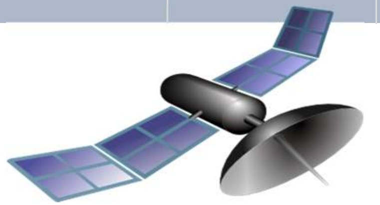


유도제어 실험 # 3

MOTOR & PENDULUM CONTROL

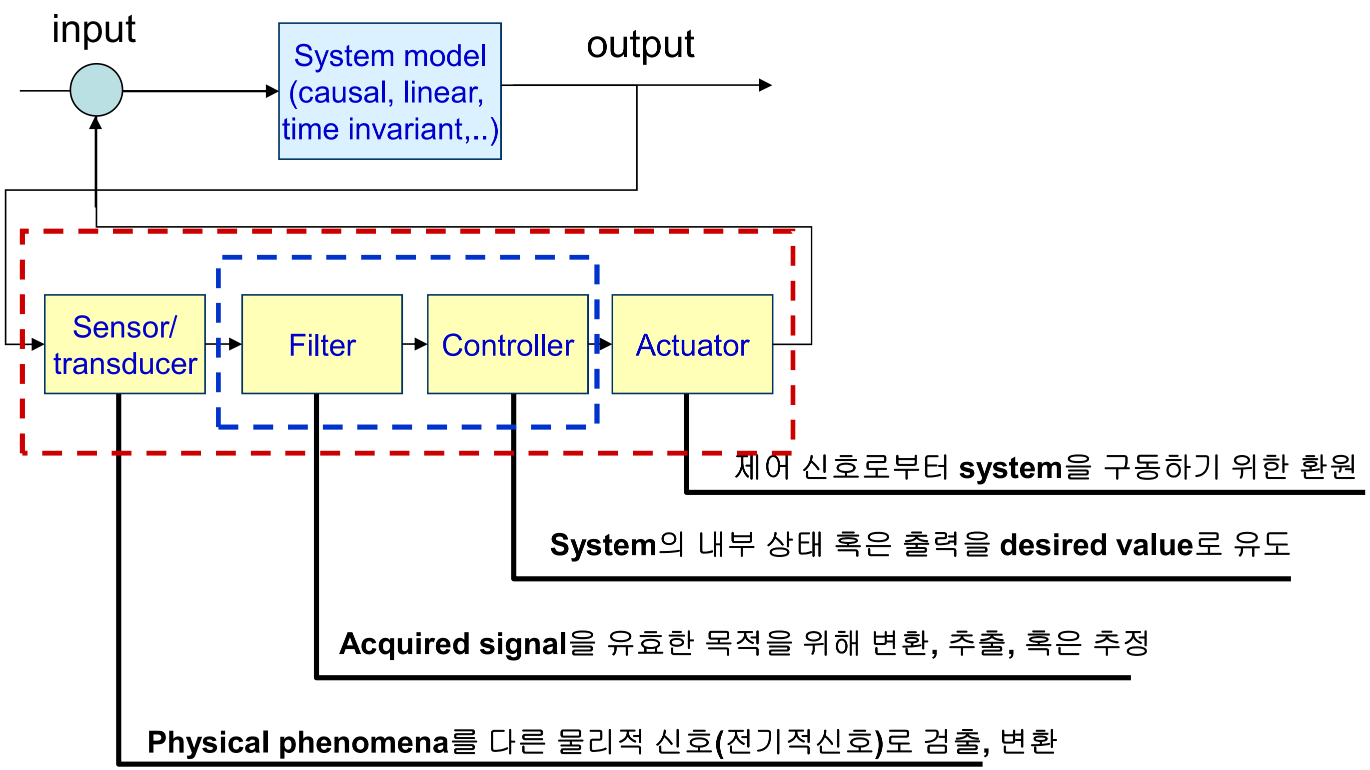
실험 소개

2025. 11. 18

KONKUK UNIVERSITY

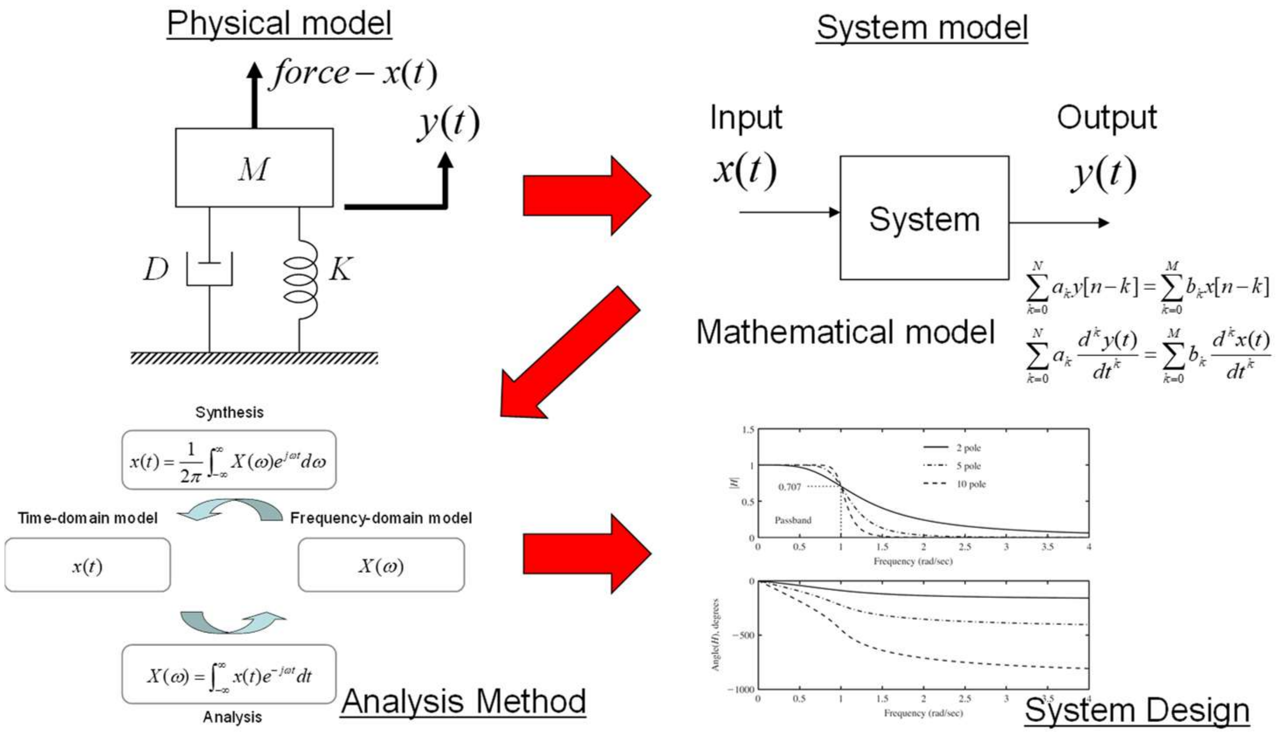
Control System Design

