & ends in south

B [magnetic field density]

$$\boxed{B = \mu H}$$

'No. of flux lines Eminated from North' = South → Resultant = 0.

$$\boxed{\therefore \nabla \cdot B = 0}$$

↓

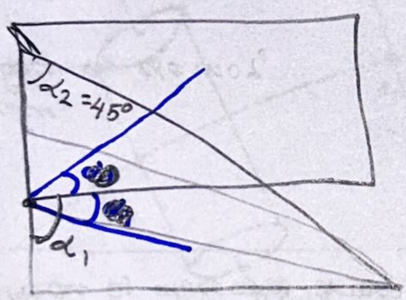
Gauss theorem of magnetism.

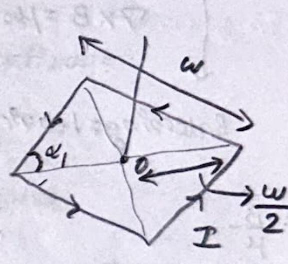
Find mag flux density at the centre of a planar square loop with side carrying a direct current 'I'.

Soln:

Finite conductor = 4

$B = \mu H$

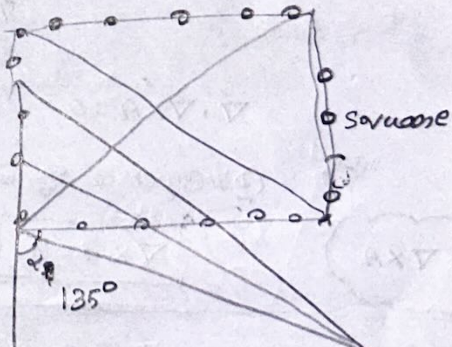


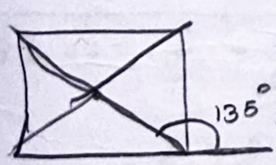


$\alpha_1 = 45°$

$\alpha_2 = 45°$

$\alpha_1 = ? = 135°$.





α_2 – outer angle

α_1 – Inner angle

$$H = \frac{+I}{4\pi \rho}[\cos\alpha_2 - \cos\alpha_1]\, a_\phi$$

$$= \frac{I}{4\pi\rho}\left[\frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}}\right)\right]$$

$$= \frac{I}{4\pi\rho}\left[\frac{2}{\sqrt{2}}\right] = \frac{I}{4\pi \frac{w}{2}}\sqrt{2} = \frac{I}{2\pi w}\sqrt{2}$$

$$\boxed{H = \frac{I}{\sqrt{2}\,\pi w}}$$

Superposition:

$$H_t = \frac{4I}{\sqrt{2}\,\pi w} = \frac{2\sqrt{2}\,I}{\pi w}$$

$B = \mu I$

$$\boxed{B = \frac{2\sqrt{2}\,I\mu}{\pi w}}$$

3/05/2021

vector mag potential

Postulates of magnetostatics in non magnetic field.

$\nabla \cdot B = 0 \rightarrow$ Diff form [Gauss] $\qquad \oint B \cdot ds = 0 \rightarrow$ Int form

$\nabla \times B = \mu_0 J \qquad \oint B \cdot dl = \mu_0 I$

Entering = Leaving

$H = \frac{B}{\mu}$

$\boxed{B = H\mu}$

'Low on no magnetic materials'

$\nabla \cdot B = 0 \rightarrow$ Gauss theorem of magnetostatics

$\nabla \cdot \nabla \times A = 0$

(Divergence of curl of a vector = 0)

$B = \nabla \times A \qquad \boxed{\nabla \times A = B}$

$\nabla \times B = \mu_0 J$

$\nabla \times \nabla \times A = \mu_0 J$

$\nabla \times \nabla \times A = \nabla(\nabla \cdot A) - \nabla^2 A \longrightarrow$ poisson & Laplace

