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Monetary Policy and Exchange Rate Volatility in a Small Open Economy

JORDI GALÍ
CREI, UPF, CEPR and NBER

and

TOMMASO MONACELLI
IGIER, Università Bocconi and CEPR

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We lay out a *small open economy* version of the Calvo sticky price model, and show how the equilibrium dynamics can be reduced to a simple representation in domestic inflation and the output gap. We use the resulting framework to analyse the macroeconomic implications of three alternative rule-based policy regimes for the small open economy: domestic inflation and CPI-based Taylor rules, and an exchange rate peg. We show that a key difference among these regimes lies in the relative amount of exchange rate volatility that they entail. We also discuss a special case for which domestic inflation targeting constitutes the optimal policy, and where a simple second order approximation to the utility of the representative consumer can be derived and used to evaluate the welfare losses associated with the suboptimal rules.

1. INTRODUCTION

Much recent work in macroeconomics has involved the development and evaluation of monetary models that bring imperfect competition and nominal rigidities into the dynamic stochastic general equilibrium structure that for a long time had been the hallmark of real business cycle (RBC) theory. In the resulting models—often referred to as New Keynesian—changes in monetary settings generally have non-trivial effects on real variables. Monetary policy may thus become a potential stabilization tool, as well as an independent source of economic fluctuations. Not surprisingly, the study of the properties of alternative monetary policy rules (*i.e.* specifications of how the central bank changes the settings of its policy instrument in response to changes in macroeconomic conditions) has been a fruitful area of research in recent years and a natural application of the new generation of models.[1]

In the present paper we lay out a *small open economy* version of a model with Calvo-type staggered price-setting, and use it as a framework for analysing the properties and macroeconomic implications of alternative monetary policy regimes.[2] The use of a staggered price-setting structure allows for richer dynamic effects of monetary policy than those found in the models with one-period advanced price-setting that are common in the recent open economy literature.[3] Most importantly, and in contrast with most of the existing literature—where monetary policy is introduced by assuming that some monetary aggregate follows an exogenous

1. The volume edited by Taylor (1999) contains several significant contributions to that literature. See, *e.g.* Clarida, Galí and Gertler (1999) for a survey.
2. See, *e.g.* King and Wolman (1996), Yun (1996), and Woodford (2003, Chapter 4), for an analysis of the canonical Calvo model in a closed economy.
3. See, *e.g.* Obstfeld and Rogoff (1995, 1999), Bacchetta and van Wincoop (2000), Betts and Devereux (2000), and Corsetti and Pesenti (2001).

stochastic process—we model monetary policy as endogenous, with a short-term interest rate being the instrument of that policy.[4] For this very reason our framework allows us to model alternative monetary regimes. Furthermore, we believe that our approach accords much better with the practice of modern central banks, and provides a more suitable framework for policy analysis than the traditional one.

Our framework differs from much of the literature in that it models the small open economy as one among a continuum of (infinitesimally small) economies making up the world economy. Our assumptions on preferences and technology, combined with the Calvo price-setting structure and the assumption of complete financial markets, give rise to a highly tractable framework and to simple and intuitive log-linearized equilibrium conditions for the small open economy. In fact, the latter can be reduced to a first order, two-equation dynamical system for domestic inflation and the output gap whose structure, consisting of a new Keynesian Phillips curve and a dynamic IS-type equation, is identical to the one associated with the workhorse sticky price model of a closed economy, often used in monetary policy analysis.[5] Of course, as we show below, the coefficients in the open economy's equilibrium conditions also depend on parameters that are specific to the open economy (in our case, the degree of openness and the substitutability among goods of different origin), while the driving forces also include world output fluctuations (which are taken as exogenous to the small open economy). As in its closed economy counterpart, the two equations must be complemented with a description of how monetary policy is conducted, in order to close the model.

We then address the issue of a welfare evaluation of alternative policy regimes. Under a particular parameterization of household's preferences we can derive a second order approximation to the consumer's utility, which can be used for policy evaluation purposes.[6] In the particular case considered (which entails log utility and a unit elasticity of substitution between bundles of goods produced in different countries), we show that the optimal policy requires that the domestic price level is fully stabilized.

We employ our framework to analyse the macroeconomic implications and the relative welfare ranking of three simple monetary policy rules for the small open economy. Two of the simple rules considered are stylized Taylor-type rules. The first has the domestic interest rate respond systematically to *domestic inflation* (*i.e.* inflation of domestic goods prices), whereas the second assumes that *CPI inflation* is the variable the domestic central bank reacts to. The third rule we consider is one that pegs the effective nominal exchange rate.

We show that these regimes can be ranked in terms of their implied volatility for the nominal exchange rate and the terms of trade. Hence, a policy of strict domestic inflation targeting, which in our framework can achieve a simultaneous stabilization of the output gap and domestic inflation, implies a substantially greater volatility in the nominal exchange rate and

4. See Lane (2001) for a survey of the new open economy macroeconomics literature. The introduction of price staggering in an open economy model follows the lead of Kollmann (2001) and Chari, Kehoe and McGrattan (2002), though both papers specify monetary policy as exogenous, restricting their analysis to the effects of a monetary shock. A recent exception is given by Obstfeld and Rogoff (1999), who solve for the optimal money supply rule in the context of a model with one-period sticky wages. A more similar methodological approach can be found in Svensson (2000), in which optimal policy is derived from the minimization by the central bank of a quadratic loss function. His model, however, differs from the standard optimizing sticky price model analysed here in that it assumes a predetermined output and inflation (resulting from their dependence on lagged variables, with a somewhat arbitrary lag structure), and introduces an *ad hoc* cost-push shock in the inflation equation (which creates a trade-off between the output gap and inflation). Since we wrote and circulated the first version of the present paper there have been many significant contributions to the literature on monetary policy regimes in open economies, including McCallum and Nelson (2000), Corsetti and Pesenti (2001, 2005), Clarida, Galí and Gertler (2001, 2002), Schmitt-Grohé and Uribe (2001), Kollmann (2002), Parrado and Velasco (2002), and Benigno and Benigno (2003), among others.

5. See, *e.g.* Clarida *et al*. (1999) and Woodford (2003, Chapter 4) among others.

6. Benigno and Benigno (2003) obtain a similar result in the context of a two-country model.

terms of trade than the one achieved under the two Taylor rules and/or the exchange rate peg. The excess smoothness in the nominal exchange rate implied by those simple rules (relative to the optimal policy), combined with the assumed inertia in nominal prices, prevents relative prices from adjusting sufficiently fast in response to changes in relative productivity shocks, causing thus a significant deviation from the first best allocation. In particular, a CPI-based Taylor rule is shown to deliver equilibrium dynamics that allow us to characterize it as a hybrid regime, somewhere between a domestic inflation-based Taylor rule and an exchange rate peg.

The ranking based on the terms of trade volatility translates one-for-one into a welfare ranking. Thus, and for a broad range of parameter configurations, a domestic inflation-based Taylor rule is shown to dominate a CPI-based Taylor rule; the latter in turn dominates an exchange rate peg. More generally, we show that, across regimes, the higher the implied equilibrium terms of trade volatility, the lower the volatility of inflation and output gap, and therefore the higher the resulting welfare score.

The remainder of the paper is organized as follows. In Section 2 we lay out the basic model. Section 3 derives the equilibrium in log-linearized form, and its canonical representation in terms of output gap and inflation. Section 4 analyses the macroeconomic implications of alternative monetary policy regimes. Section 5 analyses optimal monetary policy in both the world and the small economy under a particular parameterization in the latter, and conducts a welfare evaluation of the alternative monetary policy regimes. Section 6 concludes.

2. A SMALL OPEN ECONOMY MODEL

We model the world economy as a *continuum of small open economies* represented by the unit interval. Since each economy is of measure zero, its domestic policy decisions do not have any impact on the rest of the world. While different economies are subject to imperfectly correlated productivity shocks, we assume that they share identical preferences, technology, and market structure.

Next we describe in detail the problem facing households and firms located in one such economy. Before we do so, a brief remark on notation is in order. Since our focus is on the behaviour of a single economy and its interaction with the world economy, and in order to lighten the notation, we will use variables *without* an i-index to refer to the small open economy being modelled. Variables with an $i \in [0, 1]$ subscript refer to economy i, one among the continuum of economies making up the world economy. Finally, variables with a *star superscript* correspond to the world economy as a whole.

2.1. Households

A typical small open economy is inhabited by a representative household who seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \, U(C_t, N_t) \tag{1}$$

where N_t denotes hours of labour, and C_t is a composite consumption index defined by

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} \, (C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \, (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{2}$$

where $C_{H,t}$ is an index of consumption of domestic goods given by the CES function

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $j \in [0, 1]$ denotes the good variety.[7] $C_{F,t}$ is an index of imported goods given by

$$C_{F,t} \equiv \left(\int_0^1 (C_{i,t})^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}$$

where $C_{i,t}$ is, in turn, an index of the quantity of goods imported from country i and consumed by domestic households. It is given by an analogous CES function:

$$C_{i,t} \equiv \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Notice that parameter $\varepsilon > 1$ denotes the elasticity of substitution between varieties (produced within any given country).[8] Parameter $\alpha \in [0, 1]$ is (inversely) related to the degree of home bias in preferences, and is thus a natural index of openness. Parameter $\eta > 0$ measures the substitutability between domestic and foreign goods, from the viewpoint of the domestic consumer, while γ measures the substitutability between goods produced in different foreign countries.

The maximization of (1) is subject to a sequence of budget constraints of the form

$$\int_0^1 P_{H,t}(j)C_{H,t}(j)dj + \int_0^1 \int_0^1 P_{i,t}(j)C_{i,t}(j)dj\,di + E_t\{Q_{t,t+1}D_{t+1}\} \leq D_t + W_t N_t + T_t \quad (3)$$

for $t = 0, 1, 2, \ldots$, where $P_{i,t}(j)$ is the price of variety j imported from country i (expressed in domestic currency, *i.e.* the currency of the importing country whose economy is being modelled). D_{t+1} is the nominal pay-off in period $t+1$ of the portfolio held at the end of period t (and which includes shares in firms), W_t is the nominal wage, and T_t denotes lump-sum transfers/taxes. All the previous variables are expressed in units of domestic currency. $Q_{t,t+1}$ is the stochastic discount factor for one-period ahead nominal pay-offs relevant to the domestic household. We assume that households have access to a complete set of contingent claims, traded internationally. Notice that money does not appear in either the budget constraint or the utility function: throughout we specify monetary policy in terms of an interest rate rule (directly or indirectly); hence, we do not need to introduce money explicitly in the model.[9]

The optimal allocation of any given expenditure within each category of goods yields the demand functions:

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t}; \qquad C_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}} \right)^{-\varepsilon} C_{i,t} \quad (4)$$

for all $i, j \in [0, 1]$, where $P_{H,t} \equiv \left(\int_0^1 P_{H,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$ is the *domestic* price index (*i.e.* an index of prices of domestically produced goods) and $P_{i,t} \equiv \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$ is a price index for goods imported from country i (expressed in domestic currency), for all $i \in [0, 1]$. It follows from (4) that $\int_0^1 P_{H,t}(j)\, C_{H,t}(j)dj = P_{H,t}\, C_{H,t}$ and $\int_0^1 P_{i,t}(j)\, C_{i,t}(j)dj = P_{i,t}\, C_{i,t}$.

