CS 6501 Natural Language Processing

Feed-forward Neural Networks

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Overview

Introduction

Classification

Decision function

$$\Psi(\boldsymbol{x}, y) = \boldsymbol{w}_y^\top \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{\theta}) \tag{1}$$

- $\boldsymbol{x}$: data point
- y: label
- $\boldsymbol{w}_y$: classification weights with respect to label y
- $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{\theta})$: feature function
- $\boldsymbol{\theta}$: parameter of feature function

Example: Feature engineering

How to construct $f(x; \theta)$?

Example sentence

I love drinking coffee

- Unigram: `I`, `love`, `drinking`, `coffee`
- Bigram: `I love`, `love drinking`, ...
- POS tags: $\langle$ `I`, IN$\rangle$, ...
- Production rules: S $\rightarrow$ NP VP, ...
- ...

Example: Feature engineering

How to construct $f(x; \theta)$?

Example sentence

I love drinking coffee

Vocab	I	love	drinking	hate	coffee	tea
$x^\top$	[1	1	1	0	1	0]

$$\begin{aligned} f(x; \theta) &= \mathbf{V}x \\ \Psi(x, y) &= w_y^\top f(x; \theta) = w_y^\top(\mathbf{V}x) \end{aligned}$$

Each column of $\mathbf{W}$ is a corresponding word embedding

An Alternative View

Vocab	`I`	`love`	`drinking`	`hate`	`coffee`	`tea`
$\boldsymbol{x}^\top$	[1	1	1	0	1	0]
$\mathbf{V}$	[$\boldsymbol{v}_{\text{I}}$	$\boldsymbol{v}_{\text{love}}$	$\boldsymbol{v}_{\text{drinking}}$	$\boldsymbol{v}_{\text{hate}}$	$\boldsymbol{v}_{\text{coffee}}$	$\boldsymbol{v}_{\text{tea}}$]

An Alternative View

$$\begin{array}{c|cccccc} \text{Vocab} & \texttt{I} & \texttt{love} & \texttt{drinking} & \texttt{hate} & \texttt{coffee} & \texttt{tea} \\ \boldsymbol{x}^\top & [1 & 1 & 1 & 0 & 1 & 0] \\ \mathbf{V} & [v_{\text{I}} & v_{\text{love}} & v_{\text{drinking}} & v_{\text{hate}} & v_{\text{coffee}} & v_{\text{tea}}] \end{array}$$

$$f(\boldsymbol{x}, \boldsymbol{\theta}) = v_{\text{I}} + v_{\text{love}} + v_{\text{drinking}} + v_{\text{coffee}} \tag{2}$$

Linear Functions

Looking for a more powerful model then $f(\boldsymbol{x}, \boldsymbol{\theta}) = \mathbf{V}\boldsymbol{x}$?

How about

$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \mathbf{U}\mathbf{V}\boldsymbol{x}$$

Linear Functions

Looking for a more powerful model then $f(x, \theta) = \mathbf{V}x$?

How about

$$\begin{aligned} f(x, \theta) &= \mathbf{U}\mathbf{V}x \\ &= \underbrace{(\mathbf{U}\mathbf{V})}_{\mathbf{V}'} x \end{aligned}$$

Not really, maybe a little. Essentially, it is still a linear function with a single matrix decomposed as $\mathbf{U}\mathbf{V}$.

Nonlinearity

Add a nonlinear function $\boldsymbol{h}$

$$\begin{aligned} f(\boldsymbol{x}, \boldsymbol{\theta}) &= \boldsymbol{h}(\mathbf{V}\boldsymbol{x}) \\ \Psi(\boldsymbol{x}, y) &= \boldsymbol{w}_y^\top \boldsymbol{h}(\mathbf{V}\boldsymbol{x}) \end{aligned}$$

Now, it is a neural network!

Nonlinearity

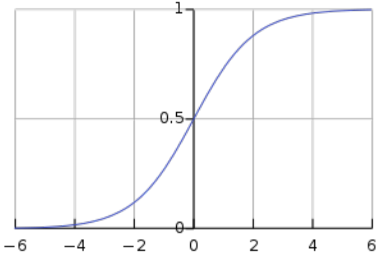
Add a nonlinear function $\boldsymbol{h}$

$$\begin{aligned} f(\boldsymbol{x}, \boldsymbol{\theta}) &= \boldsymbol{h}(\mathbf{V}\boldsymbol{x}) \\ \Psi(\boldsymbol{x}, y) &= \boldsymbol{w}_y^\top \boldsymbol{h}(\mathbf{V}\boldsymbol{x}) \end{aligned}$$

Now, it is a neural network!

