Control Systems Engineering

Prof. Sangkyung Sung

Term Project

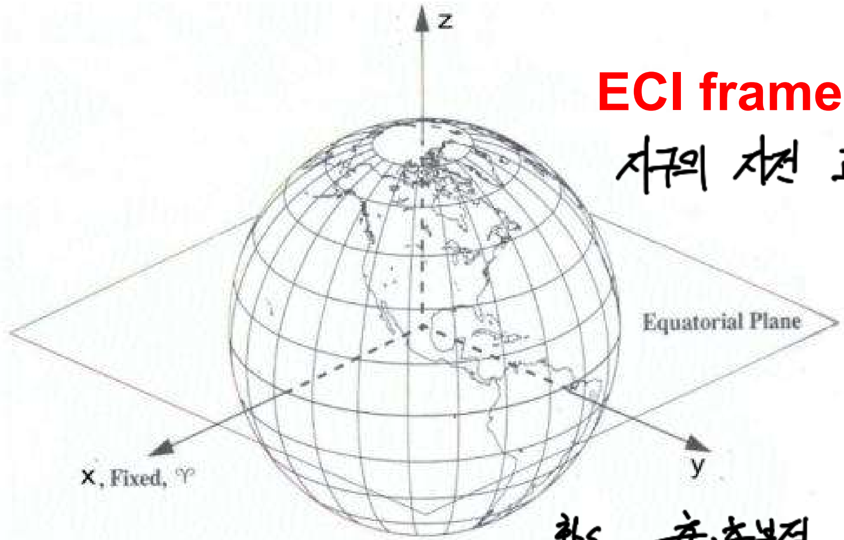
Fall Semsester

Coordinate Frames(좌표계)

■ Coordinate frame

ECI frame

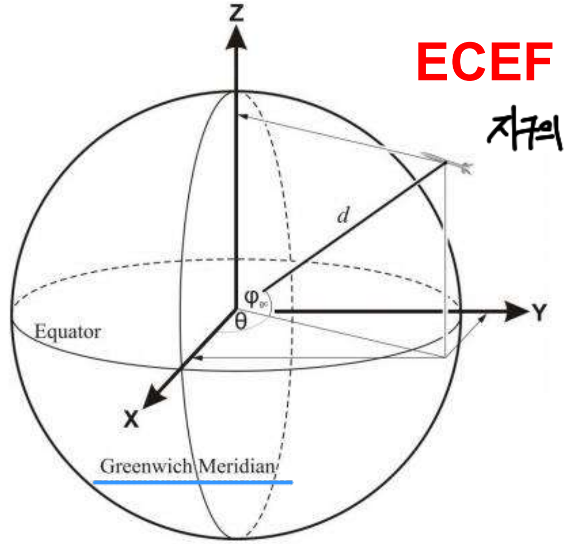
지구의 자전 고려 X.



황도. 춘·추분점.

