14.310x: Data Analysis for Social Scientists
Applications to Auctions

In the first part of the problem set, we will delve more deeply into **auction theory**, which Sara introduced in lecture. We will demonstrate some auction theory properties by performing simulations of data. In these simulations, we will compare different schemes for auctions by varying the number of bidders and valuations. At an auction, **bidders** make offers to buy the goods, and a bidder's **valuation** is how much the bidder offers to pay for the good.

To start, try to understand the following R code. Run the code to test your understanding. We will start with the assumption that there are 2 bidders. We will simulate the auction 1000 times, resulting in 1000 valuations for these 2 bidders. Imagine that you are the person trying to sell a particular good, and that you are using R to figure out the perfect pricing and allocation scheme.

```
#Preliminaries
rm(list = ls())
setwd("---------------------")

#Uniform Valuations
number_of_bidders <- 2
number_of_simulations <- 1000

set.seed(1)
valuations1 <- matrix(runif(
   number_of_bidders*number_of_simulations, min=0, max=1),
   nrow = number_of_simulations)
```

Question 1

Try to figure out what the `set.seed()` command is doing, and then answer the following true or false question:

True or False: Even though we are simulating random numbers, the use of the `set.seed()` function allows us to have the same valuations each time we re-run this code.

True

False

Question 2

Let us consider the posted price model. IN lecture, we saw that the expected revenue when there are posted prices is given by: $pPr(V_i \geq p)$ for at least one i, where p is the posted price, and v_i is individual i's valuation of the good. Then, the expected revenue is equal to: $p(1 - \Pr(v_N < p)) = p(1 - F(p)^N)$

What is the optimal price in case of two bidders, and a $U[0,1]$ distribution for valuations?

- $\frac{1}{\sqrt{2}}$
- $\frac{1}{\sqrt[3]{3}}$
- $\frac{1}{\sqrt{3}}$
- $\frac{1}{\sqrt[3]{2}}$

Question 3
Using the same scenario from Question 2, what is the expected revenue for the seller?

- $\frac{3}{2\sqrt[3]{2}}$
- $\frac{2}{3\sqrt[3]{2}}$
- $\frac{2}{3\sqrt[3]{3}}$
- $\frac{2}{3\sqrt{3}}$

Question 4
Now, we will use the R code above to test whether these predictions hold.

First, let's find the maximum valuation among all 1000 simulations among the two bidders. Name the function in R that allows you to get this value. The function you use should return the maximum valuation when you run `funcName(valuations1)`.

Please enter on the function name (what you typed for `funcName` – no arguments or parentheses!)

Question 5
Take a look at the following R code that calculates the analytic solution to the expected revenue, and compare it with the one coming from the simulation.

```
#Uniform Valuations
number_of_bidders <- 2
N <- number_of_bidders
V <- 10000
set.seed(5)
valuations <- matrix(runif(
N*V, min = 0, max = 1),
nrow = V)
```

```
maximum_valuation <- apply(valuations, 1, max)
optimal_price <- 1/((N+1)^(1/N))
expected)_revenue <- (N/(N+1)) * 1/((N+1)^(1/N))
revenue <- optimal_price*(maximum_valuation >= optimal_price)
mean(revenue)
expected_revenue
```

What variable captures the number of simulations we are using in the code?

Please enter ONLY the name of the variable without any additional text. Make sure that the capitalization matches the code!

Question 6
Now, perform this exercise for different numbers of simulations: 10, 100, 1000, and 10000. As you increase the number of simulations, does the mean of the numeric revenue vector coincide more or less with the analytic solution?

- It coincides more
- It coincides less

Question 7
Now, we will compare the results we just computed, which hold for the posted price model, with the results we would get from an auction. Let's consider an English auction, where the buyers' optimal strategy is to stay in the bidding until $p = V_i$ and then leave once $p > V_i$. As shown in lecture, the equilibrium price in this case is the second highest valuation.

What is the expected revenue when there are two bidders ($N = 2$)? Again, assume that bidders' valuations follow a uniform [0,1] distribution.

- $\frac{1}{4}$
- $\frac{1}{3}$
- 1
- $\frac{2}{3}$

Question 8
What is the minimum number of bidders such that a buyer prefers to sell the good in an English Auction rather than at a posted price auction?

Note: You can do this in two different ways: one is to solve the question mathematically (difficult!), and the other is to use the simulation in R to answer the question. To use the simulation in R, you will need to write code that computes the expected revenue in an English Auction and the expected revenue in a posted price auction given some number of bidders. You can then compare the two expected revenues for different numbers of bidders.

- 1 bidder
- 2 bidders
- 3 bidders
- 4 bidders

Question 9
On the website www.modelingonlineauctions.com, you will find a number of data sets from actual auctions conducted on eBay.

Download one involving the sale of Cartier watches: Cartier+3-day+auctions.csv. There are data on auctions of 18 different watches. For each auction, there is an auction ID, bids, time of each bid, bidder name, bidder rating, minimum bid for the auction, and winning bid for the auction. (Note: the winning bid is not the maximum bid submitted by the highest bidder, but rather the second-highest bid plus an increment.)

Load the data into R. How many auctions are in this data?

Now clean the data set and create the following variables:

- The id of the auction
- The ratio of the second highest bid to the third highest bid.
- The number of bidders.
- The number of bids.

We can provide you with the R-code here to create these variables, but some information is missing (marked with XXX). You will need to either fill the information or create your own code.

Question 10
What is the mean of the ratio of the second highest bid to the third highest bid?

Question 11
What is the median of the number of bidders?

Question 12
What is the maximum value of the number of bids?

Question 13
We can think of ordered bids as being order statistics from some underlying distribution of valuations. Using this perspective, would you expect the number of bidders and the number of bids to inform the ratio between the 2nd and 3rd highest bids?

(Note: We are not looking for a precise, mathematical answer here, just a bit of informed speculation.)

- Yes
- No