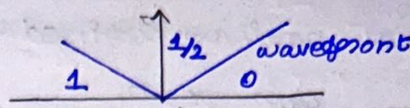
(So the upper half of the plane is divided by a V-shaped wavefront from the graph $|x|$). Into three regions. with the values 1 on the left, $\frac{1}{2}$ in the middle, and 0 on the right.



Remark 9.1

* In one dimension, a disturbance created wave fronts moving to the left & right, and the space-time diagram of the wave front was shaped like a V.

* In two dimensions, the disturbance caused by a pebble dropped in a still pond creates a circular wave front that moves outward in all directions. The space-time diagram of this wave front is shaped like an ice-cream cone (without the icecream).

* In three dimensions, the wave front created by a disturbance at a point is an expanding sphere.

D-Alembert on half-infinite integral.

Consider an infinitely long string that is fixed at one end.

model this by the wave eqn on the half-line $x>0$,

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \qquad x>0,\ t>0$$

where the fixed end is at $x=0$.

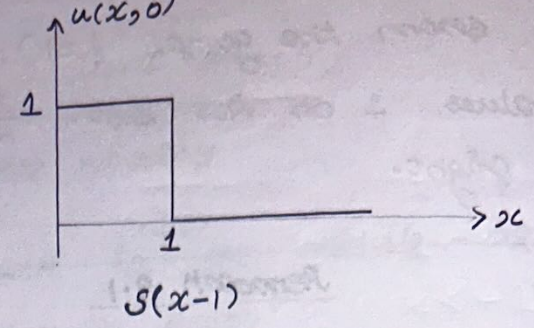
Soln:

The boundary condition at $x=0$ $u(0,t)=0$.

Suppose the initial conditions are

$$u(x,0) = S(x-1) \quad \text{and} \quad \frac{\partial u}{\partial t}(x,0)=0.$$

a step function pulse centered at $x=1$ with zero initial velocity, defined for $x>0$.



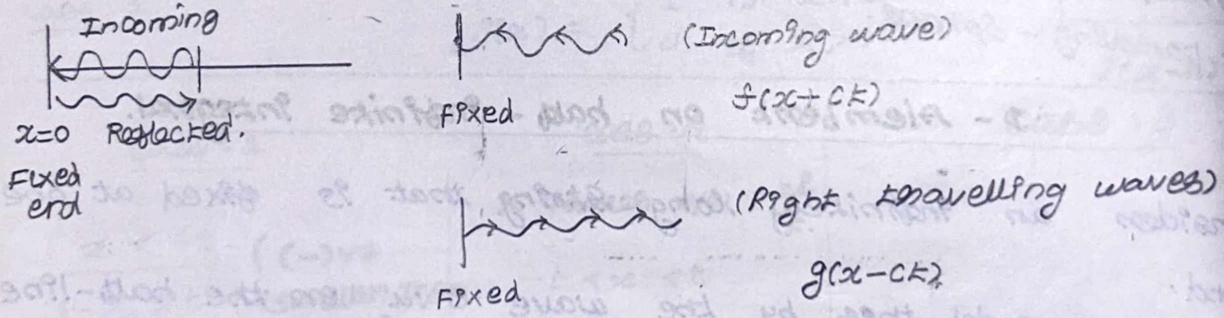
Initial position defined for $x>0$.

Let $f(x) = S(x-1)$ for $x>0$, then the incoming wave left travelling wave

$f(x+ct)$ obeys the PDE and the IC $(t=0)$, but doesn't obey the B.C $(x=0)$.

Determine the proper reflected wave (right travelling wave) $g(x-ct)$ that needs to be added to the incoming wave in order to satisfy the Neumann boundary condition.

Draw the corresponding wavefronts in the (x,t) plane.



Let,

$$u(x,t) = f(x+ct) + g(x-ct)$$

At $x=0$, the boundary condition gives $0 = f(ct) + g(-ct)$. hence we need to relate

$$g(z) = -f(-z).$$

Therefore $g(x) = -S(-x-1)$ for $x<0$

Thus in order to solve this BVP: we must extend the IC $U(x,0)$ to be defined for all x. The first step is to extend f and g to functions F and G defined for all x

Soln:
Draw the corresponding wavefronts in the (x,t) condition.

Apply BV :

$u(0,t)=0$

$g(-ct) = -f(ct)$

$g(z) = -f(-z)$

$\therefore \boxed{g(x) = -f(-x)}$

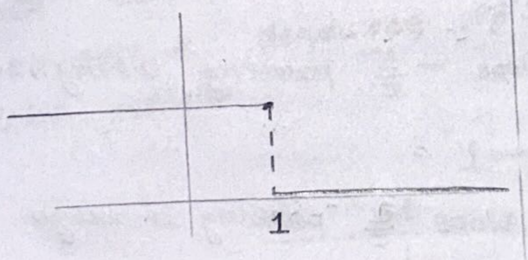
$f(x) = S(x-1)$

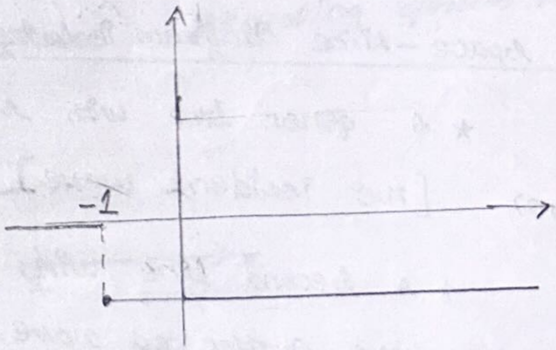
$\boxed{f(-x) = S(-x-1)}$

$\therefore u(x,t) = f(x+ct) + g(x-ct) \longrightarrow$ By superposition.

Thus,

Extend $U(x,0)$ for all x.



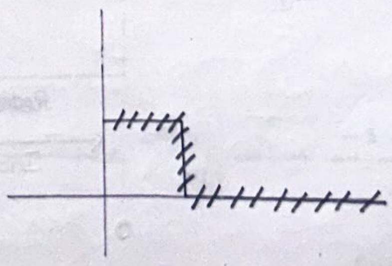


$$-S(-x) = \begin{cases} -1 & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

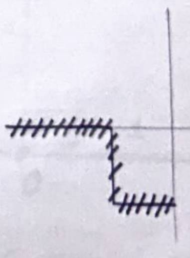
$-S(-x-1) = ?$

$x=0$	$-S(-1) = -1$
$x=1$	$-S(-2) = -1$
$x=-2$	$-S(1) = 0$
$x=-1$	$-S(0) = -1$

$F(x) = S(x-1) \cdot S(-x)$



$G(x) = -S(-x-1)\, S(x)$



why we are doing this?

To have a constant value.

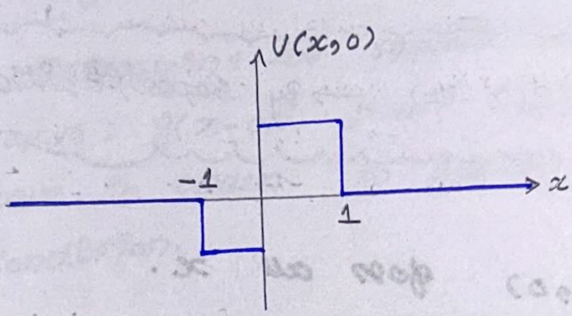
<u>Now</u>

$$F(x) = \begin{cases} 0 & x < 0 \\ 1 & 0 < x < 1 \\ 0 & x > 1 \end{cases}$$

$$G(x) = \begin{cases} 0 & x > 0 \\ 1 & -1 < x < 0 \\ 0 & x < -1. \end{cases}$$

$U(x,0) = F(x) + G(x)$

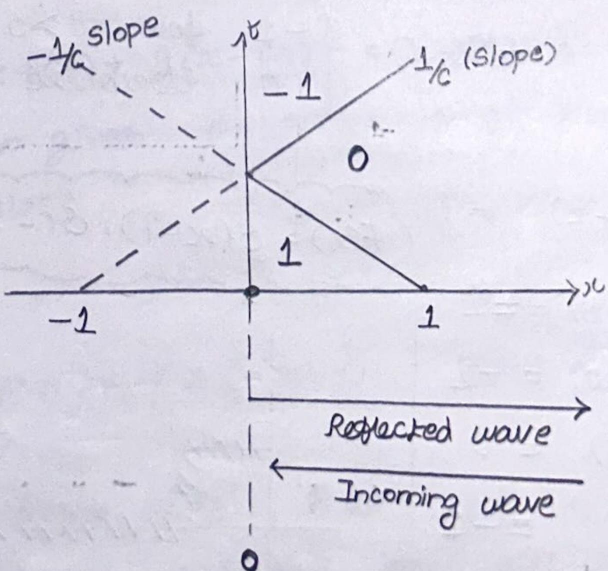
$= S(x-1)\,S(-x) - S(-x-1)\,S(x)$ is defined for all x, and is an odd extension of $S(x-1)$ defined for $x > 0$.



