Area of cylinder (surface) = $2\pi r l$

$\therefore \quad \phi = \varepsilon(0+0+2\pi r l\, E)$

$\phi = \varepsilon E(2\pi r l) \Rightarrow q_{enc} = \phi = \varepsilon E(2\pi r l)$

$\frac{\lambda \cdot l}{2\pi r l \varepsilon} = E$

$$\boxed{E = \frac{\lambda}{2\pi r \varepsilon}}$$

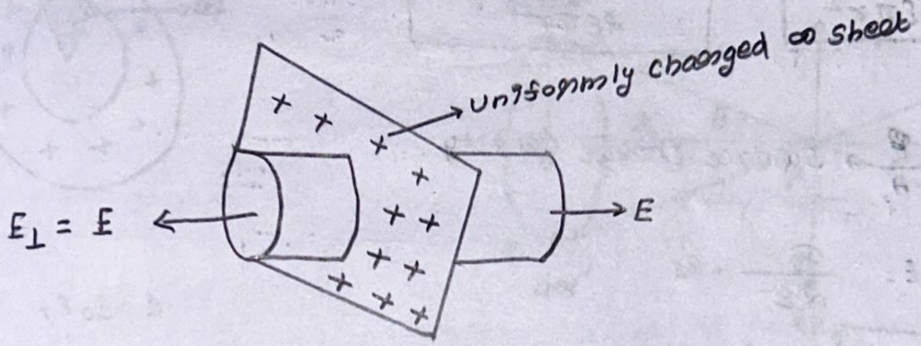
$D = \varepsilon E$

$$\boxed{D = \frac{\lambda}{2\pi r}}$$

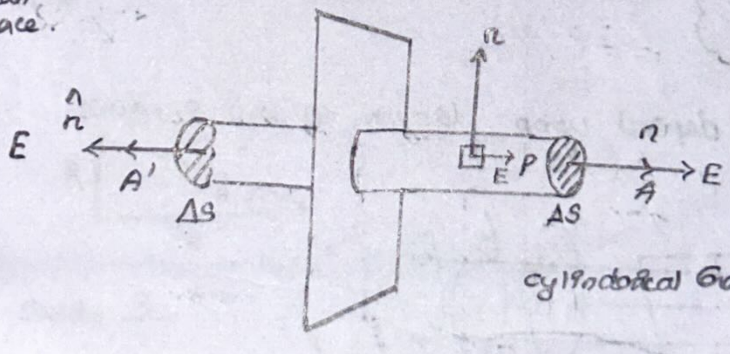
$\vec{D} = \frac{\lambda}{2\pi r}\vec{a_r}$

→ Independent of l

Infinite sheet of charge - pill box



$\hat{n}$ → normal to Gaussian surface.



cylindrical Gaussian surface.

$\phi = \oint \vec{D}\cdot\vec{dS}$

(or)

$\phi = \oint \varepsilon \vec{E}\cdot\vec{dS}$

$\phi = \varepsilon\left[\oint_T \vec{E}\cdot\vec{dS} + \oint_B \vec{E}\cdot\vec{dS} + \oint_C \vec{E}\cdot\vec{dS}\right]$

$\phi = \varepsilon\oint_T \vec{E}\cdot\vec{dS}$

$\phi = \varepsilon\oint |\vec{E}||\vec{dS}|\cos\theta \cdot \hat{a_z}$

$\phi = \varepsilon\oint \vec{E}\cdot\hat{a_z}\cdot\vec{dS}\cos\theta\ \hat{a_z}$

$\phi = \varepsilon E \cdot \int ds \cos\theta \quad [0^\circ]$

$= \varepsilon E(\pi r^2)$

Area of o'le

$\phi_B = \varepsilon\oint_B \vec{E}\cdot\vec{dS}$

$= \varepsilon\oint |\vec{E}|(-\hat{a_z})\cdot\vec{dS}\ (-\hat{a_z})\cos\theta$

$= \varepsilon E(\pi r^2)$

$\phi = \varepsilon\oint_C \vec{E}\cdot\vec{dS}$

$\phi = \varepsilon\oint |\vec{E}||\vec{dS}|\cos\theta$ $\quad 90^\circ$

$\phi_C = 0$

$\Phi_T = \oint |\vec{E}||\vec{dS}|\cos\theta$, $\phi_B = \oint |\vec{E}||\vec{dS}|\cos\theta$

0° $\qquad\qquad$ 0°

$\therefore$ $\underbrace{\vec{a_z} \quad \vec{a_z}}_{\text{along}}$ $\qquad$ $\underbrace{-\hat{a_z} \quad -\hat{a_z}}_{\text{along}}$

So mul becomes +ve.

$\therefore \phi = 0 + \varepsilon E(\pi r^2) + \varepsilon E(\pi r^2)$

$\boxed{\phi = q_{enc}}$

$\phi = 2\varepsilon E(\pi r^2)$

$\boxed{E = \dfrac{q_{enc}}{2\varepsilon\pi r^2}}$ $\qquad$ $\boxed{\vec{E} = \dfrac{q_{enc}}{2\varepsilon\pi r^2} \cdot \hat{a_z}}$

$\sigma_s = \dfrac{q}{A} \rightarrow$ surface charge density

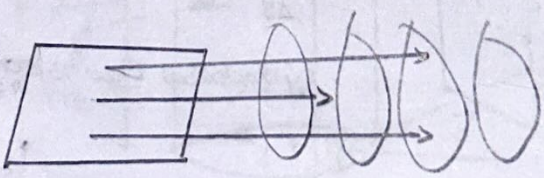
$E = \dfrac{\sigma_s}{2\varepsilon} \cdot \hat{a_z}$

$\varepsilon = \varepsilon_0 \varepsilon_r$

$\boxed{D = \varepsilon E}$

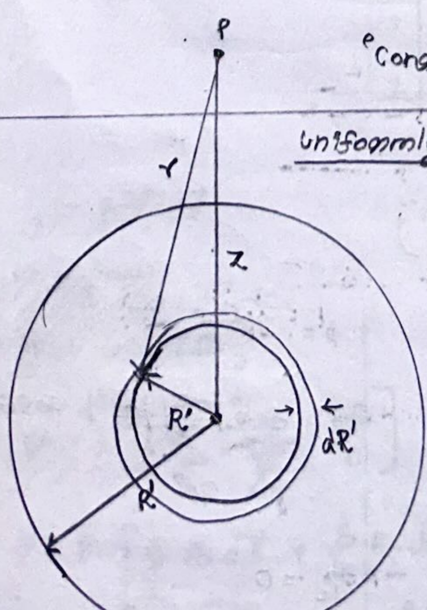
$D = \dfrac{\sigma_s}{2}$

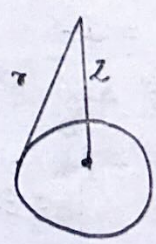
$\therefore$ Doesn't depend upon length of the surface.

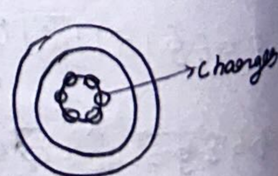


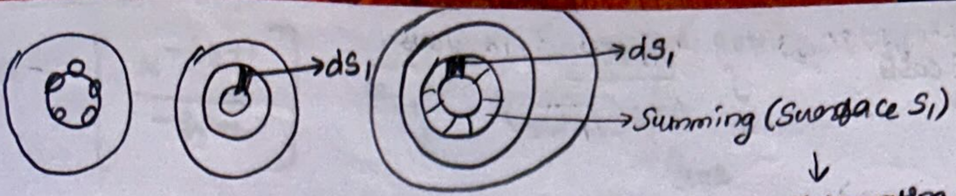
'constant no. of lines passing through'

Uniformly charged disc

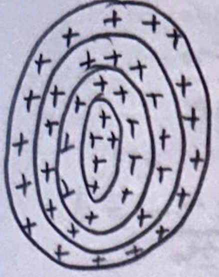


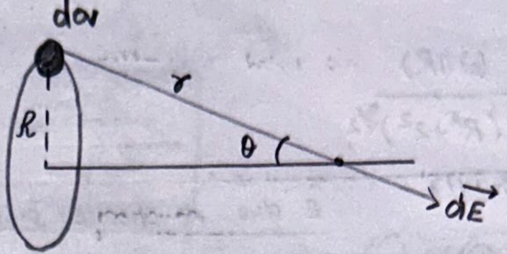




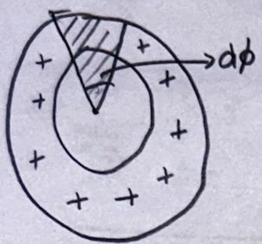


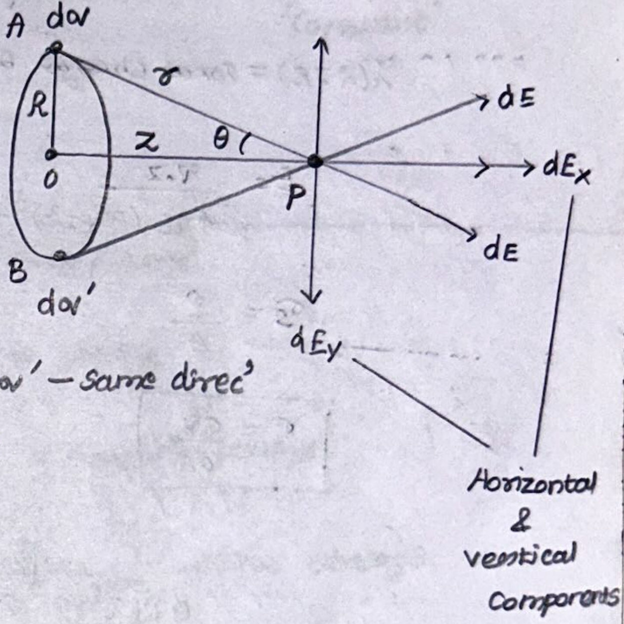
↓ Integrating to get total surface





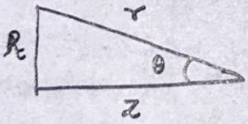
Inner circles (alone)





