Elastic Spectrum Allocation for Time-Varying Traffic in FlexGrid Optical Networks

Mirosław Klinkowski, Marc Ruiz, Luis Velasco, Davide Careglio, Victor Lopez, and Jaume Comellas

Abstract—**Elastic flexgrid optical networks (FG-ON) are considered a very promising solution for next-generation optical networks. In this article we focus on lightpath adaptation under variable traffic demands in FG-ON. Specifically, we explore the elastic spectrum allocation (SA) capability of FG-ON and, in this context, we study the effectiveness of three alternative SA schemes in terms of the network performance. To this end, we formulate a Multi-Hour Routing and Spectrum Allocation (MH-RSA) optimization problem and solve it by means of both Integer Linear Programming (ILP) and efficient heuristic algorithms. Since, as numerical results show, the effectiveness of SA schemes highly depends on the traffic demand profile, we formulate some indications on the applicability of elastic SA in FG-ON.**

Index Terms—**Elastic Optical Networks, Network Optimization, Routing and Elastic Spectrum Allocation.**

I. INTRODUCTION

THE CURRENTLY deployed dense wavelength division multiplexing (DWDM) optical networks operate within a rigid frequency grid and with single-line-rate transponders making use of single carrier modulation techniques. The evolution path of optical transport networks can be translated to the application of advanced single-carrier modulation formats (such as k-PSK, k-QAM) in mixed-line-rate networks [2], the introduction of multi-carrier modulation techniques (such as O-OFDM) [3], and the elastic access to spectral resources within a flexible frequency grid (*flexgrid*) [4]. Thanks to these advances, future elastic optical networks will utilize the spectral resources more efficiently, according to the transmission path characteristics and bandwidth requirements [5].

In a flexgrid optical network (FG-ON) the optical frequency spectrum is divided into narrow frequency *slices*. The optical path (*lightpath*) is determined by its routing path and a *channel*, which consists of a flexibly assigned subset of slices. ITU-T has recently revised the G.694.1 recommendation and included the definition of a flexible DWDM grid [4]. Concurrently, IETF is working on flexgrid extensions to signalling protocols [6][7][8][9]. Both ITU-T and IETF flexgrid definitions are very alike and they are backward compatible with the fixed DWDM grid (Fig. 1-A).

According to [4], an optical channel has flexibly (ad-hoc) assigned spectrum, which covers both the frequency range occupied by the optical signal and the guard band required for the roll-off filters. A channel is characterized by:

- its nominal *central frequency* (CF); the CF is the center of the channel and the spectrum is occupied symmetrically around the CF (see Fig. 1-B); in [4], the CF granularity, i.e., the spacing between neighboring CFs, is $\Delta_f = 6.25$GHz;
- the *channel size*, which is expressed in terms of a number ($n \in \mathbb{Z}^+$) of basic channel *segments*, where the segment width Δ_c is twice the CF spacing (i.e., $\Delta_c = 12.5$GHz); the spectrum occupied by a channel is equal to $n\Delta_c$.

In the flexgrid, each slice occupies the space between two nominal CFs (see Fig. 1-B), accordingly, its width is equal to Δ_f. In this paper we focus on a network which implements such a discrete frequency grid.

Two components are essential for FG-ON architectures: (i) bandwidth-variable transponders (BV-T) and (ii) bandwidth-variable wavelength cross-connects (BV-WXC). The role of BV-T it to adapt the client data signal to be sent to/received from the optical network using a flexibly allocated channel with a certain CF and just enough spectrum [10][11]. Concurrently, BV-WXC allow to create an optical routing path through the network by switching transmitted signals within their frequency bandwidth to appropriate switch output ports.

In a network, each optical connection has allocated a channel, which size is a function of the requested bit-rate, the modulation technique applied, the (fixed) slice width, and the guard band introduced to separate two spectrum adjacent connections, among others. Once the requested frequency resources are allocated on each link belonging to the routing path, the optical connection can be used to convey single-carrier (k-PSK, k-QAM) or multicarrier (O-OFDM) modulated signals [12]. Due to space limitations, we refer to [5],[13], and [14] for more details on flexgrid optical network and BV-WXC architectures and for reports on proof-of-concept experiments.

The spectrum allocation (SA) in FG-ON differs with the DWDM channel assignment in that the channel width is not rigidly defined but it can be tailored to the actual width of the transmitted signal. Due to this difference, Routing and Wavelength Assignment (RWA) algorithms are not appropriate for FG-ON. In FG-ON, the problem of finding unoccupied spectrum resources so that to establish a lightpath is called the Routing and Spectrum Allocation (RSA) problem. RSA

Manuscript received 13 December 2011; revised 12 September 2012. A part of this work was presented at the OFC 2012 Conference [1]. This work has been supported by the Polish National Science Centre under grant agreement DEC-2011/01/D/ST7/05884 and by the Spanish Ministry of Science and Innovation through the TEC2011-27310 ELASTIC project. Moreover, the research leading to these results has received funding from the European Community's Seventh Framework Programme FP7/2007-2013 under grant agreement n° 247674 STRONGEST project.

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Digital Object Identifier 10.1109/JSAC.2013.130104.

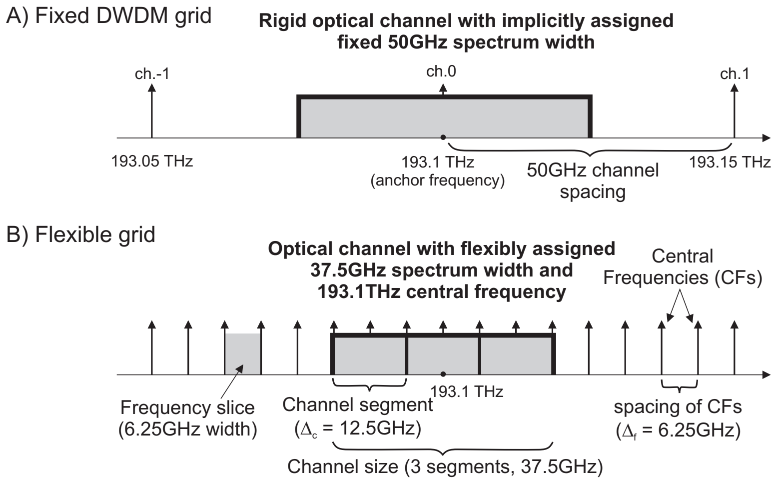

Fig. 1. A) Fixed DWDM grid (with 50GHz channel spacing) and B) flexible grid (with 6.25GHz spacing between central frequencies).

concerns assigning a contiguous fraction of frequency spectrum to a connection request - we refer to it as the *spectrum contiguity constraint* - subject to the constraint of no frequency overlapping in network links.

Offline RSA has been initially addressed with an heuristic approach in [12] and, afterwards, with different ILP formulations [15][16][17] and their extensions accounting for the selection of signal modulation levels [18][19]. The RSA optimization problem is $\mathcal{NP}$-hard [15][17][18] and it is more difficult than RWA due to the existence of the spectrum contiguity constraint. RSA algorithms make use of both meta-heuristic [18][20] and greedy [18][19] algorithms.

Regarding time-varying traffic demands, two classes of problems can be distinguished: (i) offline multi-hour (MH) or periodic design problems, and (ii) online connection establishment and resource allocation problems. In DWDM networks, both problems concern the provisioning of lightpath connectivity whenever significant traffic changes are observed. FG-ONs, in addition, bring new opportunities to optical networks, such as the adaptation of lightpaths through elastic SA in response to bandwidth variations, in particular, expansion/reduction of the spectrum when the required bit rate of a demand increases/decreases [21]. A service provider may request more bandwidth from the network in some time periods, for instance, to support the data backup service during night hours or to support video-on-demand services during evening hours. On the contrary, during the hours of low user activity, when the bandwidth requirements are lower, the unused resources are released and assigned to other network services. In this context, adaptive SA with a known a priori 24-hour traffic pattern and online adaptation of OFDM subcarriers have been addressed in [22] and [23], respectively.

Recently, the authors of [24] have shown that using the flexgrid technology aggregation can be done, in part, at the optical layer, thus extending the core network towards metro domains. As a consequence of that, and the general trend of higher bandwidth accessible to customers, flows incoming to the optical layer would show higher variability in the amount of data-rate along the day (i.e., in hourly scale) compared to the current situation. Elasticity in spectrum allocation could be used to cope with that variability thus improving network efficiency. Since elastic SA under time-varying demands puts additional requirements on network equipment and network control (as discussed in Section II), by these means increasing the network complexity, it is important to assess the performance benefits of such a feature implemented in the network. To the best of our knowledge, this important issue has not been addressed in the literature. Our main goal is to fill this gap.