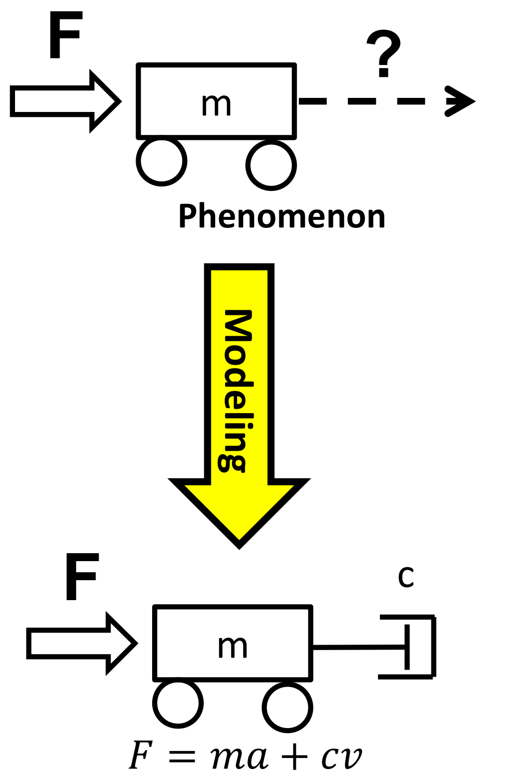
- Design Elements of Control System
  - (Dynamics) Modeling
  - Sensor & Instrumentation
  - Filter/Data Processing
  - Controller
  - Actuator
    - Motor, Engine…



