Introduction to Random Variables - Quiz

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Finger Exercises due Sep 21, 2020 19:30 EDT Past Due

Question 1

0.0/1.0 point (graded)

Which of the following are examples of discrete random variables? (Select all that apply)

- [ ] Number of customers at a grocery store on a certain day in the future ✔
- [ ] Whether someone will vote for a Democratic presidential candidate in the next election ✔
- [ ] The exact weight of a box of books someone hands you
- [ ] The weight reading on a scale with precision to the nearest kilogram, of a small box of books that someone hands you ✔

The first option involves the number of people, a discrete quantity. The second involves a binary choice, and is hence also discrete. The third involves the exact weight of an object, and this quantity is continuous as it can take any value on the real line. Finally, the last option involves the weight to the nearest whole number so it is indeed discrete.

Question 2

0.0/1.0 point (graded)

Which of the following are examples of continuous random variables? (Check all that apply)

- [ ] The weight of a fish from the ocean ✔
- [ ] Next year's inflation rate ✔
- [ ] When a bird will next pass by your window ✔
- [ ] How many people will vote for the Democratic presidential candidate

Weight, inflation rate, and time can take on any real value within their respective possible ranges and are hence continuous. While the number of people that will vote for the democratic presidential candidate must be integer, which means it is a discrete quantity. Note a technicality, which is that sometimes when we report the value a continuous random value takes, we have to report it on a discrete scale. But if this scale is highly precise, the variable is approximately continuous, which suffices to a consider as a continuos random variable.

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Show Answer

Question 3

0.0/1.0 point (graded)

Consider a discrete random variable X, and the set of outcomes in Ω. Then $X^{-1}(x)$ is the inverse function of X where $X^{-1}(x)$ is the set of outcomes s in which $X(s) = x$.

Which of the following are always true about the two sets $X^{-1}(x)$ and $X^{-1}(y)$, both subsets of Ω, when $x \neq y$?

- ◯ They are exhaustive
- ◯ They are disjoint ✔
- ◯ They are identical

Explanation

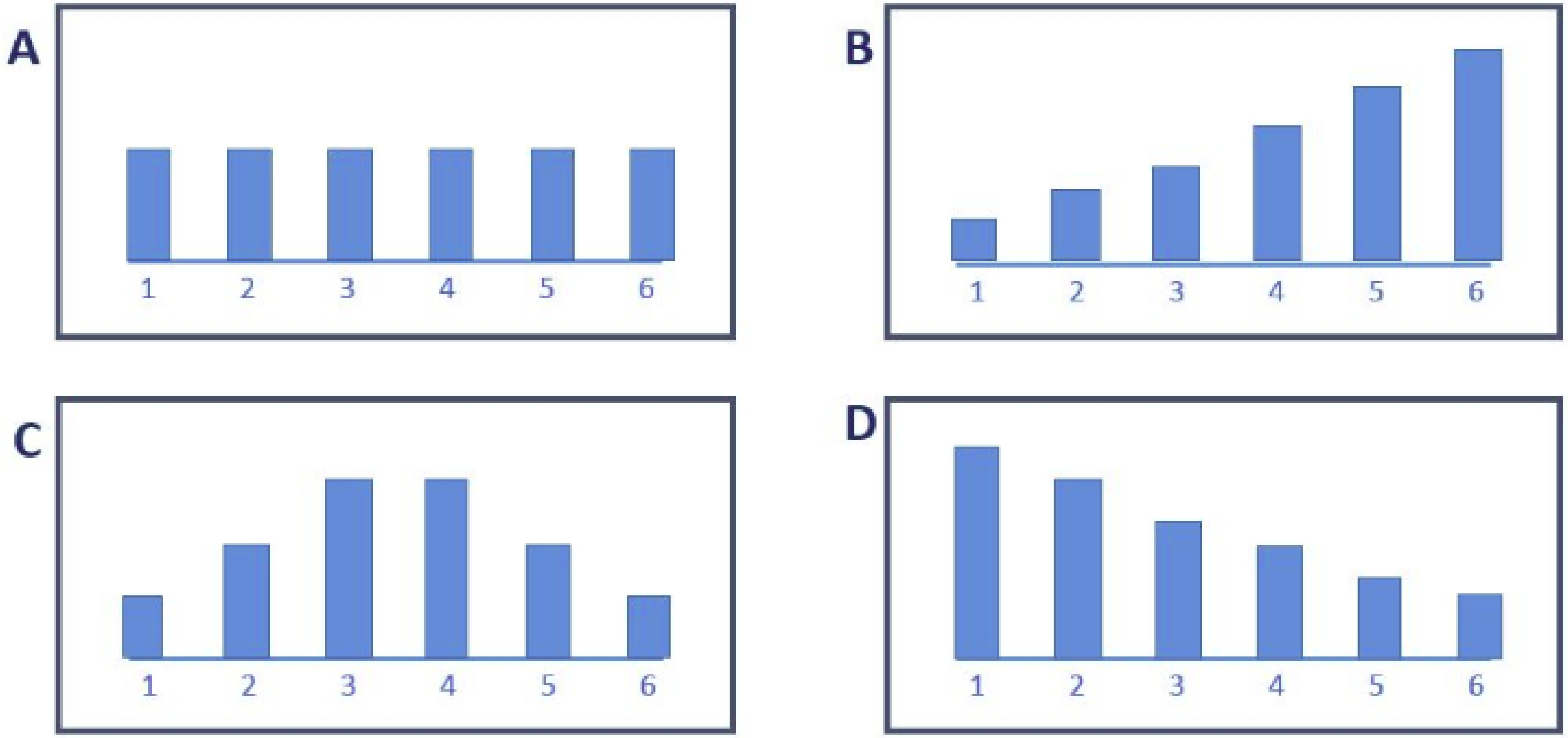
$X^{-1}(x)$ is the set of outcomes s in which $X(s) = x$. This means that the two sets must be disjoint, since if s was in both sets we would have $x = X(s) = y$, but x, y are distinct. They are not necessarily exhaustive, because there could be other possible values of $X(s)$ other than x or y. They are only identical if they are both empty, but this is not always true.

