Exploring the consequences of an alternative reality can lead to creative insights.

Seek the essential.

•

Ignore the irrelevant.

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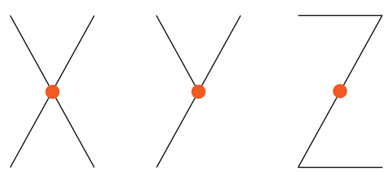
Seek novel ways to compare and contrast objects and issues.

Mindscapes *Invitations to Further Thought*

In the section, Mindscapes marked **(H)** *have hints for solutions at the back of the book. Mindscapes marked* **(ExH)** *have expanded hints at the back of the book. Mindscapes marked* **(S)** *have solutions.*

I. Developing Ideas

1. **Describing distortion.** What does it mean to say that two things are *equivalent by distortion*?
2. **Your last sheet.** You're in your bathroom reading the liner notes for a newly purchased CD. Then you discover that you've just run out of toilet paper. Is a toilet paper tube equivalent by distortion to a CD?
3. **Rubber polygons.** Find a large rubber band and stretch it with your fingers to make a triangle, then a square, and then a pentagon. Are these shapes equivalent by distortion? What other equivalent shapes can you make with the rubber band? Can you stretch it to make a rubber disk?
4. **Out, out red spot.** Remove the red spot from the letters at right. For each letter, how many pieces result? Are the original letters equivalent by distortion?



5. **De-vesting.** Try "The Divestment" challenge. Find a stretchy vest or tank top, put it on, then put on a roomy jacket or sweatshirt. Now see if you can remove the vest without removing the jacket.

II. Solidifying Ideas

6. **That theta (S).** Does there exist a pair of points on the theta curve whose removal breaks the curve into three pieces? If so, the existence of those two points would provide another proof that the circle is not equivalent by distortion to the theta curve. Why?

7. **Your ABCs (H).** Consider the following letters made of 1-dimensional line segments:

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Which letters are equivalent to one another by distortion? Group equivalent letters together.

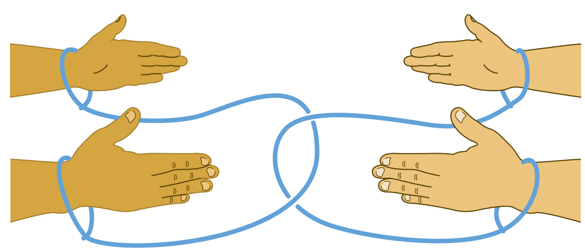
8. **Puzzled?** Find the topology puzzle in your kit. Read the kit instructions for the puzzle and then solve the puzzle. Describe your solution in words and pictures.

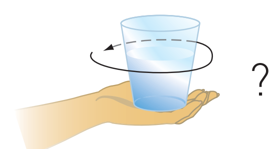
9. **Half dollar and a straw.** Suppose we drill a hole in the center of a silver dollar. Would that coin with a hole be equivalent by distortion to a straw? Explain why or why not.

10. **Drop them.** Is it possible to take off your underwear without taking off your pants? You may assume you are wearing rubber undies. Explain why or why not. Include pictures.

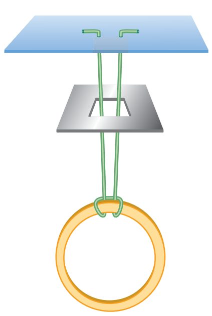
11. **Coffee and doughnuts (H).** Is a standard coffee mug equivalent by distortion to a solid doughnut? Explain why or why not.

12. **Lasting ties.** Tie a thin rope around a friend's wrists, and then tie another one around your own, with the ropes linked as pictured. Can you unlink yourselves without cutting the ropes? Why or why not? (This challenge is one you should definitely make physical!)