Horizontal comp : two dq, dq' – same direc'

vert comp : cancels out



$$E = \frac{q}{4\pi\varepsilon r^2}$$

↳ point charge

$$\cos\theta = \frac{z}{r}$$

$$\boxed{z = r\cos\theta}$$

Objective: Find $dE\cos\theta$

$$dE = \frac{dq}{4\pi\varepsilon r^2}$$

$$\boxed{\therefore \lambda = \frac{q}{l}}$$

$$\lambda = \frac{dq}{dl}$$

$$dE = \frac{\lambda dl}{4\pi\varepsilon r^2}$$

$$dE = \frac{\lambda dl}{4\pi\varepsilon r^2}$$

$$r = \sqrt{R^2+z^2}$$

$$dE = \frac{\lambda dl}{4\pi\varepsilon\left(\sqrt{R^2+z^2}\right)^2}$$

$$\cos\theta = \frac{z}{r} = \frac{z}{\sqrt{R^2+z^2}}$$

$$dE = \frac{\lambda dl}{4\pi\varepsilon\left(\sqrt{R^2+z^2}\right)^2}$$

$$dE.\cos\theta = \frac{\lambda \cdot dl}{4\pi\varepsilon\left(\sqrt{R^2+z^2}\right)^2} \cdot \frac{z}{\sqrt{R^2+z^2}}$$

$$E = \int_0^{2\pi R} dE \cos\theta = \int_0^{2\pi R} \frac{\lambda . dl}{4\pi\varepsilon(\sqrt{R^2+z^2})^2} \cdot \frac{z}{R^2+z^2}$$

$$E = \frac{\lambda \cdot z}{4\pi\varepsilon (R^2+z^2)^{3/2}} \int_0^{2\pi R} dl$$

$$E = \frac{\lambda \cdot z \,(2\pi R)}{4\pi\varepsilon(R^2+z^2)^{3/2}}$$

E due to charged ring

E -

Given the distribution of charges on a disc, the students will be able to det. the E at a point in space.

$\lambda(2\pi R)$ = Total charge on the ring q

$$E = \frac{q . z}{4\pi\varepsilon (R^2+z^2)^{3/2}}$$

$$\boxed{dq = \sigma . dA}$$

$$\sigma_S = \frac{q}{A}$$

$$A = \pi R^2$$

$$\boxed{\sigma = \frac{dq}{dA}}$$

$$\frac{dA}{dR} = 2\pi R$$

$$dA = 2\pi R . dR$$

$$\boxed{dq = \sigma . (2\pi R . dR)}$$

$$dE = \frac{dq . z}{4\pi\varepsilon(R^2+z^2)^{3/2}}$$

$$dE = \frac{\sigma(2\pi R\, dR)\, z}{4\pi\varepsilon (R^2+z^2)^{3/2}}$$

$$E = \int dE = \int_0^{R_1} \frac{\sigma(2\pi R)\, dR . z}{4\pi\varepsilon (R^2+z^2)^{3/2}}$$

$$E = \frac{\sigma z}{4\varepsilon} \int_0^{R_1} \frac{2r\, dR}{(R^2+z^2)^{3/2}}$$

$$E = \frac{\sigma z}{4\varepsilon} \int_0^{R_1} (R^2+z^2)^{-3/2} . 2R\, dR$$

$$\int x^m dx = \frac{x^{m+1}}{m+1}$$

$$E = \int_0^{R_1} (R^2 \ldots)^{-3/2}$$

$$R^2+z^2 = x$$

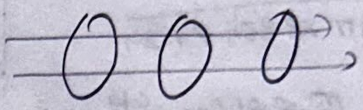
$$dx = 2R . dR.$$

$$\int x^n dx = \left[\frac{x^{-3/2+1}}{-3/2+1}\right] = \left[\frac{(R^2+z^2)^{-1/2}}{-1/2}\right]_0^{R_1}$$

$$E = \frac{-\sigma z}{4\varepsilon}\left(2(R^2+z^2)^{-1/2}\right)_0^{R_1}$$

$$E = \frac{-\sigma z}{2\varepsilon}\left[\frac{1}{\sqrt{R_1^2+z^2}} - \frac{1}{\sqrt{z^2}}\right]$$

$$E = \frac{\sigma z}{2\varepsilon}\left[\frac{1}{\sqrt{z^2}} - \frac{1}{\sqrt{R_1^2+z^2}}\right]$$



'Constant'

$$E = \frac{\sigma}{2\varepsilon}\left[1 - \frac{z}{\sqrt{R_1^2+z^2}}\right]$$

$E = \frac{q}{4\pi\varepsilon r^2}$ → point charge

$E = \frac{\lambda}{2\varepsilon\pi r}$ → line charge

$E = \frac{\sigma_s}{2\varepsilon}\cdot \hat{a_z}$ → surface charge

$E = \frac{\sigma}{2\varepsilon}\left[1 - \frac{z}{\sqrt{R_1^2+z^2}}\right]$ → Disc charge.

A S.L is parallel to z axis & passes through $(3,-3,5)$. The wire carries the uniform line charge of density $0.4\mu C/m$. Evaluate E field at $(-3,0,5)$ m

Solu:

$$E = \frac{\lambda}{2\pi\varepsilon r}$$

$$r = \sqrt{(3+3)^2+9+(5-5)^2}$$

$$= \sqrt{45}$$

$$\vec{r} = -6\hat{a_x} + 3\hat{a_y}$$

$$= \frac{0.4\mu}{2\pi \times \frac{1}{36\pi}\times 10^{-19} \times 6\sqrt{2}}$$

$m = qd$

$= 2(2)$

$= 4$

$$= \frac{0.4\mu}{\frac{2\sqrt{45}}{36}\ p}$$

$$\vec{E} = -959.16\,\hat{a_x} + 479.59\,\hat{a_y}$$

$$= 1.0733\ \text{K V/m}$$

$$\vec{E} = 1.0733\ \vec{a_r}$$

$$= \frac{1.0733(-6\hat{a_x} + 3\hat{a_y})}{\sqrt{45}}$$

A $2\mu C$ charge at (0,3,0) m & $4\mu C$ charge at (4,0,0) determine E at (0,0,5)

solu:

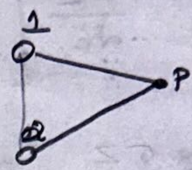
$$E = \frac{q}{4\pi\varepsilon r^2} \rightarrow \text{Point charge}$$

$$|\vec{r_1}| = \sqrt{0+9+25}$$

$$r_1^2 = 34$$

$$|\vec{r_2}| = \sqrt{16+25}$$

$$r_2^2 = 41$$



$$E_1 = \frac{q_1}{4\pi\varepsilon r_1^2}$$

$$= \frac{2\mu}{4\pi \times 8.854p \times 34}$$

$$= 5.2869 \times 10^{-4} \times 10^6$$

$$= 528.69 \text{ V/m}$$

$$\vec{E_2} = \frac{q_2}{4\pi\varepsilon r_2^2}$$

$$= \frac{4\mu}{4\pi\varepsilon \times 41}$$

$$= 876.852 \text{ V/m}$$

$$\vec{E_1} = 528.69\,\vec{a_r}$$

$$= \frac{528.69}{\sqrt{34}}(-3\vec{a_y} + 5\vec{a_z})$$

$$= -272\vec{a_y} + 453.348\,\vec{a_z}$$

$$\vec{E_2} = 876.852\,\vec{a_r}$$

$$= \frac{876.852}{\sqrt{41}}(-4\hat{a_x} + 5\hat{a_z})$$

$$= -547.765\,\hat{a_x} + 684.7063\,\hat{a_z}$$

$$\vec{E} = \vec{E_1} + \vec{E_2}$$

$$\vec{E} = (-0.547\hat{a_x} - 0.272\hat{a_y} + 1.1389\hat{a_z})K \text{ V/m}$$

$$E = 1291.97 \text{ V/m}$$

Potential

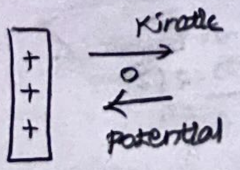
work done per unit charge in bringing the charge from ∞ to that point against the electrostatic force.

