18/02/2021

Electromagnetic fields

Transducer (Antenna): one form of Energy into another form (converts)

Transmitting end: Electrical → Electromagnetic

Receiving end: EM → Electrical (Electromagnetic induction)

Ampere's circuit law

(steady)

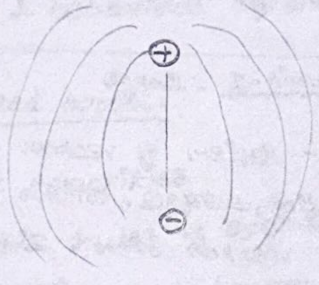
* A current carrying conductor - produces magnetic field.
* Faraday - Charging current - produces EMF.

Maxwell

An oscillating Electric field produces oscillating magnetic field.

(keep changing fields)

Dipole



Dipole

wave: Energy that moves not molecules (Disturbance that keep the Energy moving)

Electromagnetic field

* vibrations b/w an Electric field & a magnetic field

(Right angles to each other)

Em Spectrum - Range of frequencies

(Based on f or λ) (Through frequencies)

As λ decreases, frequency increases

Radio → Gamma waves

Radio waves - Broadcast

microwave - Radar, Cooking, Telephone

Infrared - heat from sun, fires, radiators

visible light - make things seen

UV - Absorbed by skin (Fluorescent lamp)

X-rays - (view bones)

Gamma - medicine (killing cancer cells)

Em wave: propagate even in vacuum.

Infrared radiation: (Pain relieving therapy)

why this sem?

Electrostatic - stationary

* RF comm
* microwave Engg
* Antennas
* Electrical machines
* Satellite Comm, Atomic & nuclear research
* EMI EMC, VLSI, quantum electronics

Unit-I - Intro

* Em model - units & constants - Review of vector algebra - Rect, cylindrical, spherical coordinate systems - line, surface, volume integrals, gradient of a scalar field, Divergence of a vector field, Divergence theorem, curl of a vector field, Strokes theorem, Null identities, Helmholtz's theorem.

Unit-II - Electrostatics

EF, Coloumb's law, Gauss law & app, Electric potential - conductors in static EF, Dielectrics in S.E.F - Electric flux density - Dielectric constant, Boundary Condt - capacitance, parallel plate, Coaxial & Spherical capacitors - Electrostatic energy - poisson's eqn, laplace's eqn, uniqueness of Electrostatic sol - current density & ohm's law - Electromotive force, KVL - Eqn of continuity - KCL

Unit-III - magnetostatics

Lorentz force eqn, law of no magnetic monopoles - Ampere's law, vector-magnetic potential - Biot-Savart law & app - MF intensity & Idea of relative permeability - magnetic ckts, Behaviour of magnetic material - Boundary conditions, Inductance & Inductors, mag energy, mag forces & torques.

Unit-IV - Time varying fields & maxwell's eqn

Faraday's laws, Displacement current & maxwell's ampere law - max eqn -

Potential functions - EM boundary condt - wave equ & solutions - Time harmonic fields.

unit - V - Plane EM waves

Plane waves in loss less, lossy media (low-loss dielectrics & good conductors) - group velocity - EM power flow & poynting vector - Normal incidence at a plane conducting boundary - Normal incidence at a plane dielectric boundary.

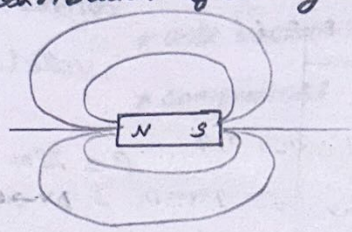
19/02/2021

Introduction

EM model:

* Study of Electric & magnetic phenomen caused by charges either in rest or motion.

Field: 'spatial distribution of charges' (quantity)



'Structure & behaviour of a system - model'

Essential steps - Idealized model:

1) Define basic quantities
2) specify the rules of operation. (vector algebra, PDE, calculus)
3) postulates some fundamental relations (Deductive approach)

Deductive approach:

* start with fundamental postulates
* Derive particular laws & theorems

Example

Theory: 'Low cost airlines always have delays'

Hypothesis:

'Low cost airlines - Always experience delays'

* Analyze the data (results): 5 out of 100 of low-cost airlines are not delayed = Reject hypothesis.

1) quantity

↙ Field ↘ Source

Field quantities

Electric → E. Field intensity (E) → V/m

→ E. flux density (D) → C/m^2

magnetic → M.F. Intensity (H) → T

↳ M.F. Density (B) → A/m

Length → meter → m

mass → kg → Kilogram

Time → second → s

current → ampere - A

'mksA - system'

Universal constants

* velocity of EM wave (C)
* permittivity of free space (ε_0)
* permeability " (μ_0)

$C = \sim 3\times10^8$ (m/s)

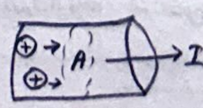
$\mu_0 = 4\pi\times10^{-7}$ (H/m)

$\varepsilon_0 = \sim (1/36\pi)\times10^{-19}$ (F/m)

$= 8.854\times10^{-12}$ (F/m)

$\mu_0 H = B$

Current density.



* Electric charge - (+ve) (or) (-ve)

Integral multiple of e^-

$\boxed{e^- = 1.6\times10^{-19}\,C}$

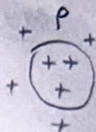
* volume charge density

(spatially distributed)

- Random (or) line or some order.

(charges distributed)

$\rho = \frac{\text{charge}}{\text{volume}}$ = Volume charge den



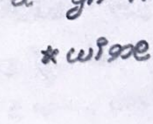
unit (SI) = $\frac{C}{m^3}$

$\rho = \lim_{\Delta V\to 0} \frac{\Delta q}{\Delta V}$ (C/m^3)

* Surface charge density

* determines charge in the given area.

$\rho_s = \lim_{\Delta S\to 0} \frac{\Delta q}{\Delta S}$ (C/m^2)



Surface charge

* Line charge density

(Present in a gn line)

* wire.

$\rho_l = \lim_{\Delta l\to 0} \frac{\Delta q}{\Delta l}$ (C/m)

Current

* Rate of change of charge w.r.t time

$I = \frac{dq}{dt}$ (C/s (or) A)

Current density

$J = \frac{I}{A}$ (current per Area)

A/m^2

Amount of current flow through a unit area normal to the direction of current flow

2) Rules of operation

vector algebra:

Scalar - magnitude (Length)
vector - Magnitude & direction. (Force)

Electrostatic potential, Electric flux, magnetic flux, Electric charge - Scalar

Electric field intensity $= \frac{\vec{F}}{q} = E$

20/2/2020.

Direction $\longrightarrow$

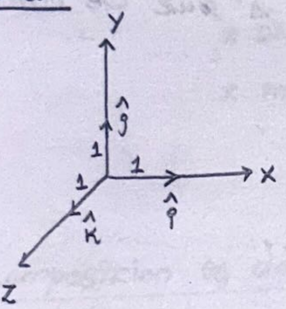
$(\longleftrightarrow)$ magnitude

} vector.

Representation of a vector

* unit vector $\rightarrow$ magnitude = 1 (Acts along the vector taken into consideration)
* components of a vector

unit vector:



$\rightarrow$ we can represent any vectors by unit vectors.

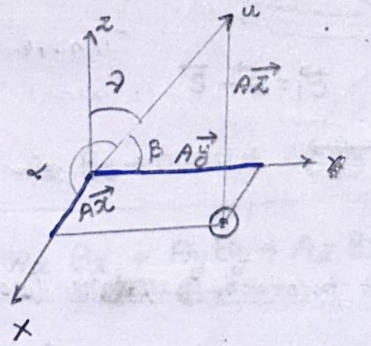
5 = magnitude along $x \longrightarrow 5\hat{i} = a$

$\hat{i}$ —
$\hat{j}$ —
$\hat{k}$ —
} unit vectors along x, y, z axes.

$\hat{i} = \frac{\vec{A}}{5}$

$\hat{i} = \frac{\vec{A}}{|\vec{A}|}$

Components of a vector



$\vec{ut} = 6\vec{i} + 7\vec{j} + 10\vec{k}$

$\vec{u}$ can be written in terms of components of axes.

$\vec{S} = a\hat{i} + b\hat{j} + c\hat{k} = a\vec{i} + b\vec{j} + c\vec{k}$

$\vec{a_s} \rightarrow$ unit vector along $\vec{S}$ (suffix)

$\vec{A} = \vec{A_x} + \vec{A_y} + \vec{A_z}$

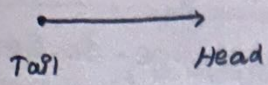
$= A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$

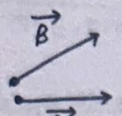
$$\vec{a_s} = \frac{\vec{S}}{|\vec{S}|} = \frac{a\vec{i} + b\vec{j} + c\vec{k}}{\sqrt{a^2+b^2+c^2}}$$

vector operations

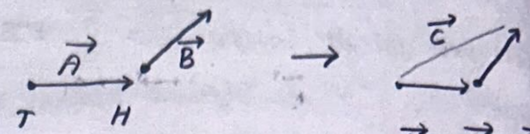
1) Addition → Head tail / Triangle rule
→ parallelogram rule

Head tail rule:



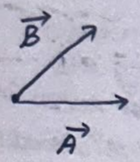


$\vec{A} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$

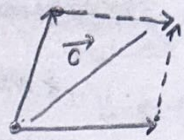


$\vec{C} = \vec{A} + \vec{B}$

Parallelogram rule:

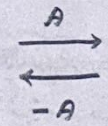


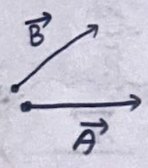
→ Draw parallel line →



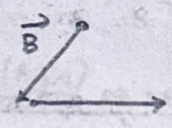
'By Δ^r rule we can find $\vec{C}$'

2) subtraction



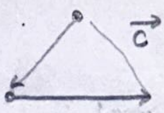


→



changing direction of $\vec{B}$

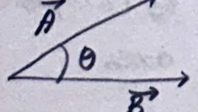
↓



$\vec{C} = \vec{A} - \vec{B}$

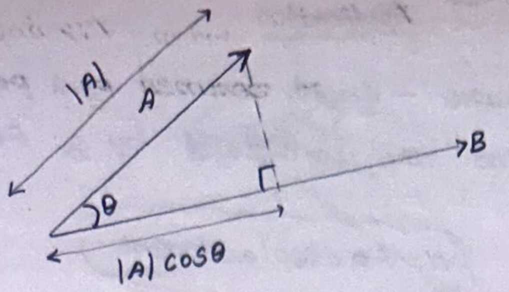
Dot product of vectors (scalar product)

'Result is a scalar'



$\vec{A}\cdot\vec{B} = |A||B|\cos\theta.$

Product of magnitudes of the two vectors & the cosine of the angle b/w them



$$\vec{A}\cdot\vec{B} = |A||B|\cos\theta$$

$\cos\theta = \frac{adj}{hyp}$

adj.s = $\cos\theta \cdot hyp$

∴ hyp side = $|A|$

adj.s = $|A| \cdot \cos\theta$.

