Setting Up the St. Petersburg Paradox - Quiz

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Finger Exercises due Oct 5, 2020 19:30 EDT Past Due

Question 1

0.0/1.0 point (graded)

The Bernoulli distribution is a special case of the _______ distribution where __________.

- Binomial ; the support ranges from 0 to 1
- Binomial ; probability is 0.5
- Binomial ; n=1 ✔
- Uniform ; the support ranges from 0 to 1

Explanation

The Bernoulli distribution is a special case of the binomial distribution where n=1. In other words, whereas the binomial distribution might describe the outcomes of a series of coin flips, for example, the Bernoulli distribution would describe the outcome of a single coin flip.

Question 2

0.0/1.0 point (graded)

Which of the following describes the geometric distribution?

- The number of successes out of a given number of independent trials or attempts
- Number of identical trials repeated until a “success” is reached ✔
- A distribution where each of the outcomes are equally likely
- A normal or “bell curve” distribution

Explanation

The geometric distribution describes the distribution of the number of trials or attempts until a “success” is reached. For example, if you flip a coin until the coin lands heads, the number of flips that tails that would land before the first heads in repeated trials could be characterized by a geometric distribution. We will learn more about the geometric and other special distributions in the next lecture.

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In the St. Petersburg paradox discussed in class, we know the following, where Y represents winnings and X represents the number of flips required until the coin comes up heads. Winnings are defined as $Y = 2^X, X \sim G(0.5)$, and $E(X) = 2$. What is the expectation of Y?

- ( ) ∞ ✔
- ( ) x
- ( ) $2x$
- ( ) $2X$

Explanation

The expectation of Y is infinity.

$$E(Y) = \sum_{n=1}^{\infty} 2^n 0.5^{n-1} * 0.5$$

$$= \sum_{n=1}^{\infty} 2^n * \left(\frac{1}{2}\right)^n$$

$$= \sum_{n=1}^{\infty} 1$$

Answers are displayed within the problem

Question 2

0.0/1.0 point (graded)

True or False: For most people, the utility (or benefit) they derive from playing the St Petersburg paradox is exactly equivalent to the expected winnings from playing the game.

- ◯ True
- ◯ False ✔

Explanation

As discussed in class, the expectation of winnings is infinity. You can imagine that people probably do not place an infinite value and probably not willing to pay an infinite amount to play this game. Instead, people likely have a certain utility function that describes how much they value playing the game. This utility function most likely exhibits diminishing marginal utility of money.

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Question 3

0.0/1.0 point (graded)

Which of the following correctly describes the concept of diminishing marginal utility?

- ◯ Extra money does not bring you any utility at all after a certain point.
- ◯ Someone who has $10,000 dollars values an additional dollar less than someone who has $100 ✔
- ◯ Additional money brings you more value up to a certain point, when additional money actually makes you less happy.
- ◯ Someone who has $100 dollars values an additional dollar less than someone who has $10,000

Explanation

The concept of diminishing marginal utility is a common concept in economics. This concept captures the way that people value, or derive utility, from money. The idea is that the marginal utility from earning one extra dollar is different for a given individual depending on how much income they have. When a person has little income, each additional dollar of income brings them a certain amount of utility. As that person's income increases, each additional dollar of income brings them less and less utility.

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Finger Exercises due Oct 5, 2020 19:30 EDT **Past Due**

Question 1

0.0/1.0 point (graded)

True or False: The expectation of a sum of random variables is equal to the sum of the expectations of each of those random variables.

- True ✔
- False

Explanation

This is true. Given that Y is represented as a sum of several random variables, $X_1, X_2, \ldots, X_n$, then the expectation of Y is given by the sum of the expectations of $X_1, X_2, \ldots, X_n$. For example, if $Y = X_1 + X_2 + X_3$, then $E[Y] = E[X_1] + E[X_2] + E[X_3]$. This is one of the useful properties of expectation given in class.

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Question 2

0.0/1.0 point (graded)

Suppose that you have a function, $Y = 6X_1 + 3X_2 + 2X_3$ and you know that $E[X_1] = 3, E[X_2] = 4$ and $E[X_3] = 1$. Using what you know about the properties of expectation, what is the expectation of Y?

Answer: 32

Explanation

We know that the expectation of a sum is equal to the sum of expectations. The expectation of Y can be calculated as follows:

$$E[Y] = 6 * E[X_1] + 3 * E[X_2] + 2 * E[X_3] = 6 * 3 + 3 * 4 + 2 * 1 = 32.$$

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Answers are displayed within the problem

Question 3

0.0/1.0 point (graded)

True or False: If $Y = X_1 * X_2$, then it is always true that $E[Y] = E[X_1] * E[X_2]$.

