Simple Linear Regression (Supervised ML Algo)

Introduction

Linear Regression

- Simple LR
- Multiple LR
- Polynomial LR
- Regularization

Simple LR : 1 input | 1 output

cgpa	Package

cgpa	Gender	12th Marks	State	Package

→ Multiple LR

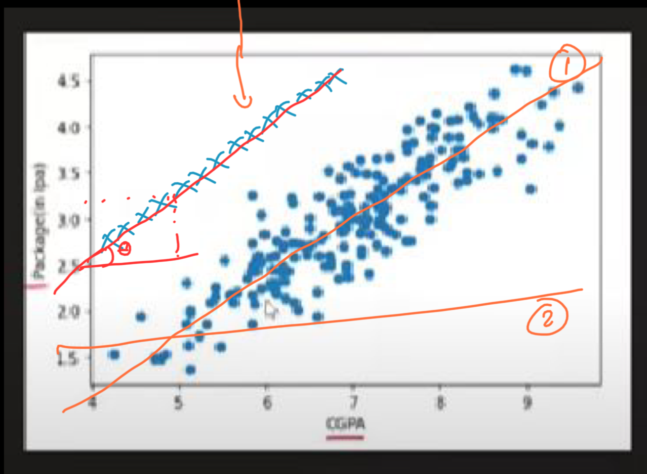
Simple Linear Regression

cgpa	Package
7.1	3.5
4.7	1.2
8.9	4.2
8.1	3.9
⋮	⋮

cgpa → Model → Package

Completely linear

Sort of linear data



Not completely linear: why

why : (i) Real world data
↳ Stochastic error

⇒ Linear regression a very Simple algorithm which work is to draw best fit line over linear or Sort of linear data.

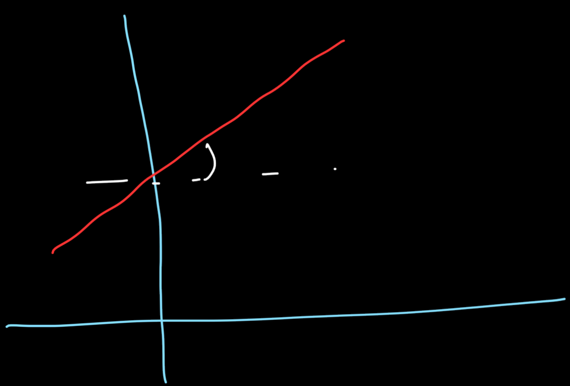
↳ Close to each and every point (Error = Actual − Predicted) should minimum

Best fit line ⇒ Algorithm give m & c

$\boxed{y = mx + c}$

c − intercept
m − slope

Intuition



$y = mx + c$

$\boxed{Package = m \times cgpa + c}$

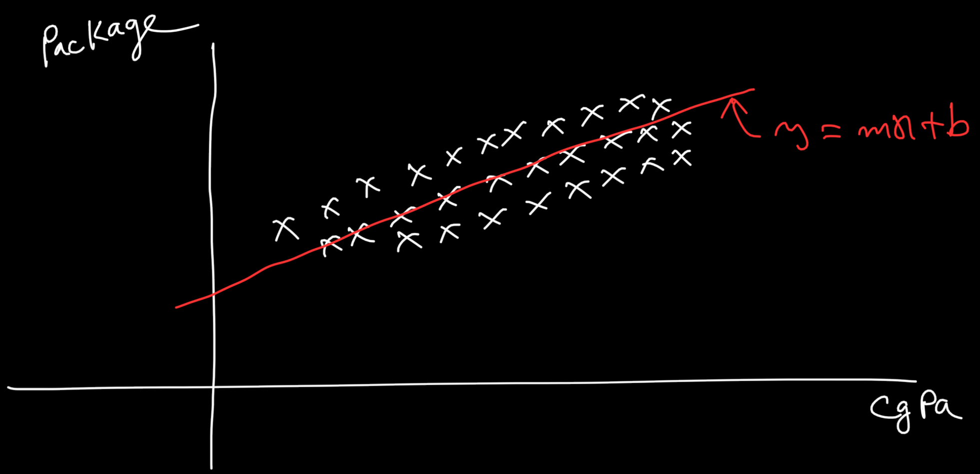
m → slope (weightage)
c → intercept (offset)