odd extension of initial condition defined for all $-\infty < x < \infty$

<u>The space-time diagram includes:</u>

* A first line with slope $-\frac{1}{c}$ passing through $(1,0)$ [the incident wave]
* A second line with slope $\frac{1}{c}$ passing through $(-1,0)$, (the reflected wave)



∴ we don't know about t:

For $x > 0$

$u(x, t) = 0$

For $x < 0$

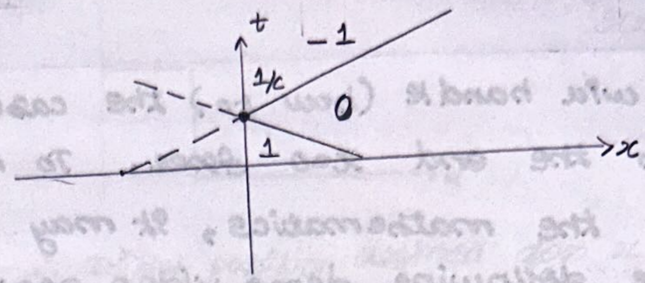
$u(x, t) = 0$

For $0 < x < 1$,

$u(x, 0) = 1$

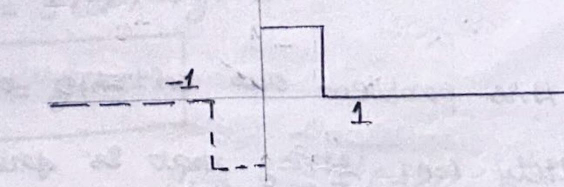
$-1 < x < 0$.

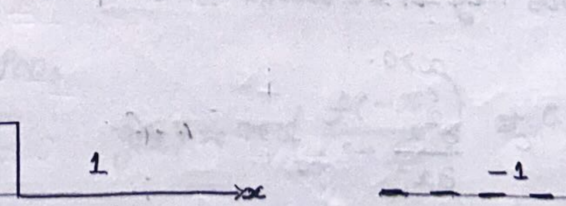
$u(x, 0) = -1$.

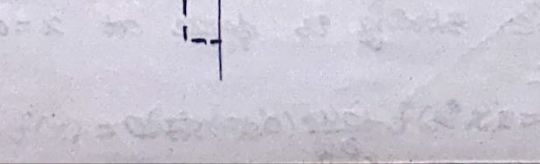


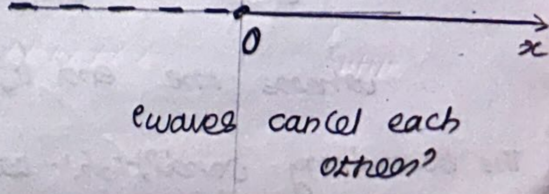
The two lines meet at $(0, \frac{1}{c})$, where the 2 waves cancel out each other exactly. The physical wave first travels as a left-going step function along the first line toward the origin, reflects because of the boundary condition, and then re-emerges as a right-going step function with amplitude -1.

To help understand the process, the image below shows a series of photographs of the wave in the string for $x > 0$ with the extended even wave shown as a dotted line for a sequence of times starting from the initial position.







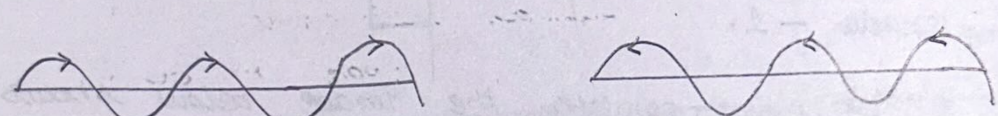


(waves cancel each other)

Now, we will handle (how to) the case of a half-line $x>0$ with the end $x=0$ free. To help you anticipate the mathematics, it may be helpful to watch the following demo video again & observe what's happening at the end points in the boundary conditions, free & fixed.

Solu:

Free end



Incoming wave   Reflected wave.

$$\boxed{\begin{array}{l}\text{Free end} \rightarrow \text{Same polarity} \\ \text{Fixed end} \rightarrow \text{polarity changed}\end{array}}$$

Example: 2:

(Try working this problem out on this problem).

Consider an infinitely long string that is free at one end and modeled this by the wave equation on the half line $x>0$.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad x>0, \ t>0.$$

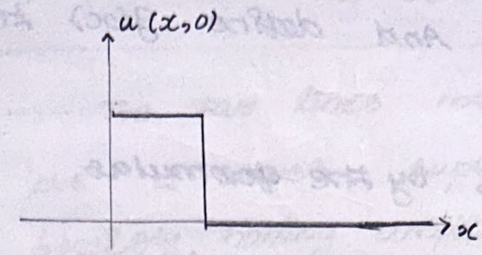
where the end of the string is free at $x=0$.

The boundary condition at $x=0$ is, $\frac{\partial u}{\partial x}(0,t) = 0$.

Suppose the initial conditions are

$$u(x,0) = S(x-1) \quad \text{and} \quad \frac{\partial u}{\partial t}(x,0) = 0$$

a step function pulse centered at $x=1$ with zero initial velocity.



$$S(x) = \begin{cases} 1 & x<0 \\ 0 & x>0 \end{cases}$$

∴ Initial position defined for $x>0$.

$f(x) = S(x-1)$, $x>0$, then the (left travelling incoming wave) $f(x+ct)$ obeys the PDE and the initial condition $(t=0)$, but doesn't obey the boundary condition $(x=0)$.

Determine the proper reflected wave (right travelling wave) $g(x-ct)$ that need to be added to the incoming wave in order to satisfy the Dirichlet boundary condition.

Draw the corresponding wavefronts in the (x,t) plane

$$u(x,t) = f(x+ct) + g(x-ct)$$

$$\frac{\partial u}{\partial x}(x,t) = f'(x+ct) + g'(x-ct)$$

$$\frac{\partial u}{\partial x}(0,t) = f'(ct) + g'(-ct)$$

$$0 = f'(ct) + g'(-ct)$$

$$\boxed{-f'(ct) = g'(-ct)}$$

Thus $g'(x) = -f'(-x)$

By integration,

$$g(x) = -\left(\frac{f(-x)}{-1}\right) + c \qquad (c=0)$$

$$g(x) = f(-x)$$

$f(x) = S(x-1)$, $+f(-x) = (-x-1)$.

