Final Exam Review - Answer Key

M302

1. 14 cards

2. No, not necessarily. Since there are 26 letters in the alphabet, all 13 people may have a name that starts with a different letter.

3. $3p$ is never prime.

4. $660 = 2 \times 2 \times 3 \times 5 \times 11$

5. Yes, the sum of two rational numbers is always rational. If a and b are rational, then each of them can be written as a fraction of whole numbers:

$$a = \frac{n}{m} \text{ and } b = \frac{s}{t}$$

where the denominators m and t are not equal to 0.
Then

$$a + b = \frac{n}{m} + \frac{s}{t} = \frac{nt + ms}{mt},$$

which is also a fraction of whole numbers. Note that the denominator mt cannot be equal to 0 since m and t are not equal to 0.

6. False

7. True

8. 1, 0, 50, and -2 are all integers. 2.5 is the only number given that is not an integer.

9. Many possible answers. Here are some examples:

(**a**) The set of all integers
(**b**) The set of all real numbers
(**c**) Any finite set. For example, $\{1, 2, 3\}$.

10. OXXXX...

11. Never

12. There is a one-to-one correspondence between the set S and the set of all natural numbers:

$$\begin{array}{ccccc} 1 & 2 & 3 & 4 & \cdots \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \\ 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \cdots \end{array}$$

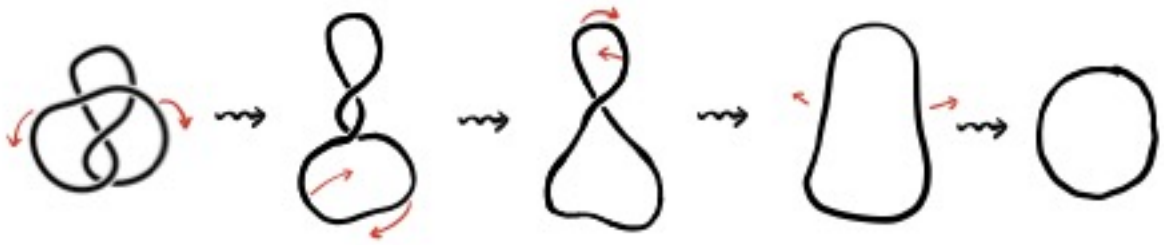
Therefore, the two sets have the same cardinality.

13. A regular polyhedron is a polyhedron with faces that are all congruent, regular polygons and with the same number of faces coming together at every vertex.

14. Many possible answers. Here are a couple examples:

Note that both figures pictured above are hollow.

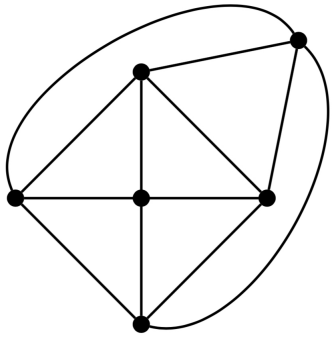
15. To show that two objects are equivalent by distortion, we must show how one can be deformed into the other without cutting or gluing.



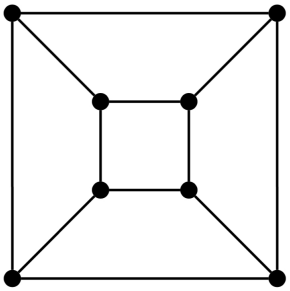
Grab the piece on top and pull it down. Then untwist the string by holding the left piece on the bottom and pulling it up and to the left. Repeat with the top to finish untwisting. Then pull out the sides to make the shape more round, forming a circle.

16.

(**a**) Many possible answers. Every vertex of the graph must have degree that is an even number. Here is one example:



(**b**) Many possible answers. At least one vertex of the graph must have degree that is an odd number. Here is one example:



17. This graph has 4 vertices, 5 edges, and 3 faces, so the Euler characteristic is $4 + 3 - 5 = 2$.

18. After 3 years you would owe $\$8,215.16$.

After 1 year: $(5000)(1.18) = \$5,900$
After 2 years: $(5000)(1.18)(1.18) = (5000)(1.18)^2 = \$6,962$
After 3 years: $(5000)(1.18)(1.18)(1.18) = (5000)(1.18)^3 = \$8,215.16$

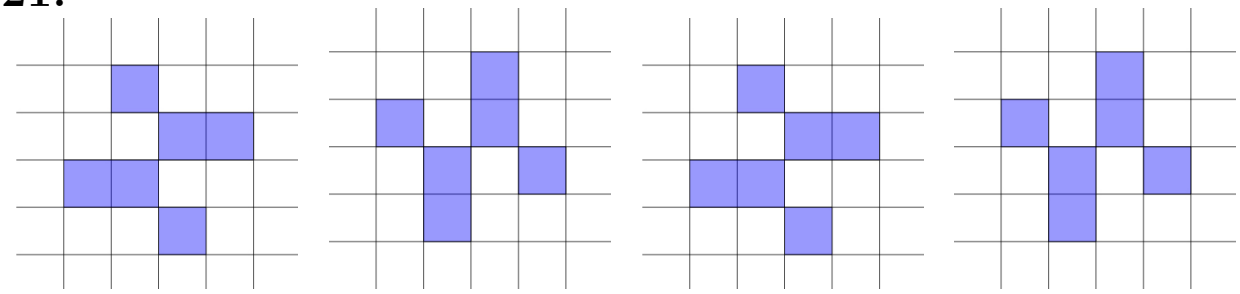
19.

(**a**) $\dfrac{200}{250} = 0.8$

(**b**) $\dfrac{300}{250} = 1.2$

20. The population will increase slowly.

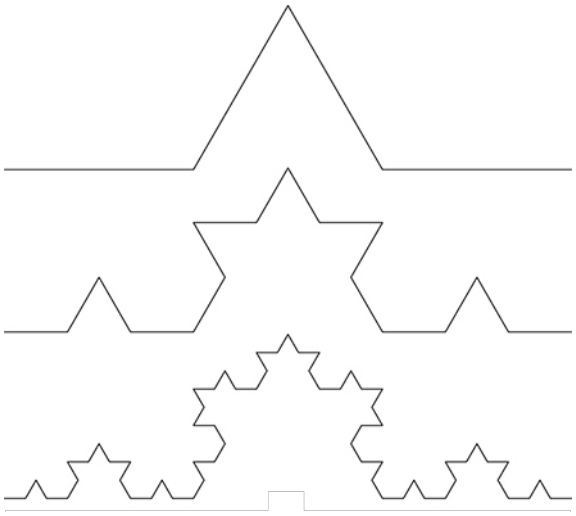
21.



Initial Population | Generation 1 | Generation 2 | Generation 3

22.

(**a**)



(**b**)

Stage 1 has 4 line segments.
Stage 2 has 4^2 line segments.
Stage 3 has 4^3 line segments.
Stage n will have 4^n line segments.

23. Answers will vary. Some examples in nature are broccoli, ferns, and tree branches.