"magnitude of $\vec{B}$ and the projection of $\vec{A}$ on to $\vec{B}$ or vice versa"

Properties of dot product

* Commutative $\quad \vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$
* Distributive $\quad (\vec{A}+\vec{B})\cdot\vec{C} = \vec{A}\cdot\vec{C} + \vec{B}\cdot\vec{C}$
* multiplication with a scalar

$$\vec{C}\cdot(\vec{A}\cdot\vec{B}) = \vec{A}\cdot(\vec{C}\cdot\vec{B})$$

vector decomposition of dot product:

$$\vec{A} = A_x\,\vec{a_x} + A_y\,\vec{a_y} + A_z\,\vec{a_z}$$

$$\vec{B} = B_x\,\vec{a_x} + B_y\,\vec{a_y} + B_z\,\vec{a_z}$$

$$\vec{A}\cdot\vec{B} = (A_x\,\vec{a_x} + A_y\,\vec{a_y} + A_z\,\vec{a_z})\cdot(B_x\,\vec{a_x} + B_y\,\vec{a_y} + B_z\,\vec{a_z})$$

$$= A_xB_x + 0 + 0 + A_yB_y + 0 + 0 + A_zB_z + 0 + 0$$

$$\boxed{\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z}$$

∴ $\cos 0 = 1$

$\cos 90^\circ = 0$

$a_x\,a_x = |a_x|\,|a_x|\cos 0$

$= (1\cdot 1)(1)$

$= 1$

$a_x\,a_y = |a_x||a_y|\cos 90^\circ$

$= (1)\cdot(1)(0)$

$= 0$

$$\underbrace{\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z}$$ $\longrightarrow$ Result (scalar).

scalars $\rightarrow A_x, B_x, A_y, B_y, A_z, B_z$

Application

* measure - angle formed by a pair of vectors
* To find the work done by a Force for a displacement done.

$$\boxed{W = F \cdot \text{displacement}}$$

* To cal- Electric potential b/w two points in the Electric field
* To calc the total Charge enclosed by a surface in Electric field.

Cross product of vectors

water bottle

Anticlockwise - open (come upwards +ve direction)
clockwise - close (downwards -ve direction)) screw drivers

$\boxed{\text{vector product of vectors}}$

$\vec{A} = -7\vec{a_x} + 12\vec{a_y} + 3\vec{a_z}$

$\vec{B} = 4\vec{a_x} - 2\vec{a_y} + 16\vec{a_z}$

Dot product & angle b/w the two

$\vec{a_x} \cdot \vec{a_x} = \vec{a_y} \cdot \vec{a_y} = \vec{a_z} \cdot \vec{a_z} = 1$

$= |a_x|\,|a_x| \cos 0$

$= (1)(1)(1) = 1.$

Solu:

$\vec{A} \cdot \vec{B} = (-7)(4) + (12)(-2) + 3(16)$

$= -28 - 24 + 48$

$= -4$

$|\vec{A}||\vec{B}| \cos\theta = \vec{A} \cdot \vec{B}$

$\cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}$ → 'orthogonal or $\perp^r$'
angle = 90°

$\cos\theta = \dfrac{-4}{\sqrt{202}\sqrt{276}}$

$\theta = \cos^{-1}\left(\dfrac{-4}{\sqrt{202}\sqrt{276}}\right) = \cos^{-1}\left(\dfrac{-4}{236.1186}\right) = 90.970$

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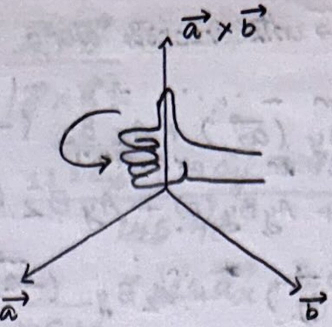


$$\boxed{|\vec{a}||\vec{b}| \sin\theta \cdot \hat{n} = \vec{a} \times \vec{b}}$$

$\hat{n} \rightarrow$ unit vector acting along the vector which is $\perp^r$ to the $\vec{a}$ and $\vec{b}$. (normal to the plane.

'Anti - commutative'

$\boxed{\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}} \longrightarrow \boxed{\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})}$ (opposite direction)

$\vec{a} \times \vec{b}$ $\quad$ $\vec{b} \times \vec{a}$

'Not associative'

'Distributive'

$\vec{A} = A_x \vec{a_x} + A_y \vec{a_y} + A_z \vec{a_z}$

$\vec{B} = B_x \vec{a_x} + B_y \vec{a_y} + B_z \vec{a_z}$

$\sin 0 = 0$

$\sin 90^\circ = 1$

$\vec{A} \times \vec{B} = ?$

'$x, y, z \rightarrow$ orthogonal to each other'

$\vec{A} \times \vec{B} = A_x B_x (\vec{a_x} \times \vec{a_x}) + A_y B_y (\vec{a_x} \times \vec{a_y}) + A_x B_z (\vec{a_x} \times \vec{a_z})$

$+ A_y B_x (\vec{a_y} \times \vec{a_x}) + A_y B_y (\vec{a_y} \times \vec{a_y}) + \cdots$

$+ A_z B_z (\vec{a_z} \times \vec{a_z})$

$$\boxed{\vec{a_x} \times \vec{a_x} = \vec{a_y} \times \vec{a_y} = \vec{a_z} \times \vec{a_z} = 0}$$

$\sin 0 = 0$

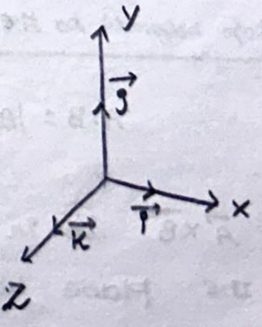
$|a_x||a_x| \cdot 0 = 0$

$\vec{a_x} \times \vec{a_y} = |a_x||a_y| \sin 90^\circ \cdot (\vec{a_z})$ unit vector

$= \vec{k}$ (or) $\vec{a_z}$

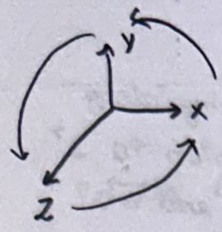
$\vec{a_x} \times \vec{a_z} = \vec{j}$ (or) $\vec{a_y}$

$a_y \times \vec{a_z} = \vec{i}$ (or) $\vec{a_x}$



$a_x, a_y, a_K \rightarrow$ unit vector along x, y and z.

$$\vec{A}\times\vec{B} = A_x B_x (0) + A_x B_y (\vec{a_z}) + A_x B_z (-\vec{a_y}) +$$
$$A_y B_x (-\vec{a_z}) + A_y B_y (0) + A_y B_z (\vec{a_x}) +$$
$$A_z B_x (\vec{a_y}) + A_z B_y (-\vec{a_x}) + A_z B_z (0)$$



$$= \vec{a_z}(A_x B_y - A_y B_x) + \vec{a_y}(-A_x B_z + A_z B_x)$$
$$+ \vec{a_x}(A_y B_z - A_z B_y)$$

$$\vec{A}\times\vec{B} = \vec{a_x}(A_y B_z - A_z B_y) + \vec{a_y}(A_z B_x - A_x B_z) + \vec{a_z}(A_x B_y - A_y B_x)$$

$$\begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_K} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \vec{a_x}(A_y B_z - A_z B_y) + \vec{a_y}(A_z B_x - A_x B_z) + \vec{a_z}(A_x B_y - A_y B_x)$$

<u>Applications</u>

$\vec{A} = 8\vec{a_x} + 3\vec{a_y} - 10\vec{a_z}$

$\vec{B} = -15\vec{a_x} + 6\vec{a_y} + 17\vec{a_z}$

$\vec{A}\times\vec{B}$, unit vector normal to the plane having the two vectors A and B.

<u>Solu</u>:

$$\begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ 8 & 3 & -10 \\ -15 & 6 & 17 \end{vmatrix} = \vec{a_x}(17\times 3 + 60) - \vec{a_y}(8\times 17 - 150) + \vec{a_z}(48 + 45)$$
$$= 111\,\vec{a_x} + 14\,\vec{a_y} + 93\,\vec{a_z} \quad \text{(or)}$$
$$= 111\,\hat{a}_x + 14\,\hat{a}_y + 93\,\hat{a}_z$$

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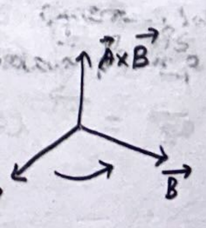
'A×B' ⟶ Result ⟶ vectors.