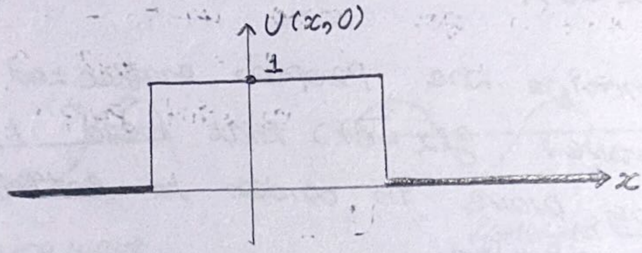
$g(x) = S(-x-1)$. In order to combine ... we must extend the function defined for all x, we must extend the functions to both be defined for all real numbers. The easiest way to do this is to define $f(x)$ to be 0 for $x<0$. And define $g(x)$ to be 0 for $x>0$.

we can do this by the formulas

$$F(x) = f(x)\, S(-x)$$
$$G(x) = g(x)\, S(x)$$

By summing these two functions, we get an initial condition to be an even function defined everywhere

$$U(x,0) = S(x-1)\, S(-x) + S(-x-1)\, S(x).$$



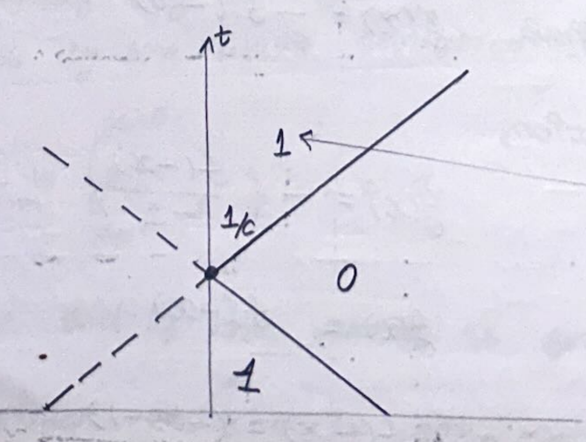
(Even extension of IC defined for all $-\infty < x < \infty$)

This tells us that the reflected wave bounced back at the free end with the same direction.

The space-time diagram includes:

$\star$ A first-line with slope $-\frac{1}{c}$ passing through $(1,0)$ (the incident wave).

$\star$ A second line with slope $\frac{1}{c}$ passing through $(-1,0)$ (the incident wave).

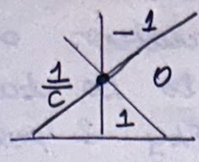


$\therefore$

$\boxed{f(x) = g(-x)}$

So 1 on the top

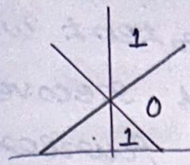
previous case:



At $(0, \frac{1}{c})$ the two waves, cancel each other.

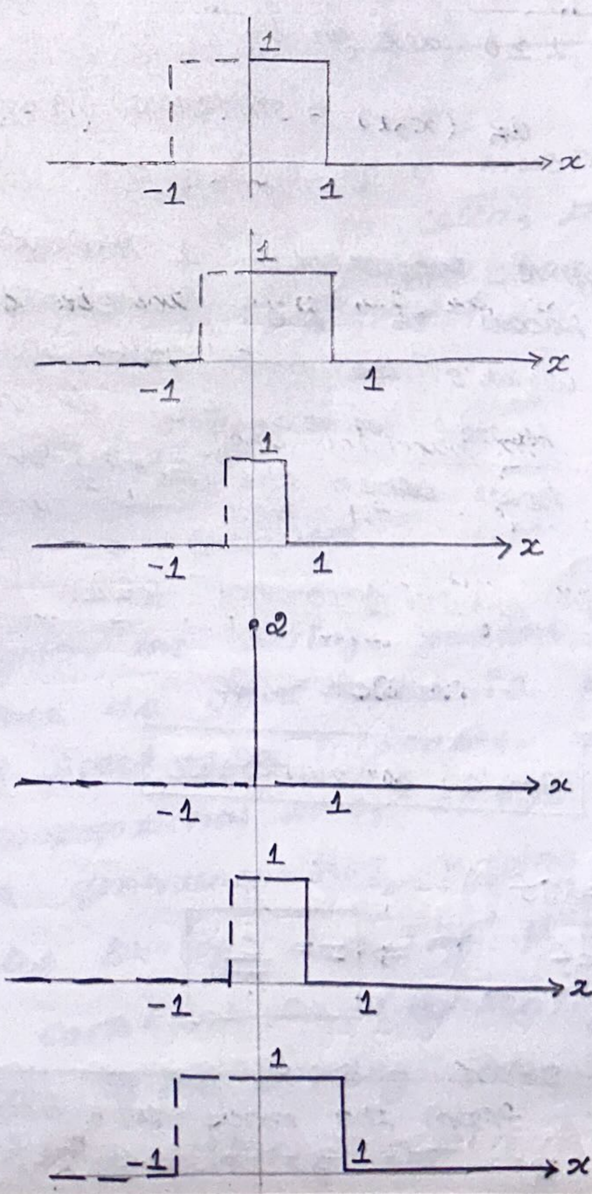
$\therefore f(x) = -g(-x)$

Here



The two lines meet at $(0, \frac{1}{c})$, where the 2 waves superpose & add to briefly having amplitude 2. The physical wave first travels as a left-going step function along the first line toward the origin, reflects because of the boundary conditions, and then re-emerges as a right-going step function with amplitude 1.

wave in the string for $x > 0$, with the extended even wave shown as a dotted line for a sequence of times starting from the initial position.



Extending the D'Alembert's solution to finite ...

It turns out that you can use the d'Alembert idea to solve the wave equation on finite interval [0, L] as well. The main idea is to take the initial velocity & position, initial conditions for the initial which are only defined for $0 < x < L$ and extend these to either even or odd $2L$ periodic functions defined for all $-\infty < x < \infty$ that are consistent with the specified boundary conditions. You will recover exactly the functions we found using Fourier's method (but expressed in a slightly different form)

Concept checking

The normal modes of the wave equation

$$\frac{\partial^2}{\partial t^2} u = c^2 \frac{\partial^2}{\partial x^2} u,$$

$$u(0,t) = u\left(\frac{\pi}{2}, t\right) = 0, \quad t > 0$$

on $0 \le x \le \frac{\pi}{2}$, $t \ge 0$ are gn by

$$u_k(x,t) = \sin(2kx)\left(A\cos(2ckt) + B\sin(2ckt)\right)$$

If the equation above represents a vibrating string, then the main note heard is the lowest frequency standing wave, what's the main note heard? what's the frequency & angular frequency you hear when the string is plucked?

Lowest angular frequency $k = 1$.

$$\therefore 2ck = 2c$$

$$\boxed{\text{Angular frequency} = 2c}$$

Frequency: $\omega = 2\pi f$

$$\frac{2c}{2\pi} = f \Rightarrow \boxed{f = \frac{c}{\pi}}$$

The main note, from the mode u_1, has the angular frequency $2c$. The frequency is $2c/2\pi = c/\pi$.

we will have higher harmonics at the frequencies

$$\frac{ck}{\pi}, \quad k = 2, 3, \ldots$$

(The sound waves induced by the vibrating string depend on the frequencies int of the modes.)

If the length of the string is longer what happens to the sound?

Longer strings have lower frequencies, lower notes. Shorter strings have higher frequencies, higher notes.

If the length of the string is L, then the equations

$v''(x) = \lambda v(x)$, $\quad v(0) = v(L) = 0$ lead to solutions

$$v_K(x) = \sin\left(\frac{K\pi x}{L}\right)$$

The associated angular frequencies in the t-variable are $\frac{Kc\pi}{L}$, so the larger the L, the smaller the $\frac{Kc\pi}{L}$ and the lower the note.

---

wave equation?

When you tighten the string of a musical instrument such as a guitar, piano or cello, the note gets higher. How does the parameter change in the differential equation

$$u_{tt} = c^2 u_{xx}$$

Soln:

c increases

When you tighten the string, you are increasing the tension T. Since the wave speed is proportional to $\sqrt{T}$. The wave speed also increases. The frequencies heard are proportional to c, so the only way to increase the frequency $\frac{Kc\pi}{L}$ keeping the length L fixed and the string density μ fixed is to increase the constant c. (Tightening the string ↑ the tension of the spring also ↑ c)

(i.e. spring constant corresponds to c).