Finger Exercises due Sep 21, 2020 19:30 EDT **Past Due**

Question 1

0.0/1.0 point (graded)

Suppose that you roll one six-sided, fair die once. Which of the following diagrams represents the associated probability function of observing each of the faces (1-2-3-4-5-6)?



A ✔

B

C

D

Explanation
A is correct, where the probability distribution depicted shows that rolling each of 1, 2, 3, 4, 5, or 6 are equally likely.

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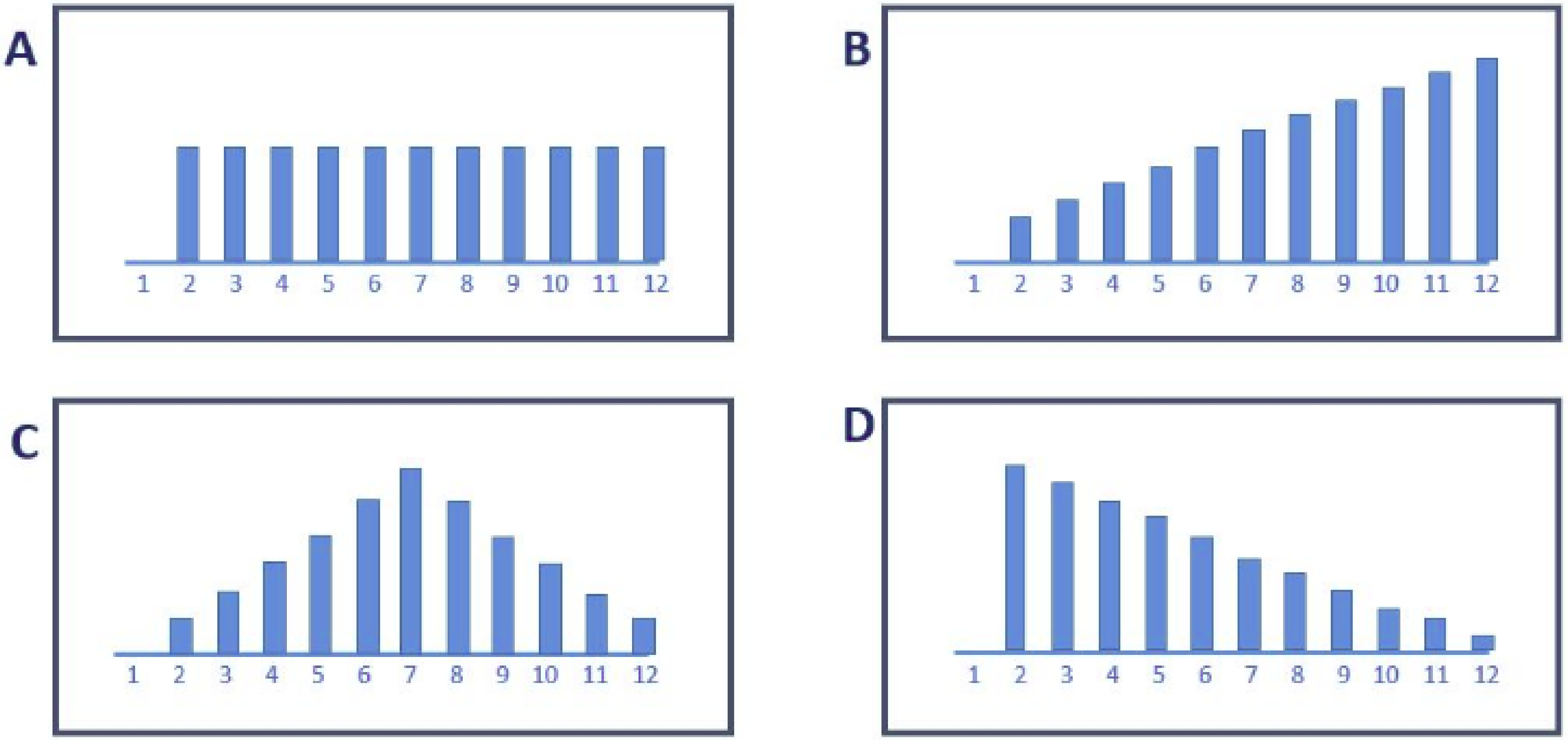
Show Answer

Answers are displayed within the problem

Question 2

0.0/1.0 point (graded)

Suppose that you will roll two six-sided, fair dice one at a time, and then add up the two values rolled. Which of the following diagrams approximately represents the associated probability of the die adding up to each of $1, 2, 3 \ldots 12$?