Suppose:
Package = m × Experience + c
Let c = 0
Now P = m × E
for fresher, Experience = 0
then acc to P = m × E, Package for fresher should be 0 (zero).
Which is not right. That's why c (intercept) is there. It is offset. It make sure that if input is zero then also output should not be zero.

m → weightage
(How much output depends over input)

c → offset
(If input is zero then also there should be some output)

How to find m and b ?



(m, b)

direct formula

Closed form Solution

↓

$\boxed{\text{OLS}}$

Ordinary Least Square

Non-Closed form solution

↳ $\boxed{\text{Gradient descent}}$

Y → Pack

X → cgpa

$$m = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

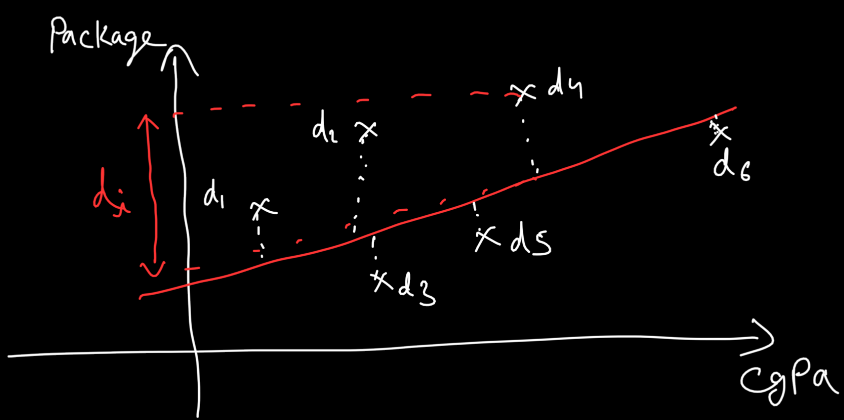
$$b = \bar{y} - m\bar{x}$$

$\bar{x}, \bar{y}$ → Mean of x & y

Given linear or sort of linear data

↓

find Best fit line (find m & c)



$$E = d_1 + d_2 + d_3 + \cdots + d_n$$

$$E = d_1^2 + d_2^2 + d_3^2 + \cdots + d_n^2$$

why not

$$E = |d_1| + |d_2| + \cdots + |d_n|$$

why not

$E = |d_1| + |d_2| + \cdots + |d_n|$

Reason : → we want to penalize outliers
→ Mod is continuous but not differentiable at origin.

$\boxed{\text{Square is differentiable at every point}}$

$$\boxed{E = \sum_{i=1}^{n} d_i^2} \leftarrow \text{Error function}$$

we want m and b such that error function should be minimized.

Actual error is happening in terms of y (Package)

$$\therefore \quad d_i = (y_i - \hat{y}_i)$$

Error will now become

$$\boxed{E = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2}$$

$$\hat{y}_i = m x_i + b$$

$$\boxed{E(m,b) = \sum_{i=1}^{n} (y_i - m x_i - b)^2}$$

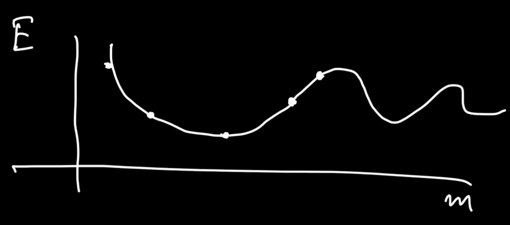
find m and b such that E(m, b) should be minimum.

