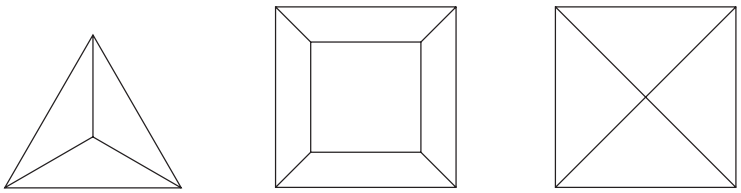
Mindscapes Invitations to Further Thought

In this section, Mindscapes marked **(H)** *have hints for solutions at the back of the book. Mindscapes marked* **(ExH)** *have expanded hints at the back of the book. Mindscapes marked* **(S)** *have solutions.*

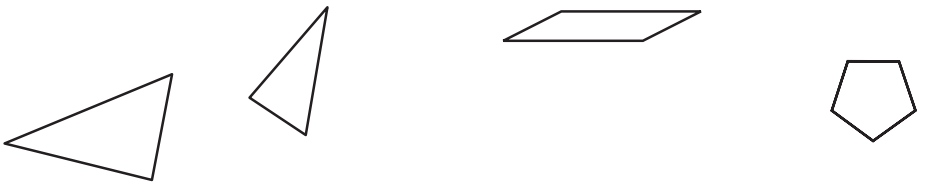
I. Developing Ideas

1. **It's nice to be regular.** What makes a polygon a *regular* polygon? Sketch six different regular polygons and three nonregular ones.
2. **Keeping it Platonic.** What makes a solid a *regular* (*Platonic*) solid?
3. **Count 'em up.** How many faces, edges, and vertices are there in a cube? How many are in a tetrahedron?
4. **Defending duality.** Explain why the cube and the octahedron are duals of each other.
5. **The eye of the beholder.** Suppose you have models of the Platonic solids that are not transparent. Below are sketches of three such solids drawn from different viewpoints. Identify each one and explain the perspective from which each solid was viewed.

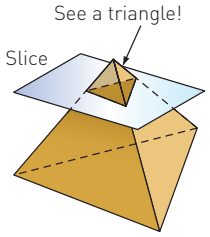


II. Solidifying Ideas

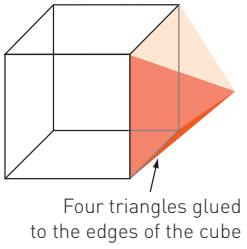
6. **Build them.** Using toothpicks and pieces of plastic hose, strings and straws, posterboard, mahogany, or your kit, make a complete set of the Platonic solids for yourself.
7. **Unfold them.** For each of the Platonic solids, draw a picture showing how to unfold it in various ways so it lies flat on the plane, and how to refold it to create the solid again.
8. **Edgy drawing (H).** Draw pictures in the plane that show the edges of each of the regular solids in the sense that each edge in your drawing corresponds to an edge on the solid and each one joins another one whenever the corresponding edges on the solid touch. For example, you could inflate each regular solid until it becomes a sphere with curved edges on it and then draw the stereographic projection.
9. **Drawing solids.** Draw each solid by completing the beginnings of the drawings shown here.



10. **Life drawing.** Draw the regular solids using your physical models. Include shading and perspective. You may title your work *Regular Still Life*.
11. **Count.** For each of the regular solids, take the number of vertices, subtract the number of edges, add the number of faces. For each regular solid, what do you get?
12. **Soccer counts (ExH).** Look at a soccer ball. Take the number of vertices, subtract the number of edges, add the number of faces. What do you get? This counting can be tricky, so think of a systematic method of accomplishing it.
13. **Golden Rectangles.** Take your toothpick or straw model of an icosahedron and place it on the table. Now prop it up so that only one edge is resting on the table. Locate the Golden Rectangle spanning the edge on the table and the top edge. Locate the pair of vertical edges that form two sides of a Golden Rectangle. Locate a pair of horizontal edges halfway up from the table to the top edge that form two sides of a Golden Rectangle. If you are dexterous, carefully weave pieces of string to construct those three Golden Rectangles. Measure the base and height of one of those rectangles and then divide the height into the base and see how it compares to the Golden Ratio.

