PMTH332 Assignment 5

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Question 1

Consider the two non-zero matrices $A, B \in M(2; R)$ defined by

$$A = \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 0 \\ 0 & a \end{pmatrix}$$

then $AB = 0$, so $M(2; R)$ has zero divisors. Consider matrix C given by

$$C = \begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix}$$

then $AC = \begin{pmatrix} 0 & a^2 \\ 0 & 0 \end{pmatrix}$ and $CA = 0$, so $M(2; R)$ is not commutative.

Question 2

Let F be a finite integral domain and take non-zero $a \in F$. Define F^* to be the set of non-zero elements of F, and define the map $\phi : F^* \to F^*$ by $\phi(x) = ax$.

Suppose that for $x, y \in F$, $\phi(x) = \phi(y)$. Then

$$ax = ay \iff ax - ay = 0 \iff a(x - y) = 0$$

As F is an integral domain, it has no zero-divisors. Therefore the above implies that either $a = 0$ or $x - y = 0$. As a is non-zero by choice, it must hold that $x = y$. Therefore ϕ is injective. Further, as F is finite, ϕ must be surjective. Therefore, as $1 \in F^*$, there exists $x \in F^*$ so that $\phi(x) = ax = 1$. Hence, every non-zero element of F has multiplicative inverse, and F is a field.

Question 3

Question 4