Assignment 7

1. Show that every solution of the d.e.

$$y'' + (3 - \sin x)y = 0$$

has an infinite sequence of zeros.

Find upper and lower bounds for the number of zeros in $0 \leq x \leq 2\pi$.

2. Write the *Hermite* equation

$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2\alpha y = 0 \tag{1}$$

in the form

$$\frac{d}{dx}(p(x)\frac{dy}{dx}) + q(x)y = 0$$

Hence show that the function $u = p^{\frac{1}{2}}y$ satisfies

$$u'' + (2\alpha + 1 - x^2)u = 0 \tag{2}$$

3. Letting $\alpha = 3$ in the Hermite equation show

   (i) a solution has a finite number of zeros spaced at least $\frac{\pi}{\sqrt{7}}$ apart.

   (ii) a solution with $y(0) = 0$ has at least two more zeros. [Use the fact that, for $x \leq 2, \quad (7 - x^2) \geq 3$]

4. Whenever α is a positive integer, (1) has a solution which is a polynomial of degree α. Find this solution in the case $\alpha = 3$, and prove that $\pi < \sqrt{\frac{21}{2}}$.