<u>unit vector normal to the plane:</u>

$$\vec{a_n} = \frac{\vec{a}}{|\vec{a}|} \quad \text{(unit vector)}$$

$$A\times B = |A||B| \sin\theta\, \hat{n}$$

∴ $\vec{A}\times\vec{B}$ is in perpendicular (normal) to the plane having the vectors $\vec{A}$ and $\vec{B}$.



$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

$$= \frac{111\vec{a_x} + 14\vec{a_y} + 93\vec{a_z}}{145.48}$$

$$= \frac{1}{145.48}(111\vec{a_x} + 14\vec{a_y} + 93\vec{a_z}) = 0.763\vec{a_x} + 0.096\vec{a_y} + 0.639\vec{a_z}$$

Two vectors $\vec{A} = 2\vec{i} + 2\vec{j}$

$\vec{B} = 3\vec{i} + 4\vec{j} - 2\vec{k}$

Find dot, cross, angle, $\vec{A} \times \vec{B}$ is at right angles to $\vec{A}$

solu:

$\vec{A} \cdot \vec{B} = 6 + 8 = 14$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & 0 \\ 3 & 4 & -2 \end{vmatrix} = \hat{a_x}(-4) - \hat{a_y}(-4) + \hat{a_z}(8-6)$$

$$= -4\hat{a_x} + 4\hat{a_y} + 2\hat{a_z}$$

$$\theta = \cos^{-1}\left(\frac{14}{\sqrt{8}\sqrt{29}}\right) = 23.19859^\circ$$

$$\hat{n} = \frac{-4\hat{a_x} + 4\hat{a_y} + 2\hat{a_z}}{\sqrt{36}} = -\frac{4}{6}\hat{a_x} + \frac{4}{6}\hat{a_y} + \frac{2}{6}\hat{a_z}$$

$$= -0.667\hat{a_x} + 0.667\hat{a_y} + 0.333\hat{a_z}.$$

---

* Torque about a point P
* Force experienced by a current carrying conductor placed in magnetic field
* Construct a vector $\perp^r$ to a plane if we have two vectors in a line

---

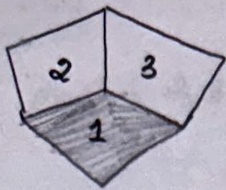
Coordinate system
- orthogonal (axes $\perp^r$ to each other)
- nonorthogonal

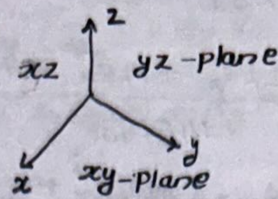
Google map - Longitude - Longitude → 'coordinate system'

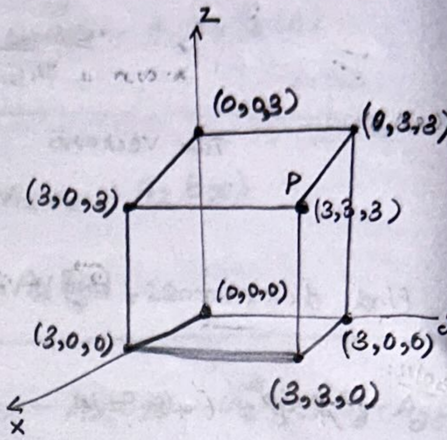
orthogonal → spherical, cartesian, cylindrical

* planes
* Differential elements
* relationships b/w systems.

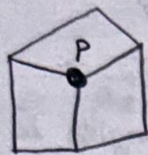
Cartesian Coordinate System



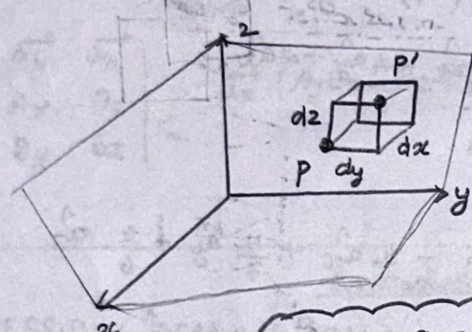


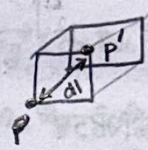


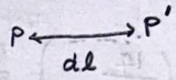
lifting xy, xz - yz (moving)



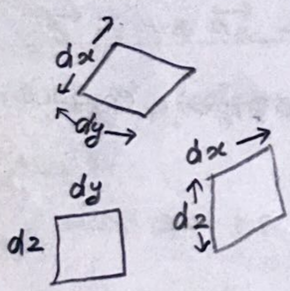
Differential - Very small change







Areas = xy plane, xz plane, yz plane



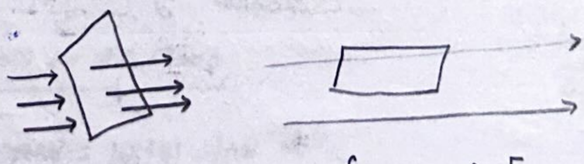
$$\left.\begin{array}{l} dx\,dy \\ dx\,dz \\ dy\,dz. \end{array}\right\}$$ Differential areas = DA

Differential volume = l×b×h

$$\boxed{DV = dx\,dy\,dz}$$

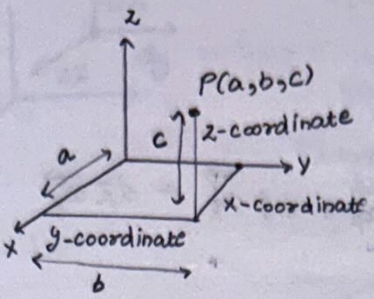
Area - scalar
(40m²)

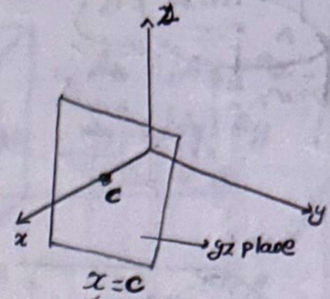
Area - vector - In Electromagnetics



'perpendicular' 'parallel' ['Based on the Area - the number of lines passing through will be determined]

'Direction as well as magnitude matters' — Area [vector] more area as well as $\perp^r$ direction → more no. of lines passing through.





23/02/2021

$$\vec{dl} = dx\,\vec{a_x} + dy\,\vec{a_y} + dz\,\vec{a_z}$$

$$\boxed{dv = dx\,dy\,dz}$$

$\hat{a_x} = \vec{a_x}$

$\hat{a_y} = \vec{a_y}$

$\hat{a_z} = \vec{a_z}$

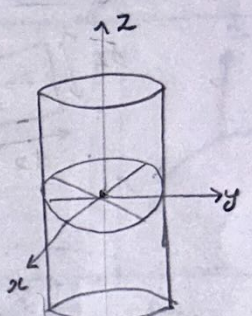
cylindrical coordination system

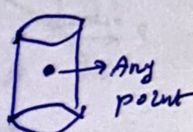
Radius → r-plane, height plane, angle plane

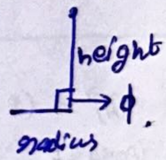
ρ - plane (radius plane)

ϕ - plane (Azimuth angle plane)

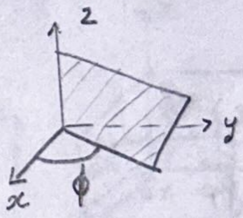
z-plane (height plane)



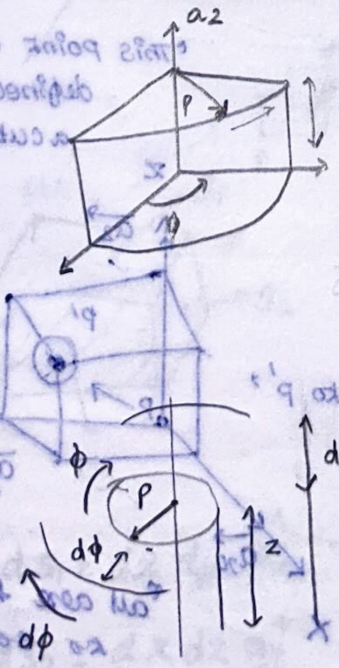


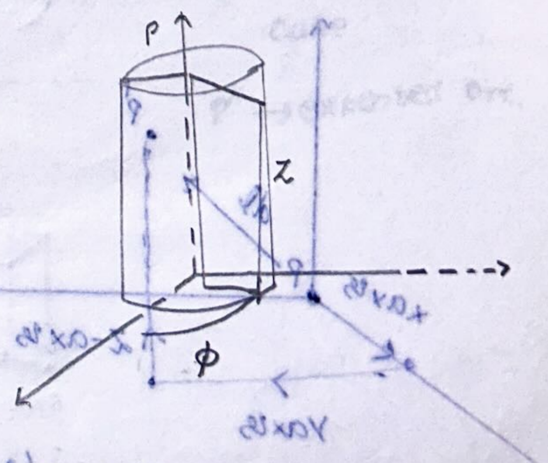


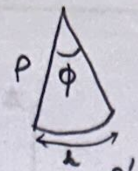
$-\infty < x < \infty$	0 to ∞ (radius)
$-\infty < y < \infty$	0 to 360° (0 to 2π)
$-\infty < z < \infty$	$-\infty$ to ∞ (height)
cartesian	cylindrical



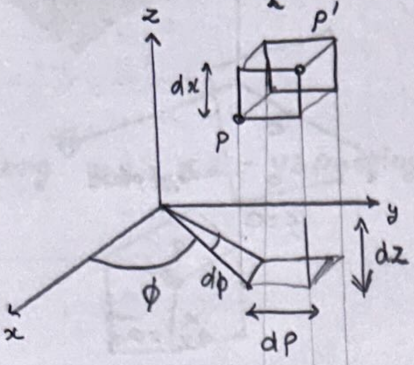
$a_\rho, a_\phi, \hat{a_z}$ → unit vectors



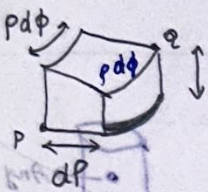




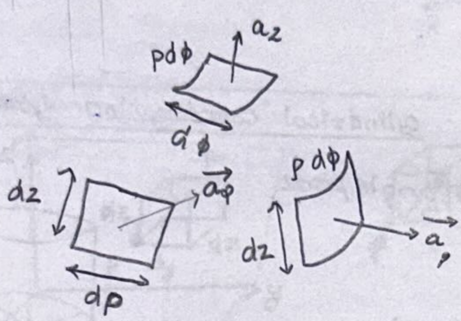
= length of the sector = $\rho\phi$.



