

```
In [18]: import time

import numpy
import matplotlib.pyplot as plt
import numpy as np
import scipy.special as sp_spec
import scipy.stats as sp_stats

import random
```

Assignment 2A. Problem 2.2.8 SVI.

Generate data

The cell below generates data for the LDA model. Note, for simplicity, we are using $N_d = N$ for all d .

```
In [19]: import torch
import torch.distributions as t_dist

def generate_data(D, N, K, W, eta, alpha):
    """
    Torch implementation for generating data using the LDA model.\
    Faster for larger datasets.
    D = number of documents
    N = number of words in each document
    K = number of topics
    W = number of words in vocabulary
    """
    # sample K topics
    beta_dist = t_dist.Dirichlet(torch.from_numpy(eta))
    beta = beta_dist.sample([K]) # size K x W

    # sample document topic distribution
    theta_dist = t_dist.Dirichlet(torch.from_numpy(alpha))
    theta = theta_dist.sample([D]) # size D x K

    # sample word to topic assignment
    z_dist = t_dist.OneHotCategorical(probs=theta)
    z = z_dist.sample([N])
    z = torch.einsum("ndk->dnk", z)

    # sample word from selected topics
    beta_select = torch.einsum("kw, dnk -> dnw", beta, z)
    w_dist = t_dist.OneHotCategorical(probs=beta_select)
    w = w_dist.sample([1])

    w = w.reshape(D, N, W)

    return w.numpy(), z.numpy(), theta.numpy(), beta.numpy()

torch.manual_seed(1)

D_sim = 500
N_sim = 5000
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K_sim = 2
W_sim = 10

eta_sim = np.ones(W_sim)
eta_sim[3] = 0.0001      # Expect word 3 to not appear in data
eta_sim[1] = 3.          # Expect word 1 to be most common in data
alpha_sim = np.ones(K_sim) * 1.0
w0, z0, theta0, beta0 = generate_data(D_sim, N_sim, K_sim, W_sim,
                                       eta_sim, alpha_sim)

w_cat = w0.argmax(axis=-1) # remove one hot encoding
unique_z, counts_z = numpy.unique(z0[0, :], return_counts=True)
unique_w, counts_w = numpy.unique(w_cat[0, :], return_counts=True)

# Sanity checks for data generation
print(f"Average z of each document should be close to theta of document.\
\n Theta of doc 0: {theta0[0]}")
    f" \n Mean z of doc 0: {z0[0].mean(axis=0)}")
print(f"Beta of topic 0: {beta0[0]}")
print(f"Beta of topic 1: {beta0[1]}")
print(f"Word to topic assignment, z, of document 0: {z0[0, 0:10]}")
print(f"Observed words, w, of document 0: {w_cat[0, 0:10]}")
print(f"Unique words and count of document 0: {[f'{u}: {c}'] for u, c in
zip(unique_w, counts_w)}")

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Average z of each document should be close to theta of document.
Theta of doc 0: [0.140 0.860]
Mean z of doc 0: [0.146 0.854]
Beta of topic 0: [0.135 0.309 0.036 0.000 0.009 0.068 0.043 0.092 0.103 0.
206]
Beta of topic 1: [0.351 0.217 0.081 0.000 0.014 0.099 0.105 0.046 0.016 0.
072]
Word to topic assignment, z, of document 0: [[1.000 0.000]
[1.000 0.000]
[0.000 1.000]
[0.000 1.000]
[1.000 0.000]
[0.000 1.000]
[0.000 1.000]
[1.000 0.000]
[0.000 1.000]
[0.000 1.000]]
Observed words, w, of document 0: [0 9 1 1 8 1 4 1 0 2]
Unique words and count of document 0: ['0: 1587', '1: 1143', '2: 384', '4:
70', '5: 467', '6: 449', '7: 265', '8: 140', '9: 495']

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Helper functions

```

In [20]: def log_multivariate_beta_function(a, axis=None):
    return np.sum(sp_spec.gammaln(a)) - sp_spec.gammaln(np.sum(a,
axis=axis))

```

CAVI Implementation, ELBO and initialization

```

In [21]: def initialize_q(w, D, N, K, W):
    """
    Random initialization.
    """
    phi_init = np.random.random(size=(D, N, K))

