PMTH332/432 **Abstract Algebra**

Assignment 1

Question 1.

Show that every equivalence relation on the set X determines a unique partition of X, and conversely.

Question 2.

Let $* : G \times G \to G$ be an associative binary operation on the set G such that

(G2R) There is an element $e \in G$ such that, for all $a \in G$,

$$a * e = a.$$

(G3R) To each $a \in G$ there is an $\overline{a} \in G$ such that

$$a * \overline{a} = e.$$

Show that $(G, *)$ is a group.

This shows that axioms (G2R), (G3R) and (G1) together imply (G1), (G2) and (G3).

Question 3.

Take $m \in \mathbb{Z}$ and let $\mathbb{Z}_m$ be the set of residue classes modulo m.

Show that

$$+ : \mathbb{Z}_m \times \mathbb{Z}_m \longrightarrow \mathbb{Z}_m; \quad ([l], [k]) \longmapsto [l + k]$$

is a well–defined binary operation, that is, $[l + k]$ does not depend on the choice of representatives l and k of the equivalence classes $[l]$ and $[k]$.

Prove that $(\mathbb{Z}_m, +)$ is a group.

Question 4.

Let G be a group such that $(a * b)^2 = a^2 * b^2$ for all $a, b \in G$.

Show that $(G, *)$ is abelian.