Assignment 4

1. Find a power series solution of the form

$$u = \sum_{k=0}^{\infty} a_k x^k$$

for the d.e.

$$u'' + x^2 u = 0$$

with the initial conditions $u(0) = 1, \quad u'(0) = 0$.

2. * Let x_0 be the smallest positive number such that $u(x_0) = 0$, where u is the solution is Question 1.

Prove that $1 - \dfrac{x^4}{12} \leq u(x) \leq 1$, for $0 \leq x \leq x_0$. Does this have any consequence for Question 5* of Assignment 1?

3. Find T_5, the fifth Chebyshev polynomial.

4. The Legendre equation

$$(1 - x^2)y'' - 2xy' + \alpha(\alpha + 1)y = 0$$

has a polynomial solution P_n when α is a non-negative integer n.

(a) Find P_0, P_1, P_2, P_3, given that $P_n(1) = 1$.

(b) Find the general solution of the d.e. when $\alpha = 1$.

5. Find a power series solution for the first order linear d.e.

$$(1 - x)y' - 2y = 0$$

of the form $\sum a_n x^n$, with the initial condition $y(0) = 1$. Comment on your answer.