

MACROECONOMICS  
N. GREGORY MANKIW

A Dynamic Model of Economic  
Fluctuations

# **IN THIS CHAPTER, YOU WILL LEARN:**

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- how to incorporate dynamics into the AD-AS model we previously studied
- how to use the dynamic AD-AS model to illustrate long-run economic growth
- how to use the dynamic AD-AS model to trace out the effects over time of various shocks and policy changes on output, inflation, and other endogenous variables

# Introduction

- The dynamic model of aggregate demand and aggregate supply gives us more insight into how the economy works in the short run.
- It is a simplified version of a DSGE model, used in cutting-edge macroeconomic research.  
(DSGE = Dynamic, Stochastic, General Equilibrium)

# Introduction

- The dynamic model of aggregate demand and aggregate supply is built from familiar concepts, such as:
  - the IS curve, which negatively relates the real interest rate and demand for goods & services
  - the Phillips curve, which relates inflation to the gap between output and its natural level, expected inflation, and supply shocks
  - adaptive expectations, a simple model of inflation expectations

# How the dynamic AD-AS model is different from the standard model

- Instead of fixing the money supply, the central bank follows a monetary policy rule that adjusts interest rates when output or inflation change.
- The vertical axis of the DAD-DAS diagram measures the inflation rate, not the price level.
- Subsequent time periods are linked together: Changes in inflation in one period alter expectations of future inflation, which changes aggregate supply in future periods, which further alters inflation and inflation expectations.

# Keeping track of time

- The subscript “ $t$ ” denotes the time period, e.g.
  - $Y_t$  = real GDP in period  $t$
  - $Y_{t-1}$  = real GDP in period  $t - 1$
  - $Y_{t+1}$  = real GDP in period  $t + 1$
- We can think of time periods as years.  
*E.g., if  $t = 2010$ , then*
  - $Y_t = Y_{2010}$  = real GDP in 2010
  - $Y_{t-1} = Y_{2009}$  = real GDP in 2009
  - $Y_{t+1} = Y_{2011}$  = real GDP in 2011

# The model's elements

- The model has five equations and five endogenous variables: output, inflation, the real interest rate, the nominal interest rate, and expected inflation.
- The equations may use different notation, but they are conceptually similar to things you've already learned.
- The first equation is for output...

# Output: The Demand for Goods and Services

$$Y_t = \bar{Y}_t - \alpha(r_t - \rho) + \varepsilon_t$$

*output*      |      *natural level of output*      |      *real interest rate*       $\alpha > 0, \rho > 0$

Negative relation between output and interest rate,  
same intuition as *IS* curve.

# Output: The Demand for Goods and Services

$$Y_t = \bar{Y}_t - \alpha(r_t - \rho) + \varepsilon_t$$

*measures the interest-rate sensitivity of demand*

*“Natural rate of interest.”*

*In absence of demand shocks,*

$$Y_t = \bar{Y}_t \text{ when } r_t = \rho$$

*demand shock,  
random and zero on average*

# The Real Interest Rate: The Fisher Equation

$$r_t = i_t - E_t \pi_{t+1}$$

ex ante  
(i.e. expected)  
real interest  
rate

nominal  
interest  
rate

expected  
inflation rate

$\pi_{t+1}$  = increase in price level from period  $t$  to  $t + 1$ ,  
not known in period  $t$

$E_t \pi_{t+1}$  = expectation, formed in period  $t$ ,  
of inflation from  $t$  to  $t + 1$

# Inflation: The Phillips Curve

$$\pi_t = E_{t-1} \pi_t + \phi(Y_t - \bar{Y}_t) + \nu_t$$

*current  
inflation*

/  
*previously  
expected  
inflation*

*supply  
shock,  
random and  
zero on  
average*

$\phi > 0$  indicates how much  
inflation responds when  
output fluctuates around  
its natural level

# Expected Inflation: Adaptive Expectations

$$E_t \pi_{t+1} = \pi_t$$

For simplicity, we assume people expect prices to continue rising at the current inflation rate.

# The Nominal Interest Rate: The Monetary-Policy Rule

$$i_t = \pi_t + \rho + \theta_\pi (\pi_t - \pi_t^*) + \theta_Y (Y_t - \bar{Y}_t)$$

*nominal  
interest rate,  
set each period  
by the central  
bank*

*natural  
rate of  
interest*

*central  
bank's  
inflation  
target*

$$\theta_\pi > 0, \theta_Y > 0$$

# The Nominal Interest Rate: The Monetary-Policy Rule

$$\dot{i}_t = \pi_t + \rho + \theta_\pi (\pi_t - \pi_t^*) + \theta_Y (Y_t - \bar{Y}_t)$$

*measures how much  
the central bank  
adjusts the interest  
rate when inflation  
deviates from its target*

*measures how much the  
central bank adjusts the  
interest rate when  
output deviates from  
its natural rate*

# CASE STUDY

## The Taylor rule

- Economist John Taylor proposed a monetary policy rule very similar to ours:

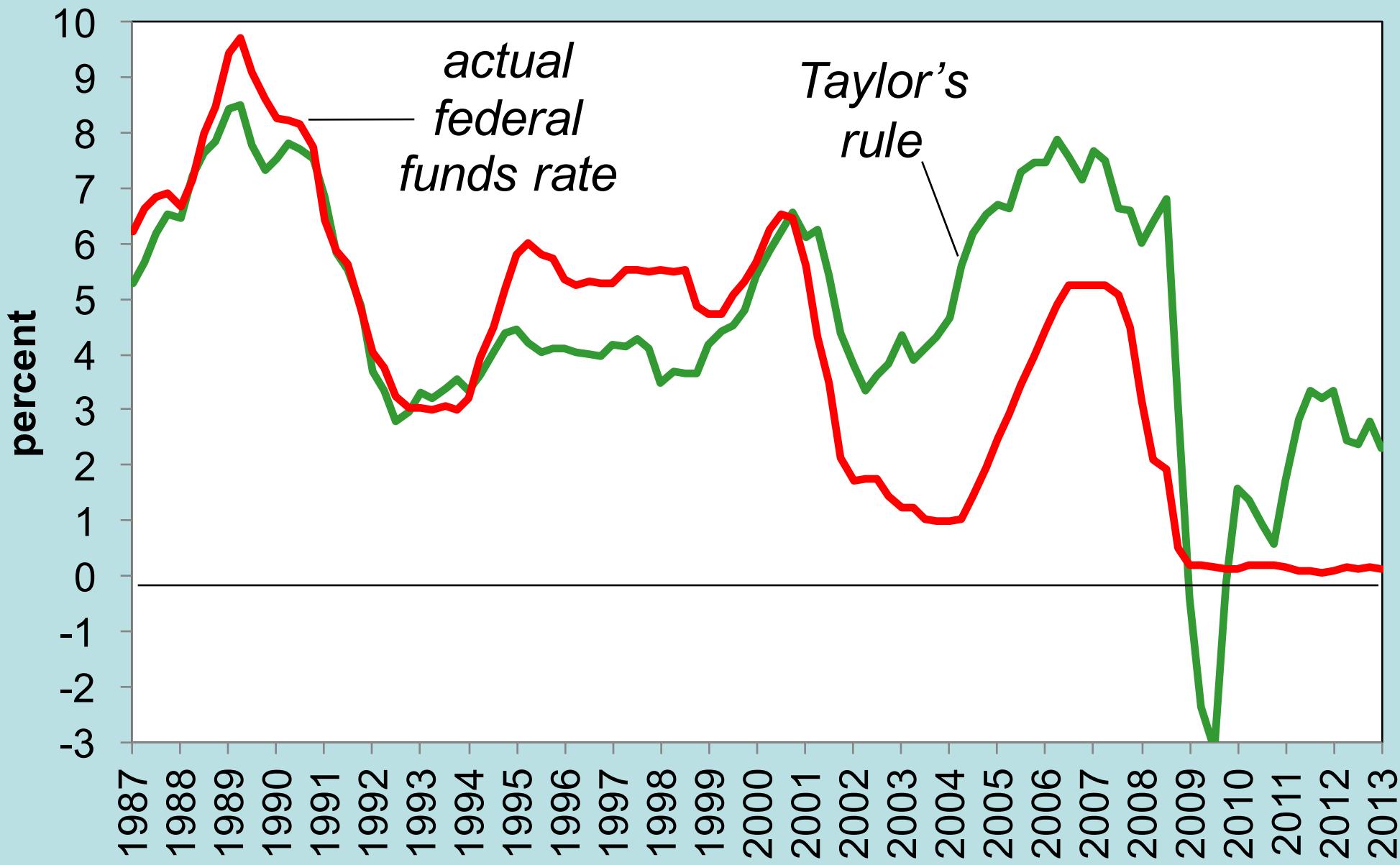
$$i_{ff} = \pi + 2 + 0.5(\pi - 2) - 0.5(\text{GDP gap})$$

where

- $i_{ff}$  = nominal federal funds rate target
- GDP gap =  $100 \times \frac{\bar{Y} - Y}{\bar{Y}}$   
= percent by which real GDP is below its natural rate
- The Taylor rule matches Fed policy fairly well....

# CASE STUDY

## The Taylor rule



# The model's variables and parameters

- Endogenous variables:

$Y_t$  = Output

$\pi_t$  = Inflation

$r_t$  = Real interest rate

$i_t$  = Nominal interest rate

$E_t \pi_{t+1}$  = Expected inflation

# The model's variables and parameters

- Exogenous variables:

$\bar{Y}_t$  = Natural level of output

$\pi_t^*$  = Central bank's target inflation rate

$\varepsilon_t$  = Demand shock

$\nu_t$  = Supply shock

- Predetermined variable:

$\pi_{t-1}$  = Previous period's inflation

# The model's variables and parameters

- Parameters:

$\alpha$  = Responsiveness of demand to the real interest rate

$\rho$  = Natural rate of interest

$\phi$  = Responsiveness of inflation to output in the Phillips Curve

$\theta_\pi$  = Responsiveness of  $i$  to inflation in the monetary-policy rule

$\theta_Y$  = Responsiveness of  $i$  to output in the monetary-policy rule

# The model's long-run equilibrium

- The normal state around which the economy fluctuates.
- Two conditions required for long-run equilibrium:
  - There are no shocks:  $\varepsilon_t = \nu_t = 0$
  - Inflation is constant:  $\pi_{t-1} = \pi_t$

# The model's long-run equilibrium

- Plugging the preceding conditions into the model's five equations and using algebra yields these long-run values:

$$Y_t = \bar{Y}_t$$

$$r_t = \rho$$

$$\pi_t = \pi_t^*$$

$$E_t \pi_{t+1} = \pi_t^*$$

$$i_t = \rho + \pi_t^*$$

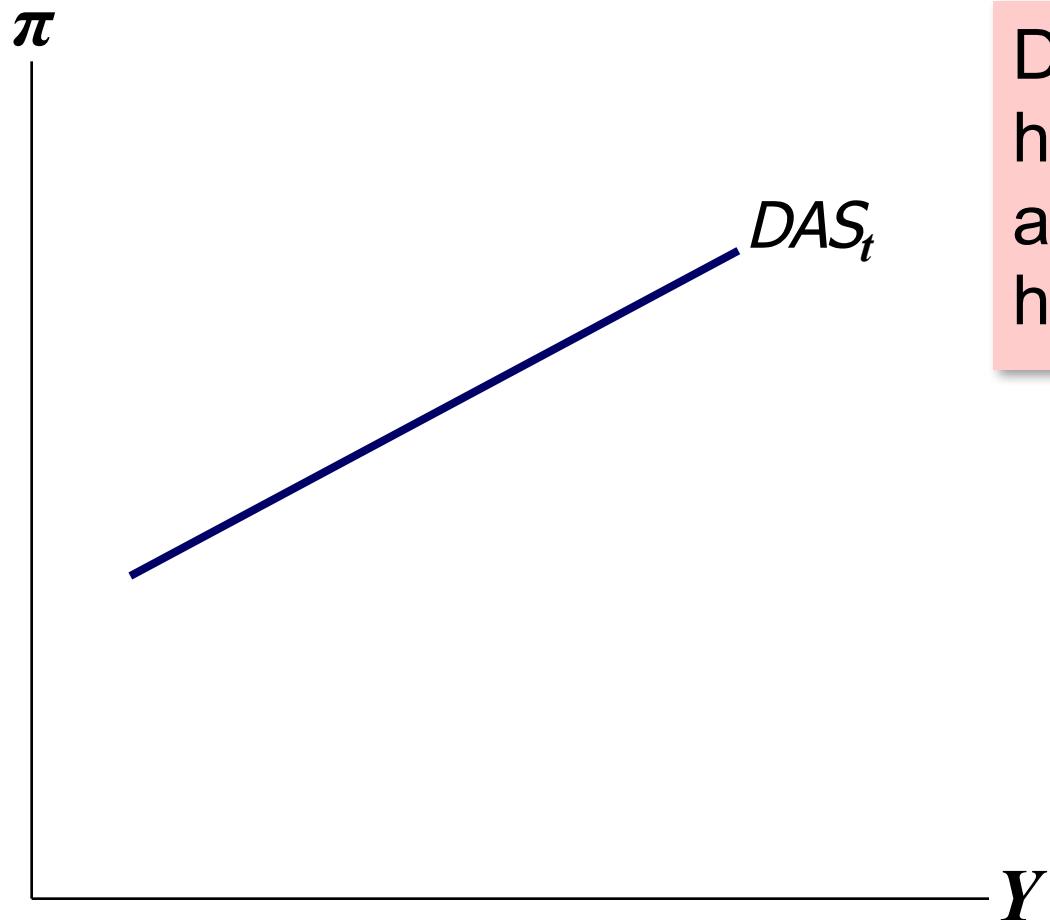
# The Dynamic Aggregate Supply Curve

- The DAS curve shows a relation between output and inflation that comes from the Phillips Curve and Adaptive Expectations:

$$\pi_t = \pi_{t-1} + \phi(Y_t - \bar{Y}_t) + \nu_t \quad (DAS)$$

# The Dynamic Aggregate Supply Curve

$$\pi_t = \pi_{t-1} + \phi(Y_t - \bar{Y}_t) + \nu_t$$



DAS slopes upward:  
high levels of output  
are associated with  
high inflation.

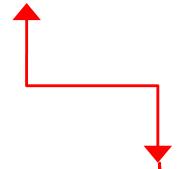
DAS shifts in response  
to changes in the  
natural level of output,  
previous inflation,  
and supply shocks.

# The Dynamic Aggregate Demand Curve

- To derive the DAD curve, we will combine four equations and then eliminate all the endogenous variables other than output and inflation.

