PMTH332/432 **Abstract Algebra**

Assignment 2

Question 1.

Let $H = (\mathbb{R}^+, \times)$ be the group consisting of the set of all strictly positive real numbers with the binary operation given by multiplication.

Show that H is isomorphic to $(\mathbb{R}, +)$, the group consisting of all real numbers with the binary operation given by addition.

Question 2.

Given two isomorphisms $\varphi : G \longrightarrow H$ and $\psi : H \longrightarrow K$, show that their composition $\psi \circ \varphi : G \longrightarrow K$ is an isomorphism.

Question 3.

Let H be a subset of the set underlying the group G. Show that

$$x \sim_H y \quad :\Longleftrightarrow \quad xy^{-1} \in H$$

is an equivalence relation if and only if H is a subgroup of G.

Question 4.

An *automorphism* of the group G is an isomorphism $\varphi : G \to G$. We put

$$\mathrm{Aut}(G) := \{\varphi : G \to G \,|\, \varphi \text{ is an isomorphism}\}$$

(a) Show that $\mathrm{Aut}(G)$ is a subgroup of $S(G)$, the group of invertible functions from the set underlying G to itself with the binary operation given by composition of functions.

(b) Given $g \in G$ show that

$$\varphi_g : G \longrightarrow G,\ x \longmapsto gxg^{-1}$$

is an automorphism. It is called the *inner automorphism* defined by g.

(c) Show that

$$\mathrm{Inn} : G \longrightarrow \mathrm{Aut}(G),\ g \longmapsto \varphi_g$$

is a homomorphism.