8/3/2021

Spherical in terms of cylin

$$\begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix}$$

cartesian - cylindrical

$$\begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

cartesian - spherical

$$\begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\theta \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix}$$

cylindrical spherical

$$\begin{bmatrix} \frac{\rho}{\sqrt{\rho^2+z^2}} & \frac{z}{\sqrt{\rho^2+z^2}} & 0 \\ 0 & 0 & 1 \\ \frac{z}{\sqrt{\rho^2+z^2}} & \frac{-\rho}{\sqrt{\rho^2+z^2}} & 0 \end{bmatrix}$$

8/3/2021 Given point P $\rightarrow P(-2,6,3)$ & vector $A = y\vec{a_x} + (x+z)\vec{a_y}$

express P both in cylindrical & spherical coordinates

Soln:

$$A = y\vec{a_x} + (x+z)\vec{a_y}$$

cartesian to cylindrical:

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\cancel{\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}} =$$

Point

$$\rho = \sqrt{x^2+y^2}$$

$$= \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) = -\tan^{-1}\left(\frac{6}{+2}\right) + \pi$$

$z = z$

$\boxed{z = 3}$

$$\boxed{\phi = 108.43^\circ}$$

1) $\boxed{\begin{array}{l} \rho = 6.325 \text{ units} \\ \phi = 108.43^\circ \\ z = 3 \end{array}}$ $P(\rho = 6.325, \phi = 108.43^\circ, z = 3)$

Cartesian to Spherical

$$r = \sqrt{x^2+y^2+z^2}$$

$$\theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

$$r = \sqrt{4+36+9}$$

$$= \sqrt{49} = 7 \text{ units.}$$

$$\theta = \cos^{-1}\left(\frac{3}{7}\right)$$

$$\boxed{\theta = 64.62^\circ}$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) = 108.43^\circ$$

$$P(r = 7, \theta = 64.62^\circ, \phi = 108.43^\circ)$$

Transform a vector $\vec{A} = y\vec{a_x} - x\vec{a_y} + z\vec{a_z}$ in to cylindrical coordinates

solu:

2) $$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ -x \\ z \end{bmatrix}$$

$$\vec{A} = A_x\vec{a_x} + A_y\vec{a_y} + A_z\vec{a_z}$$

$$A_\rho = y\cos\phi - x\sin\phi$$

$$A_\phi = -y\sin\phi - x\cos\phi$$

$$\boxed{A_z = z}$$

$x = \rho\cos\phi$

$y = \rho\sin\phi$

$$\vec{A} = (y\cos\phi - x\sin\phi)\vec{a_\rho} - (y\sin\phi + x\cos\phi)\vec{a_\phi} + z\vec{a_z}$$

$$\vec{A} = (\rho\sin\phi\cancel{\cos\phi} - \rho\cancel{\cos\phi}\sin\phi)\vec{a_\rho} - (\rho\sin^2\phi + \rho\cos^2\phi)\vec{a_\phi} + z\vec{a_z}$$

$\cancel{\vec{A}} = (\cancel{\rho\sin\phi\cos\phi} -$

$$\vec{A} = -\rho\vec{a_\phi} + z\vec{a_z}$$

3) Obtain the spherical coordinates of $10\vec{a_x}$ at the point $P(x=-3, y=2, z=4)$.

solu:

$$r = \sqrt{x^2+y^2+z^2} = \sqrt{9+4+16}$$

$$= \sqrt{29} = 5.3851$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{2}{-3}\right) = 146.30^\circ$$

$$\theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right) = \cos^{-1}\left(\frac{4}{5.3851}\right) = 42.03034^\circ.$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

Vreatty done good

$A_r = 10\sin\theta\cos\phi$

$A_\theta = 10\cos\theta\cos\phi$

$A_\phi = -10\sin\phi$

$\vec{A} = (10\sin\theta\cos\phi)\vec{a_r} + (10\cos\theta\cos\phi)\vec{a_\theta} + (-10\sin\phi)\vec{a_\phi}$

$\vec{A} = -5.57\vec{a_r} + (-6.179)\vec{a_\theta} - 5.5484\vec{a_\phi}$

$$\boxed{\vec{A} = -5.57\vec{a_r} - 6.18\vec{a_\theta} - 5.55\vec{a_\phi}}$$

Express $\vec{B} = \frac{10}{r}\vec{a_r} + r\cos\theta\,\vec{a_\theta} + \vec{a_\phi}$ in cart and cyl CS

Find $\vec{B}$ at $(-3, 4, 0)$ and $(5, \pi/2, -2)$

Solu:

Spherical to cartesian

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

$$= \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} 10/r \\ r\cos\theta \\ 1 \end{bmatrix}$$

$A_x = \frac{10}{r}\sin\theta\cos\phi + r\cos^2\theta\cos\phi - \sin\phi$

$A_y = \frac{10}{r}\sin\theta\sin\phi + r\cos^2\theta\sin\phi + \cos\phi$

$A_z = \frac{10}{r}\cos\theta - r\sin\theta\cos\theta$

$r = \sqrt{9+16}$

$= 5$

$\theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$, $\phi = \tan^{-1}\left(\frac{y}{x}\right)$

$\phi = \tan^{-1}\left(\frac{4}{-3}\right)$

$= \cos^{-1}(0)$

$\boxed{\phi = 126.86}$

$\boxed{\theta = 90^\circ}$

$A_x = 2\sin 90^\circ\cos(126.86) + 5\cos^2 90^\circ\cos(126.86) - \sin(126.86)$

$= 2(-0.599) + 5(0) - (0.8)$

$A_x = -1.998$

$\boxed{A_x \approx -2}$

$A_y = 2\sin 90^\circ \sin(126.86) + 5\cos^2 90^\circ \sin(126.86) + \cos(126.86)$

$A_y = 1.0003$

$\boxed{A_y \approx 1}$

$A_z = 2\cos 90^\circ - 5\sin 90^\circ \cos 90^\circ$

$\boxed{A_z = 0}$

$$\boxed{\vec{A} = -2\vec{a_x} + \vec{a_y}}$$

<u>spherical to cylindrical</u>

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \rho/\sqrt{\rho^2+z^2} & \frac{z}{\sqrt{\rho^2+z^2}} & 0 \\ 0 & 0 & 1 \\ \frac{z}{\sqrt{\rho^2+z^2}} & \frac{-\rho}{\sqrt{\rho^2+z^2}} & 0 \end{bmatrix} \begin{bmatrix} 10/r \\ r\cos\theta \\ 1 \end{bmatrix}$$

$\boxed{A_\phi = 1}$

$$A_\rho = \frac{10}{r}\left(\frac{\rho}{\sqrt{\rho^2+z^2}}\right) + r\cos\theta \frac{z}{\sqrt{\rho^2+z^2}}$$

$$A_z = \frac{10}{r}\left(\frac{z}{\sqrt{\rho^2+z^2}}\right) - r\cos\theta \frac{\rho}{\sqrt{\rho^2+z^2}}$$

$P(5, \pi/2, -2)$

$\rho \quad \phi \quad z$

$r = \sqrt{\rho^2+z^2}$

$= \sqrt{25+4}$

$= \sqrt{29}$

$$A_\rho = \frac{10}{\sqrt{29}}\left(\frac{5}{\sqrt{29}}\right) + \sqrt{29}\cos\left(\frac{\pi}{2}\right)\left(\frac{-2}{\sqrt{29}}\right)$$

$A_\rho = \frac{50}{29}$

$\boxed{A_\rho = 1.724}$

$$A_z = \frac{10}{\sqrt{29}}\left(\frac{-2}{\sqrt{29}}\right)$$

$$= \frac{-20}{(\sqrt{29})^2} = -0.689$$

$$\vec{A} = 1.72\vec{a_\rho} - 0.69\vec{a_z} + \vec{a_\phi}$$

$$\vec{A} = 1.72\vec{a_\rho} + \vec{a_\phi} - 0.69\vec{a_z}$$

9/3/2021

Integral relation

Line Integral (straight line or curved path)

Curved path - can be open or closed.

Distance - scalar

Displacement - vector.

W = Force x displacement

+ + + + + + + + +

$W = \int_L \vec{F} \cdot \vec{dl}$

dl

L (over the line)

'open-ended line'

'closed line integral'

'open-ended curve'

$\oint_L$ → closed line.

'contour' - path trace along the circle'

closed line integral (or) circular integral (line)

↓

'Represents circulation of a vector field along a line'

circulation - move along a closed path

$\boxed{\text{Rotation - Along its own axis}}$

Use:

* Find total charge - if line charge density is known $\left(\rho_L = \frac{Q}{dl}\right)$

$\rho_S = \frac{Q}{ds}$ (Surface charge density).

