Finger Exercises due Sep 21, 2020 19:30 EDT **Past Due**

For the following problems, consider an experiment involving **two independent** tosses of a coin. For each coin toss, let H denote the event 'heads' and let T denote the event 'tails'

Question 1

0.0/1.0 point (graded)

Which of the following sets represents the entire sample space Ω of outcomes of this experiment?

- ◯ $\Omega = \{HH, HT, TH, TT\}$ ✔
- ◯ $\Omega = \{HH, TT\}$
- ◯ $\Omega = \{H, T\}$
- ◯ $\Omega = \{HT, TH\}$

Explanation

Each outcome looks like XY, where X is the result of the first coin, and Y is the result of the second coin. Each of X, Y can be H or T, independently of each other.

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Question 2

0.0/1.0 point (graded)

Which of the following sets Ω_1 represents all the outcomes which contain at least one head?

- $\Omega_1 = \{HH, HT\}$
- $\Omega_1 = \{HH, HT, TH\}$ ✔
- $\Omega_1 = \{HT, TH\}$

Explanation

We are looking for all the outcomes in the sample space where an 'H' appears

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Question 3

0.0/1.0 point (graded)

Which of the following expresses the relationship between Ω_1 and Ω above? (Select all that apply)

- [ ] $\Omega \subset \Omega_1$
- [ ] $\Omega_1 \subset \Omega$ ✔
- [ ] $\Omega_1 = \Omega \cap \{\text{H appears in the outcomes}\}$ ✔

Explanation

Ω is the set of all outcomes, so any subset of outcomes such as Ω_1 will satisfy $\Omega_1 \subset \Omega$. The third equality captures the definition of Ω_1 in set theory notation.

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Question 4

0.0/1.0 point (graded)

Which of the following sets Ω_2 represents all the outcomes which contain at least one tail?

- $\Omega_2 = \{HT, TH, TT\}$ ✔
- $\Omega_2 = \{TH, TT\}$
- $\Omega_2 = \{HT, TT\}$

Explanation

We are looking for all the outcomes in the sample space where an 'T' appears

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Question 5

0.0/1.0 point (graded)

With possible reference to the previous sets, which of the following sets Ω_3 represents all the outcomes which contain at least one head and at least one tail? (Select all that apply)

- [ ] $\Omega_3 = \{HT, TH\}$ ✔
- [ ] $\Omega_3 = \Omega_1 \cap \Omega_2$ ✔
- [ ] $\Omega_3 = \Omega_1 \cup \Omega_2$

Explanation

If we go through the elements of Ω sequentially to find the outcomes that meet the condition, we see that only $\{HT, TH\}$ meet it.

However, $\Omega_1 \cap \Omega_2$ also captures the same outcomes, which makes same sense conceptually: Ω_1 captures all the outcomes that include a head, and Ω_2 captures all outcomes that include a tail, so their intersection captures the outcomes that include both.

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Question 6

0.0/1.0 point (graded)

Which of the following expresses the relationship between the two sets
$\Omega_3 = \{\text{outcomes which contain at least one heads and one tail}\}$ and
$\Omega_4 = \{\text{outcomes where both coin throws are identical}\}$? (Select all that apply)

- [ ] They are mutually exclusive ✔
- [ ] They are identical
- [ ] They are exhaustive ✔
- [ ] They have non-empty intersection

Explanation

Since $\Omega_3 = \{HT, TH\}$, and $\Omega_4 = \{HH, TT\}$, we know that (i) they are mutually exclusive since they have empty intersection (i.e. $\Omega_3 \cap \Omega_4 = \emptyset$), and they are (ii) exhaustive since their union comprises the whole sample space (i.e. $\Omega = \Omega_3 \cup \Omega_4$)

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Question 1

0.0/1.0 point (graded)

Which of the following set properties hold generally? (Select all that apply)

- [ ] $A \cap \emptyset = \emptyset$ ✔
- [ ] $A \cap \emptyset = A$
- [ ] $A \cap B = A \cap (B \cap A^c)$
- [ ] $A \cup B = A \cup (B \cap A^c)$ ✔

Explanation

Any set intersected with the empty set is the empty set. $B \cap A^c$ is the set of elements in B but not in A; intersecting this with A gives the empty set, not A. Instead, taking this set's union with A results in $A \cup B$.

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Question 2

0.0/1.0 point (graded)

For two sets A and B, define $B \setminus A = \{x | x \in B, x \notin A\}$. Consider the following proof of the law of inclusion-exclusion, which states that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. By subtracting and rearranging, we also have that $P(A) + P(B) = P(A \cup B) + P(A \cap B)$ (make sure you see how!).