$$\frac{W}{q} = V(\text{volts}) = \frac{\text{Joules}}{\text{Columb}}$$

$$D = \varepsilon E$$

(Energy stored) → potential

During motion → Kinetic energy (Potential Energy is converted)



$$E = F/q$$

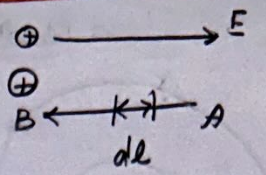
$$V = W/q$$

$$V = \frac{F \times d}{q}$$

$$V = \frac{E \times q \times d}{q}$$

$$\boxed{V = E.d} \rightarrow \text{Scalar}$$

$$dw = \vec{E}\cdot\vec{dl}$$



potential → From ∞ to a point

potential diff → From A to B.

$$\vec{E} = \frac{q}{4\pi\varepsilon r^2}\,\hat{a_r} \quad \text{(point charge)}$$

$$V = -\int_A^B \frac{q}{4\pi\varepsilon r^2}\,\hat{a_r}\cdot\vec{dr}$$

$$V = -\int_\infty^r \frac{q}{4\pi\varepsilon r^2}\,\vec{a_r}\cdot\vec{dr}\cdot\vec{a_r}$$

(r → distance b/w two charges)

$$V = -\frac{q}{4\pi\varepsilon}\int_\infty^r \frac{dr}{r^2}$$

$$V = -\frac{q}{4\pi\varepsilon}\left[\frac{r^{-1}}{-1}\right]_\infty^r$$

$$= \frac{q}{4\pi\varepsilon}\left[\frac{1}{r}-\frac{1}{\infty}\right] = +\frac{q}{4\pi\varepsilon r}$$

Electric potential (Spherical Shells)

$$V = -\int_{\text{Initial pos}}^{\text{Final pos}} \frac{q}{4\pi\varepsilon r^2}\,dr \quad \text{(potential diff)}$$

$$V = -\int_b^a \frac{q}{4\pi\varepsilon r^2}\,dr$$

$$V = \frac{q}{4\pi\varepsilon}\left[\int_b^a \frac{dr}{r^2}\right]$$

If $b = \infty$ (V = Electrostatic potential)

$$V = \frac{q}{4\pi\varepsilon}\left[\frac{1}{a}-\frac{1}{b}\right]$$

$$V = \frac{q}{4\pi\varepsilon}\left[\frac{b-a}{ab}\right]$$

Relation b/w E and V

$$dw = dv = -\vec{E}\cdot\vec{dl}$$

$$\frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy + \frac{\partial v}{\partial z}dz = -\vec{E}\cdot\vec{dl}$$

$$\left(\frac{\partial v}{\partial x}\hat{i} + \frac{\partial v}{\partial y}\hat{j} + \frac{\partial v}{\partial z}\hat{k}\right) = -\vec{E}\cdot\vec{dl}$$

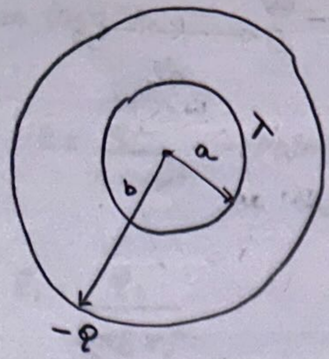
$$\nabla V\cdot\vec{dl} = -\vec{E}\cdot\vec{dl}$$

$$\boxed{\nabla V = -\vec{E}}$$

$$-\nabla V = \vec{E}$$

↳ Gradient of potential

Coaxial cylinders



Coaxial

$$E = \frac{\lambda}{2\pi r \varepsilon}$$ (λ - line charge density)

$$V = -\int_b^a E \cdot dr \cdot \hat{a_r} \cdot \hat{a_r}$$

$$V = -\int_b^a \frac{\lambda}{2\pi r \varepsilon} dr$$

$$= -\int_b^a \frac{\lambda}{2\pi r \varepsilon} dr$$

$$= \frac{-\lambda}{2\pi r} (\ln(r))_b^a$$

$$= \frac{-\lambda}{2\pi r} (\ln a - \ln b)$$

$$\boxed{V = \frac{\lambda}{2\pi r} \left(\ln\left(\frac{b}{a}\right)\right)}$$

Relation b/w E and V

$-E = \nabla V$ → Gradient of potential is Electric field intensity

$-\int E\, dl = V$

point form of Gauss law

$$\text{div}\,\vec{D} = \lim_{\Delta V \to 0} \frac{\oint \vec{D} \cdot \vec{dS}}{\Delta V}$$ → As flux lines comes out volume → 0

Also comes out Volume goes to zero.

$\boxed{\psi = \oint_S \vec{D} \cdot \vec{dS}}$ → Flux $\psi = \phi = Q_{enc}$

$$\text{div}\,\vec{D} = \lim_{\Delta V \to 0} \frac{Q}{\Delta V}$$ (volume charge density)

$$= \rho_v$$

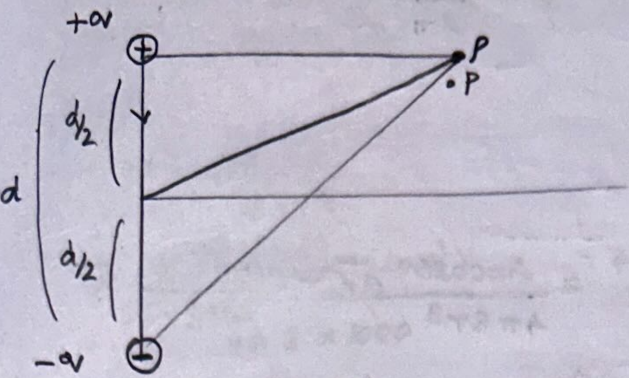
$\boxed{\nabla \cdot \vec{D} = \rho_v}$ → Scalar

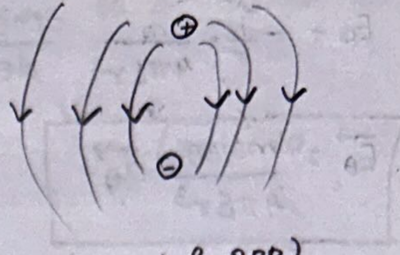
point form of Gauss law
Maxwells I eq

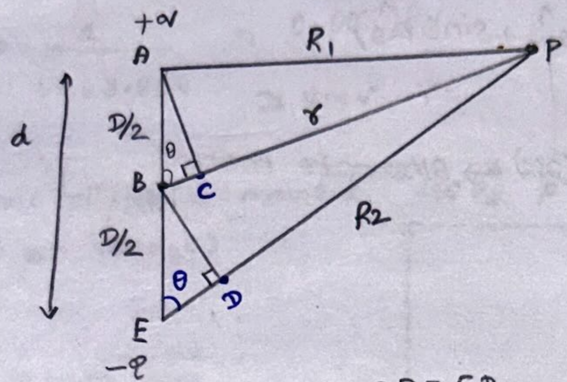
Gauss law in differential form.

Electric field intensity of a dipole

E due to Electric dipole







$BP = r$

$AP = CP$

$r - BC = CP$

$BP - BC = CP = AP$

$r - BC = R_1$

$ED + DP = EP$

$ED = \frac{D}{2}\cos\theta$

$ED = EP - DP$

$\cos\theta = \frac{adj.s}{hyp}$

$\boxed{ED = R_2 - r}$

$\frac{D}{2}\cos\theta = ED$

$R_1 = r - \frac{d}{2}\cos\theta$

$R_2 = r + \frac{d}{2}\cos\theta$

Potential at P due to A and B

$\vec{E} = E_r \hat{a}_r + E_\theta \hat{a}_\theta + E_\phi \hat{a}_\phi$

$\vec{dl} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,a_\phi$

$dv = (E_r \hat{a}_r + E_\theta \hat{a}_\theta + E_\phi \hat{a}_\phi) \cdot (dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi)$

$dv = E_r\,dr + E_\theta\,r\,d\theta$

$\frac{dv}{dr} = E_r$ & $\frac{1}{r}\frac{dv}{d\theta} = E_\theta$

$E_r = -\frac{d\left(\frac{m\cos\theta}{4\pi\varepsilon r^2}\right)}{dr}$

$E_r = \frac{m\cos\theta}{4\pi\varepsilon}\left(-\frac{d(1/r^2)}{dr}\right)$

$E_r = \frac{-(-2)\,m\cos\theta}{4\pi\varepsilon r^3}$

$\boxed{\vec{E_r} = \frac{2m\cos\theta}{4\pi\varepsilon r^3}\hat{a}_r}$

$V = +\frac{m\cos\theta}{4\pi\varepsilon r^2}$

$(V = E_r \cdot \int dr)$

$V = E_r \cdot r \cdot \hat{a}_r$

$$E_\theta = -\frac{1}{r}\,\frac{d\left(\frac{m\cos\theta}{4\pi\varepsilon r^2}\right)}{d\theta}$$

$$\vec{E_r} = \frac{2m\cos\theta}{4\pi\varepsilon r^3}\,\vec{a_r}$$

$$E_\theta = -\frac{1}{r}\cdot\frac{m}{4\pi\varepsilon r^2}\,\frac{d\cos\theta}{d\theta}$$

$$\vec{E_\theta} = \frac{m\sin\theta}{4\pi\varepsilon r^3}\,\vec{a_\theta}$$

$$\boxed{\vec{E_\theta} = \frac{m\sin\theta}{4\pi\varepsilon r^3}\,\vec{a_\theta}}$$

$$E = \vec{E_r} + \vec{E_\theta} = \frac{2m\cos\theta}{4\pi\varepsilon r^3}\,\vec{a_r} + \frac{m\sin\theta}{4\pi\varepsilon r^3}\,\vec{a_\theta}$$

$$\vec{E} = \frac{m}{4\pi\varepsilon r^3}\left(2\cos\theta\,\hat{a_r} + \sin\theta\,\hat{a_\theta}\right)$$