7. As discussed below, each country produces a continuum of differentiated goods, represented by the unit interval.

8. Notice that it is irrelevant whether we think of integrals like the one in (2) as including or not the corresponding variable for the small economy being modelled, since its presence would have a negligible influence on the integral itself (in fact each individual economy has a zero measure). The previous remark also applies to many other expressions involving integrals over the continuum of economies (*i.e.* over i) that the reader will encounter below.

9. That modelling strategy has been adopted in much recent research on monetary policy. In it money can be thought of as playing the role of a unit of account only.

Furthermore, the optimal allocation of expenditures on imported goods by country of origin implies

$$C_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\gamma} C_{F,t} \tag{5}$$

for all $i \in [0,1]$, and where $P_{F,t} \equiv \left(\int_0^1 P_{i,t}{}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}$ is the price index for *imported* goods, also expressed in domestic currency. Notice that (5) implies that we can write total expenditures on imported goods as $\int_0^1 P_{i,t}\, C_{i,t} di = P_{F,t}\, C_{F,t}$.

Finally, the optimal allocation of expenditures between domestic and imported goods is given by

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t; \qquad C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{6}$$

where $P_t \equiv \left[(1-\alpha)\,(P_{H,t})^{1-\eta} + \alpha (P_{F,t})^{1-\eta}\right]^{\frac{1}{1-\eta}}$ is the consumer price index (CPI).[10] Notice that, when the price indexes for domestic and foreign goods are equal (as in the steady state described below), parameter α corresponds to the share of domestic consumption allocated to imported goods. It is also in this sense that α represents a natural index of openness.

Accordingly, total consumption expenditures by domestic households are given by $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_t C_t$. Thus, the period budget constraint can be rewritten as

$$P_t\, C_t + E_t\{Q_{t,t+1}\, D_{t+1}\} \leq D_t + W_t N_t + T_t. \tag{7}$$

In what follows we specialize the period utility function to take the form

$$U(C,N) \equiv \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi}.$$

Then we can rewrite the remaining optimality conditions for the household's problem as follows:

$$C_t^{\sigma}\; N_t^{\varphi} = \frac{W_t}{P_t} \tag{8}$$

which is a standard intratemporal optimality condition, and

$$\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) = Q_{t,t+1}. \tag{9}$$

Taking conditional expectations on both sides of (9) and rearranging terms we obtain a conventional stochastic Euler equation:

$$\beta R_t\, E_t \left\{ \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) \right\} = 1 \tag{10}$$

where $R_t = \frac{1}{E_t\{Q_{t,t+1}\}}$ is the gross return on a riskless one-period discount bond paying off one unit of domestic currency in $t+1$ (with $E_t\left\{Q_{t,t+1}\right\}$ being its price).

For future reference it is useful to note that (8) and (10) can be respectively written in log-linearized form as:

10. It is useful to note, for future reference, that in the particular case of $\eta = 1$, the CPI takes the form $P_t = (P_{H,t})^{1-\alpha}(P_{F,t})^{\alpha}$, while the consumption index is given by $C_t = \frac{1}{(1-\alpha)^{(1-\alpha)}\alpha^{\alpha}} C_{H,t}{}^{1-\alpha} C_{F,t}{}^{\alpha}$.

$$w_t - p_t = \sigma\, c_t + \varphi\, n_t$$
$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\,(r_t - E_t\{\pi_{t+1}\} - \rho) \tag{11}$$

where lower case letters denote the logs of the respective variables, $\rho \equiv \beta^{-1} - 1$ is the time discount rate, and $\pi_t \equiv p_t - p_{t-1}$ is CPI inflation (with $p_t \equiv \log P_t$).

2.1.1. Domestic inflation, CPI inflation, the real exchange rate, and the terms of trade: some identities. Before proceeding with our analysis of the equilibrium we introduce several assumptions and definitions, and derive a number of identities that are extensively used below.

We start by defining the *bilateral terms of trade* between the domestic economy and country i as $\mathcal{S}_{i,t} = \frac{P_{i,t}}{P_{H,t}}$, *i.e.* the price of country i's goods in terms of home goods. The *effective terms of trade* are thus given by

$$\begin{aligned}\mathcal{S}_t &\equiv \frac{P_{F,t}}{P_{H,t}}\\ &= \left(\int_0^1 \mathcal{S}_{i,t}^{1-\gamma}\, di\right)^{\frac{1}{1-\gamma}}\end{aligned}$$

which can be approximated (up to first order) by the log-linear expression

$$s_t = \int_0^1 s_{i,t}\, di. \tag{12}$$

Log-linearization of the CPI formula around a symmetric steady state satisfying the purchasing power parity (PPP) condition $P_{H,t} = P_{F,t}$ yields[11]

$$\begin{aligned}p_t &\equiv (1-\alpha)\; p_{H,t} + \alpha\; p_{F,t}\\ &= p_{H,t} + \alpha\; s_t\end{aligned} \tag{13}$$

where $s_t \equiv p_{F,t} - p_{H,t}$ denotes the (log) *effective terms of trade*, *i.e.* the price of foreign goods in terms of home goods. It is useful to note, for future reference, that (12) and (13) hold *exactly* when $\gamma = 1$ and $\eta = 1$, respectively.

It follows that *domestic inflation*—defined as the rate of change in the index of domestic goods prices, *i.e.* $\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}$—and *CPI inflation* are linked according to

$$\pi_t = \pi_{H,t} + \alpha\; \Delta s_t \tag{14}$$

which makes the gap between our two measures of inflation proportional to the per cent change in the terms of trade, with the coefficient of proportionality given by the index of openness α.

We assume that the *law of one price* holds for individual goods at all times (both for import and export prices), implying that $P_{i,t}(j) = \mathcal{E}_{i,t}\; P_{i,t}^i(j)$ for all $i, j \in [0, 1]$, where $\mathcal{E}_{i,t}$ is the bilateral nominal exchange rate (the price of country i's currency in terms of the domestic currency), and $P_{i,t}^i(j)$ is the price of country i's good j expressed in the producer's (*i.e.* country i's) currency. Plugging the previous assumption into the definition of $P_{i,t}$ one obtains $P_{i,t} = \mathcal{E}_{i,t}\; P_{i,t}^i$, where $P_{i,t}^i \equiv \left(\int_0^1 P_{i,t}^i(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}$. In turn, by substituting into the definition of $P_{F,t}$

11. Below we discuss the conditions under which PPP holds in our model.

and log-linearizing around the symmetric steady state we obtain

$$p_{F,t} = \int_0^1 (e_{i,t} + p_{i,t}^i)\, di$$
$$= e_t + p_t^*$$

where $e_t \equiv \int_0^1 e_{i,t}\, di$ is the (log) *nominal effective exchange rate*, $p_{i,t}^i \equiv \int_0^1 p_{i,t}^i(j)\, dj$ is the (log) domestic price index for country i (expressed in terms of its currency), and $p_t^* \equiv \int_0^1 p_{i,t}^i\, di$ is the (log) *world price index*. Notice that for the world as a whole there is no distinction between CPI and domestic price level, nor for their corresponding inflation rates.

Combining the previous result with the definition of the terms of trade we obtain the following expression:

$$s_t = e_t + p_t^* - p_{H,t}. \tag{15}$$

Next, we derive a relationship between the terms of trade and the real exchange rate. First, we define the *bilateral real exchange rate* with country i as $\mathcal{Q}_{i,t} \equiv \frac{\mathcal{E}_{i,t} P_t^i}{P_t}$, *i.e.* the ratio of the two countries' CPIs, both expressed in domestic currency. Let $q_t \equiv \int_0^1 q_{i,t}\, di$ be the (log) *effective real exchange rate*, where $q_{i,t} \equiv \log \mathcal{Q}_{i,t}$. It follows that

$$q_t = \int_0^1 (e_{i,t} + p_t^i - p_t)\, di$$
$$= e_t + p_t^* - p_t$$
$$= s_t + p_{H,t} - p_t$$
$$= (1-\alpha)\, s_t$$

where the last equality holds only up to a first order approximation when $\eta \neq 1$.[12]

2.1.2. International risk sharing. Under the assumption of complete securities markets, a first order condition analogous to (9) must also hold for the representative household in any other country, say country i:

$$\beta \left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\sigma} \left(\frac{P_t^i}{P_{t+1}^i}\right) \left(\frac{\mathcal{E}_t^i}{\mathcal{E}_{t+1}^i}\right) = Q_{t,t+1}. \tag{16}$$

Combining (9) and (16), together with the real exchange rate definition, it follows that

$$C_t = \vartheta_i C_t^i\, \mathcal{Q}_{i,t}{}^{\frac{1}{\sigma}} \tag{17}$$

for all t, and where ϑ_i is a constant which will generally depend on initial conditions regarding relative net asset positions. Henceforth, and without loss of generality, we assume symmetric initial conditions (*i.e.* zero net foreign asset holdings and an *ex ante* identical environment), in which case we have $\vartheta_i = \vartheta = 1$ for all i. As shown in Appendix A, in the symmetric perfect foresight steady state we also have that $C = C^i = C^*$ and $\mathcal{Q}_i = \mathcal{S}_i = 1$ (*i.e.* purchasing power parity holds), for all i.

12. The last equality can be derived by log-linearizing $\frac{P_t}{P_{H,t}} = \left[(1-\alpha) + \alpha\, \mathcal{S}_t^{1-\eta}\right]^{\frac{1}{1-\eta}}$ around a symmetric steady state, which yields

$$p_t - p_{H,t} = \alpha\, s_t.$$

Taking logs on both sides of (17) and integrating over i we obtain

$$\begin{aligned} c_t &= c_t^* + \frac{1}{\sigma}\, q_t \\ &= c_t^* + \left(\frac{1-\alpha}{\sigma}\right) s_t \end{aligned} \tag{18}$$

where $c_t^* \equiv \int_0^1 c_t^i di$ is our index for world consumption (in log terms), and where the second equality holds only up to a first order approximation when $\eta \neq 1$. Thus we see that the assumption of complete markets at the international level leads to a simple relationship linking domestic consumption with world consumption and the terms of trade.[13]

2.1.3. Uncovered interest parity and the terms of trade. Under the assumption of complete international financial markets, the equilibrium price (in terms of domestic currency) of a riskless bond denominated in foreign currency is given by $\mathcal{E}_{i,t}\ (R_t^i)^{-1} = E_t\{Q_{t,t+1}\ \mathcal{E}_{i,t+1}\}$. The previous pricing equation can be combined with the domestic bond pricing equation, $(R_t)^{-1} = E_t\{Q_{t,t+1}\}$ to obtain a version of the *uncovered interest parity* condition:

$$E_t\{Q_{t,t+1}\,[R_t - R_t^i\ (\mathcal{E}_{i,t+1}/\mathcal{E}_{i,t})]\} = 0.$$

Log-linearizing around a perfect foresight steady state, and aggregating over i, yields the familiar expression[14]

$$r_t - r_t^* = E_t\{\Delta e_{t+1}\}. \tag{19}$$

Combining the definition of the (log) terms of trade with (19) yields the following stochastic difference equation:

$$s_t = (r_t^* - E_t\{\pi_{t+1}^*\}) - (r_t - E_t\{\pi_{H,t+1}\}) + E_t\{s_{t+1}\}. \tag{20}$$

As we show in Appendix A, the terms of trade are pinned down uniquely in the perfect foresight steady state. That fact, combined with our assumption of stationarity in the model's driving forces and a convenient normalization (requiring that PPP holds in the steady state), implies that $\lim_{T\to\infty} E_t\{s_T\} = 0$.[15] Hence, we can solve (20) forward to obtain

$$s_t = E_t\left\{\sum_{k=0}^{\infty} [(r_{t+k}^* - \pi_{t+k+1}^*) - (r_{t+k} - \pi_{H,t+k+1})]\right\} \tag{21}$$

i.e. the terms of trade are a function of current and anticipated real interest rate differentials.

