

## AI & Pattern Recognition Classwork 06 Autoencoder

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1. (a) Compare the MSE when the coding size is used as 10, 20, and 40 in the stacked autoencoder.

(b) Test whether the reconstruction is improved if an additional hidden layer with 100 neurons was used in the stacked autoencoder.

b).

Code size = 10 → MSE = 0.16476

Code size = 20 → MSE = 0.16745

Code size = 40 → MSE = 0.16100

Yes the reconstruction is improved.

a). Code size = 10 → MSE = 0.16776

Code size = 20 → MSE = 0.16902

Code size = 40 → MSE = 0.16275

2. (a) Compare the MSE when the strength (standard deviation) of the Gaussian noise in the denoised autoencoder is set to 0.25 and 0.75 respectively.

(b) Compare the MSE when the percentage of dropout in the denoised autoencoder is set as 0.25 and 0.75.

3. Modify the variational autoencoder that uses  $\sigma$  instead of log variance.

2.

a. Gaussian Noise stddev = 0.25 → Test MSE = 0.11079

Gaussian Noise stddev = 0.75 → Test MSE = 0.13052

b. Dropout rate = 0.25 → Test MSE = 0.12190

Dropout rate = 0.75 → Test MSE = 0.11567

# 1.Stacked AutoEncoder

a. MSE when the coding size is used as 10, 20, and 40 in the stacked autoencoder.

*#Using Keras to load the dataset.*

```
import tensorflow as tf
import pandas as pd
import numpy as np
from tensorflow import keras
fashion_mnist = keras.datasets.fashion_mnist
(x_train, y_train), (x_test, y_test) = fashion_mnist.load_data()

x_valid, x_train_n = x_train[:5000]/255.0, x_train[5000:]/255.0
x_test = x_test / 255.0
y_val = y_train[:5000]
y_train_n = y_train[5000:]
```

```
2025-05-21 02:49:45.070071: E
external/local_xla/xla/stream_executor/cuda/cuda_fft.cc:477] Unable to
register cuFFT factory: Attempting to register factory for plugin
cuFFT when one has already been registered
WARNING: All log messages before absl::InitializeLog() is called are
written to STDERR
E0000 00:00:1747795785.564294      35 cuda_dnn.cc:8310] Unable to
register cuDNN factory: Attempting to register factory for plugin
cuDNN when one has already been registered
E0000 00:00:1747795785.700145      35 cuda_blas.cc:1418] Unable to
register cuBLAS factory: Attempting to register factory for plugin
cuBLAS when one has already been registered
```

```
Downloading data from https://storage.googleapis.com/tensorflow/tf-
keras-datasets/train-labels-idx1-ubyte.gz
29515/29515 _____ 0s 0us/step
Downloading data from https://storage.googleapis.com/tensorflow/tf-
keras-datasets/train-images-idx3-ubyte.gz
26421880/26421880 _____ 0s 0us/step
Downloading data from https://storage.googleapis.com/tensorflow/tf-
keras-datasets/t10k-labels-idx1-ubyte.gz
5148/5148 _____ 0s 0us/step
Downloading data from https://storage.googleapis.com/tensorflow/tf-
keras-datasets/t10k-images-idx3-ubyte.gz
4422102/4422102 _____ 0s 0us/step
```

```
assert x_train.shape == (60000, 28, 28)
assert x_test.shape == (10000, 28, 28)
assert y_train.shape == (60000,)
assert y_test.shape == (10000,)
```

```

#Building the Stacked Autoencoder:
from sklearn.metrics import mean_squared_error
def build_stacked(code_size):
    encoder = keras.models.Sequential([
        keras.layers.Flatten(input_shape=[28,28]),
        keras.layers.Dense(128, activation="selu"),
        keras.layers.Dense(code_size, activation="selu"),
    ])
    decoder = keras.models.Sequential([
        keras.layers.Dense(128,
activation="selu",input_shape=[code_size]),
        keras.layers.Dense(28*28, activation="sigmoid"),
        keras.layers.Reshape([28,28])
    ])

    autoencoder = keras.models.Sequential([encoder,decoder])
    #Compile the model
    autoencoder.compile(loss="mse",
optimizer=keras.optimizers.SGD(learning_rate=0.0001))
    history = autoencoder.fit(x_train_n, x_train_n, epochs=50,
validation_data=(x_valid,x_valid), verbose=1)
    x_test_recon = autoencoder.predict(x_test)
    mse = mean_squared_error(x_test.flatten(),x_test_recon.flatten())

    return mse

for i in [10,20,40]:
    print(f"Code size = {i}")
    mse = build_stacked(code_size=i)
    print(f"Code size = {i} → MSE = {mse:.5f}")

```

Code size = 10

Epoch 1/50

1719/1719 ————— 5s 2ms/step - loss: 0.1730 - val\_loss: 0.1740

Epoch 2/50

1719/1719 ————— 3s 2ms/step - loss: 0.1729 - val\_loss: 0.1738

Epoch 3/50

1719/1719 ————— 3s 2ms/step - loss: 0.1731 - val\_loss: 0.1737

Epoch 4/50

1719/1719 ————— 3s 2ms/step - loss: 0.1726 - val\_loss: 0.1735

Epoch 5/50

1719/1719 ————— 4s 2ms/step - loss: 0.1727 - val\_loss: 0.1734

Epoch 6/50

1719/1719 ————— 3s 2ms/step - loss: 0.1724 - val\_loss:

```
0.1732
Epoch 7/50
1719/1719 _____ 3s 2ms/step - loss: 0.1720 - val_loss:
0.1731
Epoch 8/50
1719/1719 _____ 3s 2ms/step - loss: 0.1722 - val_loss:
0.1730
Epoch 9/50
1719/1719 _____ 3s 2ms/step - loss: 0.1717 - val_loss:
0.1728
Epoch 10/50
1719/1719 _____ 3s 2ms/step - loss: 0.1719 - val_loss:
0.1727
Epoch 11/50
1719/1719 _____ 3s 2ms/step - loss: 0.1716 - val_loss:
0.1726
Epoch 12/50
1719/1719 _____ 3s 2ms/step - loss: 0.1715 - val_loss:
0.1725
Epoch 13/50
1719/1719 _____ 3s 2ms/step - loss: 0.1718 - val_loss:
0.1724
Epoch 14/50
1719/1719 _____ 4s 2ms/step - loss: 0.1713 - val_loss:
0.1723
Epoch 15/50
1719/1719 _____ 3s 2ms/step - loss: 0.1712 - val_loss:
0.1722
Epoch 16/50
1719/1719 _____ 4s 2ms/step - loss: 0.1711 - val_loss:
0.1721
Epoch 17/50
1719/1719 _____ 3s 2ms/step - loss: 0.1707 - val_loss:
0.1720
Epoch 18/50
1719/1719 _____ 3s 2ms/step - loss: 0.1708 - val_loss:
0.1719
Epoch 19/50
1719/1719 _____ 3s 2ms/step - loss: 0.1709 - val_loss:
0.1718
Epoch 20/50
1719/1719 _____ 3s 2ms/step - loss: 0.1707 - val_loss:
0.1717
Epoch 21/50
1719/1719 _____ 3s 2ms/step - loss: 0.1706 - val_loss:
0.1716
Epoch 22/50
1719/1719 _____ 3s 2ms/step - loss: 0.1707 - val_loss:
0.1715
```

```
Epoch 23/50
1719/1719 _____ 4s 2ms/step - loss: 0.1707 - val_loss:
0.1715
Epoch 24/50
1719/1719 _____ 4s 2ms/step - loss: 0.1707 - val_loss:
0.1714
Epoch 25/50
1719/1719 _____ 3s 2ms/step - loss: 0.1703 - val_loss:
0.1713
Epoch 26/50
1719/1719 _____ 3s 2ms/step - loss: 0.1702 - val_loss:
0.1712
Epoch 27/50
1719/1719 _____ 4s 2ms/step - loss: 0.1702 - val_loss:
0.1712
Epoch 28/50
1719/1719 _____ 3s 2ms/step - loss: 0.1702 - val_loss:
0.1711
Epoch 29/50
1719/1719 _____ 3s 2ms/step - loss: 0.1699 - val_loss:
0.1710
Epoch 30/50
1719/1719 _____ 3s 2ms/step - loss: 0.1701 - val_loss:
0.1709
Epoch 31/50
1719/1719 _____ 3s 2ms/step - loss: 0.1700 - val_loss:
0.1709
Epoch 32/50
1719/1719 _____ 3s 2ms/step - loss: 0.1699 - val_loss:
0.1708
Epoch 33/50
1719/1719 _____ 4s 2ms/step - loss: 0.1697 - val_loss:
0.1707
Epoch 34/50
1719/1719 _____ 3s 2ms/step - loss: 0.1696 - val_loss:
0.1706
Epoch 35/50
1719/1719 _____ 3s 2ms/step - loss: 0.1697 - val_loss:
0.1706
Epoch 36/50
1719/1719 _____ 4s 2ms/step - loss: 0.1693 - val_loss:
0.1705
Epoch 37/50
1719/1719 _____ 3s 2ms/step - loss: 0.1694 - val_loss:
0.1704
Epoch 38/50
1719/1719 _____ 3s 2ms/step - loss: 0.1697 - val_loss:
0.1703
Epoch 39/50
```

```

1719/1719 _____ 3s 2ms/step - loss: 0.1694 - val_loss:
0.1703
Epoch 40/50
1719/1719 _____ 3s 2ms/step - loss: 0.1693 - val_loss:
0.1702
Epoch 41/50
1719/1719 _____ 4s 2ms/step - loss: 0.1692 - val_loss:
0.1701
Epoch 42/50
1719/1719 _____ 4s 2ms/step - loss: 0.1691 - val_loss:
0.1700
Epoch 43/50
1719/1719 _____ 3s 2ms/step - loss: 0.1691 - val_loss:
0.1700
Epoch 44/50
1719/1719 _____ 3s 2ms/step - loss: 0.1690 - val_loss:
0.1699
Epoch 45/50
1719/1719 _____ 3s 2ms/step - loss: 0.1689 - val_loss:
0.1698
Epoch 46/50
1719/1719 _____ 3s 2ms/step - loss: 0.1689 - val_loss:
0.1697
Epoch 47/50
1719/1719 _____ 3s 2ms/step - loss: 0.1687 - val_loss:
0.1696
Epoch 48/50
1719/1719 _____ 3s 2ms/step - loss: 0.1686 - val_loss:
0.1696
Epoch 49/50
1719/1719 _____ 4s 2ms/step - loss: 0.1685 - val_loss:
0.1695
Epoch 50/50
1719/1719 _____ 4s 2ms/step - loss: 0.1686 - val_loss:
0.1694
313/313 _____ 1s 3ms/step
Code size = 10 → MSE = 0.16776
Code size = 20

