Advanced Macroeconomics

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Midterm Exam (Open-Book)

Undergraduate Program in Economics, HUST

Wednesday, April/27/2022

Name: ______________________ **Student ID:**_____________________

1. ($10' + 10' + 10' + 5' + 5' + 10' = 50$ points) In a Solow growth model, let us consider a Variable Elasticity of Substitution (VES) Production Function:

$$Y_t = F(K_t, A_t L_t) = K_t^{\alpha}(A_t L_t + b\alpha K_t)^{1-\alpha}, \tag{1}$$

where $b > 0, \alpha \in (0, 1)$.

(a) Does the production function in (1) exhibits constant returns to scale (CRS) in capital and labor?

(b) Derive the intensive form of the production, $f(k_t)$, where k_t stands for the capital per effective labor at time t.

(c) Does the property $MPK_t \equiv \frac{\partial F(K_t, A_t L_t)}{\partial K_t} = f'(k_t)$ still hold?

(d) Consider the first two Inada conditions: (i) $f(0) = 0$, (ii) $\lim_{k_t \to 0} f'(k_t) = +\infty$. Are they satisfied?

(e) Consider the third Inada condition: (iii) $f''(k_t) < 0$. Is it satisfied?

(f) Consider the last Inada condition: (iv) $\lim_{k_t \to +\infty} f'(k_t) = 0$. It is said that this condition CANNOT be satisfied. But as we discussed in class, we can find a weaker requirement on $\lim_{k_t \to +\infty} f'(k_t)$ to ensure the existence of a unique global steady state. Try to find one. (Hint: find an inequality between a function of (α, b, s) and $(n+g+\delta)$.)

2. ($10' + 20' + 20' = 50$ points) Consider an infinite-horizon growth model with the objective function for a representative household being set as:

$$U = \sum_{t=0}^{\infty} \beta^t u(C_t) \frac{L(t)}{H} = \sum_{t=0}^{\infty} \beta^t [\alpha - e^{-\theta C_t}] \frac{L(t)}{H}, \tag{2}$$

where $u(\cdot)$ stands for the instantaneous utility function, $\beta \in (0,1)$ represents the discount rate, and $\alpha > 0, \theta > 0$.

(a) Derive the expression for the marginal utility of consumption at t, $u'(C_t)$.

(b) Give the Euler equation in terms of $(C_{t+1} - C_t) = \Gamma(\beta, r_{t+1}, \theta)$, where $\Gamma(\cdot)$ is a smooth function.

(c) Rephrase your result above using $ln(1+r_{t+1}) \approx r_{t+1}$ and $ln\beta = ln(e^{-\rho}) = -\rho$, what do you get? In the textbook, we have the Euler equation as $\frac{\dot{C}_t}{C_t} = \frac{r_t - \rho}{\theta}$, it talks about the growth rate of consumption, but why is the Euler equation here discussing the magnitude of increase in consumption?