

## 1.10 - Complexity Analysis

Wednesday, September 10, 2025 8:07 PM

↳ Time Analysis  
↳ Space complexity

### # Time Complexity Analysis

↳ A normal PC can do  $10^8$  operations in 1 sec  
[lines of code]

→ Our goal in DSA is to write a code that can execute in approx 1 sec

ops/sec {  
    cout ("Hello");  
    cout ("Hello");  
    cout ("Hello");  
}

loops {  
    for (i=0; i<10; i++) {  
        cout ("Hello");  
    }  
}

$10^{10}$

1 →  $10^8$

$10^8 \rightarrow 1 \text{ sec}$   
 $1 \rightarrow \frac{1}{10^8}$

$10^{11}$   $10^{12}$   $10^{13}$   $10^{14}$

✓ 1 sec

$$1 \rightarrow \frac{1}{10^8}$$

$$10^{10} \rightarrow \frac{1}{10^8} \times 10^{10} = 10^2$$

$$10 \rightarrow 20$$

$$1 \rightarrow \frac{20}{10}$$

$$40 \rightarrow \frac{20 \times 40}{10}$$

$$10^8 \rightarrow 1$$

$$1 \rightarrow \frac{1}{10^8}$$

$$10^{10} \rightarrow \frac{1}{10^8} \times 10^{10} = 100$$

# Big O  $\rightarrow$  notation

$\rightarrow$  It tells you the worst case complexity of an algo.

$\textcircled{n}$   $\rightarrow$   $\left\{ \begin{array}{l} \text{for } (i=0; i < n; i++) \\ \text{sort ("Hello")} \end{array} \right\} \rightarrow O(n)$

$\rightarrow$   $n \leq 10^8$

$\left\{ \begin{array}{l} \text{for } (i=0; i < 10; i++) \\ \text{sort ("Hello")} \end{array} \right\} \rightarrow O(10)$

$\rightarrow$   $O(1)$  (constant)

# # Rules of Big O

1)  $O(10) = O(80) = O(k) \Rightarrow \boxed{O(1)}$  constant

2)  $O(n) = O(2n) = O(3n) = O(k \times n) \Rightarrow \underline{O(n)}$

$[O(\underline{n^2})] \rightarrow 10^8$   
 $n^2 \leq 10^8 \Rightarrow \boxed{n \leq 10^4}$   
 $O(n^4) \quad n^4 \leq 10^8 \quad n \leq 10^2$   
 $n^2 \leq 10^8 \Rightarrow \boxed{n \leq 10^4}$

[Order of speed]

$O(1) < O(\sqrt{n}) < O(n) < O(n^2) < O(n^3) < O(n^4) \dots$

$O(\sqrt{n})$   
 $\sqrt{n} \leq 10^8$   
 $\boxed{n \leq 10^{16}}$

$i \rightarrow i$   
 $1 \rightarrow 2 \rightarrow 4$   
 $1 + 2^1 + 2^2 + \dots + 2^{n-1}$   
 slow of bro.  
 $(\log 2^4)$

\* log

$\log a b \Rightarrow$  a ki power kya kare  
 kin answer b ho

$\log_2(8)$   
 $\Rightarrow 3$

$\log_2 2 \Rightarrow 3 \times 2 \Rightarrow 6$

$\log_2 N$   $L=10$

$N = 2^{108}$

$O(1) \subset O(\log n) \subset O(\sqrt{n}) \subset O(n) \dots$

How to analyse an entire program

$O(n)$   
 $+$   
 $O(n^2)$   
 $\Rightarrow$   
 $O(n^2)$

```

[ for (i=0; i<n; i++)
    sort("Hello")
]

[ for (i=0; i<n; i++)
    for (j=0; j<n; j++)
        sort("Hello")
    ]
    
```

$n < 10^8$   
 $n < 10^6$

Time Complexity

$$O(n + n^4)$$

$\Rightarrow$

$$n < 10^8$$

$$n < 10^9$$

$$O(n)$$

$$+ O(n^4)$$

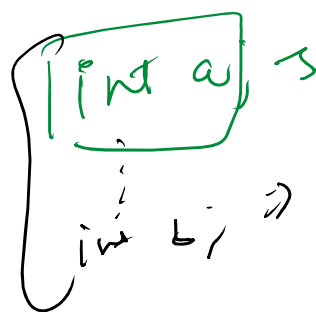
⊕ It's the worst component that decides the overall complexity of a prog.

$$O(n + \sqrt{n} + n^2 + n + n^3) \Rightarrow O(n^3)$$

$$\Rightarrow O(n + n) = O(2n) \Rightarrow O(n)$$

# Space Complexity  $\rightarrow$  Big O

$\Rightarrow$  Worst case space that my program is going to take.



$$4 \text{ Bytes} \Rightarrow O(4) \Rightarrow O(1)$$

$$400 \Rightarrow O(1)$$

int arr [ ] = new int [n]

$$O(n)$$

int arr [ ] = new int [100]

int arr[] = new int [100];  
 $\Downarrow$   
 $O(1)$

4 byte  
 (int a = sc.next Int());

int arr[] = new int[n];

space  
 $O(n)$

for (i = 0; i < arr.length; i++)  
 arr[i] = sc.nextInt();  
 }

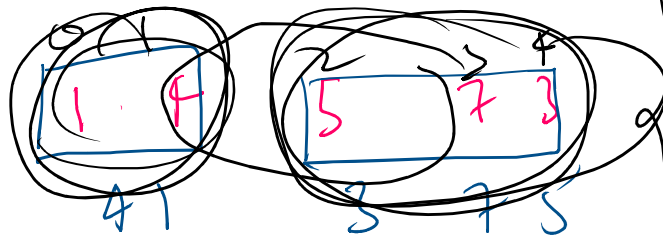
[3, 7, 5, 4, 1] 2)

$\Downarrow$

k=2

4 1 3 7 5

1 3 7 5 4  
 4 1 3 7 5



rev(0, n-1)  
 rev(0, k-1)  
 rev(k, n-1)

reverse(arr, 0, n-1)

1 3

$$\left\{ \begin{array}{l} k \in K \cap h \\ f(k, \omega) \\ k \in K \cap h \end{array} \right\}$$

$\rightarrow 0 - u_i$

$$u_1 = u_2$$