The contributions of this paper are the following:

1) In Section II, we define a general spectrum allocation framework for time-varying traffic demands in FG-ON. Namely, we discern three SA schemes of different levels of elasticity: a fixed (non-reconfigurable) SA scheme and two elastic (reconfigurable) SA schemes. These schemes put some restrictions on the accessibility of spectrum resources within the flexgrid for bandwidth-variable connections. The restrictions are applicable for both offline (planning) and online (operation) RSA problems concerning lightpath adaptation.
2) In order to assess the performance gain of elastic SA, we study an offline multi-hour RSA (MH-RSA) problem in an FG-ON implementing the SA schemes. In such a scenario, the demands either correspond to deterministic volumes of aggregated traffic, as a result of the service level agreement between the network operator and network users, or they are predicted/estimated based on the historically observed traffic patterns. MH-RSA concerns the resolution of the RSA problem, jointly, for each time period in which meaningful bandwidth variations are observed. Apart from the common RSA constraints, such as spectrum continuity and spectrum contiguity (see Sec. III-A), additional constraints resulting from the degree of flexibility of each SA scheme arise in MH-RSA. In Section III, we formulate MH-RSA as an Integer Linear Programming (ILP) problem and, to this end, we make use of a novel channel assignment modeling approach, which is simpler and computationally more efficient in comparison to the RSA problem formulations presented in the literature (see comparative results in [25]).
3) Since ILP formulations of MH-RSA are difficult to be

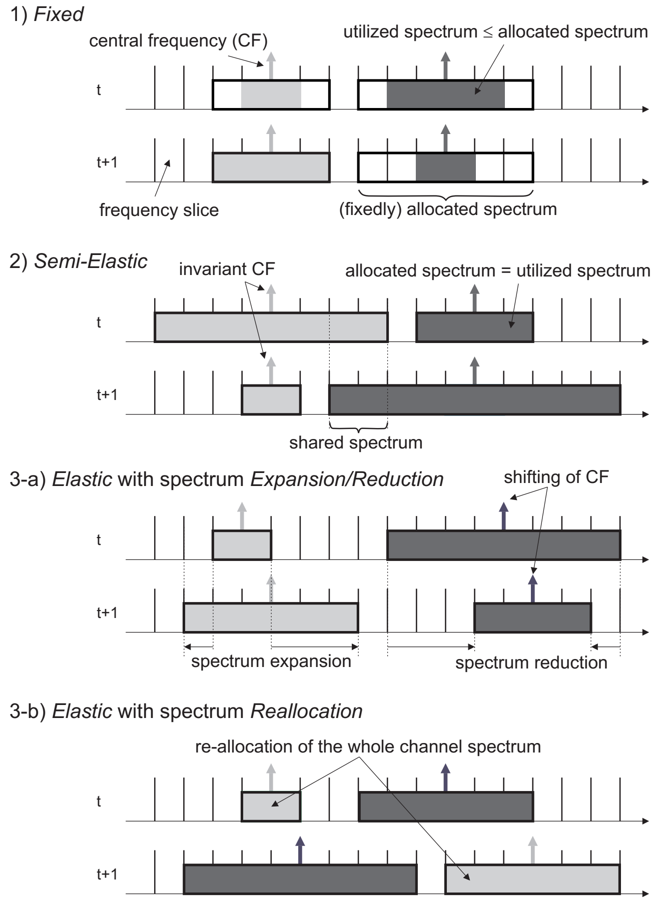

Fig. 2. SA schemes for time-dependent spectrum allocation in a flexgrid network (two time intervals are observed, t and $t+1$).

solved, as an alternative, in Section IV we propose heuristic algorithms based on the BRKGA meta-heuristic [26]. The algorithms are very efficient and they allow us to find near optimal solutions of MH-RSA for large network instances.

The discussion is supported by numerical results presented in Section V. The evaluation is performed for a large set of network and traffic scenarios. The main conclusion, presented in Section VI, is that the gain of elastic SA highly depends on the traffic profile. Accordingly, such a mechanism should be implemented only when appropriate since it involves additional hardware and control plane complexity.

II. Spectrum Allocation for Variable Traffic

One of the main advantages of FG-ON is the capability to allocate spectrum resources elastically, according to traffic demands. Indeed, the resources may be used efficiently, firstly, because of the higher granularity of the flexgrid which allows to fit closely the allocated spectrum and the signal bandwidth, secondly, due to the elastic (adaptive) allocation of spectrum in response to traffic variations.

Considering that the elastic optical network operates within a flexgrid, we discern three alternative schemes for the spectrum allocation with time-varying demands. We illustrate these schemes in Fig. 2 and, in particular, we show the changes in the spectrum allocation in two consecutive time intervals. These schemes put the following restrictions on the assigned CF and spectrum width:

1) *Fixed* (see Fig. 2-1): both the assigned CF and spectrum width do not change in time. At each time period, demands may utilize either whole or only a fraction of the allocated spectrum to convey the bit-rate requested for that period.
2) *Semi-Elastic* (see Fig. 2-2): the assigned CF is fixed but the allocated spectrum may vary. In this scheme, in each time interval, the allocated spectrum corresponds to the utilized spectrum. Here, spectrum increments/decrements are achieved by allocating/releasing frequency slices at each end of the already allocated spectrum while keeping invariant the CF. The frequency slices can be shared between neighboring demands, but used by, at most, one demand in a time interval.
3) *Elastic*: both the assigned CF and the spectrum width can be subject to change in each time interval. Furthermore, we discern two cases which differ in the grade of flexibility of the CF movements:
   a) spectrum *Expansion/Reduction* (Fig. 2.3-a): the intersection of two consecutive in-time spectrum allocations is equal to the lesser of them; here, the CF movement is short and limited within the range of the larger of the allocations;
   b) spectrum *Reallocation* (Fig. 2.3-b): the less restrictive case where any part of the spectrum can be allocated to a connection and the CF movements are not limited;

The complexity of elastic network operation increases with the increasing SA flexibility. In the following we discuss some hardware and control plane requirements.

The *Fixed* SA approach allocates for each demand a fixed number of slices which remains invariant from one period to the other. As a result, the only requirements of this approach concern control plane extensions so that to allocate a fixed channel consisting of the required number of slices - such extensions have been already proposed for GMPLS [7], RSVP-TE [8], and OSPF [9]. However, the *Fixed* scheme is spectrally the less efficient one since the utilized spectrum may be narrower than the allocated spectrum in certain time intervals.

Both the *Semi-Elastic* and *Elastic* schemes will require additional mechanisms, apart from the above discussed extensions, so that to allow modifying the existing lightpaths.

The *Semi-Elastic* approach provides elasticity in the amount of slices assigned to each demand. Therefore, relatively simple extensions to the RSVP-TE protocol, which is used for Label Switched Path (LSP) signaling in the GMPLS control plane, need to be defined to allow incrementing/decrementing the number of slices allocated to an LSP. The slice allocation decisions have to be supported by the OSPF protocol which should provide information about the occupancy of slices in network links. In addition, BV-T and optical filters must allow to increse/decrease the utilized spectrum on-demand. In particular, the spectrum tunability of both BV-T and BV-WSS should be in accordance with the flexgrid granularity (e.g., with 6.25GHz frequency step). Nonetheless, even with such extended functionality, the *Semi-Elastic* SA approach does not allow for the adaptation of CF and, therefore, its performance is expected to be moderate.

The *Elastic* approach adds a new degree of freedom allowing for the adaptation of the nominal CF, and hence, its performance is expected to be the highest one. However, control plane protocols need to be extended to allow dynamically modifying both the allocated spectrum and the CF. Moreover, since several LSPs might request to adapt their allocated resources in the same period, the Network Management System (NMS) or state-full Path Computation Element (PCE) need to implement appropriate algorithms in order to prevent conflicts due to the simultaneous access to spectrum resources. Although such algorithms will be more complex than as for *Semi-Elastic* SA, due to the higher flexibility of the asymmetric spectrum adaptation, still, they should offer better network performance. Regarding hardware functionalities, the tunability of lasers in BV-T and filters in BV-WSS must be implemented (e.g., as in [10]). The *Expansion/Reduction* approach has lower requirements for the range of laser tunability than in *Reallocation* due to the limited CF movements, however, at the cost of slightly limited flexibility. The performance gap of these approaches is studied in Sec. V-D.

Regarding the dynamicity of changes in spectrum adaptation we consider that it will be performed in an hourly scale and not in a second/subsecond scale. An important issue that will need dedicated mechanisms are possible traffic disruptions which may occur when adapting lightpaths in both *Semi-Elastic* and *Elastic* schemes. *Semi-Elastic* SA might be less prone to this effect since the CF is fixed in this scheme and only the channel width is subject to changes. For instance, when using a multi-carrier modulation format, such as O-OFDM, the adaptation of signal bit-rate might be performed by means of activating/desactivating exterior subcarriers without breaking the connectivity of the interior ones. A similar mechanism can be considered in *Elastic* SA with spectrum *Expansion/Reduction*. Here, the subcarriers that occupy the invariant part of allocated spectrum will be saved from disruptions. Although promising experiments on combining spectral slices were presented in [27] and [28], still proper solutions will have to be investigated in the view of dynamic spectrum adaptation.