- True
- False ✔

Explanation

This is only true in cases where X_1 and X_2 are independent.

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Properties of Variance, Part I - Quiz

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Finger Exercises due Oct 5, 2020 19:30 EDT Past Due

Question 1

0.0/1.0 point (graded)

True or False: Variance can be positive or negative, depending on the random variable.

- True
- False ✔

Explanation

This is false. The first property described in class holds that the variance of any random variable must necessarily be non-negative. This is intuitive: variance is a measure of spread of a random variable, at minimum 0 (which means the entire PDF of the random variable is concentrated at one point, i.e. a trivial probability distribution). This can also be seen by looking at the formula for variance, $Var\left(X\right) = E\left[\left(X - \mu\right)^2\right]$, where the square term dictates that the expectation be non-negative.

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Question 2

0.0/1.0 point (graded)

Suppose that you have a function, $Y = 5X + 2$, and the variance of X is 2. What is the variance of Y?

Answer: 50

Explanation

We know from the properties of variance that if $Y = aX + b$, then $Var\left(Y\right) = a^2 Var\left(X\right)$. So, if $Y = 5X + 2$ and $Var\left(X\right) = 2$, then $Var\left(Y\right) = 5^2 * Var\left(X\right) = 25 * 2 = 50$.

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Properties of Variance, Part II - Quiz

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Finger Exercises due Oct 5, 2020 19:30 EDT *Past Due*

Question 1

0.0/1.0 point (graded)

True or False: As is the case with expectation, the variance of a sum of random variables is always equal to the sum of the variances of the random variables.

- ◯ True
- ◯ False ✔

Explanation

This is false. Unlike the case for expectations, the variance of a sum of random variables is equal to the sum of the variances of the random variables only when the random variables have zero covariance (which is always the case when they are independent).

Show answer

Submit   You have used 0 of 1 attempt

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Question 2

0.0/1.0 point (graded)

Standard deviation can be a useful way to capture the __________ of a random variable ________ as the random variable itself.

- ◯ measure of dispersion ; in the same moment
- ◯ measure of centrality; in the same moment
- ◯ measure of dispersion ; in the same units ✔
- ◯ measure of centrality ; in the same units

Explanation

Standard deviation is calculated as the square root of variance, and is another way of measuring the dispersion of a random variable. Standard deviation can sometimes be a convenient measure of dispersion, since it is in the same units as the random variable. Variance is also a measure of dispersion, but is in the square of the unit of the random variable (which can be seen by the fact that calculating variances involve squaring expectations of the random variable)

Question 3

0.0/1.0 point (graded)

Suppose the maximum score on an exam is 10. Exam scores can therefore be modeled as a random variable X. Assume this variable has variance σ^2. However, in the final gradebook the exam scores will be linearly scaled so that each exam score is instead out of 100 (e.g. if one's score was originally 9, then their scaled score becomes 90). What is the variance of the random variable Y corresponding to the scaled scores?

- ◯ σ^2
- ◯ $10\sigma^2$
- ◯ $2\sigma^2$
- ◯ $100\sigma^2$ ✔

Explanation

One of the properties of variance is that, if $Y = a * X$, then $Var\,(Y) = a^2 * Var\,(X)$. To scale scores from having a maximum of 10 to having a maximum of 100, scores must be multiplied by 10. This means that $Y = a * X$ where $a = 10$. We know that $Var\,(X) = \sigma^2$. Therefore, $Var\,(Y) = 10^2 * \sigma^2 = 100\sigma^2$.

Show answer

Conditional Expectation - Quiz

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Finger Exercises due Oct 5, 2020 19:30 EDT **Past Due**

Question 1

0.0/1.0 point (graded)

The “Law of Iterated Expectations” states that:

- The expectation of the expectation of Y given X is equal to the expectation of Y multiplied by the expectation of X
- The expectation of Y given X is equal to the expectation of X given Y
- The expectation of Y given X is equal to the expectation of Y multiplied by the expectation of X
- The expectation of the expectation of Y given X is equal to the expectation of Y ✔

Explanation

The “law of iterated expectations” holds that the expectation of the expectation of Y given X is equal to the expectation of Y. The mathematical notation is: $E\left[E\left[Y|X\right]\right] = E\left[Y\right]$.

