The Bernoulli Distribution - Quiz

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Finger Exercises due Oct 12, 2020 19:30 EDT **Past Due**

Question 1

0.0/1.0 point (graded)

Suppose that you have a Bernouilli variable X with some probability of success given by p and some probability of failure given by q. The mean of X is given by:

- P^2
- $p/2$
- $(1-p)$
- p ✔

Explanation

The expectation of a Bernouilli variable X is given by p.

Question 2

0.0/1.0 point (graded)

For which value(s) $p \in [0,1]$ does Bernoulli variable with probability of success p have maximum variance? (Select all that apply)

- [ ] 0
- [ ] 0.25
- [ ] 0.5 ✔
- [ ] 0.75
- [ ] 1

Explanation

The variance of a Bernoulli variable with probability of success is $p\,(1-p) = p - p^2$. Using calculus, or noticing that this is a quadratic equation, or simply just plotting this as a function of $p\$$, we can see this has maximal variance at $p = 0.5$. Note this is intuitive: if the probability success is equal to the probability of failiure, the random variable has the "most" uncertainty.

Question 3

0.0/1.0 point (graded)

For which value(s) $p \in [0,1]$ does Bernoulli variable with probability of success p have minimum variance? (Select all that apply)

- [ ] 0 ✔
- [ ] 0.25
- [ ] 0.5
- [ ] 0.75
- [ ] 1 ✔

Explanation

The variance of a Bernoulli variable with probability of success is $p\,(1-p)$. Given that variance is nonnegative, it is at minimum 0 and this is clearly achieved when $p = 1$ or $p = 0$. Notice this makes sense: at these extreme values, there is *no* uncertainty about the probability of success, therefore no variance.

Show answer

Finger Exercises due Oct 12, 2020 19:30 EDT Past Due

Question 1

0.0/1.0 point (graded)

Which of the following is the appropriate interpretation of the Binomial distribution?

- ◯ The number of successes in a sequence of n success/failure trials, each of which has the same probability of success, p ✔
- ◯ The number of successes in a sequence of n success/failure trials, each of which may have a different associated probability, $p_1, \ldots, p_n$
- ◯ The distribution of the different probabilities of success, $p_1, \ldots, p_n$ for a series of n success/failure trials
- ◯ The distribution of a series of trials, each with any number of outcomes $1, \ldots, k$ and associated probabilities $p_1, \ldots, p_k$

Explanation

A binomial distribution refers the number of successes in a sequence of n success/failure trials (meaning, there are only two outcomes) where the probability of success is constant across trials and given by p.

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Question 2

0.0/1.0 point (graded)

Assume Anna and Brian take 80 free kicks each. What is the expected number of shots they would make?

Expectation of Anna's 80 free kicks

Answer: 40

Expectation of Brian's 80 free kicks

Answer: 16

Explanation

The expectation of Anna's 80 free kicks is given by $np = 80 * 0.5 = 40$. The expectation of Brian's 80 free kicks is given by $np = 80 * 0.2 = 16$.

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Question 3

0.0/1.0 point (graded)

What is the theoretical variance in the number of shots they would make in 80 free kicks each?

Variance of Anna's 80 free kicks

Answer: 20

Variance of Brian's free kicks

Answer: 12.8

Explanation
The variance of Anna's 80 free kicks is given by $npq = 80 * 0.5 * 0.5 = 20$ and for Brian, it would be $npq = 80 * 0.2 * 0.8 = 12.8$.

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Finger Exercises due Oct 12, 2020 19:30 EDT **Past Due**

Question 1

0.0/1.0 point (graded)

According to the description given in class, which of the following are likely to be characterized by the hypergeometric distribution?

- ◯ The number of red cards drawn from a regular deck 52 cards, where cards are replaced in successive draws
- ◯ At an ice cream shop, the number of customers out of the next 100 that choose chocolate
- ◯ The number of red cards drawn from a regular deck 52 cards, where cards are not replaced in successive draws ✔
- ◯ The number of shots a basketball player makes out of the next 50 shot attempts

Explanation

Similar to the binomial, the hypergeometric distribution models a sequence of success/failure trials. However, the key difference is that in the binomial models the case of sampling with replacement (where the probability of success remains constant over time), while the hypergeometric models the case of sampling without replacement (in other words, the probability of success changes over time depending on the number of successes and failures already drawn). A standard deck of cards contains 26 red and 26 black cards. A hypergeometric distribution models the outcome of 10 draws from the deck where cards are not replaced after each draw. If cards were replaced after each draw, then the sequence of 10 draws could be modeled using the binomial.