We must point out that while equation (20) (and (21)) provides a convenient (and intuitive) way of representing the connection between terms of trade and interest rate differentials, it does not constitute an additional independent equilibrium condition. In particular, it is easy to check that (20) can be derived by combining the consumption Euler equations for both the domestic and world economies with the risk sharing condition (18) and equation (14).

Next we turn our attention to the supply side of the economy.

13. A similar relationship holds in many international RBC models. See, *e.g.* Backus and Smith (1993).

14. This abstracts from the presence of a risk premium term. See Kollmann (2002) for a specification which includes an exogenous stochastic risk premium.

15. Our assumption of PPP holding in the steady state implies that the real interest rate differential will revert to a zero mean. More generally, the real interest rate differential will revert to a constant mean, as long as the terms of trade are stationary in first differences. That would be the case if, say, the technology parameter had a unit root or a different average rate of growth relative to the rest of the world. In those cases we could have persistent real interest rate differentials.

2.2. Firms

2.2.1. Technology. A typical firm in the home economy produces a differentiated good with a linear technology represented by the production function

$$Y_t(j) = A_t \; N_t(j)$$

where $a_t \equiv \log A_t$ follows the AR(1) process $a_t = \rho_a \; a_{t-1} + \varepsilon_t$, and $j \in [0, 1]$ is a firm-specific index. Hence, the real marginal cost (expressed in terms of domestic prices) will be common across domestic firms and given by

$$mc_t = -\nu + w_t - p_{H,t} - a_t$$

where $\nu \equiv -\log(1 - \tau)$, with τ being an employment subsidy whose role is discussed later in more detail.

Let $Y_t \equiv \left[\int_0^1 Y_t(j)^{1-\frac{1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$ represent an index for aggregate domestic output, analogous to the one introduced for consumption. It is useful, for future reference, to derive an approximate aggregate production function relating the previous index to aggregate employment. Hence, notice that

$$N_t \equiv \int_0^1 N_t(j)dj = \frac{Y_t \; Z_t}{A_t}$$

where $Z_t \equiv \int_0^1 \frac{Y_t(j)}{Y_t} dj$. In Appendix C we show that equilibrium variations in $z_t \equiv \log Z_t$ around the perfect foresight steady state are of second order. Thus, and up to a first order approximation, we have an aggregate relationship

$$y_t = a_t + n_t. \tag{22}$$

2.2.2. Price-setting. We assume that firms set prices in a staggered fashion, as in Calvo (1983). Hence, a measure $1-\theta$ of (randomly selected) firms sets new prices each period, with an individual firm's probability of re-optimizing in any given period being independent of the time elapsed since it last reset its price. As we show in Appendix B, the optimal price-setting strategy for the typical firm resetting its price in period t can be approximated by the (log-linear) rule

$$\overline{p}_{H,t} = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \; E_t\{mc_{t+k} + p_{H,t}\} \tag{23}$$

where $\overline{p}_{H,t}$ denotes the (log) of newly set domestic prices, and $\mu \equiv \log\left(\frac{\varepsilon}{\varepsilon-1}\right)$, which corresponds to the log of the (gross) mark-up in the steady state (or, equivalently, the optimal mark-up in a flexible price economy).

Hence, we see that the pricing decision in our model (as in its closed economy counterpart) is a forward-looking one. The reason is simple and well understood by now: firms that are adjusting prices in any given period recognize that the price they set will remain effective for a (random) number of periods. As a result they set the price as a mark-up over a weighted average of expected future marginal costs, instead of looking at current marginal cost only. Notice that in the flexible price limit (*i.e.* as $\theta \to 0$), we recover the familiar mark-up rule $\overline{p}_{H,t} = \mu + mc_t + p_{H,t}$.

3. EQUILIBRIUM

3.1. Aggregate demand and output determination

3.1.1. Consumption and output in the small open economy. Goods market clearing in the representative small open economy ("home") requires

$$Y_t(j) = C_{H,t}(j) + \int_0^1 C^i_{H,t}(j)di \tag{24}$$
$$= \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha\int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t}P^i_{F,t}}\right)^{-\gamma}\left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t di\right]$$

for all $j \in [0,1]$ and all t, where $C^i_{H,t}(j)$ denotes country i's demand for good j produced in the home economy. Notice that the second equality has made use of (6) and (5) together with our assumption of symmetric preferences across countries, which implies $C^i_{H,t}(j) = \alpha\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{\mathcal{E}_{i,t}P^i_{F,t}}\right)^{-\gamma}\left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t$.

Plugging (24) into the definition of aggregate domestic output $Y_t \equiv \left[\int_0^1 Y_t(j)^{1-\frac{1}{\varepsilon}}dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$, we obtain

$$Y_t = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha\int_0^1\left(\frac{P_{H,t}}{\mathcal{E}_{i,t}P^i_{F,t}}\right)^{-\gamma}\left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t di$$
$$= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha)C_t + \alpha\int_0^1\left(\frac{\mathcal{E}_{i,t}P^i_{F,t}}{P_{H,t}}\right)^{\gamma-\eta}\mathcal{Q}^{\eta}_{i,t}C^i_t di\right]$$
$$= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t\left[(1-\alpha) + \alpha\int_0^1\left(\mathcal{S}^i_t\mathcal{S}_{i,t}\right)^{\gamma-\eta}\mathcal{Q}^{\eta-\frac{1}{\sigma}}_{i,t}di\right] \tag{25}$$

where the last equality follows from (17), and where $\mathcal{S}^i_t$ denotes the effective terms of trade of country i, while $\mathcal{S}_{i,t}$ denotes the bilateral terms of trade between the home economy and foreign country i. Notice that in the particular case of $\sigma = \eta = \gamma = 1$ the previous condition can be written exactly as[16]

$$Y_t = C_t\,\mathcal{S}^{\alpha}_t. \tag{26}$$

More generally, and recalling that $\int_0^1 s^i_t di = 0$, we can derive the following first order log-linear approximation to (25) around the symmetric steady state:

$$y_t = c_t + \alpha\gamma\, s_t + \alpha\left(\eta - \frac{1}{\sigma}\right) q_t$$
$$= c_t + \frac{\alpha\omega}{\sigma}s_t \tag{27}$$

where $\omega \equiv \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$. Notice that $\sigma = \eta = \gamma = 1$ implies $\omega = 1$.

A condition analogous to the one above will hold for all countries. Thus, for a *generic country* i it can be rewritten as $y^i_t = c^i_t + \frac{\alpha\omega}{\sigma}s^i_t$. By aggregating over all countries we can derive a world market clearing condition as follows:

$$y^*_t \equiv \int_0^1 y^i_t di \tag{28}$$
$$= \int_0^1 c^i_t di \equiv c^*_t$$

16. Here one must use the fact that under the assumption $\eta = 1$, the CPI takes the form $P_t = (P_{H,t})^{1-\alpha}(P_{F,t})^{\alpha}$ thus implying $\frac{P_t}{P_{H,t}} = \left(\frac{P_{F,t}}{P_{H,t}}\right)^{\alpha} = \mathcal{S}^{\alpha}_t$.

where y_t^* and c_t^* are indexes for world output and consumption (in log terms), and where the main equality follows, once again, from the fact that $\int_0^1 s_t^i di = 0$.

Combining (27) with (17) and (28), we obtain

$$y_t = y_t^* + \frac{1}{\sigma_\alpha} s_t \tag{29}$$

where $\sigma_\alpha \equiv \frac{\sigma}{(1-\alpha)+\alpha\omega} > 0$.

Finally, combining (27) with Euler equation (11), we get

$$\begin{aligned} y_t &= E_t\{y_{t+1}\} - \frac{1}{\sigma}\,(r_t - E_t\{\pi_{t+1}\} - \rho) - \frac{\alpha\omega}{\sigma} E_t\{\Delta s_{t+1}\} \\ &= E_t\{y_{t+1}\} - \frac{1}{\sigma}\,(r_t - E_t\{\pi_{H,t+1}\} - \rho) - \frac{\alpha\Theta}{\sigma} E_t\{\Delta s_{t+1}\} \\ &= E_t\{y_{t+1}\} - \frac{1}{\sigma_\alpha}\,(r_t - E_t\{\pi_{H,t+1}\} - \rho) + \alpha\Theta E_t\{\Delta y_{t+1}^*\} \end{aligned} \tag{30}$$

where $\Theta \equiv (\sigma\gamma - 1) + (1-\alpha)(\sigma\eta - 1) = \omega - 1$.

3.1.2. The trade balance. Let $nx_t \equiv \left(\frac{1}{Y}\right)\left(Y_t - \frac{P_t}{P_{H,t}} C_t\right)$ denote net exports in terms of domestic output, expressed as a fraction of steady state output Y. In the particular case of $\sigma = \eta = \gamma = 1$, it follows from (25) that $P_{H,t}Y_t = P_t C_t$ for all t, thus implying a balanced trade at all times. More generally, a first order approximation yields $nx_t = y_t - c_t - \alpha\, s_t$ which combined with (27) implies

$$nx_t = \alpha\left(\frac{\omega}{\sigma} - 1\right) s_t. \tag{31}$$

Again, in the special case of $\sigma = \eta = \gamma = 1$ we have $nx_t = 0$ for all t, though the latter property will also hold for any configuration of those parameters satisfying $\sigma(\gamma - 1) + (1 - \alpha)\,(\sigma\eta - 1) = 0$. More generally, the sign of the relationship between the terms of trade and net exports is ambiguous, depending on the relative size of σ, γ, and η.[17]

3.2. The supply side: marginal cost and inflation dynamics

3.2.1. Marginal cost and inflation dynamics in the small open economy. In the small open economy, the dynamics of *domestic* inflation in terms of real marginal cost are described by an equation analogous to the one associated with a closed economy. Hence,

$$\pi_{H,t} = \beta\; E_t\{\pi_{H,t+1}\} + \lambda\; \widehat{mc}_t \tag{32}$$

where $\lambda \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$. Details of the derivation can be found in Appendix B.