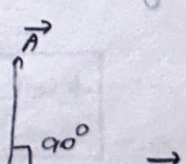
$$\boxed{Q = \int_L \rho_L \, dl}$$

* Electric potential - when Electric field intensity is known.

$$\boxed{V = -\int E.dl}$$

* Total current along a circular path if magnetic field intensity is known.

$$\boxed{I = \oint_L H.dl}$$



$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$

$= 1(1) \cdot \cos 90°$

$= 0$

conservative or laminar field - when the angular distance is 90° b/w two vectors (dot product will be zero).

Surface Integral

surface - vector in Electromagnetics → 'orientation matters'

'Distribution along surface'

$$\boxed{\rho_S = \frac{q}{ds}}$$

'Surface charge density'

$q = \oint_S \rho_s \, ds \rightarrow$ Total S.C.D.

Flux → (Flow) → (ϕ) → Total no. of Electric lines passing through the surface.

$$\phi = \oint_S \vec{B} \cdot \vec{ds}$$

$\vec{B}$ → 'magnetic field intensity'

'Total amount of current passing through a surface (Density)

$I = \oint \vec{J} \cdot \vec{dA}$ → differential Area

$$\boxed{\begin{matrix} \text{Area} \rightarrow \iint \\ \text{Volume} \rightarrow \iiint \end{matrix}}$$

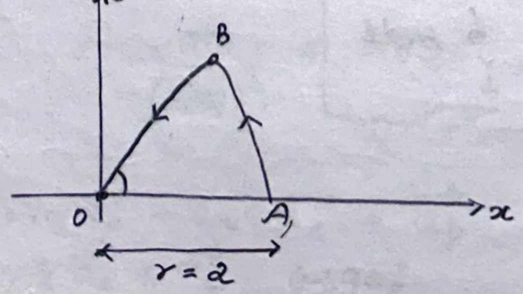
Volume Integral:

'No. of charges - in a Volume'

$$\boxed{Q = \int_V \rho_v \, dv}$$

Calculate the circulation of $\vec{A} = r\cos\phi \, \vec{a_r} + z\sin\phi \, \vec{a_z}$ around the edge L of the wedge defined by $0 \le r \le 2$, $0 \le \phi \le 60°$, $z = 0$

Soln.



Sol:

$\vec{A} = r\cos\phi\,\vec{a_r}$ $(z=0)$

$$\text{circulation of } \vec{A} = \int_{OP} + \int_{PB} + \int_{Oy}$$

$$① \int_{OP} = \int_0^2 r\cos\phi\,\vec{a_r}\, \vec{dl}$$

$$\boxed{\vec{dl} = dr\,\vec{a_r} + r\,d\phi\,\vec{a_\phi} + dz\,\vec{a_z}}$$

$$= \int_0^2 (r\cos\phi\,\vec{a_r} \cdot dr\,\vec{a_r}) + 0 + 0$$

$$= \int_0^2 r\cos\phi\, dr$$

Angle b/w P axis & ϕ axis $\to 90^\circ$

$$= \cos\phi \left[\frac{r^2}{2}\right]_0^2 = \cos\phi\left[\frac{4}{2}\right]$$

$[\phi = 0^\circ]$

$$= 2\cos\phi$$

$$= 2\cos(0)$$

$$\boxed{\text{circulation of } \vec{A} = 2}$$

$$\boxed{\therefore \phi = 0 \text{ in } x \text{ axis}}$$

$$② \int_{PB} = \int_0^2 r\cos\phi\,\vec{a_r}\cdot(\vec{a_r})\,dr \qquad \phi = 90^\circ$$

$dr = 0$ (For PB)

$$= \int_0^2 r(0)\cos\phi = 0$$

$$③ \int_{BO} = \int_2^0 r\cos 60^\circ\, dr$$

$$= \left[\frac{r^2}{2}\times\frac{1}{2}\right]_2^0$$

$$= \frac{1}{2}\left[-\frac{4}{2}\right]$$

$$= -1$$

$$\therefore \text{circ of } \vec{A} = 2 + 0 - 1$$

$$\boxed{\text{circ of } \vec{A} = 1}$$

$$\boxed{\therefore \text{circulation of } \vec{A} = 1}$$

10/3/2021

Vector differentiation

$\nabla \rightarrow$ vector differentiation operator (nabla)

Gradient, curl, Divergence

Gradient:

∇ operating on a scalar gives the vector $\longrightarrow$ Gradient.

The gradient at a point gives the direction of maximum change in a physical quantity.

Resulting gradient will be towards the maximum.

Properties:

1) Gradient is an operation performed on a scalar function which results in a vector function.

2) The magnitude of the gradient of a scalar function is the maximum rate of change of the function per unit distance.

3) The direction of the gradient of the scalar function is the direction in which the function changes most rapidly

Represent 3 coordinate systems

$$\boxed{\nabla F = \frac{1}{h_1}\frac{\partial F}{\partial u_1}\hat{a}_1 + \frac{1}{h_2}\frac{\partial F}{\partial u_2}\hat{a}_2 + \frac{1}{h_3}\frac{\partial F}{\partial u_3}\hat{a}_3}$$

Cartesian: $h_1 = h_2 = h_3 = 1$

Cylindrical: $h_1 = 1,\ h_2 = \rho,\ h_3 = 1$

Spherical: $h_1 = 1,\ h_2 = r,\ h_3 = r\sin\theta$

Divergence.

$\nabla \cdot \vec{F} = \text{Scalar}$ (operates on a vector - gives the scalar quantity)

Source - +ve divergence

Sink - (-)ve divergence

Zero divergence = [(Incoming) = Outgoing]

Divergence of $\vec{F}$

$$\operatorname{div} \vec{F} = \lim_{v \to 0} \left[\frac{\oint_s \vec{F} \cdot \vec{ds}}{v} \right]$$

Vector field = $\vec{F}$

Scalar field = F

Properties:

1) The result of divergence of a vector is a scalar.
2) Divergence of a scalar field (F) has no meaning
3) Divergence may be +ve, (–)ve or zero.

when $\Delta \cdot \vec{F} = 0$ [solenoidal] vector field.

4) Divergence gives the magnitude & nature of the field.

Source
sink
solenoidal

$$\nabla \cdot \vec{F} = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial u_1} (F_1 h_2 h_3) + \frac{\partial}{\partial u_2} (F_2 h_1 h_3) + \frac{\partial}{\partial u_3} [F_3 h_2 h_1] \right]$$

In spherical system: $(u_1, u_2, u_3) = (r, \theta, \phi)$

$F = \vec{F_1} + \vec{F_2} + \vec{F_3}$

$\vec{F_1} = F \hat{a_1}$

10/3/21

Curl (Rotation)

'About rotation on a point'

$\nabla \times \vec{F}$ = vector (Amount of twisting)

$$\text{curl } \vec{F} = \lim_{v \to 0} \left[\frac{\oint_s \vec{F} \times \vec{ds}}{v} \right]$$

$$\nabla \times \vec{F} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 a_1 & h_2 a_2 & h_3 a_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ F_1 h_1 & F_2 h_2 & F_3 h_3 \end{vmatrix}$$

For spherical system

$$\nabla \times \vec{F} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} a_r & r a_\theta & r\sin\theta \, a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ F_1 & rF_2 & r\sin\theta \, F_3 \end{vmatrix}$$

Properties of curl:

1) Result – vector
2) Curl of a scalar → has no meaning
3) $\nabla \times \vec{F} = 0$ (Irrotational)

Right hand cock rule

Direction of the resulting vector - Normal to the plane.

1) The temp in an auditorium is gn by x^2+y^2-z. A mosquito at (1, 1, 2) in the auditorium decides to fly in such a direction it will get warm as soon as possible. In what direction it must fly.

Solu:

$x^2+y^2-z \rightarrow$ scalar quantity.

Gradient:

$$\nabla F = \frac{\partial F}{\partial x}\hat{a_x} + \frac{\partial F}{\partial y}\hat{a_y} + \frac{\partial F}{\partial z}\hat{a_z}$$

$$\nabla F = 2x \cdot \hat{a_x} + 2y\,\hat{a_y} - \hat{a_z}$$

At (1, 1, 2)

$$\boxed{\nabla F = 2\hat{a_x} + 2\hat{a_y} - \hat{a_z}}$$

$\vec{F}(r,\phi,z) = 2r\sin\phi$, $\vec{F}(r,\theta,\phi) = 2r\cos\theta - 5\pi + 2$

Find the gradient.