Question 2

0.0/1.0 point (graded)

Let's look more closely at the example of a deck of 52 cards, where 13 are clubs, 13 are diamonds, 13 are spades, and 13 are hearts. Suppose that you sample 10 cards from the deck without replacing the cards. What is the probability that exactly five of the cards are hearts?

Please round your answer to three decimal places, i.e. if it is 0.5677, please round to 0.568.

Answer: 0.047

Explanation

Using the formula given in class,$(X|A,B,n) = \frac{\binom{A}{X}\binom{B}{n-x}}{\binom{A+B}{n}}$, where $x = 5$ (5 hearts), $n = 10$(10 draws), $A = 13$ (number of hearts in the full deck), and $B = 39$ (number of non-hearts in the full deck). The probability that five of the ten draws are hearts is 0.047.

You can also perform the calculation in R, using the command **dhyper(x,m,n,k)** where the arguments are as follows: x=number of successes in trials, m=number of successes in population, n=number of failures in population, and k=number of trials conducted.

Show answer

The Poisson Distribution: Example - Quiz

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Finger Exercises due Oct 12, 2020 19:30 EDT Past Due

Question 1

0.0/1.0 point (graded)

Which of the following are requirements for a series of events to be effectively modeled according to the Poisson distribution? (Select all that apply)

- [ ] The probability of the occurrence versus not happening is 50/50
- [ ] Occurrences of the event must be countable and measureable ✔
- [ ] Each of the events are independent ✔
- [ ] The average frequency of occurrences is known for a certain time period ✔

Explanation

The Poisson distribution characterizes a series of events where occurrences can be counted in whole numbers, the occurrences are independent, and the average frequency of occurrences for a given time period is known.

Finger Exercises due Oct 12, 2020 19:30 EDT **Past Due**

Question 1

0.0/1.0 point (graded)

In the Poisson Distribution discussed in class, arrivals or occurrences can be characterized by the parameters gamma (γ) or lambda (λ), where γ represents the propensity to arrive per unit of time (the arrival rate) and λ represents the propensity of arriving within some number of time units, an interval t.

Fill in the blanks with the correct interpretation: In the period of length ______, we can expect there to be _______ arrivals, or occurrences of the event. Select all that apply.

- [ ] t ; γ
- [ ] t ; λ ✔
- [ ] λ ; γ*t
- [ ] t ; γ*t ✔

Explanation

Gamma (γ) represents the propensity to arrive for a given unit of time, while lambda (λ) represents the propensity of arriving within a given time period t. In a period of time of length t, the number of arrivals or occurrences that we can expect is thus given by λ, which is equal to the arrival rate multiplied by the length of the period or, γ multiplied by t.

Question 2

0.0/1.0 point (graded)

Suppose that there is a one lane road where only one bicycle can pass through at any given point. Suppose that you know that the propensity to arrive in any given minute is 0.2. What is the expectation of the number of bicycles that will pass on the road in a 30 minute period?

Answer: 6

Explanation

We are given in the question that the propensity to arrive in any given minute (γ) is 0.2 and that the time period of interest is 30 minutes, so $t = 30$. Using the fact that $E[Nt] = \lambda = \gamma * t$, we can calculate the expectation of the number of bicycles as $E[N_t] = \lambda = \gamma * t = 0.2 * 30 = 6$. Alternatively, can consider $\lambda = 0.2$ for $t = 1$ minute. $E[N_t] = \lambda$. So $E[30 \text{ minutes}] = E[30 * N_t] = 30 * 0.2 = 6$.

Show answer

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Answers are displayed within the problem

Question 3

0.0/1.0 point (graded)

Suppose a soccer team's goal-scoring X in each of their games follows a (fixed) Poisson distribution. What does the $P(X = 2\lambda)$ capture?