1. Modeling

- Physical phenomenon(true model)에 대한 수학적 표현





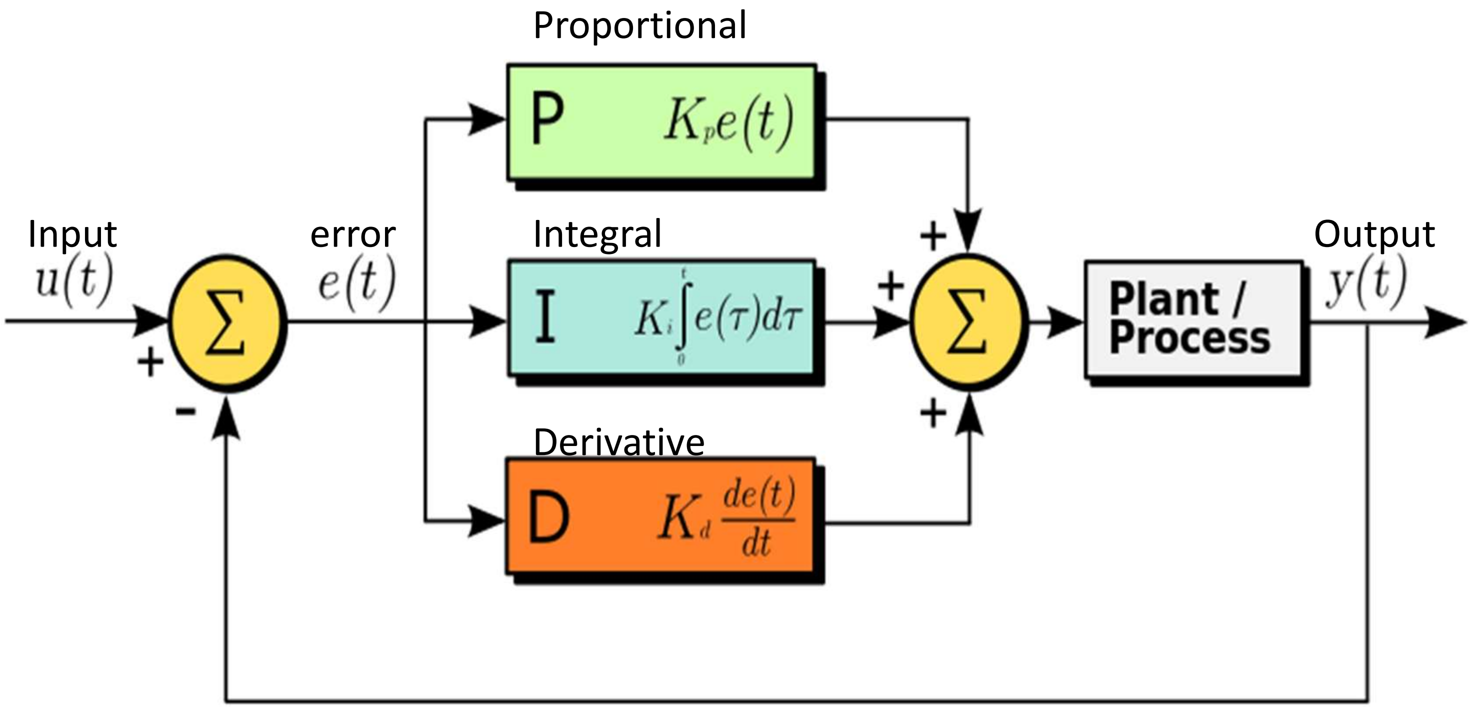
2. Transfer function

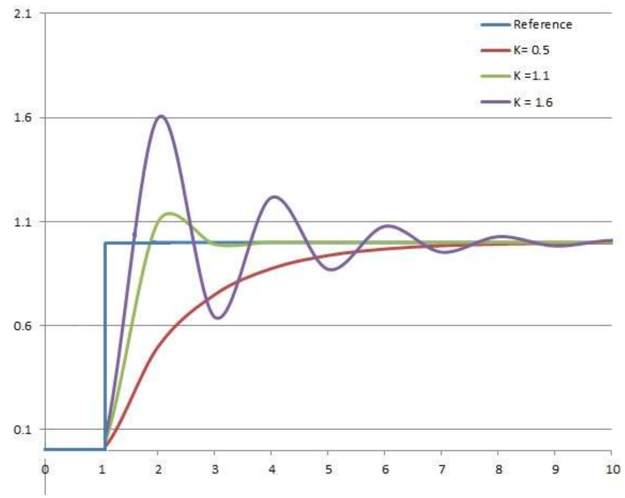
- Classical controller design(고전제어기법)
  - S-domain design
    - Laplace Transform
    - PID controller

$$G(s) = \frac{Y(s)}{U(s)} \quad \text{->} \quad \text{전달함수}(Transfer\ \ function) \quad \text{즉} \quad G(s) = \left.\frac{\mathcal{L}\{\text{출력}\}}{\mathcal{L}\{\text{입력}\}}\right|_{\text{초기조건} = 0}$$