Show answer

Question 2

0.0/1.0 point (graded)

The "Law of Total Variance" states that:

- ◯ The variance of Y is equal to the variance of the expectation of Y given X added to the expectation of the variance of Y given X ✔
- ◯ The variance of Y is equal to the variance of Y given X added to the variance of X given Y
- ◯ The variance of the expectation of Y is equal to the variance of Y given X added to the variance of X given Y
- ◯ The variance of the expectation of Y is equal to the variance of Y given X added to the expectation of the variance of X given Y

Explanation

The "law of total variance" states that variance of Y is equal to the variance of the expectation of Y given X added to the expectation of the variance of Y given X. The mathematical notation is: $Var\left(Y\right) = Var\left(E\left[Y|X\right]\right) + E\left[Var\left(Y|X\right)\right]$.

Show answer

Use the following information for each of the following questions: Suppose that you are the swim coach for athletes that will be going to the Olympic Games to compete in swimming. To qualify for the final race, your athletes must swim a fast enough time in a qualifying event. Suppose that the probability of making the final race, p_F, is 0.4 or 40% for each athlete and each athlete's outcome is independent.

Question 1

0.0/1.0 point (graded)

Your team has 8 people. What is the probability that 2 athletes make it into the final race?

Note: Please review our guidelines on precision regarding rounding answers here.

Answer: 81648/390625

Explanation

Similar to the example given in class, the key piece of information given here is that "making it to the final" follows a binomial distribution, where $F|N = n \sim B(n, 0.4)$, where F refers to the number "making it to the final," and N refers to the size of your team. To calculate the probability that 2 of the athletes make it into the final race, use the following:

$$P(F = 2|N = 8) = \binom{n}{x} {p_F}^x (1 - p_F)^{n-x} = \frac{8!}{2!6!}(0.4)^2(0.6)^6 = 0.209$$

You can also calculate this probability in R, using the command dbinom(2,8,0.4).

Show answer

Submit You have used 0 of 2 attempts

Answers are displayed within the problem

Question 2

0.0/1.0 point (graded)

What is the expected number of athletes that make it into the finals? **Please round your answer to the nearest whole number.**

Answer: 3.2 **or** 3

Explanation
The expectation for a binomial distribution is given by:

$$E[F|N=8] = np_F = 8 * 0.4 = 3.2$$

Use the following information for each of the following questions: Suppose that you are the swim coach for athletes that will be going to the Olympic Games to compete in swimming. To qualify for the final race, your athletes must swim a fast enough time in a qualifying event. Suppose that the probability of making the final race, p_F, is 0.4 or 40% for each athlete and each athlete's outcome is independent.

Question 1

1 point possible (graded)

Suppose that the team size N is a random variable with $E[N] = 2$ and $Var(N) = 1.5$. What is the expectation of the number of swimmers that make it to the final?

Note: Please review our guidelines on precision regarding rounding answers here.

Answer: 0.80

Explanation

We will use the following notation: F is the number of finalists, and N is the number of swimmers per team. N is a binomial random variable with probability 0.4, and the expectation of a binomial is $N * p$. Using this fact and the law of iterated expectations, we have:

$$E[F] = E[E[F|N]] = E[Np_F] = p_F E[N] = 0.4 * E[N] = 0.4 * 2 = 0.8$$

Question 2

0.0/1.0 point (graded)

Recall that team size N is a random variable with $E[N] = 2$ and $Var(N) = 1.5$. What is the variance of the number of swimmers that make it to the final?

Note: Please review our guidelines on precision regarding rounding answers here.