Start with the demand for goods and services:

$$Y_t = \bar{Y}_t - \alpha(r_t - \rho) + \varepsilon_t$$



using the Fisher eq'n

$$Y_t = \bar{Y}_t - \alpha(i_t - E_t \pi_{t+1} - \rho) + \varepsilon_t$$

# The Dynamic Aggregate Demand Curve

result from previous slide

$$Y_t = \bar{Y}_t - \alpha(i_t - E_t \pi_{t+1} - \rho) + \varepsilon_t$$

using the  
expectations eq'n

$$Y_t = \bar{Y}_t - \alpha(i_t - \pi_t - \rho) + \varepsilon_t$$

using monetary  
policy rule

$$Y_t = \bar{Y}_t - \alpha[\cancel{\pi_t} + \cancel{\rho} + \theta_\pi(\pi_t - \pi_t^*) + \theta_Y(Y_t - \bar{Y}_t) - \cancel{\pi_t} - \cancel{\rho}] + \varepsilon_t$$

$$Y_t = \bar{Y}_t - \alpha[\theta_\pi(\pi_t - \pi_t^*) + \theta_Y(Y_t - \bar{Y}_t)] + \varepsilon_t$$

# The Dynamic Aggregate Demand Curve

result from previous slide

$$Y_t = \bar{Y}_t - \alpha[\theta_\pi(\pi_t - \pi_t^*) + \theta_Y(Y_t - \bar{Y}_t)] + \varepsilon_t$$

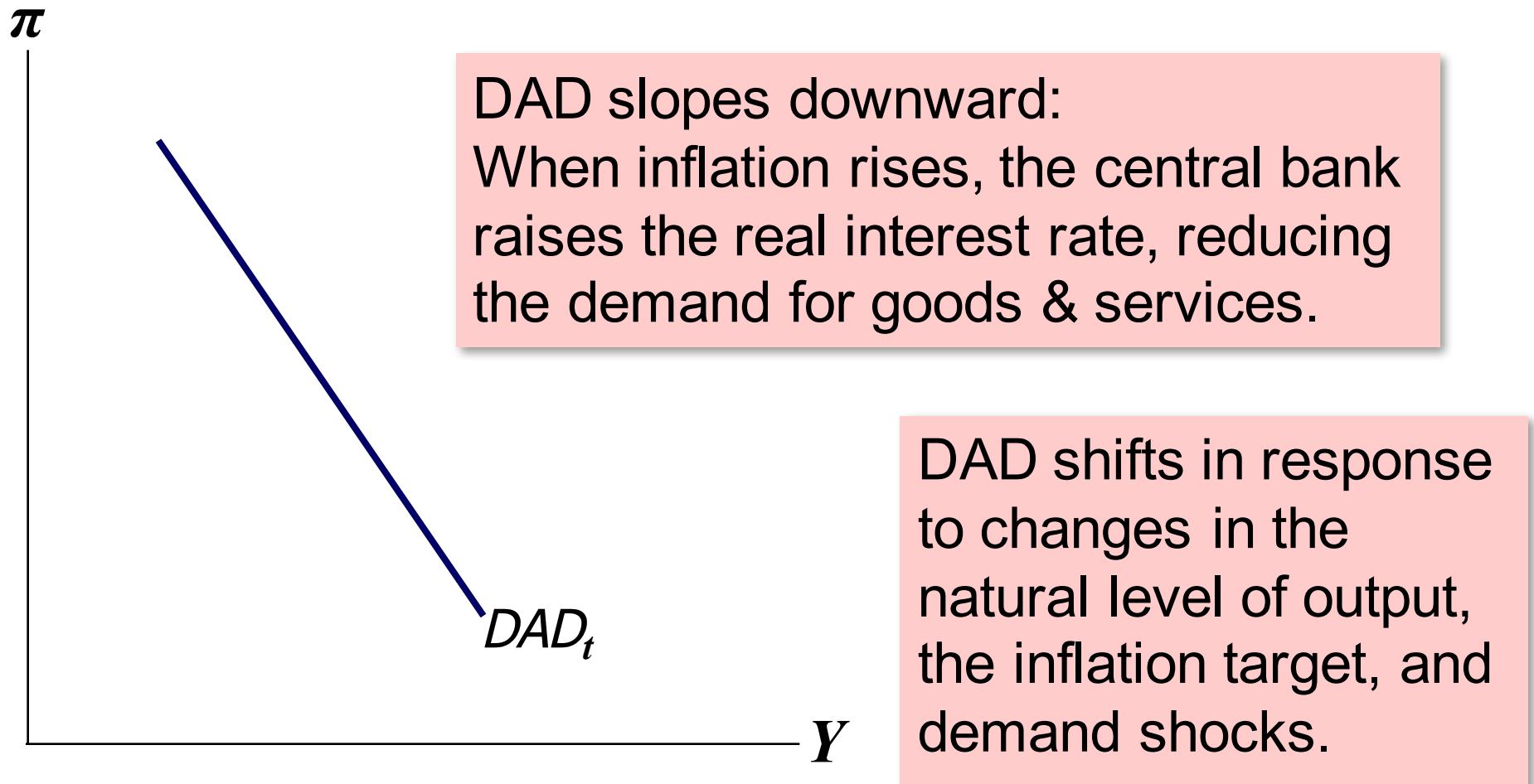
combine like terms, solve for  $Y$

$$Y_t = \bar{Y}_t - A(\pi_t - \pi_t^*) + B\varepsilon_t, \quad (DAD)$$

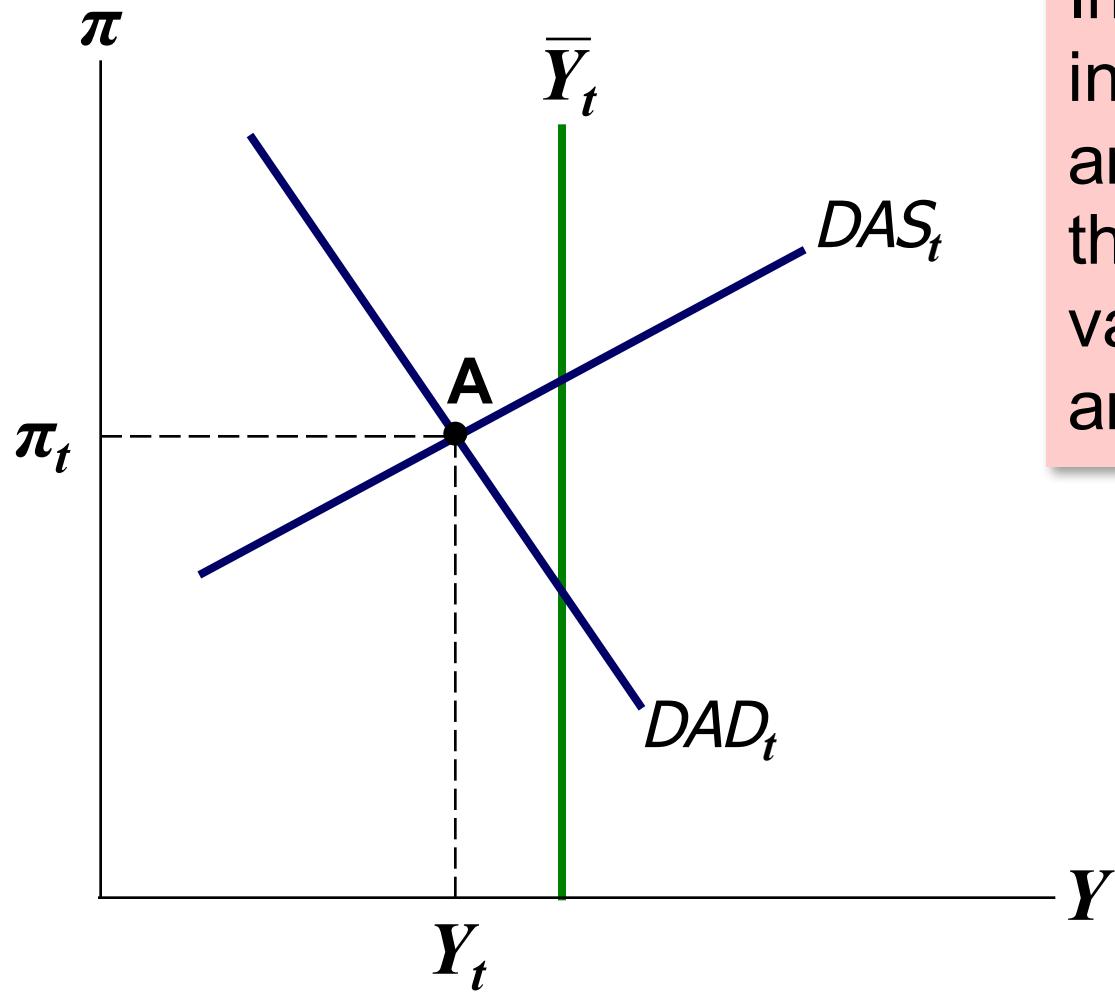
where  $A = \frac{\alpha\theta_\pi}{1+\alpha\theta_Y} > 0$ ,  $B = \frac{1}{1+\alpha\theta_Y} > 0$

# The Dynamic Aggregate Demand Curve

$$Y_t = \bar{Y}_t - A(\pi_t - \pi_t^*) + B\epsilon_t$$



# The short-run equilibrium

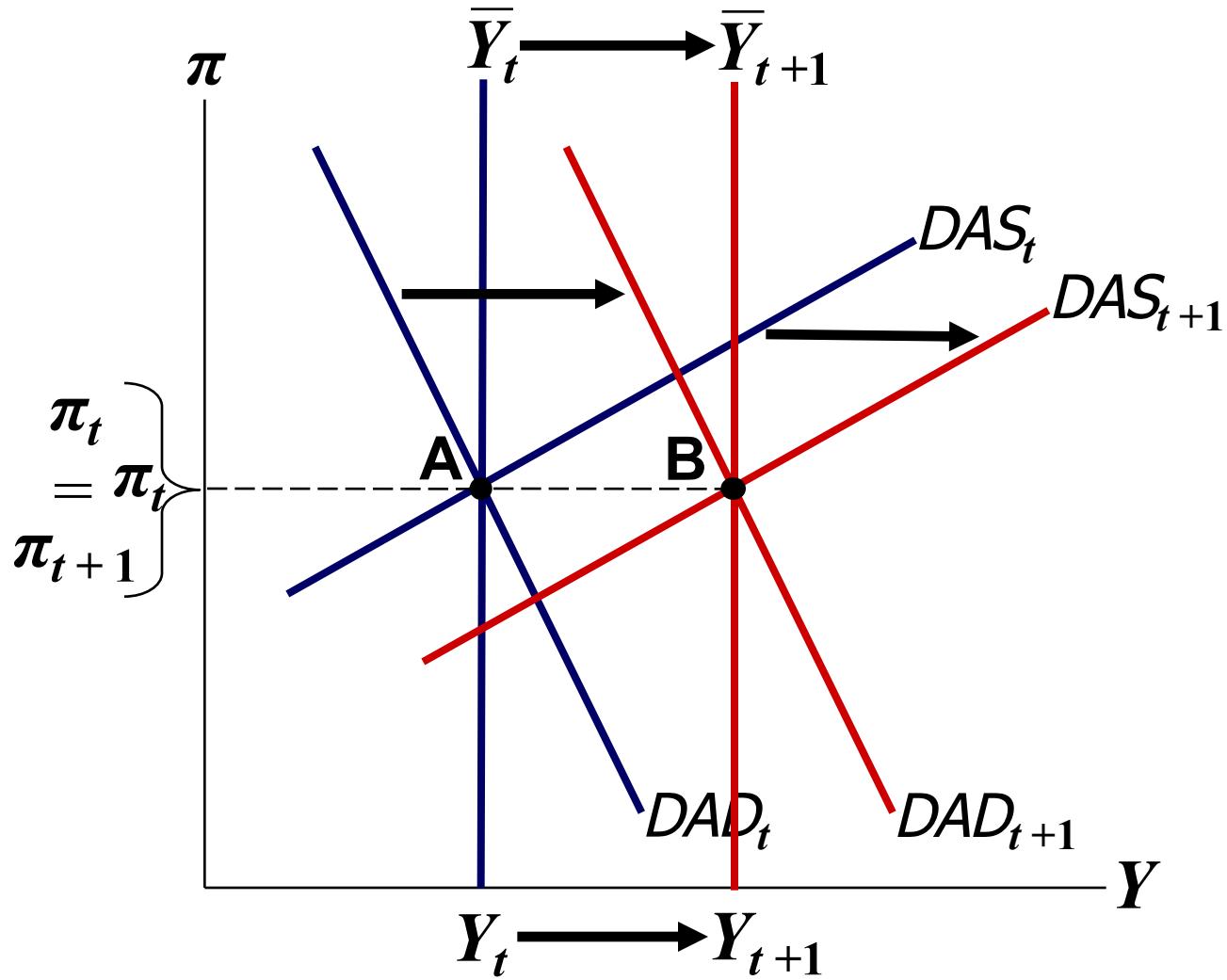


In each period, the intersection of  $DAD$  and  $DAS$  determines the short-run eq'm values of inflation and output.

In the eq'm shown here at **A**, output is below its natural level.

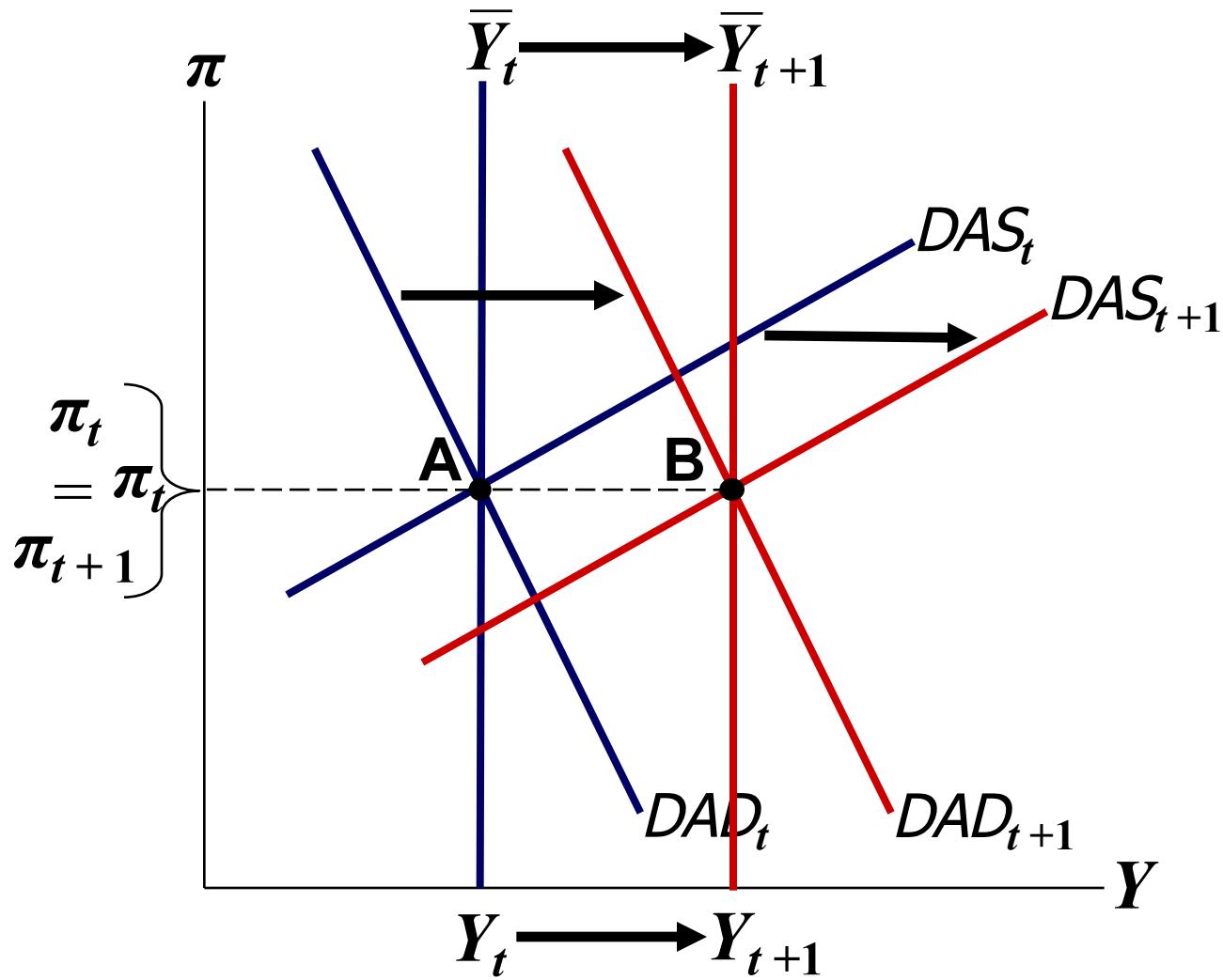
# Long-run growth

Period  $t$ :  
initial eq'm at A



Period  $t + 1$ :  
Long-run  
growth  
increases the  
natural rate  
of output.

# Long-run growth

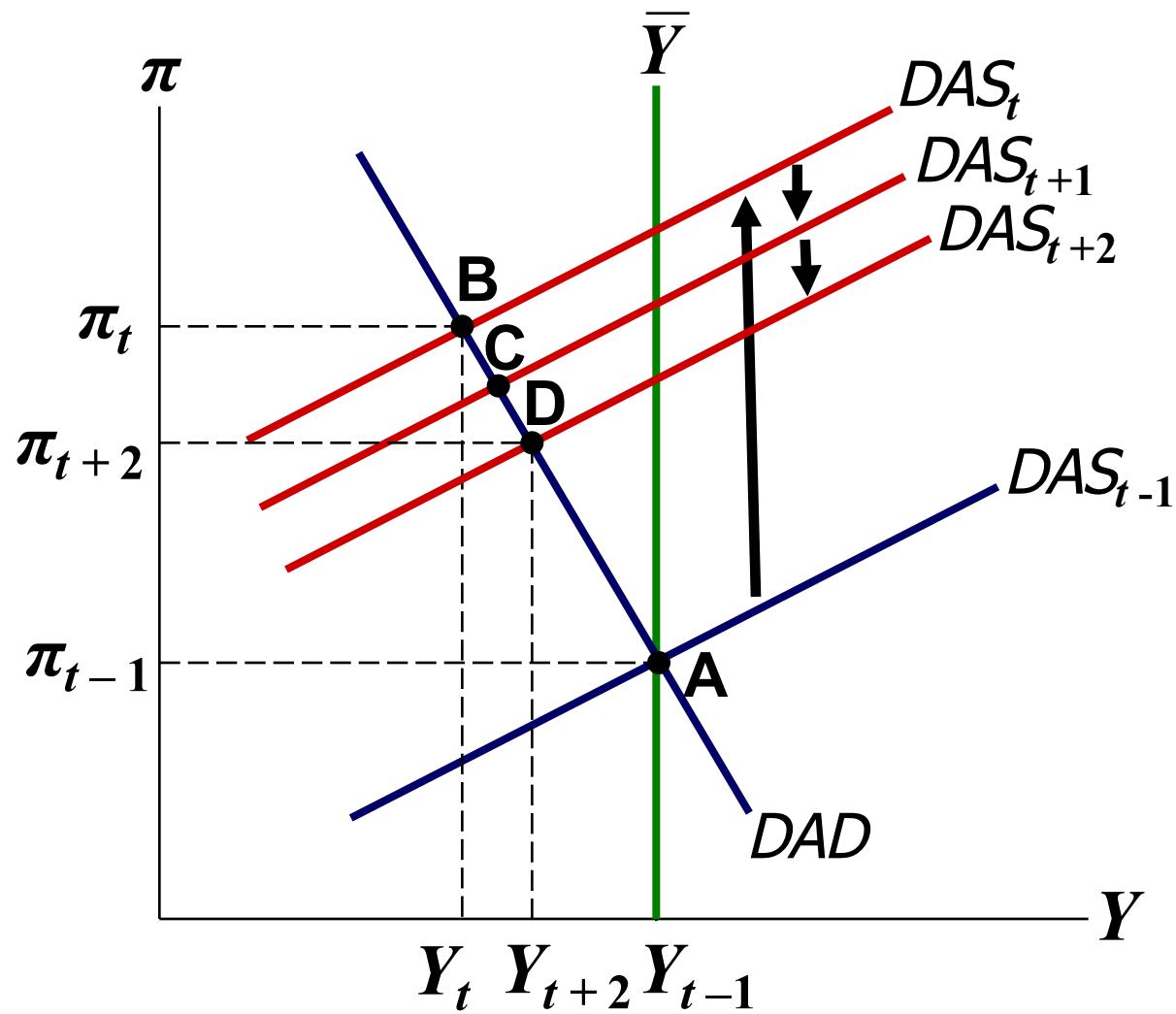


DAS shifts because economy can produce more g&s.