The reduction of traffic disruptions is more complicated in *Elastic* SA with spectrum *Reallocation*, especially, if a number of lightpath reconfigurations with changes of CFs has to be performed at once. A possible, yet challenging, solution is to employ a centralized NMS/PCE element to look for a sequence of lightpath reconfigurations which might be performed one by one while minimizing the need to release the spectral resources of other working connections. Also, such a mechanism should employ a make-before-break strategy for lightpath adaptation so that to avoid interruptions, however, it will result in additional delays.

III. OFFLINE MH-RSA

In this Section, we focus on ILP modeling of SA schemes in an FG-ON network with periodic (multi-hour) traffic demands. The goal is to meet optimization objectives jointly for a set of varying traffic demands that are predictable and periodic along some time period (such as the day or week) [29]. Afterwards, in Section V, we use the models to analyse the impact of spectrum adaptation on network performance.

A. Problem statement

The offline Multi-Hour Routing and Spectrum Allocation (MH-RSA) problem can be formally stated as:

Given:

1) an FG-ON represented by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where the set of nodes is denoted as $\mathcal{V}$, and the set of bidirectional fiber links connecting two nodes in $\mathcal{V}$ is denoted as $\mathcal{E}$;
2) a frequency spectrum, the same for each link in $\mathcal{E}$ (without loss of generality), with the flexgrid represented by ordered sets of frequency slices $\mathcal{S} = \{s_1, s_2, \ldots, s_{|\mathcal{S}|}\}$ and central frequencies $\mathcal{F} = \{f_1, f_2, \ldots, f_{|\mathcal{F}|}\}$;
3) an ordered set of time intervals $\mathcal{T}$; $\mathcal{T} = \{t_1, t_2, \ldots, t_{|\mathcal{T}|}\}$;
4) a set $\mathcal{D}$ of demands to be transported; each demand d is represented by a tuple $\left(s_d, t_d, \{h_d^t : t \in \mathcal{T}\}, h_d^{min}\right)$, where s_d and t_d are source and destination nodes respectively, h_d^t is the requested bit-rate in time interval t, and h_d^{min} is the minimum bit-rate to be satisfied in every time interval if demand d is accepted to the network; without loss of generality, we consider $h_d^{min} \leq h_d^t, \forall d \in \mathcal{D}, t \in \mathcal{T}$. Let $h_d^{max} = \max\{h_d^t : t \in \mathcal{T}\}$.

Find both the spectrum allocation (i.e., a subset of slices), for each time period, and the route over the FG-ON for every transported demand subject to the following *constraints*:

1) *spectrum contiguity*: for each demand, the allocated slices should be selected each next to the other;
2) *spectrum continuity*: for each demand, the allocated channel (i.e., the CF with the allocated slices) should be the same for each link on the selected routing path;
3) *slice capacity*: each slice in each network link can be allocated to one demand at most;
4) *time-varying spectrum allocation*: the allocated CF and channel width meet the restrictions defined in Sec. II;

and with *objective* to minimize the number of rejected demands (primary objective) and the amount of un-served bit-rate (secondary objective).

In this paper, we make use of a basic model of the requested bit-rate $h_d^{(\cdot)}$ to channel size $n_d^{(\cdot)}$ (i.e., the number of segments) transformation, according to the formula:

$$n_d^{(\cdot)} = \left\lceil (h_d^{(\cdot)}/I + G)/\Delta_c \right\rceil, \quad (1)$$

where I is the spectral efficiency of modulation format and G is the frequency guard band; here, superscript $(\cdot)$ can be substituted with t, min, or max; without loss of generality, we neglect G. Note that the model can be extended to account for different modulation formats (as in [18] and [19]). However, for ease of presentation and since our main focus is on the efficiency of various lightpath adaptation schemes, such extension is left out of the scope of the paper. Eventually, we assume that the bit-rate that exceeds the transponder capacity is split into several demands in the demand set.

B. ILP formulations

Proposed ILP formulations of MH-RSA differ from the ILP formulations of RSA presented in the literature in that the

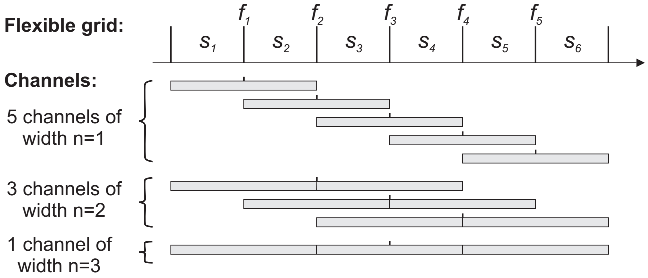

Fig. 3. A set of candidate channels in a flexgrid consisting of $|\mathcal{S}| = 6$ slices and $|\mathcal{F}| = 5$ central frequencies.

spectrum contiguity constraint is not involved explicitly into the problem constraints. Instead, a set of candidate optical channels is considered, where each channel consists of a subset of adjacent slices, and the RSA problem concerns finding a route and assigning a channel to a demand. Thanks to this novelty, our RSA formulation is simpler and more efficient (see [25]).

Channel c is defined as a contiguous (adjacent) subset of slices in ordered set $\mathcal{S}$, i.e., $c \subseteq \mathcal{S}$. Let $n_c \in \mathbb{Z}^+$ denote the number of segments forming channel c. Assuming the flexgrid definition in [4], each channel covers an even number of slices (see Fig. 3) and hence we have $|c| = 2n_c$.

Let $\mathcal{C}_d$ denote the set of admissible candidate channels for demand d, i.e., $\mathcal{C}_d = \{c : n_d^{\min} \leq n_c \leq n_d^{\max}\}$. In this paper, we consider that each $\mathcal{C}_d$ consists of all candidate channels that can be defined in $\mathcal{S}$; since spectrum contiguity is ensured then, by counting, $|\mathcal{C}_d| = (|\mathcal{S}| - n_d^{\max} - n_d^{\min} + 1)(n_d^{\max} - n_d^{\min} + 1)$ (we skip the proof due to space limitations). Let $\mathcal{C}$ denote the set of all channels, i.e., $\mathcal{C} = \bigcup_{d\in\mathcal{D}}\mathcal{C}_d$, and let $\mathcal{C}_f \subseteq \mathcal{C}$ denote the set of channels which have $f \in \mathcal{F}$ as the central frequency.

Routing path p is identified with a subset $p \subseteq \mathcal{E}$. Let $\mathcal{P}_d$ denote the (non-empty) set of predefined candidate paths for demand $d \in \mathcal{D}$. Each set $\mathcal{P}_d$ comprises paths that have the origin in s_d and the termination in t_d. Let $\mathcal{P}$ denote the set of all paths, i.e., $\mathcal{P} = \bigcup_{d\in\mathcal{D}}\mathcal{P}_d$.

We introduce a set of **problem variables**:
$x_d \in \{0, 1\}$ - equal to 1 if demand $d \in \mathcal{D}$ is rejected, and equal to 0 otherwise;
$x_p \in \{0, 1\}$ - equal to 1 if path $p \in \mathcal{P}$ is selected, and equal to 0 otherwise;
$x_{pf} \in \{0, 1\}$ - equal to 1 if central frequency $f \in \mathcal{F}$ is assigned to path $p \in \mathcal{P}$, and equal to 0 otherwise;
$x_{pc} \in \{0, 1\}$ - equal to 1 if channel $c \in \mathcal{C}$ on path $p \in \mathcal{P}$ is selected, and equal to 0 otherwise;
$x_{pc}^t \in \{0, 1\}$ - equal to 1 if channel $c \in \mathcal{C}$ on path $p \in \mathcal{P}$ is selected in interval $t \in \mathcal{T}$, and equal to 0 otherwise;
$v_d^t \in \mathbb{Z}_+$ - the un-served bit-rate of demand d in $t \in \mathcal{T}$;
$y_d^t \in \mathbb{Z}_+$ - the served bit-rate for demand d in interval t.

We formulate a set of common **problem constraints**:

1) Path selection: For demand d, whenever it is accepted (i.e., $x_d = 0$), a path is selected from set $\mathcal{P}_d$

$$\sum_{p\in\mathcal{P}_d} x_p + x_d = 1, \qquad \forall d \in \mathcal{D}. \tag{2}$$

2) Slice occupancy: in a time interval, a slice can be allocated to one demand at most

$$\sum_{d\in\mathcal{D}} \sum_{p\in\mathcal{P}_d:p\ni e} \sum_{c\in\mathcal{C}_d:c\ni s} x_{pc}^t \leq 1, \quad \forall t \in \mathcal{T}, e \in \mathcal{E}, s \in \mathcal{S}. \tag{3}$$

3) Served bit-rate: the amount of served bit-rate does not exceed the offered bit-rate and the channel capacity

$$\sum_{p\in\mathcal{P}_d} \sum_{c\in\mathcal{C}_d} \beta_{dtc} x_{pc}^t = y_d^t, \qquad \forall d \in \mathcal{D}, t \in \mathcal{T}. \tag{4}$$

where β_{dtc} coefficient represents the realized bit-rate of demand d at time interval t if channel c is selected, and calculated as:

$$\beta_{dtc} = \begin{cases} h(n_c), n_c < n_d^t, \\ h_d^t, n_c \geq n_d^t. \end{cases}$$

and where $h(n_c)$ is the channel capacity (it can be calculated using the inverse of formula (1)).