Because the sets $A \cap B$ and $B \setminus A$ are disjoint, and their union is B, $P(B) = P((A \cap B) \cup (B \setminus A)) = P(A \cap B) + P(B \setminus A)$.

Furthermore, the sets A and $B \setminus A$ are disjoint and their union is $A \cup B$.

Which of the following is true?

- ◯ $P(A \cup B) = P(A \setminus B) + P(B \setminus A)$
- ◯ $P(A \cup B) = P(A) + P(B \setminus A)$ ✔
- ◯ $P(B \setminus A) = P(A) + P(A \cup B)$

Explanation

We need a statement of the form $Pr(X \cup Y) = Pr(X) + Pr(Y)$, where $X = A$ and $Y = B \setminus A$.

Question 3

0.0/1.0 point (graded)

Which of the following demonstrates the law of inclusion-exclusion applied to find the probability that in two coin flips, at least one heads occurs?

- $P(\text{at least one head}) = P(\text{first throw is a head}) + P(\text{second throw is a head})$
- $P(\text{at least one head}) = P(\text{first throw is a head}) + P(\text{second throw is a head}) - P(\text{both throws are heads})$ ✔
- $P(\text{at least one head}) = P(\text{first throw is a head}) + P(\text{second throw is a head}) + P(\text{both throws are heads})$

Explanation

You can think of the inclusion-exclusion principle here as correcting for overcounting. To find the probability of at least one head in the throws, we first independently find the probabilities that the first throw is a head, and the second is a head, and add them. However, these events are not disjoint, so we subtract their intersection (the probabilities that both the first and second throw are heads).

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Question 1

0.0/1.0 point (graded)

If someone told you the setting for a coin toss thrown twice was a simple sample space, what do you immediately know about it? (Select all that apply)

- [ ] All outcomes are equally likely ✔
- [ ] The coin is unbiased ✔
- [ ] Each outcome $\{HH, HT, TH, TT\}$ has probability 1/4. ✔
- [ ] Each outcome $\{HH, HT, TH, TT\}$ has probability 1/2.

Explanation

A simple sample space is defined as a sample space where each of the possible outcomes is equally likely. Since there are four outcomes exhausting the sample space, they each have probability 1/4. Furthermore, if p denotes the probability of getting a heads in each coin toss, then $P(HH) = p^2 = 1/4$, so $p = 1/2$ and the coin is unbiased.

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Question 2

0.0/1.0 point (graded)

For this simple sample space problem, which of these probabilities captures the inclusion-exclusion principle statement $P\left(\text{at least one head}\right) = P\left(\text{first toss is a head}\right) + P\left(\text{second toss is a head}\right) - P\left(\text{both coin tosses are heads}\right)$

- ○ $\frac{2}{3} = \frac{1}{2} + \frac{1}{2} - \frac{1}{3}$
- ○ $\frac{3}{4} = \frac{1}{2} + \frac{1}{2} - \frac{1}{4}$ ✔
- ○ $\frac{1}{2} = \frac{1}{4} + \frac{1}{4}$

Explanation

The probability that the first toss is a head is $\frac{1}{2}$, as is the probability that the second toss is a head. The probability that they are both heads are $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$. Following the inclusion exclusion principle resolves the probability of at least one heads as $\frac{1}{2} + \frac{1}{2} - \frac{1}{4} = \frac{3}{4}$

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Probability: Another Example - Quiz

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Finger Exercises due Sep 21, 2020 19:30 EDT Past Due

Question 1

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For positive integers n, k with $n > k$, which of the following also expresses $n!/(n-k)!$?

- n^k
- $n(n-1)\ldots(n-k)$
- $n(n-1)\ldots(n-k+1)$ ✔
- n^{n-k}

Explanation

The numerator equals $n(n-1)\ldots(2)(1)$ while the denominator equals $(n-k)(n-k-1)\ldots(2)(1)$, causing all the lower terms through $n-k$ to be cancelled out, leaving $n(n-1)\ldots(n-k+1)$.

Question 2

0.0/1.0 point (graded)

Which of these enumerates sampling k items from n items with replacement?

- $n!/k!$
- $n\left(n-1\right)\ldots\left(n-k+1\right)$
- $n!/\left(n-k\right)!$
- n^k ✔

Explanation

There are n choices for each sample, and k samples, for n^k total enumerations.

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Question 3

0.0/1.0 point (graded)

Which of these enumerates sampling k items from n items without replacement? (Select all that apply)

- [ ] n^k
- [ ] $n(n-1)\ldots(n-k+1)$ ✔
- [ ] $n!/(n-k)!$ ✔
- [ ] $n!/k!$

Explanation

There are n choices for the first sample, $n-1$ for the second (can't choose the same as the first sample), $n-2$ for the third, and so on until $n-k+1$ choices for the kth sample. Taking the product of the choices gives the enumerations above.