```

```

/usr/local/lib/python3.11/dist-packages/keras/src/layers/reshaping/
flatten.py:37: UserWarning: Do not pass an `input_shape`/`input_dim`
argument to a layer. When using Sequential models, prefer using an
`Input(shape)` object as the first layer in the model instead.

```

```

    super().__init__(**kwargs)

```

```

/usr/local/lib/python3.11/dist-packages/keras/src/layers/core/dense.py
:87: UserWarning: Do not pass an `input_shape`/`input_dim` argument to
a layer. When using Sequential models, prefer using an `Input(shape)`
object as the first layer in the model instead.

```

```

    super().__init__(activity_regularizer=activity_regularizer,
**kwargs)

```

```
Epoch 1/50
1719/1719 _____ 5s 3ms/step - loss: 0.1771 - val_loss:
0.1779
Epoch 2/50
1719/1719 _____ 4s 2ms/step - loss: 0.1769 - val_loss:
0.1776
Epoch 3/50
1719/1719 _____ 3s 2ms/step - loss: 0.1765 - val_loss:
0.1773
Epoch 4/50
1719/1719 _____ 3s 2ms/step - loss: 0.1760 - val_loss:
0.1770
Epoch 5/50
1719/1719 _____ 3s 2ms/step - loss: 0.1759 - val_loss:
0.1768
Epoch 6/50
1719/1719 _____ 3s 2ms/step - loss: 0.1756 - val_loss:
0.1765
Epoch 7/50
1719/1719 _____ 3s 2ms/step - loss: 0.1755 - val_loss:
0.1763
Epoch 8/50
1719/1719 _____ 4s 2ms/step - loss: 0.1753 - val_loss:
0.1761
Epoch 9/50
1719/1719 _____ 4s 2ms/step - loss: 0.1749 - val_loss:
0.1759
Epoch 10/50
1719/1719 _____ 3s 2ms/step - loss: 0.1749 - val_loss:
0.1756
Epoch 11/50
1719/1719 _____ 3s 2ms/step - loss: 0.1747 - val_loss:
0.1754
Epoch 12/50
1719/1719 _____ 3s 2ms/step - loss: 0.1744 - val_loss:
0.1753
Epoch 13/50
1719/1719 _____ 3s 2ms/step - loss: 0.1739 - val_loss:
0.1751
Epoch 14/50
1719/1719 _____ 3s 2ms/step - loss: 0.1740 - val_loss:
0.1749
Epoch 15/50
1719/1719 _____ 3s 2ms/step - loss: 0.1739 - val_loss:
0.1747
Epoch 16/50
1719/1719 _____ 3s 2ms/step - loss: 0.1736 - val_loss:
0.1746
Epoch 17/50
1719/1719 _____ 3s 2ms/step - loss: 0.1733 - val_loss:
```

```
0.1744
Epoch 18/50
1719/1719 _____ 4s 2ms/step - loss: 0.1733 - val_loss:
0.1743
Epoch 19/50
1719/1719 _____ 3s 2ms/step - loss: 0.1731 - val_loss:
0.1742
Epoch 20/50
1719/1719 _____ 3s 2ms/step - loss: 0.1733 - val_loss:
0.1740
Epoch 21/50
1719/1719 _____ 3s 2ms/step - loss: 0.1732 - val_loss:
0.1739
Epoch 22/50
1719/1719 _____ 3s 2ms/step - loss: 0.1729 - val_loss:
0.1738
Epoch 23/50
1719/1719 _____ 3s 2ms/step - loss: 0.1728 - val_loss:
0.1736
Epoch 24/50
1719/1719 _____ 3s 2ms/step - loss: 0.1724 - val_loss:
0.1735
Epoch 25/50
1719/1719 _____ 3s 2ms/step - loss: 0.1724 - val_loss:
0.1734
Epoch 26/50
1719/1719 _____ 3s 2ms/step - loss: 0.1724 - val_loss:
0.1733
Epoch 27/50
1719/1719 _____ 4s 2ms/step - loss: 0.1721 - val_loss:
0.1732
Epoch 28/50
1719/1719 _____ 3s 2ms/step - loss: 0.1718 - val_loss:
0.1731
Epoch 29/50
1719/1719 _____ 3s 2ms/step - loss: 0.1718 - val_loss:
0.1730
Epoch 30/50
1719/1719 _____ 3s 2ms/step - loss: 0.1718 - val_loss:
0.1729
Epoch 31/50
1719/1719 _____ 3s 2ms/step - loss: 0.1720 - val_loss:
0.1727
Epoch 32/50
1719/1719 _____ 3s 2ms/step - loss: 0.1716 - val_loss:
0.1726
Epoch 33/50
1719/1719 _____ 3s 2ms/step - loss: 0.1717 - val_loss:
0.1725
Epoch 34/50
```



```
1719/1719 ————— 3s 2ms/step - loss: 0.1714 - val_loss: 0.1724
Epoch 35/50
1719/1719 ————— 3s 2ms/step - loss: 0.1712 - val_loss: 0.1723
Epoch 36/50
1719/1719 ————— 4s 2ms/step - loss: 0.1714 - val_loss: 0.1722
Epoch 37/50
1719/1719 ————— 3s 2ms/step - loss: 0.1713 - val_loss: 0.1721
Epoch 38/50
1719/1719 ————— 3s 2ms/step - loss: 0.1711 - val_loss: 0.1720
Epoch 39/50
1719/1719 ————— 3s 2ms/step - loss: 0.1709 - val_loss: 0.1719
Epoch 40/50
1719/1719 ————— 3s 2ms/step - loss: 0.1707 - val_loss: 0.1718
Epoch 41/50
1719/1719 ————— 3s 2ms/step - loss: 0.1707 - val_loss: 0.1717
Epoch 42/50
1719/1719 ————— 4s 2ms/step - loss: 0.1707 - val_loss: 0.1716
Epoch 43/50
1719/1719 ————— 3s 2ms/step - loss: 0.1705 - val_loss: 0.1715
Epoch 44/50
1719/1719 ————— 3s 2ms/step - loss: 0.1701 - val_loss: 0.1714
Epoch 45/50
1719/1719 ————— 4s 2ms/step - loss: 0.1700 - val_loss: 0.1712
Epoch 46/50
1719/1719 ————— 3s 2ms/step - loss: 0.1705 - val_loss: 0.1711
Epoch 47/50
1719/1719 ————— 3s 2ms/step - loss: 0.1701 - val_loss: 0.1710
Epoch 48/50
1719/1719 ————— 3s 2ms/step - loss: 0.1701 - val_loss: 0.1709
Epoch 49/50
1719/1719 ————— 3s 2ms/step - loss: 0.1700 - val_loss: 0.1708
Epoch 50/50
1719/1719 ————— 3s 2ms/step - loss: 0.1697 - val_loss: 0.1707
```