Solu:

$$\nabla F = \frac{\partial F}{\partial r}\vec{a_r} + \frac{1}{r}\frac{\partial F}{\partial \phi}\vec{a_\phi} + \frac{\partial F}{\partial z}\vec{a_z}$$

$$\nabla F = 2\sin\phi\,\vec{a_r} + \frac{2r\cos\phi}{r}\vec{a_\phi}$$

$$\nabla F = 2\sin\phi\,\vec{a_r} + \frac{2r\cos\phi}{r}\vec{a_\phi}$$

$$\nabla F = 2\sin\phi\,\vec{a_r} + 2\cos\phi\,\vec{a_\phi}$$

$$\nabla F = \frac{\partial F}{\partial r}\vec{a_r} + \frac{1}{r}\frac{\partial F}{\partial \theta}\vec{a_\theta} + \frac{1}{r\sin\theta}\frac{\partial F}{\partial \phi}\vec{a_\phi}$$

$$= 2\cos\theta\,\vec{a_r} - 2\sin\theta\,\vec{a_\theta} - \frac{5}{r\sin\theta}\vec{a_\phi}$$

$\rightarrow$ If 5ϕ not 5π.

Theorem → Stokes, Divergence

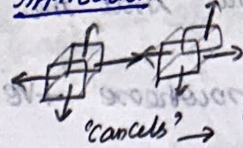
11/3/2021

Divergence theorem

For any volume V surrounded by a closed surface S, the $\iiint$ (volume integral) of the divergence of the vector field is equal to the total output flux of the vector through the surface that bounds the volume.

$$\boxed{\int_V \nabla\cdot\vec{A}\,dv = \oint_S \vec{A}\cdot d\vec{S}}$$

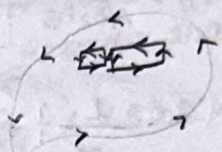
Application



Stokes theorem

It states that the surface integral of the curl of the vector field over an open surface is equal to the closed line integral of the vector along the contour (path) bounding the surface.

$$\boxed{\int_S \nabla\times\vec{A}\,dS = \oint_L \vec{A}\cdot d\vec{l}}$$



S → 'open surface'

$A = r\sin 2\phi\,\vec{a_r} + r\cos 2\phi\,\vec{a_\phi} + z^2\vec{a_z}$ → Divergence.

Solu.

$$\nabla\cdot\vec{F} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}(F_1 h_2 h_3) + \frac{\partial}{\partial u_2}(F_2 h_1 h_3) + \frac{\partial}{\partial u_3}[F_3 h_2 h_1]\right]$$

$$= \frac{1}{r}\left[\frac{\partial}{\partial r}(F_1 r) + \frac{\partial}{\partial\phi}(F_2) + \frac{\partial}{\partial z}(F_3 r)\right]$$

$$\cancel{= \frac{1}{r}\left[\frac{\partial}{\partial r} r\sin 2\phi\,\vec{a_r}\right.}$$

$$= \frac{1}{r}\left[\frac{\partial}{\partial r}(r^2\sin 2\phi) + \frac{\partial}{\partial\phi}(r\cos 2\phi) + \frac{\partial}{\partial z}(z^2 r)\right]$$

$$\cancel{= \frac{1}{r}\left[\frac{\partial}{\partial r}\right[}$$

$$= \frac{1}{r}\left[2r\sin 2\phi - r\sin 2\phi(2) + r(2z)\right]$$

$$\nabla\cdot\vec{F} = 2\sin 2\phi - 2\sin 2\phi + 2z.$$

$$\boxed{\nabla\cdot\vec{F} = 2z}$$

Divergence A at $(100, \phi, 100)$ of $A = r\sin^2\phi\,\vec{a_r} + z^2\cos^2\phi\,\vec{a_z}$

Solu.

$$\operatorname{div} A = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}(F_1 h_2 h_3) + \frac{\partial}{\partial u_2}(F_2 h_1 h_3) + \frac{\partial}{\partial u_3}[F_3 h_2 h_1]\right]$$

$$\operatorname{div} A = \frac{1}{r}\left[\frac{\partial}{\partial r}(r^2\sin^2\phi) + \frac{\partial}{\partial z}(r z^2\cos^2\phi)\right]$$

$$= \frac{1}{r}\left[2r\sin^2\phi + 2rz\cos^2\phi\right] \Rightarrow = \left[2\sin^2\phi + 2z\cos^2\phi\right]$$

2) Show that the $\operatorname{div}\vec{A} = r^{-2}\sin\theta\,\vec{a_r} + r\cos\theta\,\vec{a_\theta} + r\sin\theta\cos\phi\,\vec{a_\phi}$ (cosθ)

is nowhere +ve.

soln:

1) $A = r\sin^2\phi\,\vec{a_r} + z^2\cos^2\phi\,\vec{a_z}$

$$\nabla\cdot\vec{A} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}[F_1h_2h_3] + \frac{\partial}{\partial u_2}[F_2h_1h_3] + \frac{\partial}{\partial u_3}[F_3h_1h_2]\right]$$

$$= \frac{1}{r}\left[\frac{\partial}{\partial r}[F_1 r] + \frac{\partial}{\partial\phi}[F_2] + \frac{\partial}{\partial z}[F_3 r]\right]$$

$$= \frac{1}{r}\left[\frac{\partial}{\partial r}[r^2\sin^2\phi] + \frac{\partial}{\partial z}[rz^2\cos^2\phi]\right]$$

$$= \frac{1}{r}\left[2r\sin^2\phi + 2zr\cos^2\phi\right]$$

$$\nabla\cdot\vec{A} = 2\sin^2\phi + 2z\cos^2\phi$$

$r = \sqrt{x^2+y^2} = \sqrt{100^2+25} = 100.12$ (100, 5, 100)

$\phi = \tan^{-1}(y/x) = \tan^{-1}(5/100) = 2.8624$

$z = z$

$\boxed{z=100}$

$$\nabla\cdot\vec{A} = 2\sin^2(2.8624) + 2(100)(\cos^2 2.8624)$$

$\boxed{\nabla\cdot\vec{A} = 199.506}$ → when (100, 5, 100).

2) $$\operatorname{div}\vec{A} = \frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}[F_1h_2h_3] + \frac{\partial}{\partial\theta}[F_2h_1h_3] + \frac{\partial}{\partial\phi}[F_3h_1h_2]\right]$$

$$= \frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}[F_1r^2\sin\theta] + \frac{\partial}{\partial\theta}[F_2r\sin\theta] + \frac{\partial}{\partial\phi}[F_3r]\right]$$

$$= \frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}[\sin\theta] + \frac{\partial}{\partial\theta}[r^2\sin\theta\cos\theta] + \frac{\partial}{\partial\phi}[r^2\sin\theta\cos\phi]\right]$$

$$= \frac{1}{r^2\sin\theta}\left[r^2\left(\frac{\sin2\theta}{2}\right)' + r^2\frac{\partial}{\partial\phi}[\cos\phi\sin\theta]\right]$$

$$= \frac{1}{\sin\theta}\left[\frac{1}{2}\cdot\cos2\theta\cdot2 - \sin\phi\sin\theta\right]$$

$$= \frac{1}{\sin\theta}\left[\cos 2\theta - \sin\phi \sin\theta\right]$$

$$= \frac{1-2\sin^2\theta}{\sin\theta} - \sin\phi$$

$$= \frac{1}{\sin\theta} - 2\sin\theta - \sin\phi$$

$$\operatorname{div}\vec{A} = \operatorname{cosec}\theta - 2\sin\theta - \sin\phi$$

$$\boxed{\operatorname{div}\vec{A} = \frac{1}{\sin\theta} - 2\sin\theta - \sin\phi}$$

$$= \frac{1}{\sin\theta}\left[r^2(\cos\theta)' - \sin\phi\sin\theta\right]$$

$$= \frac{1}{\sin\theta}\left[-\sin\theta - \sin\phi\sin\theta\right]$$

$$= -1 - \sin\phi$$

$$= -(1+\sin\phi)$$

sin → never be greater than 1

$$-1 < \sin x < 1$$

$$\boxed{0 < \phi < 2\pi}$$

∴ Always negative.

Nowhere be positive. $-(1+\sin\phi) \leq 0$.

