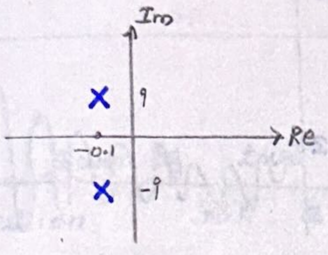
'Dominant pole approximation'.

It approximates that the slowest part of the system dominates the response, and the fastest parts of the system can be ignored. (Right most part of the pole dominates).

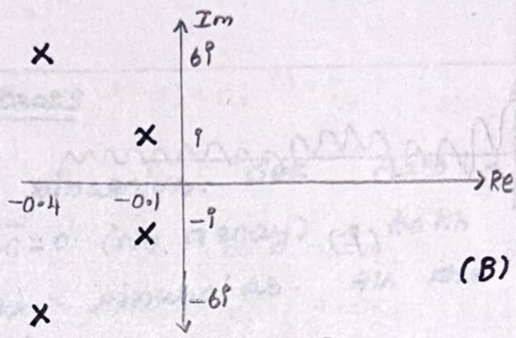
$\therefore$ (Right most part

The following are the pole diagrams of 6 functions of the form

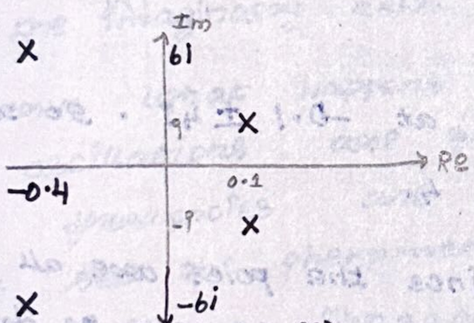
$$x(t) = Ae^{at}\cos(bt) + Be^{ct}\cos(dt)$$



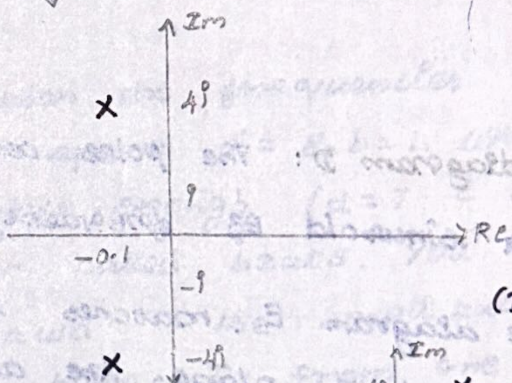
(A)



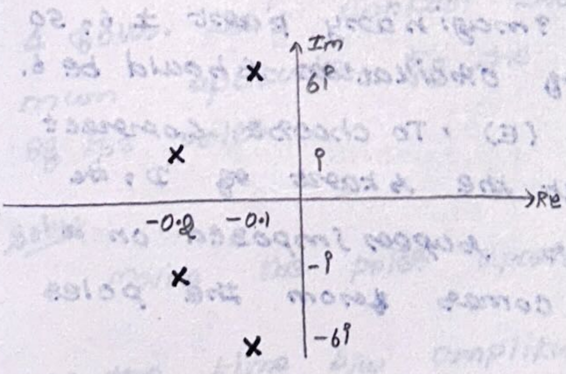
(B)



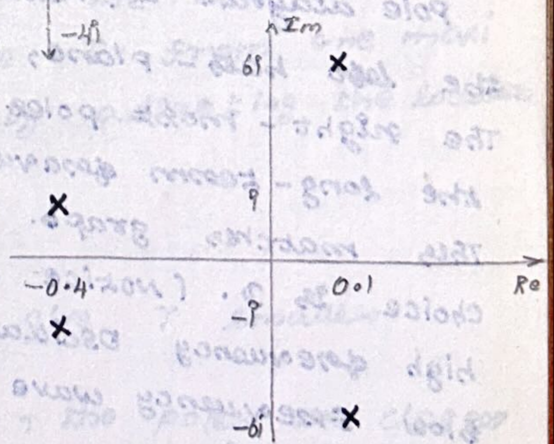
(C)



(D)

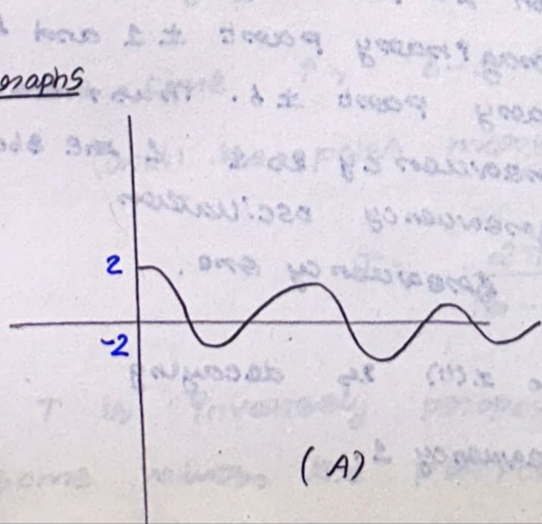


(E)

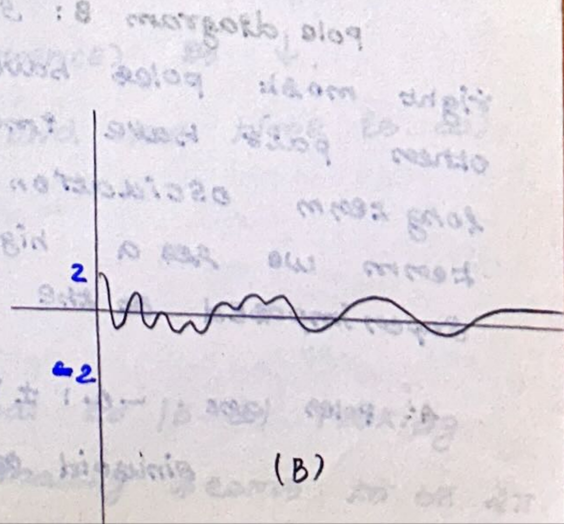


(F)

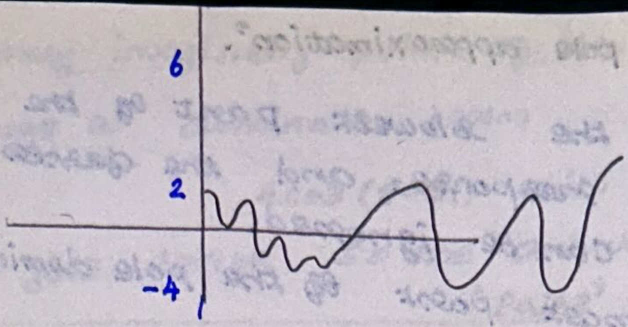
Graphs



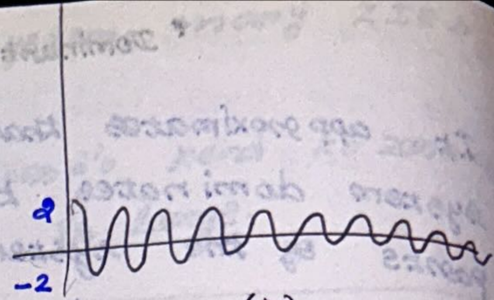
(A)



(B)



(C)



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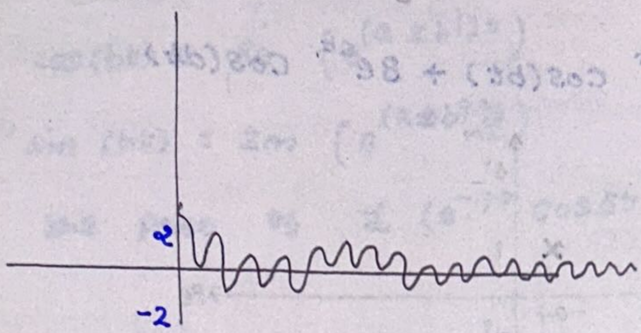
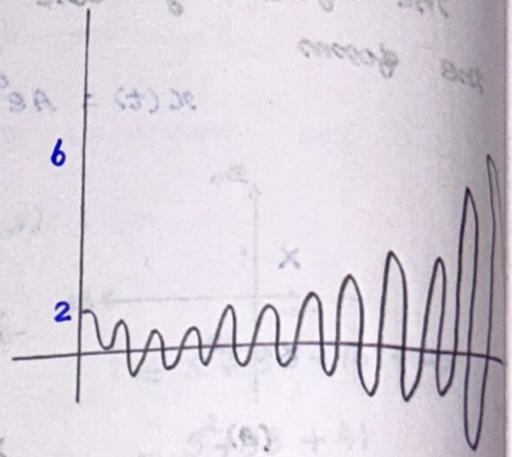


(E)



(F)

Pole diagram D: The poles are at $-0.1 \pm 4i$. So $x(t)$ is a sinusoid of frequency. 4.