- The probability that a game, the soccer team will have to play at least twice the expected number of games they will have to play to score their first goal
- The probability that in a game, the soccer team scores at least twice their expected number of goals
- The probability that a game, the soccer team will have to play twice the expected number of games they will have to play to score their first goal
- The probability that in a game, the soccer team scores twice their expected number of goals ✔

Explanation

λ captures their expected number of goals in a game. Thus $P(X = 2\lambda)$ captures the probability that the team scores twice their expected number of goals.

Show answer

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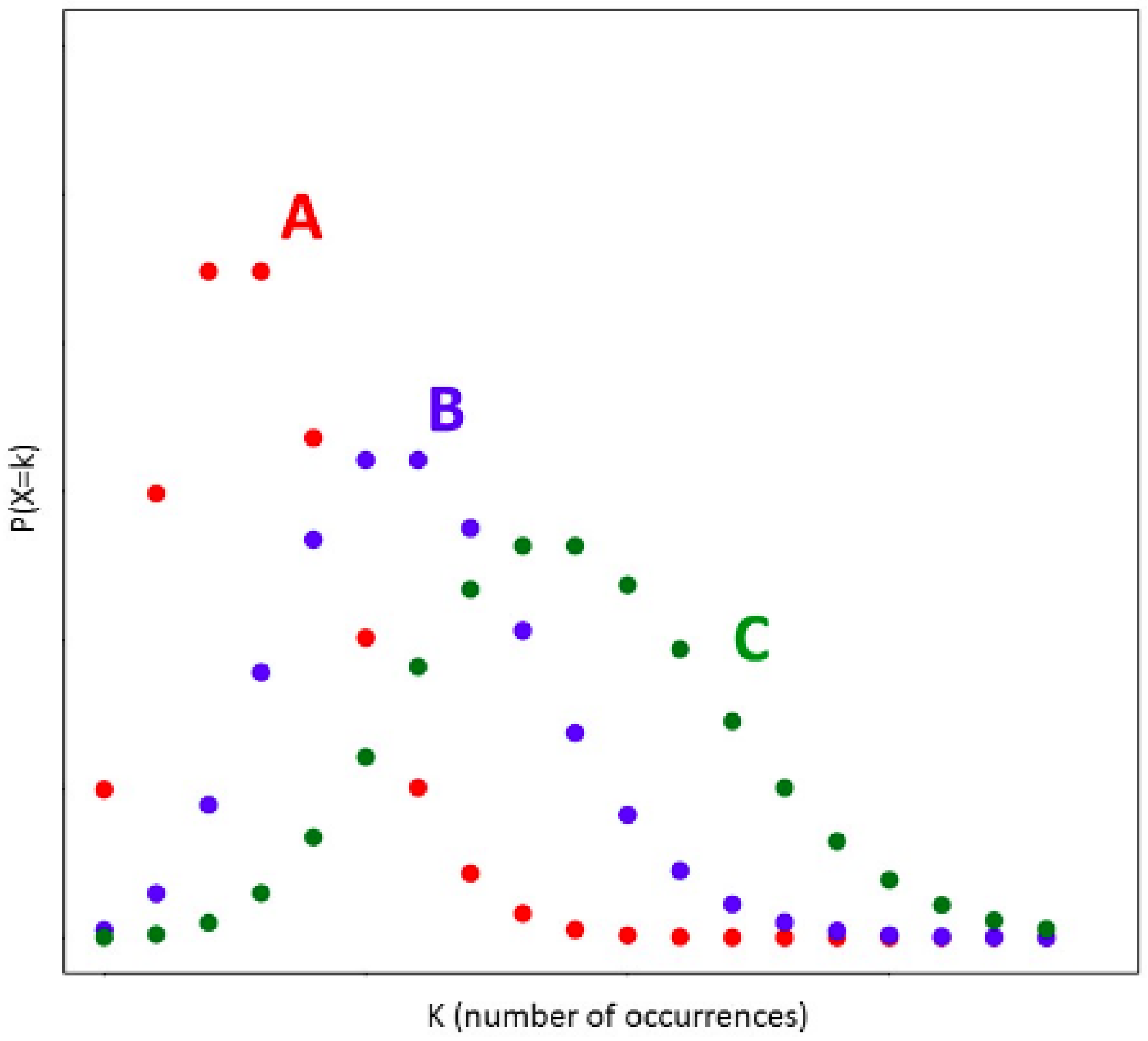
Question 1

0.0/1.0 point (graded)

For the next question, take a look at the following three Poisson variables. Based on what you know about the visual representation of the probability distribution of a Poisson distribution, which of the three distributions has the highest lambda?