Answer: 0.72

Explanation

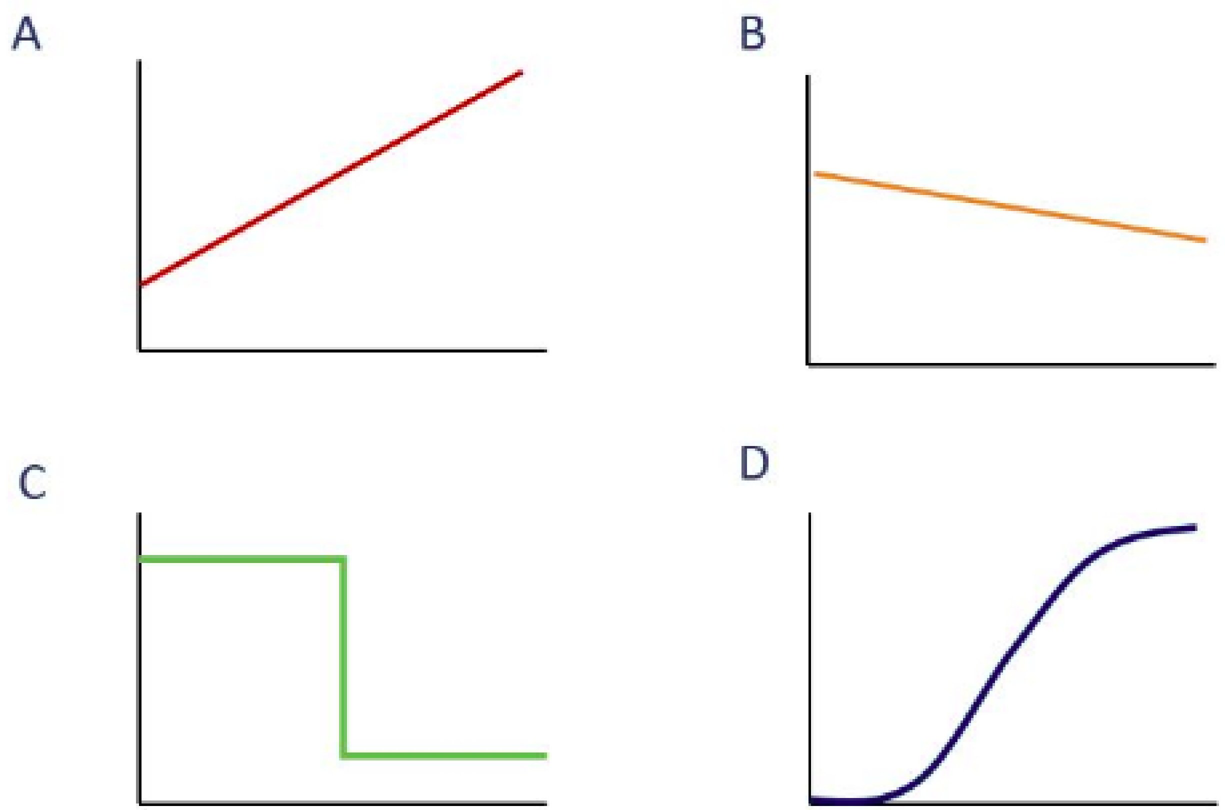
We will again use the following notation: F is the number of finalists, and N is the number of swimmers per team. N is a binomial random variable with probability 0.4; its expectation is $N * p$ and its variance is $N * p(1-p)$. Using these facts and the law of total variance, we have:

$$Var(F) = Var(E[F|N]) + E[Var(F|N)] = Var(Np_F) + E[Np_F(1-p_F)] =$$

$$p_F{}^2 Var(N) + p_F * (1-p_F) * E(N) = 0.4^2 * 1.5 + 0.4 * 0.6 * 2 = 0.72$$

Show answer

Suppose X is a random variable and Y is a function of X depicted in the graphs below (where X is on the horizontal axis and Y is on the vertical). For which of the following graphs would X and Y be positively correlated? (Select all that apply)



- [ ] Graph A ✔
- [ ] Graph B
- [ ] Graph C

Graph D ✔

Explanation

A positive correlation between X and Y means that X and Y are overall increasing (or decreasing) together, while a negative correlation shows that one increases while the other decreases. The graphs depicted in A and D show the relationship between two variables that are positively correlated. The graphs in B and D show the relationship between two variables that are negatively correlated (weakly, in the case of C, given its flat regions).

Show answer

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Answers are displayed within the problem

Question 2

0.0/1.0 point (graded)

Which of the following statements are true? (Select all that apply)

If two variables X and Y are independent, then the covariance, σ_{XY}, and the correlation, ρ_{XY}, are equal to zero ✔

- [ ] Correlation ranges between 0 and 1
- [ ] For any positively correlated variables X and Y, ρ_{XY} is close to 1
- [ ] ρ_{XY} is greater than zero for two positively correlated variables ✔

Explanation

If two variables are independent, or uncorrelated, then their covariance and correlation are both equal to zero. If two variables are negatively correlated, then correlation is less than zero and if positively correlated, then correlation is greater than zero. The other choices are incorrect. Correlation ranges between -1 to 1. Variables that are positively correlated may be weakly positive correlated, i.e. have a ρ_{XY} that is positive but closer to 0.

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Finger Exercises due Oct 5, 2020 19:30 EDT **Past Due**

Question 1

0.0/1.0 point (graded)

Suppose random variables X, Y are such that $Cov\,(X, Y) = 0$. Then it must be the case that the variables X, Y are independent.

- ◯ True
- ◯ False ✔

Explanation

It is true that if two variables are independent, then their covariance must be equal to zero. However, the relationship does not necessarily run the other way, i.e. it does not hold that if the covariance between two variables is equal to zero, the two variables are independent.

Show answer

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Question 2

0.0/1.0 point (graded)

The property that $Cov\,(aX + b, cY + d) = ac * Cov\,(X, Y)$ implies which of the following? (Select all that apply.)

