

## Trabajo Práctico N° 3

### Integrales Múltiples

#### 1. Resultados:

- (a)  $\int_1^2 \int_0^1 (4 + 5xy) dx dy = \frac{31}{4}$
- (b)  $\int_0^1 \int_0^3 \sqrt{x+y} dx dy = -\frac{4}{15}(-31 + 9\sqrt{3})$
- (c)  $\int_3^4 \int_3^4 (\frac{x}{y} + \frac{y}{x}) dx dy = 7 \ln \frac{4}{3}$
- (d)  $\int_0^{\ln 5} \int_0^{\ln 2} e^{4x-y} dx dy = 3$
- (e)  $\int_1^3 \int_0^2 \int_0^{1-z^2} 4ze^{3y} dx dz dy = -\frac{8}{3}e^3(-1 + e^6)$
- (f)  $\int_0^1 \int_0^1 \int_0^1 \frac{1}{x+y+z+1} dx dy dz = \ln \left( \frac{4194304}{1594323\sqrt{3}} \right)$
- (g)  $\int_0^2 \int_0^{2\sqrt{x}} \int_0^{\sqrt{\frac{4x-y^2}{2}}} x dz dy dx = \frac{4}{3}\sqrt{2}\pi$
- (h)  $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} xyz dz dy dx = \frac{1}{720}$

#### 2. Resultados:

- (a)  $\iint_R xye^y dA$ , donde  $R = \{(x, y) \in \mathbb{R}^2 | 0 \leq x \leq 2, 0 \leq y \leq 1\}$   
Rta:  $\iint_R xye^y dA = 2$
- (b)  $\iint_R \frac{4+x^2}{1+y^2} dA$ , donde  $R = \{(x, y) \in \mathbb{R}^2 | 0 \leq x \leq 3, 0 \leq y \leq 1\}$   
Rta:  $\iint_R \frac{4+x^2}{1+y^2} dA = \frac{21}{4}\pi$
- (c)  $\iint_R \frac{xy^2}{x^2+4} dA$ , donde  $R = \{(x, y) \in \mathbb{R}^2 | 0 \leq x \leq 2, -2 \leq y \leq 2\}$   
Rta:  $\iint_R \frac{xy^2}{x^2+4} dA = \frac{8}{3} \ln 2$
- (d)  $\iint_R \frac{y}{x^2+y^2} dA$ , donde  $R = [1, 2] \times [-1, 1]$   
Rta:  $\iint_R \frac{y}{x^2+y^2} dA = 0$

3. Rta:  $\iint_R xy dA = \frac{2}{3}$

4. Rta:  $\iint_R (x^2 + y^2) dA = \frac{1207}{210}$

5. Rta:  $I = \int_2^4 \int_y^4 f(x, y) dx dy + \int_0^2 \int_y^{x+2} f(x, y) dx dy + \int_{-1}^0 \int_{-\sqrt[3]{y}}^{y+2} f(x, y) dx dy$

6. Rta:  $I = \int_0^1 \int_{-2+\frac{\sqrt{4-y}}{2}}^{y-1} dx dy + \int_1^4 \int_{-2+\frac{\sqrt{4-y}}{2}}^0 dx dy + \int_4^6 \int_{y-6}^0 dx dy$ .

7. Rta:  $V = \frac{5}{12}$ .

8. Rta:  $V = \frac{83}{3} - 5\pi$ .

9. Rta:  $\iiint_Q 2x \cos(y+z) dV = -48 + 12\pi^2 - \pi^4$ .
10. Rta:  $V = \frac{4}{15}$ .
11. Rta:
- (a)  $V_Q = \frac{395}{8}$
- (b)  $V_Q = 8\pi$
12. Rta:  $\iint_R (x^2 - y) dA = \frac{7}{2}$ .
13. Rta:  $\iint_R \frac{y^2}{x^2} dA = \frac{143}{12}$ .
14. Rta:  $A = \pi + \frac{3}{4}\sqrt{3}$ .
15. Rta:  $V = \pi \ln 2$
16. Rta:  $\iint_R \frac{xy}{(1+x^2+y^2)^2} dA = \frac{\ln 4 - 1}{8}$ .
17. Rta:  $\iiint_Q \frac{1}{\sqrt{x^2+z^2+1}} dV = \frac{\pi}{12}(31 - 18 \ln 3)$ .
18. Rta:  $\iiint dV = \int_0^{2\pi} \int_0^R \int_{-\sqrt{R^2-r^2}}^{\sqrt{R^2-r^2}} r dz dr d\theta = \frac{4}{3}\pi R^3$ .
19. Rta:
- (a)  $\iiint_Q 2z dV = \frac{\pi}{2}$ .
- (b)  $V_Q = \frac{\pi}{3}$ .
20. Rta:  $\iiint_Q (x^2 + y^2) dV = \frac{28}{15}\pi$ .
21. Rta:  $V = \frac{4}{21}\pi$ .
22.  $V = 16\pi^2$ .
23.  $x_{CM} = \frac{1}{2}, y_{CM} = \frac{2}{5}$ .
24.  $x_{CM} = 0, y_{CM} = \frac{45}{14\pi}$ .