A
1 2 3 4 5 6 7 8 9 10 11 12
B
1 2 3 4 5 6 7 8 9 10 11 12
C
1 2 3 4 5 6 7 8 9 10 11 12
D
1 2 3 4 5 6 7 8 9 10 11 12

- ◯ A
- ◯ B
- ◯ C ✔
- ◯ D

Explanation
C correctly represents that there are several equally likely combinations that would each add up to the middle values. For example, a total of 7 could be achieved from combinations of 1+6, 2+5, or 4+3. In contrast, there are few combinations that would add up to the high and low values. For example, achieving a 2 requires both die to roll a 1 and achieving a 12 requires both die to roll 6.

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Question 3

0.0/1.0 point (graded)

Consider the sample space Ω of pure outcomes for two dice rolls. How many elements are in Ω?

Answer: 36

Explanation
For each of the six possibilities for the first die, there are six possibilities for the second, so a total of 36 outcomes.

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Show Answer

Question 4

0.0/1.0 point (graded)

If the die are fair, then each of the outcomes in Ω has what associated probability?

Note: Please review our guidelines for rounding answers here.

Answer: 1/36

Explanation

Each outcome is equally likely, and there are 36 outcomes, so there must be 1/36 probability for each.

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Question 5

0.0/1.0 point (graded)

Now consider the random variable $X : \Omega \to \mathbb{N}$ that tracks the sum of the outcomes of two dice rolls. Which of these captures $X^{-1}(8)$, i.e. the set of pure outcomes that add to 8?

Question 5

0.0/1.0 point (graded)

Now consider the random variable $X : \Omega \to \mathbb{N}$ that tracks the sum of the outcomes of two dice rolls. Which of these captures $X^{-1}(8)$, i.e. the set of pure outcomes that add to 8?

- ◯ $\{\{1,7\},\{2,6\},\{3,5\},\{4,4\},\{5,3\},\{6,2\},\{7,1\}\}$
- ◯ $\{\{2,6\},\{3,5\},\{4,4\},\{5,3\},\{6,2\}\}$ ✔
- ◯ $\{\{2,6\},\{3,5\},\{4,4\}\}$

Explanation

We are looking for unique dice combinations that add up to 8. The second answer exhausts all the possibilities and is therefore correct. Note that a dice combination like $\{2,6\}$ which means "2 on the first dice, 6 on the second", is different from $\{6,2\}$, which means "6 on the first dice, 2 on the second". The third answer thus undercounts. Meanwhile, the first answer involves throwing a "7" on a dice roll, which is impossible.

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The Hypergeometric Distribution - Quiz

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Finger Exercises due Sep 21, 2020 19:30 EDT Past Due

Question 1

0.0/1.0 point (graded)

The hypergeometric distribution is characterized by the following equation:

$$f_X(x) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

$$x = max(0, n+k-N), \ldots, min(n,k)$$

This distribution describes the number of "successes" (commonly denoted as Select an option **Answer:** x) in a sample of size Select an option **Answer:** n that is drawn from a population of size N whose initial probability of success is $\frac{k}{N}$ Select an option **Answer:** without replacement.

Explanation

A hypergeometric distribution describes the number of successes, *x* in *n* draws without replacement from a total population of size *N*, where there is initially a fixed number, *k* of the object of interest.

Question 2

0.0/1.0 point (graded)

Consider $x = k$ in the above distribution. $\binom{k}{k} = 1$, so the hypergeometric distribution reduces to $\frac{\binom{N-k}{n-k}}{\binom{N}{n}}$. Which of these events has this probability?

- When randomly choosing n items from N possible items without replacement, the probability that none of k forbidden items are chosen.
- When randomly choosing n items from N possible items without replacement, the probability that all of k required items are indeed chosen ✔
- When randomly choosing n items from N possible items without replacement, where k of them are recommended to be chosen, the probability that at least one of the recommended items is chosen

Explanation

The number of ways to choose n items, including k of the required ones, is $\binom{N-k}{n-k}$, since we only have options in the $N - k$ non-required items, and we want to choose $n - k$ of them.

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Show Answer

Question 1

0.0/1.0 point (graded)

True or false: The binomial distribution describes the number of successes in n Bernoulli (binary outcome) trials, with the additional constraint that in each trial the probability of success has to be equal to the probability of failure.

- ◯ True
- ◯ False ✔

Explanation
This is false. The “success” and “failure” outcomes are not required to be equally likely. Note also that the Bernoulli trials must be independent for the distribution to be a binomial.

Submit You have used 0 of 1 attempt

Show Answer

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Question 2

0.0/1.0 point (graded)

Consider the sample space of n tosses of a coin. What is the size of this space? (Select all that apply)

Consider the sample space of n tosses of a coin. What is the size of this space? (Select all that apply)

- [ ] 2^n ✔
- [ ] $n!$
- [ ] $\binom{n}{0} + \binom{n}{1} \ldots \binom{n}{n}$ ✔
- [ ] $\binom{n}{1} + \binom{n}{2} \ldots \binom{n}{n}$

Explanation

The sample space is of size 2^n, because at each of the n steps there are two possibilities for the coin's outcome. However, this can also be counted in another way. $\binom{n}{0}$ counts the number of outcomes in which there are 0 heads; $\binom{n}{1}$ in which there are 1 heads; and so on until $\binom{n}{n}$ counts the number of outcomes in which all n coins result in heads. Because each of these cases is disjoint, and every outcome of n coin flips must have some number of heads from 0 to n, summing all these terms covers the whole sample space. Thus $2^n = \binom{n}{0} + \binom{n}{1} \ldots \binom{n}{n}$.