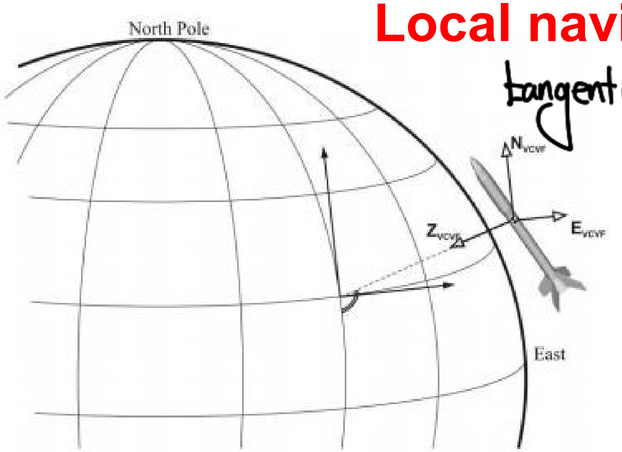
ECEF frame

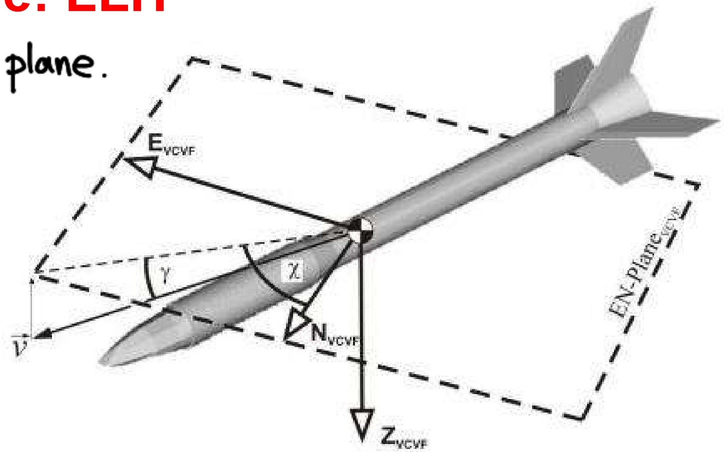
지구의 자전 고려.



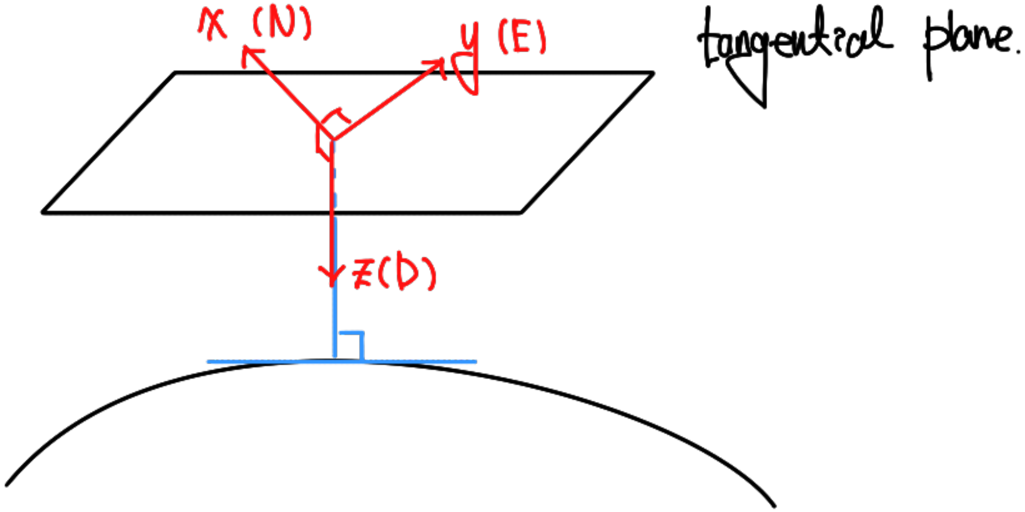
Local navigation frame: LLH

tangential plane = local plane.