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phi_init = phi_init / np.sum(phi_init, axis=-1, keepdims=True)
gamma_init = np.random.randint(1, 10, size=(D, K))
lmbda_init = np.random.randint(1, 10, size=(K, W))
return phi_init, gamma_init, lmbda_init

def update_q_Z(w, gamma, lmbda):
    D, N, W = w.shape
    K, W = lmbda.shape
    E_log_theta = sp_spec.digamma(gamma) - sp_spec.digamma(np.sum(gamma,
        axis=1, keepdims=True)) # size D x K
    E_log_beta = sp_spec.digamma(lmbda) - sp_spec.digamma(np.sum(lmbda,
        axis=1, keepdims=True)) # size K x W
    log_rho = np.zeros((D, N, K))
    w_label = w.argmax(axis=-1)
    for d in range(D):
        for n in range(N):
            E_log_beta_wdn = E_log_beta[:, int(w_label[d, n])]
            E_log_theta_d = E_log_theta[d]
            log_rho_n = E_log_theta_d + E_log_beta_wdn
            log_rho[d, n, :] = log_rho_n

    phi = np.exp(log_rho - sp_spec.logsumexp(log_rho, axis=-1,
        keepdims=True))

    return phi

def update_q_theta(phi, alpha):
    E_Z = phi
    D, N, K = phi.shape
    gamma = np.zeros((D, K))
    for d in range(D):
        E_Z_d = E_Z[d]
        gamma[d] = alpha + np.sum(E_Z_d, axis=0) # sum over N
    return gamma

def update_q_beta(w, phi, eta):
    E_Z = phi
    D, N, W = w.shape
    K = phi.shape[-1]
    lmbda = np.zeros((K, W))
    for k in range(K):
        lmbda[k, :] = eta
        for d in range(D):
            for n in range(N):
                lmbda[k, :] += E_Z[d, n, k] * w[d, n] # Sum over d and n
    return lmbda

def calculate_elbo(w, phi, gamma, lmbda, eta, alpha):
    D, N, K = phi.shape
    W = eta.shape[0]
    E_log_theta = sp_spec.digamma(gamma) - sp_spec.digamma(np.sum(gamma,
        axis=1, keepdims=True)) # size D x K
    E_log_beta = sp_spec.digamma(lmbda) - sp_spec.digamma(np.sum(lmbda,
        axis=1, keepdims=True)) # size K x W
    E_Z = phi # size D, N, K
    log_Beta_alpha = log_multivariate_beta_function(alpha)
    log_Beta_eta = log_multivariate_beta_function(eta)
    log_Beta_gamma = np.array([log_multivariate_beta_function(
        gamma[d, :]) for d in range(D)])
    dg_gamma = sp_spec.digamma(gamma)
    log_Beta_lmbda = np.array([log_multivariate_beta_function(

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        lambda[k, :]) for k in range(K)])
    dg_lambda = sp_spec.digamma(lambda)

    neg_CE_likelihood = np.einsum("dnk, kw, dnw", E_Z, E_log_beta, w)
    neg_CE_Z = np.einsum("dnk, dk -> ", E_Z, E_log_theta)
    neg_CE_theta = -D * log_Beta_alpha + np.einsum("k, dk -> ",
        alpha - 1, E_log_theta)
    neg_CE_beta = -K * log_Beta_eta + np.einsum("w, kw -> ",
        eta - 1, E_log_beta)
    H_Z = -np.einsum("dnk, dnk -> ", E_Z, np.log(E_Z))
    gamma_0 = np.sum(gamma, axis=1)
    dg_gamma0 = sp_spec.digamma(gamma_0)
    H_theta = np.sum(log_Beta_gamma + (gamma_0 - K) * dg_gamma0 -
        np.einsum("dk, dk -> d", gamma - 1, dg_gamma))
    lambda_0 = np.sum(lambda, axis=1)
    dg_lambda0 = sp_spec.digamma(lambda_0)
    H_beta = np.sum(log_Beta_lambda + (lambda_0 - W) * dg_lambda0 -
        np.einsum("kw, kw -> k", lambda - 1, dg_lambda))
    return (neg_CE_likelihood + neg_CE_Z + neg_CE_theta +
        neg_CE_beta + H_Z + H_theta + H_beta)

def CAVI_algorithm(w, K, n_iter, eta, alpha):
    D, N, W = w.shape
    phi, gamma, lambda = initialize_q(w, D, N, K, W)

