len + = 1

len - = 1.

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Example :-

Data :

Class 1 has 5 samples | Class 2 has 6 sample

$$X_1 = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 3 \\ 4 & 4 \\ 5 & 5 \end{bmatrix}_{\substack{5\times 2 \\ n_1 \times d}}, \quad X_2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 1 \\ 3 & 2 \\ 5 & 3 \\ 6 & 5 \end{bmatrix}_{\substack{6\times 2 \\ n_2 \times d}}$$

data

$$Y_1 = X_1^T = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 3 & 4 & 5 \end{bmatrix}_{2\times 5}$$

$$Y_2 = X_2^T = \begin{bmatrix} 1 & 2 & 3 & 3 & 5 & 6 \\ 0 & 1 & 1 & 2 & 3 & 5 \end{bmatrix}_{2\times 6}$$

$$\mu_1 = \frac{1}{5}\begin{bmatrix} 15 \\ 17 \end{bmatrix} = \begin{bmatrix} 3 \\ 3.6 \end{bmatrix}$$

$$\mu_2 = \frac{1}{6}\begin{bmatrix} 20 \\ 12 \end{bmatrix} = \begin{bmatrix} 3.3 \\ 2 \end{bmatrix}$$

Compute Scatter Matrix

$$S_1 = \sum_{x \in w_1} (x - \mu_1)(x - \mu_1)^T$$

$$= \left\{\begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 3 \\ 3.6 \end{bmatrix}\right\} \left\{\begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 3 \\ 3.6 \end{bmatrix}\right\}^T$$

$$+ \left\{\begin{bmatrix} 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 3 \\ 3.6 \end{bmatrix}\right\} \left\{\begin{bmatrix} 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 3 \\ 3.6 \end{bmatrix}\right\}^T + -$$

$lm - = 1.$

$m* = m+1$

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$Y_1 - \mu_1 = \begin{bmatrix} -2 & -1 & 0 & 1 & 2 \\ -1.6 & -0.6 & -0.6 & 0.4 & 1.4 \end{bmatrix}$

$S_1 = \sum_{Y \in W_1} (Y_1 - \mu_1)(Y_1 - \mu_1)^T =$

$= \left\{ \begin{bmatrix} -2 \\ -1.6 \end{bmatrix} \begin{bmatrix} -2 & -1.6 \end{bmatrix} \right\} + \begin{bmatrix} -1 \\ -0.6 \end{bmatrix} \begin{bmatrix} -1 & -0.6 \end{bmatrix}$

$+ \begin{bmatrix} 0 \\ -0.6 \end{bmatrix} \begin{bmatrix} 0, & -0.6 \end{bmatrix} + \begin{bmatrix} 1 \\ 0.4 \end{bmatrix} \begin{bmatrix} 1 & 0.4 \end{bmatrix}$

$= \begin{bmatrix} 4 & 3.2 \\ 3.2 & 0.96 \end{bmatrix} + \begin{bmatrix} 1 & +0.6 \\ +0.6 & 0.36 \end{bmatrix} + \begin{bmatrix} 2 \\ 1.4 \end{bmatrix} \begin{bmatrix} 2 & 1.4 \end{bmatrix}$

$+ \begin{bmatrix} 0 & 0 \\ 0 & 0.36 \end{bmatrix} + \begin{bmatrix} 1 & 0.4 \\ 0.4 & 0.16 \end{bmatrix}$

$+ \begin{bmatrix} 4 & 2.8 \\ 2.8 & 2.56 \end{bmatrix}$

$= \begin{bmatrix} 10 & 8 \\ 8 & 7.2 \end{bmatrix}$

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$S_2 = \begin{bmatrix} 17.3 & 16 \\ 16 & 16 \end{bmatrix}$

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Within class Scatter Matrix

$$S_W = S_1 + S_2 = \begin{bmatrix} 27.3 & 24 \\ 24 & 23.2 \end{bmatrix}_{d \times d}$$

It has full rank, don't have to solve for eigenvalues.

The inverse of S_W is $S_W^{-1} = \begin{bmatrix} 0.39 & -0.41 \\ -0.41 & 0.47 \end{bmatrix}$

Finally optimal line direction

$$W = S_W^{-1}(\mu_1 - \mu_2)$$

$$= \begin{bmatrix} -0.79 \\ 0.89 \end{bmatrix}_{d \times 1}$$

Projected vector v

$$v = W^T Y_1 = [\cdot$$

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lm* = lm + 1

lnn = lm - 1

Example

$$X_1 = \begin{bmatrix} 4 & 2 & 2 & 3 & 4 \\ 1 & 4 & 3 & 6 & 4 \end{bmatrix}$$

$$X_2 = \begin{bmatrix} 9 & 6 & 9 & 8 & 10 \\ 10 & 8 & 5 & 7 & 8 \end{bmatrix}$$

~~Within~~ class scatter Matrix

$$S_1 = \begin{bmatrix} 0.8 & -0.4 \\ -0.4 & 2.64 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 1.84 & -0.04 \\ -0.04 & 2.64 \end{bmatrix}$$

$$\mu_1 = \begin{bmatrix} 3 \\ 3.6 \end{bmatrix}, \quad \mu_2 = \begin{bmatrix} 8.4 \\ 7.6 \end{bmatrix}$$

Within class scatter Matrix or Covariance Matrix

$$S_W = S_1 + S_2 = \begin{bmatrix} 2.64 & -0.44 \\ -0.44 & 5.28 \end{bmatrix}$$

Bet^n class scatter ~~covariance~~ Matrix

$$S_B = (\mu_1 - \mu_2)(\mu_1 - \mu_2)^T$$

$$= \begin{bmatrix} -5.4 \\ -4 \end{bmatrix} \begin{bmatrix} -5.4 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 29.16 & 21.6 \\ 21.6 & 16 \end{bmatrix}$$

$m_* = m + 1$

$q_m = \underline{m - 1}$

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LDA projection is then obtained as the soln of generalized eigenvalue problem

$$S_W^{-1} S_B W = \lambda W$$

characteristic eqn

$$|S_W^{-1} S_B - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 11.89 - \lambda & 8.81 \\ 5.08 & 3.76 - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad \underline{\lambda = 15.65}$$

$$\begin{bmatrix} 11.89 & 8.81 \\ 5.08 & 3.76 \end{bmatrix} \begin{bmatrix} W_{11} \\ W_{21} \end{bmatrix} = 15.65 \begin{bmatrix} W_{11} \\ W_{21} \end{bmatrix}$$

$$\underline{W_1} = \begin{bmatrix} W_{11} \\ W_{12} \end{bmatrix} = \begin{bmatrix} 0.91 \\ 0.37 \end{bmatrix}$$

Or directly by

$$W^* = S_W^{-1}(\mu_1 - \mu_2) = \begin{bmatrix} 0.91 \\ -0.39 \end{bmatrix}$$