The currents J_{RB}, J_{pE}, and J_R are B–E junction currents only and do not contribute to the collector current. The currents J_{pc0} and J_G are B–C junction currents only. These current components do not contribute to the transistor action or the current gain.

The dc common-base current gain is defined as

$$\alpha_0 = \frac{I_C}{I_E} \tag{12.27}$$

If we assume that the active cross-sectional area is the same for the collector and emitter, then we can write the current gain in terms of the current densities, or

$$\alpha_0 = \frac{J_C}{J_E} = \frac{J_{nC} + J_G + J_{pc0}}{J_{nE} + J_R + J_{pE}} \tag{12.28}$$

We are primarily interested in determining how the collector current will change with a change in emitter current. The small-signal, or sinusoidal, common-base current gain is defined as

$$\alpha = \frac{\partial J_C}{\partial J_E} = \frac{J_{nC}}{J_{nE} + J_R + J_{pE}} \tag{12.29}$$

The reverse-biased B–C currents, J_G and J_{pc0}, are not functions of the emitter current.

We can rewrite Equation (12.29) in the form

$$\alpha = \left(\frac{J_{nE}}{J_{nE} + J_{pE}}\right)\left(\frac{J_{nC}}{J_{nE}}\right)\left(\frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}}\right) \tag{12.30a}$$

or

$$\alpha = \gamma \alpha_T \delta \tag{12.30b}$$

The factors in Equation (12.30b) are defined as:

$$\gamma = \left(\frac{J_{nE}}{J_{nE} + J_{pE}}\right) \equiv \text{emitter injection efficiency factor} \tag{12.31a}$$

$$\alpha_T = \left(\frac{J_{nC}}{J_{nE}}\right) \equiv \text{base transport factor} \tag{12.31b}$$

$$\delta = \frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}} \equiv \text{recombination factor} \tag{12.31c}$$

We would like to have the change in collector current be exactly the same as the change in emitter current or, ideally, to have $\alpha = 1$. However, a consideration of Equation (12.29) shows that α will always be less than unity. The goal is to make α as close to unity as possible. To achieve this goal, we must make each term in Equation (12.30b) as close to unity as possible, since each factor is less than unity.

The *emitter injection efficiency factor* γ takes into account the minority carrier hole diffusion current in the emitter. This current is part of the emitter current, but does not contribute to the transistor action in that J_{pE} is not part of the collector current. The *base transport factor* α_T takes into account any recombination of excess minority carrier electrons in the base. Ideally, we want no recombination in the base. The *recombination factor* δ takes into account the recombination in the forward-biased B–E junction. The current J_R contributes to the emitter current, but does not contribute to collector current.

12.3.2 Derivation of Transistor Current Components and Current Gain Factors

We now wish to determine the various transistor current components and each of the gain factors in terms of the electrical and geometrical parameters of the transistor. The results of these derivations show how the various parameters in the transistor influence the electrical properties of the device and point the way to the design of a "good" bipolar transistor.

Emitter Injection Efficiency Factor Consider, initially, the emitter injection efficiency factor. We have from Equation (12.31a)

$$\gamma = \left(\frac{J_{nE}}{J_{nE} + J_{pE}}\right) = \frac{1}{\left(1 + \frac{J_{pE}}{J_{nE}}\right)} \tag{12.32}$$

We derived the minority carrier distribution functions for the forward-active mode in Section 12.2.1. Noting that J_{nE}, as defined in Figure 12.19, is in the negative x direction, we can write the current densities as

$$J_{pE} = -eD_E \left.\frac{d[\delta p_E(x')]}{dx'}\right|_{x'=0} \tag{12.33a}$$

and

$$J_{nE} = (-)eD_B \left.\frac{d[\delta n_B(x)]}{dx}\right|_{x=0} \tag{12.33b}$$

where $\delta p_E(x')$ and $\delta n_B(x)$ are given by Equations (12.21) and (12.15), respectively.

Taking the appropriate derivatives of $\delta p_E(x')$ and $\delta n_B(x)$, we obtain

$$J_{pE} = \frac{eD_E p_{E0}}{L_E}\left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1\right] \cdot \frac{1}{\tanh\,(x_E/L_E)} \tag{12.34a}$$

and

$$J_{nE} = \frac{eD_B n_{B0}}{L_B}\left\{\frac{1}{\sinh\,(x_B/L_B)} + \frac{[\exp\,(eV_{BE}/kT) - 1]}{\tanh\,(x_B/L_B)}\right\} \tag{12.34b}$$

Positive J_{pE} and J_{nE} values imply that the currents are in the directions shown in Figure 12.19. If we assume that the B–E junction is biased sufficiently far in the forward bias so that $V_{BE} \gg kT/e$, then

$$\exp\left(\frac{eV_{BE}}{kT}\right) \gg 1$$

and also

$$\frac{\exp\,(eV_{BE}/kT)}{\tanh\,(x_B/L_B)} \gg \frac{1}{\sinh\,(x_B/L_B)}$$

The emitter injection efficiency, from Equation (12.32), then becomes

$$\boxed{\gamma = \frac{1}{1 + \dfrac{p_{E0} D_E L_B}{n_{B0} D_B L_E} \cdot \dfrac{\tanh(x_B/L_B)}{\tanh(x_E/L_E)}}} \tag{12.35a}$$

If we assume that all the parameters in Equation (12.35a) except p_{E0} and n_{B0} are fixed, then in order for $\gamma \approx 1$, we must have $p_{E0} \ll n_{B0}$. We can write

$$p_{E0} = \frac{n_i^2}{N_E} \qquad \text{and} \qquad n_{B0} = \frac{n_i^2}{N_B}$$

where N_E and N_B are the impurity doping concentrations in the emitter and base, respectively. Then the condition that $p_{E0} \ll n_{B0}$ implies that $N_E \gg N_B$. For the emitter injection efficiency to be close to unity, the emitter doping must be large compared to the base doping. This condition means that many more electrons from the n-type emitter than holes from the p-type base will be injected across the B–E space charge region. If both $x_B \ll L_B$ and $x_E \ll L_E$, then the emitter injection efficiency can be written as

$$\boxed{\gamma \approx \frac{1}{1 + \dfrac{N_B}{N_E} \cdot \dfrac{D_E}{D_B} \cdot \dfrac{x_B}{x_E}}} \tag{12.35b}$$

EXAMPLE 12.1

Objective: Calculate the emitter injection efficiency.

Assume the following transistor parameters: $N_B = 10^{15}$ cm^{-3}, $N_E = 10^{17}$ cm^{-3}, $D_E = 10$ cm^2/s, $D_B = 20$ cm^2/s, $x_B = 0.80$ μm, and $x_E = 0.60$ μm.

