

PMTH339 Assignment 7

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Question 1

$$y'' + (3 - \sin x)y = 0 \tag{1}$$

Let $a(x) = 3 - \sin x$. Then for all x , $a(x) \geq 2$. Therefore, if u is a solution to (1), Corollary 14.3 states that u has an increasing sequence of zeroes $\{\alpha_i\}$ where $0 < \alpha_1 \leq \frac{\pi}{\sqrt{2}}$ and $\alpha_{i+1} - \alpha_i \leq \frac{\pi}{\sqrt{2}}$ for all i .

Given the two inequalities above, we can iteratively determine an upper bound for the first n zeroes. Firstly, $0 < \alpha_1 \leq \frac{\pi}{\sqrt{2}}$. Then, as $\alpha_2 - \alpha_1 \leq \frac{\pi}{\sqrt{2}}$ it must hold that

$$\alpha_2 = \alpha_2 - \alpha_1 + \alpha_1 \leq \frac{\pi}{\sqrt{2}} + \frac{\pi}{\sqrt{2}} = \sqrt{2}\pi$$

Repeating the process we get that $\alpha_3 \leq \frac{3\pi}{\sqrt{2}}$. Therefore, as $\sqrt{2}\pi \leq 2\pi \leq \frac{3\pi}{\sqrt{2}}$, there are either 2 or 3 zeroes in the range $0 \leq x \leq 2\pi$.

Question 2

Question 3

Question 4