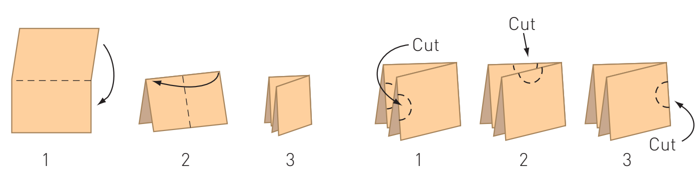
13. **Will you spill? (S).** Suppose you rest a glass of water in the palm of your hand. Is it possible to rotate the palm of your hand 360° without spilling the water or dislocating your arm?



14. **Grabbing the brass ring.** Suppose a string attached to the ceiling passes through a metal, nondistortable frame and then is tied to a brass ring, as illustrated in the picture. Is it possible to remove the brass ring without cutting the string?

15. **Hair care.** Is a regular comb equivalent by distortion to a regular hair pin? Explain why or why not.

16. **Three two-folds.** Take three pieces of paper and fold each in half and then in half again, as shown in the figure below. Suppose you cut out a semicircle from each paper as shown. Are those cut sheets of paper



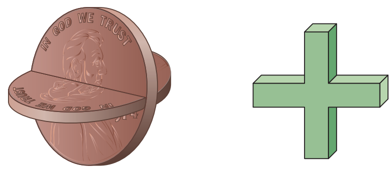
equivalent? Explain why or why not.

Cooking utensils.

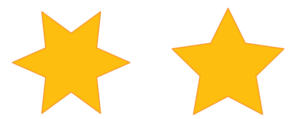
17. Equivalent objects. Group the objects in this photograph into collections that are equivalent by distortion.

18. Clips. Is a paper clip equivalent to a circle? If not, to what other small stationery products is a paper clip equivalent by distortion?

19. Pennies plus. Consider the two objects pictured at right. One is made of two pennies, cut and glued together as shown. The other is a thickened plus sign. Are these two objects equivalent by distortion?

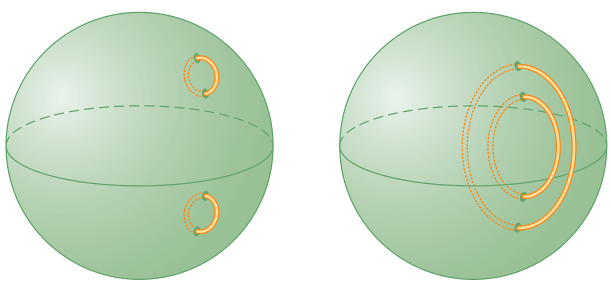


20. Starry-eyed. Consider the two stars at right. Are they equivalent by distortion?

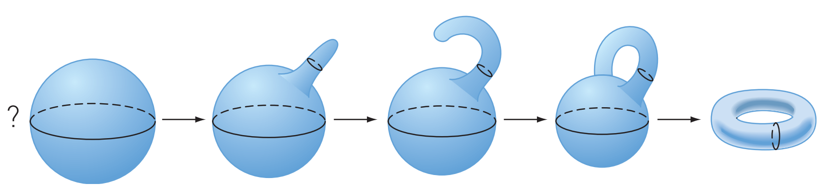


21. Learning the ropes. Pictured at right are two ropes, coiled differently. Are the ropes equivalent by distortion?

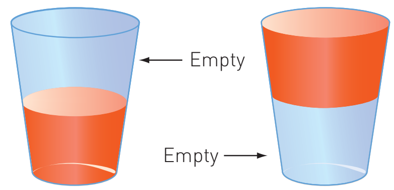
22. Holy spheres. Consider the two spheres below. Each has four holes on its surface. Ropes are looped through the holes in different ways. Is the first sphere with its two rope loops equivalent by distortion to the second sphere with its rope loops?

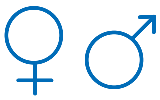


23. From sphere to torus. The following sequence of drawings takes a sphere and deforms it into a torus. Does this sequence describe an equivalence by distortion? Why or why not?



24. **Half full, half empty.** One glass is half filled with cranberry juice as illustrated. Another glass is also half filled with juice, although it is the top half (ignore such pesky issues as gravity). Are the two half-filled glasses with their contents equivalent by distortion? Explain why or why not.

