Final Exam Review

M302

1. From a standard deck of 52 cards, what is the smallest number of cards you must draw in order to guarantee that you will have at least one pair?

2. In a group of 13 people, will there be at least two people whose first names start with the same letter?

3. Suppose p is a prime number greater than 3. Is $3p$ prime?

(**a**) Always (**b**) Sometimes (**c**) Never

4. Write 660 as a product of primes.

5. Suppose a and b are rational numbers. Is $a + b$ rational? Explain.

6. **True or False**: Every rational number can be written as a terminating decimal.

7. **True or False**: Between any two distinct rational numbers, there is an irrational number.

8. Which of the following numbers is an integer? *Choose all that apply.*

(**a**) 2.5 (**b**) 1 (**c**) 0 (**d**) 50 (**e**) -2

9. Consider the set of natural numbers:

$$\mathbb{N} = \{1, 2, 3, 4, \dots\}$$

(**a**) Give an example of a set that has the same cardinality as $\mathbb{N}$.

(**b**) Give an example of a set that has a greater cardinality than $\mathbb{N}$.

(**c**) Give an example of a set that has a smaller cardinality than $\mathbb{N}$.

10. Suppose you are playing an infinite game of Dodgeball. Given the beginning of this sequence from Player 1, write the first five places of a winning sequence as Player 2.

XOXOX...
OOXOX...
XOOOO...
OXXOO...
XOXOO...

11. A triangle has sides of length 2, 5, and 6. Is the triangle a right triangle?

(**a**) Always (**b**) Sometimes (**c**) Never

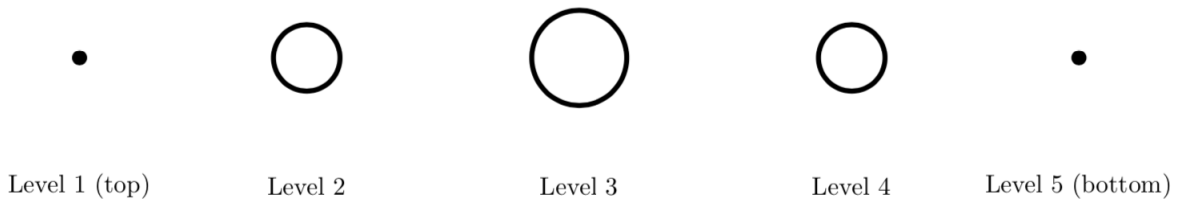
12. Consider the set

$$S = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots\right\}.$$

Prove that the set S has the same cardinality as the set of natural numbers.

13. Explain what it means for a polyhedron to be a regular polyhedron.

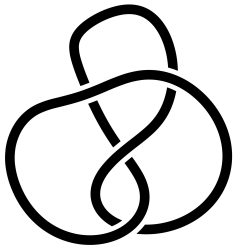
14. Pictured below are slices of a 2-dimensional object in 3-dimensional space.



Level 1 (top) Level 2 Level 3 Level 4 Level 5 (bottom)

What could this object be?

15. Show that the string pictured below is equivalent by distortion to a circle.

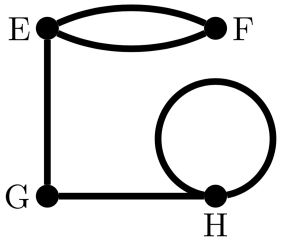


16.

(**a**) Draw a graph with at least five vertices that has an Euler circuit.

(**b**) Draw a graph with at least five vertices that does not have an Euler circuit.

17. Determine the Euler characteristic for the following graph:



18. Suppose you charge $\$5,000$ on a credit card, with an intereste rate of 18% per year. If you pay nothing for 3 years, how much would you owe?

19. A small forest can sustain a maximum of 250 of a certain species of bird.

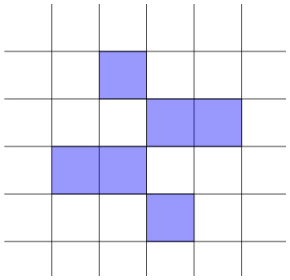
(**a**) What is the population density when there are 200 birds in the forest?

(**b**) What is the population density when there are 300 birds in the forest?

20. In the Verhulst population model, what will happen to a population that is close to the maximum sustainable population?

(**a**) The population will increase without bound.
(**b**) The population will decrease.
(**c**) The population will die out completely.
(**d**) The population will increase slowly.

21. Given the following intitial population in the Game of Life, determine the next three generations.



22. The Koch curve is constructed as follows:

1. Start with a line segment.
2. Divide the line segment into thirds.
3. Draw an equilateral triangle with the middle segment as the base.
4. Remove the middle segment.
5. Repeat with each line segment, forever!

(**a**) Start with a single line segment as stage 0, and draw the next three stages of the Koch curve.

(**b**) How many line segments will the Koch curve have at stage n?

23. Find an example of self-similarity in nature or art. Describe your example.