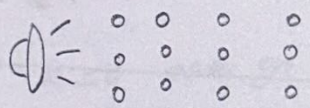
Woodwinds & pressure waves

When we looked at waves propagating in a string, we were interested in how vertical displacements of the string changed over time. That is, all displacements were transverse, and we called these transverse waves. The equation modelling this motion is the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

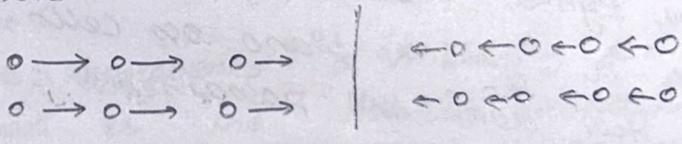
The same wave equ models longitudinal waves, which occur when all of the motion occurs along the major axis. An example of a longitudinal wave is a sound wave. (For more about how sound waves works, you may want to check out the link)

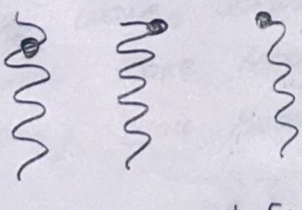
https://www.khanacademy.org/science/ap-physics-1/ap-mechanical-waves-and-sound/introduction-to-sound-waves-ap/v/sound-properties-amplitude-period-frequency-wavelength.

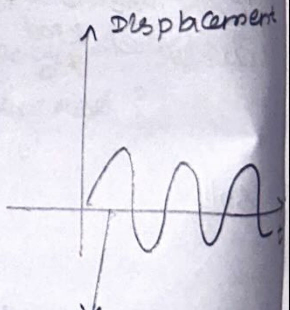


Air molecules

Air molecules move (Back & forth)

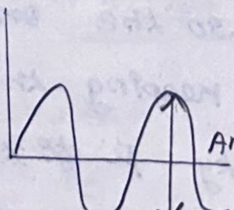




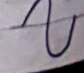


(Back and forth movement – Sine wave)

Amplitude



Not the length of Entire displacement. maximum displacement

Time period (cycle) → one back & forth movement 

Frequency = $\frac{1}{T}$ (No. of oscillations per second)

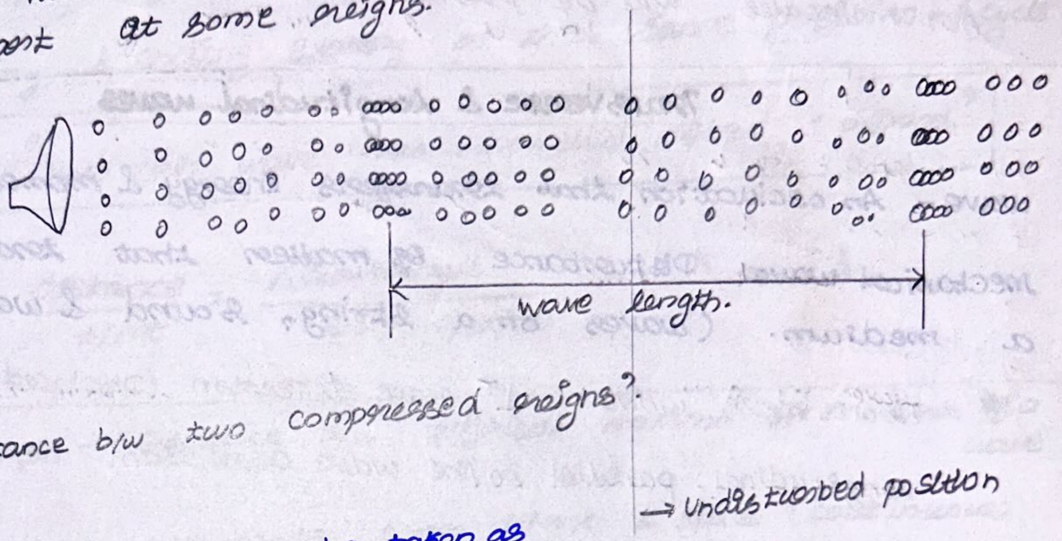
A note $\rightarrow$ 440 Hz

20 HZ – 20000 HZ (Human's ear capability).

(upto 40000 HZ $\rightarrow$ Dogs).

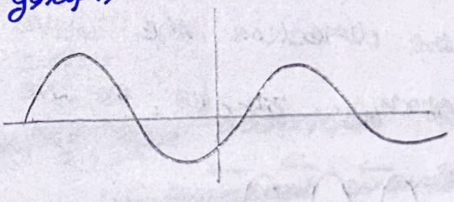
wavelength:

when the air molecules travelling through a reign of air, the air molecules looks compressed at some reign and far apart at some reigns.

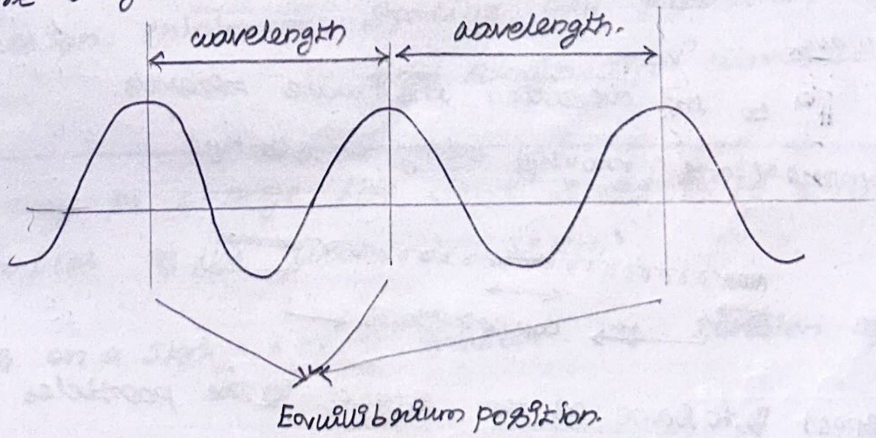


e Distance b/w two compressed reigns?

undisturbed position will be taken as a reference in the graph



In some reigns the air is displaced the most



wave - Disturbatance propogating through space

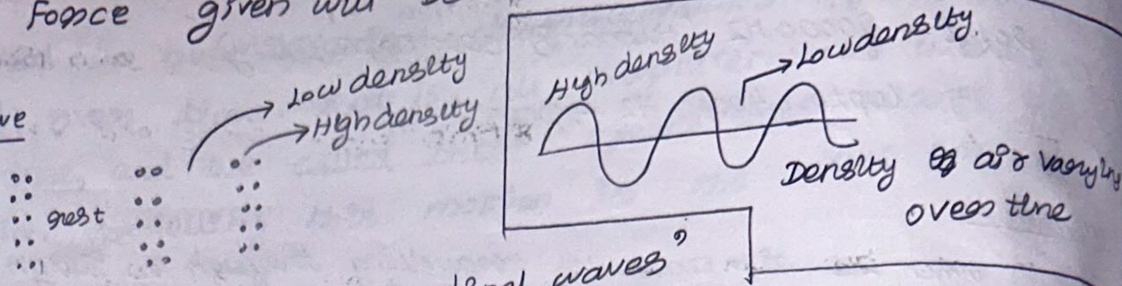
Energy " " through a medium.

∴ EMF waves travels in vacuum. so disturbance progating through space

Rope → 

Force given will be transferred

Soundwave



"Compressed or longitudinal waves"

Air molecules will be pushed". (Left & right movement)

Transverse & Longitudinal waves

wave - An osciliation that transfers Energy & momentum

mechanical wave: Disturbance of matter that travels along a medium. (waves on a string, sound & water)

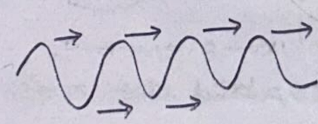
Transverse wave: $\perp^r$ wave direction (Displaced)

Longitudinal: parallel to the wave direction.

Transverse waves

$\perp^r$ to the direction the wave travel.

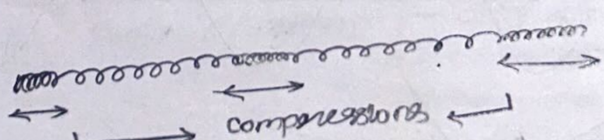
(vibrations on a string, ripples on the surface of water.



Longitudinal wave:

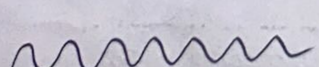
$\parallel^{el}$ to the direction the wave travels.