12/03/2021

Null Identities

$\nabla \times (\nabla . V) = 0$ — curl of gradient of a vector

$\nabla . (\nabla \times V) = 0$ — divergence of curl of a vector.

i)

$$\nabla = \left(\frac{\partial}{\partial x}\vec{a_x} + \frac{\partial}{\partial y}\vec{a_y} + \frac{\partial}{\partial z}\vec{a_z}\right)$$

$$\nabla V = \left(\frac{\partial V}{\partial x}\vec{a_x} + \frac{\partial V}{\partial y}\vec{a_y} + \frac{\partial V}{\partial z}\vec{a_z}\right)$$

$$\nabla \times (\nabla V) = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \partial V/\partial x & \partial V/\partial y & \partial V/\partial z \end{vmatrix} = \vec{a_x}\left(\frac{\partial^2 V}{\partial y \partial z} - \frac{\partial^2 V}{\partial y \partial z}\right) - \vec{a_y}\left(\frac{\partial^2 V}{\partial x \partial z} - \frac{\partial^2 V}{\partial x \partial z}\right) + \vec{a_z}\left(\frac{\partial^2 V}{\partial x \partial y} - \frac{\partial^2 V}{\partial y \partial x}\right)$$

$$= \vec{0} \quad \text{(Irrotational)}$$

$\nabla \times \vec{E} = 0$

(so E is a gradient)

ii) $\nabla . (\nabla \times V) = 0$

$$\nabla \times V = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ V_1 & V_2 & V_3 \end{vmatrix}$$

$$= \vec{a_x}\left(\frac{\partial v_3}{\partial y} - \frac{\partial v_2}{\partial z}\right) - \vec{a_y}\left(\frac{\partial v_3}{\partial x} - \frac{\partial v_1}{\partial z}\right) + \vec{a_z}\left(\frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y}\right)$$

divergence:

$$\nabla \cdot \vec{F} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial x}[F_1 h_2 h_3] + \frac{\partial}{\partial y}[F_2 h_1 h_3] + \frac{\partial}{\partial z}[F_3 h_1 h_2]\right]$$

$$\boxed{\nabla \cdot \vec{V} = \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}}$$

$h_1 = h_2 = h_3 = 1$

$$\nabla \cdot (\nabla \times \vec{F}) = \frac{\partial}{\partial x}\left(\frac{\partial v_3}{\partial y} - \frac{\partial v_2}{\partial z}\right) - \frac{\partial}{\partial y}\left(\frac{\partial v_3}{\partial x} - \frac{\partial v_1}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y}\right)$$

$$= \cancel{\frac{\partial^2 v_3}{\partial x \partial y}} - \cancel{\frac{\partial^2 v_2}{\partial x \partial z}} - \cancel{\frac{\partial^2 v_3}{\partial x \partial y}} + \cancel{\frac{\partial^2 v_1}{\partial y \partial z}} + \cancel{\frac{\partial^2 v_2}{\partial x \partial z}} - \cancel{\frac{\partial^2 v_1}{\partial z \partial y}}$$

$$\boxed{\nabla \cdot (\nabla \times \vec{F}) = 0}$$

'Divergence of a curl is zero'

↓

Solenoidal.

$F = 2\vec{a_r} + \sin\phi\ \vec{a_\phi} - z\vec{a_z}$ → curl

Soln:

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{a_r} & r\vec{a_\phi} & \vec{a_z} \\ \partial/\partial r & \partial/\partial\phi & \partial/\partial z \\ 2 & r\sin\phi & -z \end{vmatrix} \cdot \frac{1}{r}$$

$$\frac{1}{r}\left(\vec{a_r}\left(-\frac{\partial z}{\partial \phi} - \frac{\partial \sin\phi \cdot r}{\partial z}\right) - r\vec{a_\phi}\left(-\frac{\partial z}{\partial r} - \frac{\partial (2)}{\partial z}\right) + \vec{a_z}\left(\frac{\partial \sin\phi \cdot r}{\partial r} - \frac{\partial z}{\partial \phi}\right)\right)$$

$$\frac{1}{r}\left(\vec{a_r}(0-0) - \vec{a_\phi}(0) + \vec{a_z}(\sin\phi(1))\right)$$

$$= \frac{\sin\phi \cdot \vec{a_z}}{r}$$

$$= \frac{1}{r}(\sin\phi) \cdot \vec{a_z}$$

12/03/2021

<u>Helmholtz theorem - Fundamental theorem of vector calculus</u>

$\vec{T}$ → curl free vector

$\nabla \times (\nabla V) = \vec{0}$

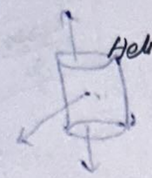
$\nabla \cdot (\nabla \times \vec{A}) = 0$

$\vec{T}$ → divergence free vector.

$\nabla \times E = 0$ | $\nabla \cdot B = 0$

$E = -\nabla \phi$ | $B = \nabla \times H$

G
D
C
DT
ST



Helmholtz theorem: solenoid + Irrotational vector

$= (\nabla \times \vec{m}) + \nabla f$

In a rapidly ↓ vector field in 3D that vector is represented by a solenoidal vector + Irrotational vector

Any sufficiently smooth - rapidly decaying vector field in 3D can be resolved in to the sum of an Irrotational vector field and a solenoidal vector field.

1) $H = \frac{Ir}{2\pi a^2} (\vec{a_\phi})$ where I and a are constants

Show that its curl is invariant in space.

solu:

$$\nabla \times H = \frac{1}{r} \begin{vmatrix} \vec{a_r} & r\vec{a_\phi} & \vec{a_z} \\ \partial/\partial r & \partial/\partial\phi & \partial/\partial z \\ 0 & r\frac{Ir}{2\pi a^2} & 0 \end{vmatrix}$$

$$= \frac{1}{r}\left[\vec{a_r}\left(-\frac{\partial}{\partial z} r\left(\frac{Ir}{2\pi a^2}\right)\right) - r\vec{a_\phi}(0) + \vec{a_z}\left(\frac{\partial}{\partial r} r\left(\frac{Ir}{2\pi a^2}\right)\right)\right]$$

$$= \frac{1}{r}\left[\left[0+0+0\right] + \left[\vec{a_z}\left(\frac{I \cdot 2r}{2\pi a^2}\right)\right] \cdot \vec{a_z}\right]$$

$$= 0 \ [\text{Irrotational}] \quad \frac{1}{r}\left[\frac{I\,2r}{2\pi a^2}\right] \cdot \vec{a_z} = \frac{I}{\pi a^2} \cdot \vec{a_z}$$

$\therefore$ curl is invariant

$\frac{I}{\pi a^2} \rightarrow$ Constant

$$\boxed{\nabla \times H = \frac{I}{\pi a^2} \cdot \vec{a_z}}$$

'No change in direction'

In a particular vector field $\vec{F} = r^2\cos^2\phi \cdot \vec{a_r} + z\sin\phi\,\vec{a_\phi}$

Find the flux emitting from the closed surface of the cylinder from $r = 4$, $0 \le z \le 1$

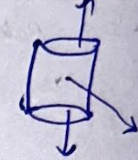
Solu:

$\int_V \nabla \cdot \vec{F}\, dv = \oint_S \vec{F} \cdot d\vec{s}$ → verify divergence theorem.

∴

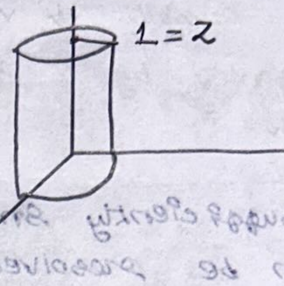
Flux emanating from the surface

$\oint_S \vec{F} \cdot d\vec{s} = \iint$ (top + bottom + curved)



In top surface 

In bottom surface 



$dS_T = r\, d\phi\, dr\, \vec{a_z}$

$dS_B = r\, d\phi\, dr\, (-\vec{a_z})$

$dS_C = r\, d\phi\, dz\, (\vec{a_r})$

$$\oint_S \vec{F} \cdot d\vec{s} = \oint_T \vec{F} \cdot d\vec{S_T} + \oint_B \vec{F} \cdot d\vec{S_B} + \oint_C \vec{F} \cdot d\vec{S_C}$$

$$= \oint_T (r^2\cos^2\phi\, \vec{a_r} + z\sin\phi\, \vec{a_\phi}) \cdot (r\, d\phi\, dr\, \vec{a_z})$$

$$+ \oint_B (r^2\cos^2\phi\, \vec{a_r} + z\sin\phi\, \vec{a_\phi}) \cdot (r\, d\phi\, dr(-\vec{a_z}))$$

$$+ \oint_C (r^2\cos^2\phi) \cdot (r\, d\phi\, dz)$$

$$= 0 + 0 \oint_C r^3\cos^2\phi\, d\phi\, dz$$

$$= \int_0^{2\pi}\int_0^{1} r^3\cos^2\phi\, d\phi\, dz$$

$$= \cancel{\int_0^{2\pi} \left[\frac{r^4}{4}\right]_0^4 \cos^2\phi\, d\phi}$$

$$= \int_0^{2\pi} \frac{1}{\cancel{64}} \cos^2\phi\, d\phi$$

$$= \frac{1}{\cancel{64}} \left[\frac{(\cos\phi)^3}{3(-\sin\phi)}\right]_0^{2\pi} = \cancel{\frac{64}{3}\left[\frac{\cos^3\phi}{\sin\phi}\right]_0^{2\pi}}$$

$= \cancel{\frac{64}{3}}$ $= \int_0^{2\pi}\int_0^1 r^3\cos^2\phi \, d\phi \, dz$

$r^2\cos^2\phi \; r\,dz\,d\phi$

$\downarrow$

$\boxed{\frac{1+\cos 2\pi}{2}}$

$= \int_0^{2\pi}\int_0^1 r^3\left(\frac{1+\cos 2\phi}{2}\right) d\phi \, dz.$

$= \cancel{\frac{r^3}{2}\int_0^{2}\left[\frac{\phi}{\phi} + \frac{\sin 2\phi}{2}\right]}$

$= \int_0^{2\pi}\left[r^3 z + \frac{r^3 z\cos 2\phi}{2}\right]_0^1 d\phi$

$= \int_0^{2\pi}\frac{r^3}{2}\left(\left(\frac{1+\cos 2\phi}{2}\right)\right) d\phi$

17/03/2021

$= r^3\left[\int_0^{2\pi}\frac{1}{2}\,d\phi + \int_0^{2\pi}\frac{\cos 2\phi}{2}\,d\phi\right]$

$= r^3\left[\frac{\phi}{2}\right]_0^{2\pi} + r^3\left[\frac{\sin 2\phi}{4}\right]_0^{2\pi}$

$r = 4$

$= r^3\left[\frac{2\pi}{2} + \frac{\sin 4\pi}{4}\right]$

$= r^3(\pi + 0)$

$= 64\pi$ units.