313/313 ————— 1s 2ms/step

Code size = 20 → MSE = 0.16902

Code size = 40

/usr/local/lib/python3.11/dist-packages/keras/src/layers/reshaping/flatten.py:37: UserWarning: Do not pass an `input\_shape`/`input\_dim` argument to a layer. When using Sequential models, prefer using an `Input(shape)` object as the first layer in the model instead.

super().\_\_init\_\_(\*\*kwargs)

/usr/local/lib/python3.11/dist-packages/keras/src/layers/core/dense.py:87: UserWarning: Do not pass an `input\_shape`/`input\_dim` argument to a layer. When using Sequential models, prefer using an `Input(shape)` object as the first layer in the model instead.

super().\_\_init\_\_(activity\_regularizer=activity\_regularizer, \*\*kwargs)

Epoch 1/50

1719/1719 ————— 5s 3ms/step - loss: 0.1743 - val\_loss: 0.1750

Epoch 2/50

1719/1719 ————— 4s 2ms/step - loss: 0.1741 - val\_loss: 0.1747

Epoch 3/50

1719/1719 ————— 4s 2ms/step - loss: 0.1739 - val\_loss: 0.1743

Epoch 4/50

1719/1719 ————— 3s 2ms/step - loss: 0.1731 - val\_loss: 0.1740

Epoch 5/50

1719/1719 ————— 4s 2ms/step - loss: 0.1730 - val\_loss: 0.1737

Epoch 6/50

1719/1719 ————— 3s 2ms/step - loss: 0.1723 - val\_loss: 0.1734

Epoch 7/50

1719/1719 ————— 3s 2ms/step - loss: 0.1723 - val\_loss: 0.1731

Epoch 8/50

1719/1719 ————— 4s 2ms/step - loss: 0.1720 - val\_loss: 0.1728

Epoch 9/50

1719/1719 ————— 3s 2ms/step - loss: 0.1717 - val\_loss: 0.1725

Epoch 10/50

1719/1719 ————— 3s 2ms/step - loss: 0.1715 - val\_loss: 0.1722

Epoch 11/50

1719/1719 ————— 3s 2ms/step - loss: 0.1710 - val\_loss: 0.1720

Epoch 12/50

```
1719/1719 ————— 4s 2ms/step - loss: 0.1711 - val_loss: 0.1718
Epoch 13/50
1719/1719 ————— 3s 2ms/step - loss: 0.1706 - val_loss: 0.1715
Epoch 14/50
1719/1719 ————— 4s 2ms/step - loss: 0.1706 - val_loss: 0.1713
Epoch 15/50
1719/1719 ————— 3s 2ms/step - loss: 0.1702 - val_loss: 0.1711
Epoch 16/50
1719/1719 ————— 3s 2ms/step - loss: 0.1701 - val_loss: 0.1708
Epoch 17/50
1719/1719 ————— 4s 2ms/step - loss: 0.1696 - val_loss: 0.1706
Epoch 18/50
1719/1719 ————— 3s 2ms/step - loss: 0.1696 - val_loss: 0.1704
Epoch 19/50
1719/1719 ————— 4s 2ms/step - loss: 0.1694 - val_loss: 0.1702
Epoch 20/50
1719/1719 ————— 3s 2ms/step - loss: 0.1691 - val_loss: 0.1700
Epoch 21/50
1719/1719 ————— 4s 2ms/step - loss: 0.1690 - val_loss: 0.1698
Epoch 22/50
1719/1719 ————— 4s 2ms/step - loss: 0.1689 - val_loss: 0.1696
Epoch 23/50
1719/1719 ————— 3s 2ms/step - loss: 0.1688 - val_loss: 0.1694
Epoch 24/50
1719/1719 ————— 3s 2ms/step - loss: 0.1683 - val_loss: 0.1692
Epoch 25/50
1719/1719 ————— 4s 2ms/step - loss: 0.1682 - val_loss: 0.1690
Epoch 26/50
1719/1719 ————— 3s 2ms/step - loss: 0.1682 - val_loss: 0.1688
Epoch 27/50
1719/1719 ————— 3s 2ms/step - loss: 0.1677 - val_loss: 0.1686
Epoch 28/50
1719/1719 ————— 4s 2ms/step - loss: 0.1677 - val_loss:
```

```
0.1685
Epoch 29/50
1719/1719 _____ 3s 2ms/step - loss: 0.1676 - val_loss:
0.1683
Epoch 30/50
1719/1719 _____ 4s 2ms/step - loss: 0.1673 - val_loss:
0.1681
Epoch 31/50
1719/1719 _____ 4s 2ms/step - loss: 0.1670 - val_loss:
0.1679
Epoch 32/50
1719/1719 _____ 3s 2ms/step - loss: 0.1668 - val_loss:
0.1677
Epoch 33/50
1719/1719 _____ 3s 2ms/step - loss: 0.1667 - val_loss:
0.1675
Epoch 34/50
1719/1719 _____ 3s 2ms/step - loss: 0.1666 - val_loss:
0.1673
Epoch 35/50
1719/1719 _____ 3s 2ms/step - loss: 0.1662 - val_loss:
0.1672
Epoch 36/50
1719/1719 _____ 4s 2ms/step - loss: 0.1661 - val_loss:
0.1670
Epoch 37/50
1719/1719 _____ 4s 2ms/step - loss: 0.1660 - val_loss:
0.1668
Epoch 38/50
1719/1719 _____ 4s 2ms/step - loss: 0.1659 - val_loss:
0.1666
Epoch 39/50
1719/1719 _____ 4s 2ms/step - loss: 0.1658 - val_loss:
0.1664
Epoch 40/50
1719/1719 _____ 4s 2ms/step - loss: 0.1653 - val_loss:
0.1662
Epoch 41/50
1719/1719 _____ 3s 2ms/step - loss: 0.1651 - val_loss:
0.1660
Epoch 42/50
1719/1719 _____ 4s 2ms/step - loss: 0.1647 - val_loss:
0.1658
Epoch 43/50
1719/1719 _____ 3s 2ms/step - loss: 0.1650 - val_loss:
0.1657
Epoch 44/50
1719/1719 _____ 3s 2ms/step - loss: 0.1649 - val_loss:
0.1655
```

```

Epoch 45/50
1719/1719 _____ 4s 2ms/step - loss: 0.1646 - val_loss: 0.1653
Epoch 46/50
1719/1719 _____ 3s 2ms/step - loss: 0.1644 - val_loss: 0.1651
Epoch 47/50
1719/1719 _____ 3s 2ms/step - loss: 0.1643 - val_loss: 0.1649
Epoch 48/50
1719/1719 _____ 4s 2ms/step - loss: 0.1638 - val_loss: 0.1647
Epoch 49/50
1719/1719 _____ 3s 2ms/step - loss: 0.1637 - val_loss: 0.1645
Epoch 50/50
1719/1719 _____ 3s 2ms/step - loss: 0.1635 - val_loss: 0.1643
313/313 _____ 1s 2ms/step
Code size = 40 → MSE = 0.16275