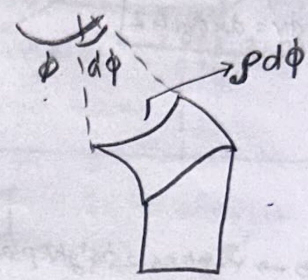
$$\vec{dl} = d\rho\,\vec{a_\rho} + d\phi\,\vec{a_\phi} + dz\,\vec{a_z}$$



Differential volume



Exploded view.



$\vec{A_1} = (\rho\, d\phi \cdot d\rho)\,\vec{a_z}$
(Area 1)

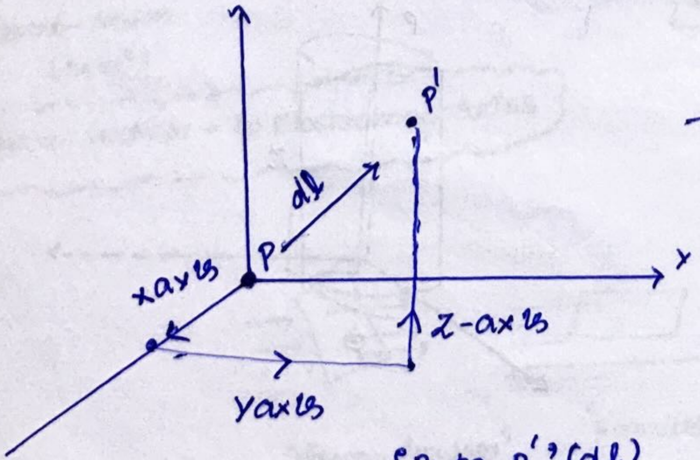
$dv = \rho\, d\phi\, d\rho\, dz$

$\vec{A_2} = (\rho\, d\phi\, dz)\,\vec{a_\rho}$

$\vec{A_3} = (dz\, d\rho)\,\vec{a_\rho}$

26/02/2021

Cartesian – x, y, z – $\perp^r$ to each others.

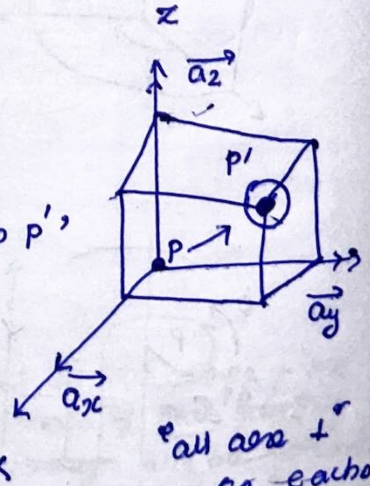


"P to P' (dl)"

"To reach that point travel along x-axis, y axis then z-axis"

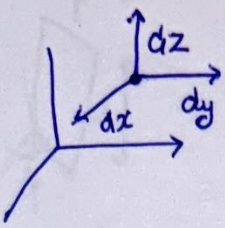
→ "P to P'"

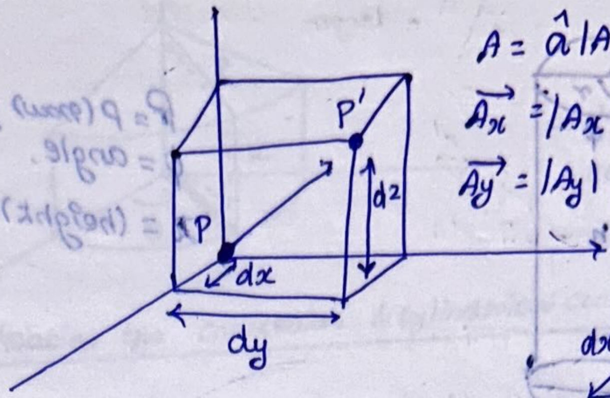
"This point can be defined by a cube"



"all are $\perp^r$ to eachother
↓
"cube""

Extending from P





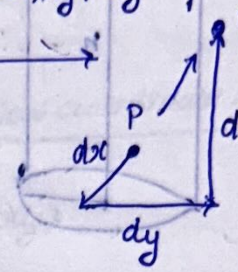
$\vec{A} = \vec{A_x} + \vec{A_y} + \vec{A_z}$

$A = \hat{a}|A|$

$\vec{A_x} = |A_x|\,\hat{a_x}$

$\vec{A_y} = |A_y|\,\hat{a_y}$

$\vec{A_z} = |A_z|\,\hat{a_z}$

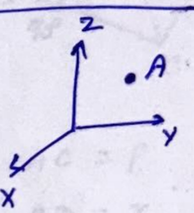


P to P' $(dx + dy + dz)$

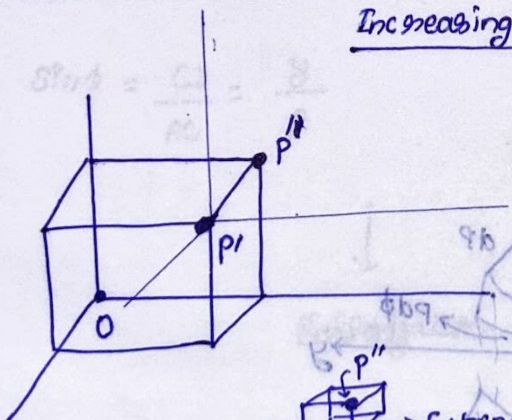
$$\vec{dl} = dx\,\hat{a}_x + dy\,\hat{a_y} + dz\,\hat{a}_z$$

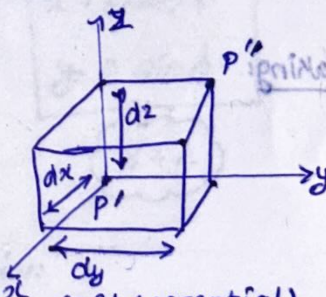
$\boxed{\text{unit vector along } dl: \ \dfrac{dl}{|dl|}}$

$\boxed{\therefore A = A_x \hat{a_x} + A_y \hat{a_y} + A_z \hat{a_z}}$



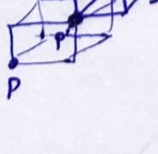
Increasing (shifting P)





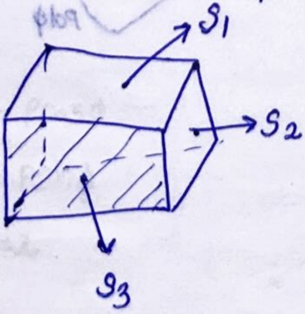
(Differential)

extented cube



→ Extented.

$P'' \rightarrow$ extented one.

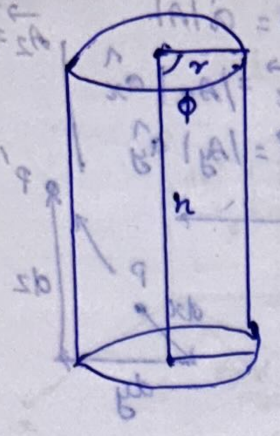


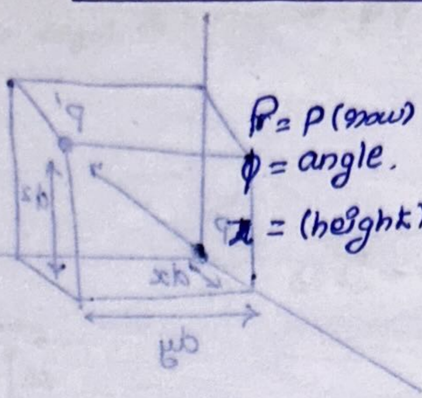
$dS_1 = dx\,dy\,\hat{a}_z$

$dS_2 = dx\,dz\,\hat{a_y}$

$dS_3 = dy\,dz\,\hat{a_x}$

Cylindrical coordinate System

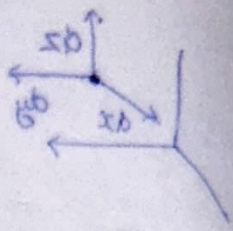




$\rho = \rho$ (raw) radius

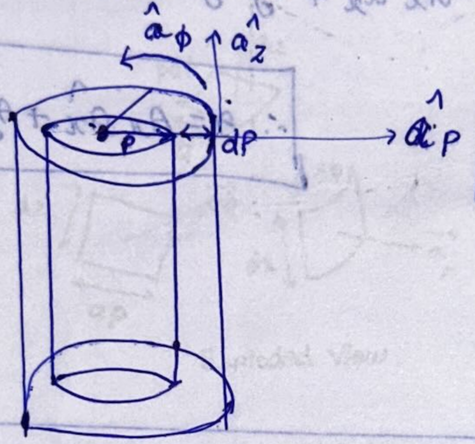
ϕ = angle.