■ Solution

From Equation (12.35b), we find

$$\gamma \cong \frac{1}{1 + \left(\dfrac{N_B}{N_E}\right)\left(\dfrac{D_E}{D_B}\right)\left(\dfrac{x_B}{x_E}\right)} = \frac{1}{1 + \left(\dfrac{10^{15}}{10^{17}}\right)\left(\dfrac{10}{20}\right)\left(\dfrac{0.80}{0.60}\right)} = 0.9934$$

■ Comment

This simple example shows a typical magnitude of the emitter injection efficiency.

■ EXERCISE PROBLEM

Ex 12.1 Repeat Example 12.1 if the base and emitter doping concentrations are $N_B = 5 \times 10^{15}$ cm^{-3} and $N_E = 10^{18}$ cm^{-3}, respectively.
(Ans. $\gamma = 0.9967$)

Base Transport Factor The next term to consider is the base transport factor, given by Equation (12.31b) as $\alpha_T = J_{nC}/J_{nE}$. From the definitions of the current directions shown in Figure 12.19, we can write

$$J_{nC} = (-)eD_B \left.\frac{d[\delta n_B(x)]}{dx}\right|_{x=x_B} \tag{12.36a}$$

and

$$J_{nE} = (-)eD_B \left. \frac{d[\delta n_B(x)]}{dx} \right|_{x=0} \tag{12.36b}$$

Using the expression for $\delta n_B(x)$ given in Equation (12.15), we find that

$$J_{nC} = \frac{eD_B n_{B0}}{L_B} \left\{ \frac{[\exp(eV_{BE}/kT) - 1]}{\sinh(x_B/L_B)} + \frac{1}{\tanh(x_B/L_B)} \right\} \tag{12.37}$$

The expression for J_{nE} is given in Equation (12.34a).

If we again assume that the B–E junction is biased sufficiently far in the forward bias so that $V_{BE} \gg kT/e$, then $\exp(eV_{BE}/kT) \gg 1$. Substituting Equations (12.37) and (12.34b) into Equation (12.31b), we have

$$\alpha_T = \frac{J_{nC}}{J_{nE}} \approx \frac{\exp(eV_{BE}/kT) + \cosh(x_B/L_B)}{1 + \exp(eV_{BE}/kT)\cosh(x_B/L_B)} \tag{12.38}$$

In order for α_T to be close to unity, the neutral base width x_B must be much smaller than the minority carrier diffusion length in the base L_B. If $x_B \ll L_B$, then $\cosh(x_B/L_B)$ will be just slightly greater than unity. In addition, if $\exp(eV_{BE}/kT) \gg 1$, then the base transport factor is approximately

$$\boxed{\alpha_T \approx \frac{1}{\cosh(x_B/L_B)}} \tag{12.39a}$$

For $x_B \ll L_B$, we may expand the cosh function in a Taylor series, so that

$$\boxed{\alpha_T \approx \frac{1}{\cosh(x_B/L_B)} \approx \frac{1}{1 + \frac{1}{2}(x_B/L_B)^2} \approx 1 - \frac{1}{2}(x_B/L_B)^2} \tag{12.39b}$$

The base transport factor α_T will be close to one if $x_B \ll L_B$. We can now see why we indicated earlier that the neutral base width x_B would be less than L_B.

EXAMPLE 12.2

Objective: Calculate the base transport factor.

Assume transistor parameters of $x_B = 0.80\ \mu$m and $L_B = 10.0\ \mu$m.

■ Solution

From Equation (12.39a), we find

$$\alpha_T \cong \frac{1}{\cosh\left(\frac{x_B}{L_B}\right)} = \frac{1}{\cosh\left(\frac{0.80}{10.0}\right)} = 0.9968$$

■ Comment

This simple example shows a typical magnitude of the base transport factor.

■ EXERCISE PROBLEM

Ex 12.2 Repeat Example 12.2 for $x_B = 1.2\ \mu$m and $L_B = 10.0\ \mu$m. (Ans. $\alpha_T = 0.9928$)

Recombination Factor The recombination factor is given by Equation (12.31c). We can write

$$\delta = \frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}} \approx \frac{J_{nE}}{J_{nE} + J_R} = \frac{1}{1 + J_R/J_{nE}} \tag{12.40}$$

We have assumed in Equation (12.40) that $J_{pE} \ll J_{nE}$. The recombination current density, due to the recombination in a forward-biased pn junction, was discussed in Chapter 8 and can be written as

$$J_R = \frac{e x_{BE} n_i}{2\tau_0} \exp\left(\frac{eV_{BE}}{2kT}\right) = J_{r0} \exp\left(\frac{eV_{BE}}{2kT}\right) \tag{12.41}$$

where x_{BE} is the B–E space charge width.

The current J_{nE} from Equation (12.34b) can be approximated as

$$J_{nE} = J_{s0} \exp\left(\frac{eV_{BE}}{kT}\right) \tag{12.42}$$

where

$$J_{s0} = \frac{eD_B n_{B0}}{L_B \tanh\,(x_B/L_B)} \tag{12.43}$$

The recombination factor, from Equation (12.40), can then be written as

$$\boxed{\delta = \frac{1}{1 + \dfrac{J_{r0}}{J_{s0}} \exp\left(\dfrac{-eV_{BE}}{2kT}\right)}} \tag{12.44}$$

The recombination factor is a function of the B–E voltage. As V_{BE} increases, the recombination current becomes less dominant and the recombination factor approaches unity.

EXAMPLE 12.3

Objective: Calculate the recombination factor.

Assume the following transistor parameters: $x_{BE} = 0.10\ \mu\text{m}$, $\tau_o = 10^{-7}$ s, $N_B = 5 \times 10^{15}\ \text{cm}^{-3}$, $D_B = 20\ \text{cm}^2/\text{s}$, $L_B = 10\ \mu\text{m}$, and $x_B = 0.80\ \mu\text{m}$. Assume $V_{BE} = 0.50$ V.