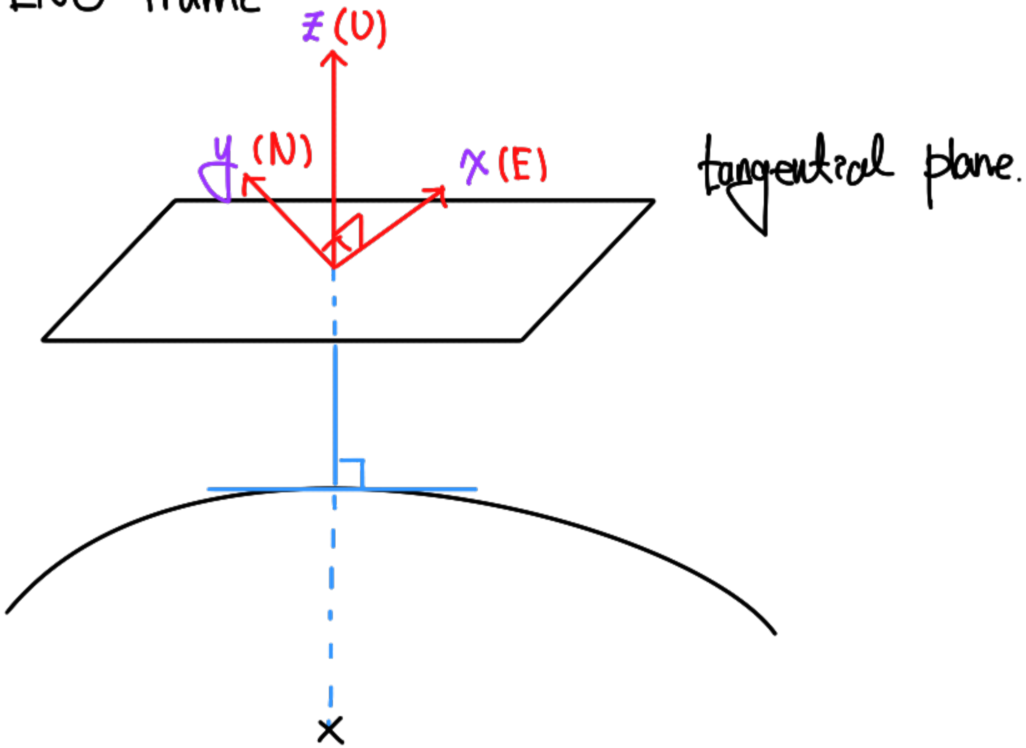




* NED Frame



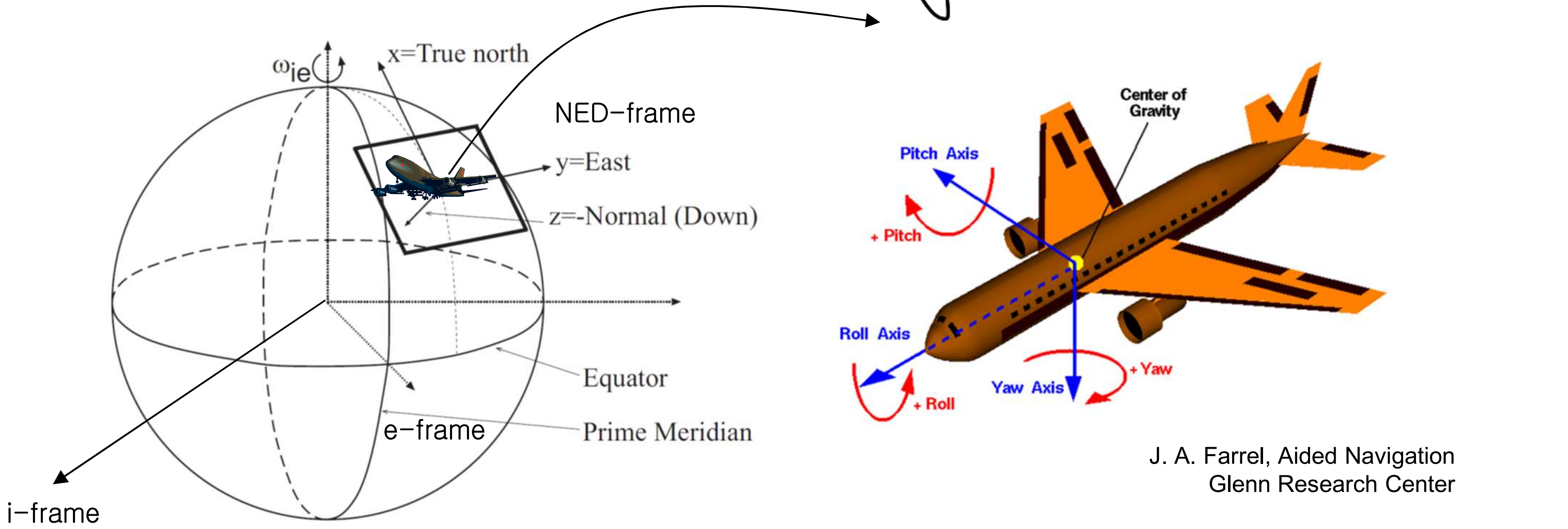
* ENU Frame



Coordinate Frames

- 4 major frames
  - ECI (Earth Centered Inertial) frame or Inertial frame (i-frame)
  - ECEF (Earth Centered Earth Fixed) frame (e-frame)
  - Local navigation frame, or navigation frame (NED frame)
  - Body frame (b-frame)

※ geocentric : 지구가 완전한 구 가정
geodetic.



