common angles

θ	0°	30°	45°	60°	90°	180°	270°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not Defined	0	Not Defined

useful identities

$$
\begin{aligned}
\sin(\alpha+\beta) &= \sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta)\\
\cos(\alpha+\beta) &= \cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)
\end{aligned}
$$

Latex snippets:

https://github.com/artisticat1/obsidian-latex-suite/blob/main/src/default_snippets.ts
https://github.com/artisticat1/obsidian-latex-suite
https://castel.dev/post/lecture-notes-1/

Taylor Series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}.(x-x_0)^n$$

geometric series

$$\sum_{k=0}^{\infty} ar^k \text{→generator term}$$
$$\frac{a}{1-r} \text{ for |r|<1}$$

Euler's number

$$e = \lim_{\delta->0} (1+\delta)^{1/\delta}$$

Think of this as the rate for constant growth.

Eulers Identity

$$e^{j\theta} = cos(\theta) + jsin(\theta)$$

Continuous

$$e^{j(\omega t+\phi)} = \cos(\omega t+\phi) + j\sin(\omega t+\phi)$$

discrete

$$e^{j(\omega nT+\phi)} = \cos(\omega nT+\phi) + j\sin(\omega nT+\phi)$$

complex sinusoid: $e^{j(\omega * n * T+\phi)}$

proof:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \ldots \text{ using taylor series}$$

$$
\begin{aligned}
e^{j\theta} &= 1 + j\theta + \frac{(j\theta)^2}{2} + \frac{(j\theta)^3}{3!} + \ldots \\
&= 1 + j\theta - \frac{\theta^2}{2} - \frac{j\theta^3}{3!} + \ldots
\end{aligned}
$$

$$
\begin{aligned}
re\{e^{j\theta}\} &= 1 - \frac{\theta^2}{2} + \frac{\theta^4}{4!} \\
im\{e^{j\theta}\} &= j\theta - \frac{j\theta^3}{3!} + \frac{j\theta^5}{5!}
\end{aligned}
$$

$$
\begin{aligned}
\left.\frac{d^n}{d\theta^n}\cos(\theta)\right|_{\theta=0} &= \begin{cases} (-1)^{n/2}, & n \text{ even} \\ 0, & n \text{ odd} \end{cases} \\
\left.\frac{d^n}{d\theta^n}\sin(\theta)\right|_{\theta=0} &= \begin{cases} (-1)^{(n-1)/2}, & n \text{ odd} \\ 0, & n \text{ even.} \end{cases}
\end{aligned}
$$

Plugging into the general Maclaurin series gives

$$
\begin{aligned}
\cos(\theta) &= \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}\theta^n \\
&= \sum_{\substack{n\geq 0 \\ n \text{ even}}}^{\infty} \frac{(-1)^{n/2}}{n!}\theta^n \\
\sin(\theta) &= \sum_{\substack{n\geq 0 \\ n \text{ odd}}}^{\infty} \frac{(-1)^{(n-1)/2}}{n!}\theta^n.
\end{aligned}
$$

Separating the Maclaurin expansion for $e^{j\theta}$ into its even and odd terms (real and imaginary parts) gives

$$
\begin{aligned}
e^{j\theta} \triangleq \sum_{n=0}^{\infty} \frac{(j\theta)^n}{n!} &= \sum_{\substack{n\geq 0 \\ n \text{ even}}}^{\infty} \frac{(-1)^{n/2}}{n!}\theta^n + j \sum_{\substack{n\geq 0 \\ n \text{ odd}}}^{\infty} \frac{(-1)^{(n-1)/2}}{n!}\theta^n \\
&= \cos(\theta) + j\sin(\theta)
\end{aligned}
$$

Mth Roots

Any real or complex number z can be represented with $re^{j\theta}$, which can represent any point in a 2D space. Here, theta would be $\tan^{-1}\left(\frac{a}{b}\right)$ where a and b are the real and imaginary coordinates.

$$
\begin{aligned}
1 &= e^{2j\pi k} = \cos 2\pi\theta + j * 0 \\
z &= re^{j\theta}e^{2j\pi k} \\
z^{1/M} &= r^{\left(\frac{1}{M}\right)}e^{j\frac{\theta}{M}}e^{\left(\frac{2\pi jk}{M}\right)}
\end{aligned}
$$

```
The formula above  has M unique answers for every positive integer k<M.
$r^\left( \frac{1}{M} \right)$ will grow into $r$ and $(e^\frac{j\theta+2\pi
```

```
k}{M})^M$ will rotate to the correct angle for every integer value of k.
```

Roots of Unity

$$1^{k/M} = e^{2\pi j \frac{k}{M}}$$

These are M equally spaced values on the unit circle. Different k values correspond the the complex sinusoids that are used in DFT for analysis of different frequencies

De Moivre Theorem

$$(\cos(x) + i\sin(x))^n = \cos(nx) + i\sin(nx)$$

Easy to prove using euler's identity.
This establishes that integer powers (and only integer powers?) of $\cos(x) + i\sin(x)$ line up on the unit circle