```

**b. Test whether the reconstruction is improved if an additional hidden layer with 100 neurons was used in the stacked autoencoder.**

```

#Building the Stacked Autoencoder:
from sklearn.metrics import mean_squared_error
def build_stacked_ad(code_size):
    encoder = keras.models.Sequential([
        keras.layers.Flatten(input_shape=[28,28]),
        keras.layers.Dense(128, activation="selu"),
        keras.layers.Dense(100, activation="selu"),
        keras.layers.Dense(code_size, activation="selu"),
    ])
    #Note that its not mandatory to have the same number of hidden
layers in the decoder part, the only thing that has to be take care is
that the output and input should match (Dimensions).
    decoder = keras.models.Sequential([
        keras.layers.Dense(100,
activation="selu",input_shape=[code_size]),
        keras.layers.Dense(128, activation="selu"),
        keras.layers.Dense(28*28, activation="sigmoid"),
        keras.layers.Reshape([28,28])
    ])

    autoencoder = keras.models.Sequential([encoder,decoder])
    #Compile the model
    autoencoder.compile(loss="mse",
optimizer=keras.optimizers.SGD(learning_rate=0.0001))
    history = autoencoder.fit(x_train_n, x_train_n, epochs=50,

```

```

validation_data=(x_valid,x_valid), verbose=1)
    x_test_recon = autoencoder.predict(x_test)
    mse = mean_squared_error(x_test.flatten(),x_test_recon.flatten())

    return mse

for i in [10,20,40]:
    print(f"Code size = {i}")
    mse = build_stacked_ad(code_size=i)
    print(f"Code size = {i} → MSE = {mse:.5f}")

```

Code size = 10

```

/usr/local/lib/python3.11/dist-packages/keras/src/layers/reshaping/
flatten.py:37: UserWarning: Do not pass an `input_shape`/`input_dim`
argument to a layer. When using Sequential models, prefer using an
`Input(shape)` object as the first layer in the model instead.
    super().__init__(**kwargs)
/usr/local/lib/python3.11/dist-packages/keras/src/layers/core/dense.py
:87: UserWarning: Do not pass an `input_shape`/`input_dim` argument to
a layer. When using Sequential models, prefer using an `Input(shape)`
object as the first layer in the model instead.
    super().__init__(activity_regularizer=activity_regularizer,
**kwargs)

```

```

Epoch 1/50
1719/1719 _____ 6s 3ms/step - loss: 0.1737 - val_loss:
0.1746
Epoch 2/50
1719/1719 _____ 4s 2ms/step - loss: 0.1737 - val_loss:
0.1742
Epoch 3/50
1719/1719 _____ 4s 2ms/step - loss: 0.1729 - val_loss:
0.1739
Epoch 4/50
1719/1719 _____ 4s 2ms/step - loss: 0.1729 - val_loss:
0.1736
Epoch 5/50
1719/1719 _____ 4s 2ms/step - loss: 0.1728 - val_loss:
0.1734
Epoch 6/50
1719/1719 _____ 4s 2ms/step - loss: 0.1723 - val_loss:
0.1731
Epoch 7/50
1719/1719 _____ 4s 2ms/step - loss: 0.1718 - val_loss:
0.1729
Epoch 8/50
1719/1719 _____ 4s 2ms/step - loss: 0.1719 - val_loss:
0.1727

```

```
Epoch 9/50
1719/1719 _____ 4s 2ms/step - loss: 0.1718 - val_loss:
0.1725
Epoch 10/50
1719/1719 _____ 4s 2ms/step - loss: 0.1715 - val_loss:
0.1723
Epoch 11/50
1719/1719 _____ 4s 2ms/step - loss: 0.1711 - val_loss:
0.1721
Epoch 12/50
1719/1719 _____ 4s 2ms/step - loss: 0.1710 - val_loss:
0.1719
Epoch 13/50
1719/1719 _____ 4s 2ms/step - loss: 0.1709 - val_loss:
0.1717
Epoch 14/50
1719/1719 _____ 4s 2ms/step - loss: 0.1707 - val_loss:
0.1716
Epoch 15/50
1719/1719 _____ 4s 2ms/step - loss: 0.1705 - val_loss:
0.1714
Epoch 16/50
1719/1719 _____ 4s 2ms/step - loss: 0.1707 - val_loss:
0.1713
Epoch 17/50
1719/1719 _____ 4s 2ms/step - loss: 0.1703 - val_loss:
0.1711
Epoch 18/50
1719/1719 _____ 4s 2ms/step - loss: 0.1702 - val_loss:
0.1710
Epoch 19/50
1719/1719 _____ 4s 2ms/step - loss: 0.1699 - val_loss:
0.1708
Epoch 20/50
1719/1719 _____ 4s 2ms/step - loss: 0.1699 - val_loss:
0.1707
Epoch 21/50
1719/1719 _____ 4s 2ms/step - loss: 0.1696 - val_loss:
0.1705
Epoch 22/50
1719/1719 _____ 4s 2ms/step - loss: 0.1694 - val_loss:
0.1704
Epoch 23/50
1719/1719 _____ 4s 2ms/step - loss: 0.1692 - val_loss:
0.1703
Epoch 24/50
1719/1719 _____ 4s 2ms/step - loss: 0.1694 - val_loss:
0.1701
Epoch 25/50
```

```
1719/1719 ————— 4s 2ms/step - loss: 0.1692 - val_loss: 0.1700
Epoch 26/50
1719/1719 ————— 4s 2ms/step - loss: 0.1689 - val_loss: 0.1698
Epoch 27/50
1719/1719 ————— 4s 2ms/step - loss: 0.1686 - val_loss: 0.1697
Epoch 28/50
1719/1719 ————— 4s 2ms/step - loss: 0.1687 - val_loss: 0.1696
Epoch 29/50
1719/1719 ————— 4s 2ms/step - loss: 0.1683 - val_loss: 0.1694
Epoch 30/50
1719/1719 ————— 4s 2ms/step - loss: 0.1686 - val_loss: 0.1693
Epoch 31/50
1719/1719 ————— 4s 2ms/step - loss: 0.1680 - val_loss: 0.1691
Epoch 32/50
1719/1719 ————— 4s 2ms/step - loss: 0.1682 - val_loss: 0.1690
Epoch 33/50
1719/1719 ————— 4s 2ms/step - loss: 0.1681 - val_loss: 0.1689
Epoch 34/50
1719/1719 ————— 4s 2ms/step - loss: 0.1677 - val_loss: 0.1687
Epoch 35/50
1719/1719 ————— 4s 2ms/step - loss: 0.1678 - val_loss: 0.1686
Epoch 36/50
1719/1719 ————— 4s 2ms/step - loss: 0.1678 - val_loss: 0.1684
Epoch 37/50
1719/1719 ————— 4s 2ms/step - loss: 0.1673 - val_loss: 0.1683
Epoch 38/50
1719/1719 ————— 4s 2ms/step - loss: 0.1675 - val_loss: 0.1682
Epoch 39/50
1719/1719 ————— 4s 2ms/step - loss: 0.1672 - val_loss: 0.1680
Epoch 40/50
1719/1719 ————— 4s 2ms/step - loss: 0.1673 - val_loss: 0.1679
Epoch 41/50
1719/1719 ————— 4s 2ms/step - loss: 0.1668 - val_loss:
```



```

0.1677
Epoch 42/50
1719/1719 _____ 4s 2ms/step - loss: 0.1667 - val_loss:
0.1676
Epoch 43/50
1719/1719 _____ 4s 2ms/step - loss: 0.1666 - val_loss:
0.1674
Epoch 44/50
1719/1719 _____ 4s 2ms/step - loss: 0.1665 - val_loss:
0.1673
Epoch 45/50
1719/1719 _____ 4s 2ms/step - loss: 0.1663 - val_loss:
0.1671
Epoch 46/50
1719/1719 _____ 4s 2ms/step - loss: 0.1663 - val_loss:
0.1670
Epoch 47/50
1719/1719 _____ 4s 2ms/step - loss: 0.1658 - val_loss:
0.1668
Epoch 48/50
1719/1719 _____ 4s 2ms/step - loss: 0.1658 - val_loss:
0.1667
Epoch 49/50
1719/1719 _____ 4s 2ms/step - loss: 0.1656 - val_loss:
0.1665
Epoch 50/50
1719/1719 _____ 4s 2ms/step - loss: 0.1656 - val_loss:
0.1664
313/313 _____ 1s 2ms/step
Code size = 10 → MSE = 0.16476
Code size = 20