$\int_V \nabla\cdot\vec{F}\,dv = \int(\nabla\cdot f)\,dv.$

$dv = \cancel{r}\,d\phi\,dr\,dz$

$\nabla\cdot F = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial r}(f_1h_2h_3) + \frac{\partial}{\partial\phi}(f_2h_1h_3) + \frac{\partial}{\partial z}(f_3h_1h_2)\right]$

$= \frac{1}{\cancel{r}}\left[\frac{\partial}{\partial r}(r^3\cos^2\phi) + \frac{\partial}{\partial\phi}(z\sin\phi)\right]$

$= \frac{1}{r}\left[3r^2\cos^2\phi + z\cos\phi\right]$

$= 3r\cos^2\phi + \frac{z}{r}\cos\phi.$

$= \int\int\int\left(3r\cos^2\phi + \frac{z}{r}\cos\phi\right) r\,d\phi\,dr\,dz.$

$$= \int_0^1 \int_0^{2\pi} \int_0^{4} ((3r^2 \cos^2\phi) + z\cos\phi)\, d\phi\, dr\, dz$$

$$= \int_0^1 \int_0^{2\pi} \left(\frac{3r^3}{3} \cos^2\phi + zr\cos\phi \right)_0^4 d\phi\, dz$$

$$= \int_0^1 \int_0^{2\pi} \left(r^3 \cos^2\phi + zr\cos\phi \right)_0^4 d\phi\, dz$$

$\int 1\, dx = x.$

$\frac{d(x)}{dx} = 1.$

$$= \int_0^1 \int_0^{2\pi} (64\cos^2\phi + 4z\cos\phi)\, d\phi\, dz$$

$$= \int_0^1 \int_0^{2\pi} \left(\frac{64}{2} \left(\cos 2\phi + 1 \right) + 4z\cos\phi \right) d\phi\, dz$$

$$= \int_0^1 32 \left(\phi + \frac{\sin 2\phi}{2} \right) + 4z\sin\phi \Big]_0^{2\pi} dz$$

$$= \int_0^1 32\, (2\pi)\, dz$$

$$= \int_0^1 64\pi\, dz$$

$$= [64\pi z]_0^1$$

$$= 64\pi$$

$$\therefore \boxed{\begin{array}{c} 64\pi = 64\pi \\ L.H.S = R.H.S \end{array}}$$

Given $\vec{a} = x^2 \vec{a_x} + xy \vec{a_y} + yz \vec{a_z}$ → verify divergence theorem over one unit cube (each side) of the Cartesian coordinate system cube: cube lies in 1st octant, with one coordinate at origin.

Solu:

$$\vec{A} = x^2 \hat{a_x} + xy \hat{a_y} + yz \hat{a_z} \quad \text{(1st octal)}.$$

$(x=1, y=1, z=1)$

$$\int_v \nabla \vec{A}\, dv = \oint_s \vec{A}\, d\vec{s}$$

$$d\vec{v} = dx\, dy\, dz$$

$$\nabla \cdot \vec{A} = \frac{\partial}{\partial x}(x^2) + \frac{\partial}{\partial y}(xy) + \frac{\partial}{\partial z}(yz)$$

$= 2x + x + y$

$= 3x + y$

$= \int_0^1 \int_0^1 \int_0^1 (3x+y)\, dx\, dy\, dz.$

$= \int_0^1 \int_0^1 \left(\frac{3x^2}{2} + xy\right)_0^1 dy\, dz$

$= \int_0^1 \int_0^1 \left(\frac{3}{2} + y\right) dy\, dz$

$= \int_0^1 \left(\frac{3y}{2} + \frac{y^2}{2}\right)_0^1 dz$

$= \int_0^1 \left(\frac{3}{2} + \frac{1}{2}\right) dz$

$= \frac{3}{2} + \frac{1}{2}$

$= \frac{4}{2}$

$= 2.$

Surface 2:

$= \int_0^1 \int_0^1 x^2\, dy\, dz.$

$= \int_0^1 \int_0^1 \left(x^2 y\right)_0^1 dz$

$= \left(x^2\right)$

$= 1$

Surface.

$\oint_S \vec{A}\, ds = \oint_{S_1} + \oint_{S_2} + \oint_{S_3}$

$\oint_S \vec{A}\, ds$

Surface: 1

$= \int_0^1 \int_0^1 (x^2 \vec{a_x} + xy \vec{a_y} + yz \vec{a_z}).$

$dx\, dy\, \vec{a_z}$

$= \int_0^1 \int_0^1 yz\, dx\, dy$

$= \int_0^1 (xyz)_0^1 dy$

$= \int_0^1 yz\, dy$

$= \left(\frac{y^2}{2} z\right)_0^1$

$= \frac{1}{2}(2)$

$= \frac{1}{2}$

Surface 3

$= \int_0^1 \int_0^1 xy\, dz\, dx$

$= \frac{1}{2}.$

$\oint_S \vec{A}\, ds = \frac{1}{2} + \frac{1}{2} + 1$

$= 2$

18/8/2021

Unit-2 - Electrostatics

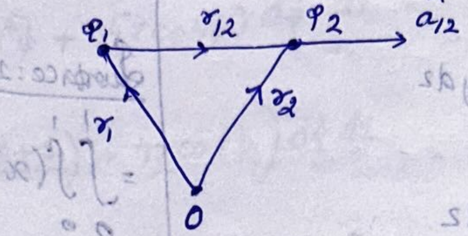
* Coloumbs
* Gauss) Laws
* Dielectric

Static charges - Stationary field.

Static Electric field → due to static charges

~~Coloumb~~

$$|F| \propto \frac{q_1 q_2}{r^2} \hat{a}_{12} \longrightarrow \text{Influence of charge 1 on 2.}$$



$\boxed{\text{Coloumbs ~~law~~}}$

Vector form

$$\boxed{F = K \frac{q_1 q_2}{r^2} \hat{a}_{12}}$$

$$\boxed{K = \frac{1}{4\pi\varepsilon_0}}$$

ε_0 - permittivity of free space.

$$\boxed{\varepsilon_0 = 8.854 \times 10^{-12}\ F/m}$$

Scalar form:

$$\boxed{F = K \frac{q_1 q_2}{r^2}}$$

Unit vector $\hat{a}_{12} = \frac{\vec{r_{12}}}{|\vec{r_{12}}|}$

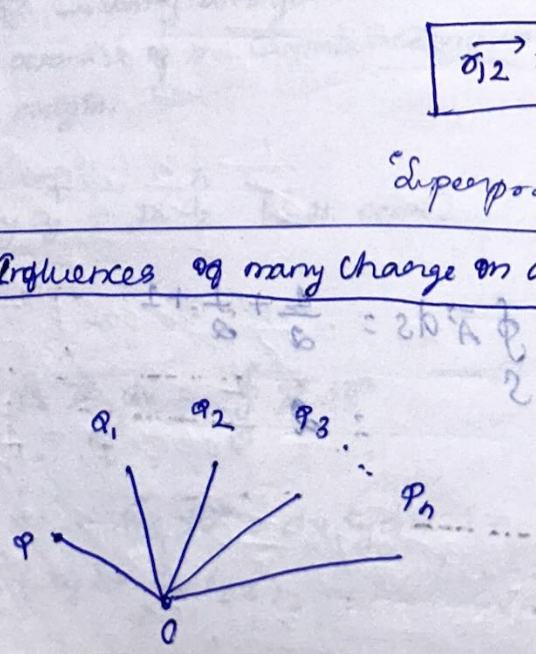
$$\vec{F} = K \frac{q_1 q_2}{r^2} \frac{\vec{r_{12}}}{|\vec{r_{12}}|}$$

$$r_1 + \vec{r_{12}} = r_2$$

$$\boxed{\vec{r_{12}} = r_2 - r_1}$$

'Superposition Principle'

Influences of many charge on a single charge



q_1 on q, q_2 on q, q_n on q.