Pole diagram E, matches since the poles are all in the left half-plane, the amplitude of $x(t)$ is decaying. The right-most poles have imaginary part ± 6, so the long-term frequency of oscillation should be 6. This matches graph. (D) or (E). To choose correct choice is D. (Notice that at the start of D, the high frequency oscillation is superimposed on a slow frequency wave. This comes from the poles at $-0.2 \pm i$.

Pole diagram B: Similar to previous, except the right most poles have imaginary part ± 1 and the other pair have imaginary part ± 6. Thus, the long term oscillation frequency is 1 & the short-term we see a high frequency oscillation superimposed on the low frequency one.

A: poles are $-0.1 \pm i$ so $x(t)$ is decaying sinusoid of frequency 1.

c: Since there are poles in the right half planes the amplitude of $x(t)$ is $\uparrow$. The right-most-poles haves imaginary part ± 1. So the long term freq of osciuations should be 1. This matches graph C.

f: $\because$ There are poles in the right half-plane, the amplitude of $x(t)$ is $\uparrow$. The right most poles have imaginary part ± 6. So the long term frequency oscillation should be 6. That it has period $\frac{\pi}{3}$.

<u>Beats</u>

Now sum of two sinusoids. Set $A = B = 2$ to get better resolution. Set $a = c = 0$ (No decay), both $f(t)$ and $g(t)$ are undamped sinusoids. All the poles are on the imaginary axis.

observe what happens when the frequencies of the two oscillations are far apart $d = 8, b = 1$. When the frequencies are close enough ($d = 7$ & $b = 6.5$) you see the phenomenon called beats. You would hear this as "waaaawawa" - the amplitude rises & falls. Let's control the time T from one maximum aplitude to the next by adjusting the location of the poles.

<u>Solu:</u>

Moving the poles apart will give T smaller

As the time b/w amplitudes $\downarrow$, the poles are closer

$b - d$ is large.

As the time $\uparrow$, $b - d$ (difference) is $\downarrow$.

(until the poles merge, and the time is ∞)

$$T = \frac{2\pi}{|b-d|}$$

T is inversely proportional to $|b-d|$. Testing some values, the proportionality seems to be 2π.

which function grows the fastest as $t \to \infty$

i) $e^{2t} \cos(400t)$

ii) $e^{2.1t} \sin(0.1t)$

iii) $t^{30} e^{1.9t}$

Solu:

∴ The growth rate is determined by the exponential term. Since $e^{2.1t}$ is the fastest growing exponential (It's the fastest growing function.

i) $e^{-2t} \cos(400t)$, $e^{-2.1t} \sin(0.1t)$, $t^{-30} e^{-1.9t}$

Solu:

∴ The decay rate is determined by the exponential term. Since $e^{-2.1t}$ is the fastest growing exponential among three. (with greatest decay right among the three, it's the fastest decaying function.

Exponential term

$f(t)$ is of exponential type K, for some real number K, if the constant $c > 0$.

$$|f(t)| \leq ce^{Kt} \quad \text{for all } t \geq 0$$

Match the smallest available exponential with functions

$f(t)$	Reign of Covergence	exponential type
e^{t}	$Re(z) > 1$	1
$e^{-t} \sin(4t)$	$Re(z) > -1$	-1
$t^{30} e^{-t} + 1$	$Re(z) > 0$	0
$\sin(t) + 1$	$Re(z) > 0$	0
$e^{-2t} + e^{-3t} \cos(t)$	$Re(z) > -2$	-2.

$\therefore$ For Laplace, we need a converging function. Incase of diverging, it blows up.

When $F(s) = \mathcal{L}(f(t); s)$ has a pole at $s = r$, the value of the integral defining it becomes ∞ & fails to converge. The right most poles in $F(s)$ mark the left edge of the region of convergence.

e^t $\quad$ ~~$e^{i\theta} = \cos\theta + i\sin\theta$~~

$f(t) \le ce^{Kt}$

Idea is that the function doesn't grow faster than e^{Kt} as $t \to \infty$.

1) e^t (No extra components)
(The function moves faster).
$\therefore$ as $Re(z) > 1$

2) Here $e^{-t}\sin(4t)$
So $Re(z) > -1$.

$f(t) \le ce^{Kt}$ only when R.H.S is moving faster than LHS

3) $t^{30}e^{-t} + 1$ $\quad$ $(Re\ z > 0)$

4) $\sin t + 1$ $\quad$ $Re\ z > 0$

5) $e^{-2t} + e^{-3t}\cos(t)$ $\quad$ $Re(z) > -2$.

$$\mathcal{L}(h_1) = \frac{s^2 - 3s + 2}{(s^2-6)(s^2+2s+5)}$$

Solu: What's the minimum (infimum) exponential type of h_1?

Poles $s = \dfrac{-2 \pm \sqrt{4-20}}{2} = -1 \pm \sqrt{-4}$

$= -1 \pm 2i$

and $s^2 = 6 \Rightarrow s = \pm\sqrt{6}$ The function h_1 is a summand of terms whose exponential type is the real part of each root of the Laplace transform.

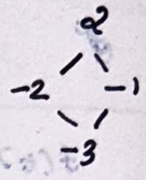
$\therefore$ The largest real part is $\sqrt{6}$.

$$\mathcal{L}(h_2) = \frac{s^2-3s+2}{(s^2-4)(s^2+2s+5)}$$

Solu:

$$s = \frac{-2 \pm \sqrt{4-4(5)}}{2} = -1 \pm 2i$$

$$s = \pm\sqrt{4} = \pm 2 \qquad \Big\} \quad s^2-3s+2=0$$



$$\therefore \mathcal{L}(h_2) = \frac{(s-2)(s-1)}{(s-2)(s+2)(s^2+2s+5)}$$

$$= \frac{s-1}{(s^2+2s+5)(s+2)}$$

$\therefore$ poles are $s = -1 \pm 2i, -2$. The largest real part is -1 (exponential type of h_2).

Conclusions on growth & decay

We can make some conclusions:

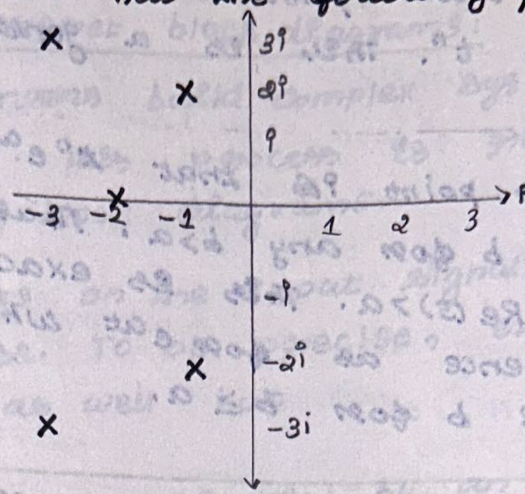
1) $f(t)$ decays exponentially to zero exactly when all of the poles of $F(s)$ have negative real part; that is, they all lie in the left half plane.

To put this in context in our analysis of LTI systems the homogeneous solutions to an LTI system are transient when the poles of the transfer function of that system all have negative real part.

The right-most poles of $F(s)$ determines the long-term behaviour of $f(t)$:

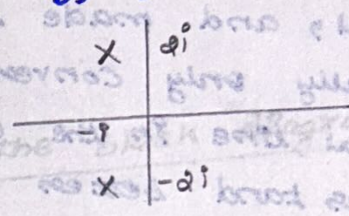
If the real part of some pole is positive, then $f(t)$ grows exponentially as $t \to \infty$. But in either case, we can be very precise about the rate of growth or decay.

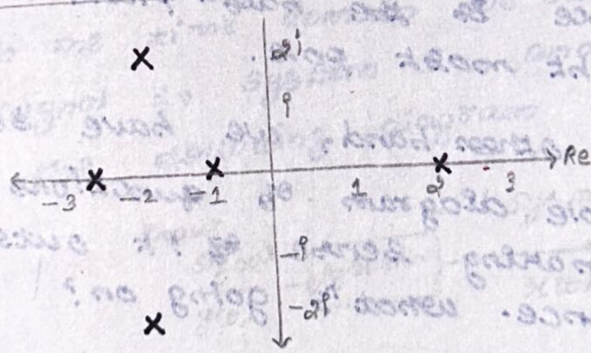
Q.1 What's the exponential type of the function whose Laplace transform has the following pole diagram?