```

```

/usr/local/lib/python3.11/dist-packages/keras/src/layers/reshaping/
flatten.py:37: UserWarning: Do not pass an `input_shape`/`input_dim`
argument to a layer. When using Sequential models, prefer using an
`Input(shape)` object as the first layer in the model instead.
  super().__init__(**kwargs)
/usr/local/lib/python3.11/dist-packages/keras/src/layers/core/dense.py
:87: UserWarning: Do not pass an `input_shape`/`input_dim` argument to
a layer. When using Sequential models, prefer using an `Input(shape)`
object as the first layer in the model instead.
  super().__init__(activity_regularizer=activity_regularizer,
**kwargs)

```

```

Epoch 1/50
1719/1719 _____ 5s 3ms/step - loss: 0.1814 - val_loss:
0.1818
Epoch 2/50
1719/1719 _____ 4s 2ms/step - loss: 0.1806 - val_loss:
0.1809

```

```
Epoch 3/50
1719/1719 _____ 4s 2ms/step - loss: 0.1797 - val_loss:
0.1800
Epoch 4/50
1719/1719 _____ 4s 2ms/step - loss: 0.1787 - val_loss:
0.1792
Epoch 5/50
1719/1719 _____ 4s 2ms/step - loss: 0.1780 - val_loss:
0.1785
Epoch 6/50
1719/1719 _____ 4s 2ms/step - loss: 0.1774 - val_loss:
0.1779
Epoch 7/50
1719/1719 _____ 4s 2ms/step - loss: 0.1766 - val_loss:
0.1774
Epoch 8/50
1719/1719 _____ 4s 2ms/step - loss: 0.1761 - val_loss:
0.1769
Epoch 9/50
1719/1719 _____ 4s 2ms/step - loss: 0.1758 - val_loss:
0.1765
Epoch 10/50
1719/1719 _____ 4s 2ms/step - loss: 0.1751 - val_loss:
0.1762
Epoch 11/50
1719/1719 _____ 4s 2ms/step - loss: 0.1751 - val_loss:
0.1758
Epoch 12/50
1719/1719 _____ 4s 2ms/step - loss: 0.1750 - val_loss:
0.1755
Epoch 13/50
1719/1719 _____ 4s 2ms/step - loss: 0.1741 - val_loss:
0.1753
Epoch 14/50
1719/1719 _____ 4s 2ms/step - loss: 0.1743 - val_loss:
0.1750
Epoch 15/50
1719/1719 _____ 4s 2ms/step - loss: 0.1739 - val_loss:
0.1748
Epoch 16/50
1719/1719 _____ 4s 2ms/step - loss: 0.1737 - val_loss:
0.1746
Epoch 17/50
1719/1719 _____ 4s 2ms/step - loss: 0.1734 - val_loss:
0.1744
Epoch 18/50
1719/1719 _____ 4s 2ms/step - loss: 0.1735 - val_loss:
0.1742
Epoch 19/50
```

```
1719/1719 ————— 4s 2ms/step - loss: 0.1731 - val_loss: 0.1740
Epoch 20/50
1719/1719 ————— 4s 2ms/step - loss: 0.1728 - val_loss: 0.1738
Epoch 21/50
1719/1719 ————— 4s 2ms/step - loss: 0.1726 - val_loss: 0.1737
Epoch 22/50
1719/1719 ————— 4s 2ms/step - loss: 0.1724 - val_loss: 0.1735
Epoch 23/50
1719/1719 ————— 4s 2ms/step - loss: 0.1725 - val_loss: 0.1733
Epoch 24/50
1719/1719 ————— 4s 2ms/step - loss: 0.1725 - val_loss: 0.1732
Epoch 25/50
1719/1719 ————— 4s 2ms/step - loss: 0.1722 - val_loss: 0.1730
Epoch 26/50
1719/1719 ————— 4s 2ms/step - loss: 0.1717 - val_loss: 0.1729
Epoch 27/50
1719/1719 ————— 4s 2ms/step - loss: 0.1717 - val_loss: 0.1727
Epoch 28/50
1719/1719 ————— 4s 2ms/step - loss: 0.1718 - val_loss: 0.1726
Epoch 29/50
1719/1719 ————— 4s 2ms/step - loss: 0.1713 - val_loss: 0.1724
Epoch 30/50
1719/1719 ————— 4s 2ms/step - loss: 0.1713 - val_loss: 0.1723
Epoch 31/50
1719/1719 ————— 4s 2ms/step - loss: 0.1712 - val_loss: 0.1721
Epoch 32/50
1719/1719 ————— 4s 2ms/step - loss: 0.1711 - val_loss: 0.1720
Epoch 33/50
1719/1719 ————— 4s 2ms/step - loss: 0.1706 - val_loss: 0.1718
Epoch 34/50
1719/1719 ————— 4s 2ms/step - loss: 0.1708 - val_loss: 0.1717
Epoch 35/50
1719/1719 ————— 4s 2ms/step - loss: 0.1706 - val_loss:
```

```
0.1715
Epoch 36/50
1719/1719 _____ 4s 2ms/step - loss: 0.1705 - val_loss:
0.1714
Epoch 37/50
1719/1719 _____ 4s 2ms/step - loss: 0.1700 - val_loss:
0.1712
Epoch 38/50
1719/1719 _____ 4s 2ms/step - loss: 0.1699 - val_loss:
0.1711
Epoch 39/50
1719/1719 _____ 4s 2ms/step - loss: 0.1699 - val_loss:
0.1709
Epoch 40/50
1719/1719 _____ 4s 2ms/step - loss: 0.1700 - val_loss:
0.1708
Epoch 41/50
1719/1719 _____ 4s 2ms/step - loss: 0.1698 - val_loss:
0.1706
Epoch 42/50
1719/1719 _____ 4s 2ms/step - loss: 0.1694 - val_loss:
0.1704
Epoch 43/50
1719/1719 _____ 3s 2ms/step - loss: 0.1693 - val_loss:
0.1703
Epoch 44/50
1719/1719 _____ 3s 2ms/step - loss: 0.1693 - val_loss:
0.1701
Epoch 45/50
1719/1719 _____ 4s 2ms/step - loss: 0.1691 - val_loss:
0.1700
Epoch 46/50
1719/1719 _____ 4s 2ms/step - loss: 0.1690 - val_loss:
0.1698
Epoch 47/50
1719/1719 _____ 4s 2ms/step - loss: 0.1687 - val_loss:
0.1696
Epoch 48/50
1719/1719 _____ 4s 2ms/step - loss: 0.1687 - val_loss:
0.1694
Epoch 49/50
1719/1719 _____ 4s 2ms/step - loss: 0.1684 - val_loss:
0.1693
Epoch 50/50
1719/1719 _____ 4s 2ms/step - loss: 0.1685 - val_loss:
0.1691
313/313 _____ 1s 2ms/step
Code size = 20 → MSE = 0.16745
Code size = 40
```

```
/usr/local/lib/python3.11/dist-packages/keras/src/layers/reshaping/flatten.py:37: UserWarning: Do not pass an `input_shape`/`input_dim` argument to a layer. When using Sequential models, prefer using an `Input(shape)` object as the first layer in the model instead.  
    super().__init__(**kwargs)  
/usr/local/lib/python3.11/dist-packages/keras/src/layers/core/dense.py:87: UserWarning: Do not pass an `input_shape`/`input_dim` argument to a layer. When using Sequential models, prefer using an `Input(shape)` object as the first layer in the model instead.  
