PMTH332/432   **Abstract Algebra**

Assignment 6

Question 1.

Show that for every integer n the number $n^{33} - n$ is divisible by 15.

Question 2.

Given a field F show that every proper non–trivial prime ideal of $F[x]$ is maximal.

Question 3.

Let F be a field and take $f(x)$, $g(x) \in F[x]$ relatively prime. Show that $f(x)|h(x)$ and $g(x)|h(x)$ implies $f(x)g(x)|h(x)$ for every $h(x) \in F[x]$.

Question 4.

(a) Find all prime ideals of $\mathbb{Z}_{12}$.

(b) Find a prime ideal of $\mathbb{Z} \times \mathbb{Z}$ which is not maximal.

Question 5.

Write $p(x) = x^3 + 2x + 3$ as a product of irreducible polynomials over $\mathbb{Z}_5$.