Prove that Language $L = \{0^n$: n is a perfect square$\}$ is irregular.

Solution:

L is infinite. Suppose L is also regular. Then according to pumping lemma there exists an integer **n** such that for every string w in where $|w| >= n$, we can break w into three strings $w = xyz$ such that:

$|y| > 0$, $|xy| <= n$ and for all $k>=0$, the string xy^kz is also in L.

Choose w to be $w = 0^s$ where $s = n^2$ that is it is a perfect square.

Let w= 00000000000000000………00000 = xyz . (The length of $w = s = n^2$ in this case.)
Let $|xy| <= n$ and $|y| = k$.

That is $w = 0000\ \underline{0^k}\ 000\ldots$
X y z

So, $|xyz| = |xz| + |y| = (n^2- k) + (k)$
From pumping lemma, I can pump y any number of times and the new string should also belong to the language. Suppose I pump y twice then, the new string should belong to the language that is it should have length that is a perfect square but,

$|xy^2z| = |xz| + 2|y| = (n^2- k) + 2k = n^2 + k$

where $n^2 + k < n^2 + n$ ($k <= n$ as $|xy| <=n$ and $|y| = k$)
$< n^2 + 2n +1$
$= (n+1)(n+1)$

and $n^2 + k > n^2$ (As $k > 0$)

=> $n^2 < n^2 + k < (n+1)^2$

- ⇨ $n^2 + k$ is not a perfect square
- ⇨ xy^2z is not in L
- ⇨ This is a contradiction to the pumping lemma
- ⇨ So, our initial assumption must have been wrong that is L is not regular.

Prove that Language L = {0^n: n is a perfect cube} is irregular.

Solution:

L is infinite. Suppose L is also regular. Then according to pumping lemma there exists an integer **n** such that for every string w in where |w| >= n, we can break w into three strings w = xyz such that:

|y| > 0, |xy| <= n and for all k>=0 , the string xy^kz is also in L.

Choose w to be w = 0^s where s = n^3 that is it is a perfect square.

Let w= 00000000000000000………00000 = xyz . (The length of w = s = n^3 in this case.)
Let |xy| <= n and |y| = k.

That is w = 0000 $\underline{0^k}$ 000…
X y z

So, $|xyz| = |xz| + |y| = (n^3 - k) + (k)$
From pumping lemma, I can pump y any number of times and the new string should also belong to the language. Suppose I pump y twice then, the new string should belong to the language that is it should have length that is a perfect cube but ,

$|xy^2z| = |xz| + 2|y| = (n^3 - k) + 2k = n^3 + k$

where $n^3 + k < n^3 + n$ (k <= n as |xy| <=n and |y| = k)
$< n^3 + 3n^2 + 3n + 1$
$= (n+1)^3$

and $n^3 + k > n^3$ (As k > 0)

=> $n^3 < n^3 + k < (n+1)^3$

- ⇨ $n^3 + k$ is not a perfect cube
- ⇨ xy^2z is not in L
- ⇨ This is a contradiction to the pumping lemma
- ⇨ So, our initial assumption must have been wrong that is L is not regular.

Steps to solve Pumping Lemma problems:

1. If the language is finite, it is regular (quiz3-section1), otherwise it might be non-regular.
2. Consider the given language to be regular
3. State pumping lemma
4. Choose a string w from language, choose smartly :`)
5. Partition it according to constraints of pumping lemma in a generic way
6. Choose a pumping factor k such that the new 'pumped' string is not part of the language given.
7. State the contradiction and end the proof.

How to remember what pumping Lemma says:

(As ahsan mentioned)

Pumping Lemma alternates between "for all" and "there is at least one" or "for every" or "there exists". Notice:

For every regular language L
There exists a constant n
For every string w in L such that |w| >= n,
There exists a way to break up w into three strings w = xyz such that |y| > 0 , |xy| <= n and
For every k>=0 , the string xy^kz is also in L.

What does this imply?

- **I can choose the language I want to** prove (this is trivial…obviously in exams you have no choice but you can pick up a language of choice during practice)
- **I cannot choose n that is I have to keep it arbitrary; I have to use a generic value**. (So I cannot choose 4 or 16 as the length of my string It has to be defined as a function of n)
- **I can choose w, with any combination of alphabets. The only constraint is that the chosen string should belong to the language under consideration.** (So I can have 000000… or 111111 … or the combination of the two even if the alphabet comprises of any number of letters in the first or second solution as long as length chosen arbitrarily is perfect square…but I cannot have all aaaaa… in a^nb^n language since it does not belong to the language.. I can choose only those strings that conform to the language's general construction)
- **I cannot choose the division of my string. It has to be arbitrary and general and should conform to the pumping lemma constraints.** (SO, I cannot choose the length of y to be 1 or 2 It has to be a generic, choose a variable k or something)
- **I can choose my pumping factor k.** (This means that I can choose to pump any number of times I want to. I can choose to pump 0 times 1 time or even put a constraint on it like I have not pumped more than n times like sir did in class in the factorial proof). **If you want to complete a proof your pumpimg factor should lead to a contradiction to pumping lemma but the point is you can pump any times you want like in prime language proof in the book you could have pumped p-m times or 2p-2m times or 3p-3m times …no constraint there.**

I hope this helps in clarifying confusions!!