■ Solution

From Equation (12.41), we find

$$J_{r0} = \frac{e x_{BE} n_i}{2\tau_o} = \frac{(1.6 \times 10^{-19})(0.10 \times 10^{-4})(1.5 \times 10^{10})}{2(10^{-7})} = 1.2 \times 10^{-7}\ \text{A/cm}^2$$

and from Equation (12.43), we find

$$J_{s0} = \frac{eD_B n_{B0}}{L_B \tanh\,(x_B/L_B)} = \frac{eD_B (n_i^2/N_B)}{L_B \tanh\,(x_B/L_B)}$$

$$= \frac{(1.6 \times 10^{-19})(20)[(1.5 \times 10^{10})^2/5 \times 10^{15}]}{(10 \times 10^{-4}) \tanh\,(0.80/10.0)} = 1.804 \times 10^{-9}\ \text{A/cm}^2$$

Then from Equation (12.44), the recombination factor is found as

$$\delta = \frac{1}{1 + \frac{J_{r0}}{J_{s0}} \cdot \exp\left(\frac{-V_{BE}}{2V_t}\right)} = \frac{1}{1 + \left(\frac{1.2 \times 10^{-7}}{1.804 \times 10^{-9}}\right) \cdot \exp\left(\frac{-0.50}{2(0.0259)}\right)}$$
$$= 0.99574$$

■ Comment

This simple example shows a typical magnitude of the recombination factor.

■ EXERCISE PROBLEM

Ex 12.3 Repeat Example 12.3 for $V_{BE} = 0.65$ V. (Ans. $\delta = 0.99976$)

The recombination factor must also include surface effects. The surface effects can be described by the surface recombination velocity as we discussed in Chapter 6. Figure 12.20a shows the B–E junction of an npn transistor near the semiconductor surface. We assume that the B–E junction is forward biased. Figure 12.20b shows the excess minority carrier electron concentration in the base along the cross section A-A′. This curve is the usual forward-biased junction minority carrier concentration. Figure 12.20c shows the excess minority carrier electron concentration along the cross section C-C′ from the surface. We have showed earlier that the excess concentration at a surface is smaller than the excess concentration in the bulk material. With this electron distribution, there is a diffusion of electrons from the bulk toward the surface where the electrons recombine with the majority carrier holes. Figure 12.20d shows the injection of electrons from the emitter into the base and the diffusion of

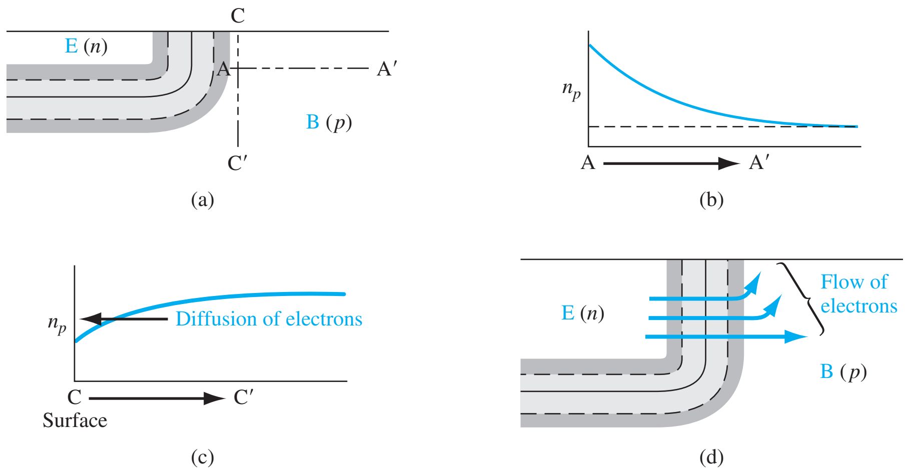

Figure 12.20 | The surface at the E–B junction showing the diffusion of carriers toward the surface.

electrons toward the surface. This diffusion generates another component of recombination current and this component of recombination current must be included in the recombination factor δ. Although the actual calculation is difficult because of the two-dimensional analysis required, the form of the recombination current is the same as that of Equation (12.41).

12.3.3 Summary

Although we have considered an npn transistor in all of the derivations, exactly the same analysis applies to a pnp transistor; the same minority carrier distributions are obtained except that the electron concentrations become hole concentrations and vice versa. The current directions and voltage polarities also change.

We have been considering the common-base current gain, defined in Equation (12.27) as $\alpha_0 = I_C/I_E$. The common-emitter current gain is defined as $\beta_0 = I_C/I_B$. From Figure 12.8 we see that $I_E = I_B + I_C$. We can determine the relation between common-emitter and common-base current gains from the KCL equation. We can write

$$\frac{I_E}{I_C} = \frac{I_B}{I_C} + 1$$

Substituting the definitions of current gains, we have

$$\frac{1}{\alpha_0} = \frac{1}{\beta_0} + 1$$

Since this relation actually holds for both dc and small-signal conditions, we can drop the subscript. The common-emitter current gain can now be written in terms of the common-base current gain as

$$\beta = \frac{\alpha}{1 - \alpha}$$

The common-base current gain, in terms of the common-emitter current gain, is found to be

$$\alpha = \frac{\beta}{1 + \beta}$$

Table 12.3 summarizes the expressions for the limiting factors in the common-base current gain assuming that $x_B \ll L_B$ and $x_E \ll L_E$. Also given are the approximate expressions for the common-base current gain and the common-emitter current gain.

12.3.4 Example Calculations of the Gain Factors

If we assume a typical value of β to be 100, then $\alpha = 0.99$. If we also assume that $\gamma = \alpha_T = \delta$, then each factor would have to be equal to 0.9967 in order that $\beta = 100$. This calculation gives an indication of how close to unity each factor must be in order to achieve a reasonable current gain.

Table 12.3 | Summary of limiting factors

Emitter injection efficiency

$$\gamma \approx \frac{1}{1 + \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}} \qquad (x_B \ll L_B),\ (x_E \ll L_E)$$

Base transport factor

$$\alpha_T \approx \frac{1}{1 + \frac{1}{2}\left(\frac{x_B}{L_B}\right)^2} \qquad (x_B \ll L_B)$$

Recombination factor

$$\delta = \frac{1}{1 + \frac{J_{r0}}{J_{s0}} \exp\left(\frac{-eV_{BE}}{2kT}\right)}$$

Common-base current gain

$$\alpha = \gamma\alpha_T\delta \approx \frac{1}{1 + \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E} + \frac{1}{2}\left(\frac{x_B}{L_B}\right)^2 + \frac{J_{r0}}{J_{s0}} \exp\left(\frac{-eV_{BE}}{2kT}\right)}$$

Common-emitter current gain

$$\beta = \frac{\alpha}{1-\alpha} \approx \frac{1}{\frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E} + \frac{1}{2}\left(\frac{x_B}{L_B}\right)^2 + \frac{J_{r0}}{J_{s0}} \exp\left(\frac{-eV_{BE}}{2kT}\right)}$$

DESIGN EXAMPLE 12.4

Objective: Design the ratio of emitter doping to base doping in order to achieve an emitter injection efficiency factor of $\gamma = 0.9967$.

