Joint PDFs - Quiz

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Finger Exercises due Sep 28, 2020 19:30 EDT *Past Due*

Question 1

0.0/3.0 points (graded)

Consider a pair of random variables X, Y defined on the same sample space. Because X, Y takes values $(x, y) \in \mathbb{R} \times \mathbb{R}$ (that is, pairs of real numbers), we can consider the joint pdf $f(x, y)$ defined on the x-y plane.

True or false: Joint pdfs must be nonzero everywhere on the x-y plane.

- True
- False ✔

True or false: Joint pdfs must be nonnegative everywhere on the x-y plane.

- True ✔
- False

True or false: Taking the double integral over the x-y plane, joint pdfs must integrate to 1.

- ◯ True ✔
- ◯ False

Explanation
These are all properties of joint PDFs discussed. Just as with single variable PDFs, joint PDFs: I) Can be zero at some points. II) Must be non-negative everywhere on the x-y plane. III) Must integrate to 1.

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Answers are displayed within the problem

Question 2

1 point possible (graded)

The joint pdf $f(x, y)$ is derived from two random variables X, Y with the following function. Solve for c.

Note: Please review our guidelines on precision regarding rounding answers here.

Question 2

1 point possible (graded)

The joint pdf $f(x, y)$ is derived from two random variables X, Y with the following function. Solve for c.

Note: Please review our guidelines on precision regarding rounding answers here.

$$f_{XY}(x, y) = \begin{cases} cxy & \text{if } 0<\text{y}<1 \text{ and } 1<\text{x}<2 \\ 0 & \text{if otherwise} \end{cases}$$

Answer: 4/3

Explanation

This example is similar to the example discussed in class. To solve for c, take the double integral over the support and set equal to 1.

$$\int_0^1 \int_1^2 cxy \, dxdy = 1$$

$$\int_0^1 cy \left(\frac{x^2}{2} \Big|_{x=1}^{x=2} \right) dy = 1$$

$$\int_0^1 cy \left(\frac{2^2}{2} - \frac{1^2}{2} \right) dy = 1$$

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Question 1

1 point possible (graded)

Suppose that you have the following joint PDF:

$$f_{XY}(x,y) = \begin{cases} \frac{xy}{9} & \text{0<y<2 and 0<x<3} \\ 0 & \text{otherwise} \end{cases}$$

What is the probability that x is greater than 2 and y is greater than 1?

Note that this is different from the previous question. Please review our guidelines on precision regarding rounding answers here.

Answer: 5/12

Explanation

To answer this question, set up the relevant double integral over the relevant support, that is, with y ranging from 1 to 2 and x ranging from 2 to 3.

$$\int_1^2 \int_2^3 \frac{1}{9} xy dx dy \int_1^2 \frac{x^2 y}{18} \Big| dy \int_1^2 \frac{5}{36} y^2 \Big|_{x=2\, y=1}^{x=3\, y=2} dy = \frac{5}{12}$$

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Question 1

0.0/1.0 point (graded)

Consider discrete random variables $X, Y : \Omega \to \mathbb{R}$ and the corresponding discrete joint probability function $f_{XY}(x, y)$. How does one derive $P(X = x)$? (Select all that apply)

- [ ] Average $f_{XY}(x, y)$ over all possible values of Y, given that $X = x$
- [ ] Sum up $f_{XY}(x, y)$ over all possible values of Y, given that $X = x$ ✔
- [ ] Integrate $f_{XY}(x, y)$ with respect to Y, where $X = x$
- [ ] Take the derivative of $f_{XY}(x, y)$ with respect to Y, where $X = x$

Explanation

In order to get the probability that $X = x$ given a joint distribution for discrete random variables X and Y, you just have to sum up the joint probabilities $f_{XY}(x, y)$ over all possible values of Y while holding $X = x$ fixed. Note that this is the discrete analogue of integrating a continuous joint distribution over all values of y in order to find the marginal PDF, $f_X(x)$, of a particular value x.

Question 2

0.0/1.0 point (graded)

Suppose that you have two discrete random variables X and Y with the following joint probability distribution, which is similar to the example in class. Fill in the marginal probabilities below.

		Possible Values of X			
		1	**2**	**3**	**4**
Possible Values of Y	**1**	0	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$
	2	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	0

Please input the exact answer in either decimal or fraction form.

Marginal Probability of $x = 1$ or $f_X(1)$:

Answer: 0.125

Marginal probability of $x = 2$ or $f_X(2)$:

Answer: 0.375

Marginal probability of $x = 3$ or $f_X(3)$:

Answer: 0.25

Marginal probability of $x = 4$ or $f_X(4)$:

Answer: 0.25

Explanation

The relevant probabilities are calculated by summing up over the possible values of Y for each $X = x$. Hence, the marginal probabilities are as below:

$$f_X(1) = \frac{1}{8} = 0.125$$

$$f_X(2) = \frac{3}{8} = 0.375$$

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Question 1

1 point possible (graded)

Let's return to the continuous example that we used earlier in this lecture,

$$f_{XY}(x,y) = \begin{cases} \frac{xy}{9} & 0<y<2 \text{ and } 0<x<3 \\ 0 & \text{otherwise} \end{cases}$$

Which of the following represents the marginal PDF of X?