    # Store output per iteration
    elbo = np.zeros(n_iter)
    phi_out = np.zeros((n_iter, D, N, K))
    gamma_out = np.zeros((n_iter, D, K))
    lambda_out = np.zeros((n_iter, K, W))

    for i in range(0, n_iter):

        ##### CAVI updates #####

        # q(Z) update
        phi = update_q_Z(w, gamma, lambda)

        # q(theta) update
        gamma = update_q_theta(phi, alpha)

        # q(beta) update
        lambda = update_q_beta(w, phi, eta)

        # ELBO
        elbo[i] = calculate_elbo(w, phi, gamma, lambda, eta, alpha)

        # outputs
        phi_out[i] = phi
        gamma_out[i] = gamma
        lambda_out[i] = lambda

    return phi_out, gamma_out, lambda_out, elbo

n_iter0 = 100
K0 = K_sim
W0 = W_sim
eta_prior0 = np.ones(W0)
alpha_prior0 = np.ones(K0)
phi_out0, gamma_out0, lambda_out0, elbo0 = CAVI_algorithm(w0,

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    K0, n_iter0, eta_prior0, alpha_prior0)
final_phi0 = phi_out0[-1]
final_gamma0 = gamma_out0[-1]
final_lambda0 = lambda_out0[-1]

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In [22]: precision = 3
print(f"----- Recall label switching - compare E[theta] and true theta\
      and check for label switching -----")
print(f"Final E[theta] of doc 0 CAVI:
      {np.round(final_gamma0[0] / np.sum(final_gamma0[0], axis=0, keepdim
print(f"True theta of doc 0:
      {np.round(theta0[0], precision)}")

print(f"----- Recall label switching - e.g. E[beta_0] could be fit to\
      true theta_1. -----")
print(f"Final E[beta] k=0: {np.round(final_lambda0[0, :] /
      np.sum(final_lambda0[0, :], axis=-1, keepdims=True), precision)}")
print(f"Final E[beta] k=1: {np.round(final_lambda0[1, :] /
      np.sum(final_lambda0[1, :], axis=-1, keepdims=True), precision)}")
print(f"True beta k=0: {np.round(beta0[0, :], precision)}")
print(f"True beta k=1: {np.round(beta0[1, :], precision)}")

----- Recall label switching - compare E[theta] and true theta and check f
or label switching -----
Final E[theta] of doc 0 CAVI: [0.271 0.729]
True theta of doc 0: [0.140 0.860]
----- Recall label switching - e.g. E[beta_0] could be fit to true theta_
1. -----
Final E[beta] k=0: [0.112 0.319 0.031 0.000 0.008 0.064 0.036 0.097 0.111
0.221]
Final E[beta] k=1: [0.392 0.200 0.089 0.000 0.015 0.105 0.117 0.037 0.001
0.046]
True beta k=0: [0.135 0.309 0.036 0.000 0.009 0.068 0.043 0.092 0.103 0.20
6]
True beta k=1: [0.351 0.217 0.081 0.000 0.014 0.099 0.105 0.046 0.016 0.07
2]

```

SVI Implementation

Using the CAVI updates as a template, finish the code below.

```

In [23]: def update_q_Z_svi(batch, w, gamma, lambda):
    """
    TODO: rewrite to SVI update
    """
    D, N, W = w.shape
    K, W = lambda.shape
    S = len(batch)
    E_log_theta = sp_spec.digamma(gamma) - sp_spec.digamma(np.sum(gamma,
        axis=1, keepdims=True)) # size D x K
    E_log_beta = sp_spec.digamma(lambda) - sp_spec.digamma(np.sum(lambda,
        axis=1, keepdims=True)) # size K x W
    log_rho = np.zeros((S, N, K))
    w_label = w.argmax(axis=-1)
    for s, d in enumerate(batch):
        for n in range(N):
            E_log_beta_wdn = E_log_beta[:, int(w_label[d, n])]
            E_log_theta_d = E_log_theta[d]
            log_rho_n = E_log_theta_d + E_log_beta_wdn