4) Un-served bit-rate calculation: The un-served bit-rate is the difference between offered and served bit-rate.

$$h_d^t - y_d^t = v_d^t, \qquad \forall d \in \mathcal{D}, t \in \mathcal{T}. \tag{5}$$

The **objective function** is defined as following:

$$\Phi = A \sum_{d\in\mathcal{D}} h_d^{\min} x_d + \frac{1}{|\mathcal{T}|} \sum_{d\in\mathcal{D}} \sum_{t\in\mathcal{T}} v_d^t. \tag{6}$$

where A is a weight that gives a priority to the primary objective; in the evaluation, we consider $A = 10$. In the primary objective, we count the number of rejected demands, each one weighted by the amount of guaranteed bit-rate. The secondary objective represents the un-served bit-rate.

Eventually, we have the following ILP formulations for SA schemes, each one comprising both the common constraints and SA specific constraints:

Fixed SA

minimize Φ (ILP1)

subject to

$$\sum_{c\in\mathcal{C}_d} x_{pc} = x_p, \qquad \forall d \in \mathcal{D}, p \in \mathcal{P}_d, \tag{7a}$$

$$x_{pc}^t = x_{pc}, \qquad \forall d \in \mathcal{D}, p \in \mathcal{P}_d, c \in \mathcal{C}_d, t \in \mathcal{T}, \tag{7b}$$

and constraints (2), (3), (4), and (5).

Constraints (7a) select a channel on the active routing path and constraints (7b) guarantee that this channel is used in each time interval.

Semi-Elastic SA

minimize Φ (ILP2)

subject to

$$\sum_{f\in\mathcal{F}} x_{pf} = x_p, \qquad \forall p \in \mathcal{P}, \tag{8a}$$

$$\sum_{c\in\mathcal{C}_f\cap\mathcal{C}_d} x_{pc}^t = x_{pf}, \qquad \forall d \in \mathcal{D}, p \in \mathcal{P}, f \in \mathcal{F}, t \in \mathcal{T}, \tag{8b}$$

and constraints (2), (3), (4), and (5).

Constraints (8a) select a central frequency on the active routing path and constraints (8b) select a channel of this CF for each time interval.

Elastic SA with Expansion/Reduction

minimize Φ (ILP3)

subject to

$$\sum_{c\in\mathcal{C}_d} x_{pc}^t = x_p, \qquad \forall d\in\mathcal{D}, p\in\mathcal{P}_d, t\in\mathcal{T}, \tag{9a}$$

$$\sum_{c\in\mathcal{C}_d:c\ni s} x_{pc}^t - \sum_{c\in\mathcal{C}_d:c\ni s} x_{pc}^{t+1} \leq \frac{n_d^t}{n_d^{t+1}+\Delta}, \quad \forall d\in\mathcal{D}, p\in\mathcal{P}_d, t\in\mathcal{T}, s\in\mathcal{S}, \tag{9b}$$

$$\sum_{c\in\mathcal{C}_d:c\ni s} x_{pc}^{t+1} - \sum_{c\in\mathcal{C}_d:c\ni s} x_{pc}^{t} \leq \frac{n_d^{t+1}}{n_d^{t}+\Delta}, \quad \forall d\in\mathcal{D}, p\in\mathcal{P}_d, t\in\mathcal{T}, s\in\mathcal{S}, \tag{9c}$$

and constraints (2), (3), (4), and (5).

where $t+1$ denotes next interval after t in $\mathcal{T}$, the next interval after $t_{|\mathcal{T}|}$ is t_1, and where $\Delta < 1$ is a number introduced so that to force the right hand side of (9b) and (9c) to be either lesser or greater than 1 (depending on n_d^t and n_d^{t+1}).

Constraints (9a) select a channel on the active routing path for each time interval. Constraints (9b) and (9c) are, respectively, expansion and reduction constraints. Whenever the allocated spectrum either is expanded or remains the same (i.e., $n_d^{t+1} \geq n_d^t$) then the left-hand side of (9b) must be equal to 0 (we remind that $n_d^t \in \mathbb{Z}^+$) and, accordingly, any slice that is allocated in interval t should be also allocated in interval $t+1$. A similar property, but for the spectrum reduction case, can be derived for Constraints (9c).

Elastic SA with Reallocation

minimize Φ (ILP4)

subject to constraints (2), (3), (4), (5) and (9a).

Numerical experiments we performed show that the above ILP problems can be solved for small networks (see Sec. V-B), and the computational complexity increases with the following order: ILP1 (the lowest), ILP4, ILP2 and ILP3 (the highest).

IV. BRKGA HEURISTIC ALGORITHMS

The complexity of the MH-RSA problem leads to the use of some meta-heuristic method to find near-optimal solutions for large problem instances. Among meta-heuristics, BRKGA, proposed in [26], has proven to effectively solve network problems such as IP routing [30], routing in single layer [31] and multilayer [32] optical networks. Essentially, BRKGA is a class of genetic algorithm where a set of individuals, called a population, evolves over a number of generations with the aim to produce high quality solutions in short running times. Each individual represents a solution of the problem to be solved (with its associated fitness value) which is encoded by a *chromosome* consisting of an array of *genes* of length m, where each gene takes a value in the real interval $[0,1]$. From a current generation, the individuals of the next generation are generated following three different mechanisms:

1) aiming at keeping track of good solutions, those individuals with the best fitness values (called the *elite set*) are copied without changes from the current generation to the next;
2) the majority of new individuals are generated by combining one elite with another non-elite individuals, both selected at random, in a process called *crossover* where the value of each gene of the elite individual is transmitted to the offspring with a certain inheritance probability;
3) some randomly generated individuals (the *mutant set*) are added to complete the population with the aim to escape from local optima.

Algorithm 1 Decoder Algorithm for SA schemes.

Require: Network $\mathcal{G}$, Demands $\mathcal{D}$, Periods $\mathcal{T}$, Chromosome $\mathcal{CH}$
Ensure: Fitness value
$sol \leftarrow \varnothing$
Initialize link metrics and demand's order with $\mathcal{CH}$
$sol[0] \leftarrow BasicRSA(\mathcal{G}, \mathcal{D})$
for all period $t \in \mathcal{T}$ **do**
  if SA scheme = Fixed SA **then**
    $sol\,[t] \leftarrow sol[0]$
  else if SA scheme = Semi-Elastic SA **then**
    $sol[t] \leftarrow AdaptiveSA$ ($\mathcal{D}$, t, $sol[0]$, 'keep CF invariant')
  else if SA scheme = Elastic SA **then**
    $sol[t] \leftarrow AdaptiveSA$ ($\mathcal{D}$, t, $sol[0]$, 'allow CF movements with either spectrum expansion/reduction or re-allocation')
return fitness value of sol

Finally, a deterministic algorithm, named *decoder*, transforms any input chromosome into a feasible solution of the optimization problem and computes its fitness value. As stated in [26], the only problem-dependent parts to specify a BRKGA heuristic are the decoder and the chromosome internal structure.

Algorithm 1 specifies the decoder pseudo-code. Essentially, the decoder is divided into three phases:

i) a *BasicRSA* procedure (Procedure 1) provides one static solution for the problem;
ii) the static solution is used to compute time-period dependent solutions. In the decoder for the *Fixed* SA scheme, the static solution is replicated for each period, whereas in the decoders for the *Semi-Elastic* and *Elastic* SA schemes an *AdaptiveSA* procedure (Procedure 2) modifies the static solution to match better the requirements for each time period;
iii) the fitness value of the solution is computed following the objective function defined in Eq. (6).

For each scheme, the decoder uses the chromosome in the BasicRSA procedure to generate a solution. Note that, since the MH-RSA problem basically consists in routing demands over a topology, the chromosome must contain information to encode that routing. With the purpose to add variability in routing, we dedicate one gen per link for the link weight. The link weight is calculated as the link distance times the value of the gene and it is then used in the shortest path computation in a routing algorithm. Moreover, since the order in which demands are processed determines the quality of the solution, we include one additional gene per demand to represent the order of demands. As a result, chromosomes consist in arrays of $m = |\mathcal{E}| + |\mathcal{D}|$ genes. It is important to remark that this encoding structure based on genes in links and in demands adds enough diversity to obtain a vast set of feasible solutions and, therefore, it increments the possibility to reach the optimal solution.