$\nabla \times \nabla \times A = \nabla(\nabla \cdot A) - \nabla^2 A \qquad$ [Assume $\nabla \cdot A = 0$]

$\mu_0 J = -\nabla^2 A$

[In electrostatics $\nabla^2 V = -\frac{\rho_v}{\varepsilon_0}$]

$\boxed{V = -\frac{1}{4\pi\varepsilon_0}\int \frac{\rho_v}{R}\,dv} \longrightarrow$ Electrostatics

$A = \frac{-\mu_0}{4\pi}\int_v \frac{J}{R}\,dv \longrightarrow J \rightarrow$ volume current density

In magnetostatics

$A \rightarrow$ vector magnetic potential

To find vector magnetic potential A from the volume current density J.

'once we know J — we know A'

$\int_S B \cdot ds = \phi_m$ → Not a closed surface. (open surface)

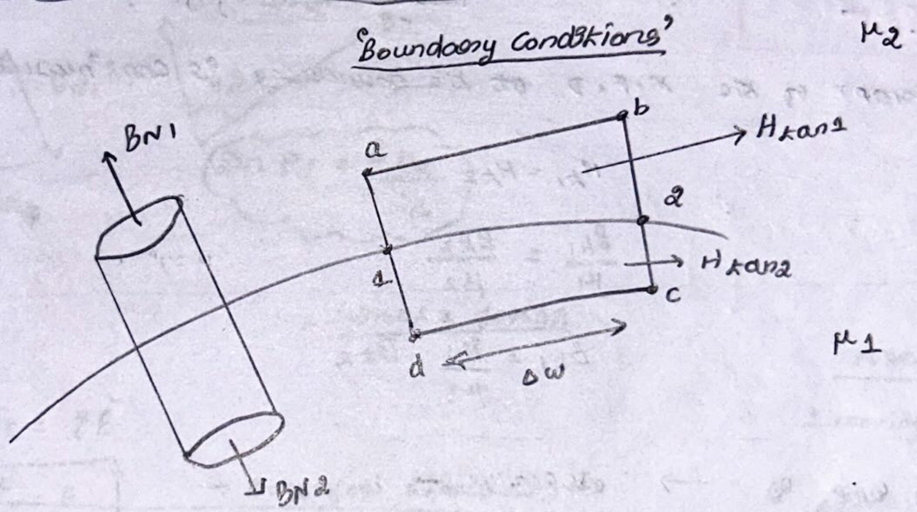
[stokes theorem]

$\phi_m = \int_S B \cdot ds = \int_S \nabla \times A \cdot ds = \oint_C A \cdot dl$ → closed line.

Physical significance of A:

* The closed line integral of vector magnetic potential is the total mag flux passing through the area enclosed by the path.

'Boundary Conditions'



Normal Component:

$\oint_S B \cdot ds = 0$

$\oint B \cdot ds = \oint_T + \oint_B + \oint_C$ $\qquad \oint_{Lateral} = 0$

$= \oint_T + \oint_B = 0$

$= B_{N_1} \Delta S - B_{N_2} \Delta S = 0$ [opp. direction]

$B_{N1} - B_{N2} = 0$

$B_{N1} = B_{N2}$

Normal Comp. of mag flux density at the boundary is continuous

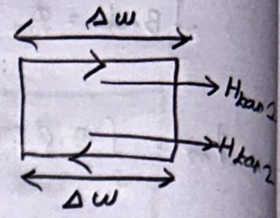
$\mu_1 H_{N_1} = \mu_2 H_{N2}$ $\qquad \boxed{B = \mu H}$

$$H_{N1} = \frac{\mu_2}{\mu_1} H_{N2}$$

Normal comp. of mag. f. d at the boundary is discontinuous.