(compressions moving along a slinky)



wave speed isn't same as the speed of the particles in the medium.

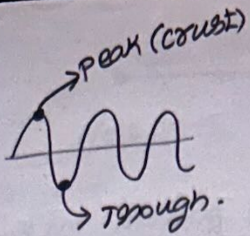
wave speed is how quickly the disturbance travels through a medium. The particle speed is how quickly a particle moves about its equilibrium position.



wave speed

particle speed.

Properties of periodic waves

1) 

* period — How many seconds per each cycle.

* Frequency → cycles per second

$\boxed{f = \frac{1}{T}}$

'Distance b/w two resting points (or) peak to peak (or) one trough to other trough → **wavelength**'

'Starting & ending exactly at that same position - 1 cycle'

'How far wave has travelled after 1 period'

$$velocity = \frac{distance}{time} = \frac{\lambda \text{(wavelength)}}{time \text{ (period)}} = \frac{\lambda}{T} = f\lambda$$

---

wavelength - Distance b/w adjacent maxima or minima of a wave

periodic wave - repeats over time & space (continuous)

crest - peak point on a transverse wave.

Trough - Lowest point

Expansion: max spacing b/w particles of a medium of longitudinal waves.

compression: minimum spacing b/w particles of a medium for longitudinal waves.

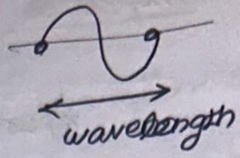
'only way to change the wave speed is to change the properties of the wave medium'.

'waves on a string travel faster - If tension of the string is ↑'

Sound waves travel faster when temperature of the air ↑.

'changing the amplitude or frequency won't change wave speed' → properties of the medium.

wave speed = $\frac{\lambda}{T}$



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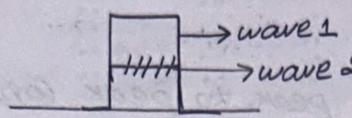
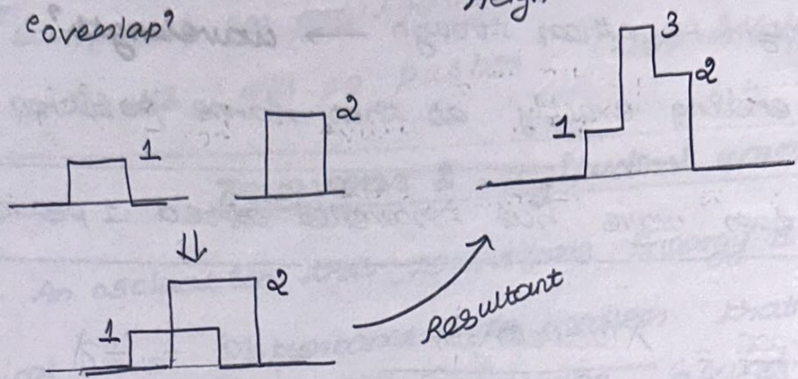
wave Interference



$y = y_1 + y_2$

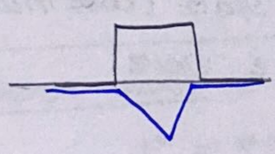
Height of 1st wave + 2nd wave

'overlap'



'Interference only happen – when two waves overlap'

else – unaffected.



$y = y_1 + y_2$.

(Resultant wave)

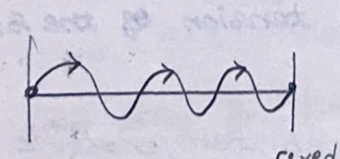
Pulse → single disturbance that moves through a medium

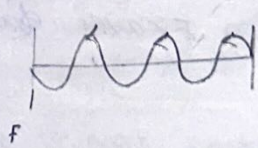
Interference → when two or more waves pass through one another. (superposition).

constructive → Resultant with amplitude (sum of causes)

Destructive → Amplitude less than their individual sum

'Standing waves'



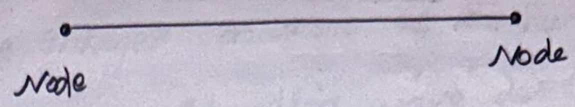


Reflected.

Incoming

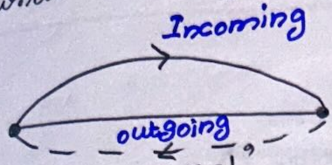
' we need to have nodes at both ends'

'set waves of particular wavelength'



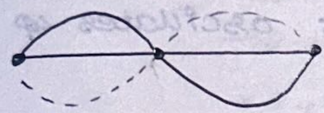
Node → No moving point (Fancy waves)

'particular wave length that set up a standing wave'
what wavelength will get each having a node at ends.

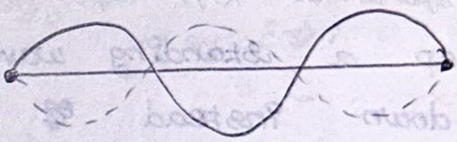


'fundamental'
Largest standing wave.

Standing wave: The peak is not going to move left & right. It will move only up & down.

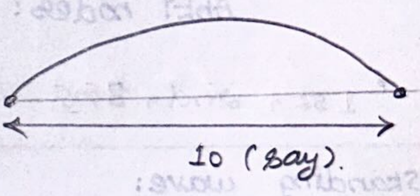


Possibility: 2 → 2nd harmonic

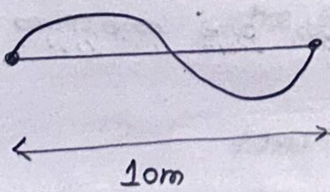


3rd harmonic

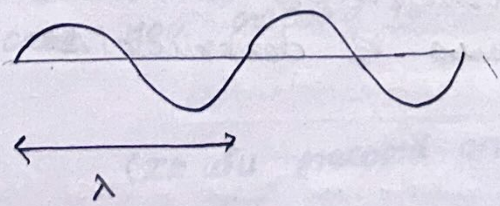
Nodes: Destructive points.
Antinodes (peak): constructive points.



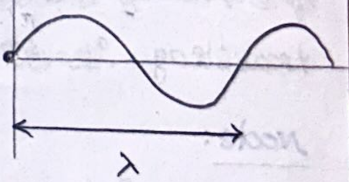
$\boxed{\lambda = 20m}$



$\lambda = 10m$

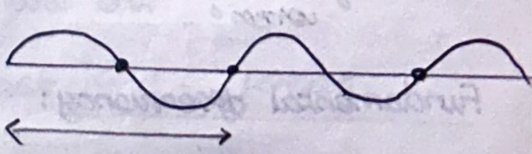


$\lambda = \frac{1}{2}(10) = 5m.$



$\lambda = \frac{2}{3} \times 10$

$= \frac{20}{3}$



$\frac{2}{5} \times 10 = 4m.$

Pattern: $\frac{2 \times 10}{1}$

2nd harmonic = $\frac{2 \times 10}{2}$

3rd harmonic = $\frac{2 \times 10}{3}$

4th harmonic = $\frac{2 \times 10}{4}$

nth harmonic = $\frac{2 \times \text{Length of String}}{n}$

For fixed ends, the waves will reflect off the boundaries & overlap itself to give constructive & destructive interference. For particular wave lengths, we can set up a standing wave which just oscillates up and down instead of left & right.

In standing waves:

Nodes: points without motion: Nodes

Anti nodes: points with maximum displacements

- 1st, 2nd, 3rd, ... n^{th} harmonic waves.

Standing wave:

waves which appear to be vibrating vertically without travelling horizontally. created from waves with identical frequency & amplitude interfering with one another while traveling in opposite directions.

Node:

positions on a standing wave where the waves stays a fixed position over time because of destructive interference.

Antinode:

- where the wave vibrates with maximum amplitude

Fundamental frequency:

Lowest frequency of a standing wave that has the fewest number of nodes & antinodes.

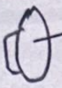
Harmonic:

+ve integer multiple of the Fundamental frequency

The length of the Standing wave depends upon the length of the string. The First harmonic is double the length of the string.