Solu: Dominant term (most rightwards)

$Ae^{-t}\cos(2t-\phi) \rightarrow$ Function is of exponential type -1.





Solu: This problem has real part alone.

Ae^{2t}. $\rightarrow$ The function is of exponential type 2.

Poles with multiplicity:

A further complication is the possibility of a repeated factor in the denominator. This results in a "double pole" (If the factor occurs squared) or higher in the pole diagram.

$$t^n \rightsquigarrow \frac{n!}{s^{n+1}}$$

$$t^n e^{at} \rightsquigarrow \frac{n!}{(s-a)^{n+1}}$$

A similar rule holds for repeated complex roots:
an $(n+1)$-fold root produces multiplication of the
expected $f(t)$ by t^n. This is a general case of
'resonance'.

The main point is that $t^n e^{at}$ is of
exponential type b for any $b>a$. Thus $\mathcal{L}(t^n e^{at}; s)$
converges for $Re(s)>a$. This is exactly the same
region of convergence as for e^{at} which has
exponential type b for $b \geq a$.

Region of convergence again

Laplace transform $F(s) = \mathcal{L}(f(t); s)$ using an improper
integral, and made the point that this integral will
generally only converge for s to the right of some
vertical line in the complex plane. Now we can
understand better why the convergence fails:
it fails when you hit a pole. Generally the
region of convergence is the half-plane to the
right of the right most pole.

On the other hand, we have been
discussing the pole diagram of functions such as
$F(s)$. Apparently making sense of it outside its
region of convergence. What's going on?

The fact is that knowledge of functions such as $F(s)$ in
some right half plane suffices to determine a
function defined everywhere in the complex plane
(except for a scattering of points, the poles).

(Complex variables - Analytic continuation)

Most of the $F(s)$ — (In this course) are rotational
functions. Their expression as a quotient of one
polynomial by another, allows computation of the
values of the function for almost every complex
number. It provides us with the analytic
continuation of the function given by the integral
expression for the Laplace transform.

Block diagrams

Generate & Interpret block diagrams:

Engineers build complex systems by combining simpler ones. This process is seconded by the symbolism of block diagrams.

A system acts on the input signal to produce the system response. To be precise, one should specify initial condt as well but

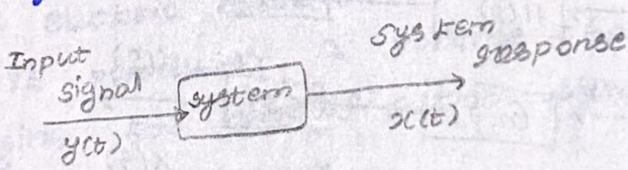
1) These are often implied by an assumption of steady state or zero initial condt.

2) They don't matter much any way if the system is stable.

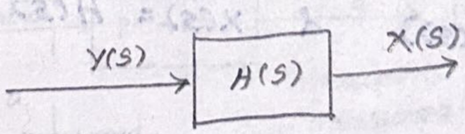
So they are usually ignored in the block diagram

notation:

In the time domain we can represent the input signal y, system S, and the system response x, by the following diagram



we can convert this to the frequency domain by applying Laplace transform to the variables x and y.



In the s-domain,

$$X(S) = H(S)\,Y(S)$$

In the frequency domain, the effect of the system is simply to multiply by the transfer function.

This makes it easy to explain what happens if you 'cascade' two system: feed a second system with transfer function G(S), say as initial signal, the output signal of a first system (with T.F H(S)).

$$Y(S) \rightarrow \boxed{H(S)} \rightarrow \boxed{G(S)} \rightarrow X(S)$$

The cascade is another LTI system, with its own transfer function, and we have just seen that the T.F of this new function is just the product of the old ones. $(G(S)\,H(S))$

∴ The input to the system having $G(S)$ as T.F is $H(S) \cdot Y(S)$

$$\therefore \quad H(S)\,Y(S) \rightarrow \boxed{G(S)} \rightarrow \text{output}$$

$$\boxed{\text{output}: \; H(S)\,G(S)\,Y(S)}$$

From the observation,

The order in which you cascade is immaterial. Feeding the output from system 1 in to Sys 2 has the same effect as 2 in to system 1. Both are equivalent to the below block diagram

$$Y(S) \rightarrow \boxed{H(S)} \rightarrow \boxed{G(S)} \rightarrow X(S)$$

$$Y(S) \rightarrow \boxed{G(S)} \rightarrow \boxed{H(S)} \rightarrow X(S)$$

$$Y(S) \rightarrow \boxed{G(S)\,H(S)} \rightarrow X(S)$$

$$X(S) = G(S)\,(H(S)\,Y(S)) \quad \& \quad X(S) = H(S)\,(G(S) \cdot Y(S))$$

$$X(S) = (H(S)\,G(S)\,Y(S))$$

Equivalently.

Intro to block diagram

Mathematically representation of complex problems.

why no Initial condt:

∴ we are mostly dealing with (mainly) stable systems where the transients die off. (ZIR signals die off in time)

And you get a single steady state response in the long run. (So no matter - What's the initial condt)

Amplification system for musical instrument

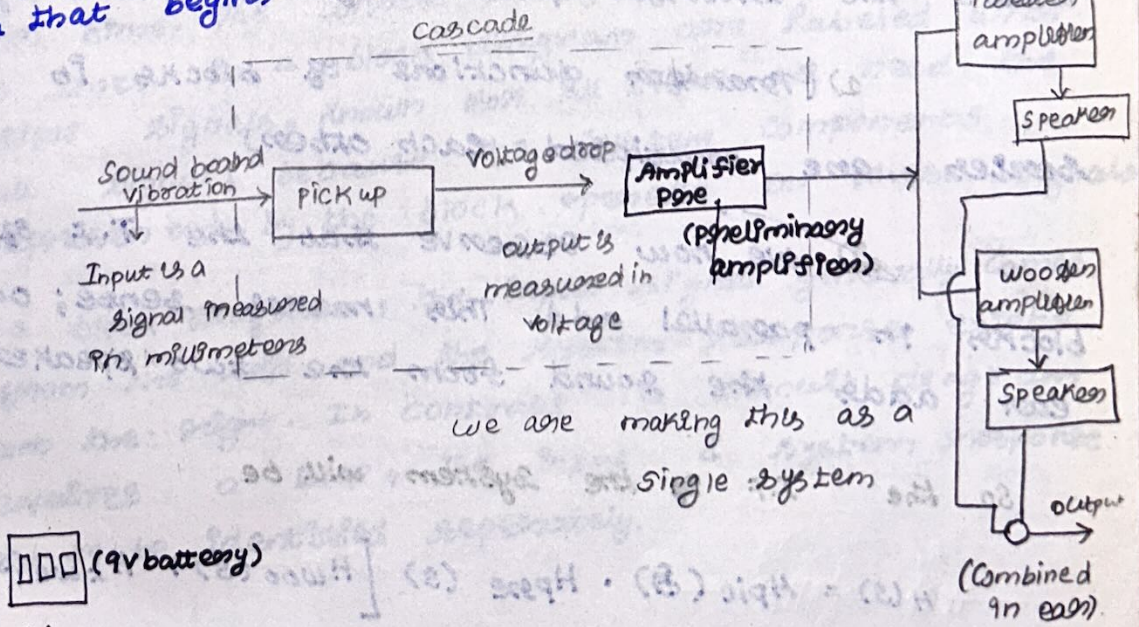
Acoustic feedback:

when the output from the speaker comes back and affects the sounding board (Instrument) (Harp - musical instrument like Yal), which is what happens and in turn affects the sounding board, and in turn goes through the amplification system. so we are getting constructive feedback loop.

Block diagram:

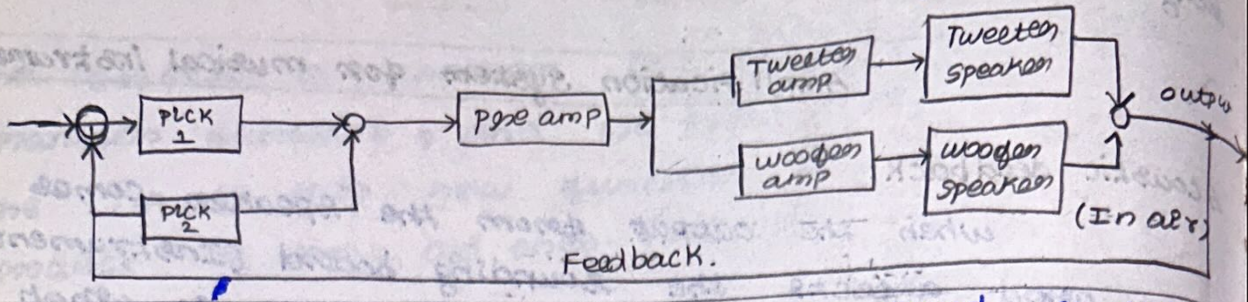
First: Beginning is strings, and the vibration of the sound board & the harp that the string produce. So the sounding board vibrates by some amount - may be measured in millimeters & that gets fed to a pickup, which is a little piece of piezo electric material that's glued to the back of the sound board.