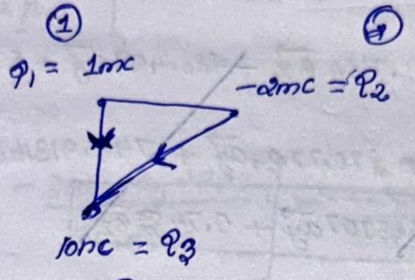
$$F = K\frac{q q_1}{r^2}\frac{\vec{r_0}-\vec{r_1}}{|r_0-r_1|} + K\frac{q q_2}{r^2}\frac{\vec{r_0}-\vec{r_2}}{|r_0-r_1|} + \cdots$$

$$= \frac{1}{4\pi\varepsilon_0 r} q \sum_{n=1}^{n} q_n \left(\frac{\vec{r_0}-\vec{r_n}}{|\vec{r_0}-\vec{r_n}|}\right).$$

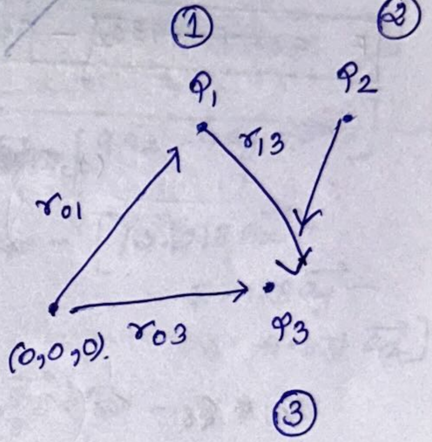
18/3/21

1) point charges 1mC and -2mC are located at $(3,2,-1)$ and $(-1,-1,4)$ respectively. calculate the force on 10nC charge located at $(0,3,1)$

soln:



$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} \cdot \frac{\vec{r}}{|\vec{r}|}$$



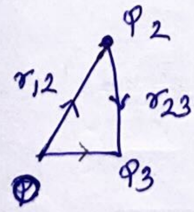
$$r_{01} + r_{13} = r_{03}$$

$$\boxed{r_{13} = r_{03} - r_{01}}$$

$$r_{13} = -3\vec{a_x} + \vec{a_y} + 2\vec{a_z}$$

$$|\vec{r_{13}}| = \sqrt{9+1+4}$$
$$= \sqrt{14}.$$

$$\boxed{r^2 = 14}$$



$$r_{23} = r_{13} - r_{12}$$

$$r_{23} = -(-\hat{a_x} - \hat{a_y} + 4\hat{a_z}) + (3\hat{a_y} + \hat{a_z})$$

$$\boxed{r_{23} = \hat{a_x} + 4\hat{a_y} - 3\hat{a_z}}$$

$$|\vec{r_{23}}| = \sqrt{1+16+9}$$
$$= \sqrt{26}.$$

$$\boxed{r^2 = 26}$$

$$F = F_1 + F_2 = \frac{q_1 q_3}{4\pi\varepsilon_0}\left[\frac{-3a_x + \vec{a_y} + 2\vec{a_z}}{14\sqrt{14}}\right] + \frac{q_2 q_3}{4\pi\varepsilon_0}\left[\frac{\vec{a_x} + 4\hat{a_y} - 3\hat{a_z}}{26\sqrt{26}}\right]$$

$\hookrightarrow |r_{13}|$

$$= \frac{q_3 m}{4\pi\varepsilon_0}\left[\frac{1(-3a_x + a_y + 2a_z)}{14\sqrt{14}} + \frac{(-q_2 m)(\vec{a_x} + 4\vec{a_y} - 3\vec{a_z})}{26\sqrt{26}}\right]$$

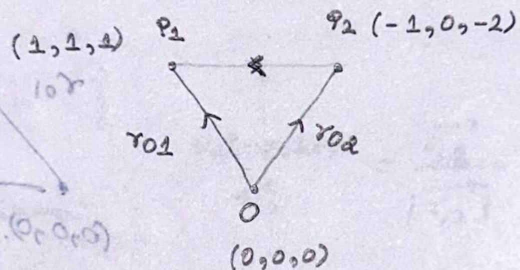
$$= \frac{10n \times m \times}{4\pi\varepsilon_0}\left[\frac{-3a_x + a_y + 2a_z}{52.3832} + \frac{-2a_x - 8a_y + 6a_z}{132.5745}\right]$$

$$= \frac{0.089877}{6944.6765}\left[-397a_x + 132.5745a_y + 265.149a_z - 104.7664a_x - 419.0656a_y + 314.2992\right] \times m$$

$$= 1.294185\left[-501.7664\vec{a_x} - 286.4911\vec{a_y} + 579.4482\vec{a_z}\right] \times m$$

$$= (-649.379\vec{a_x} - 370.7724\vec{a_y} + 749.9131687) \times m$$

$$\boxed{F = -0.6493\vec{a_x} - 0.3707\vec{a_y} + 0.749\vec{a_z}}$$



$r_{01} - r_{21} - r_{02} = 0$

$r_{21} + r_{02} - r_{01} = 0$

$$\boxed{r_{21} = r_{01} - r_{02}}$$

$$r_{21} = (\hat{a_x} + \hat{a_y} + \hat{a_z}) - (-\hat{a_x} - 2\hat{a_z})$$

$$\boxed{r_{21} = (2\hat{a_x} + \hat{a_y} + 3\hat{a_z})\, m}$$

$$r_{21} = \sqrt{(4 + 1 + 9)\, m \times m}$$

$$= \sqrt{14}\, m$$

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0}\left[\frac{2\hat{a_x} + \hat{a_y} + 3\hat{a_z}}{14\sqrt{14}\, m^2 \times m}\right]$$

$$= \frac{100\mu \times 50\mu}{4\pi \times 8.854\times10^{-12}}\left[\frac{2\hat{a_x} + \hat{a_y} + 3\hat{a_z}}{52.3832\, n}\right]$$

$$= \frac{5000}{4\pi \times 8.854 n}\left[2\hat{a_x} + \hat{a_y} + 3\hat{a_z}\right] = 4.494\times10^{10}(2\hat{a_x} + \hat{a_y} + 3\hat{a_z})$$

$$= 44.946(2\hat{a_x} + \hat{a_y} + 3\hat{a_z})$$

A $100\mu C$ located in a rect c.s at $(1,1,1)$ m and another $q_2 = 50\mu C$ $(-1,0,-2)$ m. Find the vector force on the first charge

as

$$F = \frac{100\mu \times 50\mu}{4\pi \times 8.854 \times 10^{-12}} \left[\frac{2\hat{a_x} + \hat{a_y} + 3\hat{a_z}}{52.3832}\right]$$

$= 0.8578\,(2\hat{a_x} + \hat{a_y} + 3\hat{a_z})$

$= 1.715\,\hat{a_x} + 0.86\,\hat{a_y} + 2.57\,\hat{a_z}$

$$= \frac{(2mC)(10\mu C)}{4\pi \times \frac{1}{36\pi} \times n} \left[\frac{-3a_x + a_y + 2a_z}{52.3832}\right] + \frac{(-2m)(10n)}{4\pi \times \frac{1}{36\pi}\,n} \left[\frac{9\vec{a_x} + 4\vec{a_y} - 3\vec{a_z}}{132.574}\right]$$

$$= \frac{20m}{1/9}\left[\frac{-3a_x + a_y + 2a_z}{52.3832}\right] - \frac{20m}{1/9}\left[\frac{9\vec{a_x} + 4\vec{a_y} - 3\vec{a_z}}{132.574}\right]$$

$= 1.718\,(-3a_x + a_y + 2a_z) - 1.357\,[9\vec{a_x} + 4\vec{a_y} - 3\vec{a_z}]$

$= (-5.154\,\vec{a_x} + 1.718\,\vec{a_y} + 3.436\,\vec{a_z}) - [12.213\,\vec{a_x} + 5.428\,\vec{a_y} - 4.071\,\vec{a_z}]$

$= (-17.367\,\vec{a_x} - 3.71\,\vec{a_y} + 7.507\,\vec{a_z})$ m

$= -0.017\,\vec{a_x} - 0.00371\,\vec{a_y} + 0.007507\,\vec{a_z}$

19/03/2021

Electric field intensity

→ field exp is less

will be more

Intensity varies

'Electric field intensity'

$$E = \frac{F}{q_t} \longrightarrow \text{Test charge.}$$

$$E = \frac{\dfrac{q_1 q_t}{4\pi\varepsilon_0 r^2} \cdot \dfrac{\vec{r_{1t}}}{|r_{1t}|}}{q_t}$$

$$E = \frac{\dfrac{q_1 q_t}{4\pi\varepsilon_0 |r_{1t}|^3}}{q_t} \cdot \hat{r}_{1t}$$

$$\boxed{E = \frac{q_1}{4\pi\varepsilon_0 |r_{1t}|^3} \cdot \hat{r}_{1t}} \rightarrow \text{vector.}$$