Consider an npn bipolar transistor. Assume, for simplicity, that $D_E = D_B$, $L_E = L_B$, and $x_E = x_B$.

■ Solution

Equation (12.35b) reduces to

$$\gamma = \frac{1}{1 + \frac{p_{E0}}{n_{B0}}} = \frac{1}{1 + \frac{n_i^2/N_E}{n_i^2/N_B}}$$

so

$$\gamma = \frac{1}{1 + \frac{N_B}{N_E}} = 0.9967$$

Then

$$\frac{N_B}{N_E} = 0.00331 \qquad \text{or} \qquad \frac{N_E}{N_B} = 302$$

■ Comment

The emitter doping concentration must be much larger than the base doping concentration to achieve a high emitter injection efficiency.

■ EXERCISE PROBLEM

Ex 12.4 Assume that transistor parameters are the same as described in Example 12.4. In addition, let $N_E = 6 \times 10^{18}$ cm^{-3}. Determine the base doping concentration such that the emitter injection efficiency is $\gamma = 0.9950$. (Ans. $N_B = 3.02 \times 10^{16}$ cm^{-3})

DESIGN EXAMPLE 12.5

Objective: Design the base width required to achieve a base transport factor of $\alpha_T = 0.9967$.

Consider a pnp bipolar transistor. Assume that $D_B = 10$ cm^2/s and $\tau_{B0} = 10^{-7}$s.

■ Solution

The base transport factor applies to both pnp and npn transistors and is given by

$$\alpha_T = \frac{1}{\cosh(x_B/L_B)} = 0.9967$$

Then

$$x_B/L_B = 0.0814$$

We have

$$L_B = \sqrt{D_B \tau_{B0}} = \sqrt{(10)(10^{-7})} = 10^{-3} \text{ cm}$$

so that the base width must then be

$$x_B = 0.814 \times 10^{-4} \text{ cm} = 0.814\ \mu\text{m}$$

■ Comment

If the base width is less than approximately 0.8 μm, then the required base transport factor will be achieved. In most cases, the base transport factor will not be the limiting factor in the bipolar transistor current gain.

■ EXERCISE PROBLEM

Ex 12.5 Assume that transistor parameters are the same as described in Example 12.5. Determine the minimum base width x_B such that the base transport factor is $\alpha_T = 0.9980$. (Ans. $x_B = 0.633\ \mu$m)

DESIGN EXAMPLE 12.6

Objective: Determine the forward-biased B–E voltage required to achieve a recombination factor equal to $\delta = 0.9967$.

Consider an npn bipolar transistor at $T = 300$ K. Assume that $J_{r0} = 10^{-8}$ A/cm^2 and that $J_{s0} = 10^{-11}$ A/cm^2.

■ Solution

The recombination factor, from Equation (12.44), is

$$\delta = \frac{1}{1 + \dfrac{J_{r0}}{J_{s0}} \exp\left(\dfrac{-eV_{BE}}{2kT}\right)}$$

We then have

$$0.9967 = \frac{1}{1 + \frac{10^{-8}}{10^{-11}} \exp\left(\frac{-eV_{BE}}{2kT}\right)}$$

We can rearrange this equation and write

$$\exp\left(\frac{+eV_{BE}}{2kT}\right) = \frac{0.9967 \times 10^3}{1 - 0.9967} = 3.02 \times 10^5$$

Then

$$V_{BE} = 2(0.0259) \ln (3.02 \times 10^5) = 0.654 \text{ V}$$

■ Comment

This example demonstrates that the recombination factor may be an important limiting factor in the bipolar current gain. In this example, if V_{BE} is smaller than 0.654 V, then the recombination factor δ will fall below the desired 0.9967 value.

■ EXERCISE PROBLEM

Ex 12.6 If $J_{r0} = 10^{-8}$ A/cm^2 and $J_{s0} = 10^{-11}$ A/cm^2, determine the value of V_{BE} such that $\delta = 0.9950$. (Ans. $V_{BE} = 0.6320$ V)

EXAMPLE 12.7

Objective: Calculate the common-emitter current gain of a silicon npn bipolar transistor at $T = 300$ K given a set of parameters.

Assume the following parameters:

$D_E = 10$ cm^2/s	$x_B = 0.70\ \mu$m
$D_B = 25$ cm^2/s	$x_E = 0.50\ \mu$m
$\tau_{E0} = 1 \times 10^{-7}$ s	$N_E = 1 \times 10^{18}$ cm^{-3}
$\tau_{B0} = 5 \times 10^{-7}$ s	$N_B = 1 \times 10^{16}$ cm^{-3}
$J_{r0} = 5 \times 10^{-8}$ A/cm^2	$V_{BE} = 0.65$ V

The following parameters are calculated:

$$p_{E0} = \frac{(1.5 \times 10^{10})^2}{1 \times 10^{18}} = 2.25 \times 10^2 \text{ cm}^{-3}$$

$$n_{B0} = \frac{(1.5 \times 10^{10})^2}{1 \times 10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

$$L_E = \sqrt{D_E \tau_{E0}} = 10^{-3} \text{ cm}$$

$$L_B = \sqrt{D_B \tau_{B0}} = 3.54 \times 10^{-3} \text{ cm}$$

■ Solution

The emitter injection efficiency factor, from Equation (12.35a), is

$$\gamma = \frac{1}{1 + \frac{(2.25 \times 10^2)(10)(3.54 \times 10^{-3})}{(2.25 \times 10^4)(25)(10^{-3})} \cdot \frac{\tanh (0.0198)}{\tanh (0.050)}} = 0.9944$$

The base transport factor, from Equation (12.39a), is

$$\alpha_T = \frac{1}{\cosh\left(\dfrac{0.70 \times 10^{-4}}{3.54 \times 10^{-3}}\right)} = 0.9998$$

The recombination factor, from Equation (12.44), is

$$\delta = \frac{1}{1 + \dfrac{5 \times 10^{-8}}{J_{s0}} \exp\left(\dfrac{-0.65}{2(0.0259)}\right)}$$

where

$$J_{s0} = \frac{eD_B n_{B0}}{L_B \tanh\left(\dfrac{x_B}{L_B}\right)} = \frac{(1.6 \times 10^{-19})(25)(2.25 \times 10^4)}{3.54 \times 10^{-3} \tanh\,(1.977 \times 10^{-2})} = 1.29 \times 10^{-9}\ \text{A/cm}^2$$

We can now calculate $\delta = 0.99986$. The common-base current gain is then

$$\alpha = \gamma \alpha_T \delta = (0.9944)(0.9998)(0.99986) = 0.99406$$

which gives a common-emitter current gain of

$$\beta = \frac{\alpha}{1 - a} = \frac{0.99406}{1 - 0.99406} = 167$$

■ Comment

In this example, the emitter injection efficiency is the limiting factor in the current gain.