Example: Sigmoid function

$$\boldsymbol{h}(\boldsymbol{t}) = \frac{1}{1 - e^{-t}}$$

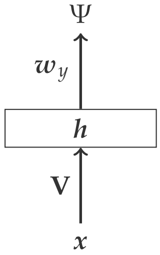


Feed-forward Neural Networks

A Simple Feed-forward Network

A fully-connected feed-forward neural network

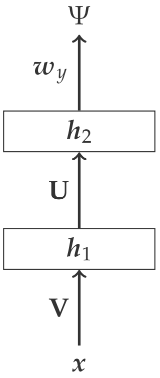
$$\Psi(\boldsymbol{x}, y) = \boldsymbol{w}_y^\top \boldsymbol{h}(\mathbf{V}\boldsymbol{x}) \tag{3}$$



Another Feed-forward Network

$$\Psi(\boldsymbol{x}, y) = \boldsymbol{w}_y^\top \cdot \underbrace{\boldsymbol{h}_2(\mathbf{U} \cdot \boldsymbol{h}_1(\mathbf{V} \cdot \boldsymbol{x}))}_{f(\boldsymbol{x},\boldsymbol{\theta})} \qquad (4)$$

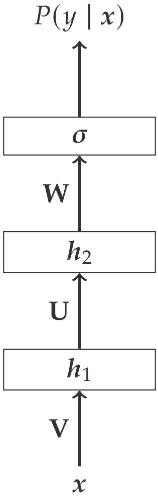
where $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ are nonlinear functions without parameters (hidden units).



Softmax Function

Normalize the score function to make a probability

$$P(y \mid \boldsymbol{x}) = \sigma(\Psi(\boldsymbol{x}, y)) = \frac{\exp(\Psi(\boldsymbol{x}, y))}{\sum_{y'} \exp(\Psi(\boldsymbol{x}, y'))} \quad (5)$$

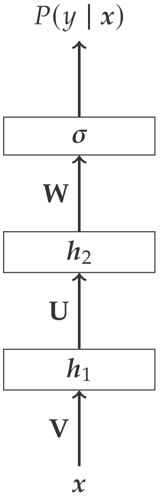


Softmax Function

Normalize the score function to make a probability

$$
\begin{aligned}
P(y \mid \boldsymbol{x}) &= \sigma(\Psi(\boldsymbol{x}, y)) \\
&= \frac{\exp(\Psi(\boldsymbol{x}, y))}{\sum_{y'} \exp(\Psi(\boldsymbol{x}, y'))}
\end{aligned} \tag{5}
$$

- This neural network model is not probabilistic — it is a determinstic transformation from $\boldsymbol{x}$ to $\Psi(\boldsymbol{x}, y)$



Softmax Function

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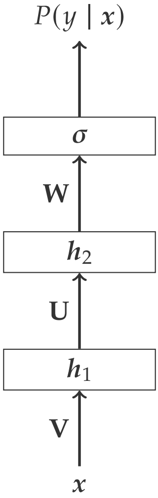
- This neural network model is not probabilistic — it is a determinstic transformation from $\boldsymbol{x}$ to $\Psi(\boldsymbol{x}, y)$
- Main advantage of the normalization term is on training



Loss Function: Cross entropy

Binary classification on a single data point $\boldsymbol{x}$ with $y \in \{0, 1\}$

$$\ell = -y \log P(y = 1 \mid \boldsymbol{x}) - (1 - y) \log(1 - P(y = 1 \mid \boldsymbol{x})) \tag{6}$$

Loss Function: Cross entropy

Binary classification on a single data point $\boldsymbol{x}$ with $y \in \{0, 1\}$

$$\ell = -y \log P(y = 1 \mid \boldsymbol{x}) - (1 - y) \log(1 - P(y = 1 \mid \boldsymbol{x})) \tag{6}$$

- if $y = 1$:
$$\ell = -\log P(y = 1 \mid \boldsymbol{x})$$
- if $y = 0$:
$$\ell = -\log(1 - P(y = 1 \mid \boldsymbol{x})) = -\log P(y = 0 \mid \boldsymbol{x})$$