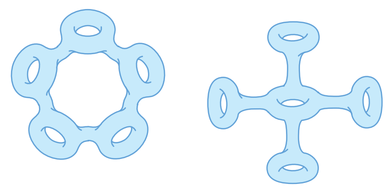




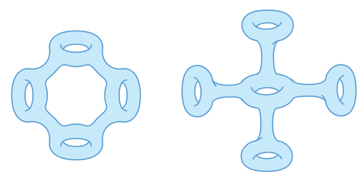
25. **Male versus female.** Consider the male and female symbols at left. Assuming they are made out of 1-dimensional lines and curves, are the symbols equivalent by distortion? Explain why or why not.

III. Creating New Ideas

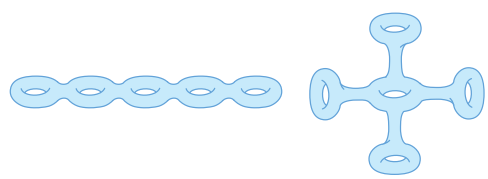
26. **Holey tori.** Are these two objects equivalent by distortion? If so, demonstrate the distortion with a sequence of pictures; if not, explain why not.

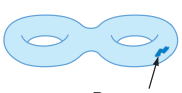


27. **More holey tori (H).** Are these two objects equivalent by distortion? If so, demonstrate the distortion with a sequence of pictures; if not, explain why not.



28. **Last holey tori.** Are these two objects equivalent by distortion? If so, demonstrate the disto-rtion with a sequence of pictures; if not, explain why not.

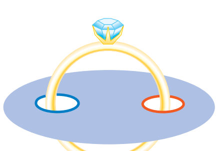




29. **Beyond the holey inner tube.** Suppose you are given a two-holed torus with a large puncture in its side. Is it possible to turn it inside out? Explain why or why not.

30. **Heavy metal.** Carefully examine this picture of metal puzzles. For each, can you unlink them using distortion? You may assume that the metal is rubber.

Metal rings.



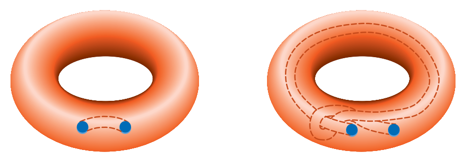
31. **Rings around the ring.** Return to “The Ring” challenge given on page 362. Draw a red circle around one of the holes on the rubber sheet and a blue circle around the other. Now redraw the sequence of moves that unlinks the ring from one of the holes, but on each figure now include

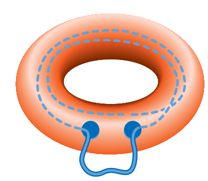
how the two colored circles get deformed. What is the result? Are both colored circles still looped around the ring?

32. **The disk and the inner tube.** Suppose you have a rubber disk with two holes and you glue a tube from one hole to another. Is that object equivalent by distortion to an inner tube with a puncture? Explain why or why not.

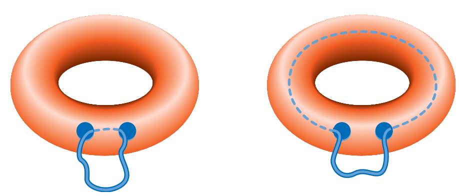
33. **Building a torus (S).** Suppose you are given a rectangular sheet of rubber. How could you glue the various edges together to build a torus? Indicate which edges get glued to which and how the edges match up.

34. **Lasso that hole.** Consider the two tori on the right. Both have two punctures on their sides. On the first torus, a rope is looped through the two holes but does not go around the hole of the torus. On the second, the rope is looped around the hole of the torus. Is it possible to distort the first torus to look like the second? How about if the rope looped around the hole twice, as shown on the right?