(No matter how long it is).

Sound

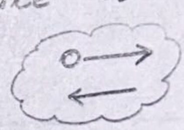
 → Diaphragm (oscillates) ←0→ ←0→ ←0→ ←0→

oscillates.

continuous oscillation ⟶ Energy will be transmitted (say: Ear)

→0 →0 → 0 →0 →0
→ 0 →0 → 0→ 0 →0
→ 0→0 → 0 → 0 →0

° only disturbance is travelled° — Not molecules they only oscillate



 → we hear two waves

* Travelling out of our mouth through the air & into our ear.

* Vibration of our sound wave travelling through flesh and bones.

Incase you record it  → mike

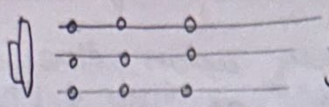
(It will record only Sound through air molecules'

when you hear it: (seems weird)

But it's the exact one what others hear when you speak.

'Haters always hate'

Speed of Sound


} Sound wave.

'Speed → Speed of the disturbance (— parallel to the axis)'

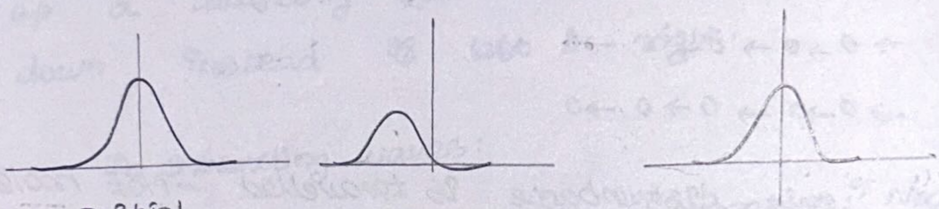
(Even if the air molecules itself moves with a speed'

$$\boxed{\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{\lambda}{T}}$$

Speed of sound = 343 m/s = 767 mph

Transverse waves: string, waves on water

After one period, the wave has to overlap with its initial shape.



$\therefore\ v = \lambda f \rightarrow$ when frequency is increased, wavelength will be decreased.

So speed is unchanged.

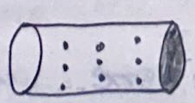
How to alter speed:

* change the medium (water, helium, etc..)
* properties (density, humidity)

Large amplitude → Louder sound (same speed).

Standing waves in open tubes

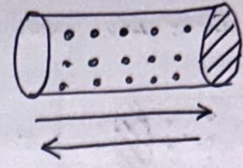
Resonance: 'when we blow air — we hear a tone (louder)'



'In closed end the air is not doing anything'

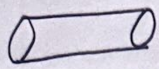
↳ Loses Energy.

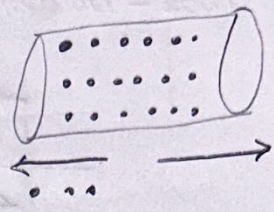
At the open end oscillates — widely
middle — little.



At closed end
↳ No oscillation.

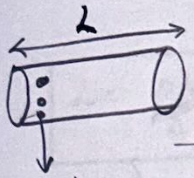
open ends:

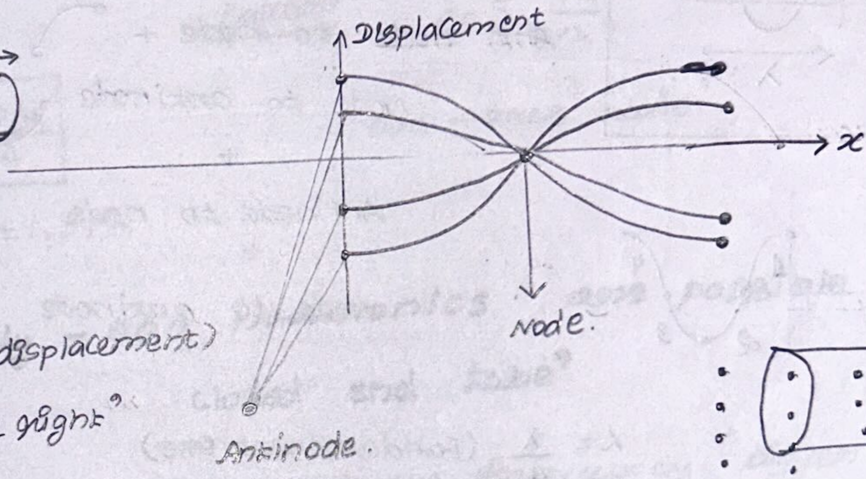


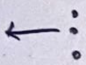


oscillates at both ends
& still at the middle'

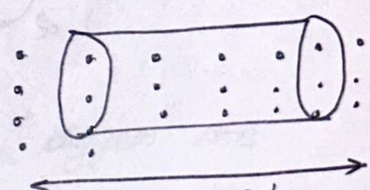
'Back & forth movement'





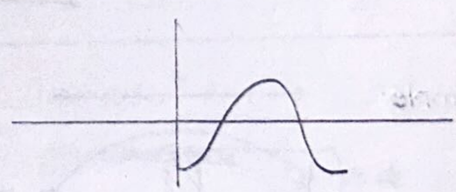


(Negative displacement)
eLeft & right'



Represent this mathematically:

1st harmonic: $\frac{1}{2}\lambda = \frac{1}{2}(2L)$



2nd harmonic $L = \lambda$

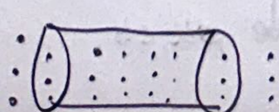
3rd harmonic $L = \frac{3}{2}\lambda$

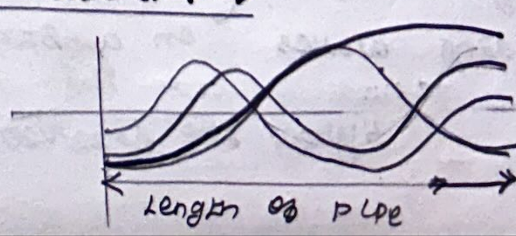
pattern: $L = \frac{n\lambda}{2}$ $(n = 1, 2, 3, \dots)$

Displacement: Free end → more displacement.
middle → relatively less.

In closed tube (pipe)

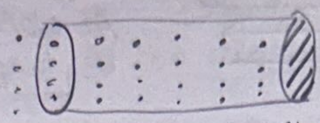
open tube





"maximum displacement at both ends" → Antinodes.

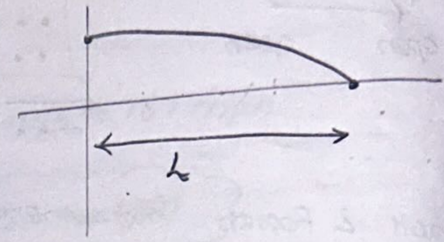
Closed end:



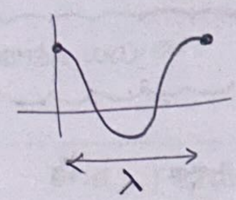
Open (Anti-node) (max displacement)

Closed.

node – No displacement.



wavelength



Anti node to node +

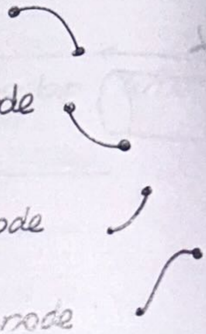
node to antinode

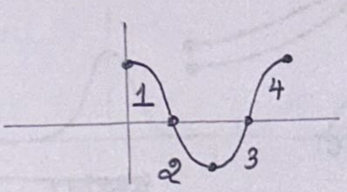
+

Antinode to node

+

node to antinode

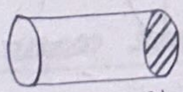




$L = \frac{\lambda}{4}$ (Fundamental one)

$\boxed{\lambda = 4L}$

Simple:



Sound wave hits a wall, it's partially absorbed or partially reflected. A person far enough will hear the sound twice. **(echo).**

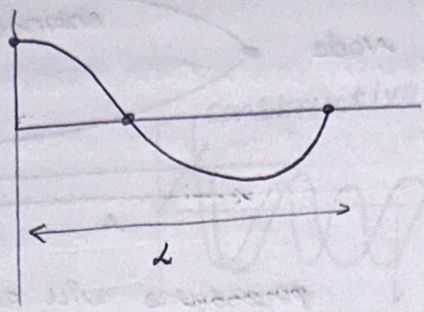
"In a small room the sound is also heard more than once, but the time difference are so small that the sound just seems to loom. This is known as reverberation"

"music is the sound produced by the instrument or voices. To play most musical instrument, we have to create standing waves on a string or in a tube or pipe.