J. A. Farrel, Aided Navigation
Glenn Research Center

body frame의 x,y,z 속도 : u,v,w.

■ Flight mechanics of fixed-wing aircraft

- ▶ Translational dynamics - force equation with thrust as given by nonlinear differential equation

$\omega_B = [p, q, r]^T$. $V_B = [u, v, w]^T$.

$$\frac{d\mathbf{v}_B}{dt} = \frac{\sum \mathbf{F}_B}{m} - \boldsymbol{\omega}_B \times \mathbf{v}_B$$

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \frac{1}{m}\begin{bmatrix} f_{B,x} \\ f_{B,y} \\ f_{B,z} \end{bmatrix} - \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

- ▶ Wind coordinate system ···바람 등으로 인해 실제 x랑 다를 수 있어....
  - ◆ Good to controller design

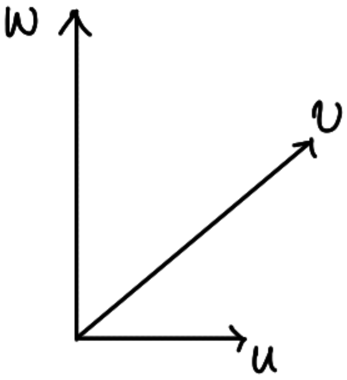
$$V = \sqrt{u^2 + v^2 + w^2}$$

$$\alpha = \tan^{-1}\left(\frac{w}{u}\right)$$

$$\beta = \tan^{-1}\left(\frac{v}{\sqrt{u^2 + w^2}}\right)$$

non-linear.

$$\begin{bmatrix} \dot{V} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} 0 \\ q - (p\cos\alpha + r\sin\alpha)\tan\beta \\ p\sin\alpha - r\cos\alpha \end{bmatrix} + \frac{1}{mV}\begin{bmatrix} V\cos\alpha\cos\beta & V\sin\beta & V\sin\alpha\cos\beta \\ -\sin\alpha/\cos\beta & 0 & \cos\alpha/\cos\beta \\ -\cos\alpha\sin\beta & \cos\beta & -\sin\alpha\sin\beta \end{bmatrix}\begin{bmatrix} f_{B,x} \\ f_{B,y} \\ f_{B,z} \end{bmatrix}$$



$$V = \sqrt{u^2 + v^2 + w^2}$$

$$\frac{dV}{dt} = \frac{\cancel{2}(u \cdot \dot{u} + v \cdot \dot{v} + w \cdot \dot{w})}{\cancel{2}V}$$

side slip angle??

angle of $\square$??

$f_{B.x,y,z}$: thrust, gravity 등 모두 포함.

$\omega_B = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$ body frame에서 각속도.

Flight mechanics of fixed-wing aircraft

- Rotational dynamics - moment equation with torque as given by nonlinear differential equation   I : xz plane에 대칭. ?