3. Feedback and PID Control

- PID controller implementation example





Video

[HW #1 – 1]

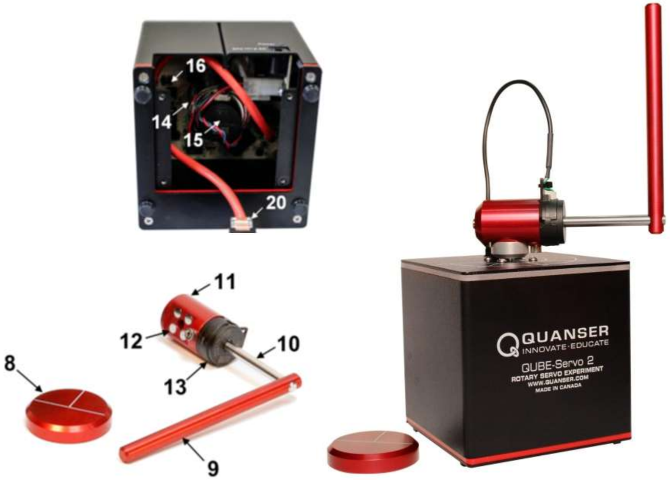
- DC motor: https://www.youtube.com/watch?v=LAtPHANEfQo

- cf. Pendulum: https://www.youtube.com/watch?v=vHd7vtadwdc

  https://www.youtube.com/watch?v=FFW52FuUODQ

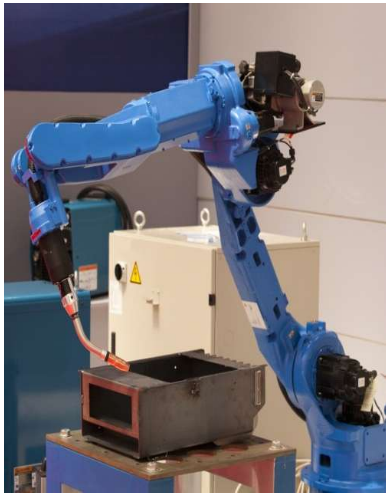
Rotary Inverted Pendulum

- 1축 모터 토크 제어를 이용한 2축 수직 운동성분 제어
- Pendulum(빨간 막대)이 위로 향하도록 하는 sustaining control