The determination of the real marginal cost as a function of domestic output in the small open economy differs somewhat from that in the closed economy, due to the existence of a wedge between output and consumption, and between domestic and consumer prices. Thus, in our model we have

17. The fact that in our economy movements in the trade balance are allowed is a key difference with respect to many models in the literature (see, *e.g.* Corsetti and Pesenti, 2001), which typically require log utility ($\sigma = 1$) and unitary elasticity of substitution ($\eta = 1$) for balanced trade to hold continuously. Notice also that our framework requires stricter conditions for balanced trade, in that it also requires $\gamma = 1$ (or any combination of η and γ such that $\frac{\omega}{\sigma} = 1$).

$$\begin{aligned} mc_t &= -\nu + (w_t - p_{H,t}) - a_t \\ &= -\nu + (w_t - p_t) + (p_t - p_{H,t}) - a_t \\ &= -\nu + \sigma\ c_t + \varphi\ n_t + \alpha\ s_t - a_t \\ &= -\nu + \sigma\ y_t^* + \varphi\ y_t + s_t - (1+\varphi)\ a_t \end{aligned} \tag{33}$$

where the last equality makes use of (22) and (18). Thus, we see that marginal cost is increasing in the terms of trade and world output. Both variables end up influencing the real wage, through the wealth effect on labour supply resulting from their impact on domestic consumption. In addition, changes in the terms of trade have a direct effect on the product wage, for any given real wage. The influence of technology (through its direct effect on labour productivity) and of domestic output (through its effect on employment and, hence, the real wage—for given output) is analogous to that observed in the closed economy.

Finally, using (29) to substitute for s_t, we can rewrite the previous expression for the real marginal cost in terms of domestic output and productivity, as well as world output:

$$mc_t = -\nu + (\sigma_\alpha + \varphi)\ y_t + (\sigma - \sigma_\alpha)\ y_t^* - (1+\varphi)\ a_t. \tag{34}$$

Notice that in the special cases $\alpha = 0$ and/or $\sigma = \eta = \gamma = 1$, which imply $\sigma = \sigma_\alpha$, the domestic real marginal cost is completely insulated from movements in foreign output.

3.3. *Equilibrium dynamics: a canonical representation*

In this section we show that the linearized equilibrium dynamics for the small open economy have a representation in terms of output gap and domestic inflation analogous to that of its closed economy counterpart. That representation, which we refer to as the canonical one, has provided the basis for the analysis and evaluation of alternative policy rules in much of the recent literature. Let us define the domestic output gap x_t as the deviation of (log) domestic output y_t from its natural level $\overline{y}_t$, where the latter is in turn defined as the equilibrium level of output in the absence of nominal rigidities (and conditional on world output y_t^*). Formally,

$$x_t \equiv y_t - \overline{y}_t.$$

The domestic natural level of output can be found after imposing $mc_t = -\mu$ for all t and solving for domestic output in equation (34):

$$\overline{y}_t = \Omega + \Gamma\ a_t + \alpha\Psi\ y_t^* \tag{35}$$

where $\Omega \equiv \frac{\upsilon-\mu}{\sigma_\alpha+\varphi}$, $\Gamma \equiv \frac{1+\varphi}{\sigma_\alpha+\varphi} > 0$, and $\Psi \equiv -\frac{\Theta\ \sigma_\alpha}{\sigma_\alpha+\varphi}$.

It also follows from (34) that the domestic real marginal cost and output gap will be related according to

$$\widehat{mc}_t = (\sigma_\alpha + \varphi)\ x_t$$

which we can combine with (32) to derive a version of the new Keynesian Phillips curve (NKPC) for the small open economy in terms of the output gap:

$$\pi_{H,t} = \beta\ E_t\{\pi_{H,t+1}\} + \kappa_\alpha\ x_t \tag{36}$$

where $\kappa_\alpha \equiv \lambda\,(\sigma_\alpha + \varphi)$. Notice that for $\alpha = 0$ (or $\sigma = \eta = \gamma = 1$) the slope coefficient is given by $\lambda\,(\sigma + \varphi)$ as in the standard, closed economy NKPC. More generally, we see that the form of the Phillips equation for the open economy corresponds to that of the closed economy, at least as far as domestic (*i.e.* producer) inflation is concerned. The degree of openness α affects the dynamics of inflation only through its influence on the size of the slope of the Phillips curve, *i.e.* the size of the inflation response to any given variation in the output gap. In the open economy, a change in domestic output has an effect on marginal cost through its impact on employment (captured by φ), and the terms of trade (captured by σ_α, which is a function of the degree of openness and the substitutability between domestic and foreign goods). In particular, under the assumption that $\sigma\eta > 1$, an increase in openness lowers the size of the adjustment in the terms of trade necessary to absorb a change in domestic output (relative to world output), thus dampening the impact of that adjustment on marginal cost and inflation.

Using (30) it is straightforward to derive a version of the so-called dynamic IS equation for the open economy in terms of the output gap:

$$x_t = E_t\{x_{t+1}\} - \frac{1}{\sigma_\alpha}\,(r_t - E_t\{\pi_{H,t+1}\} - \overline{rr}_t) \tag{37}$$

where

$$\overline{rr}_t \equiv \rho - \sigma_\alpha \Gamma(1-\rho_a)\, a_t + \alpha\sigma_\alpha(\Theta + \Psi)\, E_t\{\Delta y^*_{t+1}\}$$

is the small open economy's natural rate of interest.

Thus we see that the small open economy's equilibrium is characterized by a forward-looking IS-type equation similar to that found in the closed economy. Two differences can be pointed out, however. First, the degree of openness influences the sensitivity of the output gap to interest rate changes. In particular, if $\omega > 1$ (which obtains for "high" values of η and γ), an increase in openness raises that sensitivity (through the stronger effects of the induced terms of trade changes on demand). Second, openness generally makes the natural interest rate depend on expected world output growth, in addition to domestic productivity.

4. OPTIMAL MONETARY POLICY: A SPECIAL CASE

In this section we derive and characterize the optimal monetary policy for our small open economy, as well as its implications for a number of macroeconomic variables. Our analysis is restricted to a special case for which a second order approximation to the welfare of the representative consumer can be easily derived analytically.

Let us take as a benchmark the well-known closed economy version of the Calvo economy with staggered price-setting. As discussed in Rotemberg and Woodford (1999), under the assumption of a constant employment subsidy τ that neutralizes the distortion associated with firms' market power, it can be shown that the optimal monetary policy is the one that replicates the flexible price equilibrium allocation. That policy requires that real marginal costs (and thus mark-ups) are stabilized at their steady state level, which in turn implies that domestic prices be fully stabilized. The intuition for that result is straightforward: with the subsidy in place, there is only one effective distortion left in the economy, namely, sticky prices. By stabilizing mark-ups at their "frictionless" level, nominal rigidities cease to be binding, since firms do not feel any desire to adjust prices. By construction, the resulting equilibrium allocation is efficient, and the price level remains constant.[18]

In an open economy—and as noted, among others, by Corsetti and Pesenti (2001)—there is an additional factor that distorts the incentives of the monetary authority (beyond the presence of market power): the possibility of influencing the terms of trade in a way beneficial to domestic consumers. This possibility is a consequence of the imperfect substitutability between domestic and foreign goods, combined with sticky prices (which render monetary policy non-neutral).[19] As shown below, and similarly to Benigno and Benigno (2003) in the context of a two-country

18. See Galí (2003) for a discussion.

19. This distinguishes our analysis from that of Goodfriend and King (2001) who assume that the price of domestic goods is determined in the world market.

model, the introduction of an employment subsidy that exactly offsets the market power distortion is not sufficient to render the flexible price equilibrium allocation optimal, for, at the margin, the monetary authority would have an incentive to deviate from it to improve the terms of trade.

For the special parameter configuration $\sigma = \eta = \gamma = 1$ we can derive analytically the employment subsidy that exactly offsets the combined effects of market power and the terms of trade distortions, thus rendering the flexible price equilibrium allocation optimal.[20] That result, in turn, rules out the possibility of the existence of an average inflation (or deflation) bias, and allows us to focus on policies consistent with zero average inflation, in a way analogous to the analysis for the closed economy found in the literature.

Let us first characterize the optimal allocation from the viewpoint of a social planner facing the same resource constraints to which the small open economy is subject in equilibrium (vis-à-vis the rest of the world), given our assumption of complete markets. In that case, the optimal allocation must maximize $U(C_t, N_t)$ subject to (i) the technological constraint $Y_t = A_t N_t$, (ii) a consumption/output possibilities set implicit in the international risk sharing conditions (17), and (iii) the market clearing condition (25).

The derivation of a tractable, analytical solution requires that we restrict ourselves to the special case of $\sigma = \eta = \gamma = 1$. In that case, (18) and (26) imply the exact expression $C_t = Y_t^{1-\alpha}(Y_t^*)^\alpha$. The optimal allocation (from the viewpoint of the small open economy, which takes world output and consumption as given), must satisfy

$$-\frac{U_N(C_t, N_t)}{U_C(C_t, N_t)} = (1-\alpha)\,\frac{C_t}{N_t}$$

which, under the assumed preferences, implies a constant employment $N = (1-\alpha)^{\frac{1}{1+\varphi}}$.

Notice, on the other hand, that the flexible price equilibrium in the small open economy (with corresponding variables denoted with an upper bar) satisfies

$$\begin{aligned}
1 - \frac{1}{\varepsilon} &= \overline{MC}_t \\
&= -\frac{(1-\tau)}{A_t}\,\overline{S}_t^{\alpha}\,\frac{U_N(\overline{C}_t, \overline{N}_t)}{U_C(\overline{C}_t, \overline{N}_t)} \\
&= \frac{(1-\tau)}{A_t}\,\frac{\overline{Y}_t}{\overline{C}_t}\,\overline{N}_t^{\varphi}\,\overline{C}_t \\
&= (1-\tau)\,\overline{N}_t^{1+\varphi}.
\end{aligned}$$

Hence, by setting τ such that $(1-\tau)(1-\alpha) = 1 - \frac{1}{\varepsilon}$ is satisfied (or, equivalently, $\nu = \mu + \log(1-\alpha)$) we guarantee the optimality of the flexible price equilibrium allocation. As in the closed economy case, the optimal monetary policy requires stabilizing the output gap (*i.e.* $x_t = 0$, for all t). Equation (36) then implies that domestic prices are also stabilized under that optimal policy ($\pi_{H,t} = 0$ for all t). Thus, in the special case under consideration, (strict) *domestic inflation targeting* (DIT) is indeed the optimal policy.