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        log_rho[s, n, :] = log_rho_n

    phi = np.exp(log_rho - sp_spec.logsumexp(log_rho, axis=-1,
        keepdims=True))
    return phi

def update_q_theta_svi(batch, phi, alpha):
    """
    TODO: rewrite to SVI update
    """
    E_Z = phi
    D, N, K = phi.shape
    S = len(batch)
    gamma = np.zeros((S, K))
    for s, d in enumerate(batch):
        E_Z_d = E_Z[d]
        gamma[s] = alpha + np.sum(E_Z_d, axis=0) # sum over N
    return gamma

def update_q_beta_svi(batch, w, phi, eta):
    """
    TODO: rewrite to SVI update
    """
    E_Z = phi
    D, N, W = w.shape
    K = phi.shape[-1]
    S = len(batch)
    lmbda = np.zeros((K, W))
    for k in range(K):
        for d in batch:
            for n in range(N):
                lmbda[k, :] += E_Z[d,n,k] * w[d,n] # Sum over d and n
            lmbda[k, :] = lmbda[k, :] / S * D
        lmbda[k, :] += eta
    return lmbda

def SVI_algorithm(w, K, S, n_iter, eta, alpha):
    """
    Add SVI Specific code here.
    """
    D, N, W = w.shape
    phi, gamma, lmbda = initialize_q(w, D, N, K, W)

    # Store output per iteration
    elbo = np.zeros(n_iter)
    phi_out = np.zeros((n_iter, D, N, K))
    gamma_out = np.zeros((n_iter, D, K))
    lmbda_out = np.zeros((n_iter, K, W))

    tau = 1.0
    kappa = 0.75
    rhos = np.arange(start=0, stop=n_iter, step=1)
    rhos = (rhos + tau)**(-kappa)
    used_documents = set()
    for i in range(0, n_iter):
        # Sample batch and set step size, rho.

        batch = random.sample(range(D), S)
        rho_t = rhos[i]
        for d in batch:

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        gamma[d,:] = 1
        used_documents.add(d)

##### SVI updates #####
for _ in range(0,n_iter):
    phi_batch = update_q_Z_svi(batch, w, gamma, lambda)
    for s, d in enumerate(batch):
        phi[d,:,:] = phi_batch[s,:,:]
        gamma_batch = update_q_theta_svi(batch, phi, alpha)
        for s, d in enumerate(batch):
            gamma[d,:] = gamma_batch[s,:]

    new_lambdas = update_q_beta_svi(batch, w, phi, eta)

    lambda = (1-rho_t)*lambda + rho_t*new_lambdas

# ELBO
elbo[i] = calculate_elbo(w, phi, gamma, lambda, eta, alpha)

# outputs
phi_out[i] = phi
gamma_out[i] = gamma
lambda_out[i] = lambda

return phi_out, gamma_out, lambda_out, elbo

```

CASE 1

Tiny dataset

```

In [24]: np.random.seed(0)

# Data simulation parameters
D1 = 50
N1 = 50
K1 = 2
W1 = 5
eta_sim1 = np.ones(W1)
alpha_sim1 = np.ones(K1)

w1, z1, theta1, beta1 = generate_data(D1, N1, K1, W1, eta_sim1,
                                       alpha_sim1)

# Inference parameters
n_iter_cavi1 = 100
n_iter_svi1 = 100
eta_prior1 = np.ones(W1) * 1.
alpha_prior1 = np.ones(K1) * 1.
S1 = 5 # batch size

start_cavi1 = time.time()
phi_out1_cavi, gamma_out1_cavi, lambda_out1_cavi, elbo1_cavi = CAVI_algorithm(
    w1, K1, n_iter_cavi1, eta_prior1, alpha_prior1)
end_cavi1 = time.time()

start_svi1 = time.time()
phi_out1_svi, gamma_out1_svi, lambda_out1_svi, elbo1_svi = SVI_algorithm(
    w1, K1, S1, n_iter_svi1, eta_prior1, alpha_prior1)
end_svi1 = time.time()

```

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final_phil_cavi = phi_out1_cavi[-1]
final_gammal_cavi = gamma_out1_cavi[-1]
final_lmbdal_cavi = lmbda_out1_cavi[-1]
final_phil_svi = phi_out1_svi[-1]
final_gammal_svi = gamma_out1_svi[-1]
final_lmbdal_svi = lmbda_out1_svi[-1]