■ EXERCISE PROBLEM

Ex 12.7 Assume that $\gamma = \alpha_T = 0.9980$, $J_{r0} = 5 \times 10^{-9}$ A/cm^2, and $J_{s0} = 2 \times 10^{-11}$ A/cm^2. Determine the common-emitter current gain β for (*a*) $V_{BE} = 0.550$ V and (*b*) $V_{BE} = 0.650$ V. [Ans. (*a*) $\beta = 98.5$; (*b*) $\beta = 204$]

TEST YOUR UNDERSTANDING

NOTE: In the following Test Your Understanding questions, assume a silicon npn bipolar transistor at $T = 300$ K has the following minority carrier parameters: $D_E = 8$ cm^2/s, $D_B = 20$ cm^2/s, $D_C = 12$ cm^2/s, $\tau_{E0} = 10^{-8}$ s, $\tau_{B0} = 10^{-7}$ s, and $\tau_{C0} = 10^{-6}$ s.

TYU 12.4 If the emitter doping concentration is $N_E = 5 \times 10^{18}$ cm^{-3}, find the base doping concentration such that the emitter injection efficiency is $\gamma = 0.9950$. Assume $x_E = 2x_B = 2\ \mu$m.
(Ans. $N_B = 1.08 \times 10^{17}$ cm^{-3})

TYU 12.5 Assume that $\alpha_T = \delta = 0.9967$, $x_B = x_E = 1\ \mu$m, $N_B = 5 \times 10^{16}$ cm^{-3}, and $N_E = 5 \times 10^{18}$ cm^{-3}. Determine the common-emitter current gain β.
(Ans. $\beta = 92.4$)

TYU 12.6 Assume that $\gamma = \delta = 0.9967$ and $x_B = 0.80\ \mu$m. Determine the common-emitter current gain β.
(Ans. $\beta = 121$)

12.4 | NONIDEAL EFFECTS

In all previous discussions, we have considered a transistor with uniformly doped regions, low injection, constant emitter and base widths, an ideal constant energy bandgap, uniform current densities, and junctions that are not in breakdown. If any of these ideal conditions is not present, then the transistor properties will deviate from the ideal characteristics we have derived.

12.4.1 Base Width Modulation

We have implicitly assumed that the neutral base width x_B is constant. This base width, however, is a function of the B–C voltage, since the width of the space charge region extending into the base region varies with B–C voltage. As the B–C reverse-biased voltage increases, the B–C space charge region width increases, which reduces x_B. A change in the neutral base width will change the collector current as can be observed in Figure 12.21. A reduction in base width will cause the gradient in the minority carrier concentration to increase, which in turn causes an increase in the diffusion current. This effect is known as *base width modulation;* it is also called the *Early effect.*

The Early effect can be seen in the current–voltage characteristics shown in Figure 12.22. In most cases, a constant base current is equivalent to a constant B–E voltage. Ideally the collector current is independent of the B–C voltage so that the slope of the curves would be zero; thus, the output conductance of the transistor would be zero. However, the base width modulation, or Early effect, produces a nonzero slope and gives rise to a finite output conductance. If the collector current characteristics are extrapolated to zero collector current, the curves intersect the voltage axis at a point that is defined as the Early voltage. The Early voltage is considered to be a positive value. It is a common parameter given in transistor specifications; typical values of Early voltage are in the 100- to 300-V range.

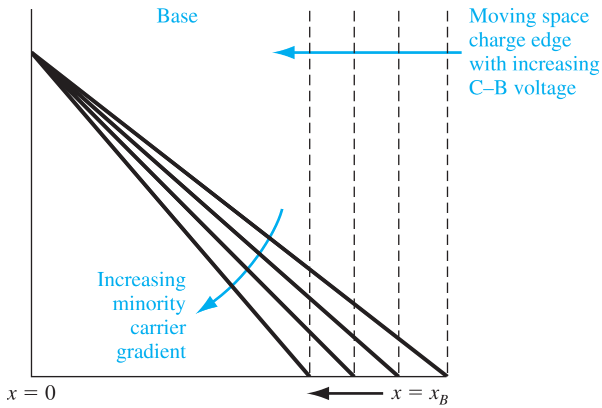

Figure 12.21 | The change in the base width and the change in the minority carrier gradient as the B–C space charge width changes.

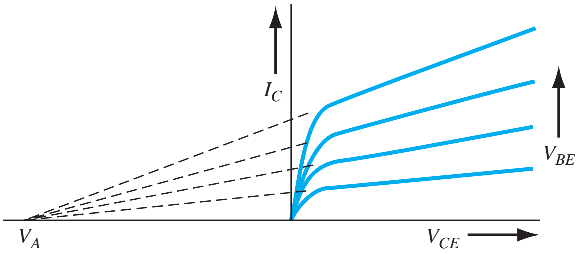

Figure 12.22 | The collector current versus collector–emitter voltage showing the Early effect and Early voltage.

From Figure 12.22, we can write that

$$\frac{dI_C}{dV_{CE}} \equiv g_o = \frac{I_C}{V_{CE} + V_A} = \frac{1}{r_o} \tag{12.45a}$$

where V_A and V_{CE} are defined as positive quantities, g_o is defined as the output conductance, and r_o is defined as the output resistance. Equation (12.45a) can be rewritten in the form

$$I_C = g_o\,(V_{CE} + V_A) = \frac{1}{r_o}(V_{CE} + V_A) \tag{12.45b}$$

showing that the collector current is now an explicit function of the collector–emitter voltage or the collector–base voltage.

EXAMPLE 12.8

Objective: Calculate the change in collector current with a change in neutral base width, and estimate the Early voltage.

Consider a uniformly doped silicon npn bipolar transistor with the following parameters: $N_B = 5 \times 10^{16}$ cm^{-3}, $N_C = 2 \times 10^{15}$ cm^{-3}, $x_{B0} = 0.70\ \mu$m, and $D_B = 25$ cm^2/s. Assume that $x_{B0} \ll L_B$ and that $V_{BE} = 0.60$ V. The collector–base voltage is in the range $2 \le V_{CB} \le 10$ V.

