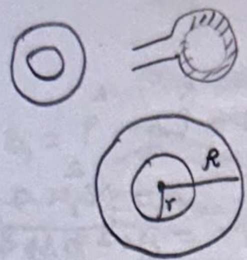
Inductance of Toroid



$\left[B = \mu H\right]$

$$B = \frac{\mu N I}{l}$$

$$H = \frac{NI}{l}$$

'N-turns', carrying I.

$\boxed{l = 2\pi R}$

circumference

$$B = \frac{\mu N I}{2\pi R}$$

$$\boxed{B = \frac{\phi}{A}}$$

$$N\phi = NBA$$

$$B = \frac{\mu N I}{2\pi R}$$

$$N\phi = \frac{\mu N^2 I}{2\pi R} A$$

$$N\phi = \frac{\mu N^2 I}{2\pi R} A$$

$$N\phi = \frac{\mu N^2 I}{2\pi R}(\pi r^2)$$

$$\boxed{L = N\frac{\phi}{I}}$$

$$L = \frac{\frac{\mu N^2 I}{2\pi R}(\pi r^2)}{I}$$

$$\phi = \frac{\mu N I}{2\pi R}(\pi r^2)$$

$$\boxed{L = \frac{\mu N^2 r^2}{2R}}$$

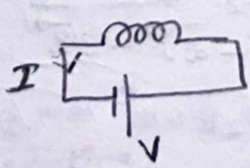
$$L = \frac{N\phi}{I} = \frac{\mu N^2}{2\pi R}(\pi r^2)$$

$$\boxed{L = \frac{\mu N^2 r^2}{2R}}$$

R - mean radiance

 $\Rightarrow$ can't say exactly.

Energy stored in Inductance & Energy density



$$\boxed{V = L\frac{di}{dt}}$$

$$P = V \times I = \frac{dW}{dt} = \frac{d\,\text{Energy}}{dt}$$

$$\int dW = \int VI\,dt$$

$$W = \int L\frac{di}{dt} I\,dt$$

$$= L\int I \cdot di$$

$$\boxed{E = \frac{LI^2}{2}}$$

E_{stored} (Ind) $= \frac{LI^2}{2}$

E_{stored} (capa) $= \frac{1}{2} CV^2$

<u>Energy density</u>

$L = \frac{N^2 \mu A}{l}$

(Inductance)

$$E.D = \frac{E}{Volume} = \frac{LI^2}{2V} = \frac{\frac{N^2 \mu A}{l} I^2}{2 A \times l}$$

$$\boxed{\text{Energy density} = \frac{N^2 \mu I^2}{2 l^2}}$$

$$\text{Energy dens} = \frac{\mu}{2}\left(\frac{NI}{l}\right)^2$$

$B = \mu H$

$H = \frac{B}{\mu}$

$H^2 = \frac{B^2}{\mu^2}$

$$E.D = \frac{\mu}{2} H^2$$

$$\boxed{E.D = \frac{1}{2} \frac{B^2}{\mu}}$$

$$(\text{Energy density})_{capacitor} = \frac{1}{2} \varepsilon E^2$$

<u>H.W.</u>: Two mag materials - sep by a surface $Z=0$, having perm $\mu_1 = 4\mu_0$ H/m ($Z>0$), $\mu_2 = 7\mu_0$ H/m ($Z<0$) → There exist surface current density $K_s = 60\,\hat{i}$ A/m at $Z=0$ for field $\vec{B_1} = 1\vec{i} - 2\hat{j} - 3\hat{k}$ mT

$\boxed{\vec{B_2} = ?}$ → find

<u>Solu</u>.