```

Evaluation

Do not expect perfect results in terms expectations being identical to the "true" theta and beta. Do not expect the ELBO plot of your SVI alg to be the same as the CAVI alg. However, it should increase and be in the same ball park as that of the CAVI alg.

```

In [25]: np.set_printoptions(formatter={'float': lambda x: "{0:0.3f}".format(x)})
print(f"----- Recall label switching - compare E[theta] and true theta\
and check for label switching -----")
print(f"E[theta] of doc 0 SVI:
      {final_gammal_svi[0] / \
      np.sum(final_gammal_svi[0], axis=0, keepdims=True)}")
print(f"E[theta] of doc 0 CAVI:
      {final_gammal_cavi[0] / \
      np.sum(final_gammal_cavi[0], axis=0, keepdims=True)}")
print(f"True theta of doc 0:
      {thetal[0]}")

print(f"----- Recall label switching - e.g. E[beta_0] could be fit to\
true theta_1. -----")
print(f"E[beta] SVI k=0:
      {final_lmbdal_svi[0, :] / \
      np.sum(final_lmbdal_svi[0, :], axis=-1, keepdims=True)}")
print(f"E[beta] SVI k=1:
      {final_lmbdal_svi[1, :] / \
      np.sum(final_lmbdal_svi[1, :], axis=-1, keepdims=True)}")
print(f"E[beta] CAVI k=0:
      {final_lmbdal_cavi[0, :] / \
      np.sum(final_lmbdal_cavi[0, :], axis=-1, keepdims=True)}")
print(f"E[beta] CAVI k=1:
      {final_lmbdal_cavi[1, :] / \
      np.sum(final_lmbdal_cavi[1, :], axis=-1, keepdims=True)}")
print(f"True beta k=0:
      {betal[0, :]}")
print(f"True beta k=1:
      {betal[1, :]}")

```

----- Recall label switching - compare E[theta] and true theta and check for label switching -----

E[theta] of doc 0 SVI: [0.863 0.137]

E[theta] of doc 0 CAVI: [0.829 0.171]

True theta of doc 0: [0.012 0.988]

----- Recall label switching - e.g. E[beta_0] could be fit to true theta_1. -----

E[beta] SVI k=0: [0.583 0.233 0.019 0.025 0.139]

E[beta] SVI k=1: [0.102 0.094 0.493 0.190 0.121]