Submit You have used 0 of 2 attempts

Show Answer

Answers are displayed within the problem

Question 3

0.0/1.0 point (graded)

Consider an unbiased coin with $P\left(Heads\right) = 0.5$. What is the probability of exactly k heads in n tosses? (Select all that apply)

- [ ] $\binom{n}{k}0.5^k(1-0.5)^{n-k}$ ✔
- [ ] $\binom{n}{k}0.5^n$ ✔
- [ ] 0.5^n
- [ ] 0.5^k

Explanation

As discussed previously, $\binom{n}{k}0.5^k(1-0.5)^{n-k}$ is the probability that k of the tosses are heads and the remaining $n-k$ are tails. This can be simplified in this case to $\binom{n}{k}0.5^n$.

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Question 4

0.0/1.0 point (graded)

Consider a biased coin with $P(head) = 0.4$. Which expression(s) capture the probability of obtaining up to $n-1$ heads in n throws? (Select all that apply)

- [ ] $\binom{n}{0}(0.4)^0(0.6)^n + \binom{n}{1}0.4^1 0.6^{n-1} + \cdots + \binom{n}{n-1}0.4^{n-1}0.6^1$ ✔
- [ ] $\binom{n}{1}0.4^1 0.6^{n-1} + \binom{n}{2}0.4^2 0.6^{n-2} + \cdots + \binom{n}{n-1}0.4^{n-1}0.6^1$
- [ ] $\binom{n}{n-1}0.4^{n-1}0.6^1$
- [ ] $1 - \binom{n}{n}0.4^n 0.6^0$ ✔
- [ ] $1 - 0.4^n$ ✔

Explanation

Each term $\binom{n}{k}.4^k.6^{n-k}$ represents the probability of there being k heads. For there to be up to $n-1$ heads, we must add these expressions from $k = 0$ through $k = n-1$.

Furthermore note that the throwing up to $n-1$ heads enumerates all the outcomes aside from throwing n heads. Thus we can calculate this probability as $1 - \binom{n}{n}.4^n.6^0$. which we can also simplify to $1 - 0.4^n$.

Properties of the Probability Distribution - Quiz

Finger Exercises due Sep 21, 2020 19:30 EDT Past Due

Question 1

0.0/1.0 point (graded)

The probability mass function f_X of a discrete random variable X has the properties: (Select all that apply)

- [ ] $0 \leq f_X(x) \leq 1$ ✔
- [ ] $\sum_x f_X(x) = 1$ ✔
- [ ] The set x where $f_X(x)$ is nonzero has to be finite.

Explanation

For each x, $f_X(x)$ captures distinct probabilities and these have to be less than 1 and add to 1 over the domain of X (the sets of values that X takes). However, the set where they are nonzero doesn't have to be finite. See the geometric distribution for an example.

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Finger Exercises due Sep 21, 2020 19:30 EDT **Past Due**

Question 1

0.0/1.0 point (graded)

True or false: The conditions for a probability density function to be valid include that the density at each point is less than 1 and that the function must integrate to 1.

- True
- False ✔

Explanation

False. The probability density function must integrate to 1 over its entire domain, but this does not mean that the probability density must be less than 1 at all points in the domain. Take for instance a uniform distribution with range $[0, 0.5]$; the probability density at every point is 2.

Submit You have used 0 of 1 attempt

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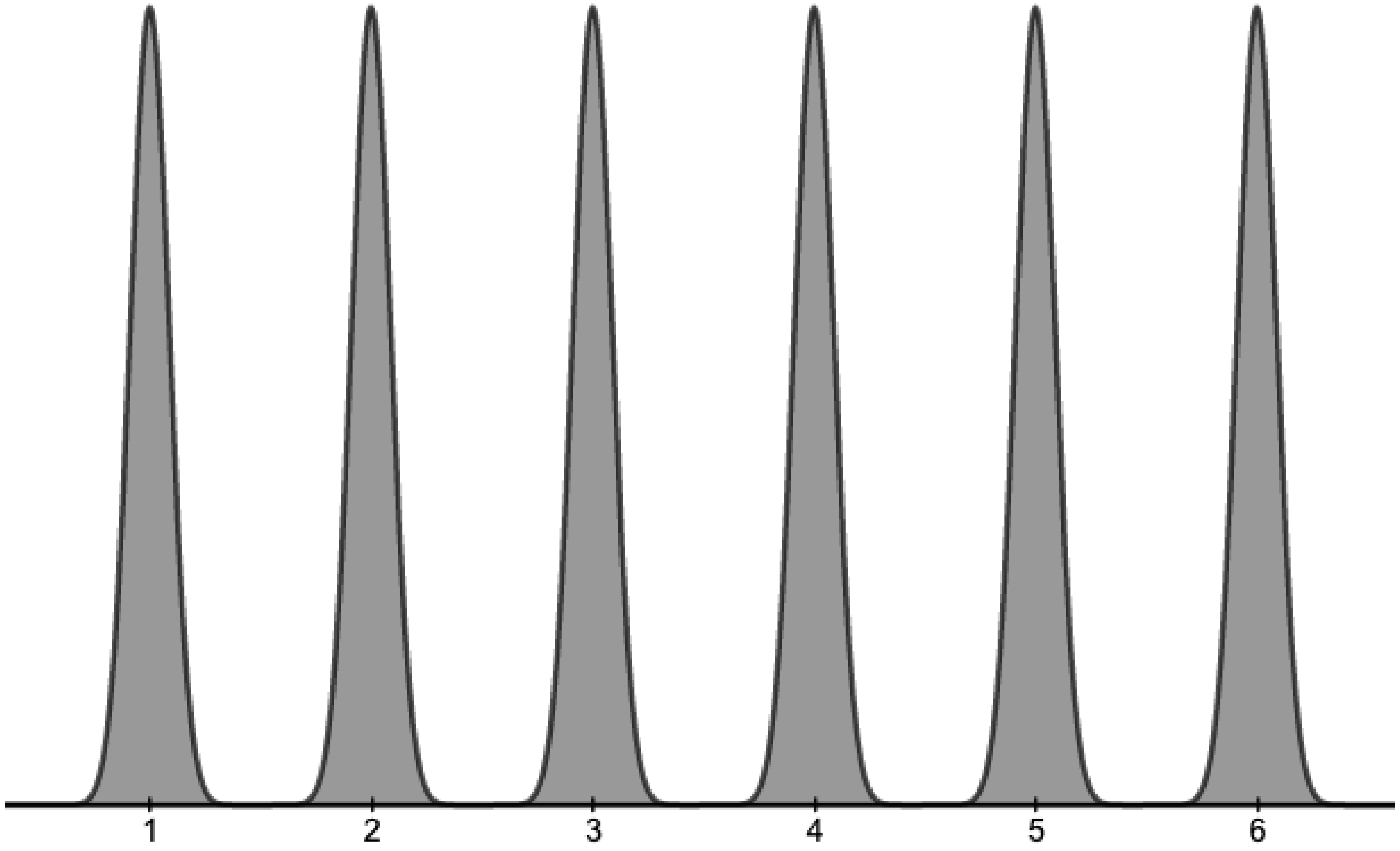
Question 2