DAD shifts because higher income raises demand for g&s.

New eq'm at B; income grows but inflation remains stable.

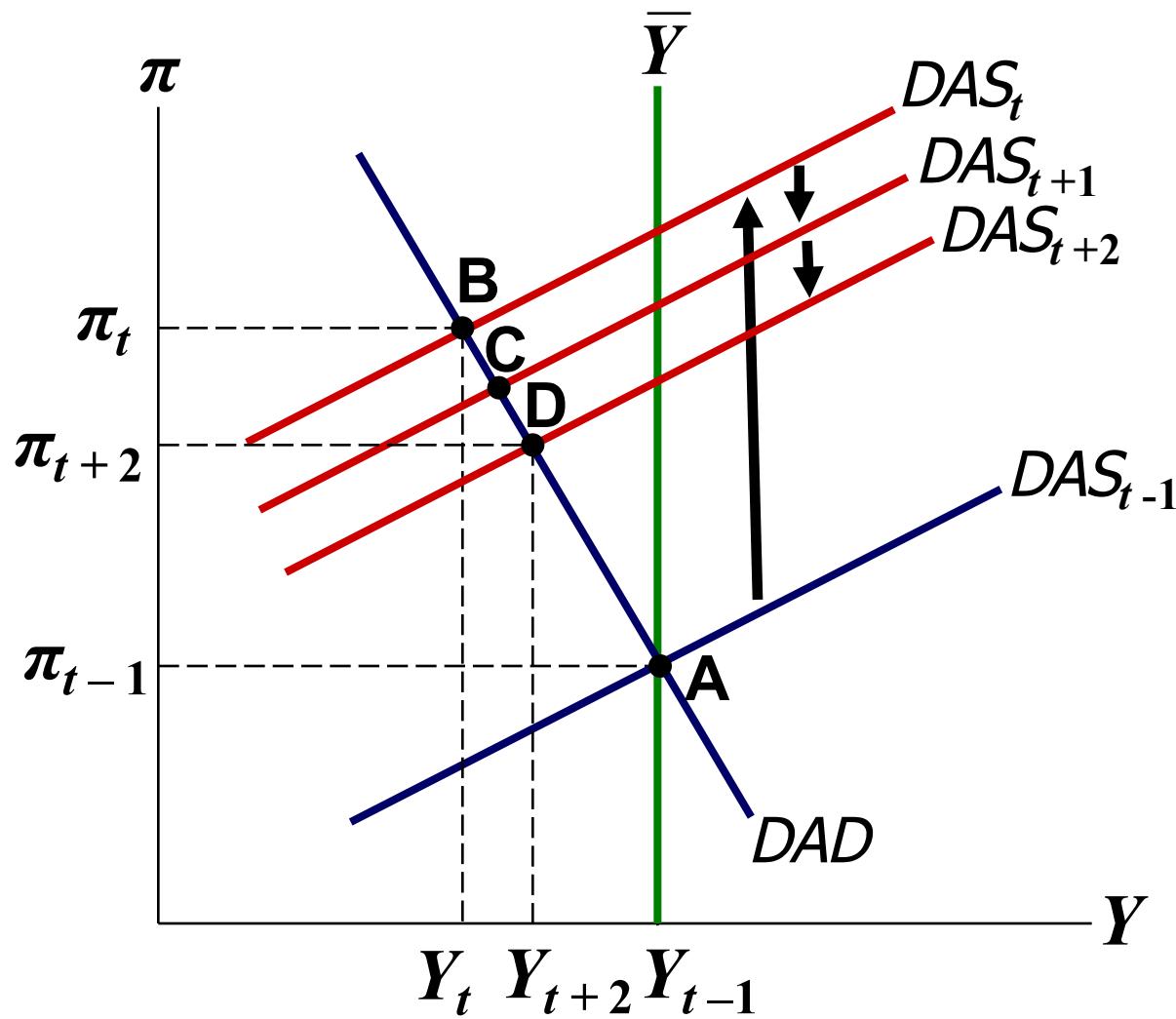
# A shock to aggregate supply



Period  $t - 1$ :  
initial eq'm at A

Period  $t$ :  
Supply shock  
( $\nu > 0$ ) shifts  
DAS upward;  
inflation rises,  
central bank  
responds by  
raising real  
interest rate,  
output falls.

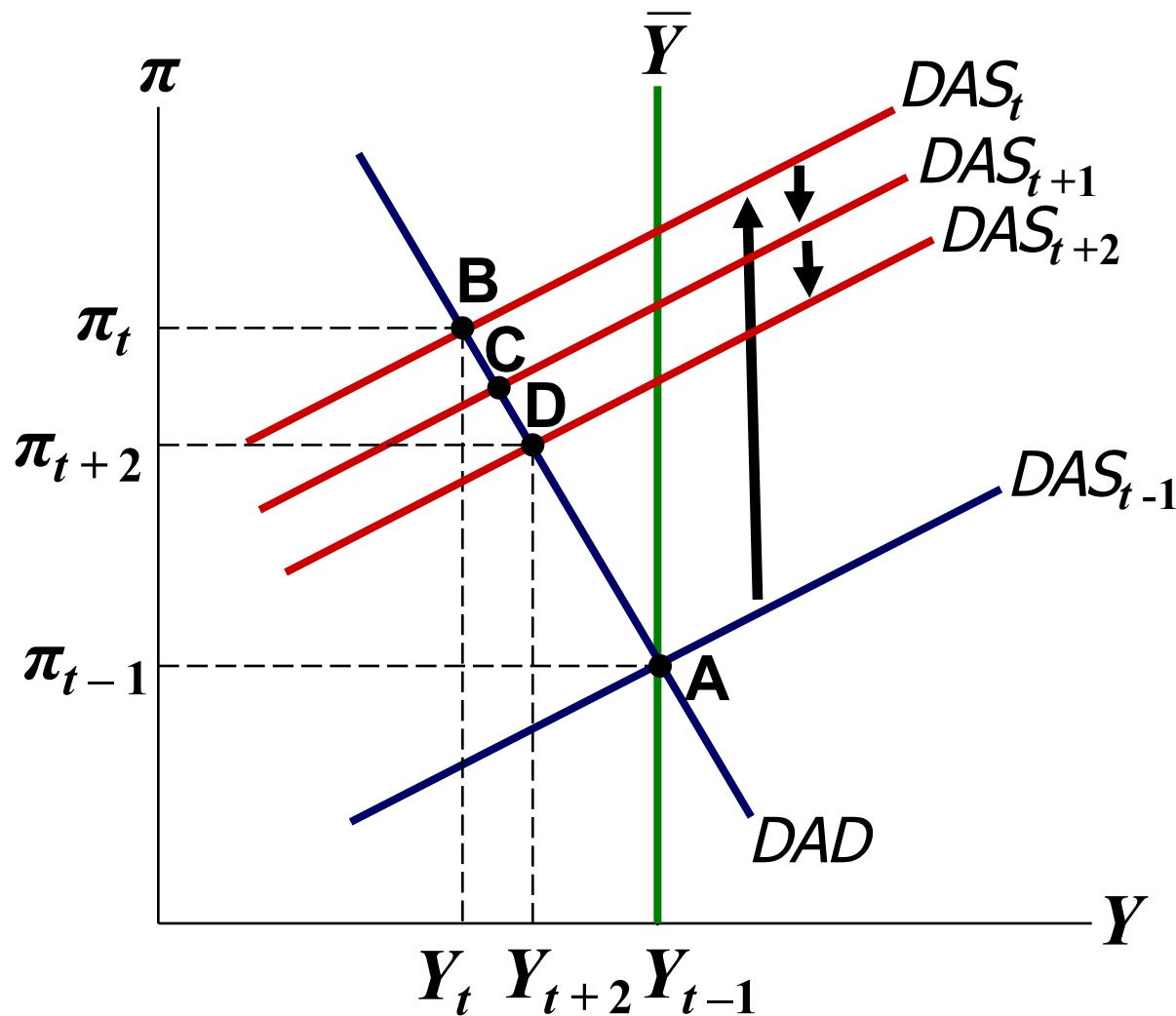
# A shock to aggregate supply



Period  $t + 1$  :  
Supply shock  
is over ( $v = 0$ )  
but DAS does not  
return to its initial  
position due to  
higher inflation  
expectations.

Period  $t + 2$ :  
As inflation falls,  
inflation  
expectations fall,  
DAS moves  
downward,  
output rises.

# A shock to aggregate supply



This process continues until output returns to its natural rate.  
LR eq'm at A.

# Parameter values for simulations

$$\bar{Y}_t = 100$$

$$\pi_t^* = 2.0$$

$$\alpha = 1.0$$

$$\rho = 2.0$$

$$\phi = 0.25$$

$$\theta_\pi = 0.5$$

$$\theta_Y = 0.5$$

Thus, we can interpret  $Y_t - \bar{Y}_t$  as the percentage deviation of output from its natural level.

Central bank's inflation target is 2 percent.

A 1-percentage-point increase in the real interest rate reduces output demand by 1 percent of its natural level.

# Parameter values for simulations

$$\bar{Y}_t = 100$$

$$\pi_t^* = 2.0$$

$$\alpha = 1.0$$

$$\rho = 2.0$$

$$\phi = 0.25$$

$$\theta_\pi = 0.5$$

$$\theta_Y = 0.5$$

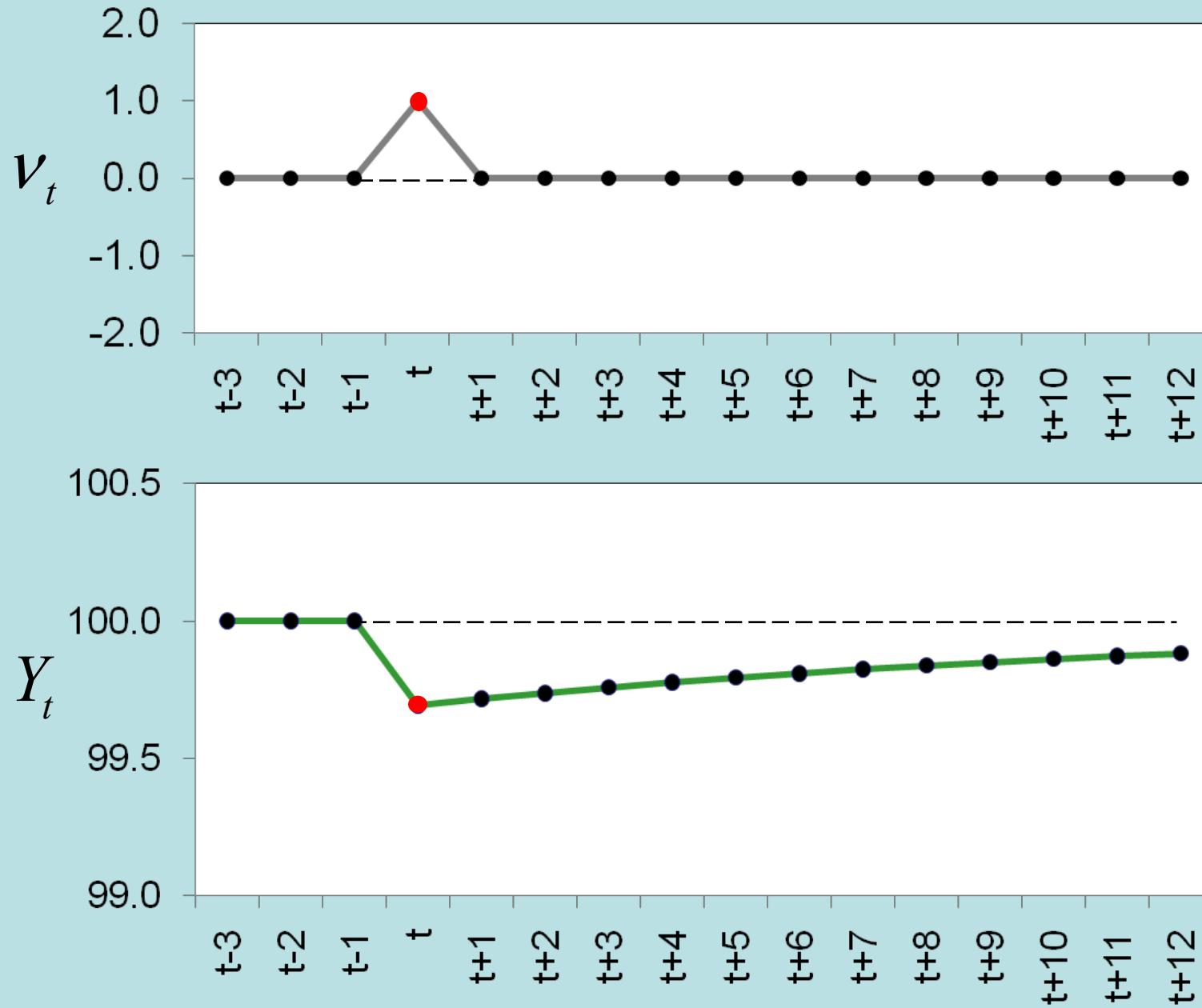
The natural rate of interest is 2 percent.

When output is 1 percent above its natural level, inflation rises by 0.25 percentage point.

These values are from the Taylor rule, which approximates the actual behavior of the Federal Reserve.

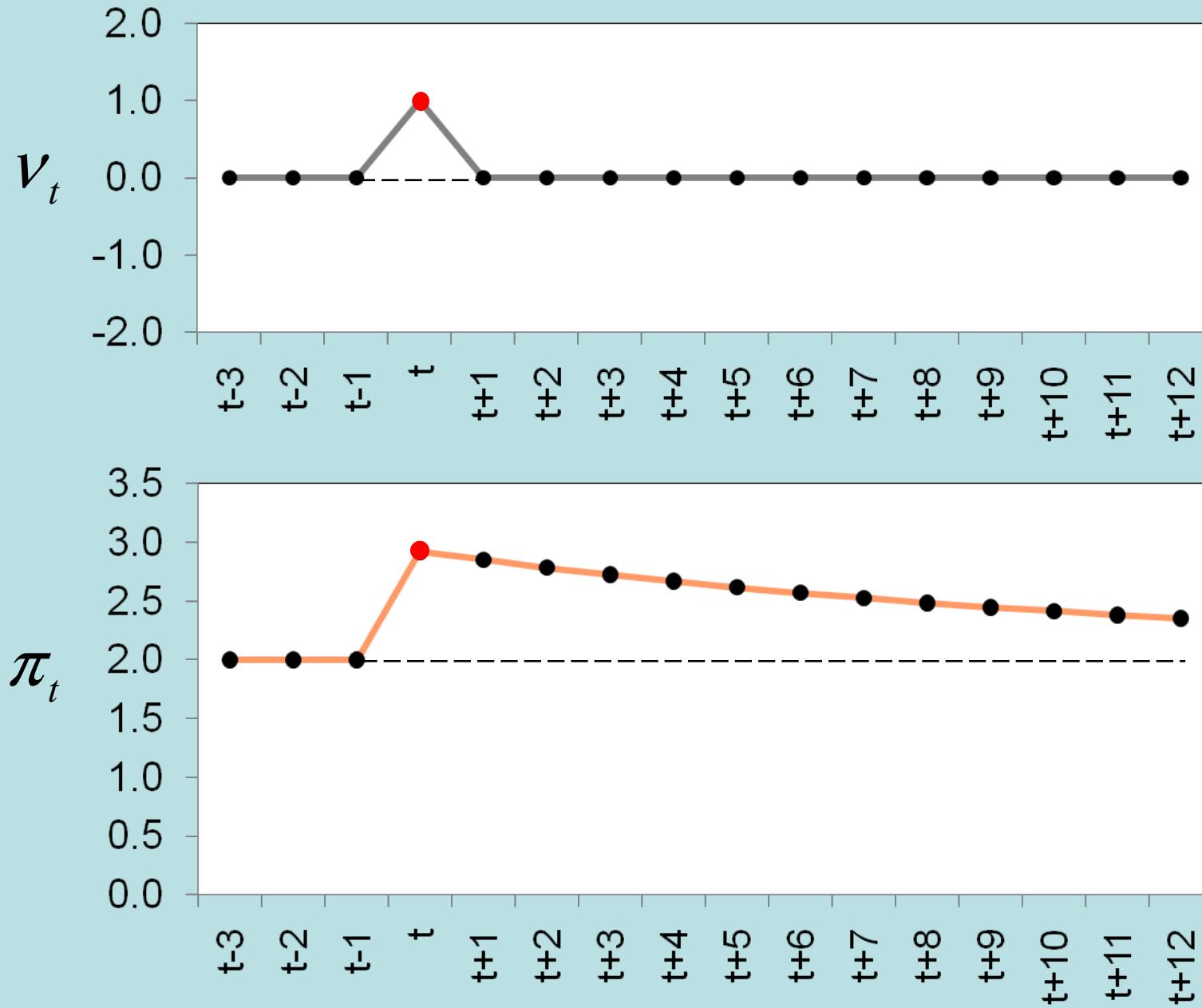
The following graphs are called ***impulse response functions***. They show the *response* of the endogenous variables to the *impulse* (the shock).