4.1. Implementation and macroeconomic implications

In this section we discuss the implementation and characterize the equilibrium processes for a number of variables of the small open economy, under the assumption of a *domestic inflation*

20. Condition $\sigma = \eta = 1$ corresponds to the one required in the two-country framework of Benigno and Benigno (2003). Our model of a small open economy in a multicountry world requires, in addition, that the substitutability (in the home consumer's preferences) between goods produced in any two foreign countries is also unity (*i.e.* $\gamma = 1$).

targeting policy (DIT). While we have shown that policy to be optimal only for the special case considered above, in this subsection we look at the implications of that policy for the general case.

4.1.1. Implementation. As discussed above full stabilization of domestic prices implies

$$x_t = \pi_{H,t} = 0$$

for all t. This in turn implies that $y_t = \overline{y}_t$ and $r_t = \overline{rr}_t$ will hold in equilibrium for all t, with all the remaining variables matching their natural level all the time.

Interestingly, however, $r_t = \overline{rr}_t$ cannot be interpreted as a "rule" that the central bank could follow mechanically in order to implement the optimal allocation. For, while $x_t = \pi_{H,t} = 0$ would certainly constitute an equilibrium in that case, the same equilibrium would not be unique; instead, multiple equilibria and the possibility of stationary sunspot fluctuations may arise. The previous result should not be surprising given the equivalence shown above between the dynamical system describing the equilibrium of the small open economy and that of the closed economy, and given the findings in the related closed economy literature. In particular, and as shown in Appendix C, we can invoke that literature to point to a simple solution to that problem. In particular, the indeterminacy problem can be avoided, and the uniqueness of the optimal equilibrium allocation restored, by having the central bank follow a rule which would imply that the interest rate should respond to domestic inflation and/or the output gap were those variables to deviate from their (zero) target values. More precisely, suppose that the central bank commits itself to the rule

$$r_t = \overline{rr}_t + \phi_\pi \, \pi_{H,t} + \phi_x \, x_t. \tag{38}$$

As shown by Bullard and Mitra (2002), if we restrict ourselves to non-negative values of ϕ_π and ϕ_x, a necessary and sufficient condition for uniqueness of the optimal allocation is given by

$$\kappa_\alpha \, (\phi_\pi - 1) + (1 - \beta) \, \phi_x > 0. \tag{39}$$

Notice that, once uniqueness is restored, the term $\phi_\pi \, \pi_{H,t} + \phi_x \, x_t$ appended to the interest rate rule vanishes, implying that $r_t = \overline{rr}_t$ for all t. Thus, we see that stabilization of the output gap and inflation requires a credible threat by the central bank to vary the interest rate sufficiently in response to any deviations of inflation and/or the output gap from the target; yet, the very existence of that threat makes its effective application unnecessary.

4.1.2. Macroeconomic implications. Under strict domestic inflation targeting (DIT), the behaviour of real variables in the small open economy corresponds to the one we would observe in the absence of nominal frictions. Hence, we see from inspection of equation (35) that domestic output always increases in response to a positive technology shock at home.

The sign of the response to a rise in world output is ambiguous, however, and it depends on the sign of Θ, as shown in (35). To obtain some intuition for the forces at work notice first that the natural level of the terms of trade is given by

$$\begin{aligned}\overline{s}_t &= \sigma_\alpha (\overline{y}_t - y_t^*) \\ &= \sigma_\alpha \Omega + \sigma_\alpha \Gamma \, a_t - \sigma_\alpha \Phi \, y_t^*\end{aligned}$$

where $\Phi \equiv \frac{\sigma+\varphi}{\sigma_\alpha+\varphi} > 0$. Thus, an increase in world output always generates an improvement in the terms of trade (*i.e.* a real appreciation). The resulting expenditure-switching effect, together with the effect of the real appreciation on domestic consumption through the risk sharing transfer of resources (see (17)), tends to reduce aggregate demand and domestic economic activity. For any given terms of trade, that effect is offset to a lesser or greater extent by a positive direct demand effect (resulting from higher exports) as well as by a positive effect on domestic consumption associated with international risk sharing (and given the implied higher world consumption). It can be easily checked that the contractionary (expansionary) effect dominates whenever $\omega > 1$ ($\omega < 1$). In the special case considered above $\omega = 1$, thus implying that a change in world output leaves the terms of trade and domestic output unchanged under DIT policy.

Given that under DIT domestic prices are fully stabilized, it follows that $\overline{e}_t = \overline{s}_t - p_t^*$, *i.e.* under the DIT regime the nominal exchange rate moves one-for-one with the (natural) terms of trade and (inversely) with the world price level. In the limiting case of constant world prices, the nominal exchange rate will inherit all the statistical properties of the (natural) terms of trade, including its stationarity (and thus its reversion to a constant mean).[21] Of course, stationarity does not necessarily imply low volatility. In particular, in the case of constant world prices, the volatility of the nominal exchange rate under DIT will be proportional to the volatility of the gap between the natural level of domestic output (in turn related to productivity) and world output. A high positive (negative) correlation between domestic productivity and world output will tend to decrease (increase) the volatility of the nominal and real exchange rates.

In addition, we can also derive the implied equilibrium process for the level of the CPI. Given the constancy of domestic prices it is given by

$$\begin{aligned}\overline{p}_t &= \alpha(\overline{e}_t + p_t^*)\\ &= \alpha\,\overline{s}_t.\end{aligned}$$

Thus, we see that under domestic inflation targeting the CPI level will vary with the (natural) terms of trade and will inherit its statistical properties. If the economy is very open, and if domestic productivity (and hence the natural level of domestic output) is not much synchronized with world output, CPI prices could potentially be highly volatile, even if the domestic price level is constant.

4.2. *The welfare costs of deviations from the optimal policy*

Under the particular assumptions for which strict domestic inflation targeting has been shown to be optimal (*i.e.* log utility and unit elasticity of substitution between goods of different origin), it is possible to derive a second order approximation to the utility losses of the domestic representative consumer resulting from deviations from the optimal policy. As we show in Appendix D, those losses, expressed as a fraction of steady state consumption, can be written as

$$\mathbb{W} = -\frac{(1-\alpha)}{2}\sum_{t=0}^{\infty}\beta^t\left[\frac{\varepsilon}{\lambda}\,\pi_{H,t}^2 + (1+\varphi)\,x_t^2\right]. \tag{40}$$

Taking unconditional expectations on (40) and letting $\beta \to 1$, the expected welfare losses of any policy that deviated from strict inflation targeting can be written in terms of the variances of inflation and the output gap:

$$\mathbb{V} = -\frac{(1-\alpha)}{2}\left[\frac{\varepsilon}{\lambda}\,\mathrm{var}(\pi_{H,t}) + (1+\varphi)\,\mathrm{var}(x_t)\right]. \tag{41}$$

Below we make use of the previous approximation to assess the welfare implications of alternative suboptimal monetary policy rules, and to rank those rules on welfare grounds.

21. The stationarity of the terms of trade is, in turn, an implication of the stationarity of the productivity differential coupled with our assumption of complete asset markets.

5. SIMPLE MONETARY POLICY RULES FOR THE SMALL OPEN ECONOMY

In the present section we analyse the macroeconomic implications of three alternative monetary policy regimes for the small open economy. Two of the simple rules considered are stylized Taylor-type rules. The first has the domestic interest rate respond systematically to domestic inflation, whereas the second assumes that CPI inflation is the variable the domestic central bank reacts to. The third rule we consider is one that pegs the effective nominal exchange rate. Formally, the *domestic inflation-based Taylor rule* (DITR, for short) is specified as follows:

$$r_t = \rho + \phi_\pi \, \pi_{H,t}.$$

The *CPI inflation-based Taylor rule* (CITR, for short) is assumed to take the form

$$r_t = \rho + \phi_\pi \, \pi_t.$$

Finally, the *exchange rate peg* (PEG, for short) implies

$$e_t = 0$$

for all t.

Below we provide a comparison of the equilibrium properties of several macroeconomic variables under the above simple rules for a calibrated version of our model economy. We compare such properties to those associated with a strict domestic inflation targeting (DIT), the policy that is optimal under the conditions discussed above, and which we assume to be satisfied in our baseline calibration. We next briefly describe the calibration strategy underlying that exercise.

5.1. A numerical analysis of alternative rules

5.1.1. Calibration. In this section we present some quantitative results based on a calibrated version of our model economy. Let us first state the main assumptions underlying our baseline calibration, which we take as a benchmark. We set $\sigma = \eta = \gamma = 1$, in a way consistent with the special case considered above. We assume $\varphi = 3$, which implies a labour supply elasticity of $\frac{1}{3}$, and a value for the steady state mark-up $\mu = 1{\cdot}2$, which implies that ε, the elasticity of substitution between differentiated goods (of the same origin), is 6. Parameter θ is set equal to 0·75, a value consistent with an average period of one year between price adjustments. We assume $\beta = 0{\cdot}99$, which implies a riskless annual return of about 4% in the steady state. We set a baseline value for α (or degree of openness) of 0·4. The latter corresponds roughly to the import/GDP ratio in Canada, which we take as a prototype small open economy. In the calibration of the interest rate rules we follow the original Taylor estimate and set ϕ_π equal to 1·5, while $\rho = (0{\cdot}99)^{-1} - 1$.

In order to calibrate the stochastic properties of the exogenous driving forces, we fit AR(1) processes to (log) labour productivity in Canada (our proxy for domestic productivity), and (log) U.S. GDP (our proxy for world output), using quarterly, HP-filtered data over the sample period 1963:1–2002:4. We obtain the following estimates (with standard errors in parantheses):

$$a_t = \underset{(0{\cdot}06)}{0{\cdot}66} a_{t-1} + \varepsilon_t^a, \qquad \sigma_a = 0{\cdot}0071$$

$$y_t^* = \underset{(0{\cdot}04)}{0{\cdot}86} y_{t-1}^* + \varepsilon_t^*, \qquad \sigma_{y^*} = 0{\cdot}0078$$

with $\mathrm{corr}(\varepsilon_t^a, \varepsilon_t^*) = 0{\cdot}3$.

