

# Quantum Mechanics

Stefan Aeschbacher

July 12, 2021



# Chapter 1

## Dirac notation

Inner product of two vectors

$$\langle a|b\rangle = \langle b|a\rangle^* \quad (1.1)$$

$$\langle a|a\rangle \geq 0 \quad (1.2)$$

### 1.1 Postulate X: time evolution

$$i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = \hat{H}(t) |\Psi(t)\rangle \quad (1.3)$$



## Chapter 2

# Postulates

### 2.1 Postulate 1: state

**Postulate 1** *The state of a physical system is described by a state vector that belongs to a complex vector space  $V$ , called the state space of the system.*



## Chapter 3

# State space

$$|\Psi_1\rangle + |\Psi_2\rangle = |\Psi_3\rangle \tag{3.1}$$

$$|\Psi_1\rangle + |\Psi_2\rangle = |\Psi_2\rangle + |\Psi_1\rangle \tag{3.2}$$





## Chapter 4

# Operators

### 4.1 Basic Properties

An operator acting on a ket creates a new ket:

$$\hat{A} |\Psi\rangle = |\Psi'\rangle \quad (4.1)$$

$$|\Psi\rangle, |\Psi'\rangle \in V \quad (4.2)$$

Operators are linear

$$\hat{A}(a_1 |\Psi_1\rangle + a_2 |\Psi_2\rangle) = (a_1 \hat{A} |\Psi_1\rangle + a_2 \hat{A} |\Psi_2\rangle) \quad (4.3)$$

$$|\Psi\rangle, |\Psi'\rangle \in V; a_1, a_2 \in \mathbb{C} \quad (4.4)$$

Operators are associative and commutative under addition

$$\hat{A} + (\hat{B} + \hat{C}) = (\hat{A} + \hat{B}) + \hat{C} \quad (4.5)$$

$$\hat{A} + \hat{B} = \hat{B} + \hat{A} \quad (4.6)$$

Multiplying operators is interpreted as applying them to kets. It is associative but NOT (in general) commutative.

$$\hat{A} \hat{B} |\Psi\rangle = \hat{A}(\hat{B} |\Psi\rangle) = \hat{A} |\Psi'\rangle \quad (4.7)$$

$$\hat{A}(\hat{B} \hat{C}) = (\hat{A} \hat{B}) \hat{C} \quad (4.8)$$

$$\hat{A} \hat{B} \neq \hat{B} \hat{A} \quad (4.9)$$

The lack of commutativity makes the "commutator" useful

$$[\hat{A}, \hat{B}] = \hat{A} \hat{B} - \hat{B} \hat{A} \quad (4.10)$$

$$(4.11)$$

+

## 4.2 Hermitian Operators

+ An operator is called Hermitian (or self-adjoint) if it is its own conjugate

$$\hat{A} = \hat{A}^\dagger$$

$$\hat{A} |A\rangle = |B\rangle \rightarrow \langle A| \hat{A}^\dagger = \langle B| \quad (4.12)$$

$$\hat{A} |A\rangle = |B\rangle \rightarrow \langle A| \hat{A} = \langle B| \quad (4.13)$$

$$(4.14)$$

A hermitian operator  $\hat{A}$  has the following properties

1.  $\hat{A} |\lambda\rangle = \lambda |\lambda\rangle \rightarrow \lambda \in \mathbb{R}$
2.  $\langle \hat{A} \rangle = \langle \Psi | \hat{A} | \Psi \rangle \in \mathbb{R}$
3. All eigenvectors with different eigenvalues are orthogonal

## 4.3 Projection Operators

A projection operator is defined

$$[\hat{A}, \hat{B}] = \hat{A} \hat{B} - \hat{B} \hat{A} \quad (4.15)$$

$$(4.16)$$

## Chapter 5

# Eigenvectors and Eigenvalues

### 5.1 Degenerate Eigenvectors

If two eigenvectors have the same eigenvalue:

$$\hat{H} |\lambda_1\rangle = \lambda |\lambda_1\rangle \quad (5.1)$$

$$\hat{H} |\lambda_2\rangle = \lambda |\lambda_2\rangle \quad (5.2)$$

their linear combination is an eigenvector as well:

$$\alpha \hat{H} |\lambda_1\rangle = \lambda \alpha |\lambda_1\rangle \quad (5.3)$$

$$\beta \hat{H} |\lambda_1\rangle = \lambda \beta |\lambda_1\rangle \quad (5.4)$$

$$\hat{H}[\alpha |\lambda_1\rangle + \beta |\lambda_2\rangle] = \lambda[\alpha |\lambda_1\rangle + \beta |\lambda_2\rangle] \quad (5.5)$$

Therefore it is possible to create two orthogonal eigenvectors for this eigenvalue.

The probability in the degenerate case is the sum of the probabilities for each eigenvector  $|\langle \hat{H} | \lambda_1 \rangle|^2 + |\langle \hat{H} | \lambda_2 \rangle|^2$