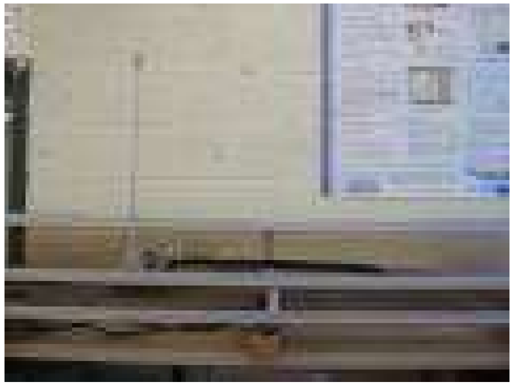
Inverted Pendulum

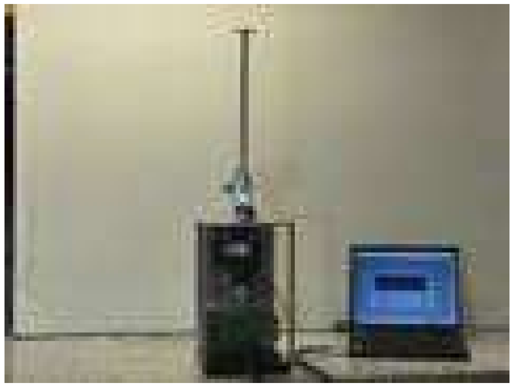
다양한
분야
응용 가능



Inverted Pendulum Control

- Inverted Pendulum- 도립진자
  - 단축 입력을 이용한 회전자 자세제어 시스템
  - https://www.youtube.com/watch?v=ZBlBbDTEF6c





DC Motor

DC Motor

- Voltage-to-angular rate transfer function

$$P_{vr}(s) = \frac{\Omega_m(s)}{V_m(s)} = \frac{K}{\tau s+1}$$

where $\Omega_m(s) = \mathcal{L}[\omega_m(t)],\ V_m(s) = \mathcal{L}[v_m(t)]$

[**K**(model's steady-state gain) = 21.9 rad/(V-s), $\boldsymbol{\tau}$(model's time constant) = 0.15 s]

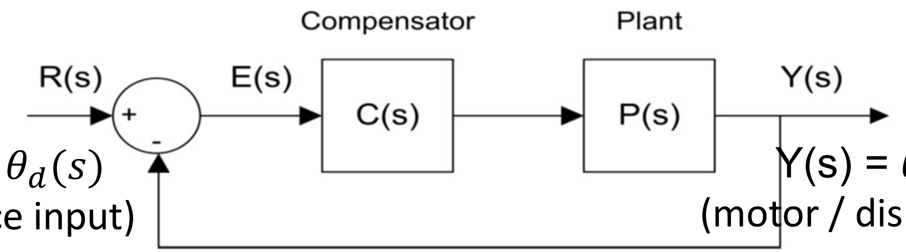
- voltage-to-angle transfer function

$$P_{va}(s) = \frac{\theta_m(s)}{V_m(s)} = \frac{K}{s(\tau s+1)}$$

where $\Theta_m(s) = \mathcal{L}[\theta_m(t)]$

DC Motor

- Unity feedback closed-loop transfer function



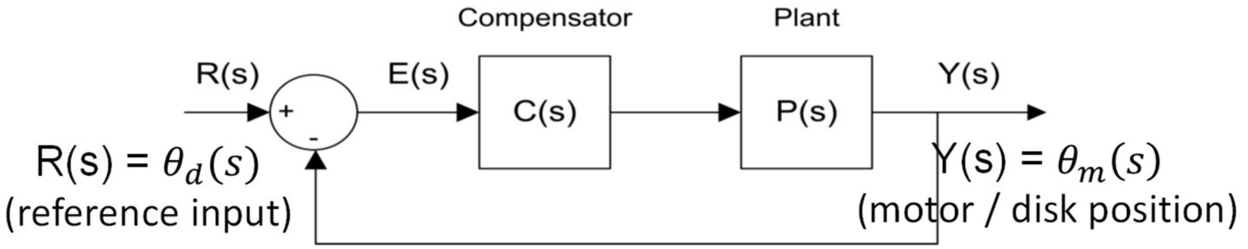
R(s) = $\theta_d(s)$
(reference input)