(or) → alternate method

$$V = V_1 + V_2$$

$$V = \frac{q_1}{4\pi\varepsilon R_1} + \frac{q_2}{4\pi\varepsilon R_2}$$

$$V = \frac{q}{4\pi\varepsilon R_1} - \frac{q}{4\pi\varepsilon R_2}$$

$$= \frac{q}{4\pi\varepsilon}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \frac{q}{4\pi\varepsilon}\left[\frac{r + \frac{d}{2}\cos\theta - r + \frac{d}{2}\cos\theta}{\left(r^2 - \frac{d^2}{4}\cos^2\theta\right)}\right]$$

$$= \frac{q}{4\pi\varepsilon}\left(\frac{d\cos\theta}{r^2 - \frac{d^2}{4}\cos^2\theta}\right) \qquad r^2 >> \frac{d^2}{4}$$

$$= \frac{q}{4\pi\varepsilon}\left(\frac{d\cos\theta}{r^2}\right)$$

$m \to$ dipole moment

$m = qd$ (C m)

columb meter

$$V = \frac{qd\cos\theta}{4\pi\varepsilon r^2} \Rightarrow V = \frac{m\cos\theta}{4\pi\varepsilon r^2}$$

$m \to$ vector

Directed from +ve to −ve

$$\boxed{\vec{m} = qd\,\vec{a_z}}$$

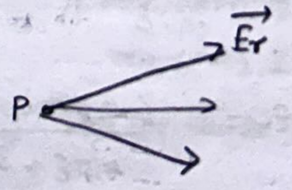
$$\vec{m}\cdot\vec{a_r} = qd\,\vec{a_z}\cdot\vec{a_r}$$

$$= qd\,|\vec{a_z}|\,|\vec{a_r}|\cos\theta$$

$$= qd\cos\theta$$

$$V = \frac{\vec{m}\cdot\vec{a_z}}{4\pi\varepsilon r^2}$$

E due to dipole



$$\vec{E} = \frac{m}{4\pi\varepsilon r^2}\left(2\cos\theta\, \hat{a_r} + \sin\theta\, \hat{a_\theta}\right)$$

Q Electric dipole of moment $100\,\vec{a_z}$ p cm is at origin. Find potential at (0,0,10)

Solu

$$r = \sqrt{100} = 10$$

$$V = \frac{\vec{m}\cdot\vec{a_r}}{4\pi\varepsilon r^3}$$

$$= \frac{(100\,\hat{a_z})\,p\,(10\,\hat{a_z})}{4\pi\varepsilon \times 1000}$$

$$= \frac{1}{4\pi \times 8.854} = 0.00898V$$

$$\approx 9mV$$

A Electric dipole of moment $100\,\hat{a_z}$ p cm is at origin. Find potential at (0,0,10)

Solu:

$$V = \frac{\vec{m}\cdot\vec{a_r}}{4\pi\varepsilon r^3}$$

$$= \frac{(100\,\hat{a_z})\,p\,(10\,\hat{a_z})}{4\pi\varepsilon \times 10^3}$$

$$= \frac{1}{4\pi \times 8.854}$$

$$= 0.00898V$$

$$\approx 0.898 \times 10^{-2}$$

$$\approx 9mV$$

2/4/2021

Conductors (up to this - we have seen materials in free space)

$\varepsilon = \varepsilon_0$ ↓ Air

Air $\varepsilon_r = 1$

$\boxed{\varepsilon = \varepsilon_0 \varepsilon_r}$

ε_0 → Permittivity of free space

ε_r → relative permittivity of a medium

Vacuum field theory - 'Electrostatics considered so far'

Conductors in static Electric field: (Free valance e^-s)

Source $\oplus$ → $\ominus$ Sink

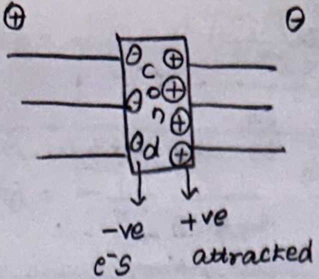
Prop 1: Net electric field inside a conductor is zero.

(+ve to -ve)

$\oplus \longrightarrow \ominus$

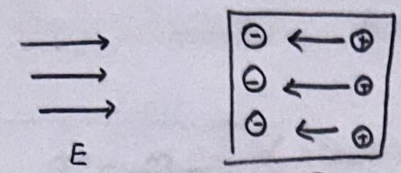
(Expose a Conductor)

'Induced Electric field'



'Net electric field $E_N = E - E_i$'

$\therefore$ $\boxed{E_N = 0}$



E_i
(Induced)

Property: 2

when E zero, potential difference will be zero.

'Net charge inside a conductor is zero

* Gauss law.

$\varepsilon \oint_S E . ds = q_{enc}$

$\oint_S \vec{D} . \vec{ds} = q_{enc}$

$\therefore E_N = 0$

$\varepsilon \oint_S 0 . ds = q_{enc}$

$q_{enc} = 0C$

$\boxed{P = \frac{q}{V}}$ $q = 0, P = 0$

$E . d = V$

$0 . d = V$ $\boxed{V = 0}$

Property: 3

charges always reside on the surface of the conductor.

condition: Conductor placed in static electric field.

LCD → uses polarization

Dielectrics → types
→ polarisation vector

(Electric susceptibility)

All dielectrics are insulators but not all insulators are dielectrics

when ever a dipole is there - there will be a dipole moment.

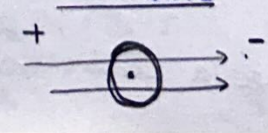
If we can polarize an insulator → we can call that insulator is dielectric

types ↗ Non-polar ( → Electron cloud's centre & +ve charge's centre aligns → Non-polar molecule

↘ polar

↳ 'Natural dipole' - There will be a shift b/w +ve & negative centres'

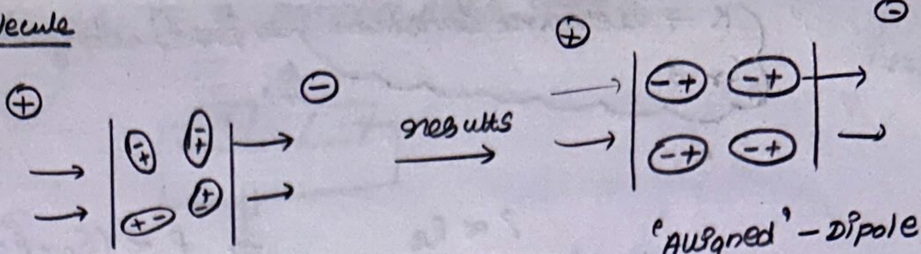
Dielectric



Non-polar molecule

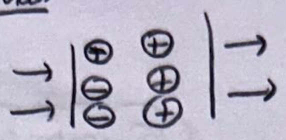
'shift' due to $E \rightarrow$ Dipole created.

Polar molecule

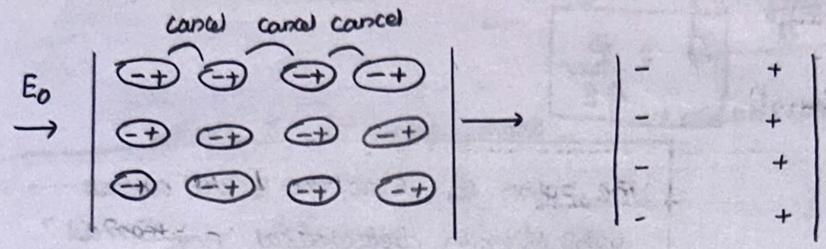


'Aligned' - Dipole.