5.1.2. Impulse responses. We start by describing the dynamic effects of a *domestic* productivity shock on a number of macroeconomic variables. Figure 1 displays the impulse

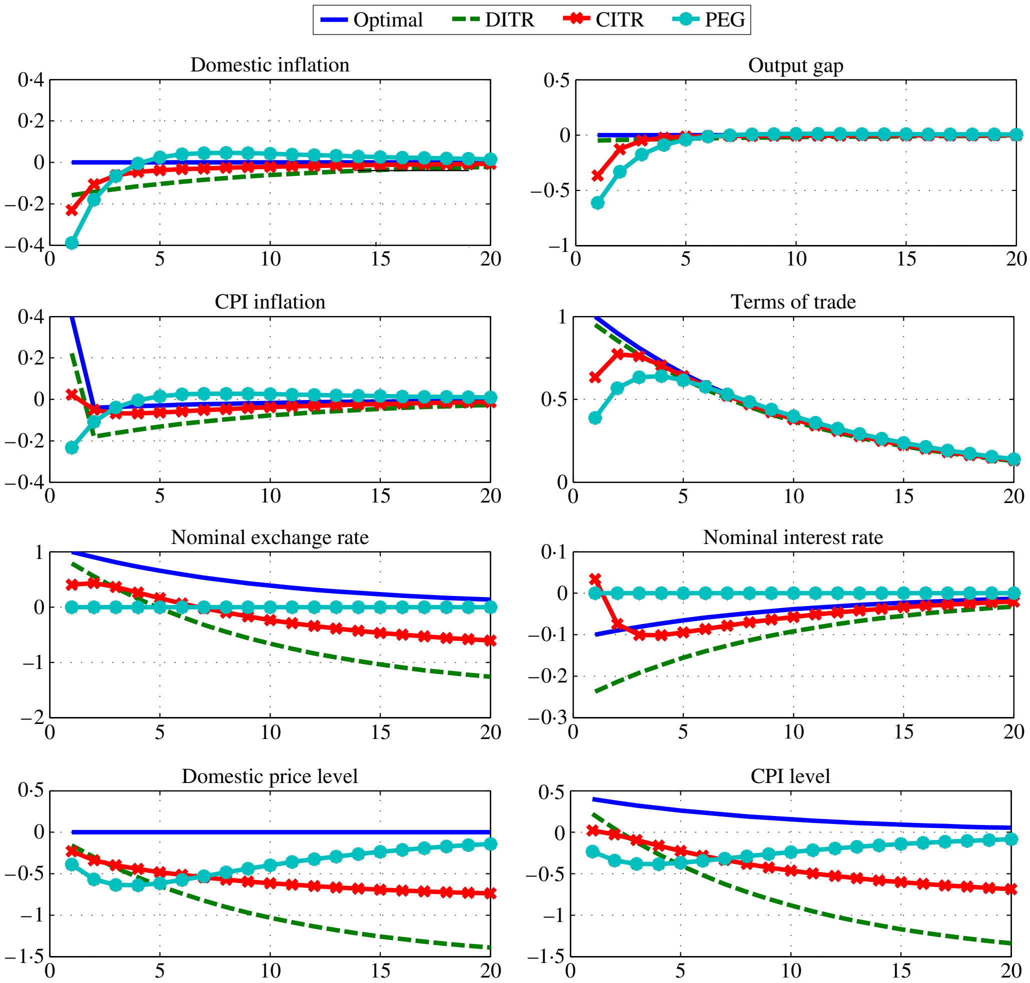

FIGURE 1

Impulse responses to a domestic productivity shock under alternative policy rules

responses to a unit innovation in a_t under the four regimes considered. By construction, domestic inflation and the output gap remain unchanged under the optimal policy (DIT). We also see that the shock leads to a persistent reduction in the domestic interest rate as it is needed in order to support the transitory expansion in consumption and output consistent with the flexible price equilibrium allocation. Given the constancy of the world nominal interest rate, uncovered interest parity implies an initial nominal depreciation followed by expectations of a future appreciation, as reflected in the response of the nominal exchange rate. Relative to all other regimes, the constancy of domestic prices accounts for a larger real depreciation and therefore for a further expansion in demand and output through a rise in net exports (not shown here). Given constant world prices and the stationarity of the terms of trade, the constancy of domestic prices implies a mean-reverting response of the nominal exchange rate.

It is interesting to contrast the implied dynamic behaviour of the same variables under the optimal policy to the one under the two stylized Taylor rules (DITR and CITR). Notice, at first, that both rules generate, unlike the optimal policy, a permanent fall in both domestic and CPI prices. The unit root in domestic prices is then mirrored, under both rules, by the unit root in the nominal exchange rate.

A key difference between the two Taylor rules concerns the behaviour of the terms of trade. While under DITR the terms of trade depreciate on impact and then start immediately to revert to steady state (mirroring closely the response under the optimal policy), under CITR the initial response of the terms of trade is more muted, and is followed by a hump-shaped pattern. The intuition is simple. Under both rules, the rise in domestic productivity and the required real depreciation lead, for *given domestic prices*, to an increase in CPI inflation. However, under CITR the desired stabilization of CPI inflation is partly achieved, relative to DITR, by means of a more muted response of the terms of trade (since the latter affect the CPI), and a fall in domestic prices. The fall in prices, in turn, requires a negative output gap and hence a more contractionary monetary policy (*i.e.* a higher interest rate). Under our calibration the latter takes the form of an initial rise in both nominal and real interest rates, with the subsequent path of the real rate remaining systematically above that implied by the optimal policy or a DITR policy.[22]

Finally, the same figure displays the corresponding impulse responses under a PEG. Notice that the responses of output gap and inflation are qualitatively similar to the CITR case. However, the impossibility of lowering the nominal rate and letting the currency depreciate, as would be needed in order to support the expansion in consumption and output required in order to replicate the flexible price allocation, leads to a very limited response in the terms of trade, and in turn to an amplification of the responses of domestic inflation and output gap. Interestingly, under a PEG, the complete stabilization of the nominal exchange rate generates stationarity of the domestic price level and, in turn, also of the CPI level (given the stationarity in the terms of trade). This is a property that the PEG regime shares with the optimal policy as specified above. The stationarity in the price level also explains why, in response to the shock, domestic inflation initially falls and then rises persistently above the steady state.

As discussed below, the different dynamics of the terms of trade are unambiguously associated with a welfare loss, relative to the optimal policy.[23]

5.1.3. Second moments and welfare losses. In order to complement our quantitative analysis, Table 1 reports business cycle properties of several key variables under alternative monetary policy regimes. The numbers confirm some of the findings that were already evident from visual inspection of the impulse responses. Thus we see that the critical element that distinguishes each simple rule relative to the optimal policy is the excess smoothness of both the terms of trade and the (first differenced) nominal exchange rate.[24] This in turn is reflected in too high a volatility of the output gap and domestic inflation under the simple rules. In particular, the PEG regime is the one that amplifies both output gap and inflation volatility to the largest extent, with the CITR regime lying somewhere in between. Furthermore, notice that the terms of trade are more stable under an exchange rate peg than under any other policy regime. That finding, which is consistent with the evidence of Mussa (1986), points to the existence of "excess smoothness" in real exchange rates under fixed exchange rates. That feature is a consequence of

22. The implied pattern for the nominal rate is still consistent with the observed depreciation of the nominal exchange rate on impact. It is, in fact, the behaviour of current and expected future interest rate differentials that matters for the current nominal exchange rate, as can be easily seen by solving the uncovered interest parity condition forward.

23. We display our results on welfare later. Notice, however, that the cost of dampening exchange rate volatility (and therefore the relative ranking between DITR and CITR) may be a function of the lags with which exchange rate movements affect prices, *i.e.* of the degree of pass-through. Intuitively, the lower the degree of pass-through, the smaller (*ceteris paribus*) the cost of short-run relative price inertia, and therefore the more desirable to pursue a policy of CITR relative to DITR.

24. We report statistics for the nominal *depreciation* rate, as opposed to the level, given that both DITR and CITR imply a unit root in the nominal exchange rate.

TABLE 1

Cyclical properties of alternative policy regimes

	Optimal sd%	DI Taylor sd%	CPI Taylor sd%	Peg sd%
Output	0·95	0·68	0·72	0·86
Domestic inflation	0·00	0·27	0·27	0·36
CPI inflation	0·38	0·41	0·27	0·21
Nominal infl. rate	0·32	0·41	0·41	0·21
Terms of trade	1·60	1·53	1·43	1·17
Nominal depr. rate	0·95	0·86	0·53	0·00

Note: Sd denotes standard deviation in %.

TABLE 2

Contribution to welfare losses

	DI Taylor	CPI Taylor	Peg
Benchmark $\mu = 1{\cdot}2, \varphi = 3$			
Var(Domestic infl.)	0·0157	0·0151	0·0268
Var(Output gap)	0·0009	0·0019	0·0053
Total	0·0166	0·0170	0·0321
Low steady state mark-up $\mu = 1{\cdot}1, \varphi = 3$			
Var(Domestic infl.)	0·0287	0·0277	0·0491
Var(Output gap)	0·0009	0·0019	0·0053
Total	0·0297	0·0296	0·0544
Low elasticity of labour supply $\mu = 1{\cdot}2, \varphi = 10$			
Var(Domestic infl.)	0·0235	0·0240	0·0565
Var(Output gap)	0·0005	0·0020	0·0064
Total	0·0240	0·0261	0·0630
Low mark-up and elasticity of labour supply $\mu = 1{\cdot}1, \varphi = 10$			
Var(Domestic infl.)	0·0431	0·0441	0·1036
Var(Output gap)	0·0005	0·0020	0·0064
Total	0·0436	0·0461	0·1101

Note: Entries are percentage units of steady state consumption.

the inability of prices (which are sticky) to compensate for the constancy of the nominal exchange rate.[25]

Table 2 reports the welfare losses associated with the three simple rules analysed in the previous section: DITR, CITR, and PEG. There are four panels in this table. The top panel reports welfare losses in the case of our benchmark parameterization, while the remaining three panels display the effects of lowering, respectively, the steady state mark-up, the elasticity of labour supply and both. All entries are to be read as percentage units of steady state consumption, and in deviation from the first best represented by DIT. Under our baseline calibration all rules are suboptimal since they involve non-trivial deviations from full domestic price stability. Also one result stands out clearly: under all the calibrations considered an exchange rate peg implies a substantially larger deviation from the first best than DITR and CITR, as one may have anticipated from the quantitative evaluation of the second moments conducted above. However, and as is usually the case in welfare exercises of this sort found in the literature, the implied welfare losses are quantitatively small for all policy regimes.

We consider next the effect of lowering, respectively, the steady state mark-up to 1·1, by setting $\varepsilon = 11$ (which implies a larger penalization of inflation variability in the loss function)

25. See Monacelli (2004) for a detailed analysis of the implications of fixed exchange rates.

and the elasticity of labour supply to 0·1 (which implies a larger penalization of output gap variability). This has a general effect of generating a substantial magnification of the welfare losses relative to the benchmark case, especially in the third exercise where the two parameters are lowered simultaneously. In the case of low mark-up and low elasticity of labour supply, the PEG regime leads to non-trivial welfare losses relative to the optimum. Notice also that under all scenarios considered here the two stylized Taylor rules, DITR and CITR, imply very similar welfare losses. While this points to a substantial irrelevance in the specification of the inflation index in the monetary authority's interest rate rule, the same result may once again be sensitive to the assumption of complete exchange rate pass-through specified in our context.[26]

6. SUMMARY AND CONCLUDING REMARKS

The present paper has developed and analysed a tractable optimizing model of a small open economy with staggered price setting *à la* Calvo. We have shown that the equilibrium dynamics for that model economy have a canonical representation (in terms of domestic inflation and the output gap) analogous to that of its closed economy counterpart. More precisely, their representations differ only in two respects: (a) some coefficients of the equilibrium dynamical system for the small open economy depend on parameters that are specific to the latter (the degree of openness, and the substitutability across goods produced in different countries), and (b) the natural levels of output and interest rates in the small open economy are generally a function of both domestic and foreign disturbances. In particular, the closed economy is nested in the small open economy model, as a limiting case.