Tangential Component → line integral

$$\oint_l H.dl = I = \int_a^b + \int_b^c + \int_c^d + \int_d^a$$



$$= H_{tan1}\,\Delta w - H_{tan2}\,\Delta w = K\,\Delta w$$

$$H_{t1} - H_{t2} = K_s$$

KΔw → line current density

"Boundary free of current" - Non magnetic media

$$H_{t1} - H_{t2} = 0$$

$$\boxed{H_{t1} = H_{t2}}$$

Tang comp of the M.F.D at the boundary is continuous.

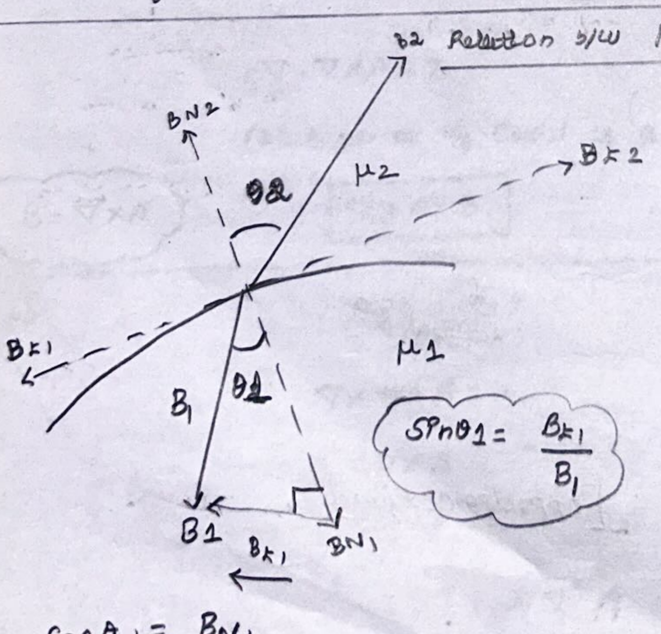
$$H_{t1} = H_{t2}$$

$$\frac{B_{t1}}{\mu_1} = \frac{B_{t2}}{\mu_2}$$

$$B_{t1} = \frac{\mu_1}{\mu_2} B_{t2}$$

Tang comp of → discontinuous

Relation b/w permeability & angles



$$\sin\theta_1 = \frac{B_{t1}}{B_1}$$

$$\cos\theta_1 = \frac{B_{N1}}{B_1}$$

$$\boxed{B_{N1} = B_1 \cos\theta_1}$$

$$\sin\theta_1 = \frac{B_{t1}}{B_1}$$

$$\boxed{B_{t1} = B_1 \sin\theta_1}$$

$$\sin\theta_2 = \frac{B_{t2}}{B_2}$$

$$\boxed{B_{t2} = B_2 \sin\theta_2}$$

$$\cos\theta_2 = \frac{B_{N2}}{B_2}$$

$$\boxed{B_{N2} = B_2 \cos\theta_2}$$

$\mu_2 B_{t1} = \mu_1 B_{t2}$

$\mu_2 \, B_1 \sin\theta_1 = \mu_1 B_2 \sin\theta_2$

$$\frac{B_1}{B_2} = \frac{\mu_1}{\mu_2} \; \frac{\sin\theta_2}{\sin\theta_1}$$

$$\boxed{\frac{B_{t1}}{\mu_1} = \frac{B_{t2}}{\mu_2}}$$

Boundary cond.

$$\boxed{B_1 \cos\theta_1 = B_2 \cos\theta_2} \rightarrow ①$$

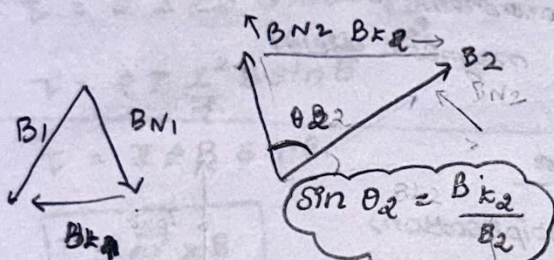
$$\boxed{B_{N1} = B_{N2}} \rightarrow ②$$

$$\boxed{B_1 \cos\theta_1 = B_2 \cos\theta_2}$$

$$\frac{\mu_2 \, B_1 \sin\theta}{B_1 \cos\theta_1} = \frac{\mu_1 \, B_2 \sin\theta_2}{B_2 \cos\theta_2}$$

① ÷ 2

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}} \rightarrow \text{Finally.}$$



Lorentz force.