The *BasicRSA* Procedure consists of two steps: in the first step (lines $2-8$), an RSA algorithm tries to serve as much demands as possible ensuring h_d^{min}; in the second step (lines $9-29$), the spectrum allocated for each demand is increased to

Procedure 1 $BasicRSA$

Require: Network $\mathcal{G}$, Demands $\mathcal{D}$
Ensure: Solution sol
1: $\mathcal{D}_{aux} \leftarrow \mathcal{D}$, $sol \leftarrow \varnothing$
2: **for all** $d \in \mathcal{D}_{aux}$ **do**
3: $d.retries \leftarrow 3$
4: Find the best RSA$(\mathcal{G}, d)$ for n_d^{min} using eq. (11) for SA
5: **if** route found **then**
6: $sol.d.n_d \leftarrow n_d^{min}$, $sol.d.y_d \leftarrow h_d^{min}$, $sol.d.RSA \leftarrow$ route + SA
7: **else**
8: $\mathcal{D}_{aux} \leftarrow \mathcal{D}_{aux} - \{d\}$
9: $k \leftarrow 1, i \leftarrow 1$
10: **while** $i \geq 0$ **do**
11: **while** $\mathcal{D}_{aux} \neq \varnothing$ **do**
12: **if** $k = 1$ **then**
13: Sort $\mathcal{D}_{aux}$ by descending order of $h_d^{max} - y_d$
14: $d \leftarrow k$-th element of $\mathcal{D}_{aux}, n \leftarrow sol.d.n_d + 1$
15: Re-allocate spectrum to fit n
16: **if** assignment not found **then**
17: Re-route d to find the best RSA$(\mathcal{G}, d)$ for n using eq. (12) for SA
18: **if** n can be allocated **then**
19: $sol.d.n_d \leftarrow n$, $sol.d.y_d \leftarrow min(h(n), h_d^{max})$
20: $sol.d.RSA \leftarrow$ route + SA
21: **if** $n = n_d^{max} - i$ **then**
22: $\mathcal{D}_{aux} \leftarrow \mathcal{D}_{aux} - \{d\}, k \leftarrow 1$
23: **else**
24: $d.retries - -$
25: **if** $d.retries = 0$ **then**
26: $\mathcal{D}_{aux} \leftarrow \mathcal{D}_{aux} - \{d\}, k \leftarrow 1$
27: **else**
28: $k + +$
29: $i - -$
30: **return** sol

fit the maximum amount of bit-rate (limited by h_d^{max}), where adjustments of the route and SA are allowed. This process is executed by increasing the assigned spectrum of each demand in steps of one channel segment till reaching, at most, $n_d^{max} - 1$. Then, the last segment of each demand (which capacity might not be completely used) is established in the last round.

The RSA algorithms contained in the *BasicRSA* procedure (lines 4 and 17) consist in a modified Dijkstra algorithm, which looks for the shortest path route based on the link weights and considering spectrum continuity and contiguity constraints, followed by a SA phase. Namely, once the route has been selected, two costs functions to choose among alternative channels can be applied, namely:

$$c^* = \underset{c \in \mathcal{C}_{p,\mathcal{G}}}{\arg\max} \left\{ \min_{e \in p} \max \left[F^e(c), ES^e_{right}(c) \right] \right\} \quad (11)$$

and

$$c^* = \underset{c \in \mathcal{C}_{p,\mathcal{G}}}{\arg\max} \left\{ \min_{e \in p} \min \left[ES^e_{left}(c), ES^e_{right}(c) \right] \right\}. \quad (12)$$

Before entering into details of the cost functions, some notation needs to be introduced. Let p be a routing path and $\mathcal{C}_{p,\mathcal{G}}$ the subset of unused channels on path p over network $\mathcal{G}$. Given channel $c \in \mathcal{C}_{p,\mathcal{G}}$, let $ES^e_{left}(c)$ and $ES^e_{right}(c)$ be the amount of unused slices at the left or right side, respectively, from channel c in link e. Moreover, let $F^e(c)$ be a value equal to $|\mathcal{S}|$ if the first or the last slice of channel c is the first or the last slice of the spectrum; otherwise, it takes the same value than $ES^e_{left}(c)$. Then, the cost function in Eq. (11) allows choosing the channel c^* so that the amount of contiguous unused slices adjacent to that channel is maximized, thus placing spectrum allocations as closer as possible. On the opposite, the cost function in Eq. (12) separates spectrum allocations as much as possible balancing the free spectrum at both sides of each channel. While the former is very useful to establish as many demands as possible during the initial routing phase, the latter is used during the reallocation phase with the aim to separate allocations in the spectrum and, thus, minimizing clashes between neighboring allocations when the assigned spectrum varies from one period to the next.

Procedure 2 $AdaptiveSA$

Require: Network $\mathcal{G}$, Demands $\mathcal{D}$, Solution $iniSol$, Period t, CF constraints CFC
Ensure: Solution sol
$\mathcal{D}_{aux} \leftarrow \mathcal{D}$, $sol \leftarrow iniSol$
for all $d \in \mathcal{D}_{aux}$ **do**
if $sol.d.y_d = 0$ **then**
$\mathcal{D}_{aux} \leftarrow \mathcal{D}_{aux} - \{d\}$
else
$dif_d \leftarrow \mathcal{D}.d.h_d^t - sol.d.y_d, d.retries \leftarrow 3$
Sort $\mathcal{D}_{aux}$ by ascending order of dif_d
for all $d \in \mathcal{D}_{aux}$ **do**
if $dif_d \leq 0$ **then**
Reduce SA to the centered n_d^t segments
$sol.d.n_d \leftarrow D.d.n_d^t$, $sol.d.y_d \leftarrow D.d.h_d^t, \mathcal{D}_{aux} \leftarrow \mathcal{D}_{aux} - \{d\}$
Update SA in $sol.d.RSA$
else
break for
$k \leftarrow 1$, $i \leftarrow 1$, $\mathcal{D}'_{aux} \leftarrow \mathcal{D}_{aux}$
while $i \geq 0$ **do**
$\mathcal{D}_{aux} \leftarrow \mathcal{D}'_{aux}$
while $\mathcal{D}_{aux} \neq \varnothing$ **do**
if $k = 1$ **then**
Sort $\mathcal{D}_{aux}$ by descending order of dif_d
$d \leftarrow k$-th element of $\mathcal{D}_{aux}, n \leftarrow sol.d.n_d + 1$
Re-allocate spectrum to fit n subject to CFC
if n can be allocated **then**
$sol.d.n_d \leftarrow n$, $sol.d.y_d \leftarrow min(h(n), h_d^{max})$
Update SA in $sol.d.RSA$
Re-compute dif_d
if $n = n_d^{max} - i$ **then**
$\mathcal{D}_{aux} \leftarrow \mathcal{D}_{aux} - \{d\}, k \leftarrow 1$
else
$d.retries - -$
if $d.retries = 0$ **then**
$\mathcal{D}_{aux} \leftarrow \mathcal{D}_{aux} - \{d\}$, $\mathcal{D}'_{aux} \leftarrow \mathcal{D}'_{aux} - \{d\}$, $k \leftarrow 1$
else
$k + +$
$i - -$
return sol

While the *Fixed* SA scheme does not allow any change in time, and thus the solution obtained in the *BasicRSA* phase is simply duplicated for each time period, both the *Semi-Elastic* and *Elastic* SA schemes use the *AdaptiveSA* procedure, which takes into account the constraints regarding changes in central frequencies. Under the *Semi-Elastic* SA scheme, the assigned spectrum is reduced or expanded keeping the CF invariant along time, whereas under the *Elastic* SA scheme the movements of the CF with either spectrum *Expansion*/*Reduction* or *Reallocation* are allowed.

TABLE I
BRKGA PARAMETER VALUES.

chromosome length (m)	$\lvert\mathcal{E}\rvert + \lvert\mathcal{D}\rvert$
population size (p)	$\min(75, m)$
elite population size	$0.2 \cdot p$
mutant population size	$0.2 \cdot p$
elite inheritance probability	0.7

For each time period, two steps can be distinguished in the *AdaptiveSA* Procedure 2: in the first step (lines $7-14$) all demands requesting lower bit-rate than the one allocated during the *BasicRSA* procedure are reduced to the value requested for the given period h_d^t; in the second step (lines $15-35$) those demands requesting higher bit-rate are expanded to match h_d^t provided that enough unused capacity is available.

Since the complexity of a BRKGA heuristic is defined by the complexity of its decoder, let us provide the complexity of each SA scheme taking into account the decoder defined in Algorithm 1. The *Fixed* SA scheme uses only the *BasicRSA* procedure, which involves an adaptation of the Dijkstra algorithm in the RSA calculation. Thus, the complexity of *Fixed* SA can be expressed as $O(\eta\,|\mathcal{D}|\,|\mathcal{V}|\,log(|\mathcal{V}|))$, where η represents the maximum difference between n_d^{min} and n_d^{max} among all demands. Additionally to this complexity, *Semi-Elastic* and *Elastic* schemes include the complexity of the *AdaptiveSA* procedure, which is slightly different depending on the SA flexibility. Particularly, both *Semi-Elastic* and *Elastic* with spectrum *Expansion/Reduction* only check for free space at neighboring spectrum, whereas the *Elastic* with spectrum *Reallocation* scheme explores the whole spectrum in the hope of finding a new SA. As a consequence of this, the complexity of the former schemes, denoted as $O(\eta\,|\mathcal{D}|\,(|\mathcal{V}|\,log(|\mathcal{V}|)+|\mathcal{T}|))$, is lower than the one of the latter scheme, represented as $O(\eta\,|\mathcal{D}|\,(|\mathcal{V}|\,log(|\mathcal{V}|)+|\mathcal{T}|\,|\mathcal{C}|))$.