~~$B_{t1} - B_{t2} = \mu_0 (K \times \hat{a})$~~ $B_1 = 1\vec{i} - 2\vec{j} - 3\vec{k}$ mT

$\mu_1 = 4\mu_0$

$Z=0$ $[K_s = 60\hat{i}]$

$\mu_2 = 7\mu_0$

$B_{N_1} = B_{N_2}$, $H_{N_1} = \frac{\mu_2}{\mu_1} H_{N_2}$, $H_{t_1} - H_{t_2} = K_s$

$H_{t_1} = \frac{B_{t_1}}{\mu_1}$

$B_1 = \hat{i} - 2\hat{j} - 3\hat{k}$

$H_{t_2} = \frac{B_{t_2}}{\mu_2}$

$\boxed{B_{N_1} = B_{N_2} = -3\hat{k}}$

$H_1 = \frac{B_1}{\mu_1} = \frac{(\hat{i} - 2\hat{j} - 3\hat{k})}{4\mu_0}$

$H_2 = \frac{B_2}{\mu_2} = \frac{B_x\hat{i} + B_y\hat{j} + B_z\hat{k}}{7\mu_0}$

$(H_1 - H_2) \times a_{n12} = K$

$H_1 a_{n12} = H_2 a_{n12} + K$

$\boxed{\frac{\hat{i} - 2\hat{j} - 3\hat{k}}{4\mu_0}} \times \hat{k} = \frac{B_x\hat{i} + B_y\hat{j} + B_z\hat{k}}{7\mu_0} \times \hat{k} + 60\hat{i}$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \vdots & \vdots & \vdots \end{vmatrix}$

$\frac{1}{4\mu_0} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -3 \\ 0 & 0 & 1 \end{vmatrix} = \frac{1}{7\mu_0} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ B_x & B_y & B_z \\ 0 & 0 & 1 \end{vmatrix} + 60\hat{i}$

$\frac{1}{4\mu_0}\left(\hat{i}(-2+0) - \hat{j}(1+0) + \hat{k}(0)\right) = \frac{1}{7\mu_0}\left[\hat{i}(B_y) - \hat{j}(B_x)\right] + 60\hat{i}$

$\frac{1}{4\mu_0}\left(-2\hat{i} - \hat{j}\right) - 60\hat{i} = \frac{1}{7\mu_0}\left[B_y\hat{i} - B_x\hat{j}\right]$

$\mu_0 = 4\pi \times 10^{-7}\ H/m$

Equating

$\frac{1}{4\mu_0}(-2\hat{i}) - 60\hat{i} = \frac{1}{7\mu_0} B_y \hat{i}$

$\frac{1}{4\mu_0}(-\hat{j}) = \frac{1}{7\mu_0}(-B_x\hat{j})$

$-\frac{2}{4\mu_0} - 60 = \frac{B_y}{7\mu_0}$

$\frac{1}{4\mu_0} = \frac{1}{7\mu_0} B_x$

$B_y = 7\mu_0\left(-\frac{2}{4\mu_0} - 60\right) = -3.5$

$B_x = \frac{7}{4} = 1.75$

$B_2 = (B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) = B_{k2} + B_{2N}$

$\boxed{B_2 = 1.75\hat{i} - 3.5\hat{j} - 3\hat{k}}$.

$B_1 = B_{1t} + B_{1n}$

$B_{1t} = B_1 - B_{1n}$

$= 1\vec{i} - 2\vec{j}$

$\boxed{B_{1n} = B_{2n} = -3K}$

App. another BC,

$$\frac{B_{1t}}{\mu_1} - \frac{B_{2t}}{\mu_2} = K_S$$

$$\left(\frac{9\vec{i} - 2\vec{j}}{4\mu_0} - \frac{B_{2t}}{7\mu_0}\right) \times \hat{k} = 60\hat{i}$$

(vectors)

$\boxed{\hat{i} \times \hat{j} = \hat{k}}$

Hemisphere → Along dR → (Area can't have dR).

10^9 → Milliyard.