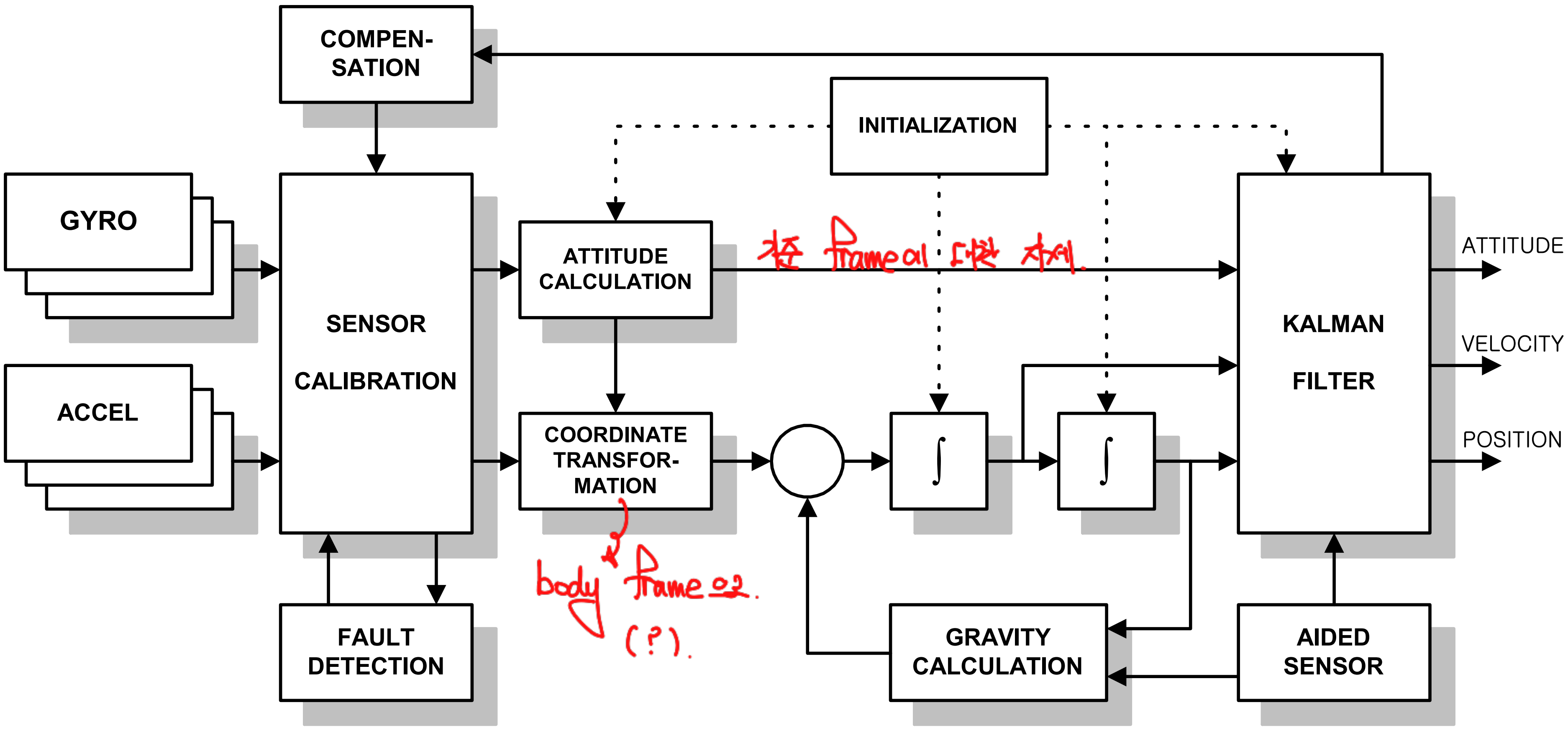
$$\sum \mathbf{M}_B = \left.\frac{d\mathbf{H}_G}{dt}\right|_I = \left.\frac{d(\mathbf{I}_B \boldsymbol{\omega}_B)}{dt}\right|_I \quad \Rightarrow \quad \frac{d\boldsymbol{\omega}_B}{dt} = \mathbf{I}_B^{-1}\left[-\boldsymbol{\omega}_B \times (\mathbf{I}_B \boldsymbol{\omega}_B) + \sum \mathbf{M}_B\right]$$

- In typical symmetric shape in XZ plane.
  - Caused by aerodynamic and thrust moments

xz. yz term 모두 0. ?

$$\mathbf{I}_B = \begin{bmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{xz} & 0 & I_{zz} \end{bmatrix} \quad \Rightarrow \quad \begin{aligned} I_{xx}\dot{p} - I_{xz}\dot{r} - I_{xz}pq + (I_{zz} - I_{yy})qr &= L = L_a + L_t \\ I_{yy}\dot{q} + (I_{xx} - I_{zz})pr + I_{xz}(p^2 - r^2) &= M = L_a + L_t \\ I_{zz}\dot{r} - I_{xz}\dot{p} + (I_{yy} - I_{xx})pq + I_{xz}qr &= N = N_a + N_t \end{aligned}$$

Strapdown INS Mechanization



*Euler Rate.