0.0/1.0 point (graded)

Consider a probability density function which has a narrow “spike” centered at each of the fixed points $1, 2, \ldots, 6$, and is 0 otherwise.



The area of each spike is $\frac{1}{6}$, so the probabilities indeed integrate to 1. Such a distribution approximates which of these discrete random variables?

- ◯ A random real number from 1 to 6
- ◯ A single roll of a fair dice ✔
- ◯ In a series of 6 dice rolls, how many dice rolls result in a "1"

Explanation

The spikes approximate a discrete uniform probability distribution where the only possible outcomes are the integers 1 through 6, hence option 1 is incorrect. Option 2 is correct: A fair dice has an equal probability, $\frac{1}{6}$, of showing any of its 6 faces when rolled, hence the probability distribution for this experiment is a discrete uniform distribution defined only at integers 1 through 6. Finally, the number of realizations of a particular outcome in a series of dice rolls follows the binomial distribution, not the uniform distribution, and is therefore not approximated by the distribution shown above.

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A Note on Terminology and the Uniform Distribution - Quiz

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Finger Exercises due Sep 21, 2020 19:30 EDT **Past Due**

Question 1

0.0/1.0 point (graded)

Assume that you have a continuous random variable which is uniformly distributed in the range: $[3, 8]$. What is the probability that the random variable takes on a value less than or equal to 7?

Note: Please review our guidelines on precision regarding rounding answers here.

Answer: 0.80

Explanation

To compute this probability we find the area of the specified region, and since this is a uniform distribution this corresponds to dividing the length of the specified range of values by the total length: $\frac{7-3}{8-3} = 0.8$.