$P_V = -0.3\ nC/mm^3$ a)

$J = -a_z\, 2.4\ A/mm^2$

$R = 5mm$

a) Total current passing through hemis cap ($R = 5$ mm)

$$I = \int J \cdot dS = -\int 2.4\,(a_z \cdot a_z)\, dS$$

$$= \int_0^{2\pi}\int_0^{\pi/2} -2.4 \cos\theta\,(5^2 \sin\theta\, d\theta\, d\phi)$$

$$= -2\pi \int_0^{\pi/2} 60 \sin\theta\, d\sin\theta$$

$$= -120\pi \left(\frac{\sin^2\theta}{2}\right)_0^{\pi/2}$$

$$= -60\pi = -188.5\ A$$

u = velocity of free charges $\quad d\sin\theta = \cos\theta$

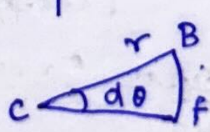
$$u = \frac{J}{P_V} = a_z \frac{-2.4}{-0.3 \times 10^{-9}}$$

$= a_z\, 8 \times 10^9$ mm/s

$= a_z\, 8$ (Mm/s)

$AB = AD = r\sin\theta$

$\boxed{BD = r\sin\theta\, d\phi}$



$CB = r$

$\angle BCF = d\theta$

$BF = r\, d\theta = DE$

$BF = DE = r d\theta$

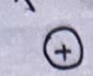
$dS = BD \cdot BF$

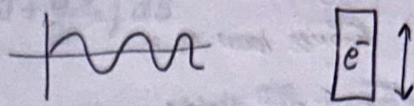
$= r\sin\theta\, d\phi \cdot r d\theta = r^2 \sin\theta\, d\theta\, d\phi$

Faraday's law - Displacement I & maxwell - Ampere law - maxwell eqn - potential functions - Em B.C

<u>unit-4</u>

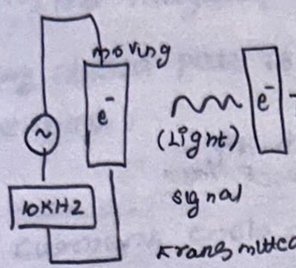
$\vec{E} = \frac{\vec{F}}{v}$





moving back & forth

$\uparrow E$



e^- $\vec{F} = -e\vec{E}$

(Light)

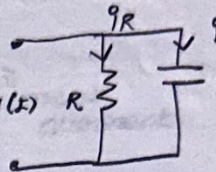
signal transmitted

Infrared radiation.

Peta Hertz $\rightarrow$ visible light (speed of light)

(Transmitted ↑)

Displacement current



mag needle deflected.

$$i_R(t) = \frac{V(t)}{R}, \quad i_C(t) = C\frac{dV(t)}{dt}$$

'Current is also due to bound charges - Since capacitor has bound charges'.

Through R - conduction current

C - Displacement current.

$$\bar{J}_d(t) = \frac{d\bar{D}(t)}{dt}$$

$$\boxed{\bar{J}_d(t) = \frac{\partial \bar{D}(t)}{\partial t}}$$

$$i_C(t) = i_d(t) = \frac{dQ}{dt} = C\frac{dV(t)}{dt}$$

$$= \frac{\varepsilon A}{d}\frac{dV(t)}{dt} = \cancel{\frac{dQ}{dt}}$$

$$= \varepsilon A \frac{d}{dt}\left[\frac{V(t)}{d}\right]$$

$$= \varepsilon A \frac{dE(t)}{dt}$$

$$= A\frac{d}{dt}\left[\varepsilon E(t)\right]$$

$$= A\frac{d\bar{D}(t)}{dt}$$

<u>Intro</u>

* Static E can exist without H
* Conductors with I has a H without E
* Time varying fields
  * E & H can't exist without each other
  * Discovered Em waves - experi of Faraday & maxwell

maxwells eqn - 4 eqn

Derived from - Amp ckt laws
Faraday's law
Gauss law of E
Gauss law of H

* used to describe EM fields

* Partial derivation eqn:

B/w B, E, D, H, J and ρ

Relation b/w changing E & H fields

Inconsistency in Ampere's law

$$\nabla \times \vec{H} = \vec{J}$$

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot \vec{J} = 0 \quad \text{[Divergence of curl of any vector = 0]}$$