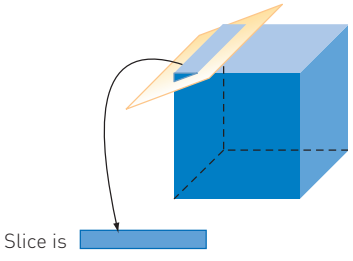


14. **A solid slice (S).** For each regular solid, imagine slicing off a vertex. What shape is the boundary of the cut? For example, slicing a vertex off a tetrahedron gives a triangular cut.
15. **Siding on the cube.** Suppose we start with the edges of a cube (that is, its skeleton). Now, on each square face, we glue four triangles together as shown at right. Sketch a drawing of this figure. Count the number of its vertices, edges, and faces.



III. Creating New Ideas

16. **Cube slices (H).** Consider slicing the cube with a plane. What are all the different-shaped slices we can get? One slice, for example, could be rectangular. What other shaped slices can we get? Sketch both the shape of the slice and show how it is a slice of the cube.
17. **Dual quads (S).** Suppose you have a cube with edges of length 1. Suppose you construct an octahedron inside it whose vertices



are in the centers of the faces of the cube. How long are the edges of that octahedron?

18. **Super dual.** Suppose you take a cube with edges of length 1 and construct an octahedron around it by making the center of each triangle of the octahedron hit at a vertex of the cube. How long are the edges of the octahedron?

19. **Self-duals.** Suppose you have a tetrahedron having each edge of length 1. Construct a tetrahedron inside whose vertices are in the centers of the faces of the original tetrahedron. How long are the edges of that inscribed tetrahedron?

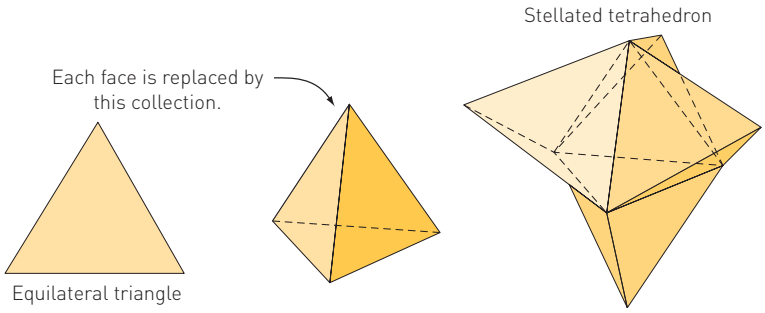
20. **Not quite regular (ExH).** Suppose you allow different numbers of triangles to come together at different vertices of a solid. Show how to produce solids with arbitrarily large numbers of triangular faces.

IV. Further Challenges

21. **Truncated solids.** Slice off all the vertices of each of the regular solids to produce new solids that have two different types of sides. Fill in the chart by counting or computing the number of vertices, edges, and faces each solid now has. Also describe how many faces of each type the truncated solid has.

Solid (pretruncated)	Number of Vertices	Number of Edges	Number of Faces
Tetrahedron			
Cube			
Octahedron			
Dodecahedron			
Icosahedron			

22. **Stellated solids.** Take each regular solid and replace each face by a collection of equilateral triangles (one attached to each face) to produce a new solid that looks like a star:



Fill in the chart by counting or computing the number of vertices, edges, and faces of each.

Solid (prestellated)	Number of Vertices	Number of Edges	Number of Faces
Tetrahedron			
Cube			
Octahedron			
Dodecahedron			
Icosahedron			

V. In Your Own Words

23. **Personal perspectives.** Write a short essay describing the most interesting or surprising discovery you made in exploring the material in this section. If any material seemed puzzling or even unbelievable, address that as well. Explain why you chose the topics you did. Finally, comment on the aesthetics of the mathematics and ideas in this section.
24. **With a group of folks.** In a small group, discuss and actively work through the idea of duality of the Platonic solids. After your discussion, write a brief narrative describing duality in your own words.
25. **Creative writing.** Write an imaginative story (it can be humorous, dramatic, whatever you like) that involves or evokes the ideas of this section.
26. **Power beyond the mathematics.** Provide several real-life issues—ideally, from your own experience—that some of the strategies of thought presented in this section would effectively approach and resolve.