(Piezo electric material: property that when it's compressed, it produces a voltage change across it.) and that begins the amplification system.



sliders (Allowing to amplify high frequency more than low frequency) → Amplifies the amplitude of the signal by a factor about four (usually) (It has its own frequency response)

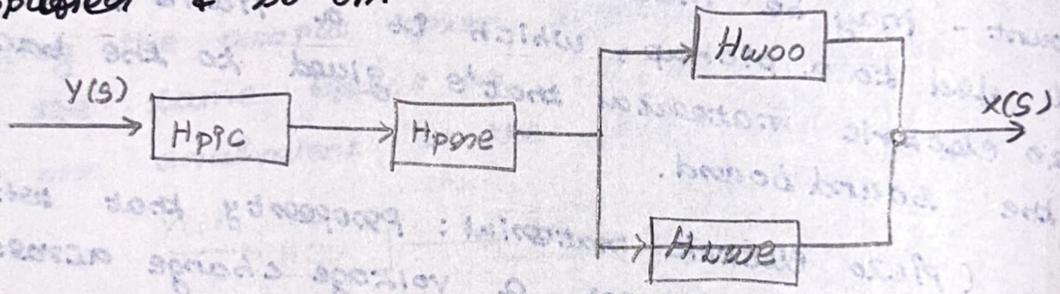
Some times we have 2 pickups 1. Low freq
2. High freq



If the speaker is placed behind the hand. so the output signal is going to contribute to the input. (A feedback loop).

We can use feedbacks for making things wiser than making worse.

Each block has a corresponding transfer function. say Hpic (s) for pick up & Hpre for the amplifier & so on.



what's the transfer function of the entire system?

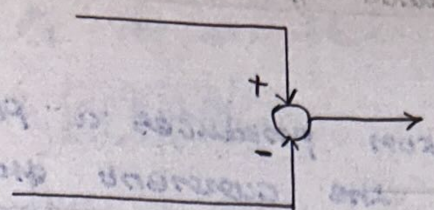
1) (Transfer functions of blocks in series are multiplied each other)

2) we now observe that the T.F of blocks in parallel add. This makes sense; our ear adds the sound from the two speakers.

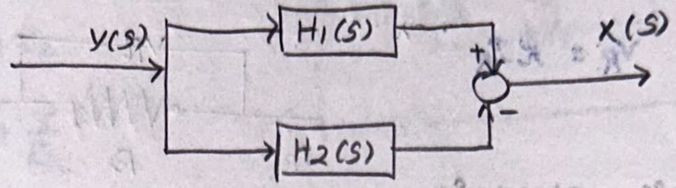
So the T.F of the system will be

$$H(S) = H_{pic}(S) \cdot H_{pre}(S) \left[H_{woo}(S) + H_{twe}(S) \right].$$

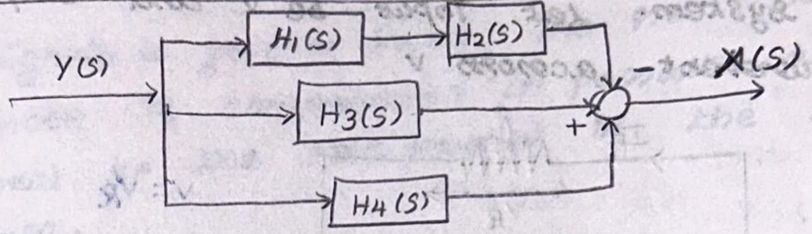
Sometimes we need to join two signals but reverse the sign of one. So form a difference rather than sum. The block diagram representation will be



3.1:



The transfer function will be $H_1(S) - H_2(S)$



Solu:

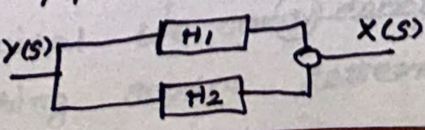
$-H_1(S) \cdot H_2(S) + H_3(S) + H_4(S)$ is the T.F.

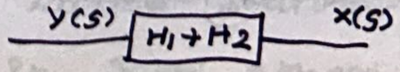
Block diagrams are not circuit diagrams

circuit diagrams and block diagrams have a superfi-cial resemblence. (But they are different.) The state of a circuit diagram (current through various components & voltage drop across them) varies with time. The wires carry electrons. In contrast, the lines in a block diagram are labeled with entire signals, known for all time. We need the full signal, because the system components represented by the block operate on these signals.

In a block diagram, the input signal generally comes in from the left and the system response exists from the right. In contrast a circuit diagram requires a loop, the input & system response need to be identified seperately.

A 'series' relationship in a block diagram is a cascade. the T.F multiply.



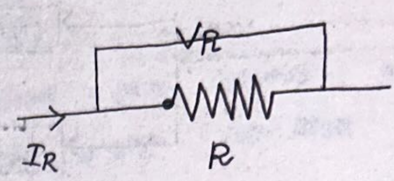


Impedance of resistors

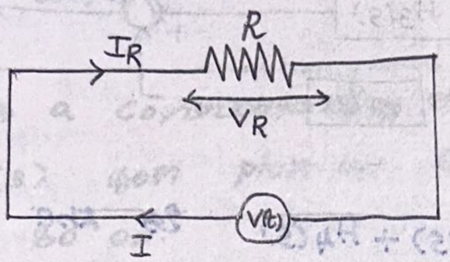
From ohm's law:

A linear resistor produces a particularly simple relationship b/w the current flowing through it & the voltage drop across it.

$$V_R = R I_R$$



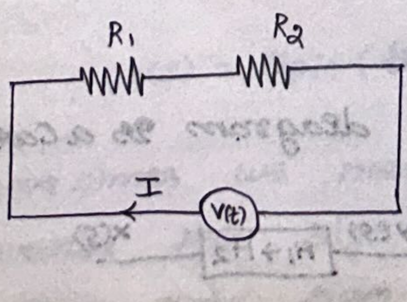
If we embed this electric component in a ckt driven by a voltage v we have a simple system. Let input be v and output to be I. The current across v



$V = V_R$

$I = \frac{V}{R}$ (The transfer Function is the constant $\frac{1}{R}$, the reciprocal of the resistance).

The effects of the other electronic components - we consider - the capacitor & the inductor coil - are more complicated, but when we consider them in the frequency domain they are similar enough to treat in a way exactly parallel with the story of resistance. The generalization of resistance is Impedance. Any configuration of resistors, Inductors & capacitors placed in a circuit with a (possibly time varying) driving voltage has an impedance. The impedance is not necessarily a constant. Rather it's a function in the frequency domain, which we will see is the reciprocal of the transfer function.



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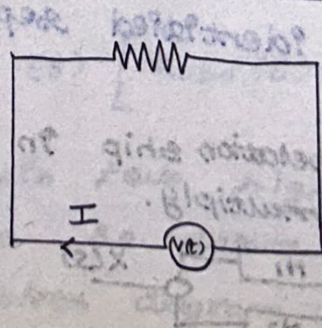
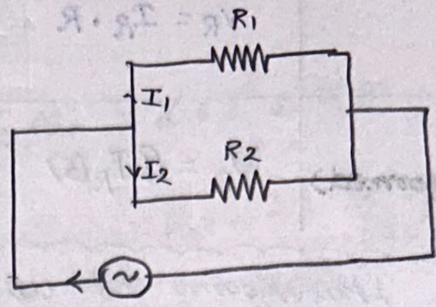


voltage drop across across R_1 is IR_1 and across R_2 it is IR_2.

$$\therefore \boxed{R = R_1 + R_2}$$



If it's parallel:

$$I_R = I_1 + I_2$$

$$I_R = \frac{V}{R_1} + \frac{V}{R_2}$$

$$I_R = \frac{R_1 + R_2}{R_1 R_2}$$

This signals a general fact: The reciprocals of impedances of components in parallel add, and is equal to the reciprocal of the total impedance.