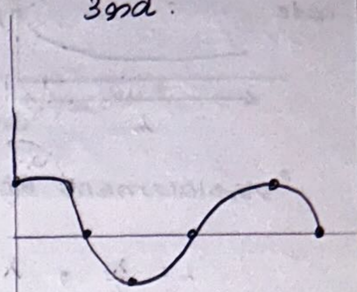
Higher the frequency – higher the pitch.

wind instruments produces sound - by means of vibrating air columns.

ii)



3rd.



$\boxed{\frac{3}{4}\lambda = L}$ $\boxed{\lambda = \frac{4L}{3}}$ $\boxed{\frac{5}{4}\lambda = L}$

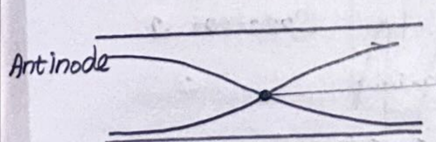
Pattern $\lambda_n = \frac{4L}{n}$ $(n = 1, 3, 5, \ldots)$

$\boxed{L = \frac{n\lambda_n}{4}}$ 'odd - ones alone'

$n = 1, 3, 5, \ldots$

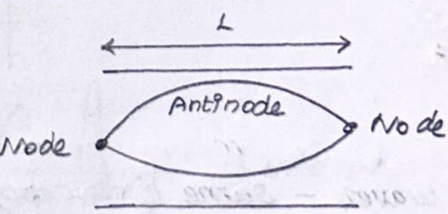
'only - odd harmonics are possible in closed end tube'

Higher the length, lower the frequency, bigger the wavelength, lower the note (or) pitch.



'open

'Displacement of air molecules represented as a standing sound wave in an open tube'



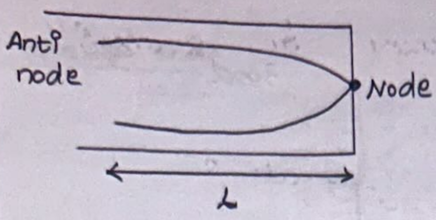
Air pressure.

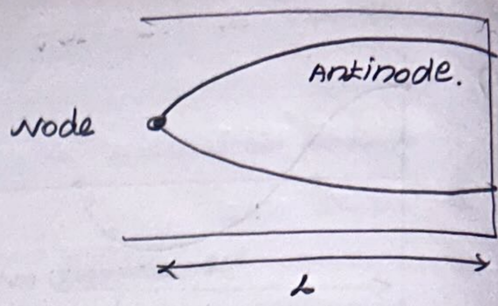
(Air pressure equalizes to atmospheric pressure at both ends (open) but stays high inside the tube)

$L = \frac{\lambda}{2}$ $f_1 = \frac{v}{\lambda}$

$\lambda = 2L$ $\boxed{f_1 = \frac{v}{2L}}$

Closed tube





'Displacement of air molecules'

$L = \frac{\lambda}{4}$, $\lambda = 4L$

$$\boxed{f_1 = \frac{v}{\lambda} = \frac{v}{4L}}$$

pressure will equalize with atmospheric pressure at open end & maximum at the closed end

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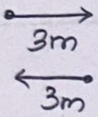
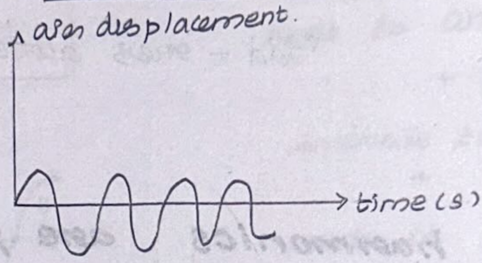
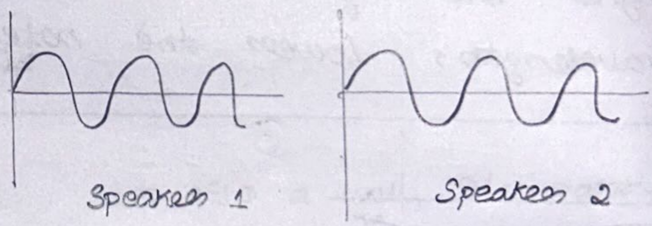
Beat frequency



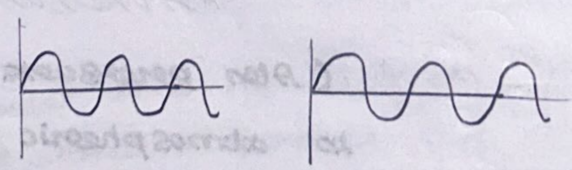
(Say)
oscillation of air molecules
(Away from the speaker)



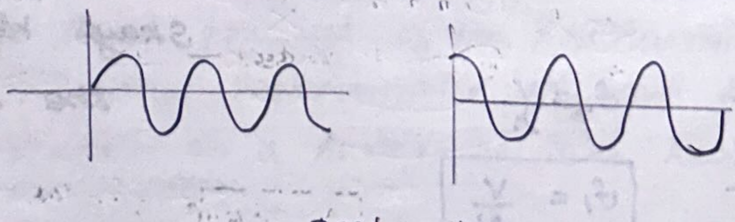
Two speakers



Two waves – same frequency (They can be constructive or destructive).



Constructive

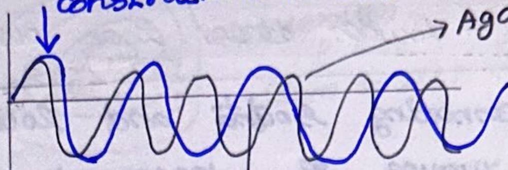


Destructive

'Different period':

'overlap two unequal frequency having waves'

↳ Beats.



'Beat Frequency: this pheonomon (Louder - Softer)

'wobble' - move unsteadily from side to side

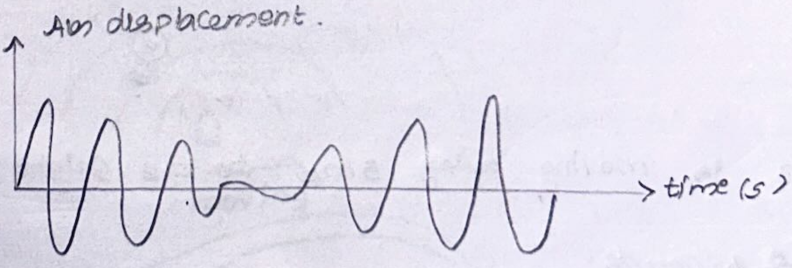
'How many wobbles per second' ⟶ (Beat frequency).

$f_1 \rightarrow$ frequency of 1st wave

$f_2 \rightarrow$ " " 2nd "

$$\boxed{f_B = |f_1 - f_2|} \rightarrow \text{No. of wobbles}$$

(From high (loud) to again loud).



Flute clariet

$f = 440 Hz$ $f_b = 5Hz$.

when you hear a wobble, you need to tune it.

(As the notes are closer & closer → wobbles / sec will be lesser). Tuned until exact sync is occured.

$f_{flute} = 440 Hz$, $\boxed{f_b = 5Hz} \rightarrow$ clariet

$f_b = 5\,Hz = |f_1 - f_2|$

clarinet may play 445 Hz or 435 Hz. So we need to fix it.

'Too Sharp – higher freq than ~~the~~ required note'

'Too flat – lower freq than our required note'

Beat → Regularly alternating ~~soft~~ and loud sound heard from two sound waves of different frequencies interfering.