    super().__init__(activity_regularizer=activity_regularizer,  
**kwargs)
```

```
Epoch 1/50  
1719/1719 _____ 5s 3ms/step - loss: 0.1796 - val_loss: 0.1801
```

```
Epoch 2/50  
1719/1719 _____ 4s 2ms/step - loss: 0.1786 - val_loss: 0.1793
```

```
Epoch 3/50  
1719/1719 _____ 4s 2ms/step - loss: 0.1782 - val_loss: 0.1785
```

```
Epoch 4/50  
1719/1719 _____ 4s 2ms/step - loss: 0.1773 - val_loss: 0.1779
```

```
Epoch 5/50  
1719/1719 _____ 4s 2ms/step - loss: 0.1767 - val_loss: 0.1772
```

```
Epoch 6/50  
1719/1719 _____ 4s 2ms/step - loss: 0.1760 - val_loss: 0.1767
```

```
Epoch 7/50  
1719/1719 _____ 4s 2ms/step - loss: 0.1754 - val_loss: 0.1762
```

```
Epoch 8/50  
1719/1719 _____ 4s 2ms/step - loss: 0.1752 - val_loss: 0.1757
```

```
Epoch 9/50  
1719/1719 _____ 4s 2ms/step - loss: 0.1744 - val_loss: 0.1753
```

```
Epoch 10/50  
1719/1719 _____ 4s 2ms/step - loss: 0.1740 - val_loss: 0.1748
```

```
Epoch 11/50  
1719/1719 _____ 4s 2ms/step - loss: 0.1739 - val_loss: 0.1745
```

```
Epoch 12/50  
1719/1719 _____ 4s 2ms/step - loss: 0.1734 - val_loss: 0.1741
```

```
Epoch 13/50  
1719/1719 _____ 4s 2ms/step - loss: 0.1730 - val_loss:
```

```
0.1737
Epoch 14/50
1719/1719 _____ 4s 2ms/step - loss: 0.1724 - val_loss:
0.1734
Epoch 15/50
1719/1719 _____ 4s 2ms/step - loss: 0.1724 - val_loss:
0.1730
Epoch 16/50
1719/1719 _____ 4s 2ms/step - loss: 0.1719 - val_loss:
0.1727
Epoch 17/50
1719/1719 _____ 4s 2ms/step - loss: 0.1717 - val_loss:
0.1724
Epoch 18/50
1719/1719 _____ 4s 2ms/step - loss: 0.1712 - val_loss:
0.1721
Epoch 19/50
1719/1719 _____ 4s 2ms/step - loss: 0.1710 - val_loss:
0.1718
Epoch 20/50
1719/1719 _____ 4s 2ms/step - loss: 0.1708 - val_loss:
0.1715
Epoch 21/50
1719/1719 _____ 4s 2ms/step - loss: 0.1706 - val_loss:
0.1712
Epoch 22/50
1719/1719 _____ 4s 2ms/step - loss: 0.1702 - val_loss:
0.1709
Epoch 23/50
1719/1719 _____ 4s 2ms/step - loss: 0.1697 - val_loss:
0.1706
Epoch 24/50
1719/1719 _____ 4s 2ms/step - loss: 0.1695 - val_loss:
0.1703
Epoch 25/50
1719/1719 _____ 4s 2ms/step - loss: 0.1694 - val_loss:
0.1700
Epoch 26/50
1719/1719 _____ 4s 2ms/step - loss: 0.1690 - val_loss:
0.1697
Epoch 27/50
1719/1719 _____ 4s 2ms/step - loss: 0.1689 - val_loss:
0.1694
Epoch 28/50
1719/1719 _____ 4s 2ms/step - loss: 0.1685 - val_loss:
0.1691
Epoch 29/50
1719/1719 _____ 4s 2ms/step - loss: 0.1680 - val_loss:
0.1689
```

```
Epoch 30/50
1719/1719 _____ 4s 2ms/step - loss: 0.1680 - val_loss:
0.1686
Epoch 31/50
1719/1719 _____ 4s 2ms/step - loss: 0.1676 - val_loss:
0.1683
Epoch 32/50
1719/1719 _____ 4s 2ms/step - loss: 0.1671 - val_loss:
0.1680
Epoch 33/50
1719/1719 _____ 4s 2ms/step - loss: 0.1669 - val_loss:
0.1677
Epoch 34/50
1719/1719 _____ 4s 2ms/step - loss: 0.1667 - val_loss:
0.1674
Epoch 35/50
1719/1719 _____ 4s 2ms/step - loss: 0.1663 - val_loss:
0.1671
Epoch 36/50
1719/1719 _____ 4s 2ms/step - loss: 0.1659 - val_loss:
0.1668
Epoch 37/50
1719/1719 _____ 4s 2ms/step - loss: 0.1657 - val_loss:
0.1665
Epoch 38/50
1719/1719 _____ 4s 2ms/step - loss: 0.1657 - val_loss:
0.1662
Epoch 39/50
1719/1719 _____ 4s 2ms/step - loss: 0.1651 - val_loss:
0.1660
Epoch 40/50
1719/1719 _____ 4s 2ms/step - loss: 0.1648 - val_loss:
0.1657
Epoch 41/50
1719/1719 _____ 4s 2ms/step - loss: 0.1645 - val_loss:
0.1654
Epoch 42/50
1719/1719 _____ 4s 2ms/step - loss: 0.1643 - val_loss:
0.1650
Epoch 43/50
1719/1719 _____ 4s 2ms/step - loss: 0.1639 - val_loss:
0.1647
Epoch 44/50
1719/1719 _____ 4s 2ms/step - loss: 0.1636 - val_loss:
0.1644
Epoch 45/50
1719/1719 _____ 4s 2ms/step - loss: 0.1637 - val_loss:
0.1641
Epoch 46/50
```

```

1719/1719 ————— 4s 2ms/step - loss: 0.1632 - val_loss:
0.1638
Epoch 47/50
1719/1719 ————— 4s 2ms/step - loss: 0.1628 - val_loss:
0.1635
Epoch 48/50
1719/1719 ————— 4s 2ms/step - loss: 0.1626 - val_loss:
0.1632
Epoch 49/50
1719/1719 ————— 4s 2ms/step - loss: 0.1621 - val_loss:
0.1629
Epoch 50/50
1719/1719 ————— 4s 2ms/step - loss: 0.1617 - val_loss:
0.1625
313/313 ————— 1s 2ms/step
Code size = 40 → MSE = 0.16100

```

## 2. Denoised Autoencoder:

a.) Compare the MSE when the strength (standard deviation) of the Gaussian noise in the denoised autoencoder is set to 0.25 and 0.75 respectively.

```

from tensorflow import keras
from sklearn.metrics import mean_squared_error

def train_denoise_gaussian(stddev):
    encoder = keras.models.Sequential([
        keras.layers.Flatten(input_shape=[28, 28]),
        keras.layers.GaussianNoise(stddev),
        keras.layers.Dense(100, activation="selu"),
        keras.layers.Dense(30, activation="selu")
    ])

    decoder = keras.models.Sequential([
        keras.layers.Dense(100, activation="selu", input_shape=[30]),
        keras.layers.Dense(28 * 28, activation="sigmoid"),
        keras.layers.Reshape([28, 28])
    ])

    model = keras.models.Sequential([encoder, decoder])
    model.compile(loss="mse",
optimizer=keras.optimizers.SGD(learning_rate=0.0005))
    model.fit(x_train_n, x_train_n, epochs=50,
validation_data=(x_valid, x_valid), verbose=0)

    x_test_recon = model.predict(x_test)