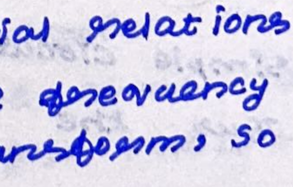
Impedance of Capacitors & Inductors

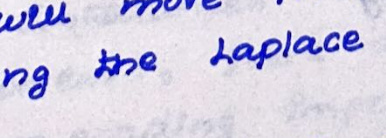
The rules relating the voltage drop to the current are

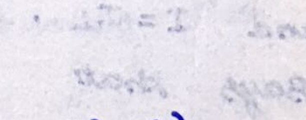
capacitor: $C\dot{V}_c = I_c$

Inductor: $V_L = L\dot{I}_L$

Here V_L, I_L, V_c, I_c are analogous to V_R and I_R across a resistor:







To convert these differential relations into ratios, we will move into the frequency domain by applying the Laplace transform, so

$$\mathcal{I}(s) = \mathcal{L}(I(t); s), \quad V_R(s) = \mathcal{L}(V_R(t); s)$$

$$V_c(s) = \mathcal{L}(V_c(t); s), \quad V_L(s) = \mathcal{L}(V_L(t); s)$$

∴ The letters in t-domain are already in capital we can denote the Laplace transform by using script letters or in some other way.

(use j for $\sqrt{-1}$).

Solu:

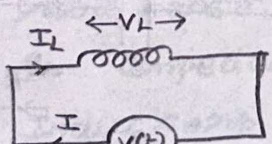
t-domain	$v_R = I_R \cdot R$	$C\dot{V_c} = I_c$	$V_L = L\dot{I_L}$
s-domain (Laplace transformed)	$V_R = R I_R(s)$	$V_c = \frac{1}{sC} I_c$	$V_L = sL I_L$

Laplace transform is written in pencil

As you can see, the role of resistance R is played by $\frac{1}{Cs}$ in the case of capacitor and Ls in the case of an inductor. The Laplace transform of voltage drop is a multiple of the transformed current, but the multiplier is no longer constant but rather depends upon s. This multiplier is the reciprocal of the system function, as we will see in the below one. It's called the impedance or complex impedance. of the component.

As usual, $s = i\omega$ is an important physical case and we will sometimes see the impedence defined interms of $i\omega$ instead of s.

Transfer functions of single component system:



For such a simple circuit, it's easy to see $V = V_L$ and $I = I_L$. Thus the voltage law of inductor says that

$$L\dot{I} = V$$

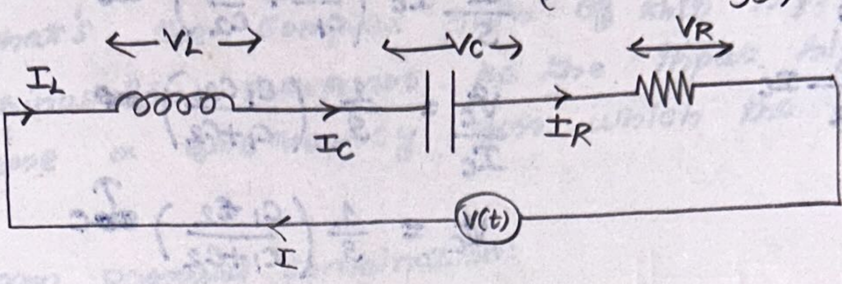
If we recall V the input, I the output of this system then the systems transfer function is $\frac{1}{sL}$. which is the reciprocal of the impedance of the inductor.

Impedance of components in series

In a series RLC, $I = I_R = I_C = I_L$ (Voltage boost by the power source equals the sum of the voltage drops across the components. In the frequency domain,

$$v = v_L + v_R + v_C(s) = LSI + RI + \frac{1}{SC} I$$

$$= \left(LS + R + \frac{1}{SC}\right) I$$



If we declare the input to be the voltage increase across the power source and the system response to be the current through the circuit, the transfer function is

$$\frac{I}{v} = \frac{1}{LS + R + \left(\frac{1}{CS}\right)} = \frac{S}{LS^2 + RS + \frac{1}{C}}$$

as a result, we have seen before. But now the reciprocal of the transfer function, the impedance,

$$LS + R + \frac{1}{CS}.$$ (is revealed to be an analogue of the resistance.)

The impedance of this series is the sum of the impedances of the three consecutive components, just as for resistances. The corresponding impedances are

Inductor: LS

Resistor: R

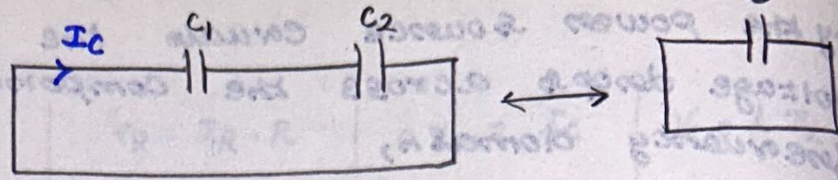
capacitor: $\frac{1}{CS}$

---

1) putting capacitors with constants c_1 and c_2 in series produces the same effect as on capacitor

with capacitance c

Solu:



$V_c = \frac{1}{sC} I_c$

$V_{C1} = \frac{1}{sC_1} I_c$

$V_{C2} = \frac{1}{sC_2} I_c$

$\therefore V_1 + V_2 = 0$

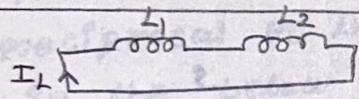
$\frac{1}{s} I_c \left(\frac{1}{C_1} + \frac{1}{C_2}\right) = 0$

$\frac{V_c}{I_c} = \frac{1}{s}\left(\frac{C_1 C_2}{C_1 + C_2}\right) = 0$

$V_c = \frac{1}{s}\left(\frac{C_1 C_2}{C_1 + C_2}\right) I_c$

$\therefore \frac{1}{sC} I_c = \frac{1}{s}\left(\frac{C_1 C_2}{C_1 + C_2}\right) I_c$

$\frac{1}{C} = \frac{C_1 C_2}{C_1 + C_2}$

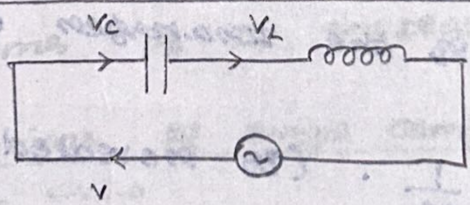


Solu:

$V_L = sLI_L$

$sLI_L = sL_1 I_L + sL_2 I_L$

$L = L_1 + L_2$



Solu:

$V_c = \frac{1}{sC} I_c \quad , \quad V_L = sL I_L \qquad (I_C = I_L)$

$\therefore V = \frac{1}{sC} I_c + sL I_L$

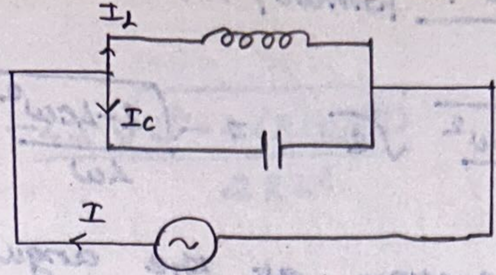
$\frac{V_T}{I_T} = \frac{1}{sC} + sL$

$Z = \frac{1}{sC} + sL$

Transfer function: $\frac{\text{output}}{\text{Input}} = \frac{I_T}{V_T} = \frac{1}{\frac{1}{sC} + sL}$

$= \frac{s}{\frac{1}{C} + s^2 L}$

Impedance of components in parallel



solu: what's the complex gain of this system. what's the sinusoidal response to the input signal cos(ωt)? Is there a frequency for which the gain is 0?

solu:
from parallel combination:

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$$ (From Impedance derivation)

$$\frac{1}{Z} = \frac{1}{LS} + CS$$

∴ The system function H(S) is the reciprocal of the Z (Impedance) $\Rightarrow \frac{1}{\frac{V_T}{I_T}} = \frac{I_T}{V_T} = \frac{\text{output}}{\text{Input}}$

$$H(S) = H(iS) = \frac{1}{LS} + CS = \frac{1+S^2LC}{LS}$$

$\boxed{S = i\omega}$

complex gain:

$$G(\omega) = H(i\omega) = \frac{1+(i\omega)^2 LC}{L(i\omega)}$$

$$= \frac{1-\omega^2 LC}{L\omega i} = \frac{i - i\omega^2 LC}{-L\omega}$$

$$= \frac{i\omega^2 LC - i}{L\omega}$$

∴ The input is sinωt

$$x_p(t) = Re\left(G(\omega)\, e^{i\omega t}\right) = Re\left(\frac{i\omega^2 LC - i}{LC}\cdot(\cos\omega t + i\sin\omega t)\right)$$