Loss Function: Cross entropy

K-class: convert label to K-dimensional one-hot vector with $\boldsymbol{y}_k = 1$, if k is the label

$$\begin{aligned}
\ell &= -\sum_{k=1}^{K} \boldsymbol{y}_k \log P(\boldsymbol{y}_k = 1 \mid \boldsymbol{x}) \\
&= -\log P(\boldsymbol{y}_k = 1 \mid \boldsymbol{x})
\end{aligned} \tag{7}$$

Loss Function: Cross entropy

K-class: convert label to K-dimensional one-hot vector with $\boldsymbol{y}_k = 1$, if k is the label

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\ell &= -\sum_{k=1}^{K} \boldsymbol{y}_k \log P(\boldsymbol{y}_k = 1 \mid \boldsymbol{x}) \\
&= -\log P(\boldsymbol{y}_k = 1 \mid \boldsymbol{x})
\end{aligned} \tag{7}
$$

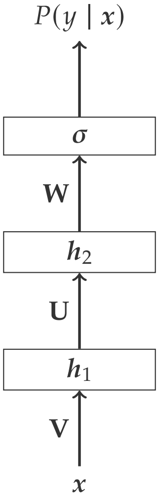
Essentially, it is the same as negative log-likelihood (NLL) in logistic regression.

Back Propagation

Online Learning

- Let $\boldsymbol{\theta} = \{\mathbf{W}, \mathbf{U}, \mathbf{V}\}$
- Training NNs with one example at a time

$$\ell(\boldsymbol{\theta}) = -\log P(\boldsymbol{y}_k = 1 \mid \boldsymbol{x}) \tag{8}$$



Gradient based Learning

Stochastic gradient descent

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \eta \cdot \frac{\partial \ell(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \tag{9}$$

For any subset of $\boldsymbol{\theta}$, e.g., $\boldsymbol{w}_k$

$$\boldsymbol{w}_k \leftarrow \boldsymbol{w}_k - \eta \cdot \frac{\partial \ell(\boldsymbol{\theta})}{\partial \boldsymbol{w}_k} \tag{10}$$

Gradient based Learning (cont.)

Recall the definition of $P(\boldsymbol{y} \mid \boldsymbol{x})$

$$\begin{aligned}\log P(\boldsymbol{y}_k \mid \boldsymbol{x}) =& \boldsymbol{w}_k^\top \cdot \boldsymbol{h}_2(\mathbf{U} \cdot \boldsymbol{h}_1(\mathbf{V} \cdot \boldsymbol{x})) \\ &- \log \sum_{k'} \exp(\boldsymbol{w}_{k'}^\top \cdot \boldsymbol{h}_2(\mathbf{U} \cdot \boldsymbol{h}_1(\mathbf{V} \cdot \boldsymbol{x})))\end{aligned} \tag{11}$$

Gradient based Learning (cont.)

Recall the definition of $P(\boldsymbol{y} \mid \boldsymbol{x})$

$$\begin{aligned}\log P(\boldsymbol{y}_k \mid \boldsymbol{x}) =& \boldsymbol{w}_k^\top \cdot \boldsymbol{h}_2(\mathbf{U} \cdot \boldsymbol{h}_1(\mathbf{V} \cdot \boldsymbol{x})) \\ & - \log \sum_{k'} \exp(\boldsymbol{w}_{k'}^\top \cdot \boldsymbol{h}_2(\mathbf{U} \cdot \boldsymbol{h}_1(\mathbf{V} \cdot \boldsymbol{x})))\end{aligned} \tag{11}$$

Gradient wrt $\boldsymbol{w}_k$

$$\begin{aligned}\frac{\partial \ell}{\partial \boldsymbol{w}_k} &= - \frac{\partial}{\partial \boldsymbol{w}_k} \log P(\boldsymbol{y} \mid \boldsymbol{x}) \\ &= - \boldsymbol{h}_2(\mathbf{U} \cdot \boldsymbol{h}_1(\mathbf{V} \cdot \boldsymbol{x}))) \\ &\quad + P(\boldsymbol{y}_k \mid \boldsymbol{x}) \cdot \boldsymbol{h}_2(\mathbf{U} \cdot \boldsymbol{h}_1(\mathbf{V} \cdot \boldsymbol{x}))\end{aligned} \tag{12}$$