- ○ $f_X(x) = \int_0^2 \frac{xy}{9} dy$ ✔
- ○ $f_X(x) = \int_0^3 \frac{xy}{9} dy$
- ○ $f_X(x) = \int_0^2 \frac{xy}{9} dx$
- ○ $f_X(x) = \int_0^3 \frac{xy}{9} dx$

Explanation

The marginal PDF of X is derived by integrating the joint PDF $f_{XY}(x,y)$ over the support of Y. Option A gives the correct integral because the joint PDF $f_{XY}(x,y)$ is integrated with respect to y over the support of Y, which is $(0,2)$.

Answers are displayed within the problem

Question 2

1 point possible (graded)

Using the same example as above, what is the value of the marginal PDF for $X = 1$?

Please review our guidelines on precision regarding rounding answers here. .

Answer: 2/9

Explanation

To calculate the value of the marginal PDF for $X = 1$, solve the integral that we set up in the previous question.

$$f_X(x) = \int_0^2 \frac{xy}{9} dy$$

$$f_X(1) = \frac{y^2}{18}\Big|_{y=0}^{y=2}$$

$$f_X(1) = \frac{2^2}{18} - \frac{0^2}{18} = \frac{2}{9}$$

Show answer

Independence of Random Variables - Quiz

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Finger Exercises due Sep 28, 2020 19:30 EDT *Past Due*

Question 1

1 point possible (graded)

Consider a pair of random variables X, Y defined on the same space. Suppose all you knew about these random variables were the marginal distributions functions of X and Y. True or false: this is sufficient to construct the joint distribution of X and Y.

- True
- False ✔

Explanation

This is false. Joint distributions contain three pieces of information: how one variable is distributed, how a second variable is distributed, and the relationship between the two variables. Having the marginal distributions gives the first two pieces of information, but it does not give us information about how the two variables are related.

Show answer

Answers are displayed within the problem

Question 2

1 point possible (graded)

Consider a pair of continuous random variables $X, Y : \Omega \to \mathbb{R}$ defined on the same space. X and Y are independent if ____________________.

- The joint PDF integrates to 1: $\iint f_{XY}(x, y)\, dx dy = 1$
- The joint PDF is equal to the product of the marginal PDFs: $f_{XY}(x, y) = f_X(x) f_Y(y)$ ✔
- The marginal PDFs are symmetrical: $f_X(x) = f_Y(y)$
- The joint PDF is equal to the sum of the marginal PDFs: $f_{XY}(x, y) = f_X(x) + f_Y(y)$

Explanation

X and Y are independent if the joint PDF is equal to the product of the marginal PDFs, so B is correct.

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Examples of Independent Random Variables - Quiz

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Finger Exercises due Sep 28, 2020 19:30 EDT **Past Due**

Question 1

1 point possible (graded)

Consider the joint distribution function $f(x, y)$ of a pair of random variables X, Y defined on the same sample space. For which of the following definitions of $f(x, y)$ are X and Y independent random variables? Assume that $f(x, y)$ is defined on some unspecified rectangular region, and that outside this region $f(x, y) = 0$. This allows $f(x, y)$ to integrate to 1. (Select all that apply)

- [ ] $f_{xy}(x, y) = \frac{2xy^2}{3}$ ✔
- [ ] $f_{xy}(x, y) = \frac{3}{x^2} + 2y$
- [ ] $f_{xy}(x, y) = x^2 + y^4$
- [ ] $f_{xy}(x, y) = \frac{2x}{3y^2}$ ✔

Explanation

Explanation

X and Y could be independent in $f_{xy}(x,y) = \frac{2xy^2}{3}$ or $f_{xy}(x,y) = \frac{2x}{3y^2}$ because those distributions both factor into non-negative functions of X and Y alone. For example, $f_{xy}(x,y) = \frac{2xy^2}{3} = \frac{2x}{3} * y^2$.

We can intuitively guess that the other two distributions are indeed not independent, because they do not obviously factor as a product of $g(x)$ and $h(y)$, where $g(x)$ is a function of X alone and $h(y)$ is function of Y alone. But we should prove this rigorously.

