Forward Kinematics

Robotics instructional lab #2

Spring 2022

In this lab, we will start building the foundation for motion planning. We will introduce the Forward Kinematics (FK) problem where the pose (position and orientation) of a robot-arm end-effector is computed based on given joint values.

1 Background

Let $\theta \in \mathbb{R}^6$ be the vector of joint angles of the arm. We search for the homogeneous transformation matrix $T_{ee} \in SE(3)$ that embed the position $\mathbf{p}_{ee} \in \mathbb{R}^3$ and orientation $R_{ee} \in SO(3)$ of the end-effector according to

$$T_{ee} = \begin{bmatrix} R_{ee} & \mathbf{p}_{ee} \\ \mathbf{0} & 1 \end{bmatrix}. \tag{1}$$

In this lab, we find T_{ee} using the Denavit-Hartenberg (DH) method. The DH convention provides instructions of how to systematically attach coordinate frames to the links of a kinematic chain and map between each two consecutive frames using four parameters. Theoretical material and instructions on how to implement DH can be read in [1, 2] and in the Introduction to Robotics (0542462101) course.

2 Prerequisites

Compute the FK of the Kinova arm using DH. Functions should be written within the template **lab02_student.py**. Do the following:

- Fill-in the enclosed DH table and in function `set_dh_table`. Note that lengths are in meters and angles are in radians. For each θ_i, simply write the corresponding angle symbolic notation plus the required rotation (for example, `q1+pi/2`).
- Write a function `dh(`α_i`,`a_i`,`d_i`,`θ_i`)` that receives the DH parameters of a row from the table and returns the corresponding homogeneous transformation matrix T^i_{i+1}.

Table 1: DH parameter table

	α_i	a_i	d_i	θ_i
T_1^0				
T_2^1				
T_3^2				
T_4^3				
T_5^4				
T_{ee}^5				

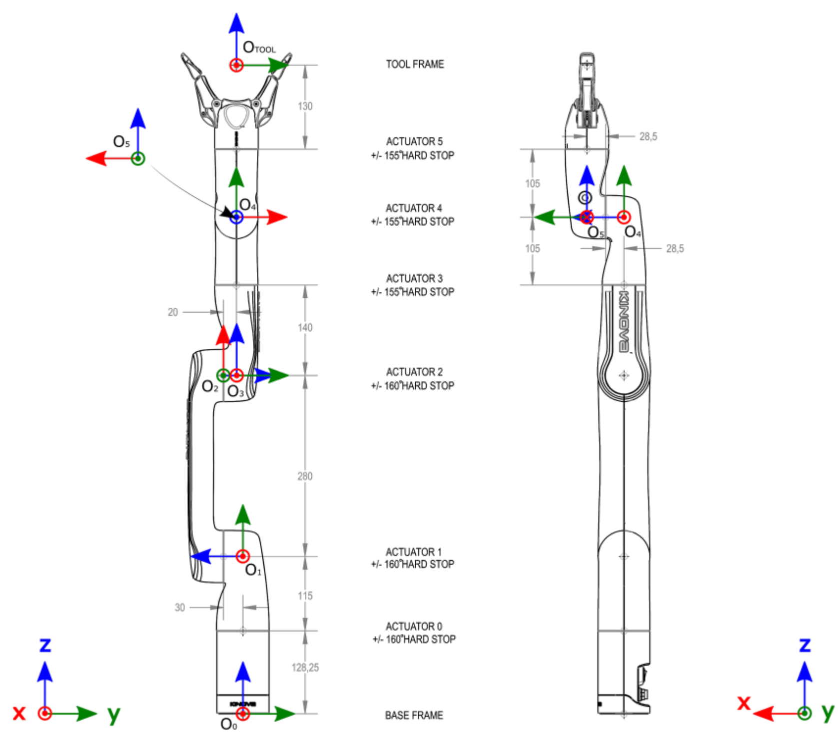

Figure 1: DH frames and dimensions (in mm).

```python
angles = {'t1': [0,-pi /7, pi /7, 0, 0, pi/7],  # deg
          't2': [np.deg2rad(300), np.deg2rad(300), 0, np.deg2rad(300), np.deg2rad(300), 0],
          't3': [0, np.deg2rad(340), np.deg2rad(75), 0, np.deg2rad(300), 0],
          't4': [pi / 10, pi / 10, pi / 10, pi / 10, pi / 10, pi / 10],
          't5': [0, 0, 0, 0, 0, 0]}
```

```
          X     Y      Z     theta_x    theta_y   theta_z
t1    [  0.53  -0.01   0.75   48.49 -19.83 106.71]
t2    [  0.37  -0.55   0.39 132.7    53.9    94.89]
t3    [  0.45   0.19   0.42  92.5    -4.33 149.91]
t4    [  0.01  -0.06   0.98  -5.73 -17.09 144.86]
t5    [ 0.06 -0.01  1.     0.     0.    90.   ]
```

Figure 2: Example of forward kinematic solution

- With the previous function, write a Python function `FK(`θ`)` that receives a joint angles vector θ and outputs the homogeneous transformation matrix T_{ee}.
- Write a function `xyz_euler` to map an homogeneous matrix to a vector of position coordinates (x, y, z) and Euler angles (XYZ) about the base axes (you may use the **scipy** package).

Prerequisites are a mandatory in order to carry out the lab .

3 Lab instructions

1. Do the following:
   - Use the GUI (from the first lab) to choose 10 random joint configuration that satisfy the joint bounds. Write down angles, and, also the position and orientation of the gripper.
   - Fill-in the list of desired joint configurations in function `angles_to_follow`.
   - Open terminal and run file **lab_02_exe.py**.
   - Compare the true gripper pose to the pose T_{ee} acquired from your FK solution.
2. Without using inverse kinematics and by using your FK function, try to find the

values of joint angles that position the robot at

$$T^*_{ee} = \begin{bmatrix} 0 & 1 & 0 & 0.41 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Hint: Look at the directions of the EE coordinate frame and manually move the robot.

4 Report requirements

The lab report should include the following:

1. Describe the process of the lab.
2. A table summarizing all results. Include Homogeneous matrices, positions and Euler angles.
3. Mean and standard deviation of the position and orientation errors for all trials (A good way to compare two rotation matrices is to take the dot product between corresponding columns. Similar orientations should yield dot products close to 1). Compare also the Euler angles.
4. Explain the difference between experimental and analytical results.
5. What are the approximated angles to reach T^*_{ee} and real T_{ee}? Compute the error. Explain your experience and difficulties reaching the desired pose.
6. Provide a summary for your results.

All angle values should be expressed in **degrees**.

References

[1] Kevin M. Lynch and Frank C. Park. *Modern Robotics: Mechanics, Planning, and Control*. Cambridge University Press, USA, 1st edition, 2017.

[2] M.W. Spong, S. Hutchinson, and M. Vidyasagar. *Robot Modeling and Control*. Wiley select coursepack. Wiley, 2005.