$F_e = qE$

$$\boxed{\frac{F}{q} = E}$$ → Both for stationary as well as mobile charges.

Force

1. moving charge in mag field
2. current element in external M.F
3. B/w current elements.

(Not stationary)

$F_{moving} = q\,u \times B$

u ↓ velocity of moving charged particle

B → Magnetic flux

$$\boxed{F = F_e + F_m}$$ → when both are there.

$= qE + q(u \times B)$

$F = q[E + u \times B]$

	E	B	Combined effect (E & B)
Stationary	qE	–	qE
moving	qE	$q(u \times B)$	$q(E + u \times B)$

"only for a moving charge - magnetic field is experienced"

"Force in a current element"

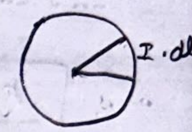
$(I \cdot dl) \rightarrow$ current element

$$I \cdot dl = \frac{dq}{dt} \, dl$$

$$= dq \cdot \frac{dl}{dt} = dq \cdot u$$

A charge q moving with velocity u is equal to the current element

$F_m = q(u \times B)$



$dF_m = dq \, (u \times B)$

$dF_m = I \cdot (dl \times B)$

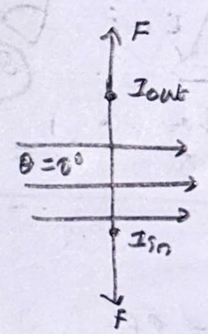
$\boxed{F_m = \oint I \cdot dl \times B} \rightarrow$ Force on a current element - (due to magnetic field).

Torque

τ (Torque) = $d \times F$ (scalar multiplication)

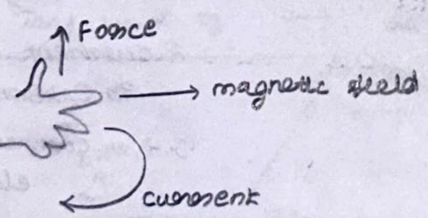
$d \rightarrow$ Length of Arm

$F \rightarrow$ Force acting on the direction of rotation.



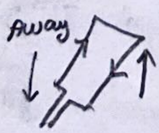
$\tau = 0$

$(\theta = 0)$

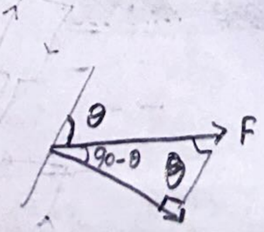


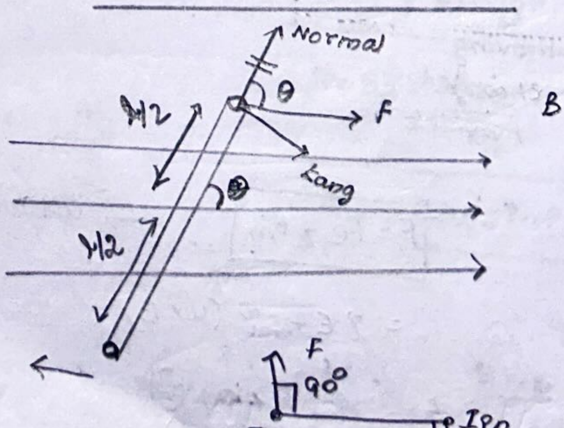
[Coil moves upwards]