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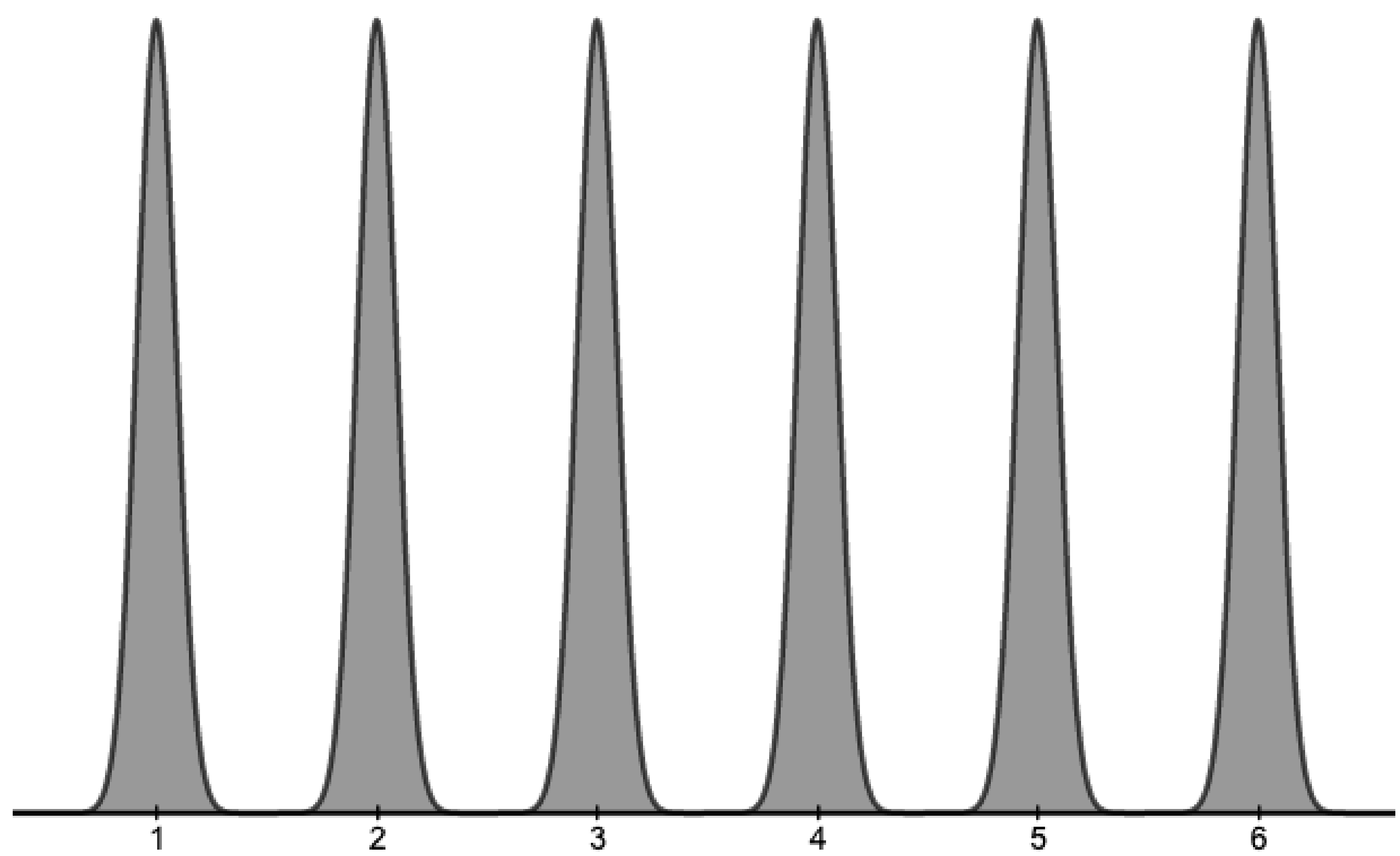
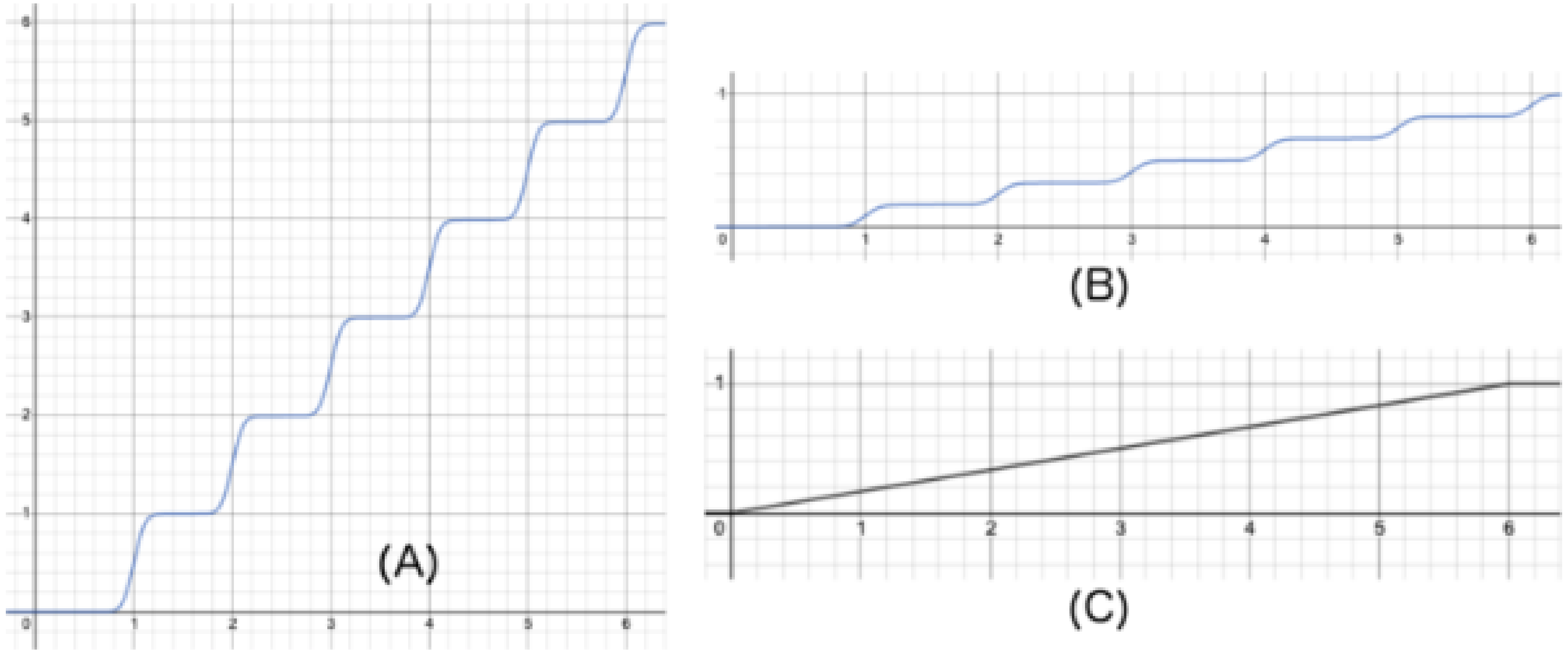
Finger Exercises due Sep 21, 2020 19:30 EDT **Past Due**

Question 1

0.0/1.0 point (graded)

Consider the probability density function which a narrow “spike” centered at fixed points 1,2,...,6, and 0 otherwise.





