Assignment 6

1. Denote by $J_0(x)$ the solution of Bessel's equation of order $\alpha = 0$ which was found in Theorem 12.1. Show that the function

$$\frac{1}{\pi}\int_0^{\pi} \cos(x \sin\phi)d\phi$$

satisfies the same d.e. and the same condition at $x = 0$ as $J_0(x)$, and is hence equal to $J_0(x)$. [Hint: an integration by parts may be required.]

2. Find two series solutions of the form

$$x^{\frac{1}{2}}\sum_{n=0}^{\infty} a_n x^n \quad \text{and} \quad x^{-\frac{1}{2}}\sum_{n=0}^{\infty} b_n x^n$$

of the Bessel equation of order $\alpha = \frac{1}{2}$. Do you recognise the series as being those of a pair of well-known functions?

3. According to Theorem 12.2 the Bessel equation with $\alpha = 0$ has a solution of the form

$$y_2(x) = J_0(x)\ln x + \sum_{k=0}^{\infty} g_k x^k.$$

Find the first few coefficients g_k either by imitating the method in the notes or by substituting the above expression in the d.e. and equating coefficients.

4. The hypergeometric equation

$$x(1-x)y'' + (c - (1+a+b)x)y' - aby = 0$$

a, b, c constant,

has a solution of the form

$$y = 1 + \sum_{n=1}^{\infty} a_n x^n$$

whenever c is not zero or a negative integer, **despite** the fact that $x = 0$ is a singular point.

Find this solution (i.e. the coefficients a_n).