N/C (or) V/m (volt/meter)

<u>Cases:</u> i) q_1 is at the origin

ii) when there are many charges starting from $q_1, q_2, \ldots$

i) $$E = \frac{q_1}{4\pi\varepsilon_0 |r_{1t}|^3} \hat{r}_{1t}$$

At origin — q_1

$\therefore r_1 = \sqrt{0+0+0} = 0$

$$= \frac{q_1}{4\pi\varepsilon_0 |r_t - r_1|^3} (\hat{r}_t - \hat{r}_1)$$

$\boxed{r_t - r_1 = r_t}$

$$E = \frac{q_1}{4\pi\varepsilon_0 |r_t|^3} (\hat{r}_t)^3$$

ii) Lot of charges.

q_1 q_2 q_3 --- q_t

$$\vec{E} = \sum \frac{q_1}{4\pi\varepsilon_0 |r_{1t}|^2} \frac{\vec{r_t} - \vec{r_1}}{|\vec{r_t} - \vec{r_1}|} + \frac{q_2}{4\pi\varepsilon_0 |r_{2t}|^2} \frac{\vec{r_t} - \vec{r_2}}{|\vec{r_t} - \vec{r_2}|} + \cdots$$

$$= \sum_{n=1}^{\infty} \frac{q_n}{4\pi\varepsilon_0 |r_{nt}|^2} \frac{\vec{r_t} - \vec{r_n}}{|\vec{r_t} - \vec{r_n}|}$$

D - Electric flux density

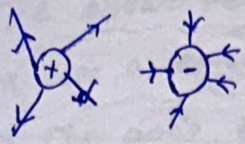
flux → flow

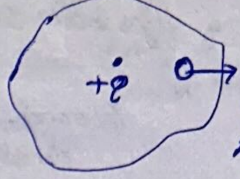
'No. of flux lines passing per unit area'

$$\boxed{D = \frac{\Psi}{A} \text{ (or) } \frac{\phi}{A}}$$

No. of field lines emanating from a closed surface passing through the surface $\perp^r$ ly.

"Divergence theorem"





$$\boxed{\psi = \oint_S D \cdot \vec{ds}}$$

→ Total surface.

Assume Spherical coordinate:

$$D = \int \frac{Q}{4\pi r^2} \, d\vec{S} \cdot \vec{a_r}$$

Area of curved surface of sphere $= 4\pi r^2$

$D =$

"Along radius"

$$\boxed{d\vec{S} = r\sin\theta \, dr \, d\phi \, \vec{a_\theta} + r^2 \sin\theta \, d\theta \, d\phi \, \vec{a_r} + r \, dr \, d\theta \, \vec{a_\phi}}$$

22/03/21 | Magnitude of Electric field ↑ → density also ↑ |

Electric flux density $(D) \longrightarrow D = \frac{Q}{A}$ (vector $\perp^r$ to the surface)

(Area of sphere $= 4\pi r^2$)

$$\vec{D} = \frac{Q}{4\pi r^2} \vec{a_r}$$

(Along the radius vector)

$$d\vec{S} = r^2 \sin\theta \, d\theta \, d\phi \, \vec{a_r} + r\sin\theta \, dr \, d\phi \, \vec{a_\theta} + r \, dr \, d\theta \, \vec{a_\phi}$$

$$\psi = \oint_S D \cdot d\vec{S}$$

$$= \oint_S \frac{Q}{4\pi r^2} \vec{a_r} \cdot d\vec{S}$$

$$= \oint_S \frac{Q}{4\pi r^2} \cdot r^2 \sin\theta \, d\phi \, d\theta$$

$$= \oint_S \frac{Q \sin\theta}{4\pi} \, d\phi \, d\theta$$

$$= \frac{Q}{4\pi} \int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta \, d\phi$$

$$= \frac{Q}{4\pi} \int_0^{\pi} \sin\theta \cdot d\theta \cdot \int_0^{2\pi} d\phi$$

$$= \frac{Q}{4\pi} \left[-\cos\theta\right]_0^{\pi} \cdot \left[\phi\right]_0^{2\pi}$$

$$= \frac{Q}{4\pi} [+1 + 1] \, (2\pi) \quad = \frac{Q}{4\pi} \cdot 4\pi$$

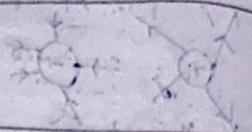
$$\boxed{\psi = Q}$$

↓

Gauss law.

The total flux crossing or eminating from a surface is equal to the algebraic sum of all the charges present in the surface. (closed)

Electric flux density $\longrightarrow$ also known as Displacement density.



Relation b/w E and D

E → Electric field intensity

D → Flux density.

$$E = \frac{F}{Q_t}$$

$$F = \frac{Q_1 Q_t}{4\pi\varepsilon_0 r^2}$$

$$\therefore D = \frac{Q_1}{4\pi r^2}$$

$$\frac{F}{Q_t} = \frac{Q_1}{4\pi\varepsilon_0 r^2}$$

$$E = \frac{Q_1}{4\pi\varepsilon_0 r^2}$$

$$\frac{E}{D} = \frac{\frac{Q_1}{4\pi\varepsilon_0 r^2}}{\frac{Q_1}{4\pi r^2}}$$

$\varepsilon_0 \rightarrow$ permittivity of free space.

$$\frac{E}{D} = \frac{1}{\varepsilon_0}$$

$$\boxed{D = \varepsilon_0 E}$$

App. of Gauss law

$$\phi = \oint_S \vec{D} \cdot ds$$

* Electric field intensity at a point at various distributions.
  - → point
  - → Line
  - → surface
  - → charge disc

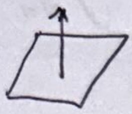
  charge

Steps: Applying Gauss law

* Symmetry (Identify) associated with charge distribution is obtained.
* Gaussian surface is identified & the direction of E is determined.
* field space is divided into different reigns.
* For each reign – the net charge enclosed by the Gauss law surface is calculated.
* ϕ is calculated for each reign
* Calculate the magnitude of E.

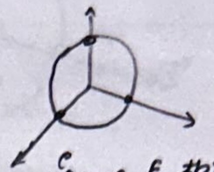
Gaussian surface – Point charge

* E should be same at all points. (E varies with radius normally but here must not)
* For a sphere – radius is equal.
* The angle b/w E and area vector should be same throughout.



surface vector

Flux



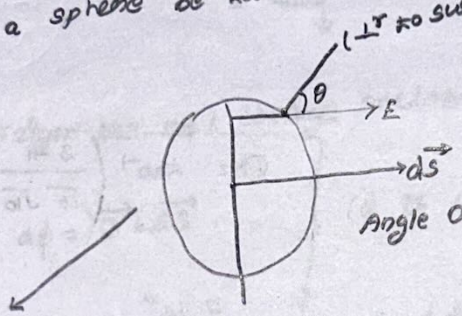
'Same E throughout'
↓
Gaussian Surface.

∴ Angle b/w E & area vector = 0° (same)

0° angle b/w E & $\vec{A}$

Gaussian surface for a line charge.

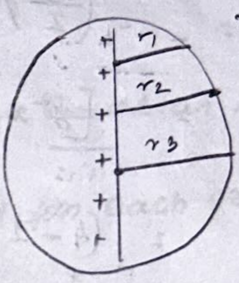
* Can a sphere be taken → ?



Angle not same.

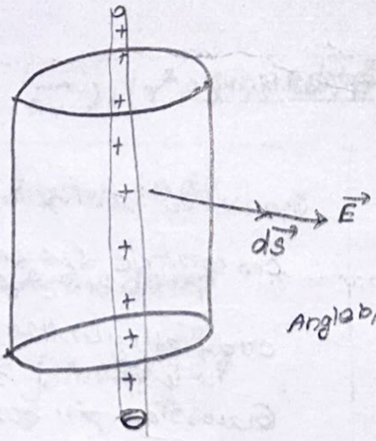
'Angle' – not same throughout.

