PMTH332/432 **Abstract Algebra**

Assignment 5

Question 1.

Let $M(2;R)$ denote the set of (2×2)–matrices with coefficients in the commutative ring $R \neq \{0\}$, that is,

$$M(2;R) := \{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a,b,c,d \in R \}$$

Then $M(2;R)$ is a ring with respect to addition and multiplication of matrices.

Show that $M(2;R)$ has zero divisors and provide an example where $M(2;R)$ is not commutative.

Question 2.

Show that a finite integral domain is a field.

Question 3.

Show that the characteristic of a non–trivial integral domain with 1 must be either zero or a prime.

Question 4.

Take an integral domain D and consider

$$\tilde{D} := \{(b,a) \mid a,b \in D, a \neq 0\}$$
$$(b,a) \sim (d,c) \ :\Longleftrightarrow \ bc \ = \ ad$$

Let F be the set of equivalence classes of this equivalence relation.

Show that $(F,+,\times)$ is a field, where

$$+ \ : \ F \times F \longrightarrow F, \quad ([(b,a)],[(d,c)]) \longmapsto [(bc+ad,ac)];$$
$$\times \ : \ F \times F \longrightarrow F, \quad ([(b,a)],[(d,c)]) \longmapsto [(bd,ac)]$$

and that

$$\iota : D \longrightarrow F, \quad a \longmapsto [(a,1)]$$

is an injective ring homomorphism.

F is the *field of quotients of* D.