Quantum Mechanics

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Chapter 1

Dirac notation

Inner product of two vectors

$$\langle a|b\rangle = \langle b|a\rangle^* \tag{1.1}$$

$$\langle a|a\rangle \geq 0 \tag{1.2}$$

1.1 Postulate X: time evolution

$$i\hbar\frac{\partial}{\partial t}\left|\Psi(t)\right\rangle = \hat{\mathrm{H}}(t)\left|\Psi(t)\right\rangle \tag{1.3}$$

Chapter 2

Postulates

2.1 Postulate 1: state

Postulate 1 *The state of a physical system is described by a state vector that belongs to a complex vector space V, called the state space of the system.*

Chapter 3

State space

$$|\Psi_1\rangle + |\Psi_2\rangle = |\Psi_3\rangle \tag{3.1}$$

$$|\Psi_1\rangle + |\Psi_2\rangle = |\Psi_2\rangle + |\Psi_1\rangle \tag{3.2}$$

Chapter 4

Operators

4.1 Basic Properties

An operator acting on a ket creates a new ket:

$$\hat{\mathrm{A}}\,|\Psi\rangle = |\Psi'\rangle \tag{4.1}$$

$$|\Psi\rangle\,, |\Psi'\rangle \in V \tag{4.2}$$

Operators are linear

$$\hat{\mathrm{A}}(a_1\,|\Psi_1\rangle + a_2\,|\Psi_2\rangle) = (a_1\,\hat{\mathrm{A}}\,|\Psi_1\rangle + a_2\,\hat{\mathrm{A}}\,|\Psi_2\rangle) \tag{4.3}$$

$$|\Psi\rangle\,, |\Psi'\rangle \in V; a_1, a_2 \in \mathbb{C} \tag{4.4}$$

Operators are associative and commutative under addition

$$\hat{\mathrm{A}} + (\hat{\mathrm{B}} + \hat{\mathrm{C}}) = (\hat{\mathrm{A}} + \hat{\mathrm{B}}) + \hat{\mathrm{C}} \tag{4.5}$$

$$\hat{\mathrm{A}} + \hat{\mathrm{B}} = \hat{\mathrm{B}} + \hat{\mathrm{A}} \tag{4.6}$$

Multiplying operators is interpreted as applying them to kets. It is associative but NOT (in general) commutative.

$$\hat{\mathrm{A}}\,\hat{\mathrm{B}}\,|\Psi\rangle = \hat{\mathrm{A}}(\hat{\mathrm{B}}\,|\Psi\rangle) = \hat{\mathrm{A}}\,|\Psi'\rangle \tag{4.7}$$

$$\hat{\mathrm{A}}(\hat{\mathrm{B}}\,\hat{\mathrm{C}}) = (\hat{\mathrm{A}}\,\hat{\mathrm{B}})\,\hat{\mathrm{C}} \tag{4.8}$$

$$\hat{\mathrm{A}}\,\hat{\mathrm{B}} \neq \hat{\mathrm{B}}\,\hat{\mathrm{A}} \tag{4.9}$$

The lack of commutativeness makes the "commutator" useful

$$[\hat{\mathrm{A}}, \hat{\mathrm{B}}] = \hat{\mathrm{A}}\,\hat{\mathrm{B}} - \hat{\mathrm{B}}\,\hat{\mathrm{A}} \tag{4.10}$$

$$\tag{4.11}$$

+

4.2 Hermitian Operators

+ An operator is called Hermitian (or self-adjoint) if it is it's own conjugate $\hat{\mathrm{A}} = \hat{\mathrm{A}}^{\dagger}$ +

$$\hat{\mathrm{A}} \left|A\right\rangle = \left|B\right\rangle \rightarrow \left\langle A\right| \hat{\mathrm{A}}^{\dagger} = \left\langle B\right| \tag{4.12}$$

$$\hat{\mathrm{A}} \left|A\right\rangle = \left|B\right\rangle \rightarrow \left\langle A\right| \hat{\mathrm{A}} = \left\langle B\right| \tag{4.13}$$

$$\tag{4.14}$$

A hermitian operator $\hat{\mathrm{A}}$ has the following properties

1. $\hat{\mathrm{A}} \left|\lambda\right\rangle = \lambda \left|\lambda\right\rangle \rightarrow \lambda \in \mathbb{R}$
2. $\langle \hat{\mathrm{A}} \rangle = \left\langle \Psi\right| \hat{\mathrm{A}} \left|\Psi\right\rangle \in \mathbb{R}$
3. All eigenvectors with different eigenvalues are orthogonal

4.3 Projection Operators

A projection operator is defined

$$[\hat{\mathrm{A}}, \hat{\mathrm{B}}] = \hat{\mathrm{A}}\,\hat{\mathrm{B}} - \hat{\mathrm{B}}\,\hat{\mathrm{A}} \tag{4.15}$$

$$\tag{4.16}$$

Chapter 5

Eigenvectors and Eigenvalues

5.1 Degenerate Eigenvectors

If two eigenvectors have the same eigenvalue:

$$\hat{\mathrm{H}} |\lambda_1\rangle = \lambda |\lambda_1\rangle \tag{5.1}$$
$$\hat{\mathrm{H}} |\lambda_2\rangle = \lambda |\lambda_2\rangle \tag{5.2}$$

their linear combination is an eigenvector as well:

$$\alpha \hat{\mathrm{H}} |\lambda_1\rangle = \lambda\alpha |\lambda_1\rangle \tag{5.3}$$
$$\beta \hat{\mathrm{H}} |\lambda_1\rangle = \lambda\beta |\lambda_1\rangle \tag{5.4}$$
$$\hat{\mathrm{H}}[\alpha |\lambda_1\rangle + \beta |\lambda_2\rangle] = \lambda[\alpha |\lambda_1\rangle + \beta |\lambda_2\rangle] \tag{5.5}$$

Therefore it is possible to create two orthogonal eigenvectors for this eigenvalue.

The probability in the degenerate case is the sum of the probabilities for each eigenvector $|\langle\hat{\mathrm{H}}| |\lambda_1\rangle|^2 + |\langle\hat{\mathrm{H}}| |\lambda_2\rangle|^2$