Non-polar



'Non-polar'



$E_P < E_0$

'Due to cancellation'

Net electric field

$E_N = E_0 - E_P$
↓
E_R (Reduced E)

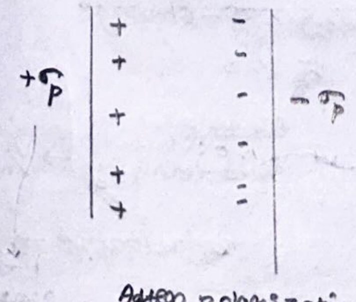
Polarization vector

$$P = \frac{\text{Dipole moment}}{\text{unit volume}} = \frac{q \cdot d}{V} = \frac{q \cdot d}{d^3} = \frac{q}{A}$$

$$\boxed{\vec{P} = \frac{q}{A}}$$

Electric susceptibility (How sensitive is the dielectric to E ?)
↓
Larger ψ - polarization large

ψ (chi) =



After polarization (Throughout the surface)

Electric field intensity of a surface charge density $= \dfrac{\sigma_s}{2\varepsilon}$ (For one surface) $\left(\begin{array}{c}\text{Air} \\ \varepsilon = \varepsilon_0\end{array}\right)$

$\therefore$ Two surfaces $\left(\dfrac{2\sigma}{2\varepsilon}\right) = \dfrac{\sigma}{\varepsilon}$

$E_R = E_0 - E_P$
↓
Polarization field. (opp. direc)

$K = \dfrac{E_0}{E_R}$ → Applied Electric field
→ Reduced E

9/8/2021

$K \rightarrow$ dielectric constant
$\varepsilon_r \rightarrow$
$\rightarrow$ Interchangable

$P \propto E_R$ (polarization vector) $\Rightarrow P \propto (E_0 - E_P)$

Prop constants

$P = \varepsilon \psi E_R$

$\boxed{E_p = \frac{\sigma_p}{\varepsilon}}$ $\rightarrow$ For two planes

$E_p = \frac{\frac{Q}{A}}{\varepsilon}$

$\boxed{E_p = \frac{Q}{A\varepsilon}}$

$\sigma_p \rightarrow$ polarization

$E_R = E_0 - E_p$

$= E_0 - \frac{\sigma_p}{\varepsilon}$

$= E_0 - \frac{\varepsilon \psi E_R}{\varepsilon}$

$E_R = E_0 - \psi E_R$

$E_0 = E_R + \psi E_R$

$E_0 = E_R(1+\psi)$

$\frac{E_0}{E_R} = 1 + \psi$

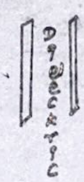
$\boxed{K = 1 + \psi}$ $\rightarrow$ Susceptibility.

$K \rightarrow$ dielectric constant

eg: mica, ceramics, silk

The point of Electric field about which the dielectrical material becomes conductor $\longrightarrow$ Dielectric breakdown.

Capacitors



"Dielectric sandwiched b/w two parallel plates"

Dielectrics (property)

* 'Store charges'
* polarized

$C = \frac{Q}{V}$ (capacitance of a capacitor) Q - charge

$C \propto \frac{A}{d}$

$\boxed{K = \varepsilon_r}$

$C = \frac{K \varepsilon_0 A}{d}$

$\varepsilon = \varepsilon_0 \varepsilon_r$

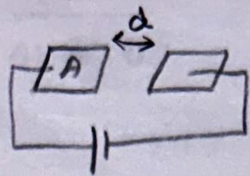
$\boxed{C = \frac{\varepsilon A}{d}}$

Electric field intensity $\rightarrow$ V/m

capacitance $\rightarrow \frac{\text{Columb}}{\text{Volts}}$

Potential $\rightarrow$ volts.

Capacitance of a parallel plate capacitor



$D = \frac{Q}{A} = \sigma$

(Surface charge density)

$D = \varepsilon E$

$\varepsilon E = \frac{Q}{A}$

$\boxed{Q = A\varepsilon E}$ $\quad E = \frac{V}{d}$

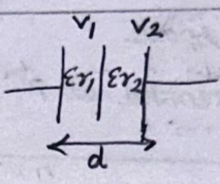
$Q = A\varepsilon \frac{V}{d}$

$C = \frac{Q}{V}$

$\boxed{C = \frac{A\varepsilon}{d}} \rightarrow$ × (unit) → Farad

$C = \frac{A\varepsilon_0 \varepsilon_r}{d}$

when d becomes half $\quad C \rightarrow$ becomes twice.



$\frac{Q}{A} = \sigma$

$\boxed{\frac{\sigma}{2\varepsilon} = E}$ → one conducting plate.

$V = V_1 + V_2$

$V_1 = E_1 d_1$

$V_2 = E_2 (d - d_1)$

$\boxed{E = \frac{\sigma}{\varepsilon} = \frac{Q}{A\varepsilon}}$

$E = \frac{Q}{\varepsilon A}$

$V = E_1 d_1 + E_2 (d - d_1)$

$V = \frac{Q d_1}{\varepsilon_0 \varepsilon_{r_1} A} + \frac{Q (d - d_1)}{\varepsilon_0 \varepsilon_{r_2} \cdot A}$

$= \frac{Q}{\varepsilon_0 A}\left[\frac{d_1}{\varepsilon_{r_1}} + \frac{(d - d_1)}{\varepsilon_{r_2}}\right]$

$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{\left(\frac{d_1}{\varepsilon_{r_1}} + \frac{(d - d_1)}{\varepsilon_{r_2}}\right)}$

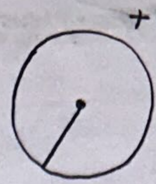
$$\boxed{C = \frac{A \varepsilon_0 \varepsilon_{r_1} \varepsilon_{r_2}}{\varepsilon_{r_2} d_1 + \varepsilon_{r_1} (d - d_1)}}$$

one medium air $\varepsilon_{r_1} = 1$

$\varepsilon_{r_2} = \varepsilon_r$

$C = \frac{A \varepsilon_0 \varepsilon_r}{\varepsilon_r d_1 + d - d_1}$

Isolated sphere



$$V = +\frac{Q}{4\pi\varepsilon r}$$

$$C = \frac{Q}{V}$$

$$4\pi\varepsilon r = \frac{Q}{V}$$

$$\boxed{C = 4\pi\varepsilon r}$$

Problems

1) Find the capacitance of a $\parallel^{el}$ plate capacitor having the plates of Area $1.5m^2 \rightarrow d = 2mm$. (potential gradient is $10^5\, V/m$) $\rho_s = 2.5\mu C/m^2$

2) 2 point charges $\rightarrow$ $1.5\,nC$ have $(0,0,0.1)$
$-1.5nC$ at $(0,0,-0.1)$ at free space.
'Dipole at origin' – potential at P $(0.3, 0, 0.4)$

---

1) $\nabla V = 10^5\, V/m$, $\rho_s = 2.5\mu C/m^2$

$-E = \nabla V$,

$$V = -\int E\, dl$$

$$C = \frac{A\varepsilon}{d}$$

$$E = \frac{V}{d}$$

$$C = \frac{Q}{V}$$

$$V = Ed$$

$$V = -10^5 \times 2\times10^{-3}$$

$$= -2\times10^2$$

$$\boxed{V = -200 \text{ volts}}$$

$$C = \frac{3.75\mu}{-200}\left(\frac{C}{V}\right)$$

$$\boxed{C = -0.1875\, nf}$$

$$D = \frac{Q}{A} = \sigma_s$$

$$C = 0.1875\, nf$$

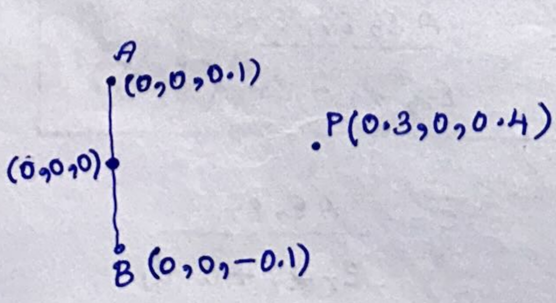
$$\boxed{Q = A\sigma_s}$$

negative sign means the capacitor is discharging (against cause)

$$Q = 1.5 \times 2.5\mu \left(\frac{C}{m^2}\times m^2\right)$$

$$\boxed{Q = 3.75\mu C}$$

2)



$$V = \frac{Qd}{4\pi\varepsilon_0 r^2}$$

$r_1 = \sqrt{0.3^2 + 0.3^2}$

$= 0.424$

$V_1 = \frac{1.5n}{4\pi \times 8.854 \times 10^{-12} \times 0.424^2}$

$\boxed{V_1 = \cancel{31.7962\,V}}$

$V_1 = \frac{1.5n}{20 \times 10^{-12}}$

$= \frac{1.5K}{20}$

$\boxed{V_1 = 75V}$

$r_2 = \sqrt{0.3^2 + 0.5^2}$

$r_2 = 0.583$

$V_2 = \frac{-1.5n}{4\pi \times 8.854 \times 10^{-12} \times 0.583^2}$

$V_2 = \frac{-1500}{37.8169}$

$\boxed{V_2 = -39.66V}$

$V = V_1 + V_2$

$= 75 - 39.66$

$\boxed{V = 35.33V}$

$\cancel{V = \frac{Qd}{4\pi\varepsilon r^2}} = \cancel{1.5 \times 10^{-8}}$

$V = \frac{\vec{m}\ \vec{a_z}}{4\pi\varepsilon r^2}$

$V = \frac{Qd}{4\pi\varepsilon r^2}(2\cos\hat{a_r} + \sin\theta\ \hat{a_\theta})$