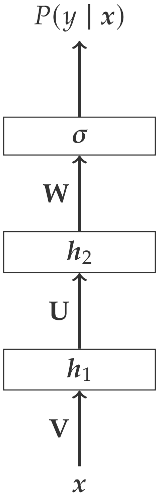
One More Example

Given

$$\begin{aligned}\ell &= -\log P(y \mid \boldsymbol{x}) \\ &= -\log \boldsymbol{\sigma}(\mathbf{W} \cdot \boldsymbol{h}_2(\mathbf{U} \cdot \boldsymbol{h}_1(\mathbf{V} \cdot \boldsymbol{x})))\end{aligned}$$

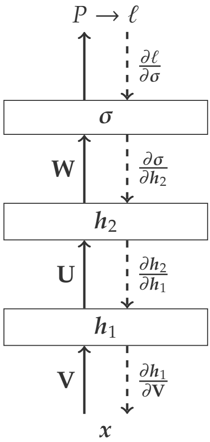
with the chain rule, we have

$$\frac{\partial \ell}{\partial \mathbf{V}} = \frac{\partial \ell}{\partial \boldsymbol{\sigma}} \cdot \frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{h}_2} \cdot \frac{\partial \boldsymbol{h}_2}{\partial \boldsymbol{h}_1} \cdot \frac{\partial \boldsymbol{h}_1}{\partial \mathbf{V}}$$



Back Propagation

$$\frac{\partial \ell}{\partial \mathbf{V}} = \frac{\partial \ell}{\partial \boldsymbol{\sigma}} \cdot \frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{h}_2} \cdot \frac{\partial \boldsymbol{h}_2}{\partial \boldsymbol{h}_1} \cdot \frac{\partial \boldsymbol{h}_1}{\partial \mathbf{V}} \tag{13}$$

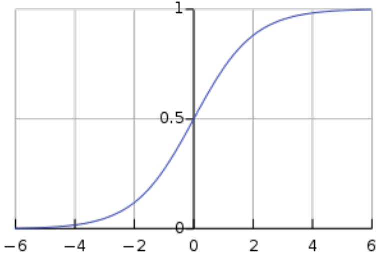


Problems of Gradients

$$\frac{\partial \ell}{\partial \mathbf{V}} = \frac{\partial \ell}{\partial \boldsymbol{\sigma}} \cdot \frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{h}_2} \cdot \frac{\partial \boldsymbol{h}_2}{\partial \boldsymbol{h}_1} \cdot \frac{\partial \boldsymbol{h}_1}{\partial \mathbf{V}} \tag{14}$$

Vanishing gradients, if $\|\frac{\partial \cdot}{\partial \cdot}\| \ll 1$

$$\|\frac{\partial \ell}{\partial \mathbf{V}}\| \to 0 \tag{15}$$



Solution: initialize the parameters carefully

Problems of Gradients (cont.)

$$\frac{\partial \ell}{\partial \mathbf{V}} = \frac{\partial \ell}{\partial \boldsymbol{\sigma}} \cdot \frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{h}_2} \cdot \frac{\partial \boldsymbol{h}_2}{\partial \boldsymbol{h}_1} \cdot \frac{\partial \boldsymbol{h}_1}{\partial \mathbf{V}} \tag{16}$$

Exploding gradients, if $\|\frac{\partial \cdot}{\partial \cdot}\| > M > 1$

$$\|\frac{\partial \ell}{\partial \mathbf{V}}\| > M^4 \tag{17}$$

Problems of Gradients (cont.)

$$\frac{\partial \ell}{\partial \mathbf{V}} = \frac{\partial \ell}{\partial \boldsymbol{\sigma}} \cdot \frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{h}_2} \cdot \frac{\partial \boldsymbol{h}_2}{\partial \boldsymbol{h}_1} \cdot \frac{\partial \boldsymbol{h}_1}{\partial \mathbf{V}} \tag{16}$$

Exploding gradients, if $\|\frac{\partial \cdot}{\partial \cdot}\| > M > 1$

$$\|\frac{\partial \ell}{\partial \mathbf{V}}\| > M^4 \tag{17}$$

Solution: norm clipping [Pascanu et al., 2013]

$$\tilde{\boldsymbol{g}} \leftarrow \lambda \cdot \frac{\boldsymbol{g}}{\|\boldsymbol{g}\|} \tag{18}$$

where $\boldsymbol{g} = \frac{\partial \ell}{\partial \mathbf{V}}$ and $1 < \lambda \leq 5$.

Summary

Reference

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