Y(s) = $\theta_m(s)$
(motor / disk position)

$$\therefore T_c(s) = \frac{Y(s)}{R(s)} = \frac{\theta_m(s)}{\theta_d(s)}$$

$$= \frac{P(s)}{1+P(s)} = \frac{K}{\tau s^2+s+K}$$

- PID controller equation
  - Mathematically: $u(t) = k_p e(t) + k_i \int_0^t e(\tau)d\tau + k_d \frac{de(t)}{dt}$
  - Transfer function: $C(s) = k_p + \frac{k_i}{s} + k_d s$

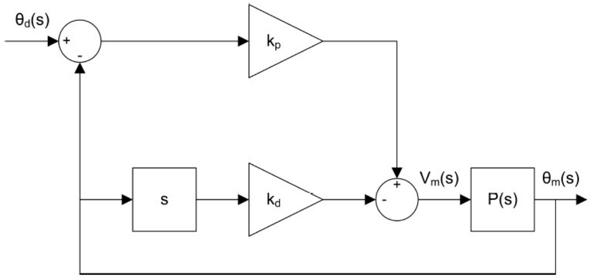


[HW #1]

$\therefore T_c(s) = \frac{Y(s)}{R(s)} = ?$

← P, PD 및 PI 제어기에 대해 전달함수 구하라.

ex) PD controller: $C(s) = k_p + k_d s$

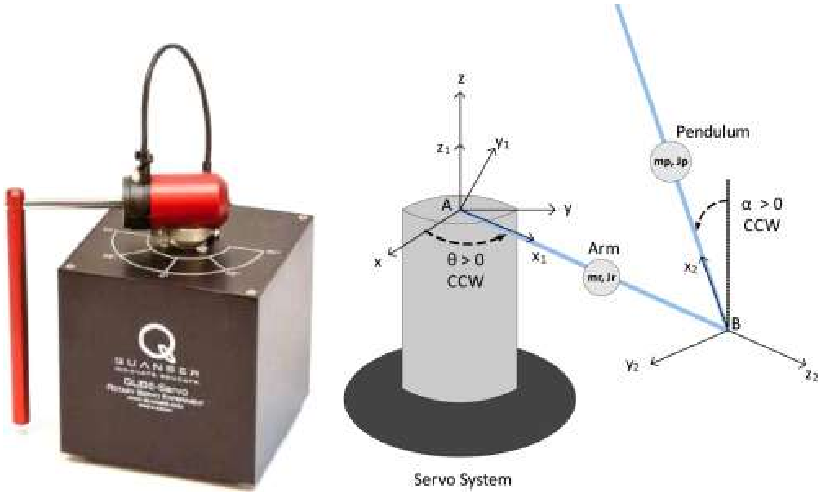


[HW #2]

Simulink 제어시스템을 완성한 후,
P, PI 및 PD 제어기에 대해 unit step response를 비교/분석하라.

Rotary Inverted Pendulum

Rotary Inverted Pendulum



***Euler-Lagrange method** 이용하여 eq. of motion(EOM) 얻은 후, nominal point linearize 시키면 다음과 같음

[HW # 3]

$$\left(m_p L_r^2 + J_r \right) \ddot{\theta} + \frac{1}{2} m_p L_p L_r \ddot{\alpha} = \tau - D_r \dot{\theta} \qquad \text{[eq. 1]}$$

$$\frac{1}{2} m_p L_p L_r \ddot{\theta} + \left(J_p + \frac{1}{4} m_p L_p^2 \right) \ddot{\alpha} + \frac{1}{2} m_p L_p g \alpha = -D_p \dot{\alpha} \qquad \text{[eq. 2]}$$