$\cancel{V = (1.5n)^2}$

$\cancel{R = 1.5n}$

$1.5nC$ • $(0,0,0.1)$

• $(0,0,0)$

$-1.5nC$ • $(0,0,-0.1)$

$$V = \frac{\vec{m} \cdot \vec{a_z}}{4\pi\varepsilon}$$

Dipole at origin

p at $(0.3, 0, 0.4)$

$(0.3\hat{a_x} + 0.4\hat{a_y})$

$|\hat{a_z}| = \sqrt{0.3^2 + 0.4^2}$

$= 0.5$

$= \sqrt{0.2^2}$

$= 0.2$

$Ap = \sqrt{0.3^2 + 0.3^2}$

$= \sqrt{0.6^2}$

$\boxed{Ap = 0.6}$

$$V = \frac{Q \cdot d}{4\pi\varepsilon r^3} \cdot \vec{a_z}$$

$$= \frac{1.5n \times (0.2)}{4\pi \times 8.854 \times 10^{-12} \times (0.5)^3} (0.3\hat{a_x} + 0.4\hat{a_y})$$

$$= \frac{0.3n}{13.9078 \times 10^{-12}} \hat{a_z}$$

$$= 0.02157 \times 10^3$$

$$= 21.57\,(0.3\hat{a_x} + 0.4\hat{a_y})$$

$$\boxed{V = 6.471\,\hat{a_x} + 8.628\,\hat{a_y} \quad V/m}$$

~~$V = 1.5n \times (0.2\hat{a_z})$~~

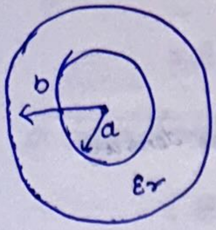
$$V = \frac{Q\,d\cos\theta}{4\pi\varepsilon r^2}$$

$$\theta = \tan^{-1}\left(\frac{opp.s}{adj.s}\right)$$

$$= \tan^{-1}\left(\frac{R_1}{\frac{D}{2}}\right)$$

$$= \tan^{-1}\Big($$

Capacitance for coaxial & spherical capacitors



$$V = +\frac{Q}{4\pi\varepsilon}\left[\frac{1}{a}-\frac{1}{b}\right]$$

spherical shells

$$C = \frac{Q}{V}$$

$$= \frac{1}{\frac{1}{4\pi\varepsilon}\left[\frac{1}{a}-\frac{1}{b}\right]}$$

$$= \frac{1}{\frac{1}{4\pi\varepsilon}\left[\frac{b-a}{ab}\right]}$$

$$\boxed{C = \frac{4\pi\varepsilon ab}{b-a}}$$

$\varepsilon = \varepsilon_0 \varepsilon_r$ (when air) $\boxed{\varepsilon_0 = \varepsilon}$

$\therefore \varepsilon_r = 1.$

co-axial cable

$$V = \frac{\lambda}{2\pi\varepsilon}\ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{V}$$

$C = \frac{\lambda}{V}$ (line charge density)

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{b}{a}\right)}$$

$$\boxed{C = \frac{2\pi\varepsilon}{\ln\left(\frac{b}{a}\right)}}$$

$$C = \frac{0.0241\,\varepsilon_r}{\log_{10}\left(\frac{b}{a}\right)}$$

$\ln \rightarrow \log_{10}$

Boundary conditions

Upto this: homogeneous medium. (Interface: when two media meet)

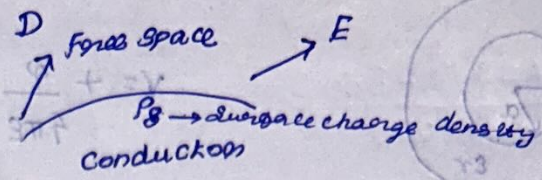
Dielectric - Dielectric

Condu - Dielectric

Conductor - free space.

Conductor – Free space

Conductor : $E_{net} = 0, V = 0$



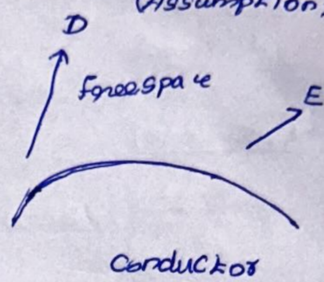
$\oint E \cdot dl = 0$

$E = 0, \; Q = 0,$

(Dot product 0 → conservative field
Solenoidal field)

$\rho_v = 0$ (Volume)

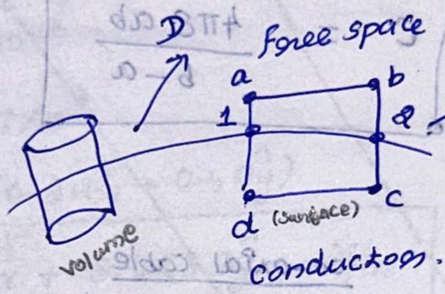
(Assumption)

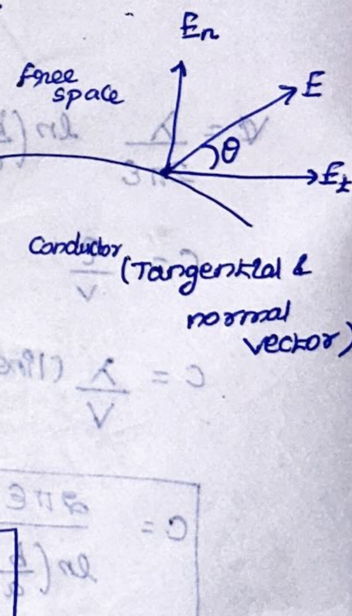


$\oint D \cdot ds = Q$ [Gauss law]

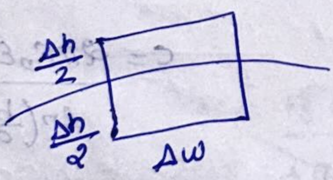
$\oint E \cdot dl = 0$ [closed path]

$\oint D \cdot ds = Q$
↳ surface





$E = E_t + E_n$



$\oint E \cdot dl = 0$

$$\boxed{\int_{ab} + \int_{bc} + \int_{cd} + \int_{da}}$$

$$\oint E \cdot dl = \int_a^b \cdot + \int_b^c + \int_c^d + \int_d^a = 0$$

$$= \int_a^b + \int_b^c + \int_d^a + 0$$

(E – assumed as constant)

$$\int_a^b E \cdot dl = E \cdot \int_a^b dl$$

Δh is small, Δw – small

(So E doesn't vary that much.

$$\int_a^b E \cdot dl = E\int_a^b dl$$

$$= E(b-a)$$

a to b (along E_t)

$$= E(\Delta w)$$

$$= E_{tan}(\Delta w)$$

$$\int_b^c E \cdot dl = E\int_b^c dl$$

$$= E\int_b^2 dl + E\int_2^c dl$$

$$= E\int_b^2 dl + 0$$

(opp to E_n)

$$= -E_n \cdot \frac{\Delta h}{2}$$

$$\int_d^a E \cdot dl = \int_d^1 E \cdot dl + \int_1^a E \cdot dl$$

$$= \int_1^a E \cdot dl$$

$$= E \cdot \frac{\Delta h}{2} \text{ (along } E_{normal})$$

$$= E_{normal} \frac{\Delta h}{2}$$

$$E_{tan}\Delta w + E_{nor}\frac{\Delta h}{2} - E_{nor}\frac{\Delta h}{2} = 0$$

$$E_{tan}(\Delta w) = 0$$

$\because \Delta w \neq 0$

$$\boxed{E_{tan} = 0}$$

'No tangential format'

$$R_1 = \sqrt{0.3^2 + 0.3^2}$$

$$R_2 = \sqrt{0.3^2 + 0.5^2}$$

$$d = \sqrt{0.2^2}$$

$$R_1 = r - \frac{d}{2}\cos\theta$$

$$R_2 = r + \frac{d}{2}\cos\theta$$

$$R_2 - R_1 = d\cos\theta$$

$$\cos\theta = \frac{R_2 - R_1}{d} = \frac{0.583 - 0.4242}{0.2} = 0.794.$$

$$\theta = \cos^{-1}\left(\frac{R_2 - R_1}{d}\right) = \cos^{-1}\left(\frac{0.583 - 0.4242}{0.2}\right)$$

$$\boxed{\theta = 37.43^\circ}$$

$$V = \frac{m\cos\theta}{4\pi\varepsilon r^2}$$

$$V = \frac{Qd\cos\theta}{4\pi\varepsilon r^2}$$

$$V = \frac{1.5n \times 0.2 \times 0.794}{4\pi \times 8.854 \times 10^{-12} \times 0.5^2}$$

$$= \frac{0.2382n}{27.81566 \times 10^{-12}}$$

$$= \frac{0.2382\,k}{27.81566}$$

$$\boxed{V = 8.5635 \ V/m}$$

$$\cos\theta = \frac{R_2 - R_1}{d}$$

$$= \frac{0.683 - 0.4242}{0.2}$$

$$= 0.794$$

$$= \sqrt{0.3^2 + 0.4^2}$$

$$= 0.5$$

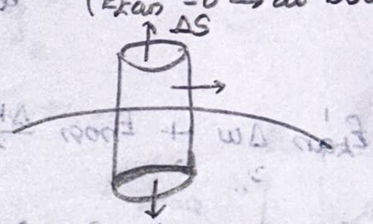
(spherical)

$$\cos\theta = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$\oint D \cdot ds = Q$$

$$\oint D \cdot ds = \underset{Top}{\oint} + \underset{Bottom}{\oint} + \underset{curved}{\oint}$$

($E_{tan} = 0 \rightarrow$ at boundary)



closed volume

$\boxed{\underset{Bottom}{\oint} = 0}$ → Conductor

$E = 0, D = 0.$

* Top
* Bottom
* curved.