Expression for torque

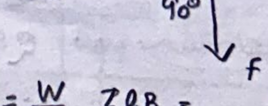


Away → Force downwards

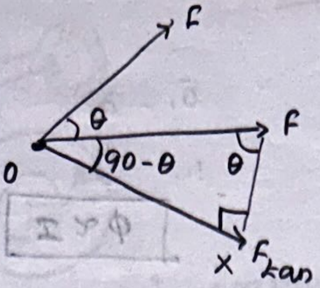




∴ $F_t \rightarrow$ is going to pull. (horizontal motion)



$$F = \frac{W}{2} \, IlB$$



$\sin\theta = \frac{Ox}{F}$

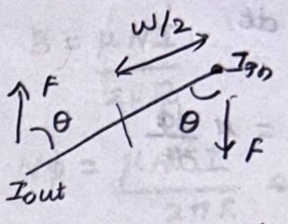
$Ox = F\sin\theta$

Total force: $F_{net} = 2F_{tan} = 2F\sin\theta$

$= 2 I l B \sin\theta$

$\boxed{F_m = \oint I \cdot dl \times B} \to$ current element

$\tau = F \times d$ (from pivot $= l/2$)



$\tau = 2 I l B \sin\theta \times \frac{l}{2}$

$\tau = \frac{w}{2} I (B\sin\theta)$

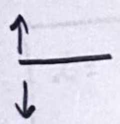
$\tau = 2 I \frac{l^2}{2} B \sin\theta$

$\boxed{\therefore l^2 = A}$

$\tau = I A B \sin\theta$

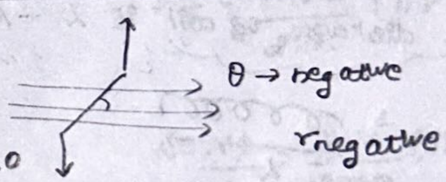
$m = IA$

$\boxed{\tau = \vec{m} \times B}$



net torque = 0

net torque $\neq 0$



EM Induction

* tendency of electric current to flow in a conductor when moved in a magnetic field. (oersted exp)

* Faraday's laws of EMI
whenever there is a change in magnetic flux an induced emf is generated.

Faraday's law: II law:

The magnitude of induced emf depends upon the rate of change of flux w.r.t time.

$\mathcal{E} \propto \frac{\Delta\phi}{\Delta t}$

$\phi = B \cdot A$

Lenz's law: opposes the cause

$$\boxed{e \propto -\frac{d\phi}{dt}}$$

Inductance:

$L = \frac{\text{total magnetic flux linkages}}{\text{current through the coil.}}$

$$\boxed{\phi \propto I}$$

$\phi = LI$
↓
Self Inductance

$$\boxed{V \propto \frac{di}{dt}}$$

$V = L\frac{di}{dt}$

∴ $V = N\frac{d\phi}{dt}$ [Faraday's law]

$L\frac{di}{dt} = N\frac{d\phi}{dt}$

∴ As numbers of turns ↓, emf ind ↓

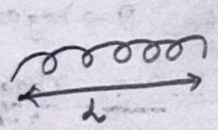
$N\frac{d\phi}{di} = L$

$$\boxed{N\frac{\phi}{I} = L}$$

$$\boxed{L = N\frac{\phi}{i}}$$

Inductance of a Solenoid - cylindrical arrange with n turns.

If distance of coil is L - Area (A)



$N\phi = NBA$

$L = N\frac{\phi}{i}$

$L = \frac{NBA}{i}$

$\boxed{\oint H\cdot dl = I}$ → we are taking one turn
↓
total turns

∴ $H\cdot l = NI$

$H = \frac{NI}{l}$

$$\boxed{B = \mu H}$$

∴ $B = \frac{\mu NI\ A\cdot N}{l}$

$L = \frac{NBA}{I}$

$= \frac{N\mu NIA}{Il} = \frac{N^2\mu A}{l}$

$\boxed{L = \frac{N^2\mu A}{l}}$ → cylindrical.