$r_1 < r_2 < r_3$

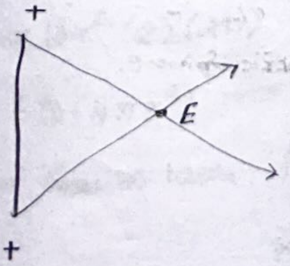


'radius changing'

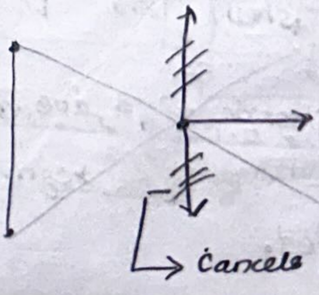
cylinder – Gaussian surface



Angle b/w them = 0° (Throughout)



Resolving in to two components

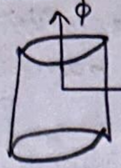


(Along radius vector) (surface)

Cancels (opp. side)

on top

ϕ



$\vec{ds}$ (Angle = 90°) $\longrightarrow$ 0.

$$D = 2xy\,\hat{i} + 3yz\,\hat{j} + 4xz\,\hat{k}$$

$$\phi = D \cdot A$$

$$= \iint (2xy\,\hat{i} + 3yz\,\hat{j} + 4xz\,\hat{k}) \cdot dy\,dz \cdot \vec{a_x}$$

$$= \int_0^4 \int_{-1}^{2} 2xy \cdot dy\,dz$$

$a \cdot b = a \cdot b \cos\theta$

$$= \left(\frac{2y^2}{2}\right)x$$

$$= \left[y^2 x\right]_{-1}^{2}$$

$$= (4-1)$$

$$= 3x$$

$$= 3x(4)$$

$$= 12 \times 3$$

$$= 36$$

$$\theta = \tan^{-1}\left(\frac{3-1}{\sqrt{6}\,\sqrt{10}}\right)$$

$$= 14^\circ$$

26/3/2021.

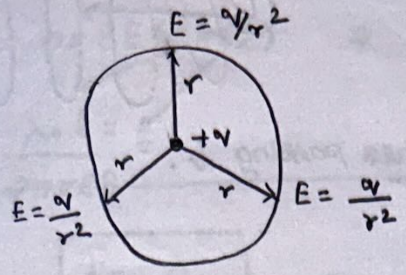
Applications of Gauss law

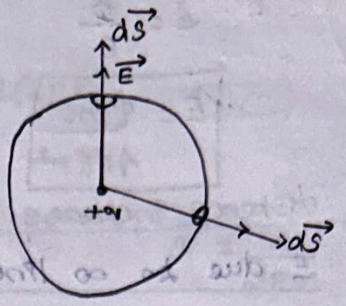
S.no	System	Symmetry	Gaussian surface
1	point charge	Spherical	Concentric surface
2	Infinite rod	cylindrical	coaxial cylinder
3	Infinite plane	planar	Gaussian pill box
4)	Sphere n spherical shell	Spherical	Concentric Sphere.

Electric field due to point charge:

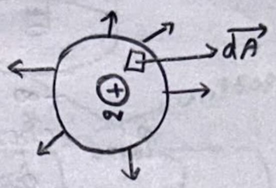
* Symmetry associated with charge distribution is obtained.

* E should be finite at all points (same)
* Angle b/w E and area vector should be same throughout





* Symmetry associated with charge distribution → spherical
* Gaussian surface & direction of E is determined → sphere
* Field space - divided in to different regions



For each region the net charge enclosed by the Gaussian surface is calculated.

$$d\phi = \vec{D} \cdot \vec{dS}$$ (ϕ is calculated for each region)

$$\vec{dS} = r^2 \sin\theta \, d\theta \, d\phi \, \vec{a_r} + r\sin\theta \, dr \, d\phi \, \vec{a_\theta} + r \, dr \, d\theta \, \vec{a_\phi}$$

$$\vec{D} = D \cdot \vec{a_r}$$

$$\phi = \oint \vec{D} \cdot \vec{dS}$$

$$\phi = \oint (D \cdot \vec{a_r}) \cdot (r^2 \sin\theta \, d\theta \, d\phi \, \vec{a_r} + r\sin\theta \, dr \, d\phi \, \vec{a_\theta} + r \, dr \, d\theta \, \vec{a_\phi})$$

$$= \oint D \cdot r^2 \sin\theta \, d\theta \, d\phi$$

$$= \iint D r^2 \sin\theta \, d\theta \, d\phi$$

$$= D r^2 \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi$$

$$= D r^2 (2)(2\pi)$$

$$= D(4\pi r^2)$$

From Gauss law. $\phi = Q_{enc}$

$$\phi = D(4\pi r^2)$$

$$Q_{enc} = D \cdot (4\pi r^2)$$

$$\boxed{D = \frac{Q_{enc}}{4\pi r^2}}$$

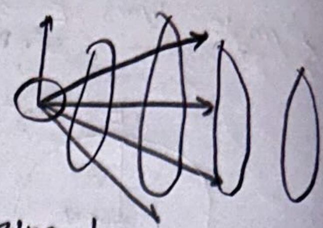
magnitude of D is deduced.

$$D = \varepsilon E$$

$$E = \frac{D}{\varepsilon}$$

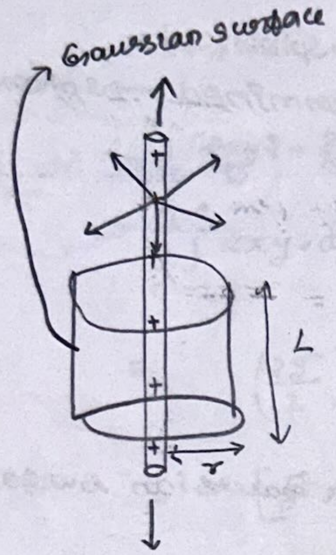
$$\boxed{E = \frac{q_{enc}}{4\pi\varepsilon r^2}}$$

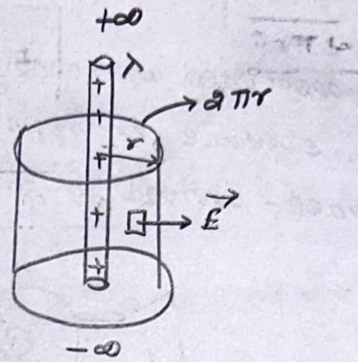
Inference



As distance increase no. of field lines passing ↓.

E due to ∞ line charge

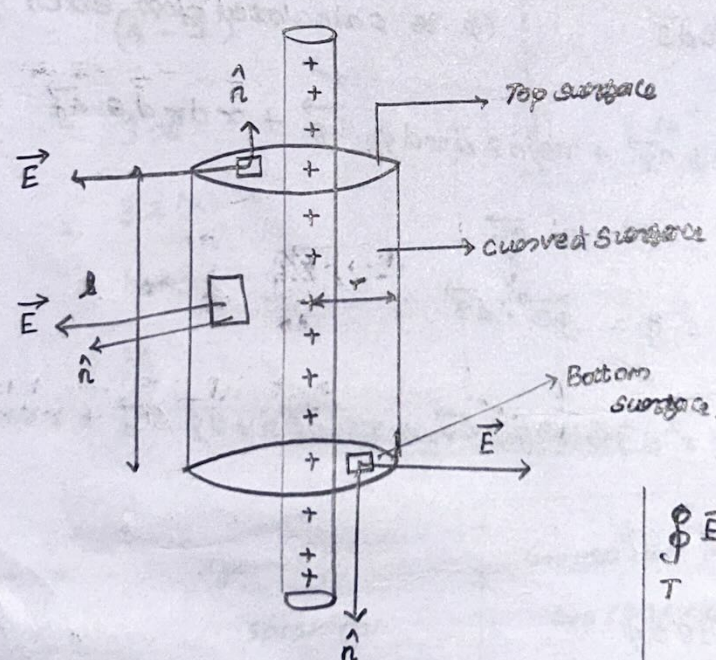


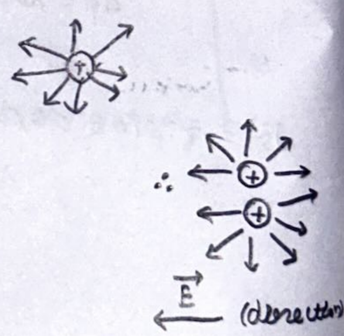


$$\boxed{\phi = q_{enc}}$$

q_{enc} = charge density × length

$$\boxed{q_{enc} = \lambda \cdot l}$$





$$\phi = \oint \vec{D} \cdot \vec{ds}$$

(or)

$$\phi = \oint \varepsilon \vec{E} \cdot \vec{ds}$$

$$\phi = \varepsilon \left[\oint_T + \oint_B + \oint_C \right]$$

$$\phi = \varepsilon \left[\oint_T E \cdot \vec{ds} + \oint_B E \cdot \vec{ds} + \oint_C E \cdot \vec{ds} \right]$$

$$\oint_T \vec{E} \cdot \vec{ds} = \oint |\vec{E}|\,|\vec{ds}| \cos\theta$$

$\therefore 90°$

$= 0$

$$\oint_B \vec{E} \cdot \vec{ds} = \oint |\vec{E}|\,|\vec{ds}| \cos\theta$$

$90°$

$= 0$

$$\oint_C \vec{E} \cdot \vec{ds} = \oint |\vec{E}|\,|\vec{ds}| \cos\theta$$

$$= E \oint \vec{ds} \quad (1)$$