Numerical optimization and large scale linear algebra (PT)

Computer Assignment 1

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Introduction

Portfolio optimization is a financial strategy where an investor selects allocations for a set of assets to maximize potential returns while minimizing risk. This involves creating a portfolio that combines risk-free and risky assets to achieve a specified level of return, considering the portfolio's volatility. This report applies the principles of the Markowitz Portfolio Optimization model and uses constrained least squares optimization.

Analysis

We will optimize a set of holdings to minimize the risk for various average returns.
We want to choose weights that will properly allocate the total amount of investment so that we achieve high return and low risk.
This is an optimization problem with 2 objectives, return and risk. The approach to the problem is to fix the return of the portfolio at a given value p and to minimize the risk over all portfolios that achieve the required return.
To minimize risk, with return value p, we must solve the linearly constrained least square problem

$$
\begin{aligned}
&minimize\ ||Rw - \rho 1||^2 \\
&subject\ to\ \begin{bmatrix} 1^T \\ \mu^T \end{bmatrix} w = \begin{bmatrix} 1 \\ \rho \end{bmatrix}
\end{aligned}
$$

The portfolio optimization problem has the solution:

$$
\begin{bmatrix} w \\ z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 2R^T R & 1 & \mu \\ 1^T & 0 & 0 \\ \mu^T & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 2\rho T\mu \\ 1 \\ \rho \end{bmatrix}
$$

By solving the above equation for each ρ, we obtain the weight for the allocation to each asset.

T: The period
R: The investment returns matrix (T x n)
μ: The n-vector for the average asset returns

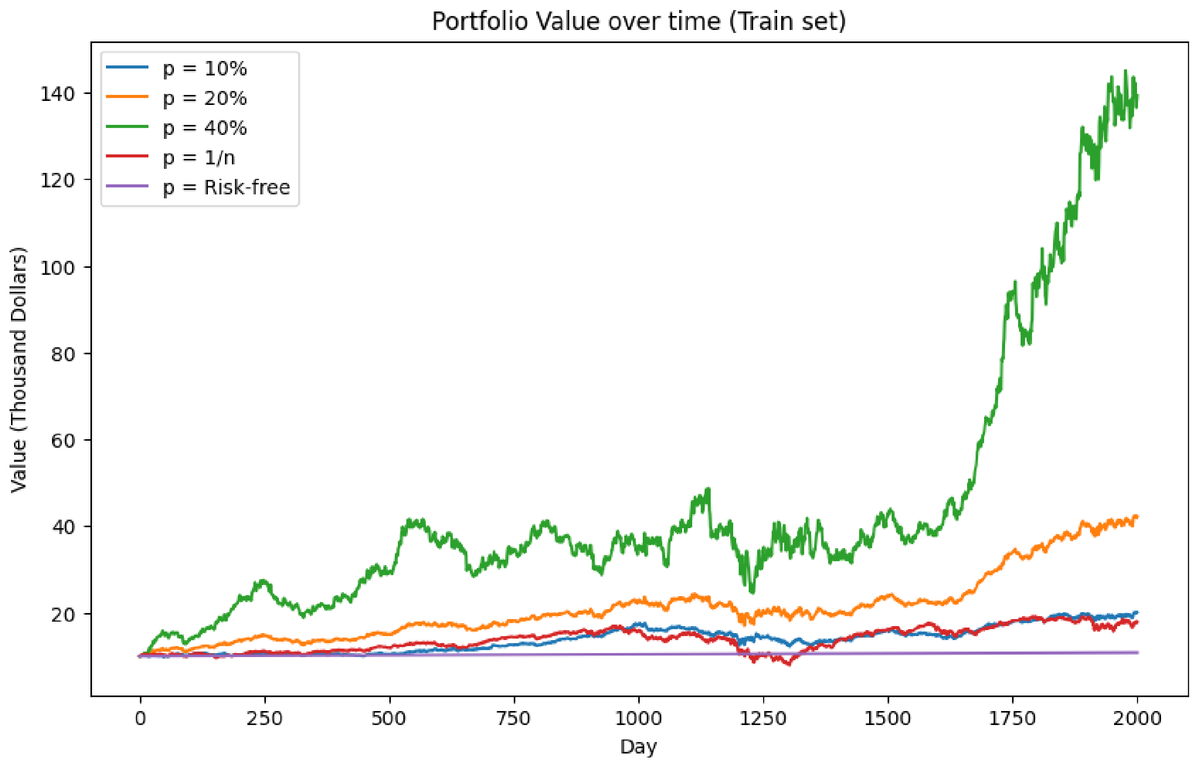
w: The weights
z: Lagrange multipliers for the equality constraints which we don't care about