Under some special—but not implausible—assumptions (log utility and unit elasticity of substitution between bundles of goods produced in different countries) we have shown how a second order approximation to the utility of the small open economy's consumer can be derived, and the welfare level implied by alternative monetary policy rules can be evaluated. In that case, the implied loss function is analogous to the one applying to the corresponding closed economy, *i.e.* it penalizes fluctuations in domestic inflation and the output gap. In particular, under our assumptions, domestic inflation targeting emerges as the optimal policy regime.

We have then used our framework to analyse the properties of three alternative monetary regimes for the small open economy: (a) a domestic inflation-based Taylor rule, (b) a CPI-based Taylor rule, and (c) an exchange rate peg. Our analysis points to the presence of a trade-off between the stabilization of both the nominal exchange rate and the terms of trade, on the one hand, and the stabilization of domestic inflation and the output gap on the other. Hence a policy of domestic inflation targeting, which achieves a simultaneous stabilization of both domestic prices and the output gap, entails a substantially larger volatility of the nominal exchange rate and the terms of trade relative to the simple Taylor rules and/or an exchange rate peg. The converse is true for the latter regime. In general, a CPI-based Taylor rule delivers equilibrium dynamics that lie somewhere between a domestic inflation-based Taylor rule and a peg. Perhaps not surprisingly, an exchange rate peg generates substantially higher welfare losses than a Taylor rule, due to the excess smoothness of the terms of trade that it entails. In all our simulations, a Taylor rule in which the monetary authority reacts to domestic inflation is shown to deliver higher welfare than a similar rule based on the CPI index of inflation.

Our framework lends itself to several extensions. First, it is important to emphasize that, in our analysis, domestic price stability (along with fully flexible exchange rates) stands out

26. In the context of a different model, with both tradable and non-tradable goods and capital accumulation, Schmitt-Grohé and Uribe (2001) point out that the welfare ranking between domestic and CPI targeting may be sensitive to the specification of other distortions in the economy, for instance, the adoption of a transaction role for money.

as the welfare maximizing policy in the particular case of log utility and unitary elasticity of substitution (both between domestic and foreign goods in general and between different foreign countries' goods), coupled with an appropriate subsidy scheme that guarantees the optimality (from the viewpoint of the small open economy) of the flexible price equilibrium allocation. The derivation of the relevant welfare function for the small open economy in the case of more general preferences as well as that of uncorrected steady state distortions would allow a more thorough analysis and quantitative evaluation of the optimal monetary policy and should certainly be the object of future research.[27]

Second, a two-country version of the framework developed here would allow us to analyse a number of issues that cannot be addressed with the present model, including the importance of spillover effects in the design of optimal monetary policy, the potential benefits from monetary policy coordination, and the implications of exchange rate stabilization agreements. Recent work by Clarida *et al*. (2002), Benigno and Benigno (2003) and Pappa (2004) has already made some inroads on that front.

A further interesting extension would involve the introduction, along with sticky prices, of sticky nominal wages in the small open economy. As pointed out by Erceg, Henderson and Levine (2000), the simultaneous presence of the two forms of nominal rigidity introduces an additional trade-off that renders strict price inflation targeting policies suboptimal. It may be interesting to analyse how that trade-off would affect the ranking across monetary policy regimes obtained in the present paper.

Finally, it is worth noticing that our analysis features complete exchange rate pass-through of nominal exchange rate changes to prices of imported (or exported) goods. Some of the implications of less than complete pass-through associated with local currency pricing by exporters and importers have already been analysed by several authors in the context of two-country models with one-period advanced price-setting (see, *e.g.* Devereux and Engel (2002), Bacchetta and van Wincoop (2003), and Corsetti and Pesenti (2005)). It would be interesting to explore some of those implications (*e.g.* for the nature of the optimal monetary policy problem and the relative performance of alternative policy regimes) in the context of the simple small open economy with staggered price-setting proposed here.

APPENDIX A. THE PERFECT FORESIGHT STEADY STATE

In order to show how the home economy's terms of trade are uniquely pinned down in the perfect foresight steady state, we invoke symmetry among all countries (other than the home country), and then show how the terms of trade and output in the home economy are determined. Without loss of generality, we assume a unit value for productivity in all foreign countries, and a productivity level A in the home economy. We show that in the symmetric case (when $A = 1$) the terms of trade for the home economy must necessarily be equal to unity in the steady state, whereas output in the home economy coincides with that in the rest of the world.

First, notice that the goods market clearing condition, when evaluated at the steady state, implies

$$\begin{aligned}
Y &= (1-\alpha)\left(\frac{P_H}{P}\right)^{-\eta} C + \alpha \int_0^1 \left(\frac{P_H}{\mathcal{E}_i P_F^i}\right)^{-\gamma} \left(\frac{P_F^i}{P^i}\right)^{-\eta} C^i \, di \\
&= \left(\frac{P_H}{P}\right)^{-\eta} \left[(1-\alpha)\, C + \alpha \int_0^1 \left(\frac{\mathcal{E}_i P_F^i}{P_H}\right)^{\gamma-\eta} \mathcal{Q}_i^{\eta} C^i \, di\right] \\
&= h(\mathcal{S})^{\eta} C \left[(1-\alpha) + \alpha \int_0^1 \left(\mathcal{S}^i \mathcal{S}_i\right)^{\gamma-\eta} \mathcal{Q}_i^{\eta-\frac{1}{\sigma}} \, di\right] \\
&= h(\mathcal{S})^{\eta} C \left[(1-\alpha) + \alpha \mathcal{S}^{\gamma-\eta} q(\mathcal{S})^{\eta-\frac{1}{\sigma}}\right]
\end{aligned}$$

27. See Benigno and Woodford (2004) for recent developments in this direction.

where we have made use of (17) and of the relationship

$$\frac{P}{P_H} = \left[(1-\alpha) + \alpha \int_0^1 (\mathcal{S}_i)^{1-\eta}\, di\right]^{\frac{1}{1-\eta}}$$
$$= \left[(1-\alpha) + \alpha \ (\mathcal{S})^{1-\eta}\right]^{\frac{1}{1-\eta}} \equiv h(\mathcal{S})$$

and we have made the substitution $\mathcal{Q} = \frac{\mathcal{S}}{h(\mathcal{S})} \equiv q(\mathcal{S})$. Notice that $q(\mathcal{S})$ is strictly increasing in $\mathcal{S}$.

Under the assumptions above, the international risk sharing condition implies that the relationship

$$C = C^* \ \mathcal{Q}^{\frac{1}{\sigma}}$$
$$= C^* \ q(\mathcal{S})^{\frac{1}{\sigma}}$$

must also hold in the steady state.

Hence, combining the two relations above, and imposing the world market clearing condition $C^* = Y^*$ we obtain

$$Y = \left[(1-\alpha)\ h(\mathcal{S})^{\eta}\ q(\mathcal{S})^{\frac{1}{\sigma}} + \alpha\ \mathcal{S}^{\gamma-\eta} h(\mathcal{S})^{\eta} q(\mathcal{S})^{\eta}\right] Y^*$$
$$= \left[(1-\alpha)\ h(\mathcal{S})^{\eta}\ q(\mathcal{S})^{\frac{1}{\sigma}} + \alpha\ h(\mathcal{S})^{\gamma} q(\mathcal{S})^{\gamma}\right] Y^* \tag{A.1}$$
$$\equiv v(\mathcal{S})\ Y^* \tag{A.2}$$

where $v(\mathcal{S}) > 0$, $v'(\mathcal{S}) > 0$, and $v(1) = 1$.

Furthermore, the clearing of the labour market in the steady state implies

$$C^{\sigma}\left(\frac{Y}{A}\right)^{\varphi} = \frac{W}{P}$$
$$= A\frac{1-\frac{1}{\varepsilon}}{(1-\tau)}\ \frac{P_H}{P}$$
$$= A\frac{1-\frac{1}{\varepsilon}}{(1-\tau)}\ \frac{1}{h(\mathcal{S})}$$

which, when combined with the sharing condition above, yields

$$Y = A^{\frac{1+\varphi}{\varphi}} \left(\frac{1-\frac{1}{\varepsilon}}{(1-\tau)\ (Y^*)^{\sigma}\ \mathcal{S}}\right)^{\frac{1}{\varphi}}. \tag{A.3}$$

Notice that, conditional on A and Y^*, (A.2) and (A.3) constitute a system of two equations in Y and S, with a unique solution, given by

$$Y = Y^* = A^{\frac{1+\varphi}{\sigma+\varphi}} \left(\frac{1-\frac{1}{\varepsilon}}{1-\tau}\right)^{\frac{1}{\sigma+\varphi}}$$

and

$$\mathcal{S} = 1$$

which in turn must imply $S_i = 1$ for all i.

APPENDIX B. OPTIMAL PRICE-SETTING IN THE CALVO MODEL

Following Calvo (1983) we assume that each individual firm resets its price with probability $1-\theta$ each period, independently of the time elapsed since its last price adjustment. Thus, each period a measure $1-\theta$ of (randomly selected) firms reset their prices, while a fraction θ keep their prices unchanged. Let $\overline{P}_{H,t}(j)$ denote the price set by a firm j adjusting its price in period t. Under the Calvo price-setting structure, $P_{H,t+k}(j) = \overline{P}_{H,t}(j)$ with probability θ^k for $k = 0, 1, 2, \ldots$. Since all firms resetting prices in any given period will choose the same price, we henceforth drop the j subscript.

When setting a new price in period t firm j seeks to maximize the current value of its dividend stream, conditional on that price being effective:

$$\max_{\overline{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k \, E_t \left\{ Q_{t,t+k} \left[Y_{t+k} \, (\overline{P}_{H,t} - MC^n_{t+k}) \right] \right\}$$

subject to the sequence of demand constraints

$$Y_{t+k}(j) \le \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \left(C_{H,t+k} + \int_0^1 C^i_{H,t+k} di \right) \equiv Y^d_{t+k}(\overline{P}_{H,t})$$

where $MC^n_t = \frac{(1-\tau) W_t}{A_t}$ denotes the nominal marginal cost.