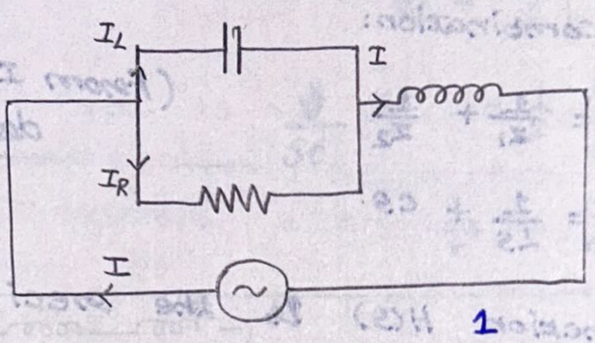
$$= Re\left(\frac{i\omega^2 LC(\cos\omega t) - i\cos\omega t}{LC} + \frac{\sin\omega t - \omega^2 LC\sin\omega t}{LC}\right)$$

$$= \left(\frac{\sin\omega t - \omega^2 LC\sin\omega t}{LC}\right) = \frac{1 - LC\omega^2}{LC}\sin(\omega t)$$

Gain $g(\omega) = \frac{|1-LC\omega^2|}{|L\omega|} |\sin\omega t|$

$$= \frac{\sqrt{1-LC\omega^2}}{L\omega} \sqrt{1} = \frac{\sqrt{1-LC\omega^2}}{L\omega} = \frac{|1-LC\omega^2|}{L\omega}$$

we are driving the system at the angular frequency $\omega_0 = \frac{1}{\sqrt{LC}}$, the gain is zero when there is no current in it. The transfer function has zeros at $s = \pm \omega_0 i$.



$$Z = \frac{1}{\frac{1}{Z_{parallel}}} + Z_{series}$$

$$\frac{1}{Z_{parallel}} = \frac{1}{Z_C} + \frac{1}{Z_R}$$

$$(Z = Z_p + Z_s)$$

Total impedance $Z = \frac{1}{\left(\frac{1}{Z_C} + \frac{1}{Z_R}\right)} + Z_L$

$$Z = \left(Cs + \frac{1}{R}\right)^{-1} + sL$$

Transfer function $H(s) = \frac{1}{Z} = \frac{1}{\left(Cs + \frac{1}{R}\right)^{-1} + sL}$

$$= \frac{1}{\frac{1}{Cs + \frac{1}{R}} + sL}$$

$$= \frac{1}{\frac{R}{RCs+1} + sL}$$

$$= \frac{1\,(RCs+1)}{R + sL + RLCs^2}$$

what are the poles

$$= \frac{1+RCs}{R+Ls+RLCs^2}$$

$-Ls \pm \sqrt{L^2s^2 - 4()}$

$RLC\,s^2 + Ls + R = P(D)$

poles:

$$s = \frac{-L \pm \sqrt{L^2 - 4(RLC)(R)}}{2RLC} = \frac{-L \pm \sqrt{L^2 - 4R^2CL}}{2RLC}$$

So the poles have

non imaginary poles when

$$L^2 - 4R^2CL > 0$$

The solution has non-zero imaginary part when

$L^2 - 4R^2CL < 0$ (That's when

$$L < 4R^2C$$

$\therefore$ oscillatory transients exists when the zero has a non-zero imaginary part.

Summary:

The complex impedance of a system is the reciprocal of its system function.

Impedances add in series; reciprocals of impedances add in parallel.

Note: Block diagrams & ckt diagrams have resemblence but they are different.

Parallel blocks: Adds the T.F s

Series blocks: multiplies the T.F S.

$$\sin\omega t = \frac{\omega}{s^2+\omega^2}, \quad \cos\omega t = \frac{s}{s^2+\omega^2}$$

$$\therefore \sin t = \frac{1}{s^2+1}, \quad \cos t = \frac{s}{s^2+1}$$

$$\sin at = \frac{a}{s^2+a^2}, \quad \cos at = \frac{s}{s^2+a^2}$$

$$= \frac{a}{a^2\left(\left(\frac{s^2}{a^2}\right)+1\right)}, \quad \cos at = \frac{1}{a^2}\cdot\frac{s}{\left(\left(\frac{s^2}{a^2}\right)+1\right)}$$

$$\mathcal{L}(\cos at; s) = \frac{1}{a}\frac{\left(\frac{s}{a}\right)}{\left(\frac{s^2}{a^2}+1\right)} = \frac{\frac{1}{a^2}\cdot s}{\frac{1}{a^2}(s^2+a^2)} = \frac{s}{s^2+a^2}$$

$$\mathcal{L}(f(at); s) = \frac{1}{a} F(s/a)$$ $\therefore$ proved

$$\therefore \cos at \rightsquigarrow \left(\frac{1}{a^2 \times a}\right) * \frac{\left(\frac{s}{a}\right)}{\left(\frac{s^2}{a^2}+1\right)} = \mathcal{L}(\cos(\cancel{at}); s)$$

$p(D) = D+2$ $\quad (p(D)x = e^t)$

soln:

$$X(s) = \frac{\frac{1}{s-1}}{s+2} = \frac{1}{(s-1)(s+2)}$$

Inverse Laplace transform:

$$\frac{1}{(s^2-s+2s-2)} = \frac{1}{s^2+s-2}$$

-2, -2, 1, 1

$$= \frac{1}{(s-1)(s+2)}$$

$$= \frac{a}{s-1} + \frac{b}{s+2}$$

$as+2a+bs-b = 1$

$a+b=0$

$2a-b=1$

$3a=1$

$a = 1/3$

$b = -1/3$

$$= \frac{1/3}{s-1} + \frac{-1/3}{s+2}$$

$\therefore$

Inverse Laplace transform:

$$x(t) = \left(\frac{1}{3}\right)e^{t} - \left(\frac{1}{3}\right)e^{-2t}$$

The smallest number K in the option such that exponential type K is $(x(t))$

$\therefore$ Right most value $= 1$ Ans: 1.

$\ddot{y} - 2\dot{y} = 12, \quad y(0) = 0, \quad \dot{y}(0) = 1$

solu:

$1 \rightsquigarrow \frac{1}{s}$.

$$(s^2 y - 0) - 2sy = \frac{12}{s}$$

$$s(s-2)y = \frac{12}{s} + 1$$

$$y = \frac{12+s}{s^2(s-2)}$$

using coversup

$$\frac{12+s}{s^2(s-2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-2}$$

$B = -6, \quad C = 7/2, \quad A = -7/2$

$$y = -\frac{7}{2s} - \frac{6}{s^2} + \frac{7}{2(s-2)}$$

plugging, we got

$$y(t) = -\frac{7}{2} - 6t + \frac{7}{2} e^{2t}$$

The smallest number k such that y(t) is exponential is 2 since $y(t) \approx (7/2)\, e^{2t}$ for larges values of t.

3) $\ddot{x} + 2\dot{x} + 2x = \cos 2t$

solu:

$$(s^2 x - s - 1) + 2(sx - 1) + 2x = \frac{s}{s^2+4}$$

$$(s^2 + 2s + 2)x - (s+3) = \frac{s}{s^2+4}$$

$$x = \frac{s}{(s^2+4)(s^2+2s+2)} + \frac{s+3}{s^2+2s+2}.$$

By coversup

$$\frac{s}{(s^2+4)(s^2+2s+2)} = \frac{As+B}{s^2+4} + \frac{C(s+1)+D}{(s+1)^2+1}$$

$A = -1/10, \quad B = 4/10, \quad C = 1/10, \quad D = -3/10$

$$X = \frac{-\frac{1}{10}s + \frac{4}{10}}{s^2+4} + \frac{\frac{1}{10}(s+1) - \frac{3}{10}}{(s+1)^2+1} + \frac{(s+1)+2}{(s+1)^2+1}$$

Using Laplace table: (Inverse Laplace transform)

$$x(t) = A\cos 2t + \left(\frac{B}{2}\right)\sin 2t + Ce^{-t}\cos t + De^{-t}\sin(t) + e^{-t}\cos t + 2e^{-t}\sin t$$

$$= -0.1\cos 2t + 0.2\sin 2t + 1.1e^{-t}\cos t + 1.7e^{-t}\sin t$$

pure sinusoid occurs when

$$x(t) = -0.1\cos(2t) + 0.2\sin(2t)$$

which has Initial conditions

$$x(0) = -0.1\cos 0 + 0.2\sin 0 = -0.1$$

$$\dot{x}(0) = 0.2\sin(0) + 0.4\cos 0 = 0.4.$$

pure sinusoid: No damping

→ $C=0, D=0$ compute in the coverup method equations.