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t} \neq 0$$

↓

[contradiction] Continuity equation

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modified - maxwell amp law

$$\nabla \times \vec{H} = \vec{J} + J_d$$

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot \vec{J} + \nabla \cdot \vec{J_d} = 0$$

$$\nabla \cdot \vec{J_d} = -\nabla \cdot \vec{J} = \frac{\partial \rho}{\partial t} = \frac{\partial}{\partial t}(\nabla \cdot \vec{D}) = \nabla \cdot \frac{\partial \vec{D}}{\partial t}$$

$$\vec{J_d} = \frac{\partial \vec{D}}{\partial t}$$

$$\boxed{\nabla \times H = J + \frac{\partial D}{\partial t}}$$

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maxwell eqn - I

From Amp's ckt law:

$$I = \oint H \cdot dl$$

$$\oint H \cdot dl = \iint_S (J_c + J_d) \cdot ds$$

$$\oint H \cdot dl = \iint_S (\text{cond current} + \text{disp current}) \cdot ds$$

obtained from Ampere's

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$\oint H\, dl = \iint \left(J + \frac{\partial D}{\partial t}\right) ds$$

$I_D = \frac{dq}{dt} = \frac{\varepsilon A}{d} \cdot \frac{dE}{dt}$

$J_D = \frac{I_D}{A}$

$= \varepsilon \frac{dE}{dt}$

Amp ckt law:

Line integral of magnetic field intensity 'H' on any closed path is equal to the current enclosed by the path.

$V = E \cdot d$

$$\oint H \cdot dl = I_{enc}$$

$\frac{dq}{dt} = \frac{\varepsilon A}{d} \cdot \frac{dE}{dt}$

Total current enclo = Conduction + Displacement current
(R) (C)

$q = E\varepsilon$

$\boxed{\frac{dq}{dt} = \varepsilon \frac{dE}{dt}}$

$$\oint H \cdot dl = \iint_S (J_c + J_D) \cdot dS$$

$\boxed{I = J \cdot A}$

J_c = conduction current density

If E is the electric field,

From ohm's law:

$V = I_c R$

$El = V$

$R = \frac{\rho l}{A}$

$I_c = \frac{V}{R}$

$\boxed{\rho = \frac{1}{\sigma}}$

$= \frac{V}{\frac{\rho l}{A}} = \frac{V \times A}{\rho l}$

$I_c = \frac{E \times l \times A \times \sigma}{l}$

$= \frac{V \times A \times \sigma}{l}$

$\boxed{I_c = EA\sigma}$

$J_c = \frac{I_c}{A} = \sigma E$

$\boxed{J_c = \sigma E}$

current through capacitor (I_D)

$I_D = \frac{dq}{dt}$ $\quad C = \frac{\varepsilon A}{d}$

$q = CV$

$\frac{dq}{dt} = C \frac{dv}{dt}$

$\frac{dq}{dt} = \frac{\varepsilon A}{d} \frac{d}{dt} (E \cdot d)$

$= \varepsilon A \frac{dE}{dt}$

$J_D = \frac{I_D}{A} = \varepsilon \frac{dE}{dt}$

$\boxed{J_D = \frac{\partial D}{\partial t}}$ $\quad \boxed{\varepsilon E = D}$

$$\oint H \cdot dl = \iint_S \left(\sigma E + \frac{\partial D}{\partial t}\right) ds$$

This is called integral form

To get the differential form, we apply Stokes theorem:

$$\oint H\cdot dl = \oint_S (\nabla \times H)\, ds$$

$$\iint_S \left(\sigma E + \frac{\partial D}{\partial t}\right) ds = \iint_S (\nabla \times H)\, ds$$

$$\nabla \times H = \sigma E + \frac{\partial D}{\partial t}$$

This is the point of differential form of maxwell's eqn I

Maxwell's equation II

* From faraday's law: emf induced in a ckt is equal to the rate of decrease of the magnetic flux linkage of the ckt.