- A
- B
- C ✔
- It could be any, it depends on other parameters

Explanation

With a higher lambda, the mass of the distribution moves right (in a more positive direction). In the diagram shown above, C shows the probability distribution with the largest lambda (λ). Note that, as discussed in class, as lambda increases (λ), the Poisson distribution begins to be approximated by the binomial distribution.

Show answer

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Answers are displayed within the problem

Question 2

Please refer to the figure in Question 1 to answer this question.

Now suppose that we want to plot the number of shots on goal during a soccer match. Suppose that there is a fixed propensity for a shot on goal in any given minute of the match. Suppose that the total match is 90 minutes, and you want to plot three probability distributions that represent the total number of shots on goal in 30 minutes, 45 minutes, and 90 minutes. Suppose that the 3 curves represent the 3 distributions.

Which one would represent the probability distribution for the 30 minute window?

- ○ A ✔
- ○ B
- ○ C
- ○ It could be either A or B, but depends on other parameters

Explanation

While gamma (γ), is fixed, lambda (λ) is a function of the window of time that you are interested in. Recall that $\lambda = \gamma * t$. λ will be 90*γ for the 90 minute window, 45*γ for the 45 minute window, and 30*γ for the 30 minute window. λ is the smallest for the 30 minute window, which would be represented by the probability mass function labeled A. Of course it makes perfect sense when you think about it: you are less likely to see a goal in 30 minutes than in 90 minutes!

Show answer

Finger Exercises due Oct 12, 2020 19:30 EDT **Past Due**

Question 1

0.0/1.0 point (graded)

In the context of the exponential distribution, what is meant by memorylessness?

- If x describes the waiting time for some event, then the probability distribution of x at $t = 0$ is the same as the probability distribution of x at time $t = 1$ or $t = 100$ when the event has not occurred, for example. ✔
- The occurrence of any event does not depend on other events
- If x describes the waiting time for some event, then $p(x)$ follows a binomial distribution
- The probability an event occurs is unrelated to the amount of time that has elapsed

Explanation

If you think of the variable X as the wait time for some event, the probability that the event occurs at time $t = 0$ is the same as the probability that the event occurs at time $t = 10$ even if the event has not already occurred. Going back to the soccer example, if x is the time between goals, the distribution of x is the same in the 15^{th} minute of the match as it is in the 50^{th} minute. If the match is scoreless in the 15^{th} minute, there is a certain probability distribution for the time until the next goal; if the match is still scoreless in the 50^{th} minute, the probability distribution for the time until the next goal is the same.

Show answer

Question 2

0.0/1.0 point (graded)

Suppose that you want to create a random variable that is exponentially-distributed in R. Which of the following methods could you use? (Select all that apply.)

- [ ] Use `runif()` to create a uniformly-distributed random variable, then plug that into the inverse PDF function
- [ ] Use `rexp()`, which creates an exponentially-distributed random variable for you ✔
- [ ] Use `runif()`, which creates a uniformly-distributed random variable, and then apply an exponential function
- [ ] Use `runif()` to create a uniformly-distributed random variable, then plug that into the inverse CDF formula given in class ✔

Explanation

B and D are correct. R comes with a function that can create an exponentially-distributed random variable with a single command, `rexp()`. However, you can also build the random variable from scratch by creating a uniformly-distributed random variable, and plugging the random variable in the inverse CDF formula for the exponential. You can check for yourself this produces the same result This is useful to know for some programming languages that can generate uniformly-distributed random variables, but cannot generate random variables of other distributions.