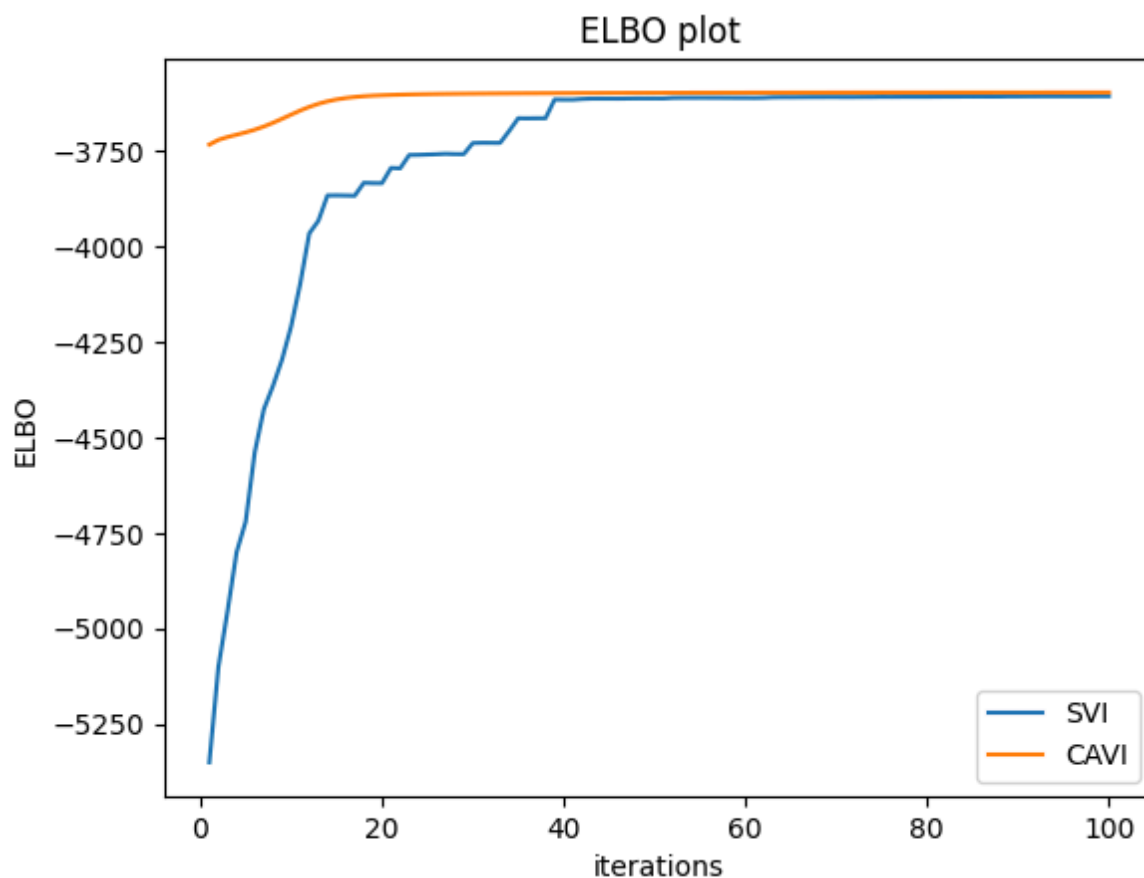
E[beta] CAVI k=0: [0.548 0.280 0.004 0.014 0.153]

E[beta] CAVI k=1: [0.256 0.056 0.415 0.172 0.101]

True beta k=0: [0.280 0.131 0.351 0.125 0.114]

True beta k=1: [0.561 0.255 0.013 0.032 0.140]


```
In [26]: plt.plot(list(range(1, n_iter_cavi1 + 1)), elbo1_svi[np.arange(
    0, n_iter_svi1, int(n_iter_svi1 / n_iter_cavi1))])
plt.plot(list(range(1, n_iter_cavi1 + 1)), elbo1_cavi)
plt.title("ELBO plot")
plt.xlabel("iterations")
plt.legend(["SVI", "CAVI"])
plt.ylabel("ELBO")
plt.show()
```



CASE 2

Small dataset

```
In [28]: np.random.seed(0)

# Data simulation parameters
D2 = 1000
N2 = 50
K2 = 3
W2 = 10
eta_sim2 = np.ones(W2)
alpha_sim2 = np.ones(K2)

w2, z2, theta2, beta2 = generate_data(D2, N2, K2, W2, eta_sim2,
    alpha_sim2)

# Inference parameters
n_iter_cavi2 = 100
n_iter_svi2 = 100
eta_prior2 = np.ones(W2) * 1.
alpha_prior2 = np.ones(K2) * 1.
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S2 = 100 # batch size

start_cavi2 = time.time()
phi_out2_cavi, gamma_out2_cavi, lambda_out2_cavi, elbo2_cavi = \
    CAVI_algorithm(w2, K2, n_iter_cavi2, eta_prior2, alpha_prior2)
end_cavi2 = time.time()

start_svi2 = time.time()
phi_out2_svi, gamma_out2_svi, lambda_out2_svi, elbo2_svi = \
    SVI_algorithm(w2, K2, S2, n_iter_svi2, eta_prior2, alpha_prior2)
end_svi2 = time.time()

final_phi2_cavi = phi_out2_cavi[-1]
final_gamma2_cavi = gamma_out2_cavi[-1]
final_lambda2_cavi = lambda_out2_cavi[-1]
final_phi2_svi = phi_out2_svi[-1]
final_gamma2_svi = gamma_out2_svi[-1]
final_lambda2_svi = lambda_out2_svi[-1]