A

B ✔

C

Explanation

The distribution approximates a probability distribution of a discrete random variable, and the CDF for a discrete random variable takes the form of a step function. This is because the probability of all points between consecutive points with non-zero probability is equal to 0. Finally, we can see that both options A and B closely resemble such a function, but the maximum value of a CDF is always equal to 1, which is why the correct answer is B.

Finger Exercises due Sep 21, 2020 19:30 EDT *Past Due*

Question 1

0.0/1.0 point (graded)

Joint probability density functions (joint PDF) for continuous random variables exhibit which of the following properties? (Select all that apply)

- [ ] The value of the joint PDF at any particular point is zero
- [ ] The value of the joint PDF at any particular point is non-negative ✔
- [ ] The joint PDF integrates to 1 over the entire domain ✔
- [ ] The joint PDF integrates over the entire domain to the number of variables. For example, for 2 variables x and y, the joint PDF integrates to 2

Explanation

Note the distinction between the **value** of the joint PDF and the **probability** of the joint PDF. Just as in the case of the PDF for a single random variable, the joint probability density, $f_{XY}(x, y)$, at any particular point is non-negative, and the joint PDF must integrate to 1 over the $x - y$ plane.

Question 2

0.0/1.0 point (graded)

Suppose the continuous random variable X has a pdf that is nonzero in the region $[-5, 5]$ and the continuous random variable Y has a pdf that is only nonzero in the region $[-10, 10]$. Then the joint distribution of (X, Y) has a pdf that is nonzero in a subset of which region?

- ◯ The interval $[-5, 5] \cup [-10, 10] = [-10, 10]$
- ◯ The ellipse $x^2/25 + y^2/100 = 1$
- ◯ The rectangle $[-5, 5] \times [-10, 10]$ ✔

Explanation

The joint distribution of (X, Y) lies in the cross product of the two random variables' dimensions (you can think of X, Y as lying in the 2D plane, one axis corresponding to X values and one axis corresponding to Y values). Thus it can only be nonzero in the region where X and Y are both nonzero, i.e. the rectangle $[-5, 5] \times [10, 10]$. If it were nonzero anywhere else non-trivially (i.e. for a region R outside this rectangle with some 'thickness'), then if we integrated the joint distribution over the plane to find the marginal probabilities, we would obtain that the marginal probabilities of either X or Y are nonzero over the x or y projections of R, violating the assumptions of the question.

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Show Answer

Show Answer

Answers are displayed within the problem

Question 3

0.0/1.0 point (graded)

In the region selected in the above question what is true of the double integral of the joint distribution?

- It equals 0
- It equals 1 ✔
- It is undefined

Explanation

All valid probability distribution functions integrate to 1 over their entire domain.

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Joint Distributions: An Example, Part I - Quiz

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Finger Exercises due Sep 21, 2020 19:30 EDT **Past Due**

Question 1

0.0/1.0 point (graded)

In the example given in class, Sara aims to calculate the probability that her headache will return if she takes a tablet of naproxen (which has an effective period of X) and a tablet of acetaminophen (which has an effective period of Y). In order to get the joint probability that her headache returns within three hours, the process is to take the Select an option

Answer: double integral of the Select an option **Answer:** joint probability density function over the regions where x is between Select an option **Answer:** 0 and 3 and y is between Select an option **Answer:** 0 and 3

Explanation

In the example given in class, we want to compute the probability that x is less than 3 and y is less than 3. To compute this joint probability, we take the double integral of the joint probability distribution function for the region where x is less than 3 **AND** y is less than 3.

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Show Answer

Answers are displayed within the problem

Question 1

0.0/1.0 point (graded)

Given the CDF of a continuous random variable, which of the following processes allows you to get the PDF of that random variable?

- You cannot recover the PDF knowing only the CDF
- Integrate from 0 to 1
- Integrate over the relevant region
- Take the derivative of the CDF ✔

Explanation

In order to compute the PDF from a CDF of a continuous random variable, you simply take the derivative of the CDF.