Beat frequency (f_B) → The magnitude of the difference b/w the two interfering frequencies. (Hz → units (scalar))

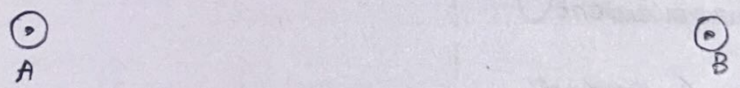
Construction → (Louder)

Destructive → (soft).

Doppler effect.

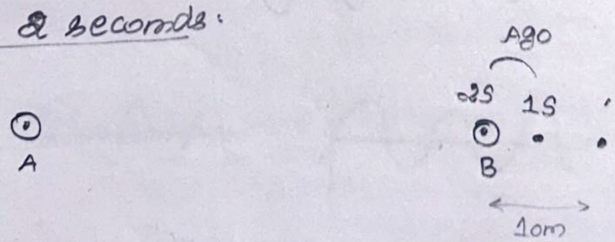
one → stationary } sources.
other → moving,
↳ 5m/s to the right.

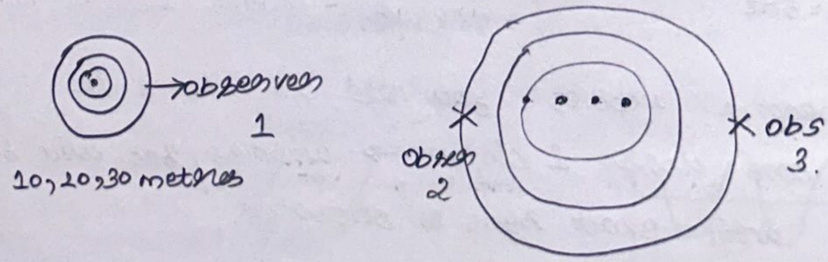
Let velocity of wave by both sources = 10 m/s



As B is moving away 5 m/s to the right from A.

Before 2 seconds.

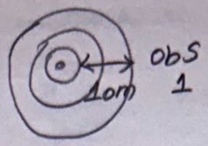




1 → hear A 1st, B 2nd

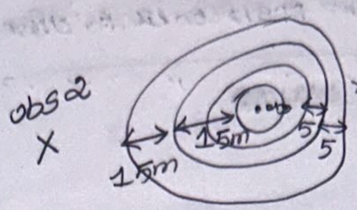
2 → Hear B 1st, A 2nd

3 → Hear B 1st, A 2nd.



period = $\frac{1 \text{ cycle}}{\text{second}}$

obs 1 → 10m apart from source.



x observer 3 (10m apart).

obs 2 → 30m from source 8 (At that instant)

(2 cycles / second)

$T = \frac{1}{2} \frac{\text{sec}}{\text{cycle}}$

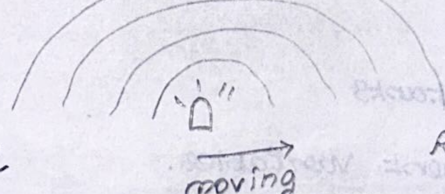
As source is also moving towards him

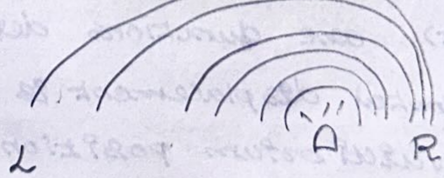
wavefront:

Imaginary surface that represents points on a distribution that all vibrate in unison, such as a ripple that forms from throwing a stone into water.

Doppler effect:

change in frequency & wavelength of a wave due to relative motion b/w the wave source & observer.





speed of waves are not changing - medium isn't changing

R → hears the wavefront more frequently.

L → hears the wavefront less frequently.

Vector - magnitude & direction

Scalar - magnitude

eg: velocity → scalar 2.5 m/s

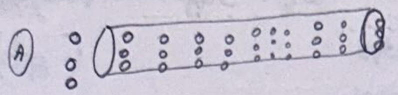
2.5 m/s right → vector. (change in position with direction)

Woodwinds & pressure waves

wave equ

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

Sound waves moving through a flute:



Variables & functions:

L - Length of the cylinder

A - cross-sectional area of cylinder

t - time

x - position along the cylinder (from 0 to L)

$u(x,t)$ - Horizontal displacement of the molecules

P_0 - Ambient air pressure.

$p(x,t)$ - Difference in pressure away from the ambient air pressure.

Here,

L, A, P_0 → constants

t, x → Independent variables.

$u = u(x,t)$ & $p = p(x,t)$ are functions defined for $x \in [0, L]$ and $t \geq 0$. The horizontal displacement is measured relative to the equilibrium position in which (statistically) air molecules are evenly spaced and at ambient pressure.

Assumptions: All statistically significant air molecule motion occurs in the x-direction, and is small. changes in pressure are small compared to the ambient air pressure

Ambient - Atmospheric

Both the pressure & the horizontal displacement satisfy the wave equation.

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad , \quad \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \cdot c^2$$

$$0 < x < L , \quad t > 0.$$

where c — speed of the sound.

To understand boundary conditions, we need to understand how the pressure and the horizontal displacement are related. The physics of sound involves three main ideas.

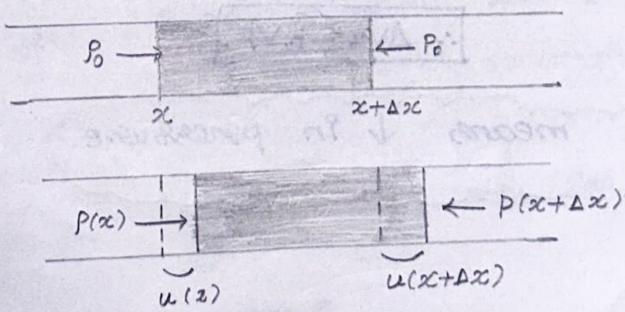
1) Moving air molecules cause changes in air density.

2) changes in density correspond to changes in pressure.

3) pressure differences generate motion of air molecules.

Let's look at what happens when we look at a small volume of air in our cylinder at equilibrium at equilibrium pressure P_0. The volume V at equilibrium pressure P_0. The volume is given by

$$\boxed{V = A\Delta x}$$



If the pressure is changed from equilibrium $P(x)$, then the change in volume from equilibrium is

$$\Delta V = A(u(x+\Delta x) - u(x)) = A\Delta u$$

All that is left to understand how changes in pressure affects changes in volume. What we know is that if we increase the pressure on a given volume, the volume decreases. The decrease is proportional to the original volume V and the vibration in pressure p. This is captured in the equation

$\Delta V = -K V p$ 		V - original volume
p - pressure.

$K \to$ physical constant.

plugging in the expressions we found from the image for V and ΔV involving Δx and Δu into our last equation, we find a relationship b/w pressure p and displacement u.

$$\Delta V = -K V p$$

$$A \, \Delta u = -K \, (A \Delta x) P$$

$$\frac{\Delta u}{\Delta x} = -KP$$

$$\frac{\partial u}{\partial x} = -KP \qquad (\text{as } \Delta x \to 0)$$

understanding Boundary conditions

(cylinder with two open ends - can't hold any pressure beyond the ambient pressure P_0)

$$P(0,t) = P(L,t) = 0 \text{ for all } t > 0$$

Soln:

Boundary conditions:

$\therefore$ larger volume $\to$ pressure will be low.

$\uparrow$ pressure $\to \uparrow$ (+ve change in pressure ΔV)

$$\boxed{\therefore \Delta V = -KVP.}$$

$\therefore$ $\uparrow$ volume means $\downarrow$ in pressure.

$$\frac{\partial u}{\partial x} = -KP$$

$$P(0,t) = P(L,t) = 0.$$

$$\therefore \quad \frac{\partial u}{\partial x}(0,t) = 0 \quad , \quad \frac{\partial u}{\partial x}(L,t) = 0.$$

$\frac{\partial u}{\partial x} \to$ speed of displacement.

Eigen values, Eigen function & normal modes

Consider a cylinder with two open ends of length L. Longitudinal air waves / pressure waves along the midline of the