(i.e) $V_{emf} = -\frac{d\phi}{dt} = -\frac{d}{dt}\left[\iint_S B\cdot ds\right]$

w.k.t

$V_{emf} = \oint E\cdot dl$

$$\oint E\cdot dl = -\frac{d}{dt}\left[\iint B\cdot ds\right] = -\iint \frac{\partial B}{\partial t} ds = -\mu \iint_S \frac{\partial H}{\partial t} ds$$

$B = \mu H$

From Stokes:

$$\oint E\cdot dl = \oint_S (\nabla \times E)\, ds$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

Emf around a closed path is equal to the magnetic flux density through that closed path

$$\boxed{\nabla \times E = -\mu \frac{\partial H}{\partial t}}$$

Maxwell equation - III

* From Gauss law: Electric flux through a closed surface is equal to the charge enclosed by the surface

$\psi = Q$

$$\oint D\cdot ds = Q = \int_V \rho_v \cdot dv$$

$$\boxed{\iint_S D\cdot ds = \iiint_V \rho_v\, dv}$$

→ this is maxwell's eqn from Gauss' law in integral form

By app. Divergence theorem

$$\iint D\cdot ds = \iiint_V (\nabla\cdot D)\, dv = \iiint_V \rho_v\, dv$$

$$\nabla\cdot D = \rho_v$$

This is the point form of diff form

The total electric displacement through the surface enclosing a volume is equal to the charge (total) within the volume'

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maxwell equ – IV

magnetic flux through any closed surface is equal to zero $\phi = 0$

$\oint B \cdot ds = 0 \longrightarrow$ Integral form

App. Divergence theorem:

$$\oint B \cdot ds = \iiint_V (\nabla \cdot B)\, dv$$

$$\oint B \cdot ds = \int_V (\nabla \cdot B) \cdot dv = 0$$

$$\nabla \cdot B = 0$$

Net magnetic flux emerging through any closed surface is zero.

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max equ: Diff & Integral form

Name of law	Diff form	Integral form
Gauss's law	$\nabla \cdot D = \rho_v$	$\int_S D \cdot dS = \int_V \rho_v \cdot dv = Q$
Faraday's law	$\nabla \times E = -\frac{\partial B}{\partial t}$	$\oint_C E \cdot dl = -\int_S \frac{\partial B}{\partial t} \cdot ds$
Gauss's law of magnetics	$\nabla \cdot B = 0$	$\int_S B \cdot ds = 0$
Ampere's law	$\nabla \times H = J + \frac{\partial D}{\partial t}$	$\oint_C H \cdot dl = \int_S \left(J + \frac{\partial D}{\partial t}\right) \cdot ds.$

Boundary conditions for time varying fields

The B.C derived in static fields remains valid for time varying fields.

B.C b/w two dielectric materials [Arun murthy - 315]

Tangential components:

we have derived the tangential components of dielectric-dielectric boundary conditions using the dielectric-dielectric relation of $\oint E \cdot dl = 0$ for a closed path in a static field.

Now we apply

$\oint E.dl = -\int \frac{\partial B}{\partial t} d\vec{S}$ around a selected rect path (closed) 1-2-3-4-1.

Here, the integral must be broken into four parts

$$\oint E.dl = \int_1^2 + \int_2^3 + \int_3^4 + \int_4^1 = -\int \frac{\partial B}{\partial t} dS$$

let the long 1-2 and 3-4 be Δw, $\Delta h \to 0$ (keeping the interface b/w them)

$$\oint E.dl = \int_1^2 E_T . \Delta w + \int_2^3 E_N \frac{\Delta h}{2} + \int_3^4 E_T \Delta w + \int_4^1 E_N \frac{\Delta h}{2} = -\int \frac{\partial B}{\partial t} dS$$

$$E_{T(1-2)} \Delta w - E_{N(2-3)} \frac{\Delta h}{2} - E_{T(3-4)} \Delta w + E_{N(4 \to 1)} \frac{\Delta h}{2} = -\int \frac{\Delta B}{\Delta t} \Delta S$$

$\Delta h \to 0$

E_N at b $\frac{1}{2}\Delta h \simeq E_N$ at a $\frac{1}{2}\Delta h$ and as the area of rectangle approaches zero, the surface integral of $\frac{\partial B}{\partial t}$ vanishes.