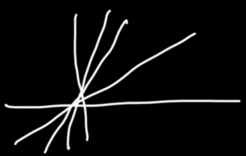
$y = f(x)$ (y is a function of x, y changes when x changes)

Here $E = f(m,b)$ (Error will change whenever m or b or m and b both changes)

If $b = 0$

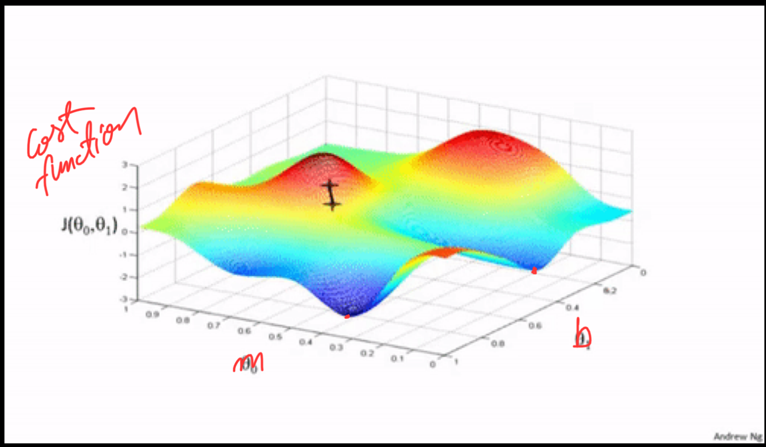
$$E(m) = \sum_{i=1}^{n} (y_i - m x_i)^2$$





suppose $m = 1$

$$E(b) = \sum_{i=1}^{n} (y_i - x_i - b)^2$$



$\boxed{\text{In minima slope} = 0}$

for slope find derivative and equate it with zero.

Suppose $E(\eta)$ $\rightarrow$ E is a function of η

then $\frac{dE}{d\eta} = 0$

Here $E(m, b)$

$$\frac{\partial E}{\partial m} = 0 \quad , \quad \frac{\partial E}{\partial b} = 0$$

$$\frac{\partial E}{\partial b} = \frac{\partial}{\partial b} \sum_{i=1}^{n} (y_i - m x_i - b)^2 = 0$$

$$\Rightarrow \sum \frac{\partial}{\partial b} (y_i - m x_i - b)^2 = 0$$

$$\Rightarrow -2(y_i - m x_i - b) = 0$$

$$\Rightarrow \sum (y_i - m x_i - b) = 0$$

$$\Rightarrow \sum y_i - \sum m x_i - \sum b = 0$$

dividing both side by n

$$\Rightarrow \frac{\sum y_i}{n} - \frac{\sum m x_i}{n} - \frac{\sum b}{n} = \frac{0}{n}$$

$$\Rightarrow \bar{y} - m \frac{\sum x_i}{n} - \frac{nb}{n} = 0$$

$$\Rightarrow \bar{y} - m \bar{x} - b = 0$$

$$\Rightarrow \boxed{b = \bar{y} - m \bar{x}}$$

If we are trying to update η by a factor epsilion x in order to minimize $f(\eta)$, and we know the derivative of f, then our Job is done: the derivative Completely describe how $f(\eta)$ evolves as we change η. If we want to reduce the value of $f(\eta)$, we just need to move η a little in the opposite direction from the derivative.

$$E = \sum (y_i - mx_i - \bar{y} + m\bar{x})^2$$

$$\frac{\partial E}{\partial m} = \sum \frac{\partial}{\partial m} (y_i - mx_i - \bar{y} + m\bar{x})^2 = 0$$

$$\Rightarrow \sum 2 (y_i - mx_i - \bar{y} + m\bar{x}) (-x_i + \bar{x}) = 0$$

$$\Rightarrow \sum -2 (y_i - mx_i - \bar{y} + m\bar{x}) (x_i - \bar{x}) = 0$$

$$\Rightarrow \sum (y_i - mx_i - \bar{y} + m\bar{x}) (x_i - \bar{x}) = 0$$

$$\Rightarrow \sum \left[(y_i - \bar{y}) - m(x_i - \bar{x}) \right] (x_i - \bar{x}) = 0$$

$$\Rightarrow \sum (y_i - \bar{y})(x_i - \bar{x}) = m \sum (x_i - \bar{x})$$

$$\Rightarrow \boxed{m = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})}}$$