(Use $\tau = \frac{k_m (V_m - k_m \dot{\theta})}{R_m}$, $J_T = J_p m_p L_r^2 + J_r J_p + \frac{1}{4} J_r m_p L_p^2$)

- α: pendulum angle
- θ: arm angle
- τ: applied torque at the base of the rotary arm
- J_T: total moment of inertia
- $k_{p,\theta}/ k_{d,\theta}$: the arm angle proportional gain/ derivative gain
- $k_{p,\alpha}/ k_{d,\alpha}$: the pendulum angle proportional gain/ derivative gain

$$\dot{x} = Ax + Bu \qquad x = [\,\theta\ \dot{\theta}\ \alpha\ \dot{\alpha}\,]^T$$
$$y = Cx + Du \qquad y = [\,\theta\ \alpha\,]^T$$

- *A, B, C, D* : 상태공간 모델의 state-space matrices

[System Parameters]

Symbol	Description	Value
R_m	Terminal resistance	8.4 Ω
k_m	Motor back-emf constant	0.042 V/(rad/s)
k_t	Torque constant	0.042 N-m/A
V_m	Motor voltage	1 V
m_p	the total mass of the pendulum assembly	0.024 kg
L_r	the length of the arm	0.085 m
L_p	the length of the pendulum	0.129 m
J_r	the moment of inertia of the arm	5.72E-5 kg-m^2
J_p	the moment of inertia of the pendulum at the pivot axis	3.33E-5 kg-m^2
D_r	rotary arm viscous damping coefficient	0.0015 N - m - s/rad
D_p	pendulum damping coefficient	0.0005 N - m - s/rad

State-space model

[HW # 3]

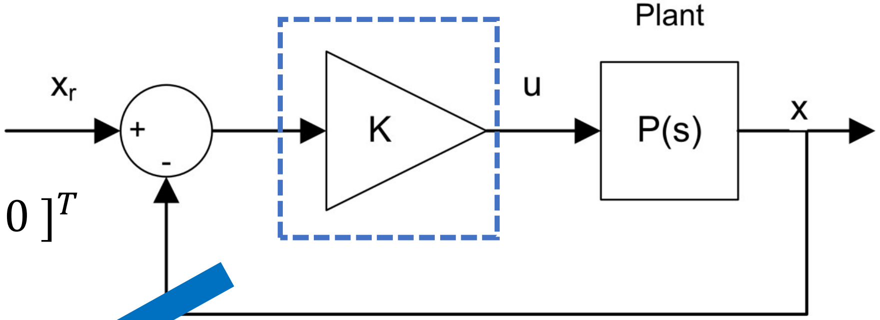
$$\boxed{\begin{aligned}\dot{x} &= \boldsymbol{A}x + \boldsymbol{B}u \\ y &= \boldsymbol{C}x + \boldsymbol{D}u\end{aligned}} \qquad \begin{aligned} x &= [\,\theta\ \dot{\theta}\ \alpha\ \dot{\alpha}\,]^T \\ y &= [\,\theta\ \alpha\,]^T \end{aligned}$$

[eq. 3]

[Block diagram of balance state-feedback control for rotary pendulum]