Thus,

$$E_{T_1} \Delta w - E_{T_2} \Delta w = 0$$

$$\boxed{E_{T_1} = E_{T_2}}$$

Boundary cond. stated above hold good for both static & time changing fields.

Normal components:

For determining the normal component, we select a Gaussian surface & apply the Gauss law on it

$$\oint D.dS = \Delta Q = \int_V \rho_v . dv$$

Gauss law is same for both static & time changing fields, so boundary conditions are

$D_{N_1} - D_{N_2} = \rho_S$ — $\rho_S = 0 \to$ at dielectric

$\boxed{D_{N_1} = D_{N_2}}$ — charge at boundary

B.C b/w two diff mag materials:

modified amp's law is used

$$\oint H.dl = \int_S \left(J_c + \frac{\partial D}{\partial t}\right) ds$$ instead of

$\oint H \cdot dl = I = \int_S J \cdot ds$

It's assumed that $\Delta h \to 0$ the $\int_S$ of $\frac{\partial D}{\partial t}$ vanishes, so the time changing $\vec{D}$ normal to rect path (half in medium) doesn't contribute.

From $\oint H \cdot dl = \int_S \left(J_c + \frac{\partial D}{\partial t}\right) ds$, the conduction current density J_c may also change with time. However its surface integral also vanishes as $\Delta h \to 0$.

$$H_{T_1} - H_{T_2} = K \times \hat{a}_{12}$$ (current sheet at the boundary)

$\boxed{H_{T_1} = H_{T_2}} \to$ Absence of K.

Normal components:

The normal comp. of B & H are same as that of static field boundary conditions

$$\boxed{B_{N1} = B_{N2}}$$

1) A $25\mu C$ charge is located at $(3,4,0)$ m. Evaluate the resulting at the origin. a) Spherical b) cylindrical

$(0,0,0) \rightarrow (0,0,0)$

Solu:

a) Spherical:

$r = \sqrt{x^2+y^2+z^2}$

$r = \sqrt{9+16+0}$

$r = 5$

$\theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$

$\theta = \cos^{-1}(0)$

$\theta = (\pi/2)^\circ$

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$

$\phi = \tan^{-1}(4/3)$

$\phi = 53.13^\circ$

$\vec{r} = -5\hat{a}_r - \frac{\pi}{2}\hat{a}_\theta - 53.13\hat{a}_\phi$

$r = \sqrt{25} = 5$

$\vec{E}_{(\text{point charge})} = \frac{Q}{4\pi\varepsilon r^2} \cdot \vec{a_r}$

$= \frac{25\mu}{4\pi\varepsilon \times 25} \cdot \frac{(-5\hat{a}_r - \frac{\pi}{2}\hat{a}_\theta - 53.13\hat{a}_\phi)}{5}$

$= \frac{25\mu}{4\pi \times 8.854 \times 10^{-12} \times 25} \times \frac{(-5\hat{a}_r - \frac{\pi}{2}\hat{a}_\theta - 53.13\hat{a}_\phi)}{5}$

$= \frac{25}{4\pi \times 25 \times 8.854 \times 10^{-6}} \times \frac{(-5\hat{a}_r - \pi/2\,\hat{a}_\theta - 53.13\hat{a}_\phi)}{5}$

$= 1797.54\,(-5\hat{a}_r - \pi/2\,\hat{a}_\theta - 53.13\hat{a}_\phi)$

$= -8987.7\hat{a}_r - 1[illegible]\hat{a}_\theta - 95503.30\hat{a}_\phi$

$$\boxed{\vec{E} = -(8.987\hat{a}_r + 161.778\hat{a}_\theta + 95.503\hat{a}_\phi)\,K \ V/m}$$

$E = \frac{Q}{4\pi\varepsilon r^2} = \frac{25\mu}{4\pi \times 8.854 \times 10^{-12} \times 25}$