# The dynamic response to a supply shock



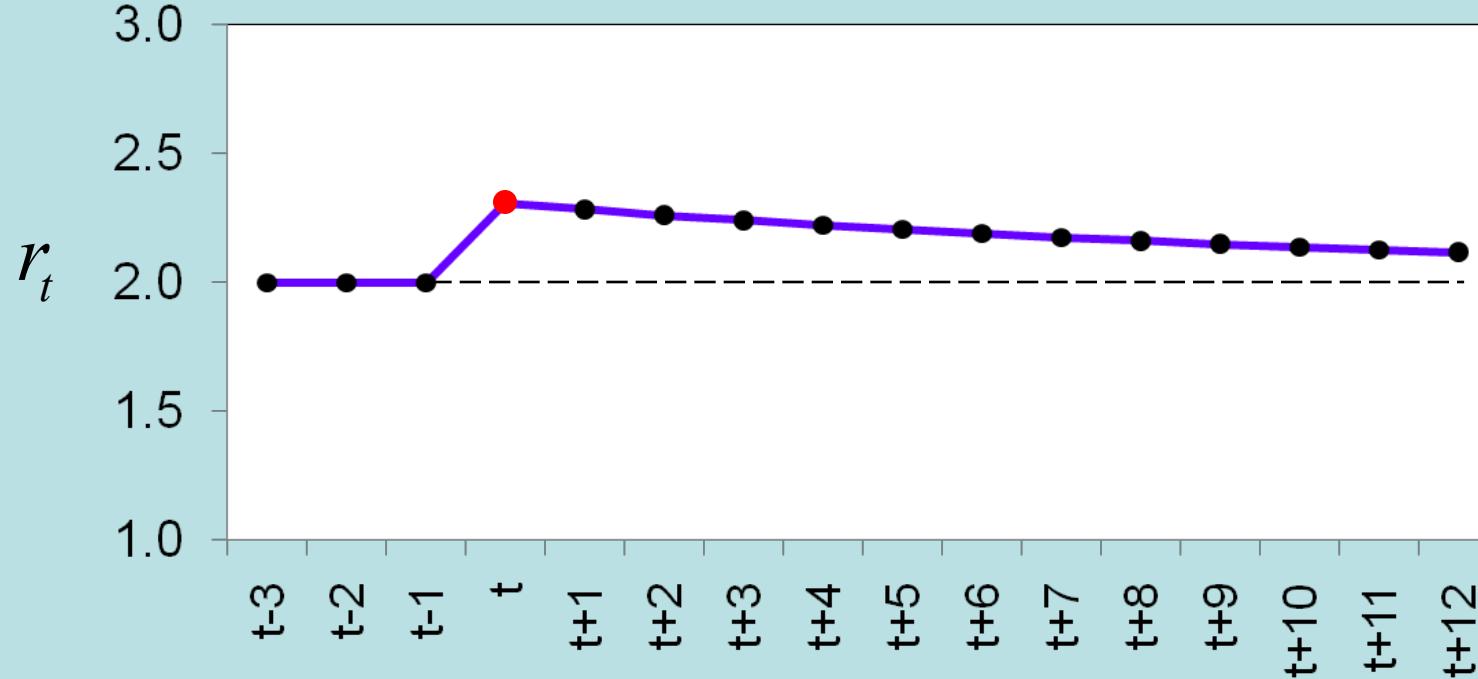
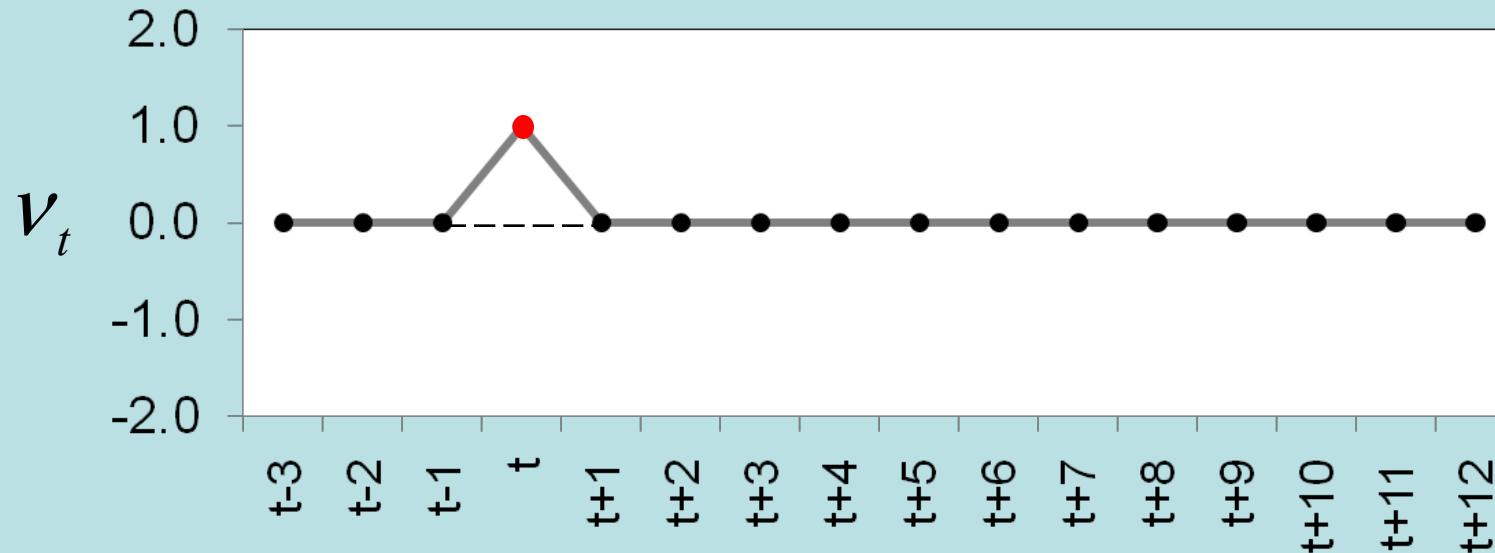
A one-period supply shock affects output for many periods.

# The dynamic response to a supply shock



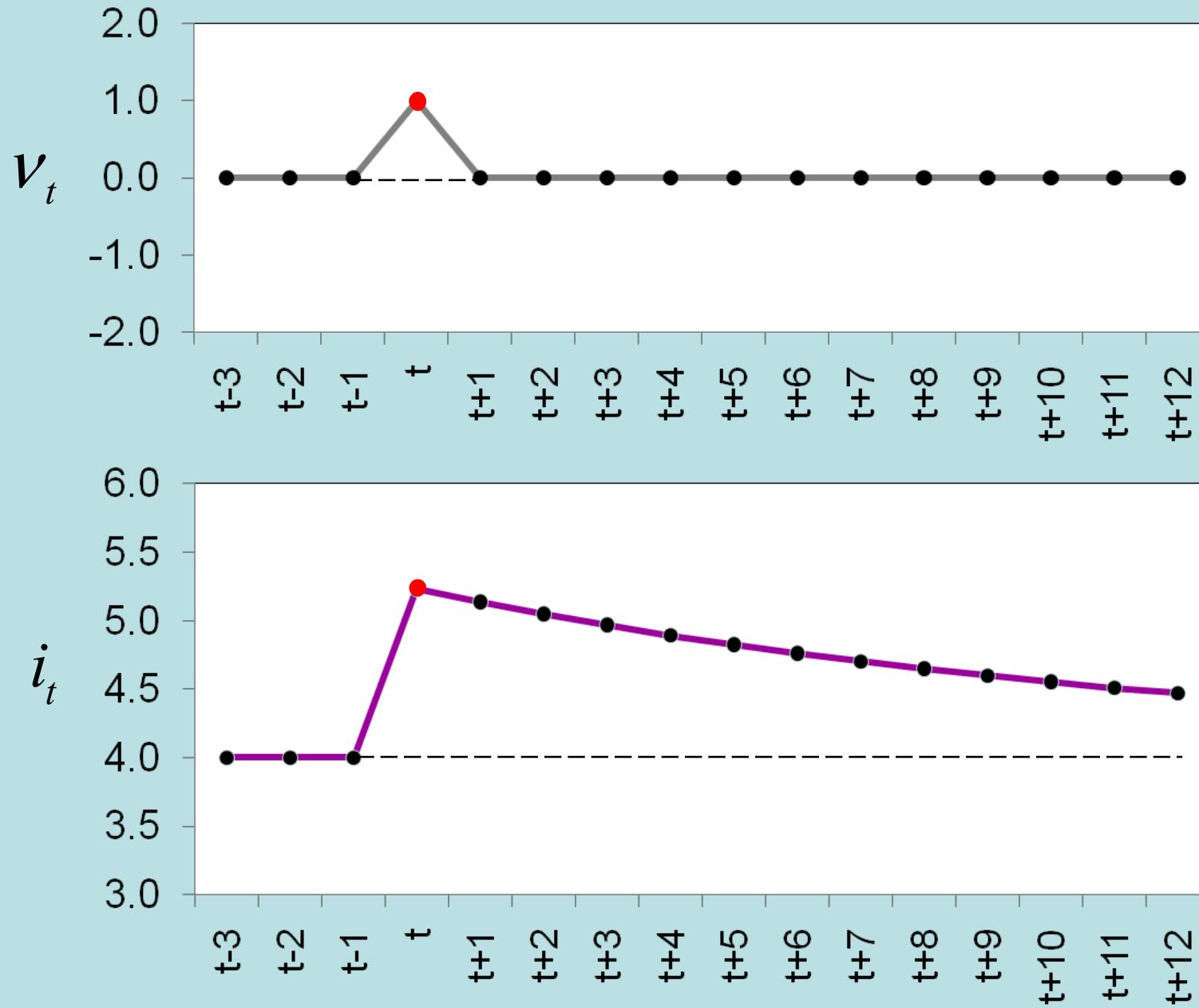
Because inflation expectations adjust slowly, actual inflation remains high for many periods.

# The dynamic response to a supply shock



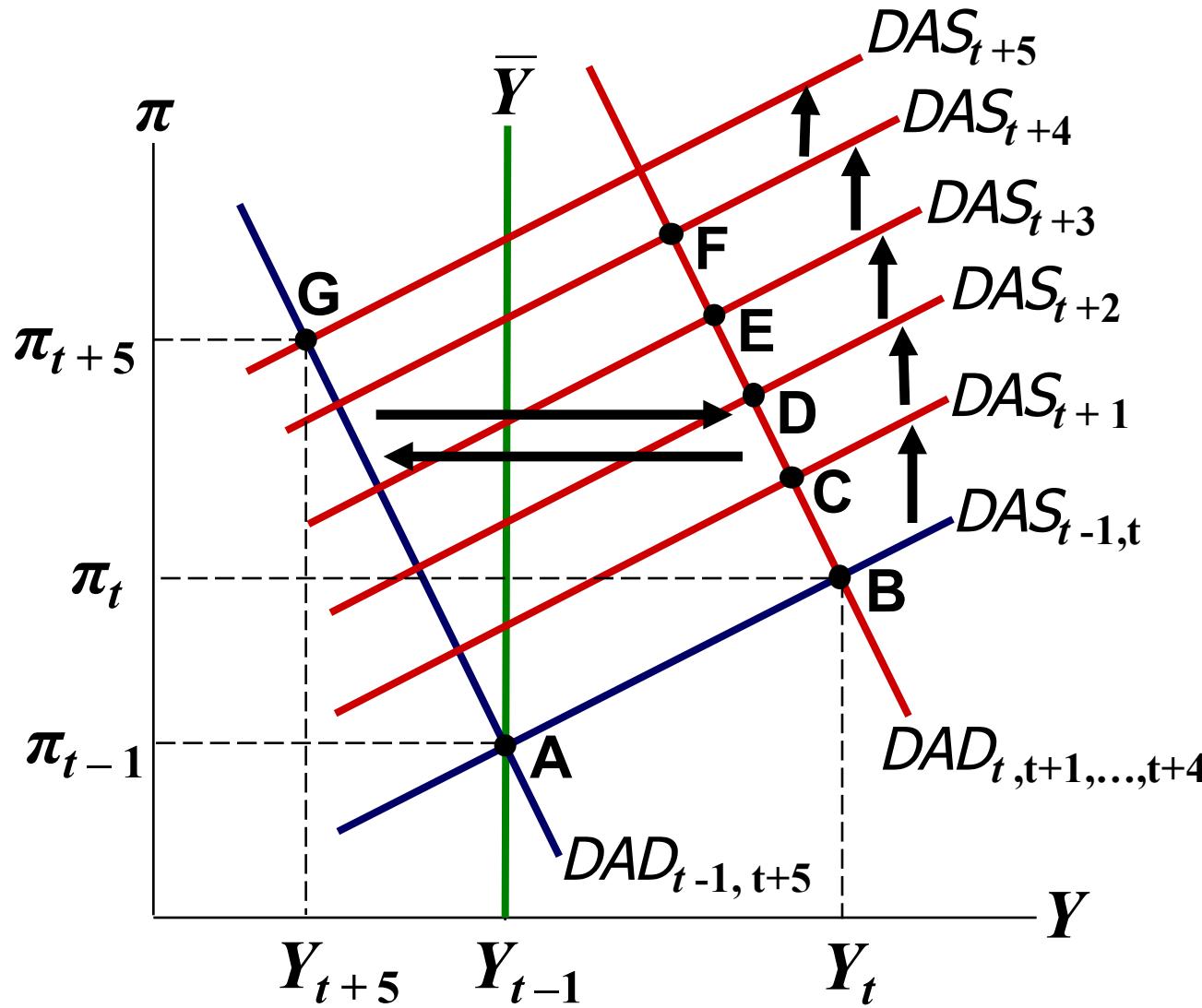
The real interest rate takes many periods to return to its natural rate.

# The dynamic response to a supply shock



The behavior of the nominal interest rate depends on that of the inflation and real interest rates.