■ Solution

Assuming $x_{B0} \ll L_B$, the excess minority carrier electron concentration in the base can be approximated by Equation (12.15b), which is

$$\delta n_B(x) \cong \frac{n_{B0}}{x_B}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right](x_B - x) - x\right\}$$

The collector current is

$$|J_C| = eD_B\frac{d[\delta n_B(x)]}{dx} \cong \frac{eD_B n_{B0}}{x_B}\exp\left(\frac{V_{BE}}{V_t}\right)$$

The value of n_{B0} is found as

$$n_{B0} = \frac{n_i^2}{N_B} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{16}} = 4.5 \times 10^3 \text{ cm}^{-3}$$

For $V_{CB} = 2$ V, we find (see the following Exercise Problem Ex 12.8)

$$x_B = x_{B0} - x_{dB} = 0.70 - 0.0518 = 0.6482\ \mu\text{m}$$

and

$$|J_C| = \frac{(1.6 \times 10^{-19})(25)(4.5 \times 10^3)}{0.6482 \times 10^{-4}} \exp\left(\frac{0.60}{0.0259}\right) = 3.195 \text{ A/cm}^2$$

For $V_{CB} = 10$ V, we find (see the following Exercise Problem Ex 12.8)

$$x_B = 0.70 - 0.103 = 0.597 \ \mu\text{m}$$

and

$$|J_C| = \frac{(1.6 \times 10^{-19})(25)(4.5 \times 10^3)}{0.597 \times 10^{-4}} \exp\left(\frac{0.60}{0.0259}\right) = 3.469 \text{ A/cm}^2$$

We now can find, from Equation (12.45a)

$$\frac{dJ_C}{dV_{CE}} = \frac{\Delta J_C}{\Delta V_{CB}} = \frac{J_C}{V_{CE} + V_A} = \frac{J_C}{V_{BE} + V_{CB} + V_A}$$

or

$$\frac{3.469 - 3.195}{8} = \frac{3.195}{0.60 + 2 + V_A}$$

The Early voltage is then determined to be

$$V_A = 90.7 \text{ V}$$

■ Comment

This example indicates how much the collector current can change as the neutral base width changes with a change in the B–C space charge width, and it also indicates the magnitude of the Early voltage.

■ EXERCISE PROBLEM

Ex 12.8 Consider a silicon npn bipolar transistor with parameters described in Example 12.8. Determine the neutral base width for a C–B voltage of (*a*) $V_{CB} = 2$ V and (*b*) $V_{CB} = 10$ V. Neglect the B–E space charge width.
[Ans. (*a*) $x_B = 0.6482\ \mu$m; (*b*) $x_B = 0.597\ \mu$m]

The previous example and exercise problem demonstrate, too, that we can expect variations in transistor properties due to tolerances in transistor-fabrication processes. There will be variations, in particular, in the base width of narrow-base transistors that will cause variations in the collector current characteristics simply due to the tolerances in processing.

12.4.2 High Injection

The ambipolar transport equation that we have used to determine the minority carrier distributions assumed low injection. As V_{BE} increases, the injected minority carrier concentration may approach, or even become larger than, the majority carrier concentration. If we assume quasi–charge neutrality, then the majority carrier hole concentration in the p-type base at $x = 0$ will increase as shown in Figure 12.23 because of the excess holes.

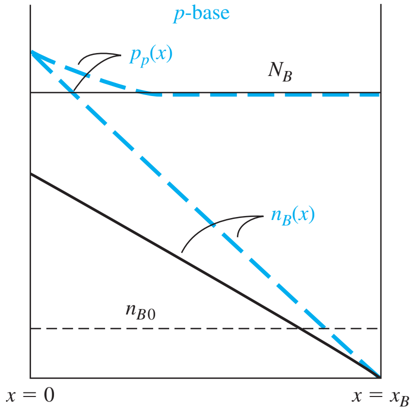

Figure 12.23 | Minority and majority carrier concentrations in the base under low and high injection (solid line: low injection; dashed line: high injection).

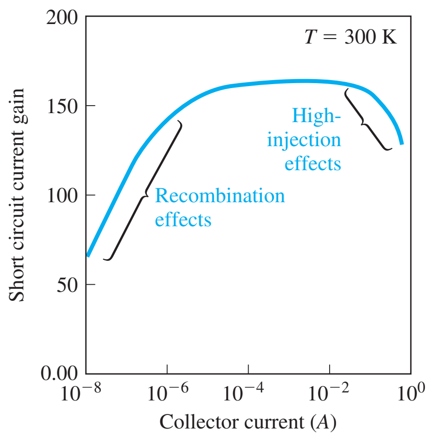

Figure 12.24 | Common-emitter current gain versus collector current. *(From Shur [14].)*

Two effects occur in the transistor at high injection. The first effect is a reduction in emitter injection efficiency. Since the majority carrier hole concentration at $x = 0$ increases with high injection, more holes are injected back into the emitter because of the forward-biased B–E voltage. An increase in the hole injection causes an increase in the J_{pE} current and an increase in J_{pE} reduces the emitter injection efficiency. The common-emitter current gain decreases, then, with high injection. Figure 12.24 shows a typical common-emitter current gain versus collector current curve. The low gain at low currents is due to the small recombination factor and the drop-off at the high current is due to the high-injection effect.

We now consider the second high-injection effect. At low injection, the majority carrier hole concentration at $x = 0$ for the npn transistor is

$$p_p(0) = p_{p0} = N_a \tag{12.46a}$$

and the minority carrier electron concentration is

$$n_p(0) = n_{p0}\exp\left(\frac{eV_{BE}}{kT}\right) \tag{12.46b}$$

The pn product is

$$p_p(0)n_p(0) = p_{p0}n_{p0}\exp\left(\frac{eV_{BE}}{kT}\right) \tag{12.46c}$$

At high injection, Equation (12.46c) still applies. However, $p_p(0)$ will also increase, and for very high injection it will increase at nearly the same rate as $n_p(0)$. The increase in $n_p(0)$ will asymptotically approach the function

$$n_p(0) \approx n_{p0}\exp\left(\frac{eV_{BE}}{2kT}\right) \tag{12.47}$$

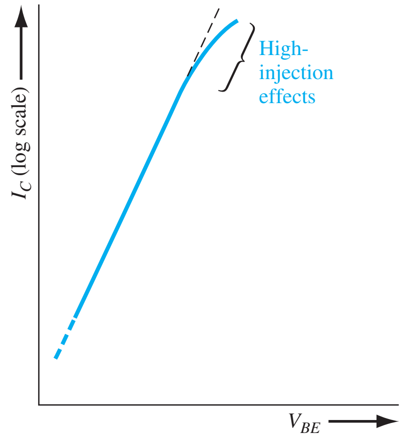

Figure 12.25 | Collector current versus base–emitter voltage showing high-injection effects.

The excess minority carrier concentration in the base, and hence the collector current, will increase at a slower rate with B–E voltage in high injection than low injection. This effect is shown in Figure 12.25. The high-injection effect is very similar to the effect of a series resistance in a pn junction diode.