```

Evaluation

Do not expect perfect results in terms expectations being identical to the "true" theta and beta. Do not expect the ELBO plot of your SVI alg to be the same as the CAVI alg. However, it should increase and be in the same ball park as that of the CAVI alg.

```

In [29]: np.set_printoptions(formatter={'float': lambda x: "{0:0.3f}".format(x)})
print(f"----- Recall label switching - compare E[theta] and true theta\
      and check for label switching -----")
print(f"E[theta] of doc 0 SVI:
      {final_gamma2_svi[0] / \
      np.sum(final_gamma2_svi[0], axis=0, keepdims=True)}")
print(f"E[theta] of doc 0 CAVI:
      {final_gamma2_cavi[0] / \
      np.sum(final_gamma2_cavi[0], axis=0, keepdims=True)}")
print(f"True theta of doc 0:
      {theta2[0]}")

print(f"----- Recall label switching - e.g. E[beta_0] could be fit\
      to true theta_1. -----")
print(f"E[beta] k=0:
      {final_lambda2_svi[0, :] / \
      np.sum(final_lambda2_svi[0, :], axis=-1, keepdims=True)}")
print(f"E[beta] k=1:
      {final_lambda2_svi[1, :] / \
      np.sum(final_lambda2_svi[1, :], axis=-1, keepdims=True)}")
print(f"E[beta] k=2:
      {final_lambda2_svi[2, :] / \
      np.sum(final_lambda2_svi[2, :], axis=-1, keepdims=True)}")
print(f"True beta k=0: {beta2[0, :]}")
print(f"True beta k=1: {beta2[1, :]}")
print(f"True beta k=2: {beta2[2, :]}")

print(f"Time SVI: {end_svi2 - start_svi2}")
print(f"Time CAVI: {end_cavi2 - start_cavi2}")

```

```

----- Recall label switching - compare E[theta] and true theta and check f
or label switching -----
E[theta] of doc 0 SVI:      [0.373 0.294 0.333]
E[theta] of doc 0 CAVI:    [0.443 0.432 0.124]

```

```

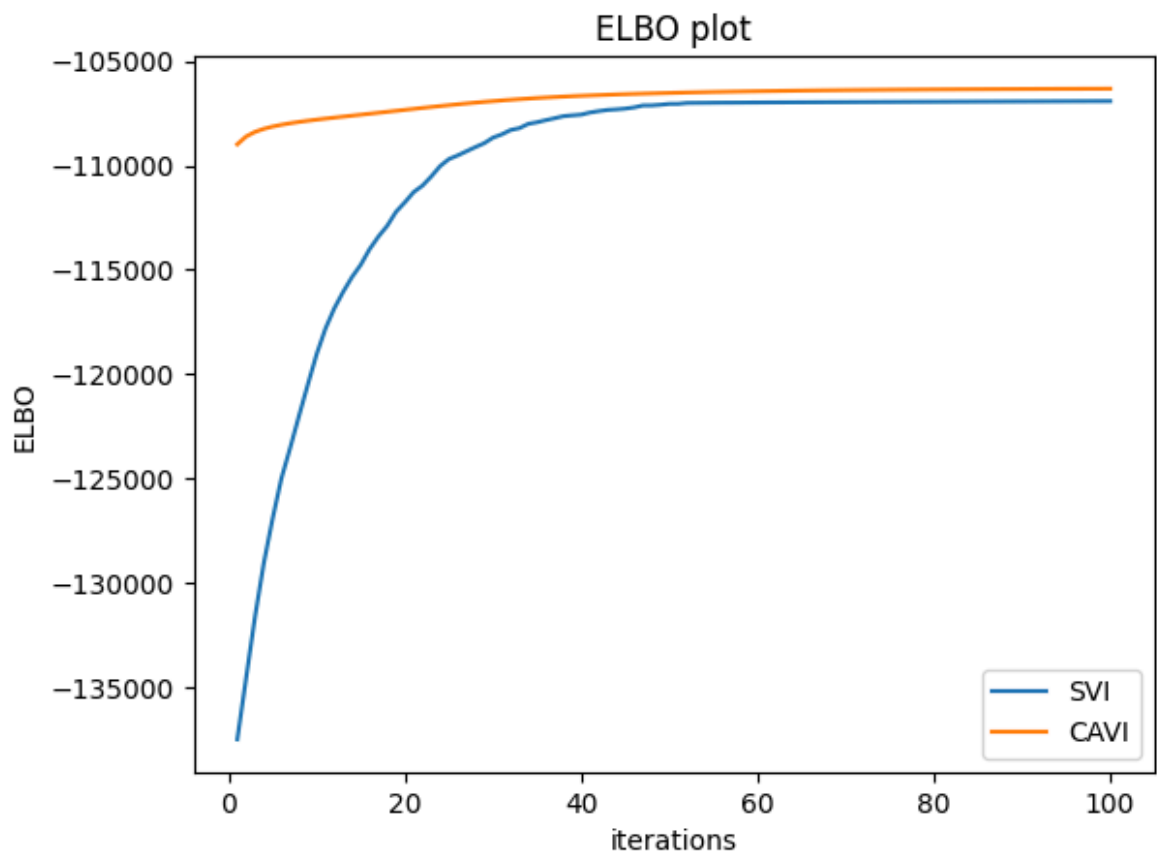
True theta of doc 0:      [0.427 0.019 0.554]
----- Recall label switching - e.g. E[beta_0] could be fit to true theta_
1. -----
E[beta] k=0:      [0.012 0.043 0.045 0.143 0.218 0.143 0.191 0.157 0.035 0.0
13]
E[beta] k=1:      [0.025 0.110 0.182 0.021 0.009 0.387 0.115 0.112 0.014 0.0
26]
E[beta] k=2:      [0.053 0.216 0.015 0.401 0.012 0.087 0.043 0.113 0.041 0.0
18]
True beta k=0:     [0.026 0.182 0.074 0.230 0.030 0.249 0.074 0.102 0.002 0.0
32]
True beta k=1:     [0.030 0.087 0.167 0.057 0.080 0.184 0.148 0.138 0.089 0.0
18]
True beta k=2:     [0.031 0.092 0.006 0.251 0.136 0.194 0.137 0.140 0.002 0.0
11]
Time SVI: 49.16213083267212
Time CAVI: 20.515984058380127