After implementing the afore-defined decoders, we performed several tests to tune the parameters used in the BRKGA main algorithm. The values are presented in Table I and they are considered for the ongoing performance analysis. As commonly accepted in BRKGA algorithms, the size of the population is assumed to be proportional to the size of the chromosome. This size, however, could rise to impractical values resulting in poor convergence [32]. For this reason, we limit the size of the population (see Table I) so as to bring convergence within acceptable convergence times.

V. NUMERICAL RESULTS

In this Section, we first introduce the network and traffic scenarios considered in our MH-RSA problem. Next, we validate the quality of the proposed heuristic algorithms against exact solutions obtained by solving the ILP formulations. Then, we use the heuristics to carry out exhaustive experiments so that to compare the effectiveness of the SA schemes. In the evaluation, *Elastic* SA implements the spectrum *Extension/Reduction* approach which is less complex than the *Reallocation* approach (see Sec. II). In order to assess the performance trade-off, finally, we compare both Elastic SA approaches.

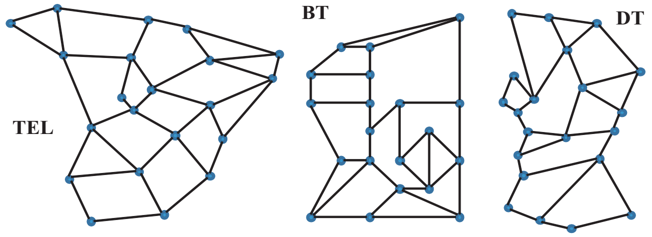

Fig. 4. Network topologies: the 21-node Spanish Telefónica (TEL), the 20-node British Telecom (BT), and 21-node Deutsche Telekom (DT).

TABLE II
CHARACTERIZATION OF TRAFFIC PROFILES.

TP	Share of services (q) Normal Business	Trans. Business	Plain Internet	Variation Coefficient (VC) Temporal	Spatial
TP1	1	0	0	0.271	0.193
TP2	0	1	0	1.361	0.852
TP3	0	0	1	0.707	0.197
TP4	0.3	0.2	0.5	0.707	0.550
TP5	0.4	0.2	0.4	0.663	0.547
TP6	0.5	0.2	0.3	0.620	0.537

A. Network scenario

With the aim to study MH-RSA in realistic network scenarios, we consider the three optical network topologies shown in Fig. 4: the Spanish Telefónica (TEL), the British Telecom (BT), and the Deutsche Telekom (DT). We consider the optical spectrum $OS = 800$GHz, the flexgrids of three different channel segment widths $\Delta_c \in (50, 25, 12.5)$ GHz, and spectral efficiency $I = 2$bit/s/Hz. If not mentioned differently, the number of considered intervals $|\mathcal{T}| = 12$.

In a multi-period planning problem, the bit rate of the demand is subject to change from one time period to the next. In this paper, we consider a set of traffic services (TS) consisting in three different services. Each one is defined by its associated profile, which is characterized by an average bit rate for each time period ($\hat{h}^t$), and a range (expressed in % w.r.t. $\hat{h}^t$) within which the bit-rate of each demand can vary randomly. In particular, we have:

a) the *normal business* service, which bit rate increases till reaching a maximum at working hours and decreases during night hours; the magnitude of change is as $1:3$;
b) the *transactional business* service with a flat bit rate but one peak period, different for each demand, during night hours. The bit rate requested for the peak is one order of magnitude higher than during the rest of the periods;
c) the *plain Internet* service where the bit rate fluctuates following the same mean and variance for all periods.

In Fig. 5 we illustrate both the service profiles and the parameter values considered in the evaluation. Note that these bit-rates can be achieved with a single transponder [11].

For the ongoing analysis we consider 6 different traffic profiles (TP), each consisting in a mix of the above defined services, as detailed in Table II. Note that TP 1-3 consist of demands implementing just one service (*pure* TPs), whereas TP 4-6 combine services in different proportion (*mixed* TPs). For each TP, each evaluated load contains a number of demands

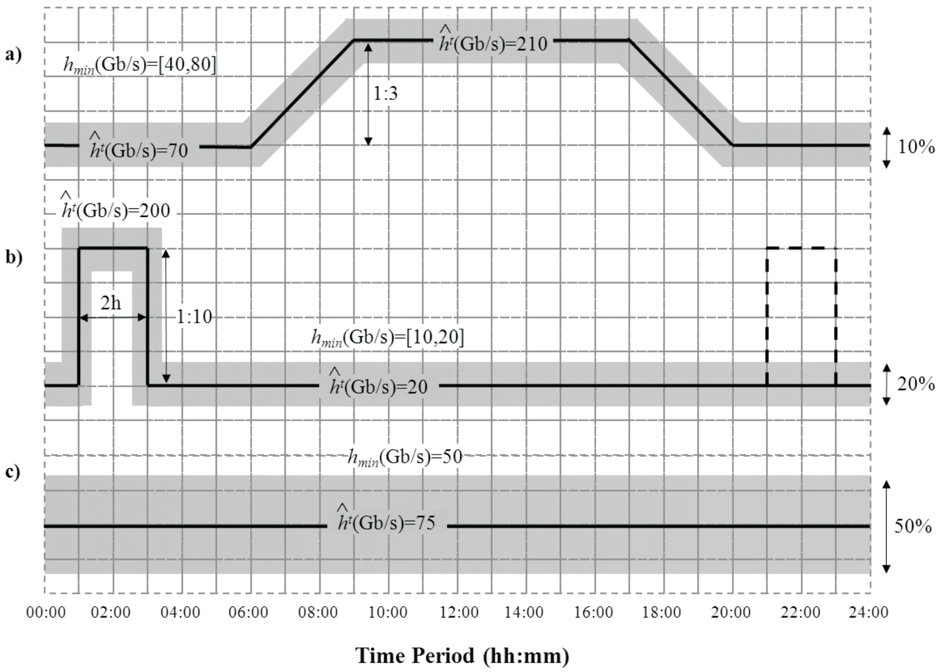

Fig. 5. Profiles of the considered services: Normal business (a), Transactional business (b), and Plain Internet (c).

belonging to each traffic service in proportion given by the share of service indicator (q). For each specific demand d, its source (s_d) and termination (t_d) nodes are chosen following an uniform distribution, whereas the specific values of h_d^t for each $t \in \mathcal{T}$ and h_d^{min} are selected at random from the profile values of the traffic service that demand belongs to. In the evaluated scenarios, there is enough capacity in the network to guarantee bit-rate h_d^{min} for each $d \in \mathcal{D}$ and, therefore, there are no rejected demands.

For characterization purposes, we compute the deviation of the traffic fluctuations. More specifically, we use two different measures of variation coefficient (the *temporal* VC and the *spatial* VC), widely used in statistics to compute the deviation (or variance) independently of the unit and the range in which the measurement has been taken [33]. Before detailing VCs' computation, two random variables need to be defined. Let X^t be the difference in terms of bit rate between a pair of demands in time period t. Additionally, let Y_i^t be the difference in terms of bit rate of a demand of service i between consecutive time periods t and $t+1$. The *temporal* VC and the *spatial* VC can then be computed as follows:

$$spatial\ VC = \frac{1}{|\mathcal{T}|}\sum_{t\in\mathcal{T}} \frac{\sqrt{\sigma^2\left(X^t\right)}}{\mu\left(X^t\right)}, \tag{13}$$

$$temporal\ VC = \frac{1}{|\mathcal{T}|-1}\sum_{t\in\mathcal{T}-\left\{t_{|\mathcal{T}|}\right\}}\sum_{i\in TS} q_i \frac{\sqrt{\sigma^2\left(Y_i^t\right)}}{\mu\left(Y_i^t\right)}, \tag{14}$$

where $\mu(\cdot)$ and $\sigma^2(\cdot)$ are the expected mean and variance, respectively. The values of $\mu(\cdot)$ and $\sigma^2(\cdot)$ can be obtained from the traffic profile values given in Fig. 5. As can be anticipated from the definition of X_t and Y_t^i random variables, the temporal VC is correlated with the bit rate increment (or decrement) that a *specific* demand can experience from one period to the next. In contrast, the spatial VC is correlated with the bit rate difference between two different demands in the same time period. High values of the spatial VC indicate the traffic with heterogeneous bit rate demands. Concurrently, high values of the temporal VC characterize demands with high bit rate fluctuations in consecutive time periods. In Table II we present the VC values for the defined TPs. We can see that TP2 represents the highest VC values as a consequence of its high traffic peaks, whereas the spatial VC is very low in both TP1 and TP3. Mixed TPs are characterized by moderate VC values.