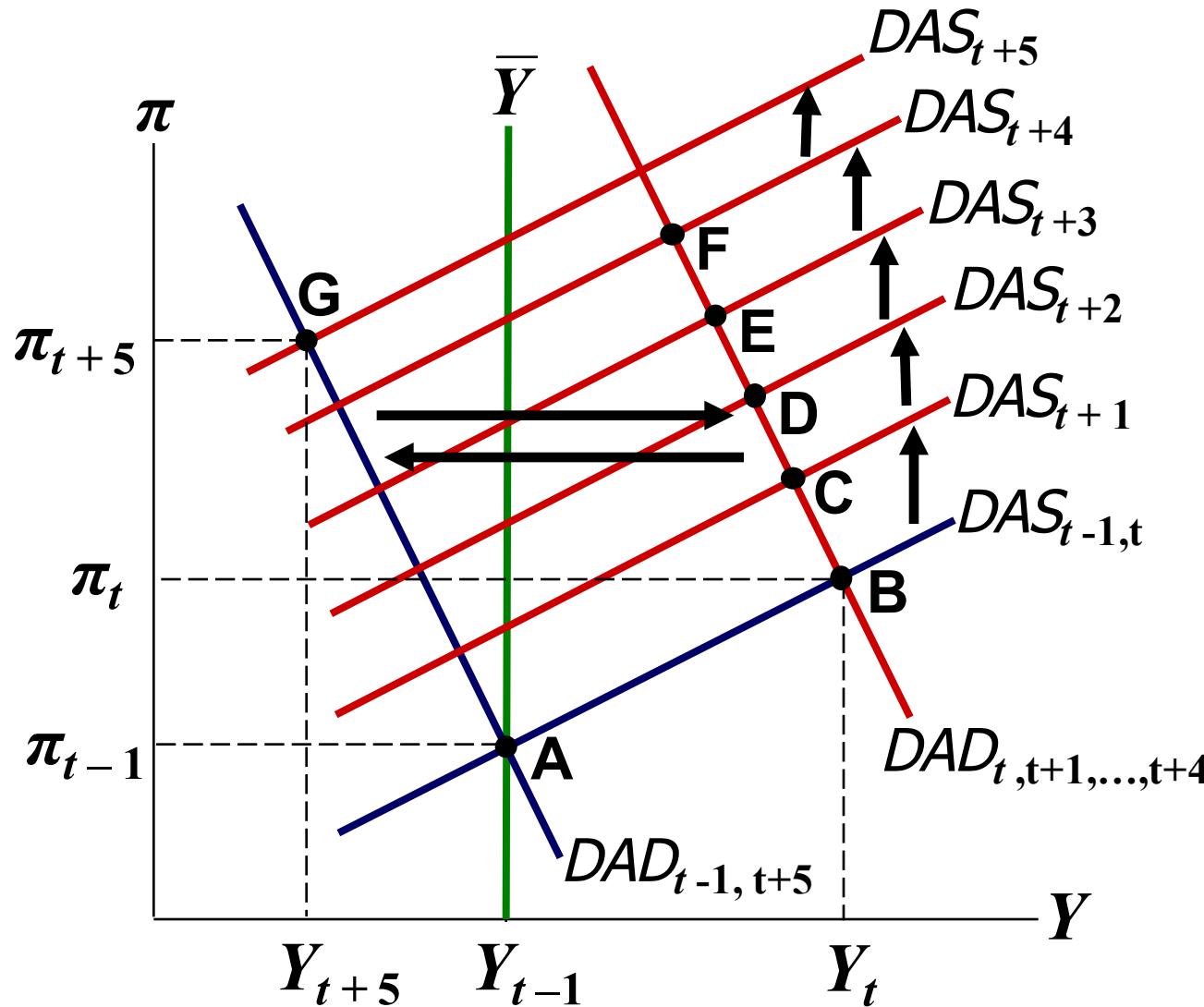
# A shock to aggregate demand



Period  $t - 1$ :  
initial eq'm at A

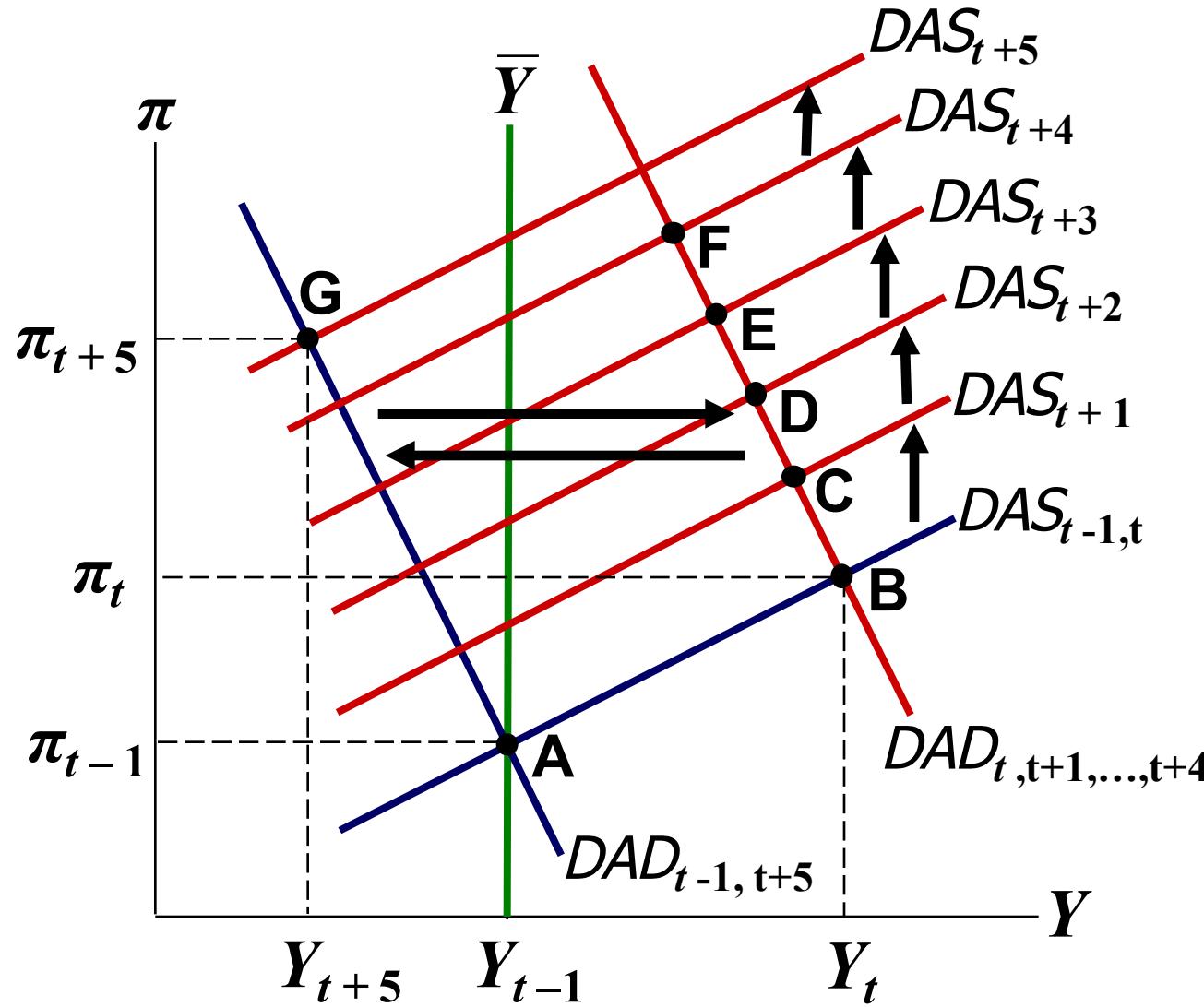
Period  $t$ :  
Positive  
demand shock  
( $\varepsilon > 0$ ) shifts  
 $DAD$  to the  
right; output  
and inflation  
rise.

# A shock to aggregate demand



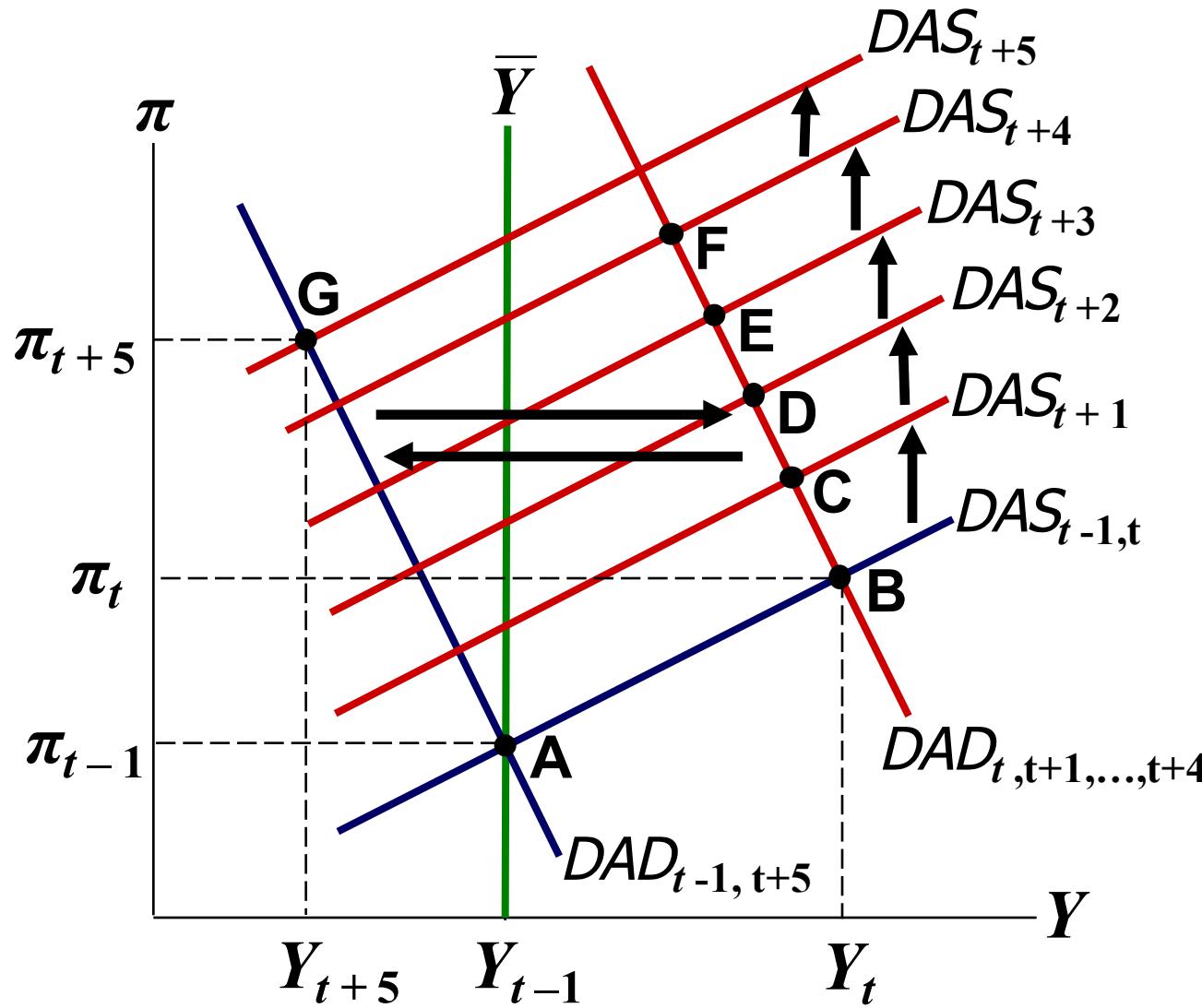
Period  $t + 1$  :  
Higher inflation in  
 $t$  raised inflation  
expectations  
for  $t + 1$ ,  
shifting DAS up.  
Inflation rises  
more, output falls.

# A shock to aggregate demand



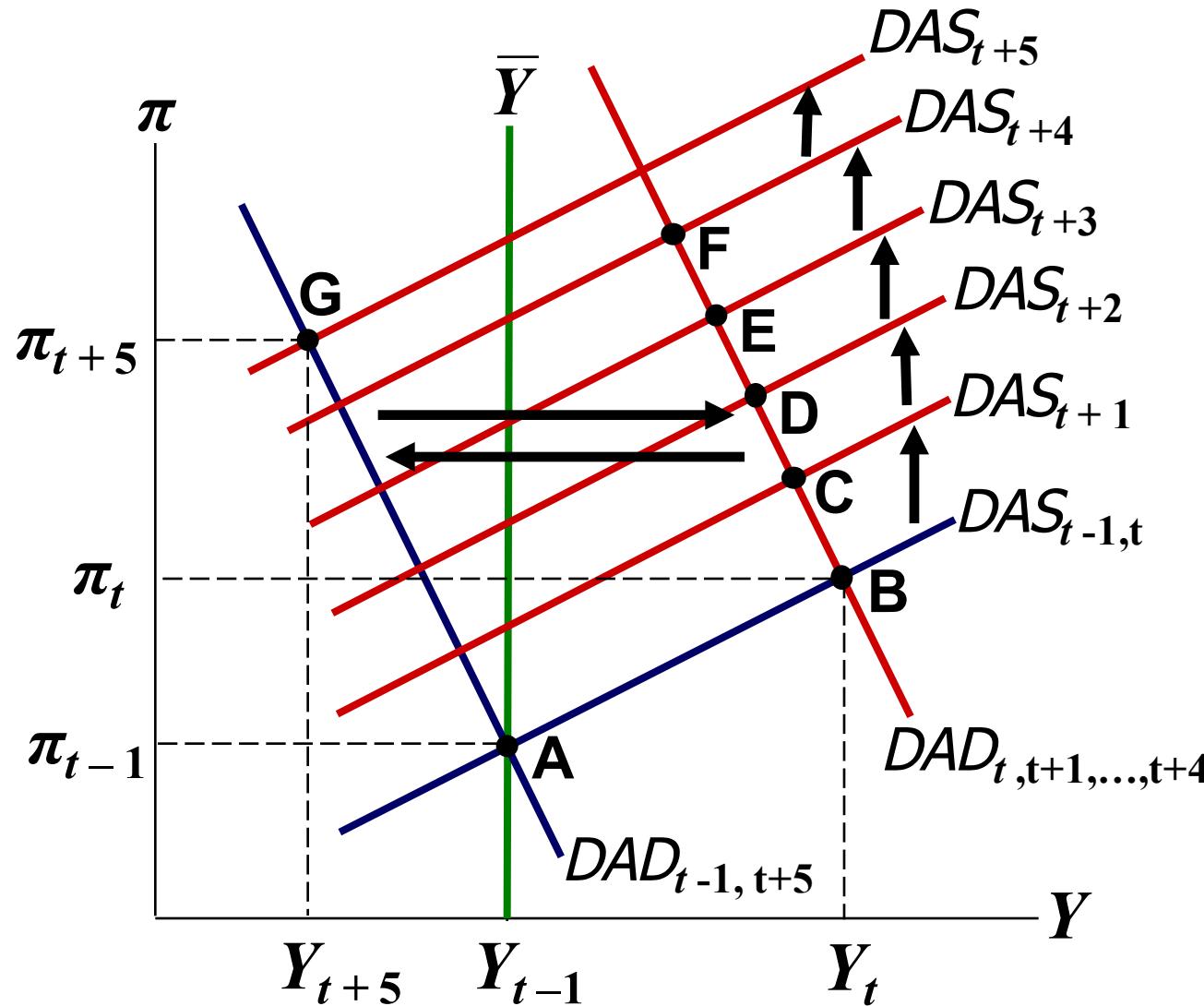
Periods  $t + 2$  to  $t + 4$  :  
Higher inflation in previous period raises inflation expectations, shifts DAS up. Inflation rises, output falls.

# A shock to aggregate demand



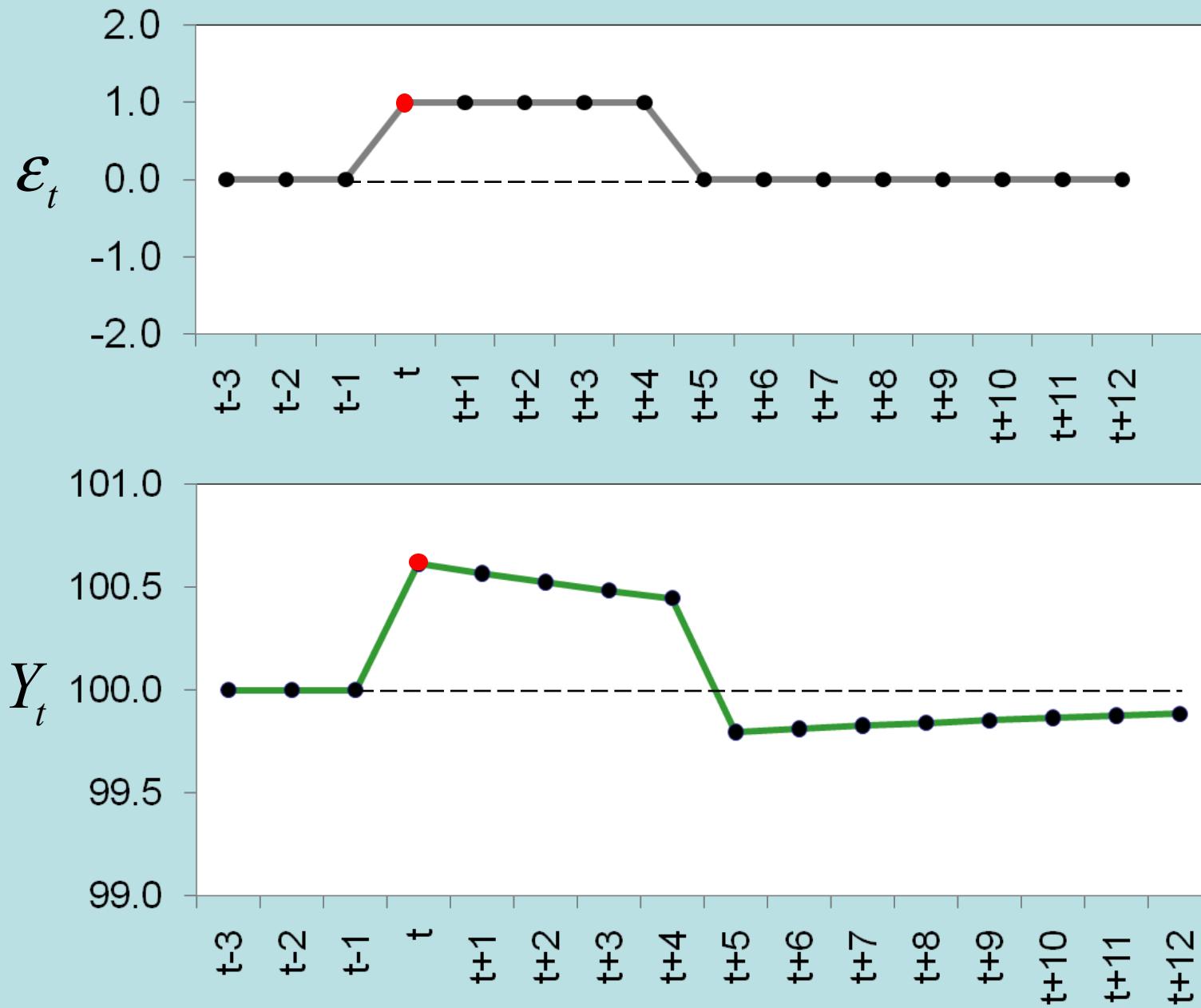
Period  $t + 5$ :  
 DAS is higher  
 due to higher  
 inflation in  
 preceding period,  
 but demand  
 shock ends and  
 DAD returns to  
 its initial position.  
 Eq'm at **G**.

