In previous chapters we saw that the transfer function for the recursive LTI filter is:

$$H(z) = \frac{g(1 + \beta_1 z^{-1} + \beta_2 z^{-2} + \cdots + \beta_M z^{-M})}{1 + \alpha_1 z^{-1} + \alpha_2 z^{-2} + \cdots + \alpha_N z^{-N}}$$

We know that the polynomials in the numerator and denominator can be factored, giving the form:

$$H(z) = \frac{g(1 - q_1 z^{-1})(1 - q_2 z^{-1}) \dots (1 - q_M z^{-1})}{(1 - p_1 z^{-1})(1 - p_2 z^{-1}) \dots (1 - p_N z^{-1})}$$

Assuming none of the terms in the numerator and denominator cancel out, poles are points where z, a complex number, is equal to any of the p_i values. Zeroes are when z is equal to the q_i values. We can think of z values lying on a 2D plane, and the H magnitude, which is a real number being the 3rd dimension that lies above the z-plane. q_i and p_i values would be points where the H magnitude is either infinitely small (zero) or large (resembling a pole).

Also note that the filter order is either M or N, or the number of poles or zeros, whichever is greater. **So the order of an LTI filter is the order of its transfer function.**

Graphical Computation of Amplitude Response

If we take the factored form of the transfer function and set z to e^{jwt}, we get the frequency response in factored form:

$$H(z) = \frac{g(1 - q_1 e^{-jwt})(1 - q_2 e^{-jwt}) \dots (1 - q_m e^{-jwt})}{(1 - p_1 e^{-jwt})(1 - p_2 e^{-jwt}) \dots (1 - p_N e^{-jwt})}$$

The amplitude response $G(w) = |H(e^{jwt})|$ ends up being:

$$G(w) = |g| \frac{|e^{jwt} - q_1||e^{jwt} - q_2| \dots |e^{jwt} - q_M|}{|e^{jwt} - p_1| \dots |e^{jwt} - p_M|}$$

proof: https://ccrma.stanford.edu/~jos/filters/Graphical_Amplitude_Response.html

Consider that the magnitude of vector $u - v$ is a point drawn from the tip of v to u.
This leads to the following conclusion:
> The frequency response magnitude (amplitude response) at frequency w is given by the product of lengths of vectors drawn from the zeros to the point e^{jwt} divided by the product of vectors drawn from the poles to the point e^{jwt}.

Further reading

analytic function: https://mathworld.wolfram.com/AnalyticFunction.html
Conformal mapping: https://mathworld.wolfram.com/ConformalMapping.html