B. Heuristics vs. ILP models

In this subsection, we validate the proposed heuristic algorithms comparing their performance against that of the ILP models. We define a test scenario consisting in a 6-node network topology with 8 bidirectional links [19], $|\mathcal{P}_d| = 2$, three different spectrum widths $OS \in \{100, 200, 300\}$ GHz, $\Delta_c = 12.5$GHz, and 50 different traffic instances with the number of demands $|\mathcal{D}| \in [3, 15]$ and the number of time periods $|\mathcal{T}| \in \{3, 6, 12\}$. The ILP models of SA schemes (*Elastic* SA implements spectrum *Reallocation*) are solved using the `CPLEX` v.12.4 optimizer [34] on a 2.4GHz Quad-core machine with 8GB RAM. BRKGA-based heuristics are implemented in `Java`.

Due to space limitations, we report that the heuristics find the optimal solution in more than 95% of the instances, with the worst gap between the heuristic and the optimal ILP solution below 3% when an optimal solution is not reached by the heuristic, and with average gap below 0.1%. Although the heuristic run-time is limited to 1 hour, best solutions are

TABLE III
RELATIVE GAIN IN OFFERED LOAD W.R.T. *Fixed* SA AT $UB = 1\%$.

	Semi-Elastic SA			Elastic SA		
TP	TEL	BT	DT	TEL	BT	DT
TP4	3.52%	8.32%	2.39%	28.31%	29.19%	16.90%
TP5	4.71%	4.67%	2.02%	30.79%	27.29%	16.70%
TP6	8.40%	15.14%	8.82%	27.30%	29.98%	16.59%

TABLE IV
RELATIVE OFFERED LOAD GAIN OF SPECTRUM REALLOCATION W.R.T. EXPANSION/REDUCTION.

	Network		
TP	TEL	BT	DT
TP4	1.45%	3.62%	4.90%
TP5	0.95%	1.29%	4.01%
TP6	0.14%	0.84%	0.54%

obtained in less than 50 sec. in average and in a few minutes for the largest problem instances.

The computation time of ILP models of Fixed SA and Elastic SA is below, respectively, 21 sec. and 502 sec. for the most demanding case. However, for Semi-Elastic SA with $OS = 200$, $|\mathcal{D}| \geq 12$, and $|\mathcal{T}| = 12$, the optimal solutions cannot be found after 10 hours of computation for 20% (4 out of 20) of problem instances. For these instances, the optimality gap is about 3.6% in average and these results are not taken into account when performing the comparison with heuristic results.

C. Performance evaluation of SA schemes

Here, we focus on the performance comparison of SA schemes in realistic networks with a large set of demands. In light of the MH-RSA complexity, in the remainder we use the heuristic algorithms to evaluate the effectiveness of SA schemes. The runtime is limited to 3 hours within which, as we have observed, the algorithms converge to stable solutions (i.e., a large number of consecutive generations without improving the best solution is observed before termination).

Firstly, we analyze the performance of the SA schemes for the considered TPs under different channel segment widths. Fig. 6 illustrates, for the TEL network topology, the accumulated percentage of un-served bit-rate (UB) over all time periods as a function of the network offered load (ρ), represented by the average load per time period, for each *pure* TP and segment width. Each point in the plots represents the average value of 10 independent runs for that load. For the sake of comparability, the demands are not blocked and h_d^{min} is ensured for all demands.

We can observe substantial differences in the performance under the *pure* TPs. For TP1 and TP3, the performance of *Fixed* SA and *Semi-Elastic* SA is very similar, whereas the *Elastic* scheme - we remaind that it implements *Expansion/Reduction* - outperforms them quite significantly. Notwithstanding, the gain in the offered load of *Elastic* SA vs. *Fixed* SA is lower than 10% at $UB = 1\%$ (which represents a realistic reference value in the absence of blocked demands). In contrast, given a UB value, the performance of the SA schemes under TP2 follows: $\rho_{Fixed} << \rho_{Semi-Elastic} << \rho_{Elastic}$, what means that the higher the flexibility of the scheme, the higher load can be supported. For instance, with slot width of 12.5GHz, the offered load at $UB = 1\%$ is 2.2, 2.7, and 3.4 Tb/s for *Fixed* SA, *Semi-Elastic* SA, and *Elastic* SA, respectively.

Recalling the VC values in Table II, a clear correlation between TP's VC and the performance of the SA schemes is thus observed; high VC entails high gains when elasticity is used. In the opposite, elastic spectrum allocation does not provide any improvement with respect to the fixed allocation when no variability exists (as for static traffic). The benefits of elasticity are as a consequence of the relation VC/segment width. In this regard, Fig. 6 confirms that the narrower the slot width the higher the benefits provided by elasticity. Indeed, for our TPs, the amount of channel segments requested by a demand at a certain time period is between 1 and 3 if the segment width is equal to 50GHz. Using 12.5GHz segments, the amount of requested segments ranges from 1 to 12. In such conditions, *Elastic* SA, and partially *Semi-Elastic* SA, reduces the amount of un-served bit-rate when comparing to *Fixed* SA.

Complementing these results, in Fig 7 we show the performance of the SA schemes for the *mixed* TPs with the 12.5GHz segment width. We can see that the performance is similar for all three TPs as it could be anticipated in view of VC values in Table II. Additionally, we can observe the same clear proportionality between VC and elasticity's gains.

Finally, in Table III we provide, for all *mixed* TPs and all topologies, the relative gain in terms of offered load at $UB = 1\%$ of the *Semi-Elastic* and *Elastic* SA schemes with respect to the *Fixed* one. Although the values differ slightly for the studied networks, the same conclusions as for the TEL network can be applied to the DT and BT networks: i) non-significant differences among *mixed* TPs, ii) low gain provided by the *Semi-Elastic* SA scheme (in the order of 6% on average), and iii) high gains when *Elastic* SA is used (an increment of 25% on average in terms of the offered load can be observed).

In light of these results, we can conclude that the *Elastic* SA scheme provides the highest gains in a bandwidth variable environment.

D. Comparison of the Elastic SA approaches

So far, we have studied the *Elastic* SA scheme with spectrum *Expansion/Reduction*. Here, we compare it with spectrum *Reallocation*. The *Reallocation* approach has the highest SA flexibility among the studied schemes since it allows moving the channel CF within the whole spectrum.

In Table IV we present the relative gain, in terms of the offered load at $UB = 1\%$, achieved with spectrum *Reallocation* vs. spectrum *Expansion/Reduction*. As we can see, this gain is rather small and does not exceed 5%.

Aiming at a deeper analysis of the *Expansion/Reduction* scheme, in Fig. 8 we depict, for the TEL network with the 12.5GHz segment width, both the probability and the cumulative probability distributions of the CF shifting value (in GHz), which is experienced by demands in time. Since we obtained similar probability values for each single TP, we depict the average probability distribution function for all TPs. Accounting for these results, we can conclude that when