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \\ 0 & \cos\phi & \\ 0 & & \end{bmatrix} \cdot \omega_b .$$

$\phi_{k+1} = \phi_k + \dot{\phi} \cdot \Delta t$ ... 이런 식으로 자세 update.

Euler Rate Equation

Step 1: Identify the equation

The image displays the matrix form of the Euler rate equation, which relates the time derivatives of the Euler angles (roll ϕ, pitch θ, and yaw ψ) to the angular velocities in the body-fixed frame (ω_b).

Step 2: Present the full equation

The complete standard equation is as follows:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \frac{\sin\phi}{\cos\theta} & \frac{\cos\phi}{\cos\theta} \end{bmatrix} \begin{bmatrix} \omega_{bx} \\ \omega_{by} \\ \omega_{bz} \end{bmatrix}$$

Answer:

The Euler rate equation shown in the image relates the rates of change of Euler angles ($\dot{\phi}, \dot{\theta}, \dot{\psi}$) to the angular velocities ($\omega_{bx}, \omega_{by}, \omega_{bz}$) in the body-fixed frame using the transformation matrix:

$$\begin{bmatrix} \dot{\boldsymbol{\phi}} \\ \dot{\boldsymbol{\theta}} \\ \dot{\boldsymbol{\psi}} \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \sin\boldsymbol{\phi}\tan\boldsymbol{\theta} & \cos\boldsymbol{\phi}\tan\boldsymbol{\theta} \\ \mathbf{0} & \cos\boldsymbol{\phi} & -\sin\boldsymbol{\phi} \\ \mathbf{0} & \frac{\sin\boldsymbol{\phi}}{\cos\boldsymbol{\theta}} & \frac{\cos\boldsymbol{\phi}}{\cos\boldsymbol{\theta}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega_{bx}} \\ \boldsymbol{\omega_{by}} \\ \boldsymbol{\omega_{bz}} \end{bmatrix}$$

Flight Control Case Study

· trim 조건.(?)

· bank turn. rudder → yaw rate.

aileron 사용.