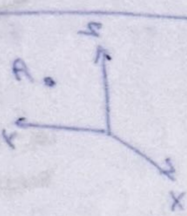
z = (height)



Increasing or decreasing ϕ plane: curvature of radius.

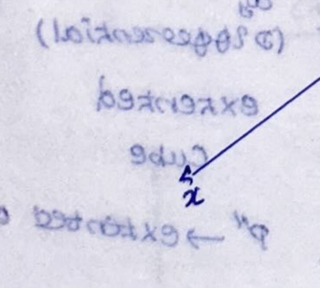
Increasing or decreasing ρ = (radius of the cylinders)

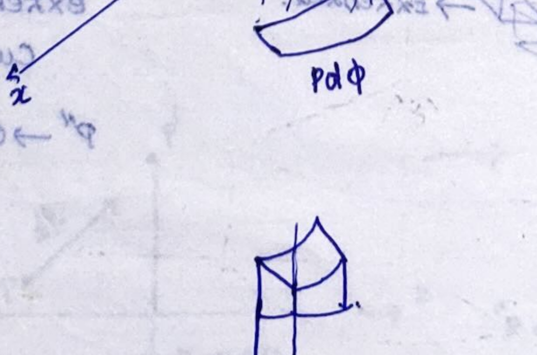


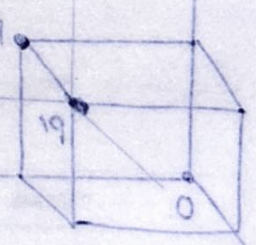


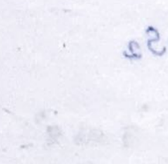
$\frac{dl}{|dl|}$: unit vector along dl

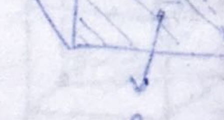
Taking:

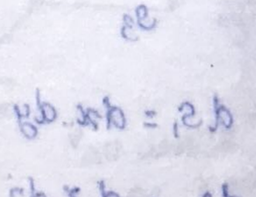


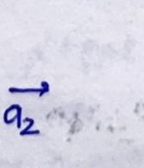












extented.

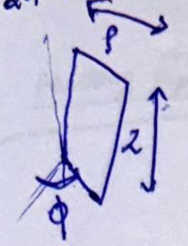
$\vec{dl} = \rho d\phi \, a_\phi + \rho \vec{a_\rho} + z \vec{a_z}$

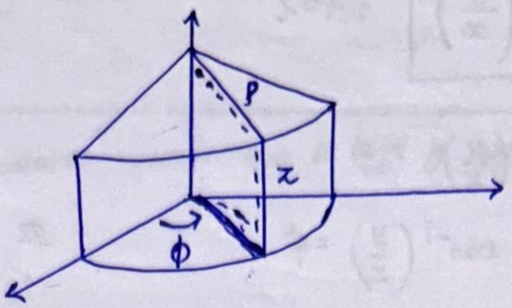
$ds_1 = dx \, dy \, \hat{a_z}$

$ds_2 = dx \, dz \, \hat{a_y}$

$ds_3 = dy \, dz \, \hat{a_x}$

20/02/2021



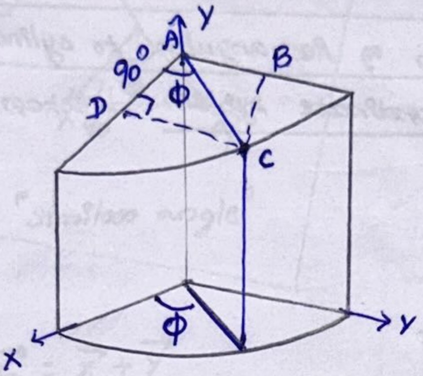


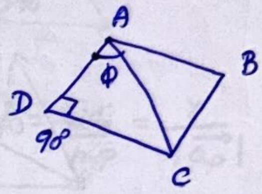
'1/4 quater'

'Section of cylindrical coordinate system'

Relation b/w cartesian & cylindrical coordinate system

* $x = \text{const}$, $y = \text{const}$, $z = \text{const}$ planes (yz, xz, xy)





$AC = \rho$

$AD = x$

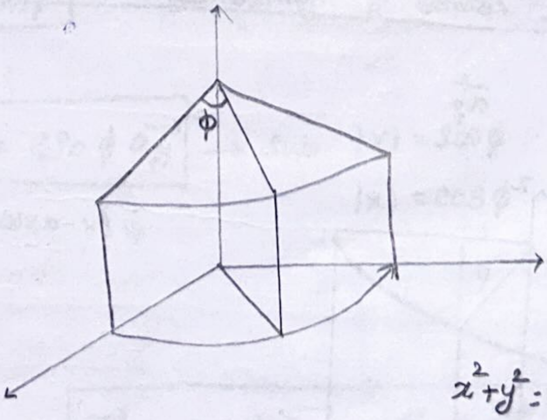
$\cos\phi = \frac{\text{Adj}}{\text{hyp}} = \frac{AD}{AC} = \frac{x}{\rho}$

$\sin\phi = \frac{CD}{AC} = \frac{y}{\rho}$

$$\boxed{\begin{aligned} x &= \rho\cos\phi \\ y &= \rho\sin\phi \end{aligned}}$$

$z = z$

Cylindrical to cartesian



$x = \rho\cos\phi$

$y = \rho\sin\phi$

$z = z$

$x^2 + y^2 = \rho^2\cos^2\phi + \rho^2\sin^2\phi$

$\therefore$ $\boxed{x^2 + y^2 = \rho^2}$

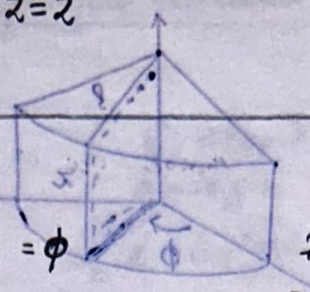
$\rho = \sqrt{x^2 + y^2}$

$\boxed{\tan\phi = \frac{\sin\phi}{\cos\phi}}$

$\therefore \frac{y}{x} = \frac{\rho\sin\phi}{\rho\cos\phi}$

$\frac{y}{x} = \tan\phi$

$\rho = \sqrt{x^2+y^2}$ $\boxed{\phi = \tan^{-1}\left(\frac{y}{x}\right)}$, $z = z$

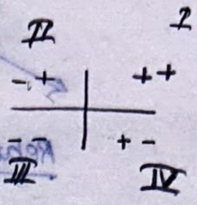


1 quadrant $\rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \phi$

2 quandrant $\rightarrow \pi - \tan^{-1}\left(\frac{y}{x}\right) = \phi$

3 quadrant $\rightarrow \pi + \tan^{-1}\left(\frac{y}{x}\right) = \phi$

4 quadrant $\rightarrow -\tan^{-1}\left(\frac{y}{x}\right) = \phi$



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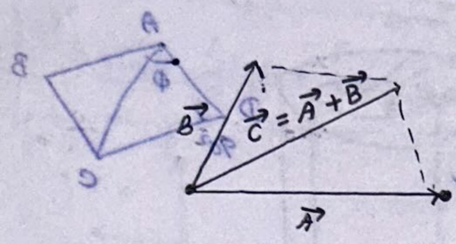
conversion of Rectangular to cylindrical Coordinate System - vector.



$\therefore$ we can write a vector in terms of two components

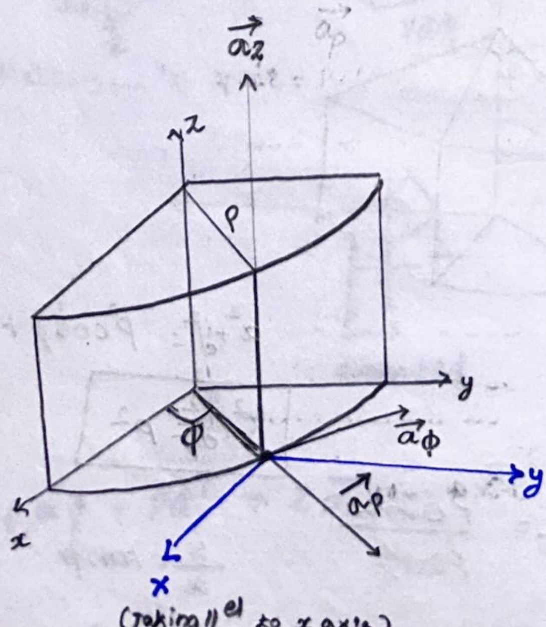
$$\boxed{A = \vec{A_x} + \vec{A_y}}$$

Rectangular or cartesian

$$\vec{A_R} = A_x \vec{a_x} + A_y \vec{a_y} + A_z \vec{a_z}$$

$$\vec{A_c} = A_\rho \vec{a_\rho} + A_\phi \vec{a_\phi} + A_z \vec{a_z}$$

Find A_R vector in terms of cylindrical Coordinate parameters:



ϕ (x-axis & ϕ plane)
(Asimuthplane)

'All three coordinates - $\perp^r$ to each others'

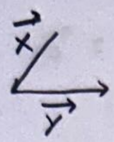
Finding $\vec{a_p}$

$\vec{a_p} \rightarrow$ resultant vector b/w x and y axes

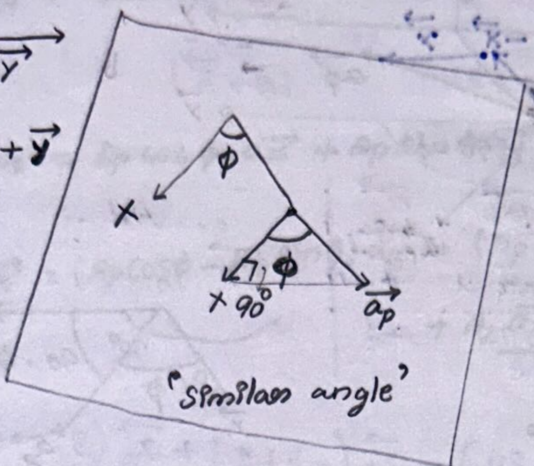
$\vec{x} = x\,\vec{a_x}$

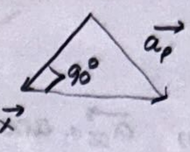
$\vec{y} = y\,\vec{a_y}$

$\vec{a_x} = \frac{\vec{x}}{|x|}$



$\vec{x} + \vec{y}$





$\cos\phi = \frac{adj}{hyp} = \frac{x}{|\vec{a_p}|}$

$\vec{a_p} = \vec{x} + \vec{y}$

'$\cos\phi \rightarrow$ Not a vector'

$|\vec{a_p}|\cos\phi = x$

$x = \cos\phi$

$\boxed{\therefore\ \vec{x} = \cos\phi\ \vec{a_x}}$

ii) $\sin\phi = \frac{opp}{hyp} = \frac{y}{|\vec{a_p}|}$

$\sin\phi \rightarrow$ Not a vector.