$\oint$ lateral $= 0$

(curved surface Area $= 2\pi r h$)

$= 2\pi r \Delta h$ ($\Delta h \rightarrow$ very small)

$\frac{\Delta h}{2} \rightarrow$ ∴ negligible

∴ $\frac{\Delta h}{2}$ of cond

$\oint$ lateral $= 0$

cond

$\boxed{\oint \text{lateral} \approx 0 \quad \text{Free space}}$

$$\oint_{top} = \underset{normal}{\oint} D \cdot ds = \underset{normal}{\oint} D_N \, ds$$

$$= D_N \underset{normal}{\oint} (ds) = D_N \cdot \Delta S = Q$$

$$Q = \rho_S \Delta S$$

$$D_N \Delta S = \rho_S \Delta S$$

$$\boxed{D_N = \rho_S}$$

$$\boxed{E_N = \frac{\rho_S}{\varepsilon_0}}$$

$\varepsilon = \varepsilon_r \varepsilon_0$

($\varepsilon_r = 1$ Air)

Inference:

$E_{tan} = 0$

$E_N = \frac{\rho_s}{\varepsilon_0}$

For a Conductor – Freespace BC

* E inside a cond is 0 $(E_{tan} = 0, V = 0)$
* Conductor surface is an equipotential surface.
* Conductor surface
* Static E is directed normal to the surface

Everywhere.

$\boxed{5V\ 5V\ 5V}$

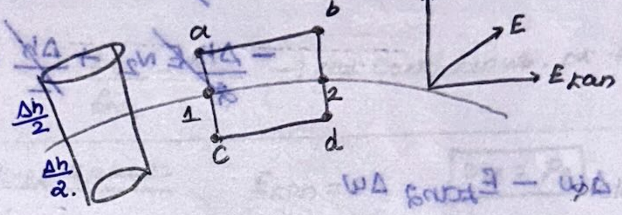
$\therefore 5V - 5V = 0$

(Equipotential)

Same potential

Conductor – Dielectric

Dielectric



Conductor.

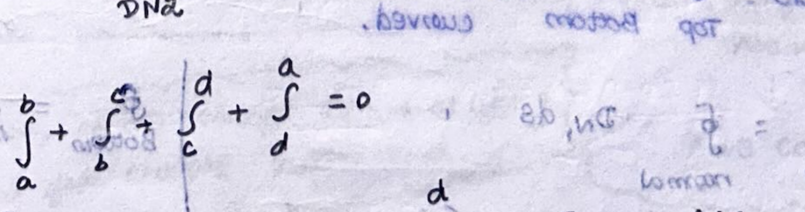
$$\boxed{\begin{aligned} E_{tan} &= 0 \\ E_n &= \frac{\rho_s}{\varepsilon_0 \varepsilon_r} \end{aligned}}$$

$D_N = \rho_s$

$E_{tan} = D_{tan} = 0$

Dielectric – Dielectric

Dielectric



Dielectric

$$E_{tan} = \int_a^b + \int_b^c + \int_c^d + \int_d^a = 0$$

$$\int_a^b E \cdot dl = E \cdot \int_a^b dl$$

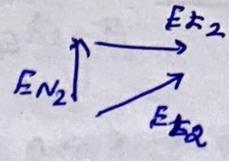
$$= E\,(b-a)$$

$$= E_{tan1}\,(\Delta w)$$

$$E \int_c^d dl = -E_{tan2}\,\Delta w$$

① $E\int_a^b dl = E_{tan1}\,\Delta w$

② $E\int_c^d dl = E_{tan2}\,(-\Delta w)$

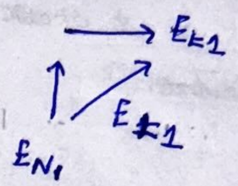


③ $E\int_b^c dl = \int_b^2 E\cdot dl + \int_2^c E\cdot dl$

$= -\frac{\Delta h}{2}E_{N1} + \frac{\Delta h}{2}E_{N2}$

$E_2 = E_{t2} + E_{N2}$

$E_1 = E_{t1} + E_{N1}$



④ $E\int_d^a = \int_d^1 E\cdot dl + \int_1^a E\cdot dl$

$= -\frac{\Delta h}{2}E_{N2} + \frac{\Delta h}{2}E_{N1}$

$E_{tan} = E_{tan1}\,\Delta w - E_{tan2}\,\Delta w - \cancel{\frac{\Delta h}{2}E_{N1}} + \cancel{\frac{\Delta h}{2}}E_{N2}$

$- \cancel{\frac{\Delta h}{2}E_{N2}} + \cancel{\frac{\Delta h}{2}}E_{N1}$

$E_{tan} = E_{tan1}\,\Delta w - E_{tan2}\,\Delta w$

$E_t = \Delta w\,(E_{t1} - E_{t2})$

$0 = \Delta w\,(E_{t1} - E_{t2})$.

$\boxed{E_{t1} = E_{t2}}$ → Both tangential forces are equal

E is continuous at the boundary. $\left[\frac{D_{tan1}}{\epsilon_1} = \frac{D_{tan2}}{\epsilon_2}\right]$

<u>Expression for D</u>

$\oint D\cdot ds = \oint_{Top} + \oint_{Bottom} + \oint_{curved}$.

(opp. direction)

$\oint_{Top} = \oint_{normal} D_{N1}\,ds$

$= D_{N1}\oint ds = D_{N1}\,\Delta S$

$\rho_{S1}\cancel{\Delta S_1} = D_{N1}\cancel{\Delta S_1}$

$\rho_{S1} \to D_{N1}$

$\oint_{Bottom} = -\oint_{nor} D_{N2}\,ds$

$= -D_{N2}\,\Delta S$.

$\oint_{curve} = \oint_{nor} \cancel{D}$

$= 0$ (As Δh is small).

$= D_N(2\pi r h)$ (As $h \to 0$)

$D_{N1}\Delta S - D_{N2}\Delta S = \rho_S \Delta S$ $\left(\frac{Q}{\text{Surface}} = \rho_S\right)$

$D_{N1} - D_{N2} = \rho_S$ ($\rho_S = 0$ for ideal dielectric)

$D_{N1} = D_{N2}$

$E_{N1}\varepsilon_1 = E_{N2}\varepsilon_2$

$$\boxed{\frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_2}{\varepsilon_1}} \qquad \frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_{r2}\cdot\cancel{\varepsilon_0}}{\varepsilon_{r1}\cancel{\varepsilon_0}} = \frac{\varepsilon_{r2}}{\varepsilon_{r1}}$$

Maxwell eqn

$\int E\,dl = 0$

$\int D \cdot ds = Q$ (gauss law)

$D_{tan1} = \frac{\varepsilon_1}{\varepsilon_2} D_{tan2}$ $\longrightarrow$ Not continuous due to $\frac{\varepsilon_1}{\varepsilon_2}$.

Note: No free charges in dielectrics

$\therefore \rho_S = 0$ in dielectrics.

$D_{N1} = D_{N2}$

($D_N \rightarrow$ Normal component D is continuous)

$\frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_2}{\varepsilon_1}$ $\rightarrow$ Not continuous at the boundary.

---

① conductor free space:

$E_{tan} = 0$

$E_N = \frac{\rho_S}{\varepsilon_0}$

$\boxed{D_N = \rho_S}$, $D_{tan} = 0$

② Conductor - dielectric:

$E_{tan} = 0$

$E_N = \rho_S / \varepsilon_0\varepsilon_r$

$D_{tan} = 0$

$D_N = \rho_S$

③ Dielectric Dielectric:

$E_{t1} = E_{t2}$, $\frac{D_{t1}}{\varepsilon_1} = \frac{D_{t2}}{\varepsilon_2}$

$\frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_2}{\varepsilon_1}$, $D_{N1} = D_{N2}$

Poisson & Laplace eqn

when?

Poisson: Boundary condt: charge dest → at bound

we can find for entire region.

when charge density is known (entire region)

↓

"we can find E & V"

① poisson's eqn

② Laplace eqn.

③ method of images

Find E & V across the region when the value of charge or potential at its boundary is known.

* Derived from point's form of Gauss law.