# A shock to aggregate demand



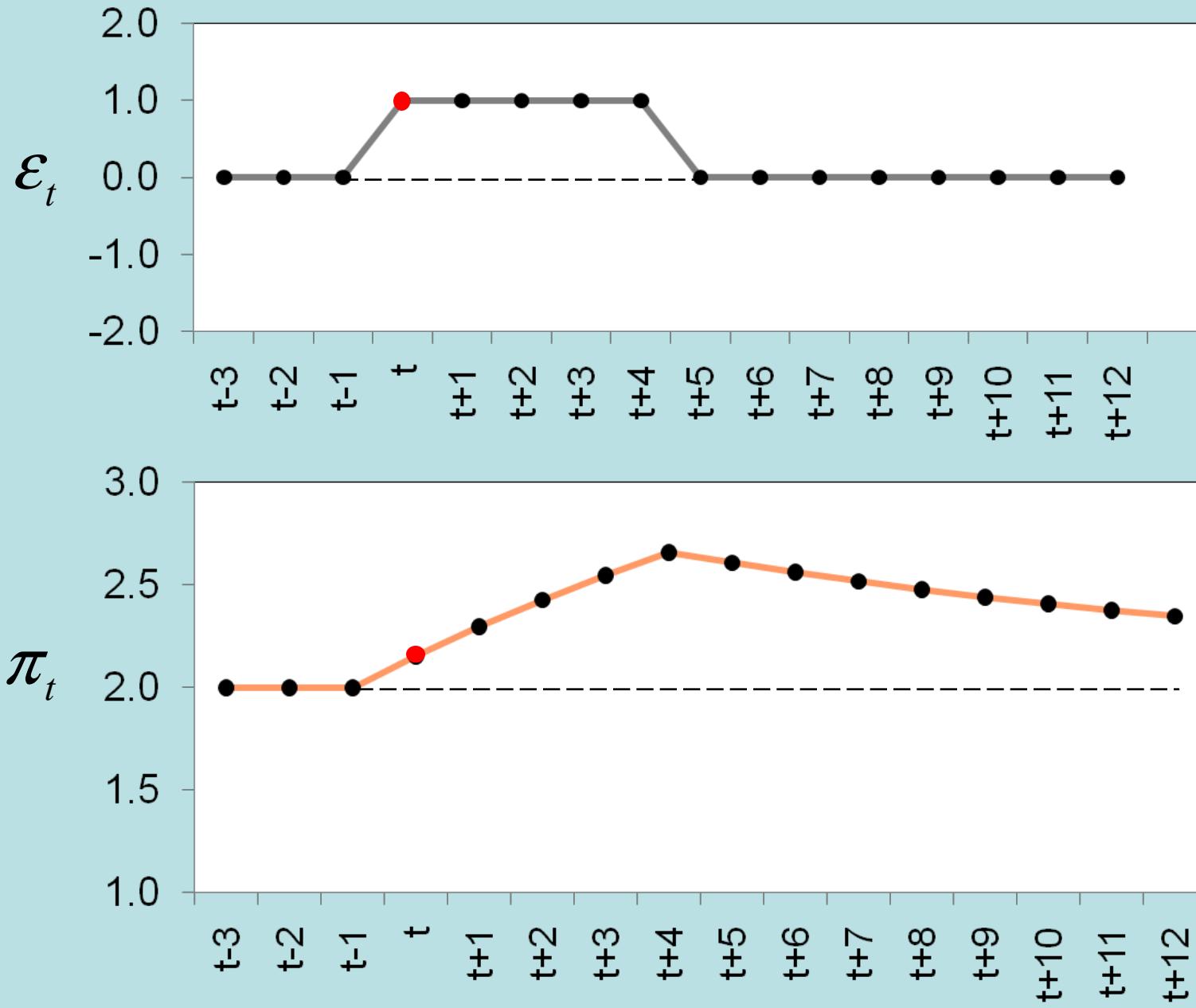
Periods  $t + 6$  and higher:  
DAS gradually shifts down as inflation and inflation expectations fall, economy gradually recovers until reaching LR eq'm at A.

# The dynamic response to a demand shock



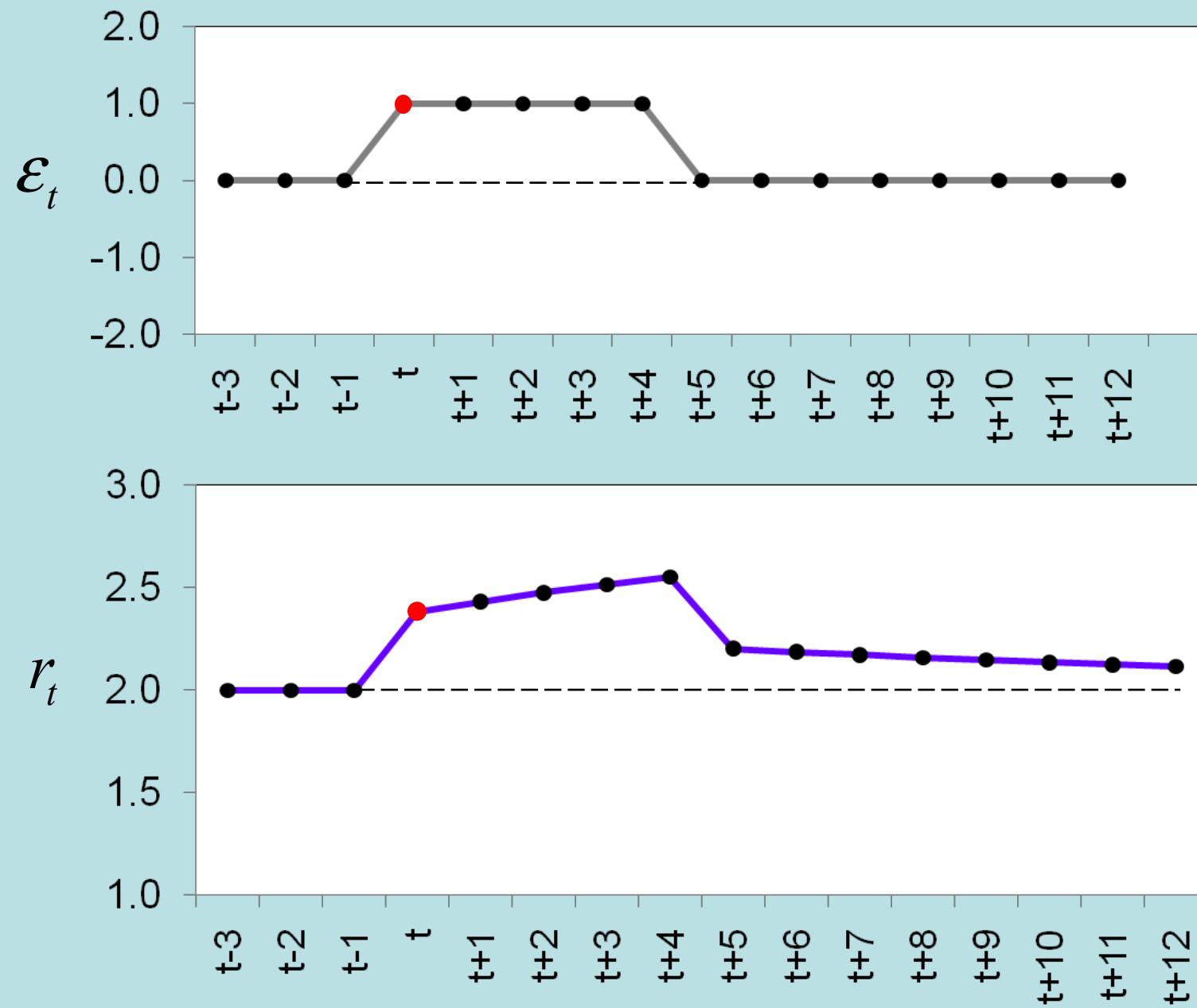
The demand shock raises output for five periods. When the shock ends, output falls below its natural level and recovers gradually.

# The dynamic response to a demand shock



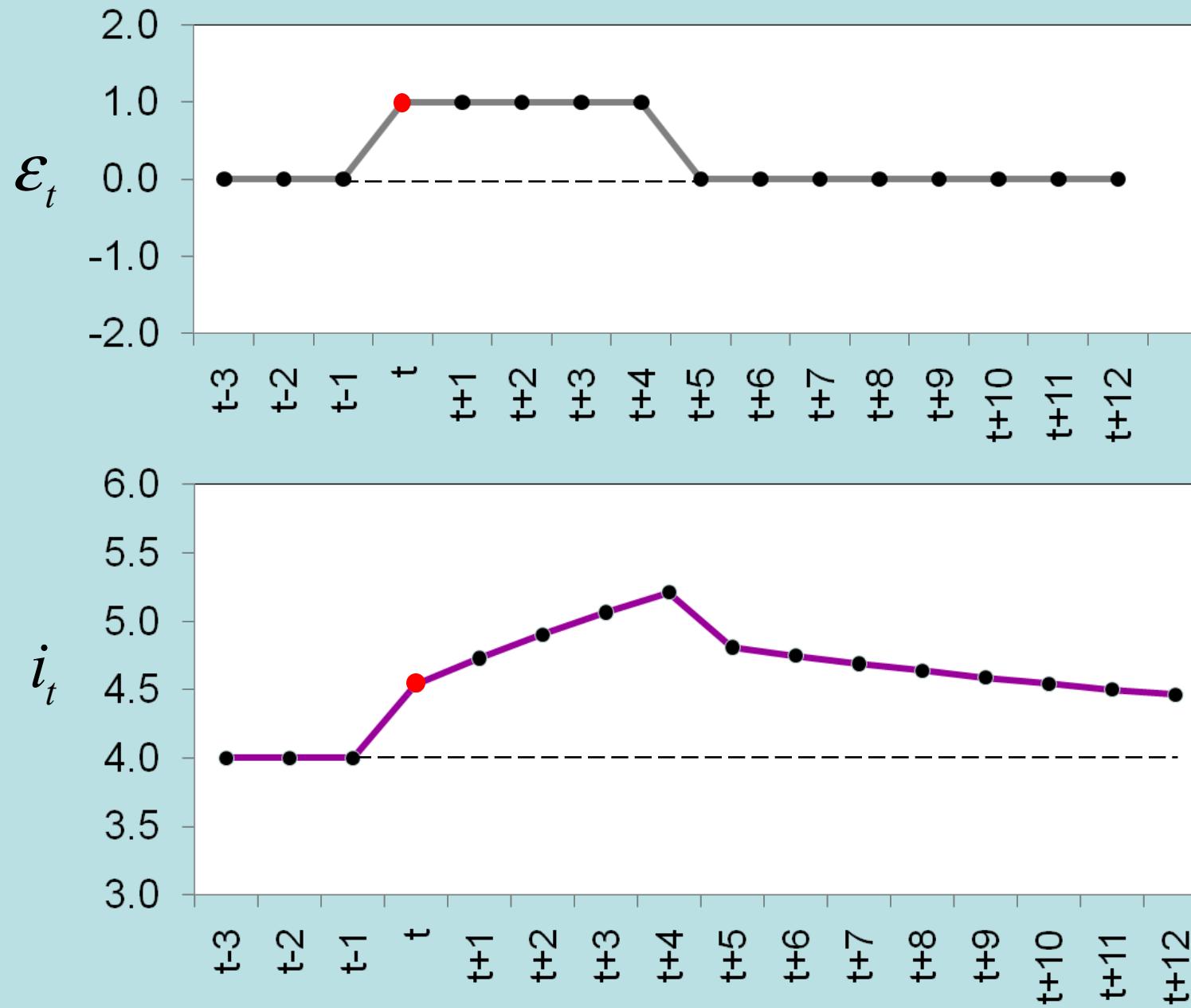
The demand shock causes inflation to rise. When the shock ends, inflation gradually falls toward its initial level.

# The dynamic response to a demand shock



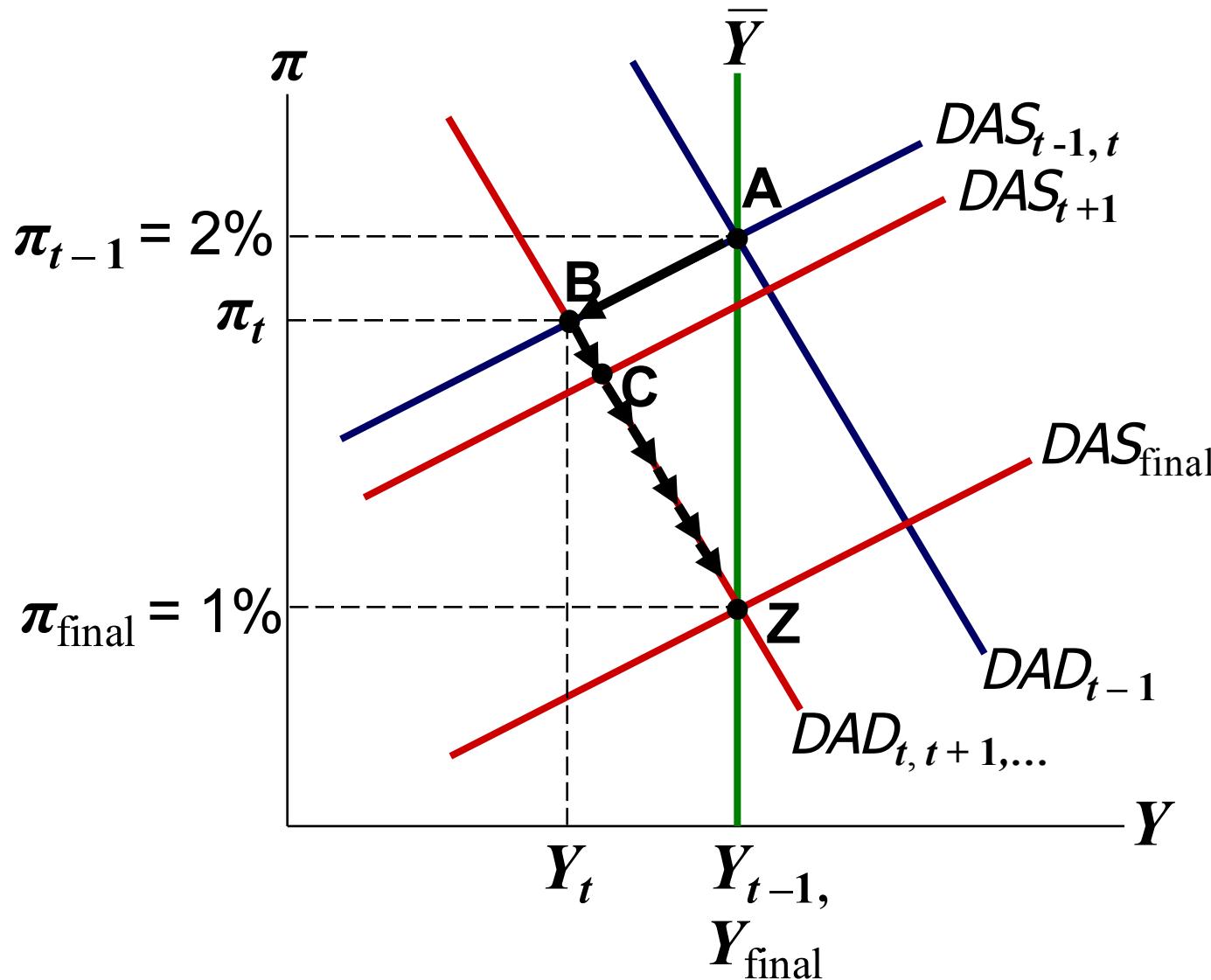
The demand shock raises the real interest rate. After the shock ends, the real interest rate falls and approaches its initial level.

# The dynamic response to a demand shock



The behavior of the nominal interest rate depends on that of the inflation and real interest rates.