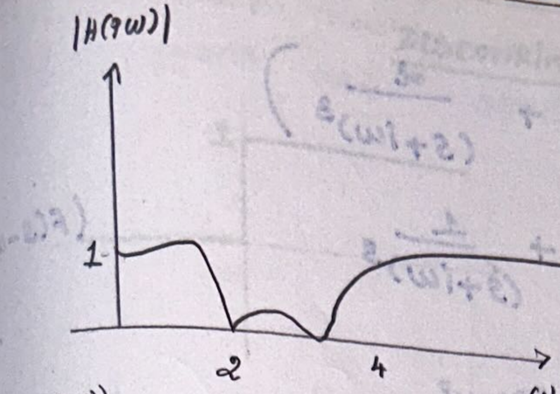
Bode plot

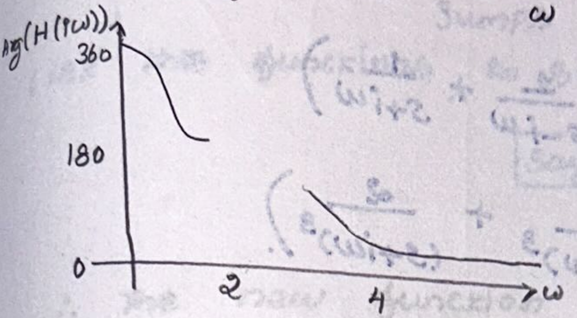
All radio spam has been restricted to frequencies in the Range $[2,3]$ (in units of megahertz say). Make a sketch of an ideal filter that has the effect of suppressing those frequencies but allowing all other frequencies to pass with gain 1.

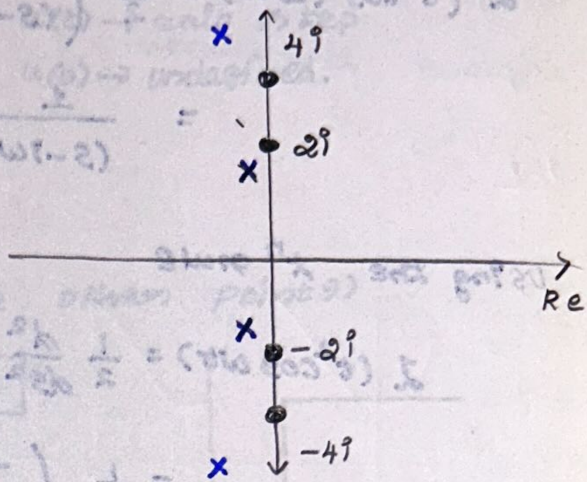
Relate pole diagram to the gain curve. (That will suppress the spam signals).

In other words, create a pole diagram that has 4 poles using the create button.

Using mathlet.







place zeros at $\pm 2i$ & $\pm 3i$ (But slightly on the outside of this $[2,3]$ interval to bring the gain up to 1 outside of this interval.

$$\mathcal{L}(\cos(\omega t)\cdot f(t)\,;s) = ?$$

Solu.

$$\cos\omega t = \frac{1}{2}\left(e^{i\omega t} + e^{-i\omega t}\right)$$

$$= \frac{1}{2}\left(\mathcal{L}(e^{i\omega t} f(t)\,;s) + \mathcal{L}(e^{-i\omega t} f(t)\,;s)\right)$$

$$= \frac{1}{2}\left(F(s-i\omega) + F(s+i\omega)\right)$$

$$\mathcal{L}(\sin\omega t \; f(t)\,;s)$$

Solu:

$$\sin\omega t = \frac{1}{2i}\left(e^{i\omega t} - e^{-i\omega t}\right)$$

$$\mathcal{L}(\sin\omega t\, f(t)\,;s) = \frac{1}{2i}\left(\mathcal{L}(e^{i\omega t} f(t)) - \mathcal{L}(e^{-i\omega t} f(t))\right)$$

$$= \frac{1}{2i}\left(F(s-i\omega) - F(s+i\omega)\right)$$

$\mathcal{L}(t^2) = \frac{2}{s^2}$ (we have)

$$\mathcal{L}(t^2\cos(\omega t)) = \frac{1}{2}\left(\frac{2}{(s-i\omega)^3} + \frac{2}{(s+i\omega)^3}\right)$$

$$= \frac{1}{(s-i\omega)^3} + \frac{1}{(s+i\omega)^3}$$

$(F(s-b))$

Using the t^n rule

$$\mathcal{L}(t^2\cos\omega t) = \frac{1}{2}\frac{d^2}{ds^2}\left(\frac{1}{s-i\omega} + \frac{1}{s+i\omega}\right)$$

$$= \frac{1}{2}\left(\frac{2}{(s-i\omega)^3} + \frac{2}{(s+i\omega)^3}\right).$$

---

1) $\mathcal{L}(t^2+1\,; s) \rightsquigarrow \frac{2}{s^3} + \frac{1}{s}$ (Linearity)

2) $\mathcal{L}((t^2+1)\sin(2t)\,; s) \rightsquigarrow \mathcal{L}(t^2\sin 2t) + \mathcal{L}(\sin 2t)$

$$\rightsquigarrow \frac{1}{2i}\left(\frac{2}{(s-2i)^3} - \frac{2}{(s+2i)^3}\right) + \frac{2}{s^2+4}$$

3) $(t^2+1)e^{-t}\sin(2t)$

$$= \frac{1}{2i}\left(\frac{2}{(s+1-2i)^3} - \frac{2}{(s+1+2i)^3}\right) + \frac{2}{(s+1)^2+4}$$

Step & Impulse response and feedback.

Piecewise - continuous functions:

1) Use the unit step function to give formulas for discontinuous functions.

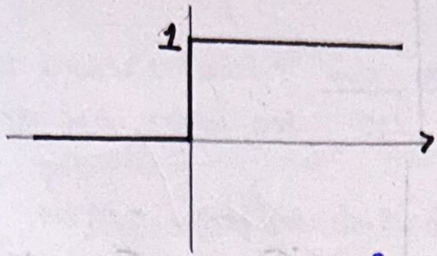
2) Recognize left and right values & piece-wise continuous functions

3) Employ t-shift rule to Laplace transforms & their inverses.

4) Describe & compute the step response

of an LTI system using laplace transform.

Discontinuous functions

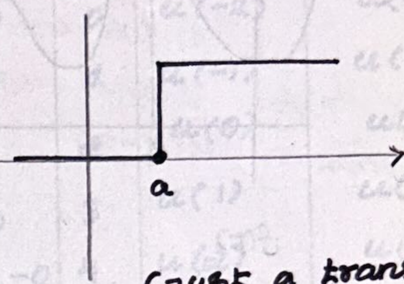


$u(t) \rightarrow$ unit step

$u(0) \rightarrow$ undefined.

(Let the functions jumps to some other points)

$\boxed{\text{Say: } a}$

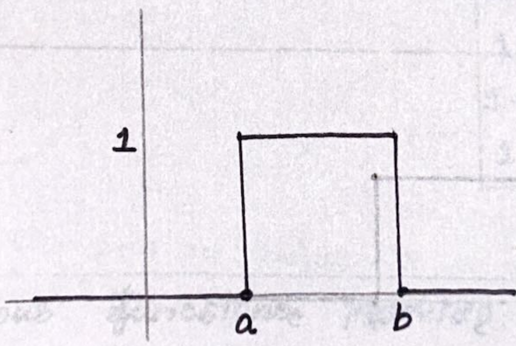


(just a translati-on)

$\therefore$ The new function is

$u(t-a) = u_a(t)$

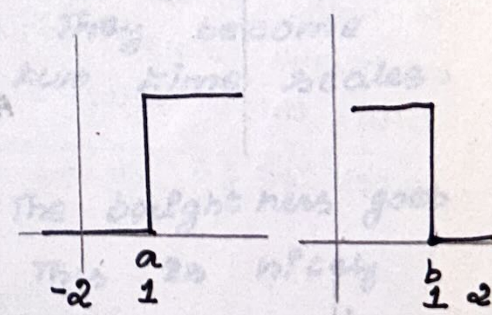
Unit-box function:



$u_{ab}(t) = u_a(t) - u_b(t)$

(Translated one)

$= u(t-a) - u(t-b)$



'Step up at a'

then

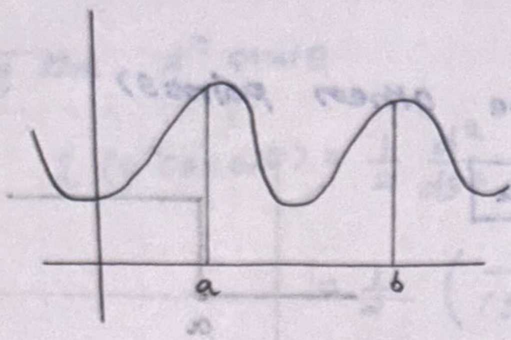
'step down at b'

$u_{ab}(t) = u(t-a) - u(t-b).$

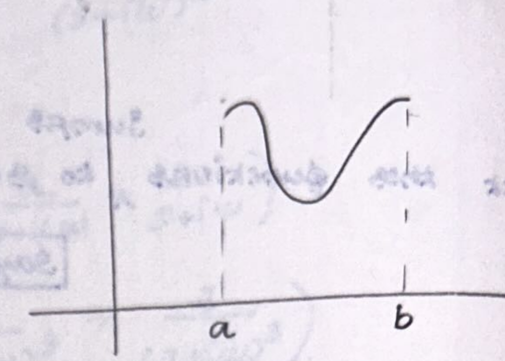
why these functions?