12.4.3 Emitter Bandgap Narrowing

Another phenomenon affecting the emitter injection efficiency is bandgap narrowing. We have implied from our previous discussion that the emitter injection efficiency factor will continue to increase and approach unity as the ratio of emitter doping to base doping continues to increase. As silicon becomes heavily doped, the discrete donor energy level in an n-type emitter splits into a band of energies. The distance between donor atoms decreases as the concentration of impurity donor atoms increases, and the splitting of the donor level is caused by the interaction of donor atoms with each other. As the doping continues to increase, the donor band widens, becomes skewed, and moves up toward the conduction band, eventually merging with it. At this point, the effective bandgap energy has decreased. Figure 12.26 shows a plot of the change in the bandgap energy with impurity doping concentration.

A reduction in the bandgap energy increases the intrinsic carrier concentration. The intrinsic carrier concentration is given by

$$n_i^2 = N_c N_v \exp\left(\frac{-E_g}{kT}\right) \tag{12.48}$$

In a heavily doped emitter, the intrinsic carrier concentration can be written as

$$n_{iE}^2 = N_c N_v \exp\left[\frac{-(E_{g0} - \Delta E_g)}{kT}\right] = n_i^2 \exp\left(\frac{\Delta E_g}{kT}\right) \tag{12.49}$$

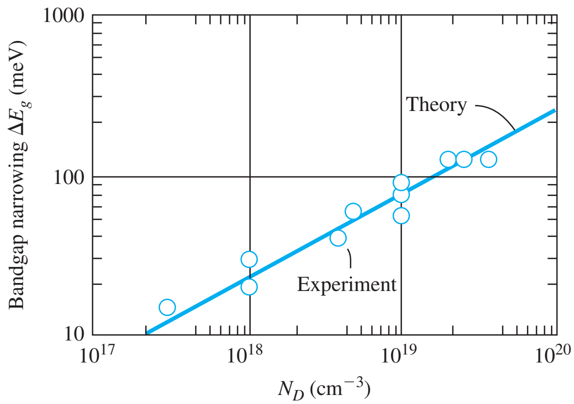

Figure 12.26 | Bandgap narrowing factor versus donor impurity concentration in silicon.
(From Sze [19].)

where E_{g0} is the bandgap energy at a low doping concentration and ΔE_g is the bandgap narrowing factor.

The emitter injection efficiency factor is given by Equation (12.35) as

$$\gamma = \frac{1}{1 + \dfrac{p_{E0} D_E L_B}{n_{B0} D_B L_E} \cdot \dfrac{\tanh(x_B/L_B)}{\tanh(x_E/L_E)}}$$

The term p'_{E0} is the thermal-equilibrium minority carrier concentration in the emitter, taking into account bandgap narrowing, and can be written as

$$p'_{E0} = \frac{n_{iE}^2}{N_E} = \frac{n_i^2}{N_E} \exp\left(\frac{\Delta E_g}{kT}\right) \tag{12.50}$$

As the emitter doping concentration increases, ΔE_g increases; thus, p'_{E0} does not continue to decrease with increasing emitter doping N_E. If p'_{E0} starts to increase because of the bandgap narrowing, the emitter injection efficiency begins to fall off instead of continuing to increase with increased emitter doping.

EXAMPLE 12.9

Objective: Determine the increase in p_{E0} in the emitter due to bandgap narrowing.

Consider a silicon emitter at $T = 300$ K. Assume the emitter doping increases from 10^{18} to 10^{19} cm^{-3}. Determine the new value of p'_{E0} and determine the ratio p'_{E0}/p_{E0}.

■ Solution

For emitter doping concentrations of $N_E = 10^{18}$ and 10^{19} cm^{-3}, we have, neglecting bandgap narrowing,

$$p_{E0} = \frac{n_i^2}{N_E} = \frac{(1.5 \times 10^{10})^2}{10^{18}} = 2.25 \times 10^2 \text{ cm}^{-3}$$

and

$$p_{E0} = \frac{n_i^2}{N_E} = \frac{(1.5 \times 10^{10})^2}{10^{19}} = 2.25 \times 10^1 \text{ cm}^{-3}$$

Taking into account the bandgap narrowing effect shown in Figure 12.26, we obtain, respectively, for $N_E = 10^{18}$ and 10^{19} cm^{-3} ,

$$p'_{E0} = \frac{n_i^2}{N_E}\exp\left(\frac{\Delta E_g}{kT}\right) = \frac{(1.5 \times 10^{10})^2}{10^{18}}\exp\left(\frac{0.020}{0.0259}\right) = 4.87 \times 10^2 \text{ cm}^{-3}$$

and

$$p'_{E0} = \frac{(1.5 \times 10^{10})^2}{10^{19}}\exp\left(\frac{0.080}{0.0259}\right) = 4.94 \times 10^2 \text{ cm}^{-3}$$

Taking the ratio of p'_{E0}/p_{E0} for $N_E = 10^{18}$ cm^{-3}, we find

$$\frac{p'_{E0}}{p_{E0}} = \exp\left(\frac{\Delta E_g}{kT}\right) = \exp\left(\frac{0.020}{0.0259}\right) = 2.16$$

and for $N_E = 10^{19}$ cm^{-3}, we find

$$\frac{p'_{E0}}{p_{E0}} = \exp\left(\frac{0.080}{0.0259}\right) = 21.95$$

■ Comment

If the emitter doping concentration increases from 10^{18} to 10^{19} cm^{-3}, the thermal-equilibrium minority carrier concentration actually increases rather than decreasing by a factor of 10 as would be expected.

■ EXERCISE PROBLEM

Ex 12.9 Determine the thermal-equilibrium minority carrier concentration for an emitter doping concentration of $N_E = 10^{20}$ cm^{-3} taking into account bandgap narrowing.

(Ans. $p_{E0} = 2.25$ cm^{-3}, $p'_{E0} = 1.618 \times 10^4$ cm^{-3})

As the emitter doping increases, the bandgap narrowing factor, ΔE_g, will increase; this can actually cause p_{E0} to increase. As p_{E0} increases, the emitter injection efficiency decreases; this then causes the transistor gain to decrease, as shown in Figure 12.24. A very high emitter doping may result in a smaller current gain than we anticipate because of the bandgap narrowing effect.

12.4.4 Current Crowding

It is tempting to neglect the effects of base current in a transistor since the base current is usually much smaller than either the collector or the emitter current. Figure 12.27 is a cross section of an npn transistor showing the lateral distribution of base current. The base region is typically less than a micrometer thick, so there can be a sizable base resistance. The nonzero base resistance results in a lateral potential difference under the emitter region. For the npn transistor, the potential decreases from the edge of the emitter toward the center. The emitter is highly doped, so as a first approximation the emitter can be considered an equipotential region.