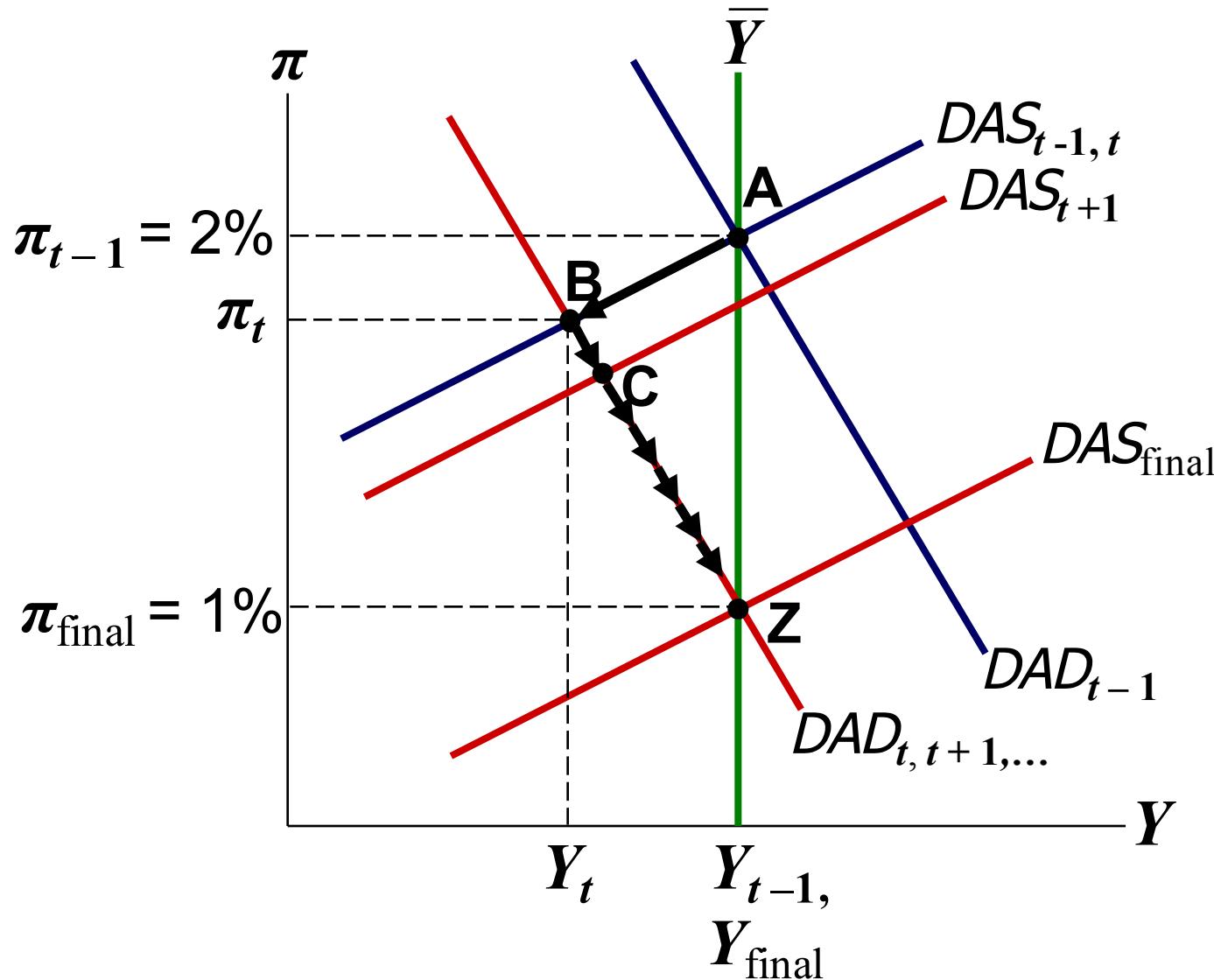
# A shift in monetary policy



Period  $t - 1$ :  
target inflation  
rate  $\pi^* = 2\%$ ,  
initial eq'm at A

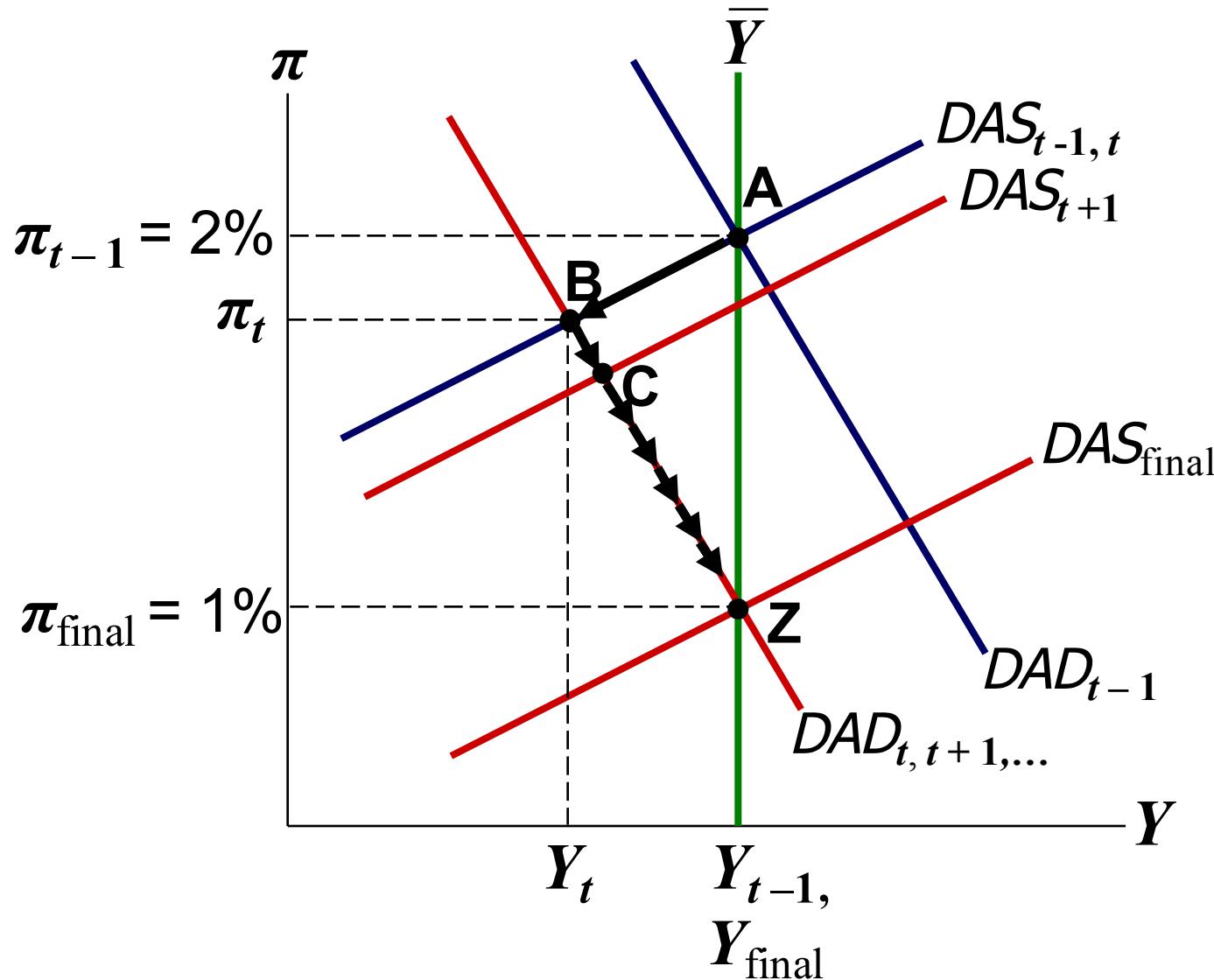
Period  $t$ :  
Central bank  
lowers target  
to  $\pi^* = 1\%$ ,  
raises real  
interest rate,  
shifts DAD  
leftward.  
Output and  
inflation fall.

# A shift in monetary policy



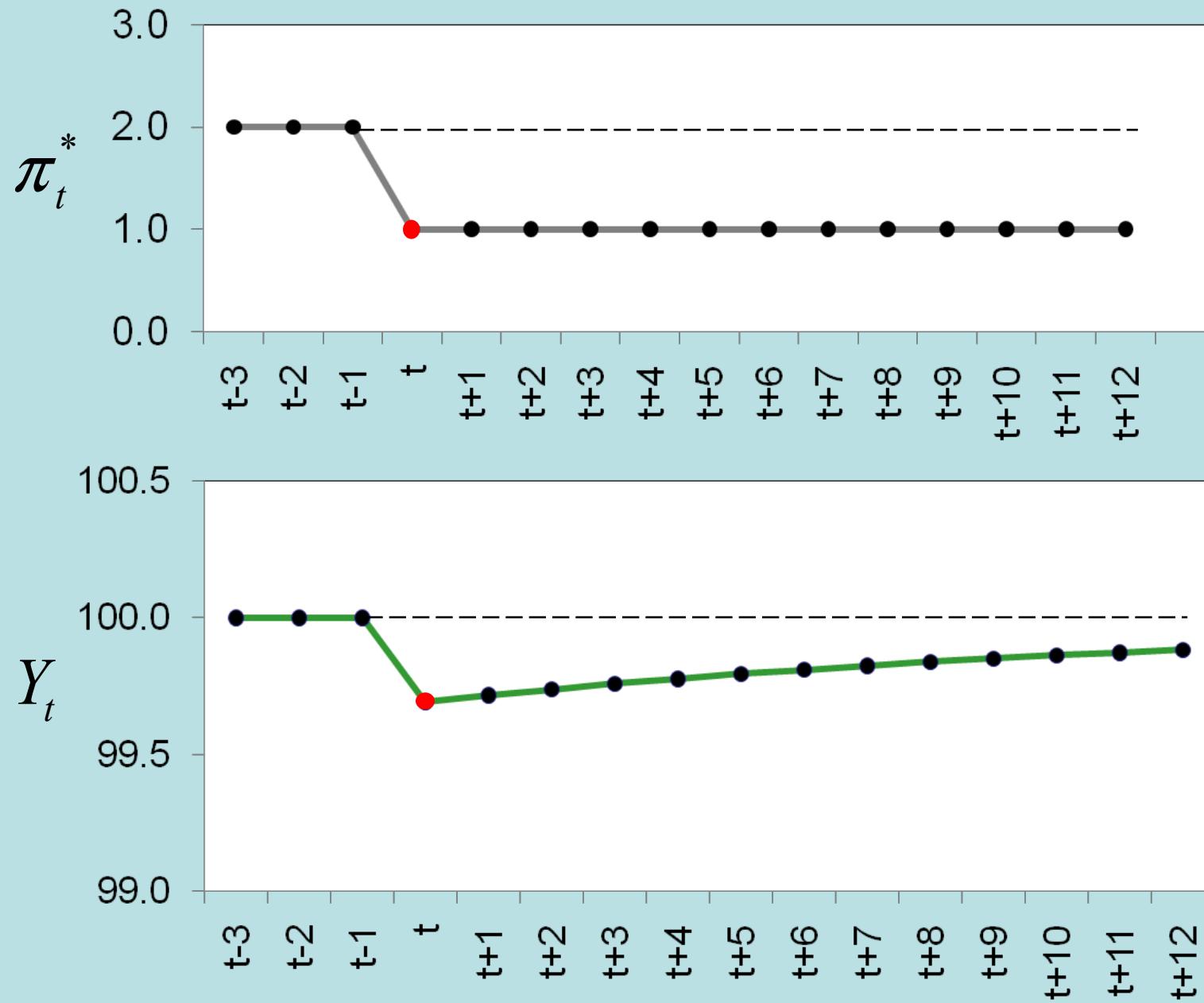
Period  $t + 1$  :  
The fall in  $\pi_t$   
reduced  
inflation  
expectations  
for  $t + 1$ ,  
shifting DAS  
downward.  
Output rises,  
inflation falls.

# A shift in monetary policy



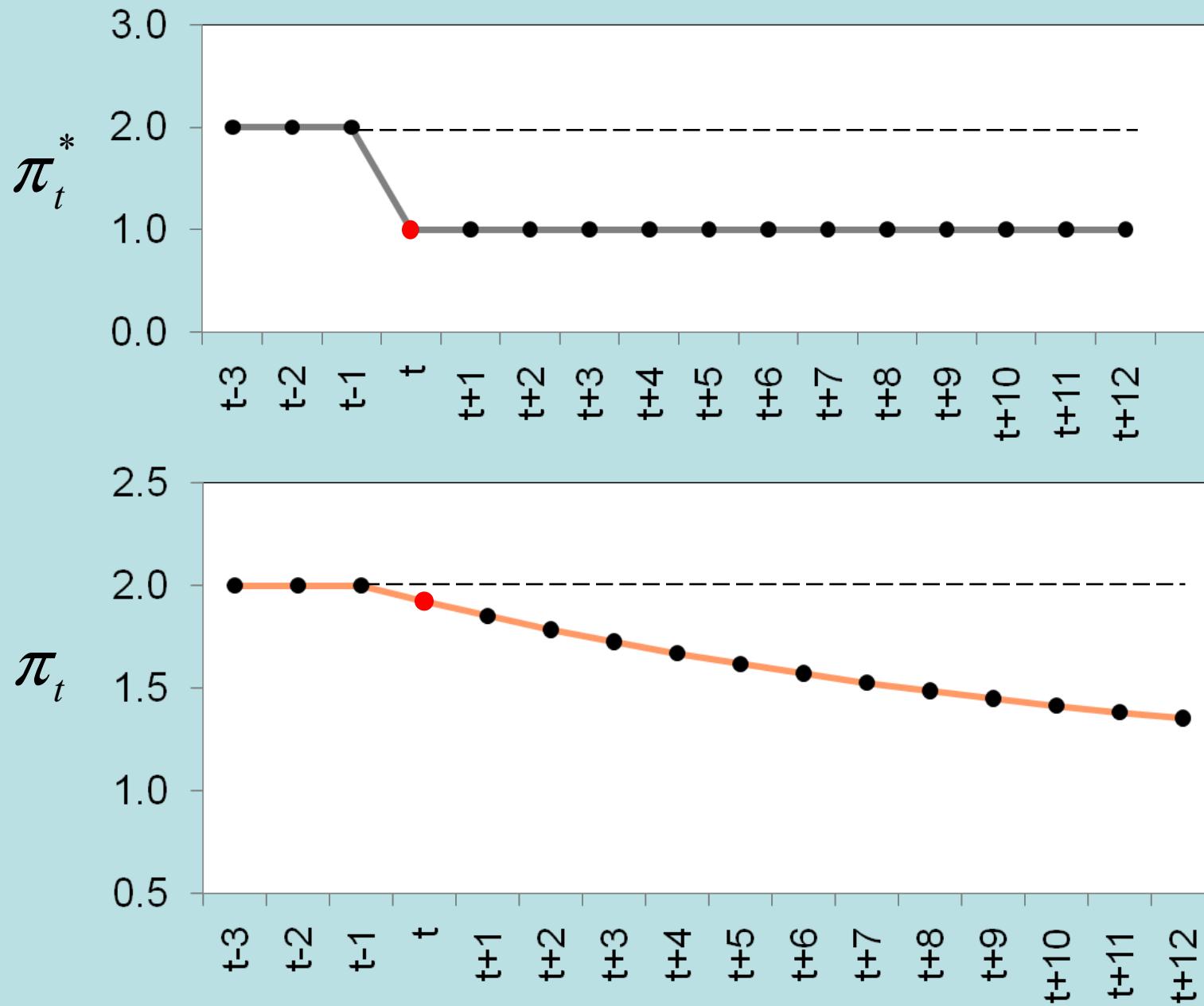
Subsequent periods:  
This process continues until output returns to its natural rate and inflation reaches its new target.