Since these functions when multiplied with other functions operates with them to give Laplace transform.

Let



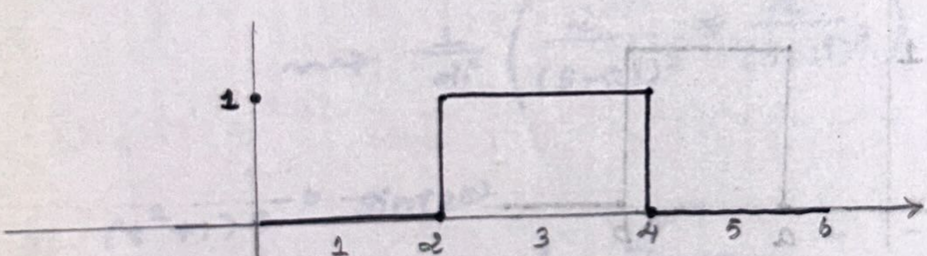
$f(t)$



$u_{ab}(t)\, f(t)$

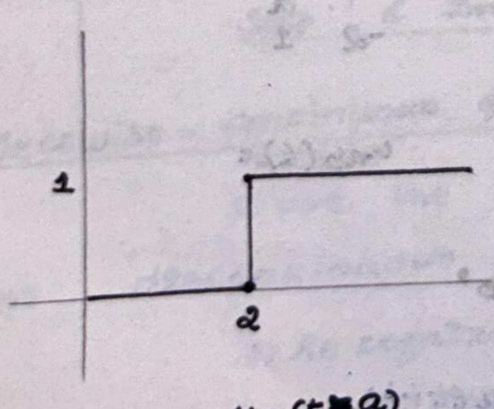
(sliced a particular part)

wiped out all of its graph except the part b/w a and b.

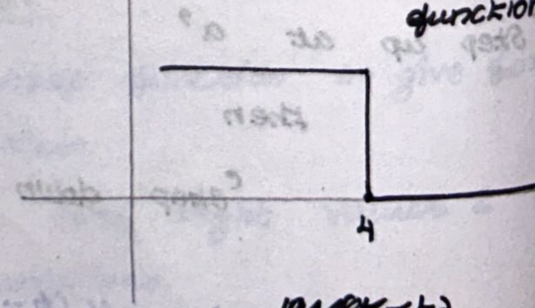


Answer function

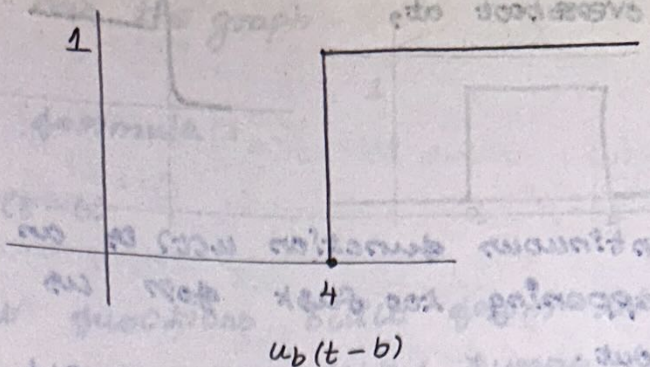
Some other functions



$u_a(t-a)$

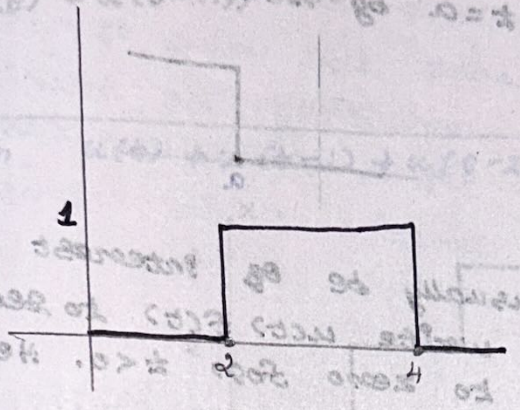


$u_b(t-b)$



Now

$u_a(t-2) - u_b(t-4)$

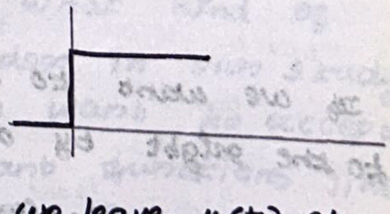


Ans	t	$u_a(t-2)$	$u_b(t-4)$
0	0	$u(-2)$	$u(-4)$
0	1	$u(-1)$	$u(-3)$
0	2	$u(0)$	$u(-2)$
0	3	$u(1)$	$u(-1)$
1-0	4	$u(2)$	$u(0)$
1-0	5	$u(3)$	$u(1)$
1-0	6	$u(4)$	$u(2)$
1-0	7	$u(5)$	$u(3)$
1-1	8	$u(6)$	$u(4)$
1-1	9	$u(7)$	$u(5)$
1-1	10	$u(8)$	$u(6)$

Discontinuous functions provide important tools in representing tools in nature. They become useful where ever there are two time scales at play.

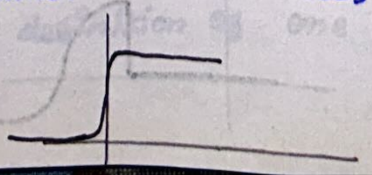
eg:- I turn on Electric light. The brightness goes from 0 to a large +ve value. This is nicely modeled by the Heaviside or unit step function

Solu:-
$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0. \end{cases}$$

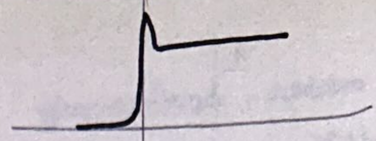


we leave $u(t)$ at $t=0$ (undefined).

If we were look at this process at finer scale (time) — milliseconds



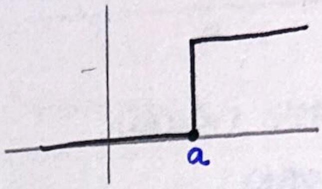
or even overshoot at,



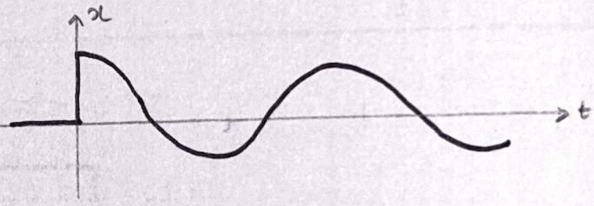
The discontinuous function u(t) is an idealization of an event happening too fast for us to see, or to care about.

The unit step can be used to build many other useful discontinuous functions.

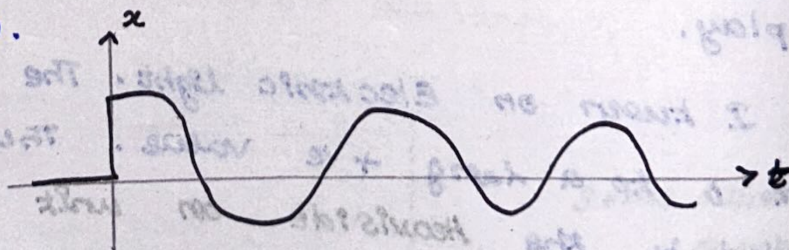
1) we can represent the light turning on at some later time $t=a$ by shifting the unit step function $u(t-a)$



2) our functions will usually be of interest only $t>0$. so its often useful to write $u(t)\,f(t)$ to set the values of the function to zero for $t<0$. Here's the graph of $u(t)\cos t$.



3) we want to shift a function to the left or right. To make sure we are still only considering $t>0$. we might shift right by a units and then clip like this: $u(t)\cdot f(t-a)$. Here's the graph of $u(t)\cos(t-\pi/6)$.



If we want to leave zeros behind after shifting to the right by a units, we can use $u(t-a)\,f(t-a)$

$u(t-\pi/2)\cos(t-\pi/2)$