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Ordered and Unordered Arrangements - Quiz

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Finger Exercises due Sep 21, 2020 19:30 EDT Past Due

Question 1

0.0/1.0 point (graded)

Let's apply what we've learned about combination counting to Vandermonde's identity, which states that any combination of k objects from a group of $m+n$ must have some r objects (where $0 \leq r \leq k$) from a group of m objects and $k-r$ objects from a group of n objects. In other words,

$$\sum_{r=0}^{k} \binom{m}{r} \binom{n}{k-r} = \binom{m+n}{k}$$

Let's illustrate this by imagining you are selecting from a menu of m fruits and n vegetables. Which of the following scenarios does this statement capture?

- ◯ The right hand side directly enumerates the selection of k items from the menu, subject to the requirement that there be exactly r fruits and the remaining be vegetables. The left hand side, meanwhile, sums over the enumeration of selecting k items with exactly r fruits, for each $r \leq k$

- ◯ The right hand side directly enumerates the selection of k items from the menu, while the left hand side sums over the enumeration of selecting k items from the menu subject to the requirement that there be exactly r vegetables, for each $r \leq k$

The right hand side directly enumerates the selection of k items from the menu, while the left hand side sums over the enumeration of selecting k items from the menu subject to the requirement that there be exactly r fruits, for each $r \leq k$ ✔

Explanation

It is clear that the right hand side directly enumerates the selection of k items from the menu. Meanwhile, each expression $\binom{m}{r}$ on the left hand side enumerates a selection of exactly r fruits from m fruits, while $\binom{n}{k-r}$ enumerates a selection of $k-r$ vegetables from n vegetables. Thus each summand $\binom{m}{r}\binom{n}{k-r}$ enumerates a selection of k items total from the basket of $m+n$ items, but with exactly r fruits (and therefore $k-r$ vegetables).

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Question 2

0.0/1.0 point (graded)

Per your answer to the previous question, what is the principle by which we can simply add the number of combinations on the left hand side to obtain the number of combinations on the right hand side?

Each summand on the left hand side enumerates events with have nonzero intersection

Each summand on the left hand side enumerates events which are disjoint ✔

Each summand on the left hand side enumerates events which are independent

Explanation

We know each summand on the left hand side sums over the enumeration of selecting k items from the menu subject to the requirement that there be exactly r fruits. For two distinct r_1, r_2 these summands are disjoint; selecting exactly r_1 fruits necessarily means not selecting exactly r_2 fruits. Thus we can just add the disjoint enumerations. Adding over all $r \leq k$ exhausts all the possibilities of enumerating exactly $1, \ldots k$ fruits when selecting k items in total.

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Question 1

0.0/1.0 point (graded)

Which of the following identities expresses the probability of choosing from m fruits and n veggies a selection of r pieces of food, where exactly $k \leq r$ of them are fruits and the rest are veggies?

- $\frac{\binom{m}{k}\binom{n}{r-k}}{\binom{m+n}{r}}$ ✔
- $\frac{\binom{m+n}{r}}{\binom{m}{k}\binom{n}{r-k}}$
- $\frac{\binom{m}{k}\binom{n}{k}}{\binom{m}{r}\binom{n}{r}}$
- $\frac{\binom{m}{k}\binom{n}{r-k}}{\binom{m}{r}\binom{n}{r}}$

Explanation

There are $\binom{m}{k}\binom{n}{r-k}$ ways to choose a set of r foods with exactly k fruits and the rest veggies. There are a total of $\binom{m+n}{r}$ ways to choose a set of r foods with no other restrictions. The probability is thus the former divided by the latter.

0.0/1.0 point (graded)

Which of these pairs of events are independent events? Hint: check your answer using the multiplicative criterion. (Select all that apply)

- [ ] A dice roll is both at most 4 and odd ✔
- [ ] A dice roll is both at most 5 and odd
- [ ] In a sequence of 5 coin flips: the first coin landing on heads and there being at least 3 heads
- [ ] In a sequence of 5 coin flips: the first coin landing heads and the total number of heads is even ✔

Explanation

For the first option, we can check that P(At most 4 and odd) = 1/3, while P(At most 4) = 2/3 and P(odd) = 1/2; so the events are independent. P(at most 5) = 5/6 while P(At most 5 and odd) is 1/2; so the second pair of events is not independent.