■ Yaw control problem

▶ Consider B-747 lateral flight dynamics

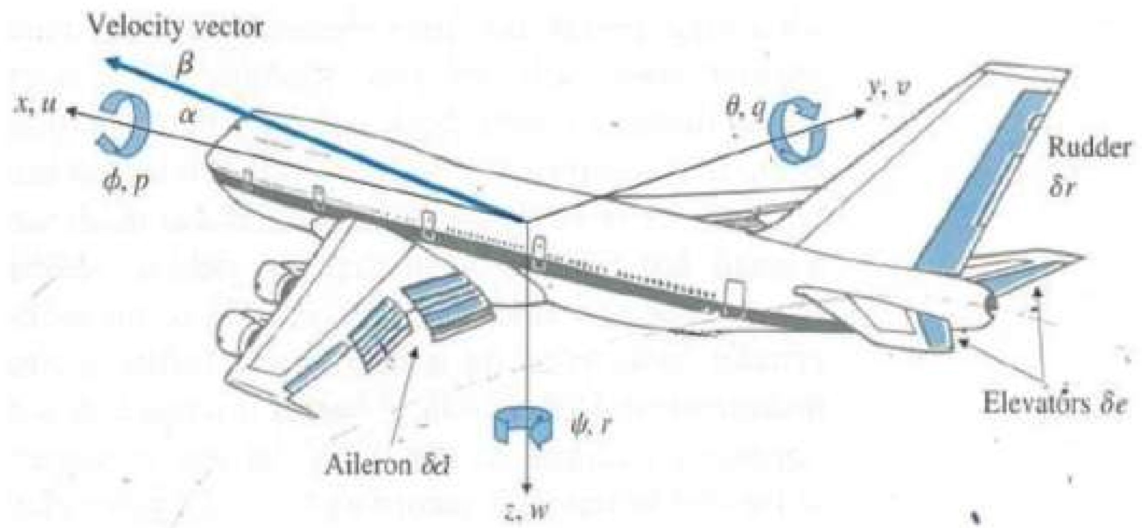


Figure 1. Definition of plane coordinates.

4 States

-Side slip angle -> β

-Bank angle -> ϕ

-Yaw Rate -> r

-Roll Rate -> p

2 Inputs

-Rudder -> δr

-Aileron -> δa

2 Outputs

-Yaw rate -> r

-Bank angle -> ϕ

⇔ roll angle.

※ rudder dynamics → 상대적으로 unstable.

trim 조건 (등속 평형 상태)에서 linearize.

· · · 등속운동상황 ??

■ State space model

state vector $x = \begin{bmatrix} \beta \\ \phi \\ r \\ p \end{bmatrix}$.

▸ Use the following G.E.

$u = \begin{bmatrix} \delta r \\ \delta a \end{bmatrix}$.

$$\dot{x} = \begin{bmatrix} \dot{\beta} \\ \dot{r} \\ \dot{p} \\ \dot{\phi} \end{bmatrix}$$

$$= \begin{bmatrix} -0.0558 & -0.9968 & 0.0802 & 0.0415 \\ 0.598 & -0.115 & -0.0318 & 0 \\ -3.05 & 0.388 & -0.465 & 0 \\ 0 & 0.0805 & 1 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ r \\ p \\ \phi \end{bmatrix} + \begin{bmatrix} 0.00729 & 0 \\ -0.475 & 0.00775 \\ 0.155 & 0.143 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta r \\ \delta a \end{bmatrix}$$

$$y = \begin{bmatrix} r \\ \phi \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \beta \\ r \\ p \\ \phi \end{bmatrix}$$

$\dot{\beta} \Rightarrow$ rudder에만 의존. bank angle 또는 side slip x.

$\dot{r} \Rightarrow$ rudder의 영향 $\uparrow$.

$\dot{p}$

$\dot{\phi} \Rightarrow$ $\delta r, \delta a$의 직접적 영향 x.

r, p의 변화로부터 변함.

settling time. rise time 등 측정. 2차보다 고차라 ζ, ωn x.

■ Assignments

1. Find each transfer function from input to output and plot the impulse and step responses.
2. Design a compensator using root locus method that removes/mitigates oscillation at non-zero bank angle.
3. Compare the performance of control system with or without compensator.
4. To avoid over-stabilized spiral mode, consider a *wash-out filter* in the model and design again the controller/compare performance
5. (open problem) 시뮬링크를 이용하여 앞서 제어시스템 설계를 구현하라. (가능한 어떤 형태도 좋으니..) 자신만의 제어기를 적용하여 앞서 비행제어의 성능을 비교/분석해 보라!

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

$$G(s) = C(sI-A)^{-1}B$$

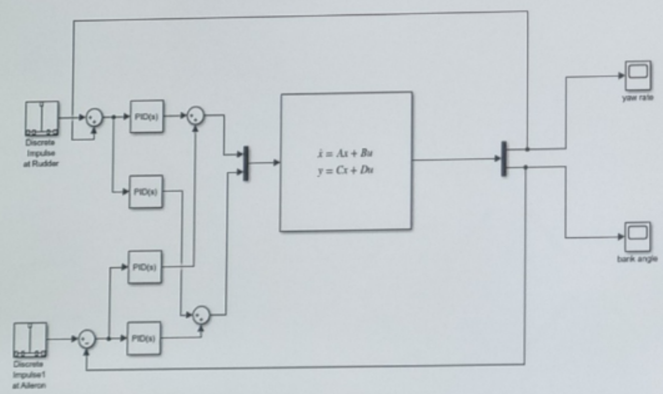
transfer matrix.