- [ ] The covariance of a linear transformation of a set of variables is equal to the covariance of the original variables
- [ ] In a linear transformation between a set of two variables, additive constants do not factor in to any changes in covariance ✔
- [ ] Covariance from a linear transformation of two variables is altered by both the coefficient of change and the additive constant
- [ ] Covariance is unchanged for a linear transformation of a set of two **independent** variables ✔

Explanation

The property $Cov\,(aX + b, cY + d) = ac * Cov\,(X, Y)$ implies that when you take a linear transformation of variables, the covariance is equal to the original covariance multiplied by the multiplicative constants of the linear transformation, but without consideration of the additive constants. Thus the additive constants in a linear transformation do not factor into changes in covariance.

In the special case of two independent random variables, the original covariance $Cov\,(X, Y)$ is 0, so any linear transformation of this necessarily leaves this unchanged.

Question 3

0.0/1.0 point (graded)

Suppose random variables X, Y are such that $Cov\left(X,Y\right) = 4$, whereas random variables A, B are such that $Cov\left(A,B\right) = 400$. Is it correct to say that A and B are therefore more "closely related" than X and Y?

- ◯ Yes
- ◯ No ✔

Explanation

The problem is that the unit of covariance is the unit of the random variable, and in general we don't know that the two sets of random variables that we find pairwise covariances for have the same units. For example, for X, Y such that $Cov\left(X,Y\right) = 4$, consider $A = 10X, B = 10Y$. Then one can see that A and B are related to each other "equally" compared to X, Y (effectively, we are just changing units: for example, X, Y could be a random variables capturing heights in centimetres, whereas A, B capture the same heights in millimetres). However,
$Cov\left(A,B\right) = Cov\left(10X, 10Y\right) = 10 * 10Cov\left(X,Y\right) = 400.$
However, correlation helps "standardize" changes of units; so for the example above, it is true that $Cor\left(A,B\right) = Cor\left(X,Y\right)$. So correlations do let us compare the relatedness of one pair of random variables with a different pair of random variables.

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A Preview of Regression - Quiz

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Finger Exercises due Oct 5, 2020 19:30 EDT **Past Due**

Question 1

0.0/1.0 point (graded)

True or False: In linear regression, where the relationship between X and Y is expressed as $Y = \alpha + \beta X + U$ (where $\beta = \rho_{XY} * \sigma_y / \sigma_X, \alpha = \mu_y - \beta * \mu_X$), $\alpha + \beta X$ refers to the variation in Y that is "explained" by X, and U refers to the variation in Y that is "unexplained," where $E[U] = 0$ and $Cov(X, U) = 0$.

- ( ) True ✔
- ( ) False

Explanation

This is true, and provides some of the useful intuition to keep in mind when we will start to go more in depth into linear regression. Two helpful things to keep in mind are that the expectation of the error term, $E[U]$, is equal to zero, and the error term and the independent variable X are uncorrelated.

Show answer

Markov Inequality and Chebyshev Inequality - Quiz

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Finger Exercises due Oct 5, 2020 19:30 EDT Past Due

Question 1

0.0/1.0 point (graded)

Suppose you have a probability distribution for a non-negative random variable X, where the expectation of X is given by $E[X] = 5$. By Markov's inequality, the probability that X is greater than or equal to 10 is ______________.

- exactly 1.25
- no more than 0.8
- no less than 1.25
- no more than 0.5 ✔

Explanation

The Markov inequality bounds the the probability that a random variable exceeds a certain threshold t. In particular, it states that for any $t > 0, P(X \geq t) \leq \frac{E[X]}{t}$. In this case we set $t = 10$ to get that the probability that X is greater than or equal to 10 is no more than $5/10 = 0.5$.

Question 2

0.0/1.0 point (graded)

Now suppose that you have a random variable X, where $E[X] = 6$ and $Var[X] = 2$. By Chebyshev's inequality, the probability that X is greater than 11 or less than 1 is no more than ______________________.

Note: Please review our guidelines on precision regarding rounding answers here.

Answer: 2/25

Explanation

Chebyshev's inequality bounds the probability of a random variable deviating from its mean by a specified amount t, i.e. it bounds $P(|X - E[X]| \geq t)$. Given that $E[X] = 6$, bounding the probability of X being greater than 11 or less than 1 is exactly bounding the probability of $P(|X - E[X]| \geq t)$ where $t = 5$. In this case, by Chebyshev's inequality,

$$P(|X - E[X]| \geq t) \leq \frac{Var(x)}{t^2}$$

$$P(|X - 6| \geq 5) \leq \frac{2}{5^2} = \frac{2}{25} = 0.08$$

Show answer