For the 5 coin flips, we can see that the probability of at least three heads is $\binom{5}{3} + \binom{5}{4} + \binom{5}{5})/2^5 = 1/2$; the probability of the first coin being heads is 1/2; but the probability of both happening is $1/2$ (the probability of the first heads) * $\left(\binom{4}{2} + \binom{4}{3} + \binom{4}{4}\right)/2^4 = 11/32$ since we need at least 2 of the remaining 4 coin flips to be heads. But $11/32 \neq 1/4$, so the events are not independent.

On the other hand, we can check that the probability that the number of heads is even is $\left(\binom{5}{0} + \binom{5}{2} + \binom{5}{4}\right)/2^5 = 1/2$. The probability of both an even number of heads and the first coin landing heads is $1/2$ (the probability of the first heads) * $\left(\binom{4}{1} + \binom{4}{3}\right)/2^4 = 1/4$, since we need either 1 or 3 heads in the remaining 4 coin flips. Thus these events are indeed independent.

Conditional Probability - Quiz

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Finger Exercises due Sep 21, 2020 19:30 EDT **Past Due**

Question 1

0.0/1.0 point (graded)

Select all the **true** statements about independent events A and B.

- [ ] $P(A \cap B) = P(A)\,P(B)$ ✔
- [ ] $P(A \cap B) = P(A) + P(B)$
- [ ] $P(A|B) = P(A)$ ✔
- [ ] $P(A|B) = P(B)$

Explanation

The first statement is the so-called multiplicative criterion that defines independent events. The third statement can be derived from Bayes's law, $P(A|B) = P(A \cap B)\,/\,P(B)$ and substituting the multiplicative criterion.

Question 2

0.0/1.0 point (graded)

Suppose the probability of getting an "A" on a test conditional on studying is 0.90 and the probability conditional on not-studying is 0.25. The unconditional probability of getting an "A" is:

- .873
- .575
- .45
- Not possible to determine ✔

Explanation

The probability of getting an "A" is $P(A) = P(A|\text{Study})\,P(\text{Study}) + P(A|\text{Doesn't study})\,P(\text{Doesn't study})$. However, we don't have $P(\text{Study})$ (or equivalently, $P(\text{Doesn't study}) = 1 - P(\text{Study})$, and different values of $P(\text{Study})$ lead to different final $P(A)$. Thus $P(A)$ as stated is not possible to determine.

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Conditional Probability in American Presidential Politics - Quiz

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Finger Exercises due Sep 21, 2020 19:30 EDT Past Due

Question 1

0.0/1.0 point (graded)

Suppose the probability of getting an "A" on a test conditional on studying is 0.90 and the probability conditional on not-studying is 0.25. The probability of studying is 0.70. What is the unconditional probability of obtaining an A?

Note: Please review our guidelines on precision regarding rounding answers here.

Answer: 0.705

Explanation

The probability is

$$P(A) = P(A|\text{Study})\,P(\text{Study}) + P(A|\text{Doesn't study})\,P(\text{Doesn't study}) = .9 * .7 + .25 * (1 - .7) = .705.$$

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0.0/1.0 point (graded)

You are a participant in a "Turing Test" experiment to identify whether the entity hidden behind a screen is a human being or a robot. Of course you don't know at the outset, but from reading campus news you know that this particular lab has been devoting a lot of research to building robots that are capable of imitating humans. So your prior guess is that they are more interested in showing you a robot - you guess that $P(\text{entity is a robot }) = 0.70$.

Then the entity, still hidden behind the screen, says "Hello!". You figure that this voice is very robotic. Before declaring what the entity is now, you want to update your probabilities as a "Bayesian updater" given this evidence. You reason through two new ingredients: Given an actual robot, it would with probability 0.90 emit a robotic noise. On the other hand, you guess that one fifth of humans are capable of very robotic voices - and you think that the researchers would indeed try to mislead you if they could!

So, after hearing the greeting, what do you estimate as the new probability that the entity is a robot?

Note: Please review our guidelines on precision regarding rounding answers here.

Answer: .913

Explanation

We are interested in the posterior guess $P(\text{entity is a robot }|\text{entity emitted a robotic noise})$, which we can call $P(A|B)$.

By Baye's theorem, $P(A|B) = P(B|A)\,P(A)\,/P(B)$. But we have that $P(B) = P(B|A)\,P(A) + P(B|A^c)\,P(A^c)$.

We can now supply our ingredients to find the denominator:

$P(B|A) = .9; P(B|A^c) = .2; P(A) = .7; P(A^c) = 1 - .7 = .3$, so $P(B) = .9 * .7 + .2 * .3 = .69$. Meanwhile the numerator is .9 * .7 = .63. Thus the answer is .63 / .69 = .913.