$= \frac{25}{4\pi \times 8.854 \times 10^{-6} \times 25}$

$$\boxed{E = 8987.742438 \ V/m}$$

Cartesian coordinates

$E = 8987.74 \ V/m$

$\vec{E} = 8987.7424\,\hat{a}_r$

$= 8987.7424\left(\frac{-3\hat{a}_x - 4\hat{a}_y}{5}\right)$

$$\vec{E} = +1797.54846\,(-3\hat{a_x} - 4\hat{a_y})$$

$$\boxed{\vec{E} = (-5.392\,\hat{a_x} - 7.190\,\hat{a_y})\ K \quad V/m}$$

2) If $V = 2x^2y + 20z - \dfrac{4}{x^2+y^2}$ volts. Find $\vec{E}$ and $\vec{D}$ at $P(6, -2.5, 3)$

solu:

i) $$V = -\int E.dl$$

$$E = -\nabla V \text{ (potential gradient)}$$

$$\nabla V = \left(\frac{\partial V}{\partial x}\hat{a_x} + \frac{\partial V}{\partial y}\hat{a_y} + \frac{\partial V}{\partial z}\hat{a_z}\right)$$

$$\nabla V = \left(4xy + \frac{4(2x)}{(x^2+y^2)^2}\right)\hat{a_x} + \left(2x^2 + \frac{4(2y)}{(x^2+y^2)^2}\right)\hat{a_y} + (20)\hat{a_z}$$

$$\nabla V = \left(4xy + \frac{8x}{(x^2+y^2)^2}\right)\hat{a_x} + \left(2x^2 + \frac{8y}{(x^2+y^2)^2}\right)\hat{a_y} + 20\hat{a_z}$$

$$\vec{E} = -\nabla V\Big|_{(6,-2.5,3)}$$

$$\vec{E} = -\left((4\times 6\times -2.5) + \frac{8(6)}{(36+(-2.5)^2)^2}\right)\hat{a_x} + \left(2\times 36 + \frac{8(-2.5)}{(36+2.5^2)^2}\right)\hat{a_y} + 20\hat{a_z}$$

$$\vec{E} = -\left(-60 + \frac{48}{1785.0625}\right)\hat{a_x} + \left(72 - \frac{20}{1785.0625}\right)\hat{a_y} + 20\hat{a_z}$$

$$\boxed{\vec{E} = 59.9731\,\hat{a_x} - 71.988\,\hat{a_y} - 20\hat{a_z} \quad V/m}$$

$$\boxed{E = 95.807\ V/m}$$

ii) $$\vec{D} = \varepsilon\vec{E}$$

$$\vec{D} = 8.854\times 10^{-12}\,(\vec{E})$$

$$= \frac{N}{C} \times \frac{C^2}{m^2 N}$$

$$\vec{D} = (531.001\,\hat{a_x} - 637.381\,\hat{a_y} - 177.08\,\hat{a_z}) \times 10^{-12}$$

$$= \frac{C}{m^2}$$

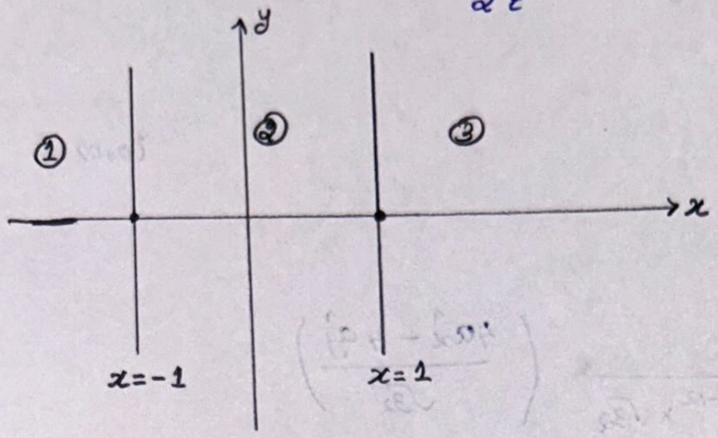
$$\vec{D} = (0.531\,\hat{a_x} - 0.637\,\hat{a_y} - 0.177\,\hat{a_z})\ n \quad \left(V/m \times \frac{F}{m}\right)$$

$$\vec{D} = (0.531\,\hat{a_x} - 0.637\,\hat{a_y} - 0.177\,\hat{a_z})\ n \quad C/m^2$$