$|\vec{a_p}|\sin\phi = y$

$\boxed{y = \sin\phi}$

$\boxed{\vec{y} = \sin\phi\ \vec{a_y}} \rightarrow$ sub $|y| = \sin\phi$

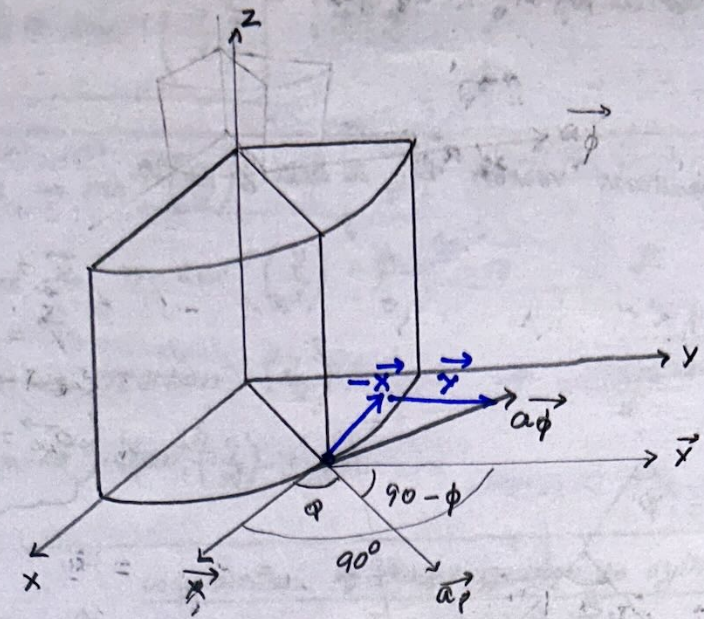
$|x| = \cos\phi$

$\vec{x} = \cos\phi\ \vec{a_x}$

$\vec{y} = \sin\phi\ \vec{a_y}$

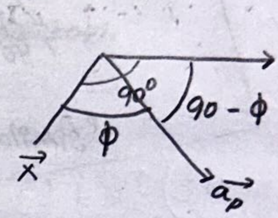
$$\boxed{\vec{a_p} = \cos\phi\ \vec{a_x} + \sin\phi\ \vec{a_y}}$$

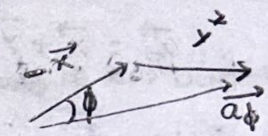
Find $\vec{a_\phi}$



Angle b/w $\vec{a_x}$ and $\vec{a_y} = 90^\circ$

$\vec{a_\phi}$ and $\vec{a_\rho} = 90^\circ$





$\vec{a_\phi} = \vec{x} + \vec{y}$

$\cos\phi = \frac{Adj}{Hyp}$

$\cos\phi = \frac{y}{|\vec{a_\phi}|}$

$|\vec{a_\phi}| \cos\phi = y$

$\cos\phi = y$

$\boxed{\vec{y} = \cos\phi\,\vec{a_y}}$

$\vec{a_\phi} = \vec{x} + \vec{y}$

$\sin\phi = \frac{OPP}{hyp}$

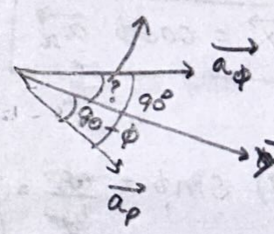
$\sin\phi = \frac{x}{|\vec{a_\rho}|}$

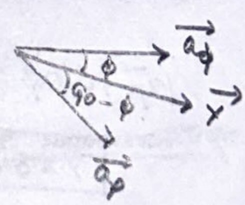
$|\vec{a_\phi}| \sin\phi = x$

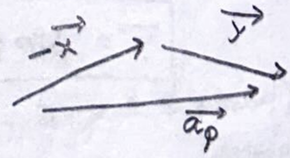
$\boxed{x = \sin\phi}$

$\boxed{\vec{x} = -\sin\phi\,\vec{a_x}}$ → (-)ve

$90-(90-\phi) = \phi$



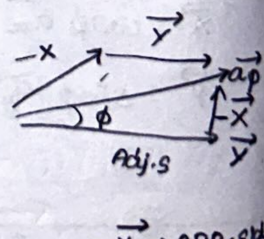




$\vec{a_\phi} = \vec{x} + \vec{y}$

$\vec{x} = \sin\phi(-\vec{a_x})$

$\boxed{\vec{a_z} = \vec{a_z}}$



∴

$\vec{a_\rho} = \cos\phi\,\vec{a_x} + \sin\phi\,\vec{a_y}$

$\vec{a_z} = \vec{a_z}$

$\vec{a_\phi} = \cos\phi\,\vec{a_y} - \sin\phi\,\vec{a_x}$

$$\vec{A_c} = A_\rho \vec{a_\rho} + A_\phi \vec{a_\phi} + A_z \vec{a_z}$$

$$\boxed{\begin{aligned} \vec{a_\rho} &= \cos\phi\, \vec{a_x} + \sin\phi\, \vec{a_y} \rightarrow (3) \\ \vec{a_\phi} &= -\sin\phi\, \vec{a_x} + \cos\phi\, \vec{a_y} \rightarrow (4) \\ \vec{a_z} &= \vec{a_z} \rightarrow (5) \end{aligned}}$$

$$\vec{A_c} = A_\rho(\cos\phi\, \vec{a_x} + \sin\phi\, \vec{a_y}) + A_\phi(-\sin\phi\, \vec{a_x} + \cos\phi\, \vec{a_y}) + A_z \vec{a_z}$$

$$\vec{A_c} = A_\rho \cos\phi \cdot \vec{a_x} + A_\rho \sin\phi\, \vec{a_y} + A_\phi \cos\phi\, \vec{a_y} - A_\phi \sin\phi\, \vec{a_x} + A_z \vec{a_z}$$

$$\vec{A_c} = \underbrace{(A_\rho \cos\phi - A_\phi \sin\phi)}_{A_x} \vec{a_x} + \underbrace{(A_\rho \sin\phi + A_\phi \cos\phi)}_{A_y} \vec{a_y} + A_z \vec{a_z}$$

$$\vec{A_c} = (A_\rho \cos\phi - A_\phi \sin\phi)\, \vec{a_x} + (A_\rho \sin\phi + A_\phi \cos\phi)\, \vec{a_y} + A_z \vec{a_z}$$

$$\begin{bmatrix} t \\ u \\ v \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} l \\ m \\ n \end{bmatrix}$$

$t = al + bm + cn$

$u = dl + em + fn$

$v = gl + hm + in$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = ?$$

Mul by $\sin\phi$ equ (3)

$$\sin\phi\, \vec{a_\rho} = \sin\phi \cos\phi \cdot \vec{a_x} + \sin^2\phi\, \vec{a_y} \rightarrow (6)$$

$$\cos\phi\, \vec{a_\phi} = \cos^2\phi\, \vec{a_y} - \sin\phi\cos\phi\, \vec{a_x} \rightarrow (7)$$

Add (5) (6)

$$\sin\phi\, \vec{a_\rho} + \cos\phi\, \vec{a_\phi} = \sin^2\phi\, \vec{a_y} + \cos^2 \vec{a_y}$$

$$\boxed{\vec{a_y} = \sin\phi\, \vec{a_\rho} + \cos\phi\, \vec{a_\phi}} \rightarrow (8)$$

1) If two vectors are expressed in cylindrical coordinates as

$$\vec{A} = 2\vec{a_r} + \pi\vec{a_\phi} + \vec{a_z}$$

$$\vec{B} = -\vec{a_r} + \frac{3\pi}{2}\vec{a_\phi} - 2\vec{a_z}$$

Compute a unit vector $\perp^r$ to the plane having A and B

Solu:

$\hat{n}$ (unit vector $\perp^r$ to the plane having A & B) $= \dfrac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \vec{a_r} & \vec{a_\phi} & \vec{a_z} \\ 2 & \pi & 1 \\ -1 & \frac{3\pi}{2} & -2 \end{vmatrix}$$

$$= \vec{a_r}\left(-2\pi - \frac{3\pi}{2}\right) - \vec{a_\phi}\left(-4+1\right) + \vec{a_z}\left(3\pi + \pi\right)$$

$$= \vec{a_r}\left(\frac{-4\pi - 3\pi}{2}\right) - \vec{a_\phi}(-3) + \vec{a_z}(4\pi)$$

$$= -\frac{7\pi}{2}\vec{a_r} + 3\vec{a_\phi} + 4\pi\vec{a_z}$$

$$|\vec{A}\times\vec{B}| = \sqrt{\left(-\frac{7\pi}{2}\right)^2 + (3)^2 + (4\pi)^2}$$

$$= \sqrt{\frac{49\pi^2}{4} + 9 + 16\pi^2}$$

$$= \sqrt{287.8163243}$$

$$= 16.965155029.$$

$$\hat{n} = \frac{-\frac{7\pi}{2}\vec{a_r} + 3\vec{a_\phi} + 4\pi\vec{a_z}}{16.965155029}$$

$$= -0.648126\,\vec{a_r} + 0.1768\,\vec{a_\phi} + 0.74071\,\vec{a_z}$$

$$\hat{n} = -0.65\vec{a_r} + 0.18\vec{a_\phi} + 0.74\vec{a_z}.$$

(Unit vector $\perp^r$ to the plane having A and B)

2) Convert the given rectangular coordinate $A(x=2, y=3, z=1)$ in to the corresponding cylindrical coordinate.