Let's examine the case $f(x,y) = x^2 + y^4$. If f is defined on the rectangle $[a,b] * [c,d]$, and it were possible to factor $f(x,y) = g(x)h(y)$ in this region, then we would have

$$f(a,c)f(b,d) = g(a)h(c)g(b)h(d) = g(a)h(d)g(b)h(c) = f(a,d)f(b,c)$$

Thus

$$0 = f(a,c)f(b,d) - f(a,d)f(b,c) = (a^2+c^4)(b^2+d^4) - (a^2+d^4)(b^2+c^4)$$

But it turns out that we can factorize this as $0 = (a^2-b^2)(d^4-c^4)$. If $f(x,y)$ were to be factorizable on this rectangle, this equality would also hold for all a,b,c,d that describe a point inside the rectangle. But this is clearly not true if $a \neq b, c \neq d$: we can pick points inside that have different x coordinate and different y coordinate, violating this equality. Thus in no rectangle can $f(x,y)$ be factorized as $g(x)h(y)$; this is not the joint pdf of two random variables, over any possible rectangle.

The same reasoning can be applied to the remaining two distributions.

Show answer

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You have used 0 of 2 attempts

Question 2

1 point possible (graded)

Suppose that you have the following joint distribution between X and Y from the discrete example we used earlier. Are X and Y independent?

		Possible Values of X			
		1	2	3	4
Possible Values of Y	1	0	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$
	2	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	0

- Yes
- No ✔

Explanation

X and Y are not independent. From before, we know that $f_X(1) = \frac{1}{8}$ and we can easily calculate that $f_Y(1) = \frac{1}{2}$. However, from the table above, $f_{XY}(1,1) = 0$, so it cannot be the case that the joint distribution is the product of the marginal distributions.

Conditional Distributions - Quiz

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Finger Exercises due Sep 28, 2020 19:30 EDT Past Due

Question 1

0.0/1.0 point (graded)

Consider the pair of continuous random variables X, Y defined on the same sample space . What is the conditional distribution $f_{Y|X}(y|x)$ in terms of the joint distribution function $f_{XY}(x, y)$, and the marginal distributions $f_X(x), f_Y(y)$? (Select all that apply)

- [ ] $\frac{f_{XY}(x,y)}{f_X(x)*f_Y(y)}$
- [ ] $\frac{f_{XY}(x,y)}{f_X(x)+f_Y(y)}$
- [ ] $\frac{f_{XY}(x,y)}{f_Y(Y)}$
- [ ] $\frac{f_{XY}(x,y)}{f_X(x)}$ ✓

- [ ] The joint PDF divided by the product of the two marginal PDFs
- [ ] The joint PDF divided by the sum of the two marginal PDFs
- [ ] The joint PDF divided by the marginal PDF of Y
- [ ] The joint PDF divided by the marginal PDF of X ✔

Explanation

The conditional PDF of Y given what you know about X would be calculated as the joint PDF divided by the marginal PDF of X, to "condition" the representation of the distribution of B based on what you know about X. In mathematical terms:

$$f_{Y|X} = \frac{f_{XY}(x, y)}{f_X(x)}$$

Show answer

Submit You have used 0 of 2 attempts

Answers are displayed within the problem

Question 1

0.0/1.0 point (graded)

Let's go back to the discrete example that we've been looking at throughout this lecture. Fill in the blanks below.

Note: Please review our guidelines on precision regarding rounding answers here.

		Possible Values of X			
		1	2	3	4
Possible Values of Y	1	0	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$
	2	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	0

What is the conditional distribution of Y give that X is equal to 2?

$$f_{Y|X}\left(Y = 1|X = 2\right) = A$$

$$f_{Y|X}\left(Y = 2|X = 2\right) = B$$

Input your response for the value for A

Answer: 1/3

Input your response for the value of B

Answer: 2/3

What is the conditional distribution of X given that Y is equal to 2?

$$f_{X|Y}\left(X = 1|Y = 2\right) = C$$

$$f_{X|Y}\left(X = 2|Y = 2\right) = D$$

$$f_{X|Y}\left(X = 3|Y = 2\right) = E$$

$$f_{X|Y}\left(X = 4|Y = 2\right) = F$$

Input your response for the value of C

Answer: 0.25

Input your response for the value of D

Answer: 0.5

Input your response for the value of E

Answer: 0.25

Input your response for the value of F

Answer: 0

Explanation
To see this, let's walk through the calculation for $f_{X|Y}\left(x = 3|y = 2\right)$. We know from before, or can equally calculate the marginal probability that Y=2, which is $\frac{1}{2}$ or 0.5 by summing up over the possible values of X. The joint probability that x=3 and y=2 comes directly from the table as $\frac{1}{8}$ or 0.125. To calculate the marginal distribution of X evaluated at x=3, divide 0.125 by 0.5 to get $\frac{1}{4}$ or 0.25.

Show answer

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Submit You have used 0 of 2 attempts

Answers are displayed within the problem

Question 2

1 point possible (graded)

True or False: The conditional distribution of a random variable X given Y is equivalent to the unconditional distribution of X so long as X and Y are independent.

- True ✔
- False

Explanation

This is true. Two variables are independent if the distribution of the first variable conditional on the second variable is equal to the unconditional distribution of the first. Intuitively, if knowing the realization of the second variable tells you nothing about the distribution of the first variable, it must be that the two variables are independent.

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