The number of electrons from the emitter injected into the base is exponentially dependent on the B–E voltage. With the lateral voltage drop in the base between the edge and center of the emitter, more electrons will be injected near the emitter edges than in the center, causing the emitter current to be crowded toward the edges. This

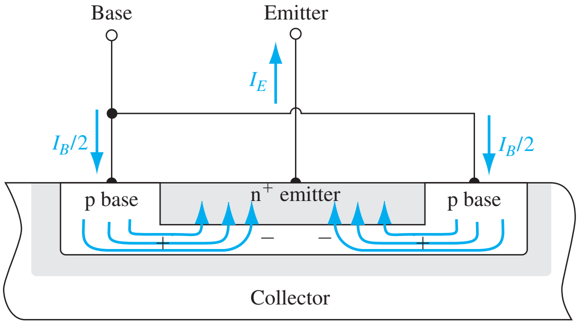

Figure 12.27 | Cross section of an npn bipolar transistor showing the base current distribution and the lateral potential drop in the base region.

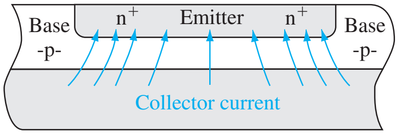

Figure 12.28 | Cross section of an npn bipolar transistor showing the emitter current crowding effect.

current crowding effect is schematically shown in Figure 12.28. The larger current density near the emitter edge may cause localized heating effects as well as localized high-injection effects. The nonuniform emitter current also results in a nonuniform lateral base current under the emitter. A two-dimensional analysis would be required to calculate the actual potential drop versus distance because of the nonuniform base current. Another approach is to slice the transistor into a number of smaller parallel transistors and to lump the resistance of each base section into an equivalent external resistance.

Power transistors, designed to handle large currents, require large emitter areas to maintain reasonable current densities. To avoid the current crowding effect, these transistors are usually designed with narrow emitter widths and fabricated with an interdigitated design. Figure 12.29 shows the basic geometry. In effect, many narrow emitters are connected in parallel to achieve the required emitter area.

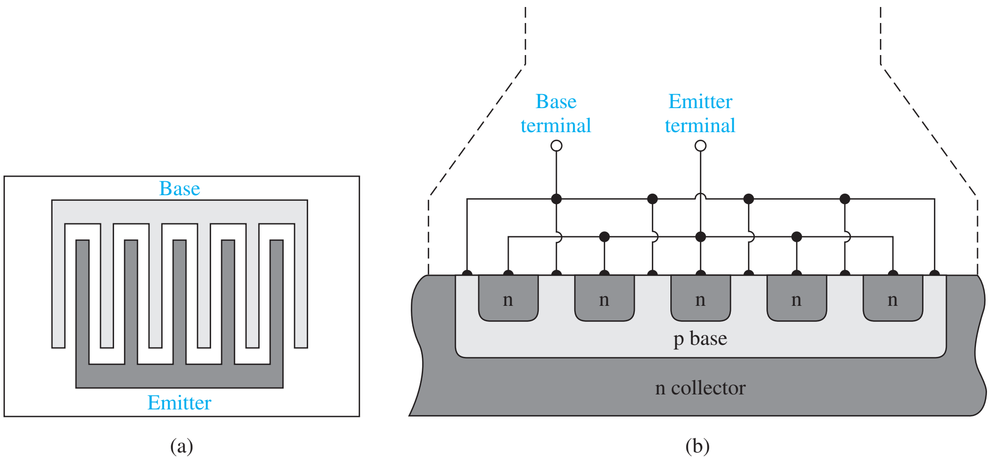

Figure 12.29 | (a) Top view and (b) cross section of an interdigitated npn bipolar transistor structure.

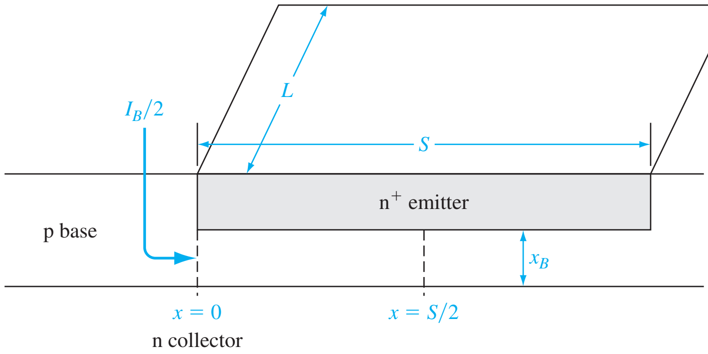

Figure 12.30 | Figure for TYU 12.7.

TEST YOUR UNDERSTANDING

TYU 12.7 Consider the geometry shown in Figure 12.30. The base doping concentration is $N_B = 10^{16}$ cm^{-3}, the neutral base width is $x_B = 0.80$ μm, the emitter width is $S = 10$ μm, and the emitter length is $L = 10$ μm. (*a*) Determine the resistance of the base between $x = 0$ and $x = S/2$. Assume a hole mobility of $\mu_p = 400$ cm^2/V-s. (*b*) If the base current in this region is uniform and given by $I_B/2 = 5$ μA, determine the potential difference between $x = 0$ and $x = S/2$. (*c*) Using the results of part (*b*), what is the ratio of emitter current density at $x = 0$ to that at $x = S/2$? [Ans. (*a*) 9.77 kΩ; (*b*) 48.83 mV; (*c*) 6.59]

*12.4.5 Nonuniform Base Doping

In the analysis of the bipolar transistor, we have assumed uniformly doped regions. However, uniform doping rarely occurs. Figure 12.31 shows a doping profile in a doubly diffused npn transistor. We can start with a uniformly doped n-type substrate, diffuse acceptor atoms from the surface to form a compensated p-type base, and then diffuse donor atoms from the surface to form a doubly compensated n-type emitter. The diffusion process results in a nonuniform doping profile.

We determined in Chapter 5 that a graded impurity concentration leads to an induced electric field. For the p-type base region in thermal equilibrium, we can write

$$J_p = e\mu_p N_a \mathrm{E} - eD_p \frac{dN_a}{dx} = 0 \tag{12.51}$$

Then

$$\mathrm{E} = +\left(\frac{kT}{e}\right)\frac{1}{N_a}\frac{dN_a}{dx} \tag{12.52}$$

According to the example of Figure 12.31, dN_a/dx is negative; hence, the induced electric field is in the negative x direction.