# The dynamic response to a reduction in target inflation



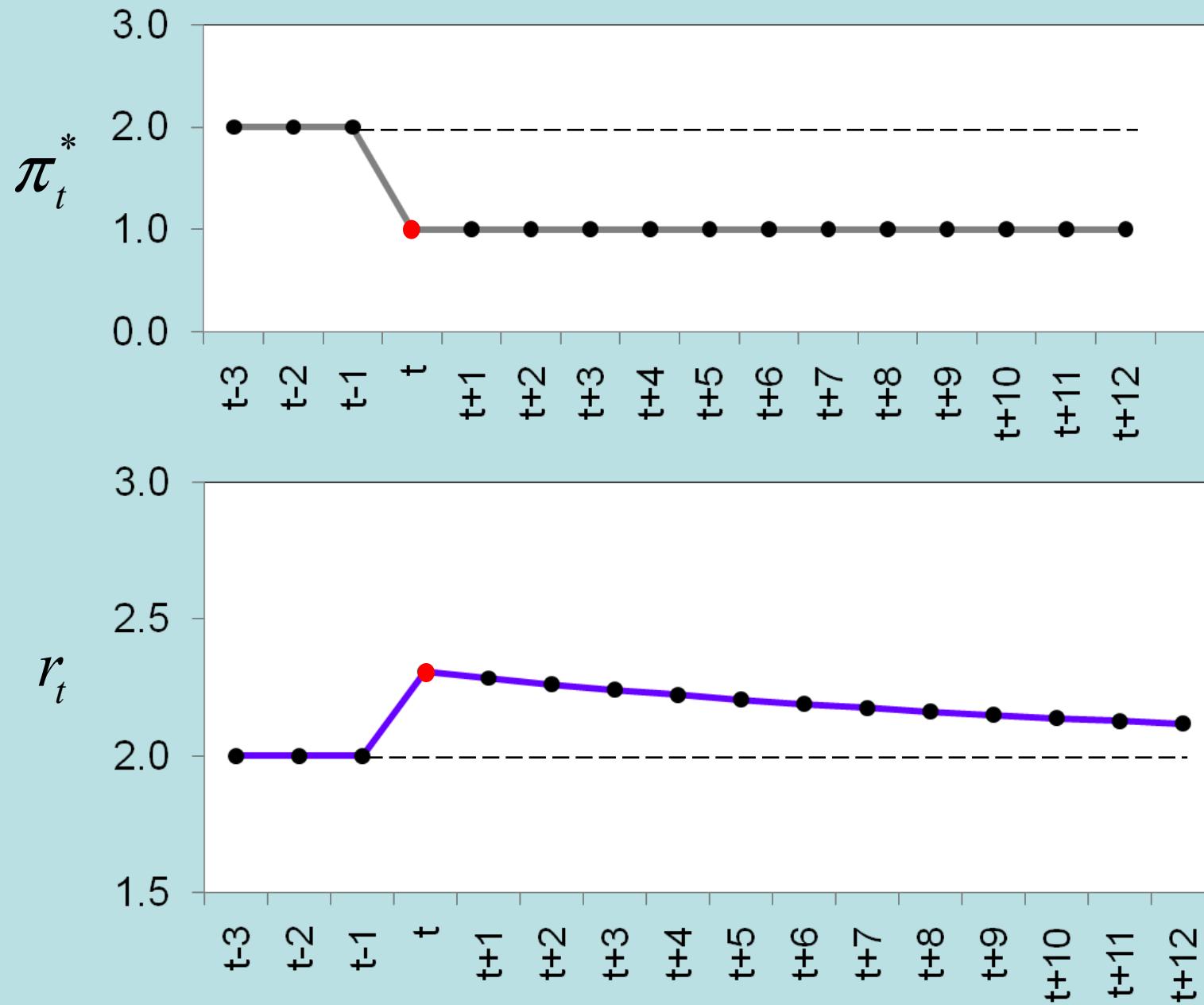
Reducing the target inflation rate causes output to fall below its natural level for a while. Output recovers gradually.

# The dynamic response to a reduction in target inflation



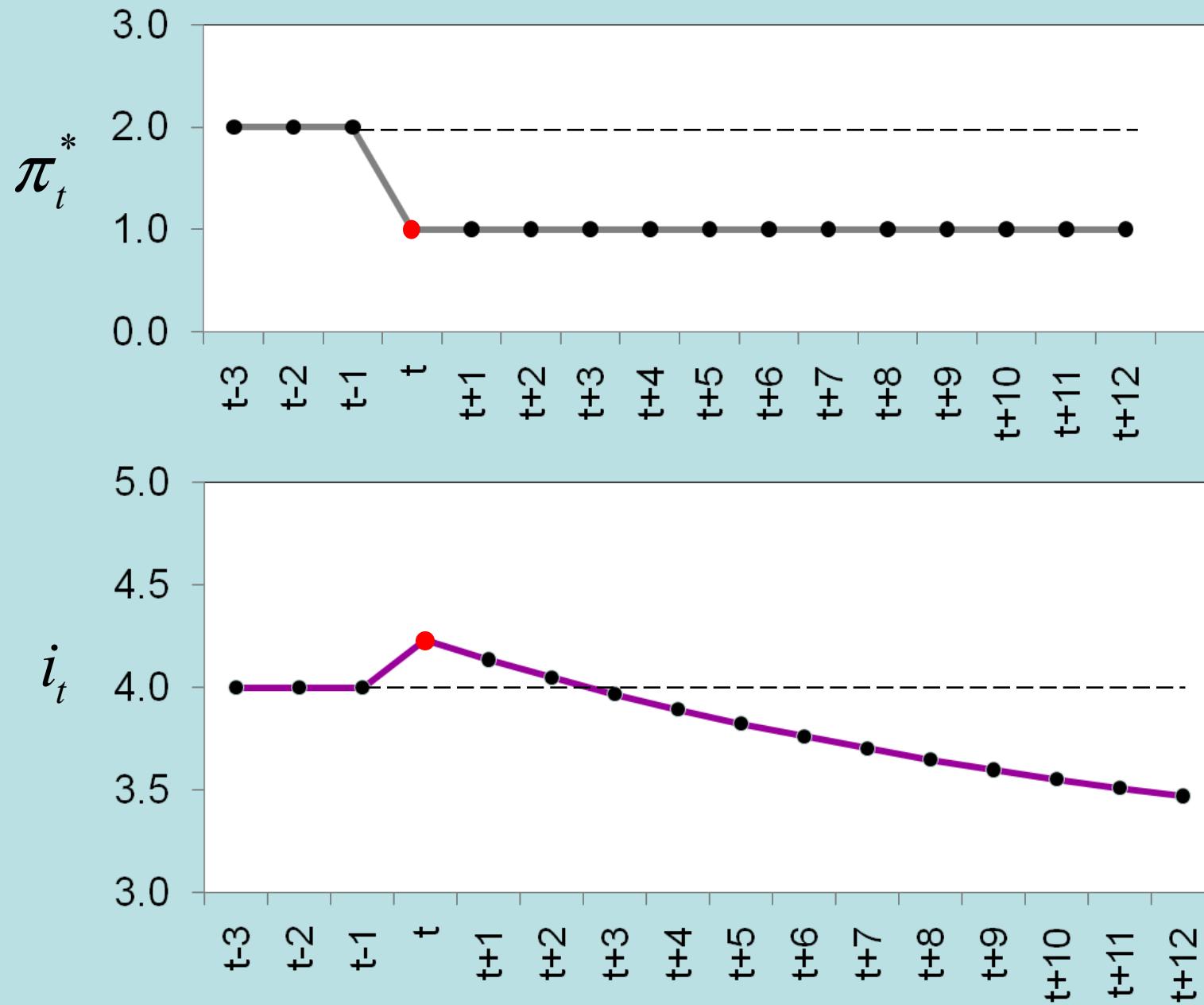
Because expectations adjust slowly, it takes many periods for inflation to reach the new target.

# The dynamic response to a reduction in target inflation



To reduce inflation, the central bank raises the real interest rate to reduce aggregate demand. The real interest rate gradually returns to its natural rate.

# The dynamic response to a reduction in target inflation



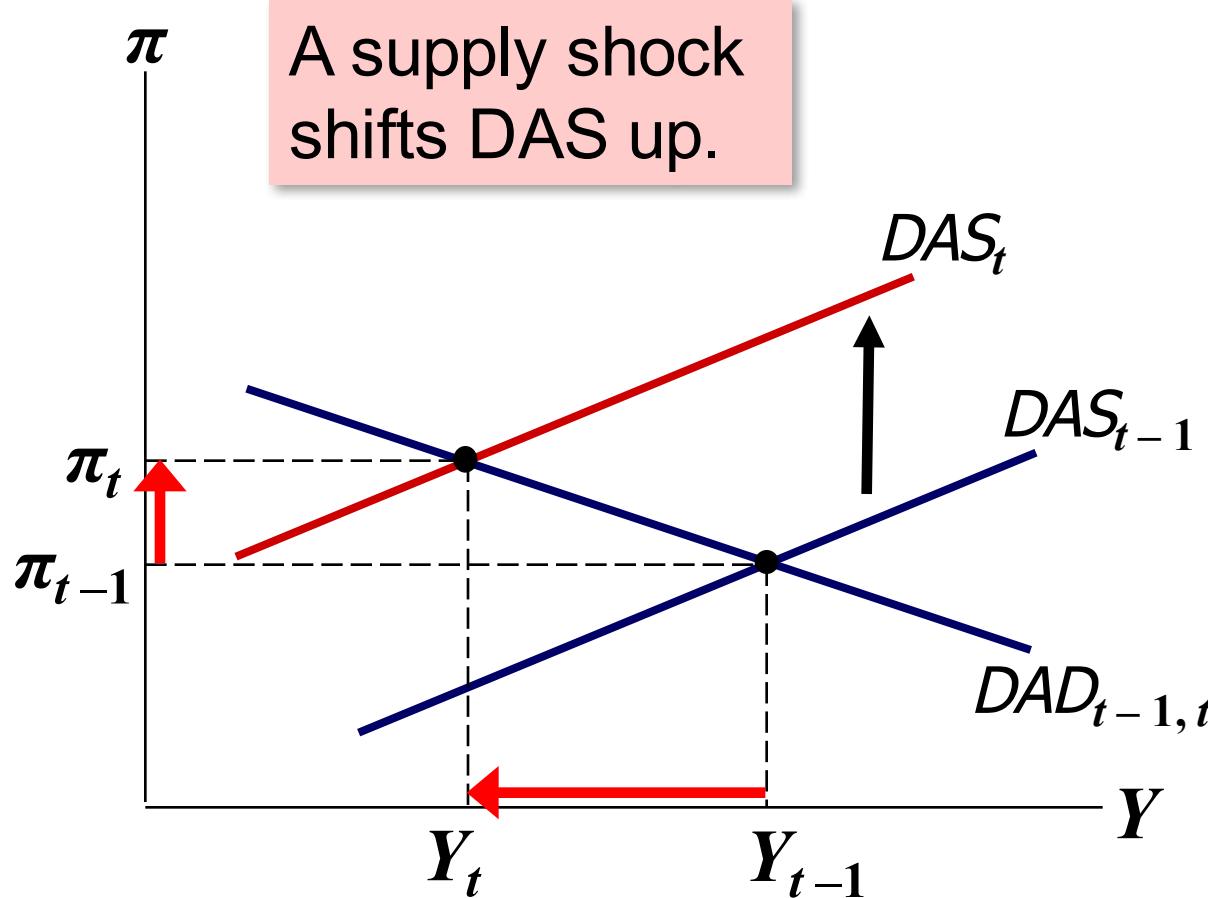
The initial increase in the real interest rate raises the nominal interest rate. As the inflation and real interest rates fall, the nominal rate falls.

## **APPLICATION: Output variability vs. inflation variability**

- A supply shock reduces output (bad) and raises inflation (also bad).
- The central bank faces a tradeoff between these “bads” – it can reduce the effect on output, but only by tolerating an increase in the effect on inflation....

# APPLICATION: Output variability vs. inflation variability

CASE 1:  $\theta_\pi$  is large,  $\theta_Y$  is small

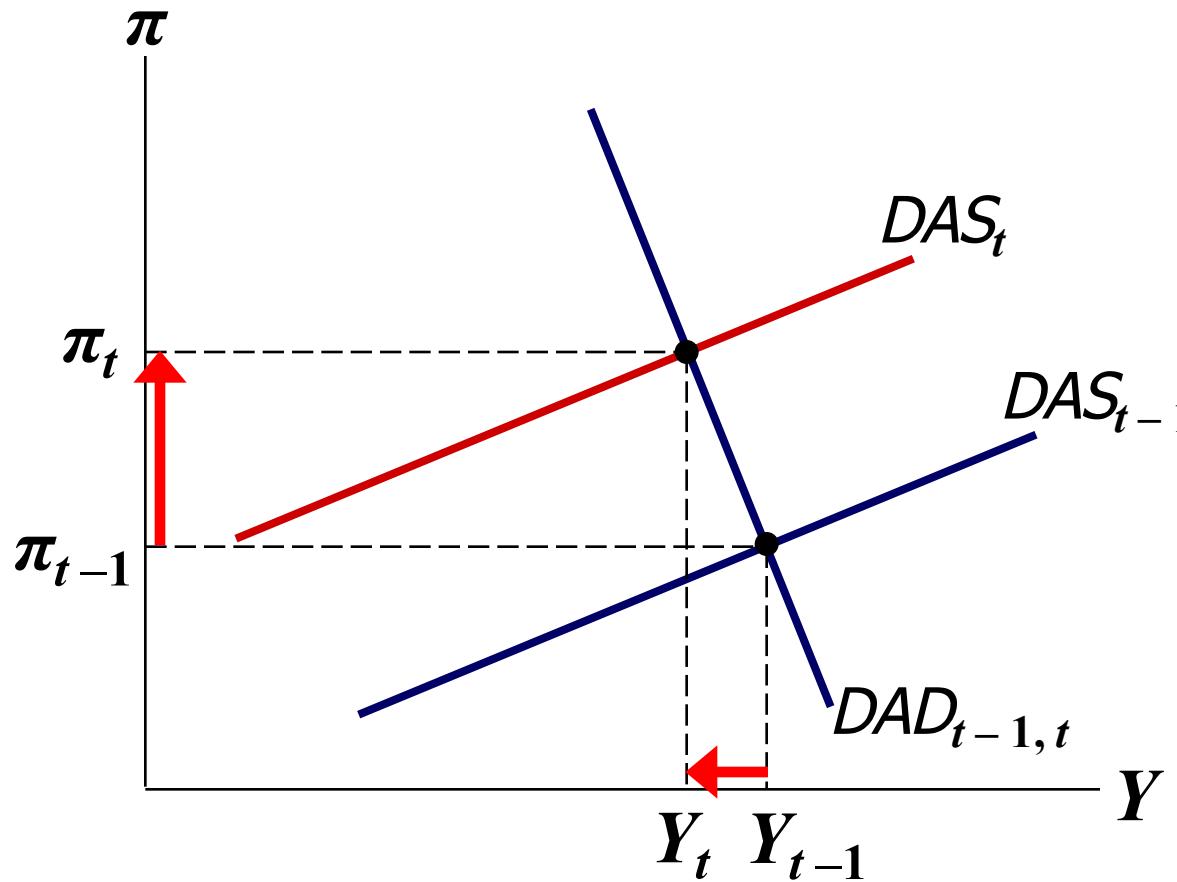


In this case, a small change in inflation has a large effect on output, so DAD is relatively flat.

The shock has a large effect on output but a small effect on inflation.

# APPLICATION: Output variability vs. inflation variability

CASE 2:  $\theta_\pi$  is small,  $\theta_Y$  is large



In this case, a large change in inflation has only a small effect on output, so DAD is relatively steep.

Now, the shock has only a small effect on output, but a big effect on inflation.

## APPLICATION: The Taylor principle

- **The Taylor principle** (named after John Taylor):  
The proposition that a central bank should respond to an increase in inflation with an even greater increase in the nominal interest rate (so that the real interest rate rises).  
*I.e.*, central bank should set  $\theta_\pi > 0$ .
- Otherwise, DAD will slope upward, economy may be unstable, and inflation may spiral out of control.

## APPLICATION: The Taylor principle

$$Y_t = \bar{Y}_t - \frac{\alpha\theta_\pi}{1+\alpha\theta_Y}(\pi_t - \pi_t^*) + \frac{1}{1+\alpha\theta_Y}\varepsilon_t \quad (DAD)$$

$$i_t = \pi_t + \rho + \theta_\pi(\pi_t - \pi_t^*) + \theta_Y(Y_t - \bar{Y}_t) \quad (MP \text{ rule})$$

If  $\theta_\pi > 0$ :

- When inflation rises, the central bank increases the nominal interest rate even more, which increases the real interest rate and reduces the demand for goods & services.
- DAD has a negative slope.

# APPLICATION: The Taylor principle

$$Y_t = \bar{Y}_t - \frac{\alpha\theta_\pi}{1+\alpha\theta_Y}(\pi_t - \pi_t^*) + \frac{1}{1+\alpha\theta_Y}\varepsilon_t \quad (DAD)$$

$$i_t = \pi_t + \rho + \theta_\pi(\pi_t - \pi_t^*) + \theta_Y(Y_t - \bar{Y}_t) \quad (MP \text{ rule})$$

If  $\theta_\pi < 0$ :

- When inflation rises, the central bank increases the nominal interest rate by a smaller amount. The real interest rate falls, which increases the demand for goods & services.
- DAD has a positive slope.

# APPLICATION: The Taylor principle

- If DAD is upward-sloping and steeper than DAS, then the economy is unstable: output will not return to its natural level, and inflation will spiral upward (for positive demand shocks) or downward (for negative ones).
- Estimates of  $\theta_\pi$  from published research:
  - $\theta_\pi = -0.14$  from 1960–78, before Paul Volcker became Fed chairman. Inflation was high during this time, especially during the 1970s.
  - $\theta_\pi = 0.72$  during the Volcker and Greenspan years. Inflation was much lower during these years.

# CHAPTER SUMMARY

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- The DAD-DAS model combines five relationships: an IS-curve-like equation of the goods market, the Fisher equation, a Phillips curve equation, an equation for expected inflation, and a monetary policy rule.
- The long-run equilibrium of the model is classical. Output and the real interest rate are at their natural levels, independent of monetary policy. The central bank's inflation target determines inflation, expected inflation, and the nominal interest rate.

# CHAPTER SUMMARY

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- The DAD-DAS model can be used to determine the immediate impact of any shock on the economy and can be used to trace out the effects of the shock over time.
- The parameters of the monetary policy rule influence the slope of the DAS curve, so they determine whether a supply shock has a greater effect on output or inflation. Thus, the central bank faces a tradeoff between output variability and inflation variability.

# CHAPTER SUMMARY

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- The DAD-DAS model assumes that the Taylor principle holds, i.e. that the central bank responds to an increase in inflation by raising the real interest rate. Otherwise, the economy may become unstable and inflation may spiral out of control.