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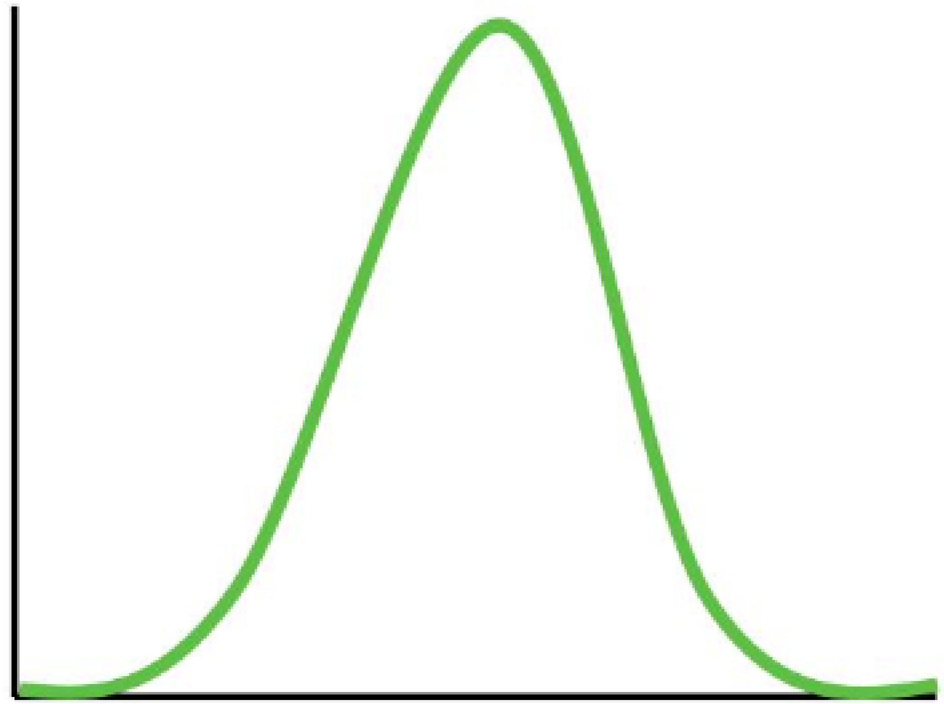
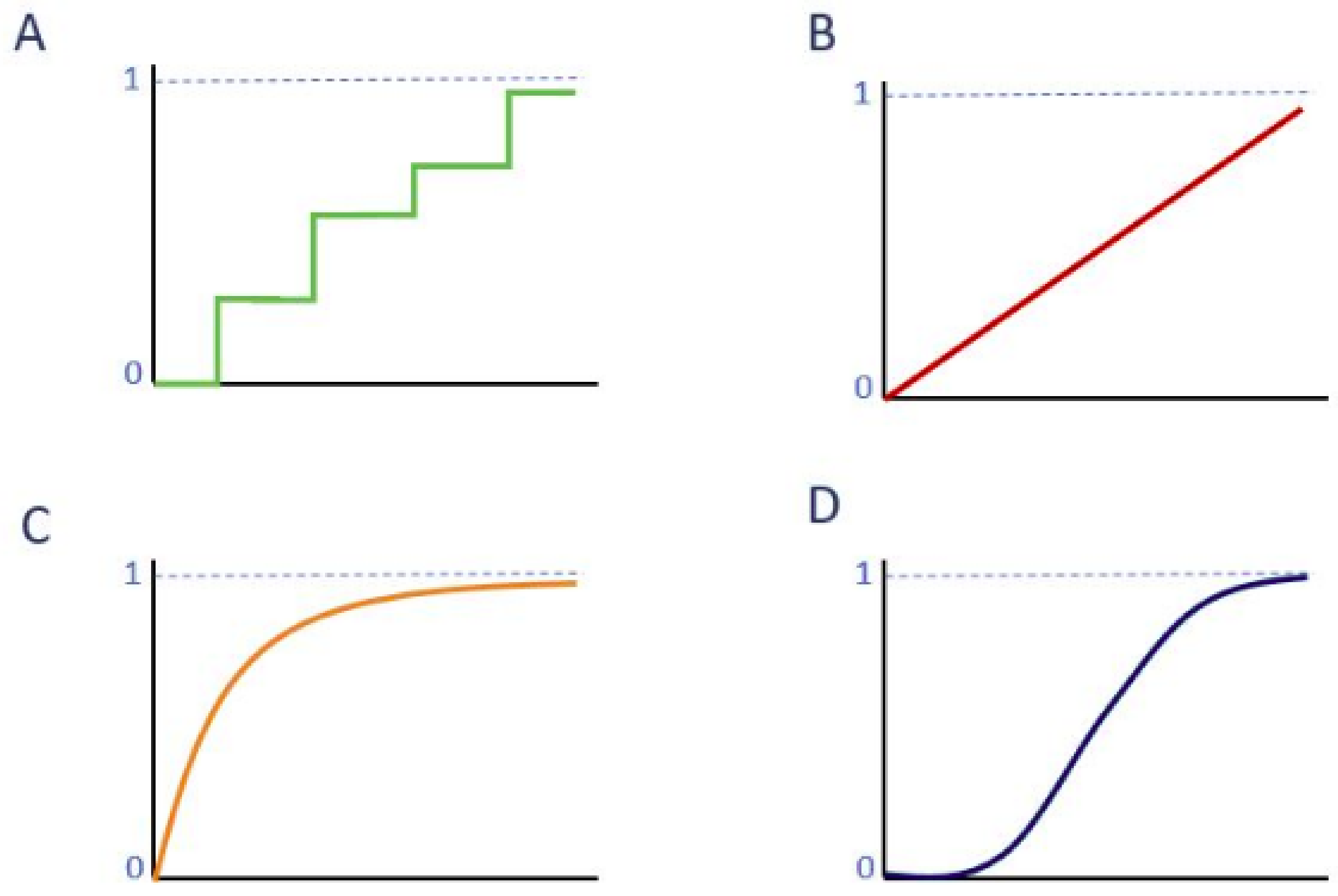
Question 2

0.0/1.0 point (graded)



Assume you have the PDF shown above. Based on what you know about the relationship between the PDF and the CDF, which of the following CDFs appears to correspond?



- A
- B
- C

D ✔

Explanation

We can immediately rule out option A as it represents the CDF of a discrete random variable. Now looking at the PDF shown, and with the knowledge that the PDF is the derivative of the CDF, we can conclude that we are looking for a CDF that has small derivatives (slopes) at first, then larger ones, then small ones again. Only option D fits this description.

Submit

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Show Answer

Answers are displayed within the problem