```



```

mse = mean_squared_error(x_test.flatten(), x_test_recon.flatten())
return mse

# Compare
for std in [0.25, 0.75]:
    mse = train_denoise_gaussian(stddev=std)
    print(f"Gaussian Noise stddev = {std} → Test MSE = {mse:.5f}")

/usr/local/lib/python3.11/dist-packages/keras/src/layers/reshaping/flatten.py:37: UserWarning: Do not pass an `input_shape`/`input_dim` argument to a layer. When using Sequential models, prefer using an `Input(shape)` object as the first layer in the model instead.
    super().__init__(**kwargs)
/usr/local/lib/python3.11/dist-packages/keras/src/layers/core/dense.py:87: UserWarning: Do not pass an `input_shape`/`input_dim` argument to a layer. When using Sequential models, prefer using an `Input(shape)` object as the first layer in the model instead.
    super().__init__(activity_regularizer=activity_regularizer,
**kwargs)

313/313 ————— 1s 2ms/step
Gaussian Noise stddev = 0.25 → Test MSE = 0.11079

/usr/local/lib/python3.11/dist-packages/keras/src/layers/reshaping/flatten.py:37: UserWarning: Do not pass an `input_shape`/`input_dim` argument to a layer. When using Sequential models, prefer using an `Input(shape)` object as the first layer in the model instead.
    super().__init__(**kwargs)
/usr/local/lib/python3.11/dist-packages/keras/src/layers/core/dense.py:87: UserWarning: Do not pass an `input_shape`/`input_dim` argument to a layer. When using Sequential models, prefer using an `Input(shape)` object as the first layer in the model instead.
    super().__init__(activity_regularizer=activity_regularizer,
**kwargs)

313/313 ————— 1s 2ms/step
Gaussian Noise stddev = 0.75 → Test MSE = 0.13052

def train_denoise_dropout(dropout_rate):
    encoder = keras.models.Sequential([
        keras.layers.Flatten(input_shape=[28, 28]),
        keras.layers.Dropout(dropout_rate),
        keras.layers.Dense(100, activation="selu"),
        keras.layers.Dense(30, activation="selu")
    ])

    decoder = keras.models.Sequential([
        keras.layers.Dense(100, activation="selu", input_shape=[30]),
        keras.layers.Dense(28 * 28, activation="sigmoid"),
        keras.layers.Reshape([28, 28])
    ])

```

```

    model = keras.models.Sequential([encoder, decoder])
    model.compile(loss="mse",
optimizer=keras.optimizers.SGD(learning_rate=0.0005))
    model.fit(x_train_n, x_train_n, epochs=50,
validation_data=(x_valid, x_valid), verbose=0)

    x_test_recon = model.predict(x_test)
    mse = mean_squared_error(x_test.flatten(), x_test_recon.flatten())
    return mse

```

*# Compare*

```

for drop in [0.25, 0.75]:
    mse = train_denoise_dropout(dropout_rate=drop)
    print(f"Dropout rate = {drop} → Test MSE = {mse:.5f}")

```

```

/usr/local/lib/python3.11/dist-packages/keras/src/layers/reshaping/
flatten.py:37: UserWarning: Do not pass an `input_shape`/`input_dim`
argument to a layer. When using Sequential models, prefer using an
`Input(shape)` object as the first layer in the model instead.

```

```

    super().__init__(**kwargs)
/usr/local/lib/python3.11/dist-packages/keras/src/layers/core/dense.py
:87: UserWarning: Do not pass an `input_shape`/`input_dim` argument to
a layer. When using Sequential models, prefer using an `Input(shape)`
object as the first layer in the model instead.
    super().__init__(activity_regularizer=activity_regularizer,
**kwargs)

```

```

313/313 _____ 1s 2ms/step
Dropout rate = 0.25 → Test MSE = 0.12190

```

```

/usr/local/lib/python3.11/dist-packages/keras/src/layers/reshaping/
flatten.py:37: UserWarning: Do not pass an `input_shape`/`input_dim`
argument to a layer. When using Sequential models, prefer using an
`Input(shape)` object as the first layer in the model instead.

```

```

    super().__init__(**kwargs)
/usr/local/lib/python3.11/dist-packages/keras/src/layers/core/dense.py
:87: UserWarning: Do not pass an `input_shape`/`input_dim` argument to
a layer. When using Sequential models, prefer using an `Input(shape)`
object as the first layer in the model instead.
    super().__init__(activity_regularizer=activity_regularizer,
**kwargs)

```

```

313/313 _____ 1s 2ms/step
Dropout rate = 0.75 → Test MSE = 0.11567

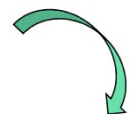
```

## Latent loss

- Kullback–Leibler (KL) divergence between the Gaussian distribution and the actual distribution of the codings

$$\mathcal{L} = -\frac{1}{2} \sum_{i=1}^n \left[ 1 + \log(\sigma_i^2) - \sigma_i^2 - \mu_i^2 \right]$$

$n$  is the codings'  
dimensionality


$$\gamma = \log(\sigma^2)$$

$$\mathcal{L} = -\frac{1}{2} \sum_{i=1}^n \left[ 1 + \gamma_i - \exp(\gamma_i) - \mu_i^2 \right]$$

3. Modify the variational autoencoder that uses  $\sigma$  instead of log variance.

```
import tensorflow as tf
from tensorflow import keras
from tensorflow.keras import backend as K

# Sampling Layer (Reparameterization Trick)
class Sampling(keras.layers.Layer):
    def call(self, inputs):
        mean, sigma = inputs
        epsilon = K.random_normal(shape=tf.shape(sigma))
        return mean + sigma * epsilon

# Custom KL Divergence Layer (adds KL loss to model)
class KLDivergenceLayer(keras.layers.Layer):
    def call(self, inputs):
        mean, sigma = inputs
        kl_loss = -0.5 * tf.reduce_sum(
            1 + tf.math.log(tf.square(sigma) + 1e-8) -
```

```

tf.square(sigma) - tf.square(mean),
    axis=1
)
self.add_loss(tf.reduce_mean(kl_loss) / 784.) # normalize by
image size
return inputs # pass forward

# Build the Variational Autoencoder
def building_vae(code_size):
    # ----- Encoder -----
    inputs = keras.layers.Input(shape=(28, 28))
    x = keras.layers.Flatten()(inputs)
    x = keras.layers.Dense(150, activation="selu")(x)
    x = keras.layers.Dense(100, activation="selu")(x)

    codings_mean = keras.layers.Dense(code_size)(x)
    codings_sigma = keras.layers.Dense(code_size,
activation="softplus")(x) #  $\sigma > 0$ 

    # Add KL loss via custom layer
    codings_mean, codings_sigma = KLDivergenceLayer()([codings_mean,
codings_sigma])
    codings = Sampling()([codings_mean, codings_sigma])

    # ----- Decoder -----
    decoder_inputs = keras.layers.Input(shape=(code_size,))
    x = keras.layers.Dense(100, activation="selu")(decoder_inputs)
    x = keras.layers.Dense(150, activation="selu")(x)
    x = keras.layers.Dense(28 * 28, activation="sigmoid")(x)
    decoder_outputs = keras.layers.Reshape((28, 28))(x)

    # Define decoder model
    decoder = keras.Model(inputs=decoder_inputs,
outputs=decoder_outputs)

    # Connect encoder and decoder
    reconstructed = decoder(codings)
    vae = keras.Model(inputs=inputs, outputs=reconstructed)

    # Compile
    vae.compile(loss="binary_crossentropy", optimizer="rmsprop")

    return vae

# Load and prepare dataset
(X_train_full, _), (X_test, _) =
keras.datasets.fashion_mnist.load_data()
X_train, X_valid = X_train_full[5000:] / 255.0, X_train_full[:5000] /
255.0
X_test = X_test / 255.0

```

```
# Build and train the model
vae = building_vae(code_size=10)
vae.summary()
vae.fit(X_train, X_train, epochs=50, batch_size=32,
validation_data=(X_valid, X_valid), verbose=1)
```

Model: "functional\_14"