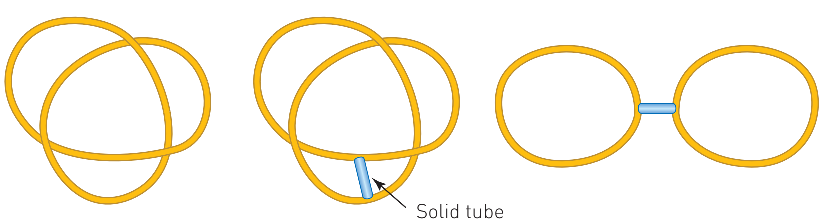


35. **Knots in doughnuts.** We are given two solid doughnuts: one with a worm hole drilled as shown and the other with a knotted worm hole drilled as shown. Are these two-holed doughnuts equivalent?

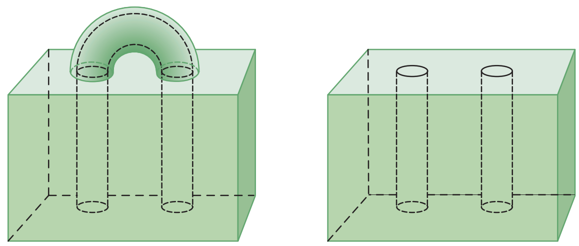


IV. Further Challenges

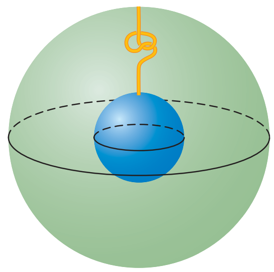
36. **From knots to glasses (ExH).** Take the thickened knot and then add a solid tube, as illustrated below. Is it possible to distort this new object into a pair of eyeglass frames?



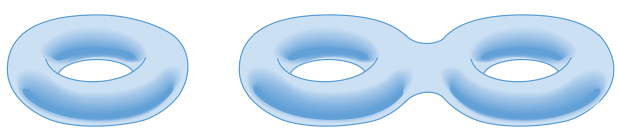
37. **More Jell-O.** Suppose we take a cube of Jell-O, drill two holes in it, and glue a Jell-O tube between them, as shown. Show that this Jell-O object is equivalent to the Jell-O cube on the right with two holes.



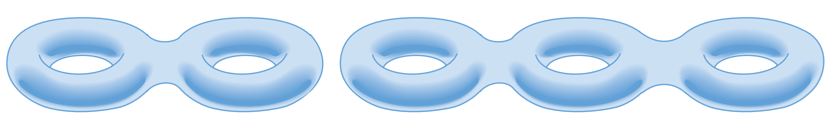
38. **Fixed spheres (H).** We are given two spheres made of glass. They are not distortable, they are rigidly fixed in space—one inside the other as indicated—and they cannot be moved. The middle sphere floats miraculously in midair without any means of support. A rubber rope hangs from the inside ceiling of the big sphere to the roof of the smaller sphere, as illustrated. The rope has a knot in it. Is it possible to unknot the rope without moving or distorting the two spheres?



39. **Holes.** Is a torus equivalent to a two-holed torus? If not, carefully justify your answer. (Consider the strategy used in "Is a Torus a Sphere?" on page 368 but with more than one cut.)



40. **More holes.** Is a two-holed torus equivalent to a three-holed torus? If not, carefully justify your answer. Can you generalize your observations and arguments?



V. In Your Own Words

41. **Personal perspectives.** Write a short essay describing the most interesting or surprising discovery you made in exploring the material in this section. If any material seemed puzzling or even unbelievable, address that as well. Explain why you chose the topics you did.

Finally, comment on the aesthetics of the mathematics and ideas in this section.

42. **With a group of folks.** In a small group, discuss and work through the reasoning for how a two-holed torus with its holes linked can be distorted into an unlinked two-holed torus. After your discussion, write a brief narrative describing the method in your own words.

43. **Creative writing.** Write an imaginative story (it can be humorous, dramatic, whatever you like) that involves or evokes the ideas of this section.

44. **Power beyond the mathematics.** Provide several real-life issues—ideally, from your own experience—that some of the strategies of thought presented in this section would effectively approach and resolve.