Thus, $\overline{P}_{H,t}$ must satisfy the first order condition

$$\sum_{k=0}^{\infty} \theta^k \, E_t \left\{ Q_{t,t+k} \, Y_{t+k} \left(\overline{P}_{H,t} - \frac{\varepsilon}{\varepsilon - 1} \, MC^n_{t+k} \right) \right\} = 0. \tag{B.1}$$

Using the fact that $Q_{t,t+k} = \beta^k \, (C_{t+k}/C_t)^{-\sigma} (P_t/P_{t+k})$, we can rewrite the previous condition as

$$\sum_{k=0}^{\infty} (\beta\theta)^k \, E_t \left\{ P^{-1}_{t+k} \, C^{-\sigma}_{t+k} \, Y_{t+k} \, \left(\overline{P}_{H,t} - \frac{\varepsilon}{\varepsilon - 1} \, MC^n_{t+k} \right) \right\} = 0$$

or, in terms of stationary variables,

$$\sum_{k=0}^{\infty} (\beta\theta)^k \, E_t \left\{ C^{-\sigma}_{t+k} \, Y_{t+k} \, \frac{P_{H,t-1}}{P_{t+k}} \left(\frac{\overline{P}_{H,t}}{P_{H,t-1}} - \frac{\varepsilon}{\varepsilon - 1} \, \Pi^H_{t-1,t+k} \, MC_{t+k} \right) \right\} = 0$$

where $\Pi^H_{t-1,t+k} \equiv \frac{P_{H,t+k}}{P_{H,t-1}}$, and $MC_{t+k} = \frac{MC^n_{t+k}}{P_{H,t+k}}$. Log-linearizing the previous condition around the zero-inflation steady state with balanced trade we obtain

$$\overline{p}_{H,t} = p_{H,t-1} + \sum_{k=0}^{\infty} (\beta\theta)^k \, E_t\{\pi_{H,t+k}\} + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \, E_t\{\widehat{mc}_{t+k}\}$$

where $\widehat{mc}_t \equiv mc_t - mc$ is the (log) deviation of real marginal cost from its steady state value $mc = -\log \frac{\varepsilon}{\varepsilon - 1} \equiv -\mu$.

Notice, that we can rewrite the previous expression in more compact form as

$$\overline{p}_{H,t} - p_{H,t-1} = \beta\theta \, E_t\{\overline{p}_{H,t+1} - p_{H,t}\} + \pi_{H,t} + (1 - \beta\theta) \, \widehat{mc}_t. \tag{B.2}$$

Alternatively, using the relationship $\widehat{mc}_t = mc^n_t - p_{H,t} + \mu$ to substitute for $\widehat{mc}_t$ in (B.2), and after some straightforward algebra, we obtain a version of the price-setting rule in terms of expected nominal marginal costs:

$$\overline{p}_{H,t} = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \, E_t\{mc^n_{t+k}\}$$

which corresponds to expression (23) in the text.

Under the assumed price-setting structure, the dynamics of the domestic price index are described by the equation

$$P_{H,t} \equiv \left[\theta \, P_{H,t-1}{}^{1-\varepsilon} + (1 - \theta) \, (\overline{P}_{H,t})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{B.3}$$

which can be log-linearized around the zero-inflation steady state to yield

$$\pi_{H,t} = (1 - \theta) \, (\overline{p}_{H,t} - p_{H,t-1}).$$

Finally, we can combine the previous expression with (B.2) above to yield, after some algebra,

$$\pi_{H,t} = \beta \, E_t\{\pi_{H,t+1}\} + \lambda \, \widehat{mc}_t$$

where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$, and which corresponds to (32) in the text.

APPENDIX C. OPTIMAL POLICY IMPLEMENTATION

After setting $r_t = \overline{rr}_t$ and plugging into (37), the equilibrium conditions can be summarized by means of the difference equation

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = \mathbf{A_O} \begin{bmatrix} E_t\{x_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} \tag{C.1}$$

where

$$\mathbf{A_O} \equiv \begin{bmatrix} 1 & \sigma^{-1} \\ \kappa & \beta + \kappa\sigma^{-1} \end{bmatrix}.$$

Clearly, $x_t = \pi_t = 0$, all t, constitutes a solution to (C.1). Yet, as shown in Blanchard and Khan (1980), a necessary and sufficient condition for the uniqueness of such a solution in a system with no predetermined variables like (C.1) is that the two eigenvalues of $\mathbf{A_O}$ lie inside the unique circle. It is easy to check, however, that such a condition is not satisfied in our case. More precisely, while both eigenvalues of $\mathbf{A_O}$ can be shown to be real and positive, the largest is always greater than one. As a result there exists a continuum of solutions in a neighbourhood of $(0, 0)$ that satisfy the equilibrium conditions (local indeterminacy) and one cannot rule out the possibility of equilibria displaying fluctuations driven by self-fulfilling revisions in expectations (stationary sunspot fluctuations).

That indeterminacy problem can be avoided, and the uniqueness of the equilibrium allocation restored, by having the central bank follow a rule which would imply that the interest rate should respond to inflation and/or the output gap were those variables to deviate from their (zero) target values. More precisely, suppose that the central bank commits itself to following the rule

$$r_t = \overline{rr}_t + \phi_\pi \, \pi_t + \phi_x \, x_t. \tag{C.2}$$

In that case, the equilibrium is described by a stochastic difference equation like (C.1), with

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_x) \end{bmatrix}$$

where $\Omega \equiv \frac{1}{\sigma + \phi_x + \kappa\phi_\pi}$. If we restrict ourselves to non-negative values of ϕ_π and ϕ_x, a necessary and sufficient condition for $\mathbf{A}_T$ to have both eigenvalues inside the unit circle (thus implying that $(0, 0)$ is the unique non-explosive solution to (C.1)) is given by[28]

$$\kappa \, (\phi_\pi - 1) + (1 - \beta) \, \phi_x > 0. \tag{C.3}$$

Notice that, once uniqueness is restored, the term $\phi_\pi \, \pi_t + \phi_x \, x_t$ appended to the interest rate rule vanishes, implying that $r_t = \overline{rr}_t$ all t. Intuitively, stabilization of the output gap and inflation requires a credible threat by the central bank to vary the interest rate sufficiently in response to any deviations of inflation and/or the output gap from target; yet the very existence of that threat makes its effective application unnecessary.

APPENDIX D. DERIVATION OF THE WELFARE LOSS FUNCTION IN A SPECIAL CASE

In the present Appendix we derive a second order approximation of representative consumer's utility about the flexible price equilibrium allocation. As discussed in the main text, we eventually restrict our analysis to the special case of $\sigma = \eta = \gamma = 1$. Below we make frequent use of the following second order approximation of per cent deviations in terms of log deviations:

$$\frac{Y_t - Y}{Y} = y_t + \frac{1}{2} \, y_t^2 + o(\|a\|^3)$$

where $o(\|a\|^n)$ represents terms that are of order higher than n-th, in the bound $\|a\|$ on the amplitude of the relevant shocks.

Notice that we can write the utility of consumption as

$$\begin{aligned} \log C_t &= \overline{c}_t + \widetilde{c}_t \\ &= \overline{c}_t + (1 - \alpha) \, x_t \end{aligned}$$

where in deriving the second equality we have made use of the fact that, under our assumptions, the exact relationships (18) and (29) can be combined to yield $c_t = (1 - \alpha) y_t + \alpha y_t^*$.

We can also approximate the disutility of labour about its level in the flexible price equilibrium, *i.e.*

$$\frac{N_t^{1+\varphi}}{1+\varphi} = \frac{\overline{N}_t^{1+\varphi}}{1+\varphi} + \overline{N}_t^{1+\varphi} \left[\widetilde{n}_t + \frac{1}{2}(1+\varphi) \, \widetilde{n}_t^2 \right] + o(\|a\|^3).$$

The next step consists in rewriting the previous expression in terms of the output gap. Using the fact that $N_t = \left(\frac{Y_t}{A_t}\right) \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} di$, we have

$$\widetilde{n}_t = x_t + z_t$$

28. See, *e.g.* Bullard and Mitra (2002).

where $z_t \equiv \log \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} di$. The following lemma shows that z_t is proportional to the cross-sectional distribution of relative prices (and, hence, of second order).

Lemma 1. $z_t = \frac{\varepsilon}{2} \text{var}_i\{p_{H,t}(i)\} + o(\|a\|^3)$.

Proof. Let $\widehat{p}_{H,t}(i) \equiv p_{H,t}(i) - p_{H,t}$. Notice that

$$\begin{aligned}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{1-\varepsilon} &= \exp\left[(1-\varepsilon)\ \widehat{p}_{H,t}(i)\right] \\ &= 1 + (1-\varepsilon)\ \widehat{p}_{H,t}(i) + \frac{(1-\varepsilon)^2}{2}\ \widehat{p}_{H,t}(i)^2 + o(\|a\|^3).\end{aligned}$$

Furthermore, from the definition of $P_{H,t}$, we have $1 = \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{1-\varepsilon} di$. Hence, it follows that

$$E_i\{\widehat{p}_{H,t}(i)\} = \frac{(\varepsilon-1)}{2}\ E_i\{\widehat{p}_{H,t}(i)^2\}.$$

In addition, a second order approximation to $\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon}$, yields

$$\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} = 1 - \varepsilon\ \widehat{p}_{H,t}(i) + \frac{\varepsilon^2}{2}\ \widehat{p}_{H,t}(i)^2 + o(\|a\|^3).$$

Combining the two previous results, it follows that

$$\begin{aligned}\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} di &= 1 + \frac{\varepsilon}{2}\ E_i\{\widehat{p}_{H,t}(i)^2\} \\ &= 1 + \frac{\varepsilon}{2}\ \text{var}_i\{p_{H,t}(i)\}\end{aligned}$$

from which it follows that $z_t = \frac{\varepsilon}{2}\ \text{var}_i\{p_{H,t}(i)\} + o(\|a\|^3)$.

We can thus rewrite the second order approximation to the disutility of labour as

$$\frac{N_t^{1+\varphi}}{1+\varphi} = \frac{\overline{N}_t^{1+\varphi}}{1+\varphi} + \overline{N}_t^{1+\varphi} \left[x_t + z_t + \frac{1}{2}(1+\varphi)\ x_t^2\right] + o(\|a\|^3).$$

Under the optimal subsidy scheme assumed, the optimality condition $\overline{N}_t^{1+\varphi} = (1-\alpha)$ holds for all t, allowing us to rewrite the period utility as

$$U(C_t, N_t) = -(1-\alpha)\ \left[z_t + \frac{1}{2}(1+\varphi)\ x_t^2\right] + \text{t.i.p.} + o(\|a\|^3)$$

where t.i.p. denotes terms independent of policy.

Lemma 2. $\sum_{t=0}^{\infty} \beta^t\ \text{var}_i\{p_{H,t}(i)\} = \frac{1}{\lambda} \sum_{t=0}^{\infty} \beta^t\ \pi_{H,t}^2$, *where* $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$.

Proof. Woodford (2003, Chapter 6).

Collecting the previous results, we can write the second order approximation to the small open economy's consumer's utility function as follows:

$$\mathbb{W} \equiv -\ \frac{(1-\alpha)}{2} \sum_{t=0}^{\infty} \beta^t\ \left[\frac{\varepsilon}{\lambda}\ \pi_{H,t}^2 + (1+\varphi)\ x_t^2\right] + \text{t.i.p.} + o(\|a\|^3)$$

which is equation (40) in the text.

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