PMTH332/432 **Abstract Algebra**

Assignment 4

Question 1.

A *maximal normal subgroup* of the group G is a proper normal subgroup of G, such that any other normal subgroup N of G containing M must be equal to M or to G.

The non–trivial group G is *simple* if and only if the only normal subgroups of G are the trivial subgroup $\{e_G\}$ and G.

Show that M is a maximal normal subgroup of G if and only if G/M is simple.

Question 2.

For $1 \leq i \leq n$ take groups G_i and elements $a_i \in G_i$ of finite order.

Consider $G := \prod_{i=1}^{n} G_i$.

Show that the order of $(a_1, a_2, \ldots, a_n)$ in G is the least common multiple of the orders of a_i in G_i, that is,

$$\operatorname{ord}(a_1, a_2, \ldots, a_n) = \operatorname{lcm}(\operatorname{ord} a_1, \operatorname{ord} a_2, \ldots, \operatorname{ord} a_n)$$

Question 3.

Take non–zero positive integers m and n.

Show that

$$\mathbb{Z}/m\mathbb{Z} \times \mathbb{Z}/n\mathbb{Z} \cong \mathbb{Z}/mn\mathbb{Z}$$

if and only if m and n are relatively prime, that is, $\gcd(m, n) = 1$.

Question 4.

Let G be a group of order 324.

Show that G has subgroups of order 2, 3, 4, 9, 27 and 81, but none of order 10.