Assignment 2

1. Find a fundamental pair of solutions of the d.e.

$$x^2y'' + 3xy' + y = 0$$

for $x < 0$. Hence find a solution satisfying the initial conditions $y(-1) = 3$, $y'(-1) = 4$.

2. For the third-order linear equation

$$y''' - 3y' + 2y = 0$$

verify that e^x, xe^x and e^{-2x} are solutions. By imitating the theory of second-order equations use these solutions to construct a solution for which

$$y(0) = 1 \quad , \quad y'(0) = 0 \quad , \quad y''(0) = 0.$$

3. State the intervals in which there are sure to be solutions of the d.e.

$$(1 - x^2)y'' - xy' + 4y = 0.$$

Show that $1 - 2x^2$ is a solution and, using the Wronskian or some other method, try to find another solution.

4. Show that the first order non-linear d.e.

$$y' = x^2 + y^2$$

is transformed into a second-order linear homogeneous equation, with new unknown function u, by means of the transformation

$$y = -\frac{u'}{u}$$

Translate Q5 of Assignment 1 into an equivalent problem for the new equation.