PMTH332 Assignment 5

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Question 1

Consider the two non-zero matrices $A, B \in M(2; R)$ defined by

$$A = \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \qquad\qquad B = \begin{pmatrix} 0 & 0 \\ 0 & a \end{pmatrix}$$

then $AB = 0$, so $M(2; R)$ has zero divisors. Consider matrix C given by

$$C = \begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix}$$

then $AC = \begin{pmatrix} 0 & a^2 \\ 0 & 0 \end{pmatrix}$ and $CA = 0$, so $M(2; R)$ is not commutative.

Question 2

Let F be a finite integral domain and take $x \in F$. As F is a domain, there are no zero divisors. Therefore x is a unit by Proposition 13.21. That is, $\exists x^{-1} \in F$ where $xx^{-1} = x^{-1}x = 1$.

Question 3

Question 4