3. Two infinite uniform sheets of charge each with density located at $x = \pm 1$ Determine E in all regions.

Solu :

E due to infinite sheet is $\vec{E} = \frac{\rho_s}{2\varepsilon} \cdot \hat{a}_z$



Region 1:

$$\vec{E} = \left[\frac{\rho_s}{2\varepsilon} + \frac{\rho_s}{2\varepsilon}\right](-\hat{a}_x)$$

$$\vec{E} = \frac{2\rho_s}{2\varepsilon}(-\hat{a}_x)$$

$$\vec{E} = -\frac{\rho_s}{\varepsilon}\hat{a}_x \quad \Big|_{x<-1}$$

Region 2:

$$\vec{E} = \frac{\rho_s}{2\varepsilon}\hat{a}_x + \frac{\rho_s}{2\varepsilon}(-\hat{a}_x)$$

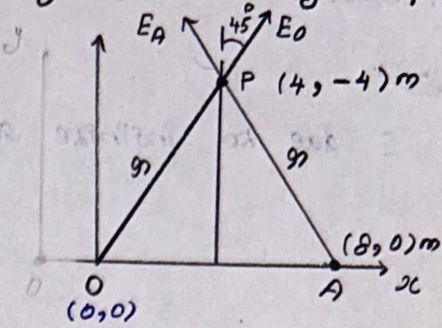
$$\vec{E} = 0 \quad \Big|_{-1<x<1}$$

Region 3:

$$\vec{E} = \frac{\rho_s}{2\varepsilon}\hat{a}_x + \frac{\rho_s}{2\varepsilon}\hat{a}_x$$

$$\vec{E} = \frac{\rho_s}{\varepsilon}\hat{a}_x \quad \Big|_{x>1}.$$

4) Two straight wires (conductors) parallel to z-axis passing through points O and A. The wires carry equal & uniform charges of density $0.4\,\mu C/m$ determine E at point P



Solu:

$$E = \frac{\lambda}{2\pi\varepsilon r}\hat{a_r} \quad \text{(Line charge)}$$

Point O:

$$\vec{r} = 4\hat{a_x} - 4\hat{a_y}$$

$$|\vec{r}| = \sqrt{32}$$

$$\vec{E_O} = \frac{0.4\mu}{2\pi\times 8.854\times 10^{-12}\times\sqrt{32}}\left(\frac{4\hat{a_x} - 4\hat{a_y}}{\sqrt{32}}\right)$$

$$\vec{E_O} = \frac{0.4}{2\pi\times 32\times 8.854\times 10^{-6}}(4\hat{a_x} - 4\hat{a_y})$$

$$\vec{E_O} = 224.69356\,(4\hat{a_x} - 4\hat{a_y})$$

$$\boxed{\vec{E_O} = 898.77\,\hat{a_x} - 898.77\,\hat{a_y} \;\; V/m}$$

$$\boxed{E_O = 1271.05\; V/m}$$

Point A:

$$\vec{r} = -4\hat{a_x} - 4\hat{a_y}$$

$$|\vec{r}| = \sqrt{32}$$

$$\boxed{E_A = 1271.05\; V/m}$$

$$\vec{E_A} = \frac{0.4\mu}{2\pi\times 8.854\times 10^{-12}\times\sqrt{32}}\left(\frac{-4\hat{a_x} - 4\hat{a_y}}{\sqrt{32}}\right)$$

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$$\vec{E_A} = -224.69356\,(4\hat{a_x} + 4\hat{a_y})$$

$$\boxed{\vec{E_A} = -898.77\,\hat{a_x} - 898.77\,\hat{a_y} \;\; V/m}$$

By superposition

$$\vec{E_p} = \vec{E_O} + \vec{E_A}$$

$$\boxed{\vec{E_p} = -1797.54\,\hat{a_y} \;\; V/m}$$