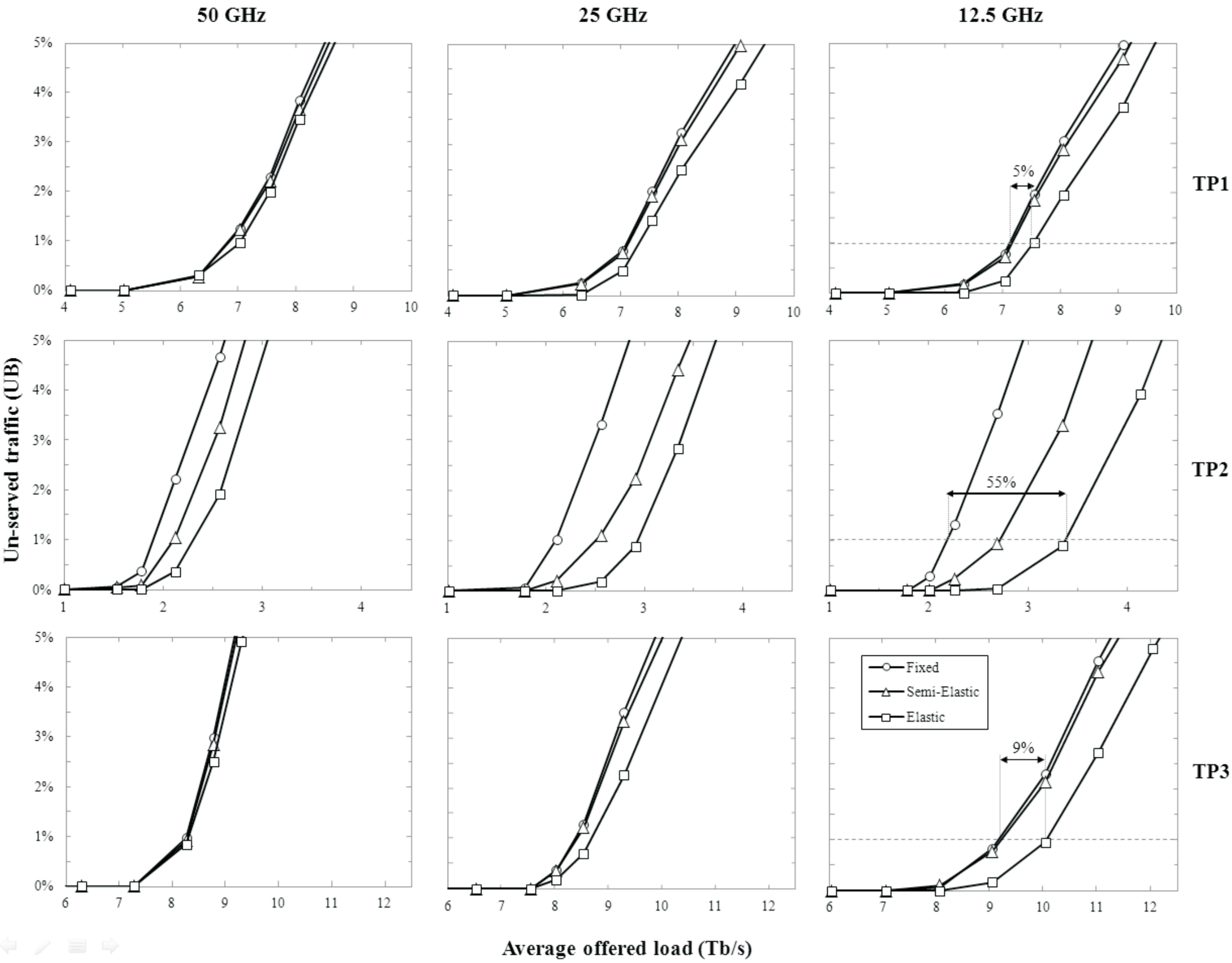

Fig. 6. Un-served bit-rate as a function of offered load and channel segment width for *pure* TPs in the TEL topology.

limiting the range of allowed CF movements to 12.5GHz, e.g., due to limited tunability of lasers in BV-Ts, the network performance should not suffer since more than 95% of the CF movements is in the range of 12.5GHz.

These results allow us to promote the *Elastic* SA with spectrum *Expansion*/*Reduction* scheme as a candidate solution to effectively exploit elasticity in flexgrid networks.

VI. Concluding Remarks

In this work we address the problem of spectrum allocation in time-varying traffic scenarios. Three SA schemes of different grades of flexibility are proposed to cope with traffic variations: i) the *Fixed* scheme which allocates for each demand a fixed fraction of spectrum; ii) the *Semi-elastic* scheme, which allocates the spectrum elastically while keeping invariant the CF; and iii) the *Elastic* scheme, which adds a new degree of freedom allowing CF movements in consecutive time intervals. Here, two SA options are considered: 1) asymmetrical *Expansion*/*Reduction* of the already occupied spectrum with proper CF movements within a short range, and 2) spectrum *Reallocation* within the whole spectrum range.

The multi-hour RSA problem is mathematically modeled by means of ILP formulations and, given its complexity, BRKGA-based heuristics are developed. For the sake of a comprehensive study, a set of realistic network topologies and traffic profiles are considered. Aiming at characterizing the variability of traffic profiles, both spatial and temporal variation coefficients are defined.

Our numerical experiments show that the *Elastic* scheme with *Expansion*/*Reduction* minimizes the amount of un-served bit-rate, since it provides gains in the range of $16.6\% - 30.8\%$ with respect to the *Fixed* scheme. It is much higher than the equivalent gain of the *Semi-elastic* scheme, which is on the level of $2\% - 15.1\%$. Apart from that, the relative gain the spectrum *Reallocation* with respect to the spectrum *Expansion*/*Reduction* in the *Elastic* scheme is in the range of $0.1\% - 4.9\%$. The *Expansion*/*Reduction* approach should involve lower hardware and control plane complexity (w.r.t. *Reallocation*) since both the spectrum adaptation decisions and the CF movements are limited to the neighborhood of the already allocated spectrum. Since the performance trade-off of this scheme is low, spectrum *Expansion*/*Reduction* can be considered as an attractive approach for elastic SA.

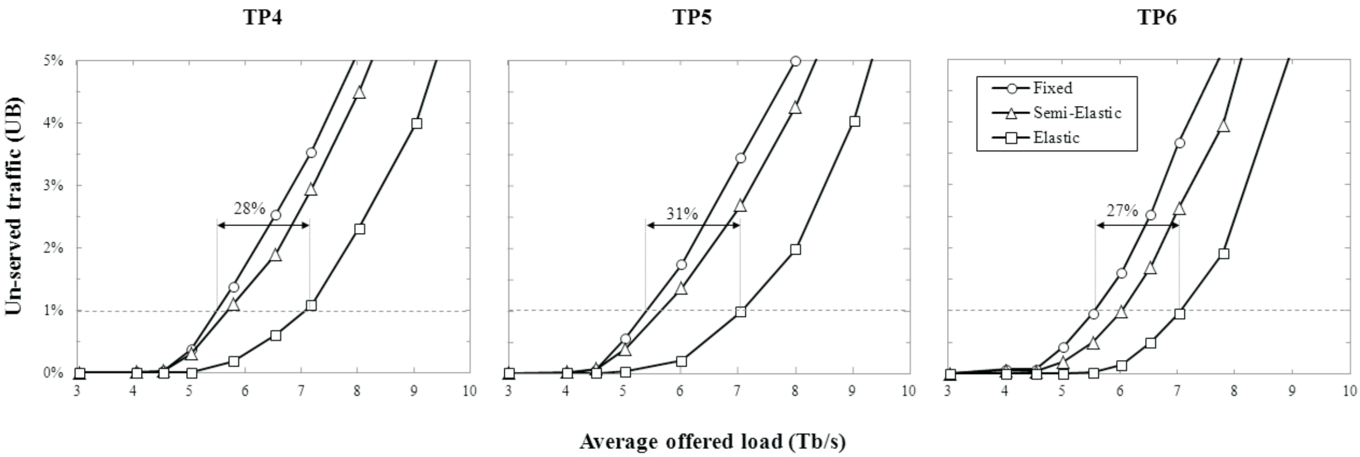

Fig. 7. Un-served bit-rate as a function of the offered load for *mixed* TPs in the TEL topology with 12.5GHz channel segment width.

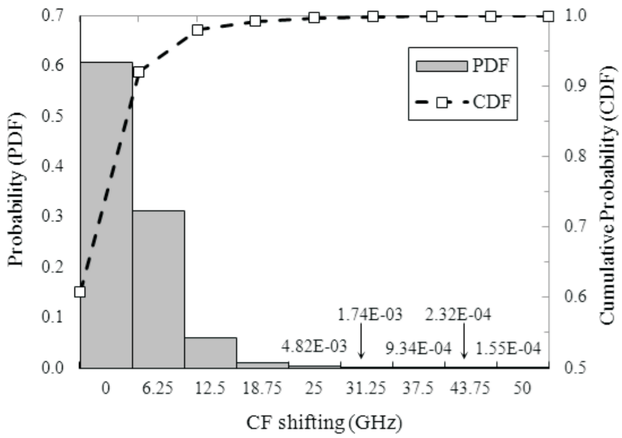

Fig. 8. Probability functions of CF movements in *Elastic* SA with *Expansion/Reduction* for the TEL topology.

TABLE V
VARIABILITY VS. OPTICAL TECHNOLOGY.

		Temporal Variability	
		Low	High
Spatial Variability	Low	Fixed grid (DWDM)	Flexgrid (no elasticity req.)
	High	Flexgrid (no elasticity req.)	Flexgrid with elasticity

The fixed grid DWDM has proved to be a cheap and efficient technology in networks in which the offered traffic does not fluctuate significantly and the transported signal bit rates are homogenous, i.e., when both *spatial* and *temporal* VCs are low. As a response to heterogeneous bandwidth requirements, there have been proposed the use of flexible grids, which are characterized by the higher spectral efficiency (vs. DWDM). Flexgrid networks can efficiently support multiple-rate demands of static bandwidth requirements, i.e., with high *spatial* VC, without need for elastic lightpath adaptation. In this paper, we have observed that, whenever the *spatial* and *temporal* variation coefficients of the offered traffic are both high (as in the TP 2 scenario) or moderate (in scenarios TP 4-6), the elastic (time-varying) spectrum allocation in flexgrid optical networks brings performance benefits. Eventually, in the case that only the *temporal* VC is high, it is enough to implement the flexgrid technology without elastic SA so that to cope with traffic variations. It is confirmed in the TP 3 scenario where the performance gain of *Elastic* SA is not prominent. In Table V we briefly summarize these conclusions.

The obtained results should foster research in suitable methods, including signaling protocols, to implement the *Elastic* SA scheme with *Expansion/Reduction*. Other important issue left for further study is the application of SA schemes in dynamic network scenarios.

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