```

```

In [30]: plt.plot(list(range(1, n_iter_cavi2 + 1)), elbo2_svi[np.arange(
0, n_iter_svi2, int(n_iter_svi2 / n_iter_cavi2))])
plt.plot(list(range(1, n_iter_cavi2 + 1)), elbo2_cavi)
plt.title("ELBO plot")
plt.legend(["SVI", "CAVI"])
plt.xlabel("iterations")
plt.ylabel("ELBO")
plt.show()

```



CASE 3

Medium small dataset, one iteration for time analysis.

```

In [32]: np.random.seed(0)

# Data simulation parameters
D3 = 10**4
N3 = 500
K3 = 5
W3 = 10
eta_sim3 = np.ones(W3)
alpha_sim3 = np.ones(K3)

w3, z3, theta3, beta3 = generate_data(D3, N3, K3, W3, eta_sim3,
                                       alpha_sim3)

# Inference parameters
n_iter3 = 1
eta_prior3 = np.ones(W3) * 1.
alpha_prior3 = np.ones(K3) * 1.
S3 = 100 # batch size

start_cavi3 = time.time()
phi_out3_cavi, gamma_out3_cavi, lmbda_out3_cavi, elbo3_cavi = CAVI_algorithm(
    w3, K3, n_iter3, eta_prior3, alpha_prior3)
end_cavi3 = time.time()

start_svi3 = time.time()
phi_out3_svi, gamma_out3_svi, lmbda_out3_svi, elbo3_svi = SVI_algorithm(
    w3, K3, S3, n_iter3, eta_prior3, alpha_prior3)
end_svi3 = time.time()

final_phi3_cavi = phi_out3_cavi[-1]
final_gamma3_cavi = gamma_out3_cavi[-1]
final_lmbda3_cavi = lmbda_out3_cavi[-1]
final_phi3_svi = phi_out3_svi[-1]
final_gamma3_svi = gamma_out3_svi[-1]
final_lmbda3_svi = lmbda_out3_svi[-1]

```

```

In [33]: print(f"Examine per iteration run time.")
print(f"Time SVI: {end_svi3 - start_svi3}")
print(f"Time CAVI: {end_cavi3 - start_cavi3}")

```

```

Examine per iteration run time.
Time SVI: 1.2486693859100342
Time CAVI: 31.095299005508423

```