Show answer

The Normal Distribution - Quiz

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Finger Exercises due Oct 12, 2020 19:30 EDT *Past Due*

Question 1

0.0/1.0 point (graded)

True or False: Taking a linear transformation of a normally-distributed random variable generates a normally-distributed random variable. In other words, if X_1 is normally-distributed and $X_2 = a + b * X_1$ for $b \neq 0$, then X_2 is also normally-distributed.

- ○ True ✔
- ○ False

Explanation

This is true. We will also see that this is another useful property of the normal distribution.

Show answer

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Answers are displayed within the problem

Question 2

0.0/1.0 point (graded)

True or False: If you have a set of i.i.d normal random variables, then any linear combination of these variables will follow a uniform distribution.

- True
- False ✔

Explanation

False. As discussed in class, any linear combination of normally distributed random variables will also follow a normal distribution. We will see that this is a very useful property.

Show answer

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Answers are displayed within the problem

Question 3

0.0/1.0 point (graded)

True or False: If $X_1, \ldots, X_n$ are i.i.d. and $X_i \sim N\left(\mu_i, \sigma_i^2\right)$ then $Y = \sum_i X_i^2 \sim N\left(\sum_i \mu_i^2, \sum_i \sigma_i^2\right)$

- True
- False ✔

Explanation

The theorem covered in class only refers to linear combinations of random variables, not arbitrary functions of random variables.

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Finger Exercises due Oct 12, 2020 19:30 EDT Past Due

Question 1

0.0/1.0 point (graded)

After receiving your graded midterm for 14.310x you are told that the exam had an approximately normal distribution. You are told by your TA that you scored one standard deviation above average. You scored higher than what percent of students?

- 50 %
- 84 % ✔
- 95 %
- 68 %

Explanation

Approximately 68% of the data in a normal is within one standard deviation from the normal distribution. If you are exactly one standard deviation above the mean, then you scored higher than 84% of students. If 68% of students scored within one standard deviation of the mean, then 32% scored either lower than one standard deviation below, or greater than one standard deviation above the mean. So, there are 32/2=16% of students that scored one deviation below the mean, added to the 68% that scored within 1 standard deviation of the mean, meaning that you scored higher than 68+16%=84% of students.

Show answer

Finding the Area Under the Curve Using R - Quiz

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Finger Exercises due Oct 12, 2020 19:30 EDT Past Due

Question 1

0.0/1.0 point (graded)

Let $x = 0.75$. Without typing into your R console, what should you get for an output of `qnorm(pnorm(0.75,lower.tail=TRUE),lower.tail=TRUE)`?

Answer: 0.75

Explanation

As discussed in class, the function `pnorm(x)` takes in a value x and returns the CDF of a normal distribution at x. The function `qnorm(x)` gives the value at which the CDF of the standard normal is x. Thus applying both functions are inverses of each other, and thus applying x to the composite function will return x, if the value is in both domains.

Show answer

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Answers are displayed within the problem

Question 2

0.0/1.0 point (graded)

Without typing into your R console, what should you get for an output of the R code `pnorm(qnorm(2.1, lower.tail=TRUE))` ?

- ◯ 0.98
- ◯ `NaN` ✔
- ◯ -2.1
- ◯ 2.1

Explanation

The function `qnorm(x)` gives the value at which the CDF of the standard normal is x. Since the cumulative distribution function can at most take the value of 1, the function is undefined when $x = 2.1$, thus your R code should produce an error `NaN` (Not a Number) when trying to compute `qnorm(2.1)` , which will produce an error for the whole function.

Show answer

Question 3

0.0/1.0 point (graded)

Malia is applying to medical school and has to take the MCAT. To get into the school of her dreams she has to score in the top 20% of all test takers for her year. Last year in 2015 the MCAT had a mean of 500 and a standard deviation of 10.6. Assuming a normal distribution and based on what was discussed in class what simple R code would you write to estimate the score she would need in to get into her dream school?

Using `qnorm(A, B, C)`, fill in what you would input for each of A, B, and C.

What is the input for A?

Answer: 0.8

What is the input for B?

Answer: 500

What is the input for C?

Answer: 10.6