Solu:

$$T: A(x,y,z) \rightarrow A(\rho, \phi, z)$$

Rectangular → cylindrical coordinate system

$$\rho = \sqrt{x^2+y^2}$$
$$\rho = \sqrt{4+9}$$
$$\rho = \sqrt{13}$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$
$$\phi = \tan^{-1}\left(\frac{3}{2}\right)$$
$$\phi = 56.309^\circ$$

$$z = z$$
$$\boxed{z = 1}$$

(Radians: $\phi = 0.9827$ radians)

$$T: A(x,y,z) \rightarrow A(\rho, \phi, z).$$

$$A(x=2, y=3, z=1) \rightarrow A(\rho = 3.606, \phi = 56.31^\circ, z=1)$$

3/3/2021 Any vector $\xrightarrow{\text{resolve}}$ (x, y components)

Continuation

mul eqn 3 by $\cos\phi$ & eqn 4 by $\sin\phi$.

$$\cos\phi\, \vec{a_\rho} = \cos^2\phi\, \vec{a_x} + \sin\phi\cos\phi\, \vec{a_y} \rightarrow Ⓐ$$

$$\sin\phi\, \vec{a_\phi} = -\sin^2\phi\, \vec{a_x} + \cos\phi\sin\phi\, \vec{a_y} \rightarrow Ⓑ$$

$\boxed{A - B}$

$$\cos\phi\, \vec{a_\rho} - \sin\phi\, \vec{a_\phi} = \cos^2\phi\, \vec{a_x} + \sin^2\phi\, \vec{a_x}$$

$$\boxed{\cos\phi\, \vec{a_\rho} - \sin\phi\, \vec{a_\phi} = \vec{a_x}}$$

Result:

$$\vec{a_x} = \cos\phi\, \vec{a_\rho} - \sin\phi\, \vec{a_\phi}$$

$$\vec{a_y} = \sin\phi\, \vec{a_\rho} + \cos\phi\, \vec{a_\phi}$$

$$\vec{a_z} = \vec{a_z}$$

$$A_{cylind} = \overset{A_x}{\cos\phi}\, \vec{a_\rho} - \overset{A_y}{\sin\phi}\, \vec{a_\phi} + \overset{A_y}{\sin\phi}\, \vec{a_\rho} + \overset{A_x}{\cos\phi}\, \vec{a_\phi} + \overset{A_z}{\vec{a_z}}$$

$$A_{cylind} = \underset{A_\rho}{\vec{a_\rho}\,(\overset{A_x}{\cos\phi} + \overset{A_y}{\sin\phi})} + \underset{A_\phi}{\vec{a_\phi}\,(\overset{A_y}{\cos\phi} - \overset{A_x}{\sin\phi})} + \underset{A_z}{\overset{A_z}{\vec{a_z}}}$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$A_{cylind} = \vec{a_\rho}\,(A_x\cos\phi + A_y\sin\phi) + \vec{a_\phi}\,(A_y\cos\phi - A_x\sin\phi) + A_z\,\vec{a_z}.$$

3/3/2021.

$$A = \vec{A_x} + \vec{A_y} + \vec{A_z}$$

$$= A_x \cdot \vec{a_x} + A_y\, \vec{a_y} + A_z\, \vec{a_z}$$

$$\vec{dl} = \vec{dx} + \vec{dy} + \vec{dz}$$

$$= dx\, \vec{a_x} + dy\, \vec{a_y} + dz\, \vec{a_z}$$

$$\vec{dl} = \underbrace{d\rho\, \vec{a_\rho}}_{radius} + \underbrace{\rho\, d\phi\, \vec{a_\phi}}_{radius \times angle} + \underbrace{dz\, \vec{a_z}}_{height}.$$

4/3/21

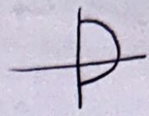
Spherical coordinate system

$P(r, \theta, \phi)$

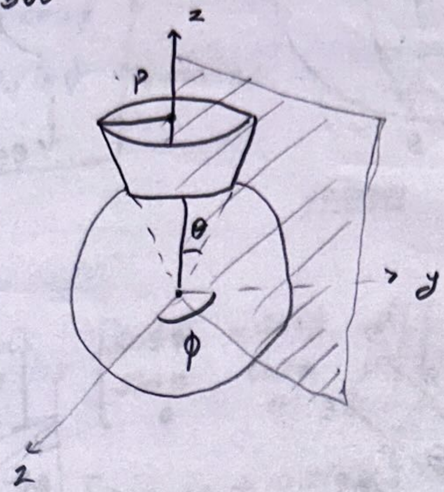
$r \to 0$ to ∞

$\theta \to 0$ to $180°$.

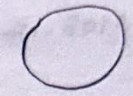
$\phi \to 0$ to $360°$



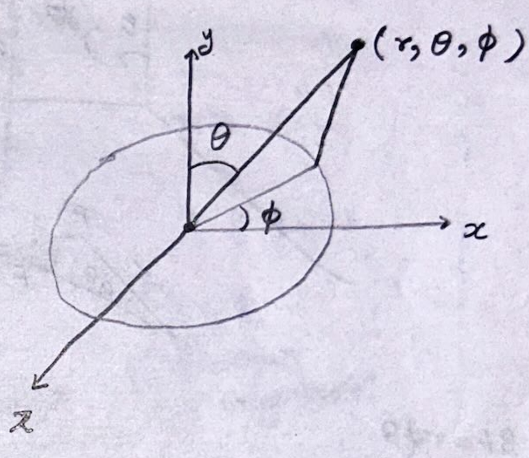
360° – surface of our sphere.



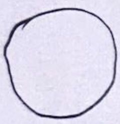
$\theta = 180°$



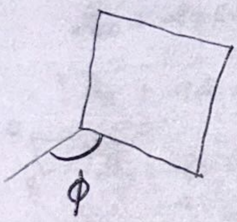
→ Flat surface.



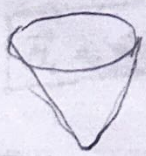
3 planes



$r =$ constant



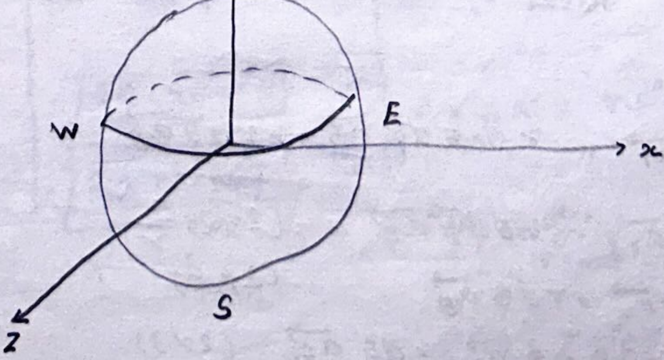
Half plane

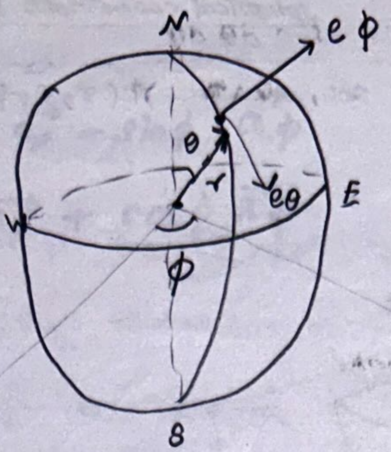


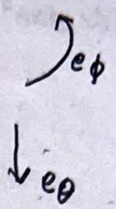
Cone.

$e_\theta, e_\phi, e_r \to \perp^r$ to each others.