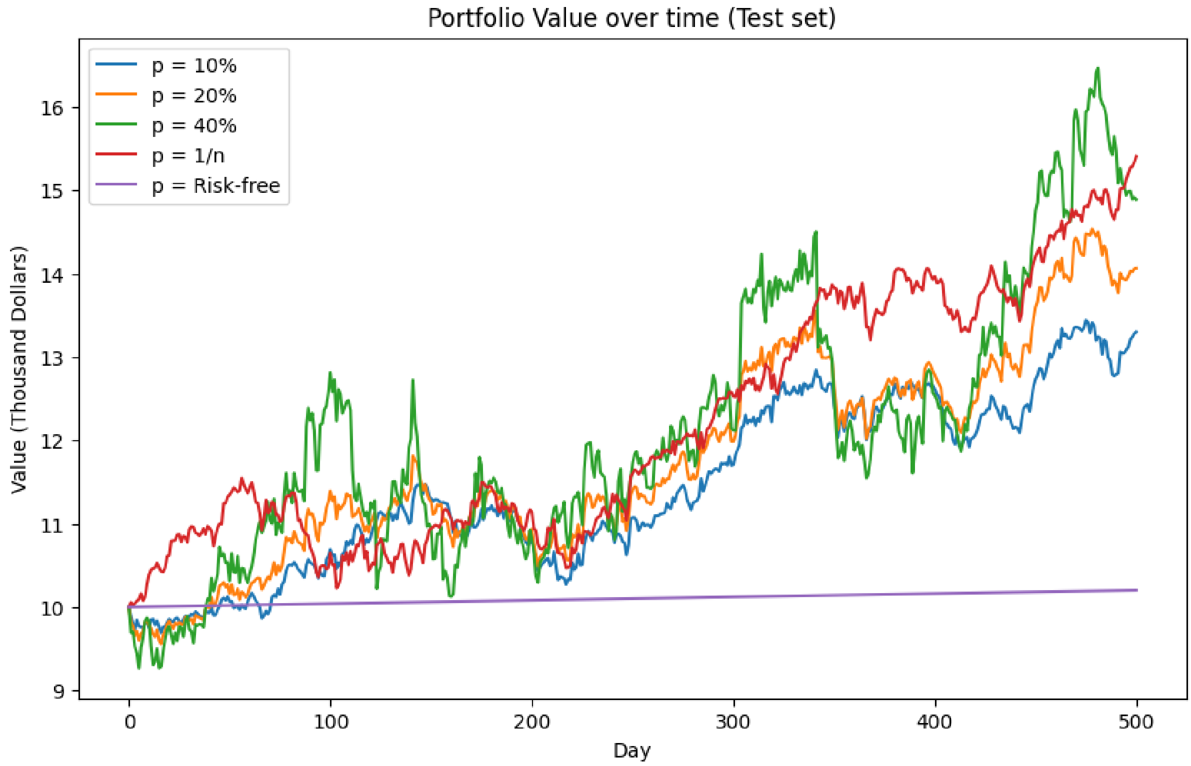
The dataset consists of 20 columns (19 stocks and one risk-free asset), and we split it into a training and a testing dataset.

First, we will calculate the investment return matrix R, by calculating the percentage difference of each line of the dataset with the previous one.

For the different values of annualized return (10%, 20%, 40%), we obtain the weights during the training phase, and we plot the total values of the portfolios over the time starting with an initial investment of $10,000.

As we have calculated the weights, we also see the total values of portfolios in the test phase set. Below we can see the plots with the performance for Train and Test set over the time.





We also need to calculate the annualized return, risk with minimum and maximum allocation, and leverage for each portfolio.

The Leverage is the sum of the absolute values of the weights.

The quantities avg(r) and std(r) give the per period return and risk and the annualized return and risk are calculated as $P\ avg(r)\ \ and\ \ \sqrt{P}std(r)$ respectively, where r is the resulting portfolio return time series r = Rw.

After the analysis, the results are shown below.

Portfolio	**Return_Train**	**Return_Test**	**Risk_Train**	**Risk_Test**	**Leverage**
10%	0.10	0.15	0.16	0.12	1.66
20%	0.20	0.18	0.20	0.15	2.40
40%	0.40	0.25	0.38	0.31	5.30
1/n	0.10	0.22	0.24	0.13	1.00
Risk Free	0.01	0.01	0.00	0.00	1.00

Conclusions

The portfolio optimization results clearly illustrate the trade-off between risk and return, with higher returns entailing higher risk and volatility. The training data shows more efficient optimization, indicating the model's effectiveness when applied to historical data. However, the testing data reveals the real-world applicability and potential challenges of the model.

The comparison between the training and testing sets' portfolio performance provides the below key insights:

Efficiency in Training Data: Portfolios achieve peak returns more quickly in the training data, suggesting efficient optimization based on historical data. This highlights the model's ability to be adjusted on known patterns and trends within the training period.

Gradual Growth in Testing Data: The testing data exhibits a slower, more gradual approach to reaching returns, indicating differences in market behavior and potential overfitting to the training data. This underlines the challenges of predicting future market movements and the importance of robustness in portfolio strategies.

Risk and Volatility: Both datasets demonstrate increased fluctuations in portfolio performance as the target return rate increases. Higher returns are associated with higher levels of risk and volatility, emphasizing the trade-offs between risk and reward. The 40% return portfolio, in particular, shows significant volatility, reflecting the high risk involved in aiming for high returns.

Consistency and Stability: The risk-free portfolio remains constant across both training and testing sets, demonstrating its stability and minimal risk. This provides a contrast to the more volatile portfolios and highlights the safety of risk-free assets, although they minimal returns.

Balanced Approach: The equal-weighted (1/n) portfolio performs reasonably well in both datasets, providing a balanced approach between risk and return. It offers a middle ground, avoiding extreme fluctuations while still achieving moderate growth.

In general, in this report we highlight the importance of understanding the risk-return trade-off in portfolio optimization. While higher returns can be achieved through efficient optimization, they come with increased risk and volatility, particularly in real-world scenarios. The differences between training and testing performances underscore the need for robust, adaptable strategies that can handle varying market conditions. This analysis provides a clear understanding of the portfolio optimization process and results, emphasizing the practical implications of these strategies.