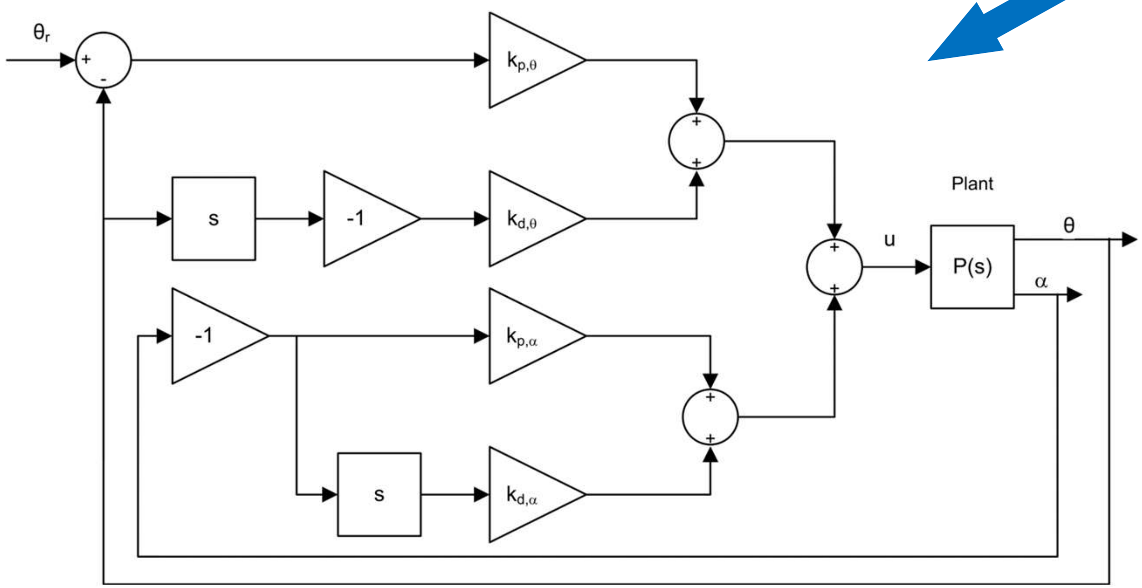
$$\boxed{u = K(x_r - x)}$$

K = [$k_{p,\theta}$ $k_{p,\alpha}$ $k_{d,\theta}$ $k_{d,\alpha}$], x = [θ $\dot{\theta}$ α $\dot{\alpha}$ $]^T$, x_r = [θ_r 0 0 0 $]^T$



[Block diagram of SS controller for balancing rotary pendulum]

[HW # 4]



$$\boxed{\begin{aligned} \mathbf{u} &= k_{p,\theta}(\theta_r - \theta) + k_{p,\alpha}(\alpha_r - \alpha) \\ &+ k_{d,\theta}\frac{d}{dt}(\theta_r - \theta) + k_{d,\alpha}\frac{d}{dt}(\alpha_r - \alpha) \end{aligned}}$$

Exp #3 예비리포트

[DC motor]

1. P, PI 및 PD controller를 설계했을 경우, 각각의 전달함수식($T_c(s)$)을 유도하라.
2. 시스템 출력을 안정화하는 P, PI 및 PD 제어기를 설계하라

   ① 시스템 출력을 안정화하기 위한 제어기 조건을 설명하라.

   ② [Simulation]Matlab Simulink 파일을 이용하여 P/PD/PI 제어기를 구성하라. 단위계단입력(step input)시, 제어 이득값에 따른 출력을 도시하고 P, PI 및 PD 제어기의 성능 차이 원인을 분석/토의해 보시오.

Exp #3 예비리포트

[Rotary Pendulum]

3. [eq. 1], [eq. 2] 를 각각 **2차항($\ddot{\theta}$, $\ddot{\alpha}$) 식으로 정리**한 후, 최종적으로 State-space model 을 **유도**하라.(eq. 3 결과와 동일한지 확인할 것)

4. [시뮬레이션] 주어진 Simulink 파일을 완성하여 모델과 PD 제어기를 구현하고 arm 입력 (θ)을 주기적 펄스로 인가했을 때, 출력 특성을 도시하라.

   - Arm 입력은 -25° -> 0° -> 25° -> 0°… 로 변화 (10초 간격으로 변화)

Summary

- 리포트: 3차실험 예비리포트
- 제공 파일:
  1. 유도제어실험 3차 설명자료
  2. 부분 완성 매트랩 Simulink 코드: DC Motor, Pendulum
  3. 각 조별 실험 데이터(모든 조 실험 끝난 후 배포)
- 제출일: 예비이론 수업후 1주 - 11/25(화) 5시까지