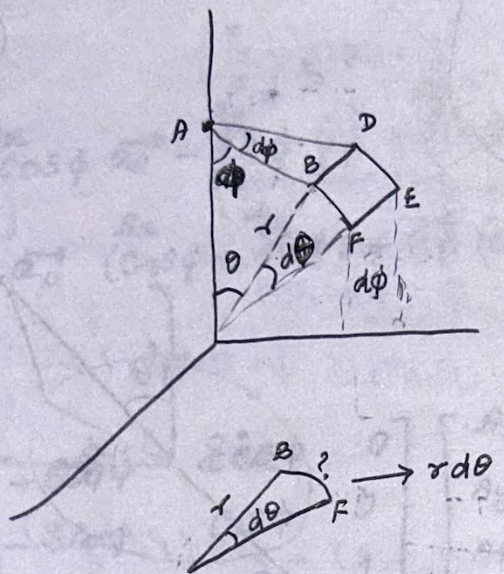
Unit vector representation

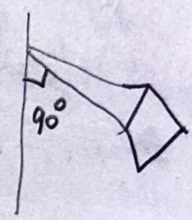






(moving)

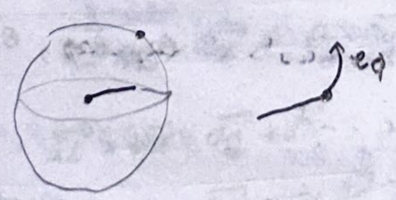




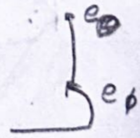
$BF = r\,d\theta$

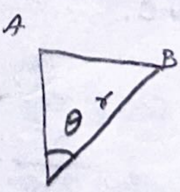
moves

(i)



(ii)





$\sin\theta = \frac{AB}{r}$

hyp = r

$AB = r\sin\theta$

$\vec{dl} = dr\,\vec{a_r} + r\sin\theta\, d\phi\, \vec{a_\phi} + r\,d\theta\, \vec{a_\theta}$

$\vec{dA_1} = dr\,\vec{a_r} \cdot r\sin\theta\, d\phi\, \vec{a_\phi}$ (1,2)

$\vec{dA_2} = dr\,\vec{a_r} \cdot r\,d\theta\, \vec{a_\theta}$ (1×3)

$\vec{dA_3} = r\sin\theta\, d\phi\, \vec{a_\phi} \cdot r\,d\theta\, \vec{a_\theta}$ (2×3)

$dv = (1\times3\times2) = r^2 dr \sin\theta \, d\theta \, d\phi.$

Find the cartesian coordinates of $\vec{A} = 20\vec{a_r} - 10\vec{a_\rho} + 3\vec{a_z}$ at P $(x=5, y=2, z=-1)$

soln.

$$\boxed{\begin{aligned} x &= \rho\cos\phi \\ y &= \rho\sin\phi \\ z &= z. \end{aligned}} \rightarrow \text{points}$$

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$

$= \tan^{-1}\left(\frac{2}{5}\right)$

$= 21.80140949.$

vector

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ -10 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} 20\cos\phi + 10\sin\phi \\ 20\sin\phi - 10\cos\phi \\ 3 \end{bmatrix}$$

$A_x = 20\cos\phi + 10\sin\phi$

$\boxed{A_x = 22.2834}$

scalar (magnitude).

$A_y = 20\sin\phi - 10\cos\phi$

$\boxed{A_y = -1.8569}$

$\boxed{A_z = 3}$

$$\boxed{\vec{A}_c = 22.3\,\vec{a_x} - 1.86\,\vec{a_y} + 3\vec{a_z}}$$

5/3/2021

$\theta \rightarrow$ Angle b/w axis & half cone

ϕ – Asimuth angle.

<u>Relation b/w spherical & rectangular</u>

$\sin\theta = \frac{AP}{r}$

$AP = r\sin\theta$

$\boxed{AP = OM}$

$\cos\theta = \frac{OA}{r}$

$\boxed{OA = r\cos\theta}$

$\boxed{z = r\cos\theta}$

$\cos\phi = \frac{ON}{r\sin\theta}$

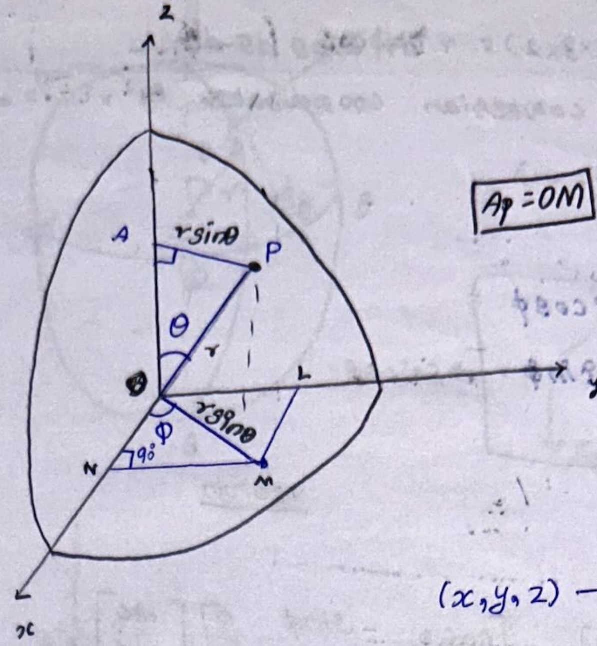
$ON = x = r\sin\theta\cos\phi$

$\sin\phi = \frac{NM}{r\sin\theta} \Rightarrow$

$NM = r\sin\theta\sin\phi$

$NM = r\sin\theta \sin\phi$

$\boxed{y = r\sin\theta \sin\phi}$



$\boxed{A_P = OM}$

$$\boxed{\begin{aligned} x &= r\sin\theta\cos\phi \rightarrow ① \\ y &= r\sin\theta\sin\phi \rightarrow ② \\ z &= r\cos\theta \rightarrow ③ \end{aligned}}$$

$(x, y, z) \rightarrow (r, \theta, \phi)$

vice versa

$$\frac{x}{y} = \frac{r\sin\theta\cos\phi}{r\sin\theta\sin\phi}$$

9) $\frac{x}{y} = \cot\phi$ (or) $\frac{y}{x} = \tan\phi$

$$\phi = \cot^{-1}\left(\frac{x}{y}\right)$$

$$\boxed{\phi = \tan^{-1}\left(\frac{y}{x}\right)} \rightarrow ④$$

$$\phi = \frac{1}{\tan^{-1}\left(\frac{x}{y}\right)}$$

(or)

$$x^2+y^2+z^2 = r^2\sin^2\theta(\sin^2\phi+\cos^2\phi) + r^2\cos^2\theta$$

$$= r^2\sin^2\theta + r^2\cos^2\theta$$

$$x^2+y^2+z^2 = r^2(1)$$

$$\boxed{r = \sqrt{x^2+y^2+z^2}} \rightarrow ⑤$$

$$z = r\cos\theta$$

$$\frac{z}{r} = \cos\theta$$

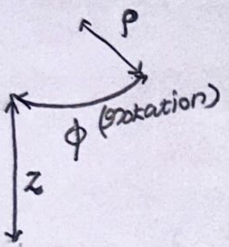
$$\boxed{\theta = \cos^{-1}\left(\frac{z}{r}\right)} \rightarrow ⑥$$

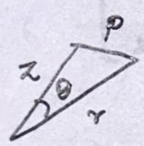
$$\theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$

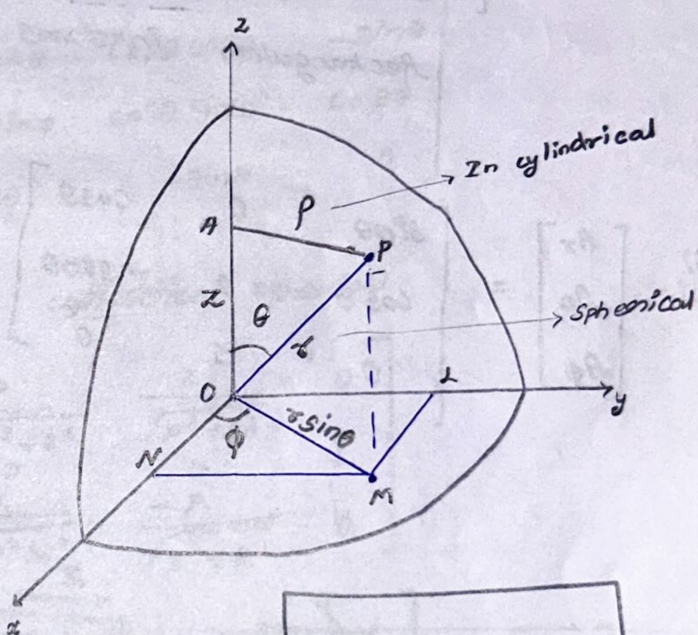
$$\boxed{\begin{aligned} r &= \sqrt{x^2+y^2+z^2} \\ \theta &= \cos^{-1}\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\right) \\ \phi &= \tan^{-1}\left(\frac{y}{x}\right) \end{aligned}}$$

Relation b/w spherical & cylindrical

cylindrical:







$$r = \sqrt{\rho^2+z^2} \rightarrow (1)$$

$$\tan\theta = \frac{opp}{adj} = \frac{\rho}{z} \rightarrow$$

$$\theta = \tan^{-1}\left(\frac{\rho}{z}\right) \rightarrow (2)$$

$$\boxed{\phi_s = \phi_c}$$

$$\boxed{\begin{aligned} r &= \sqrt{\rho^2+z^2} \\ \theta &= \tan^{-1}\left(\frac{\rho}{z}\right) \\ &\boxed{\phi_s = \phi_c} \end{aligned}}$$

vice versa

$$\boxed{\begin{aligned} \phi_c &= \phi_s \\ \rho &= r\sin\theta \\ z &= r\cos\theta \end{aligned}}$$

$$\sin\theta = \frac{\rho}{r}$$

$$\cos\theta = \frac{z}{r}$$

R-C
C-S
R-S
S-R

Transformation of vectors

1)
$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Spherical interms of rectangular.

2)
$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\theta \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

Rectangular interms of spherical.

3)
$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

4)
$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \frac{\rho}{\sqrt{\rho^2+z^2}} & \frac{z}{\sqrt{\rho^2+z^2}} & 0 \\ 0 & 0 & \\ \frac{z}{\sqrt{\rho^2+z^2}} & \frac{-\rho}{\sqrt{z^2+\rho^2}} & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

planes	Coord var	limits	Diff len	S	Volume
cart					
cyl					
sph					

cart cyl sp

Point relation	vectors (matrices).