Layer (type) Connected to	Output Shape	Param #
input_layer_13 - (InputLayer)	(None, 28, 28)	0
flatten_6 (Flatten) input_layer_13[0][0]	(None, 784)	0
dense_38 (Dense) flatten_6[0][0]	(None, 150)	117,750
dense_39 (Dense) dense_38[0][0]	(None, 100)	15,100
dense_40 (Dense) dense_39[0][0]	(None, 10)	1,010
dense_41 (Dense) dense_39[0][0]	(None, 10)	1,010
kl_divergence_layer_1 dense_40[0][0], (KLDivergenceLayer) dense_41[0][0]	[(None, 10), (None, 10)]	0
sampling_5 (Sampling) kl_divergence_layer_1...	(None, 10)	0

kl_divergence_layer_1...		
functional_13 sampling_5[0][0] (Functional)	(None, 28, 28)	134,634

Total params: 269,504 (1.03 MB)

Trainable params: 269,504 (1.03 MB)

Non-trainable params: 0 (0.00 B)

```
Epoch 1/50
1719/1719 ————— 9s 4ms/step - loss: 0.4066 - val_loss: 0.3354
Epoch 2/50
1719/1719 ————— 4s 2ms/step - loss: 0.3352 - val_loss: 0.3237
Epoch 3/50
1719/1719 ————— 4s 2ms/step - loss: 0.3270 - val_loss: 0.3231
Epoch 4/50
1719/1719 ————— 4s 2ms/step - loss: 0.3219 - val_loss: 0.3163
Epoch 5/50
1719/1719 ————— 4s 2ms/step - loss: 0.3199 - val_loss: 0.3143
Epoch 6/50
1719/1719 ————— 4s 2ms/step - loss: 0.3187 - val_loss: 0.3146
Epoch 7/50
1719/1719 ————— 4s 2ms/step - loss: 0.3159 - val_loss: 0.3133
Epoch 8/50
1719/1719 ————— 4s 2ms/step - loss: 0.3154 - val_loss: 0.3120
Epoch 9/50
1719/1719 ————— 4s 2ms/step - loss: 0.3148 - val_loss: 0.3126
Epoch 10/50
1719/1719 ————— 4s 2ms/step - loss: 0.3149 - val_loss: 0.3111
Epoch 11/50
1719/1719 ————— 4s 2ms/step - loss: 0.3135 - val_loss: 0.3104
```

```
Epoch 12/50
1719/1719 _____ 4s 2ms/step - loss: 0.3138 - val_loss:
0.3102
Epoch 13/50
1719/1719 _____ 4s 2ms/step - loss: 0.3129 - val_loss:
0.3101
Epoch 14/50
1719/1719 _____ 4s 2ms/step - loss: 0.3116 - val_loss:
0.3098
Epoch 15/50
1719/1719 _____ 4s 2ms/step - loss: 0.3115 - val_loss:
0.3093
Epoch 16/50
1719/1719 _____ 4s 2ms/step - loss: 0.3113 - val_loss:
0.3084
Epoch 17/50
1719/1719 _____ 4s 2ms/step - loss: 0.3121 - val_loss:
0.3085
Epoch 18/50
1719/1719 _____ 4s 2ms/step - loss: 0.3117 - val_loss:
0.3090
Epoch 19/50
1719/1719 _____ 4s 2ms/step - loss: 0.3108 - val_loss:
0.3084
Epoch 20/50
1719/1719 _____ 4s 2ms/step - loss: 0.3108 - val_loss:
0.3106
Epoch 21/50
1719/1719 _____ 4s 2ms/step - loss: 0.3106 - val_loss:
0.3076
Epoch 22/50
1719/1719 _____ 4s 2ms/step - loss: 0.3091 - val_loss:
0.3088
Epoch 23/50
1719/1719 _____ 4s 2ms/step - loss: 0.3098 - val_loss:
0.3075
Epoch 24/50
1719/1719 _____ 4s 2ms/step - loss: 0.3101 - val_loss:
0.3072
Epoch 25/50
1719/1719 _____ 4s 2ms/step - loss: 0.3087 - val_loss:
0.3071
Epoch 26/50
1719/1719 _____ 4s 2ms/step - loss: 0.3095 - val_loss:
0.3063
Epoch 27/50
1719/1719 _____ 4s 2ms/step - loss: 0.3101 - val_loss:
0.3090
Epoch 28/50
```

```
1719/1719 ————— 4s 2ms/step - loss: 0.3097 - val_loss: 0.3075
Epoch 29/50
1719/1719 ————— 4s 2ms/step - loss: 0.3094 - val_loss: 0.3064
Epoch 30/50
1719/1719 ————— 4s 2ms/step - loss: 0.3085 - val_loss: 0.3065
Epoch 31/50
1719/1719 ————— 4s 2ms/step - loss: 0.3096 - val_loss: 0.3058
Epoch 32/50
1719/1719 ————— 4s 2ms/step - loss: 0.3085 - val_loss: 0.3072
Epoch 33/50
1719/1719 ————— 4s 2ms/step - loss: 0.3084 - val_loss: 0.3055
Epoch 34/50
1719/1719 ————— 4s 2ms/step - loss: 0.3093 - val_loss: 0.3063
Epoch 35/50
1719/1719 ————— 4s 2ms/step - loss: 0.3078 - val_loss: 0.3068
Epoch 36/50
1719/1719 ————— 4s 2ms/step - loss: 0.3079 - val_loss: 0.3059
Epoch 37/50
1719/1719 ————— 4s 2ms/step - loss: 0.3087 - val_loss: 0.3071
Epoch 38/50
1719/1719 ————— 4s 2ms/step - loss: 0.3084 - val_loss: 0.3059
Epoch 39/50
1719/1719 ————— 4s 2ms/step - loss: 0.3078 - val_loss: 0.3051
Epoch 40/50
1719/1719 ————— 4s 2ms/step - loss: 0.3074 - val_loss: 0.3068
Epoch 41/50
1719/1719 ————— 4s 2ms/step - loss: 0.3073 - val_loss: 0.3063
Epoch 42/50
1719/1719 ————— 4s 2ms/step - loss: 0.3077 - val_loss: 0.3075
Epoch 43/50
1719/1719 ————— 4s 2ms/step - loss: 0.3080 - val_loss: 0.3058
Epoch 44/50
1719/1719 ————— 4s 2ms/step - loss: 0.3079 - val_loss:
```



```
0.3059
Epoch 45/50
1719/1719 _____ 4s 2ms/step - loss: 0.3085 - val_loss:
0.3051
Epoch 46/50
1719/1719 _____ 4s 2ms/step - loss: 0.3065 - val_loss:
0.3052
Epoch 47/50
1719/1719 _____ 4s 2ms/step - loss: 0.3080 - val_loss:
0.3049
Epoch 48/50
1719/1719 _____ 4s 2ms/step - loss: 0.3072 - val_loss:
0.3052
Epoch 49/50
1719/1719 _____ 4s 2ms/step - loss: 0.3077 - val_loss:
0.3053
Epoch 50/50
1719/1719 _____ 4s 2ms/step - loss: 0.3074 - val_loss:
0.3080
```

```
<keras.src.callbacks.history.History at 0x7956b85230d0>
```

```
import matplotlib.pyplot as plt
```

```
# Function to display images side by side: original vs reconstructed
```

```
def plot_image(image):
```

```
    plt.imshow(image, cmap="gray")
```

```
    plt.axis("off")
```

```
def show_original_and_reconstructed(originals, reconstructions,
n_images=12):
```

```
    plt.figure(figsize=(n_images * 1.5, 3))
```

```
    for i in range(n_images):
```

```
        # Original image
```

```
        plt.subplot(2, n_images, i + 1)
```

```
        plot_image(originals[i])
```

```
        if i == 0:
```

```
            plt.title("Original")
```

```
        # Reconstructed image
```

```
        plt.subplot(2, n_images, i + 1 + n_images)
```

```
        plot_image(reconstructions[i])
```

```
        if i == 0:
```

```
            plt.title("Reconstructed")
```

```
    plt.tight_layout()
```

```
    plt.show()
```

```
# Reconstruct 12 test images using the full VAE model
```

```
test_samples = X_test[:12]
```

```
reconstructed_images = vae.predict(test_samples)
```

```
# Visualize
```

```
show_original_and_reconstructed(test_samples, reconstructed_images,  
n_images=12)
```

1/1 ————— 1s 925ms/step