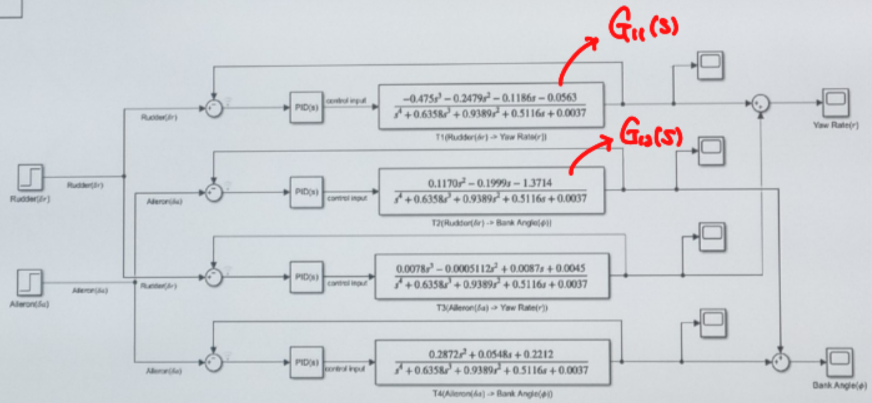
$$= \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$

$$G_{22}(s) = \frac{\phi(s)}{\delta a(s)}.$$

Tips

- 구성 사례 - simulink



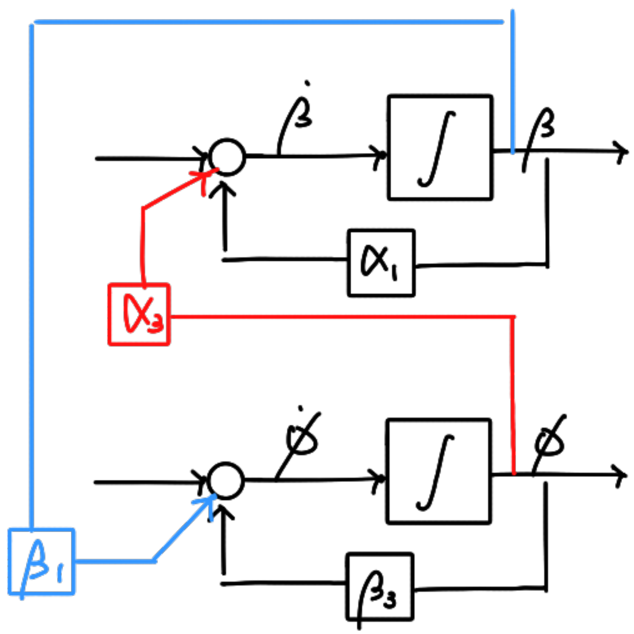


■ Matlab 내장함수(control system toolbox)

- ss - 실수 또는 복소수 값을 갖는 상태공간 모델을 만들거나 동적 시스템 모델을 상태공간 모델 형식으로 변환
  - sys = ss(A,B,C,D)
- ss2tf - 상태공간 표현식(State-Space Representation)을 전달 함수로 변환
  - [b,a] = ss2tf(A,B,C,D)
- impulse - 동적 시스템의 임펄스 응답 플롯, 임펄스 응답 데이터
  - y = impulse(sys,t)
- step - 동적 시스템의 계단 응답 플롯, 계단 응답 데이터
  - [y,tOut] = step(sys)

$$\dot{\beta} = \alpha_1 \beta + \alpha_2 r + \alpha_3 \phi.$$

$$\dot{\phi} = \beta_1 \beta + \beta_2 r + \beta_3 \phi.$$



Note

- Term Project
  - Control system design and analysis via Matlab
  - Due to 12/2, 온라인 제출

Any Question?

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