Poisson's eqn:

Relates V & charge density that generates it

derived from point form of Gauss law

$$\nabla \cdot D = \rho_v \longrightarrow \text{point form}$$

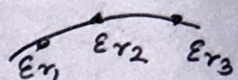
$$\boxed{D = \varepsilon E} \qquad \boxed{E = -\nabla V}$$

$$\nabla \cdot \varepsilon E = \rho_v$$

$$\nabla \cdot \varepsilon(-\nabla V) = \rho_v$$

Non-Homogeneous medium

Permittivity depends upon distance.



Homogeneous medium

$$\varepsilon \nabla(-\nabla V) = \rho_v$$

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$$

ε – constant for homogeneous medium.

Different coordinate systems:

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$$

$$\nabla V = \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z$$

$$\nabla \cdot \nabla V = \left(\frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z\right)\left(\frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z\right)$$

$$\boxed{\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho_v}{\varepsilon} \quad \text{(scalar)}}$$

Cartesian

Cylindrical

$$\nabla V = \frac{1}{h_1} \frac{\partial F_1}{\partial \rho} a_\rho + \frac{1}{h_2} \frac{\partial F_2}{\partial \phi} a_\phi + \frac{1}{h_3} \frac{\partial F_3}{a_z} a_z$$

Given a field $E = \frac{-6y}{x^2} a_x + \frac{6}{x} a_y + 5 a_z$ V/m. Find potential diff V_{AB} b/w $A(-7, 2, 1)$ & $B(4, 1, 2)$

$$V_{AB} = -\int_B^A E \cdot dl$$

Soln:

$$E = \left[\frac{-6y}{x^2} a_x + \frac{6}{x} a_y + 5 a_z\right] \text{ V/m}$$

$$dl = dx\, \hat{a_x} + dy\, \hat{a_y} + dz\, \hat{a_z}$$

$$V_{AB} = -\int_4^{-7} \left(\frac{-6y}{x^2} a_x + \frac{6}{x} a_y + 5a_z\right)(dx\, a_x + dy\, a_y + dz\, a_z)$$

$$= -\int_4^{-7} \left(\frac{-6y\,dx}{x^2} + \frac{6}{x} dy + 5dz\right)$$

$$= \int_4^{-7} \left(\frac{6y\,dx}{x^2} - \frac{6}{x} dy - 5dz\right)$$

Line equ

$$\frac{x - x_b}{x_b - x_a} = \frac{y - y_b}{y_b - y_a} = \frac{z - z_b}{z_b - z_a}$$

$$\frac{x-4}{4-(-7)} = \frac{y-1}{+1-2} = \frac{z-2}{2-1}$$

$$\frac{x-4}{11} = \frac{y-1}{-1} = \frac{z-2}{1}$$

$$\frac{x-4}{11} = \frac{y-1}{-1} = \frac{z-2}{1}$$

$\cancel{-x+4 = 11y - 11}$

$$\frac{x-4}{11} = \frac{y-1}{-1} \qquad \Bigg| \qquad \frac{x-4}{11} = \frac{z-2}{1}$$

$$-x + 4 = 11y - 11 \qquad \Bigg| \qquad x - 4 = 11z - 22$$

$$y = \frac{15 - x}{11} \qquad\qquad z = \frac{18 + x}{11}$$

$$dy = \frac{-dx}{11} \qquad\qquad dz = \frac{dx}{11}$$

$$V_{PQ} = \int_4^{-7} \left(\left(\frac{6y}{x^2}\right) dx - \left(\frac{6}{x}\right) dy - 5\,dz \right)$$

$$= \int_4^{-7} \frac{6}{x^2}\left(\frac{15-x}{11}\right) dx - \frac{6}{x}\int_4^{-7} -\frac{dx}{11} - 5\int_4^{-7}\frac{dx}{11}$$

$$= \int_4^{-7}\left(\frac{90}{11}x^{-2} - \frac{6}{11x}\right)dx + \frac{6}{11x}\int_{-7}^{4} \cancel{dx} - \frac{5}{11}\left[x\right]_4^{-7}$$

$$= \left[\frac{90}{11}\cdot\frac{x^{-1}}{-1}\right]_4^{-7} - \cancel{\left(\frac{6}{11x}\int_4^{-7}\frac{dx}{x} + \frac{6}{11}\int\frac{dx}{x}\right)} - \frac{5}{11}\left[x\right]_4^{-7}$$

$$= \left(\frac{-90}{11x}\right)_4^{-7} - \frac{5}{11}\left[x\right]_4^{-7}$$

$$= \frac{-90}{11\times -7} + \frac{90}{44} - \frac{5}{11}(-7-4)$$

$$= \frac{-90}{-77} + \frac{90}{44} - \frac{5(-11)}{11}$$

← (Against the field)

↓

Force

$$= \frac{90}{77} + \frac{90}{44} + 5$$

$$= 1.16883 + 2.04545 + 5$$

$$\boxed{V_{AB} = 8.21425\ V}$$

Electrostatic Energy density

* Derive expressions for

  * Energy density
  * Energy stored in a capacitor

$\frac{W}{Q} = V$ (Work done per unit test charge ↓ Potential)

case: 1: If there are no point charges in space: $(W=0)$

: No charges

$E=0$

• Q_1

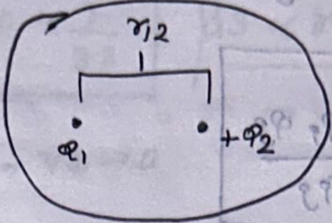
(Test charge)

No charges → $E=0$, $W=0$, $F=0$

Case:2: one point charge in space

$$V = \frac{q}{4\pi\varepsilon r}$$

$$\frac{W}{q_2} = V_1$$



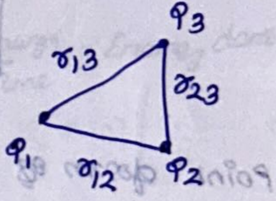
$$\frac{W}{q_2} = \frac{q_1}{4\pi\varepsilon r_{12}} = \frac{Kq_1}{r}$$

$$W = \frac{q_1 q_2}{4\pi\varepsilon r_{12}} = \frac{K q_1 q_2}{r_{12}}$$

$$\boxed{W = \frac{K q_1 q_2}{r_{12}}}$$

case:3 Two point charges in space

$q_3 \rightarrow$ work done against q_1 & q_2



$$\frac{W}{q_3} = V_1 + V_2$$

$$W = q_3 (V_1 + V_2)$$

$$W = q_3 \left(\frac{Kq_1}{r_{13}} + \frac{Kq_2}{r_{23}} \right)$$

$$W = \left(\frac{K q_1 q_3}{r_{13}} + \frac{K q_2 q_3}{r_{23}} \right)$$

$$\boxed{W = \frac{K q_1 q_4}{r_{14}} + \frac{K q_2 q_4}{r_{24}} + \frac{K q_3 q_4}{r_{34}}} \rightarrow \text{3 point charges in space}$$

__n charges__

$$W = \frac{K q_1 q_n}{r_{1n}} + \frac{K q_2 q_n}{r_{2n}} + \dots \frac{K q_{n-1} q_n}{r_{(n-1)n}}$$

__Total Energy:__

$$W = 0 + \frac{K q_1 q_2}{r_{12}} + \frac{K q_1 q_3}{r_{13}} + \frac{K q_2 q_3}{r_{23}} + \frac{K q_1 q_4}{r_{14}} + \frac{K q_2 q_4}{r_{24}} +$$

$$+ \frac{Kq_3q_4}{r_{34}} + \cdots$$

(No charge + 1 charge + 2 charge + 3 charge + ...)

$$\boxed{W_{tot} = \frac{1}{2}\sum_{p \neq q}^{n} \frac{Kq_p q_q}{r_{pq}}}$$

$$\boxed{\frac{Kq_1q_2}{r_{12}} = \frac{Kq_2q_1}{r_{21}}} \rightarrow \text{Two times coming so divide by 2}$$

$$W_{tot} = \frac{1}{2}\sum_{p=1}^{n} \frac{q_p Kq_q}{r_{pq}}$$

$$\boxed{= \frac{1}{2}\sum_{p=1}^{n} q_p V_p}$$

Point form of Gauss law $\nabla \cdot D = \rho_v$

$$W_{tot} = \frac{1}{2}\sum_{p=1}^{n} q_p \cdot V_p$$

$$W_{tot} = \frac{1}{2}\int \rho_v \cdot dv$$

$$= \frac{1}{2}\int (\nabla \cdot D)\, dv \cdot V$$

$$\boxed{\nabla \cdot (AB) = A \cdot \nabla \cdot B + B \cdot \nabla \cdot A}$$

$$W_{tot} = \frac{1}{2}\int [(\nabla \cdot VD) - D \cdot \nabla \cdot V]\, dv$$

$$\nabla \cdot (\overset{s\ v}{AB}) = \overset{s}{A} \cdot \overset{v}{\nabla} \cdot B + \overset{v}{B} \cdot \nabla \cdot \overset{s}{A}$$

s - scalar,
v - vector

Divergence theorem

Relation b/w Volume & Surface.

$$W_{tot} = \frac{1}{2}\int (\nabla \cdot VD)\, dv - \frac{1}{2}\int D \cdot \nabla V\, dv$$