Strat 1 : Line 6   / Strat 2 : Line 16

$$\sum_{i=1}^{n-1}\sum_{j=i}^{n-1} 1 \;=\; \sum_{i=1}^{n-1}\sum_{j=1}^{n-i} 1 \;=\; \sum_{i=1}^{n-1}(n-i+1)$$

$$= n\sum_{i=1}^{n-1} - \sum_{i=1}^{n-1} i + \sum_{i=1}^{n-1} 1 \;=\; n(n-1) - \frac{n(n-1)}{2} + n - 1$$

$$= \underbrace{n^2}_{} \cancel{/n} - \underbrace{n^2/2}_{} + n/2 \cancel{/n} - 1$$

$$= n^2/2 + n/2 - 1$$

$$= n(n+1)/2 - 1$$

Strat 2 : T(n) Total

$1 + 1 + n + 4n - 4 + n - 1 + 3n - 3 + 5n - 5 + 4n - 4 + 2n - 2$
$+ 2n - 2 + 1 + 1 + 1 + 1 + 1 + n + n(n+1)/2 - 1$
$+ 2n^3 - 3n^2 + n + 12n^3 - 21n^2 + 9n + 10n^3 - 15n^2 + 5n$
$+ 8n^3 - 12n^2 + 4n + 4n^3 - 6n^2 + 2n + 1$

$= -14 + n + 4n + n + 3n + 5n + 4n + 2n + 2n + n$
$+ n^2/2 + n/2 + 2n^3 - 3n^2 + n + 12n^3 - 21n^2 + 9n$
$+ 10n^3 - 15n^2 + 5n + 8n^3 - 12n^2 + 4n + 4n^3 - 6n^2 + 2n$

$= -14 + \frac{89}{2}n + n^2/2 + 2n^3 - 3n^2 + 12n^3 - 21n^3 + 18n^3 - 15n^3$
$+ 8n^3 - 12n^2 + 4n^3 - 6n^2$

$= -14 + \frac{89}{2}n - \frac{113}{2}n^2 + 36n^3$

$= \frac{1}{2}(72n^3 - 113n^2 + 89n - 28)$

Strat 1: Line 7   /   Strat 2: Line 17

$$\sum_{i=1}^{n-1} \sum_{j=i}^{n-1} \sum_{k=i}^{j} 1 = \sum_{i=1}^{n-1} \sum_{j=i}^{n-1} j - i + 1$$

$$= \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} j - i + 1$$

$$= \sum \left(\sum j - \sum i + \sum 1 \right)$$

$$= \sum \left(\sum j - i \sum 1 + \sum 1 \right)$$

$$= \sum \left(\frac{n(n-1)}{2} - i(n-i) + (n-i) \right)$$

$$= \sum \left(\frac{n^2}{2} - \frac{n}{2} - ni + i^2 + n - i + \frac{n}{2} \right)$$

$$= \frac{1}{2} \sum n^2 - n \sum i + \sum i^2 - \sum i + \frac{1}{2} \sum n$$

$$= \frac{1}{2} n^2 (n-1) - n \left(n(n-1)/2 \right) + n(n-1)(2n-1)/2$$

$$\hookrightarrow - n(n-1)/2 + \frac{1}{2} n(n-1)$$

$$= \frac{n^3}{2} - \frac{n^2}{2} - \frac{n^3}{2} + \frac{n^2}{2} + 2n^3 - 3n^2 + n - \frac{n^2}{2} + \frac{n}{2} + \frac{n^2}{2} - \frac{n}{2}$$

$$= 2n^3 - 3n^2 + n$$

Strat 1 : Line 8 / Strat 2: Line 18 *

* Worst Case

$$\sum_{i=1}^{n-1} \sum_{j=i}^{n-1} \left(1 + j - i + 1 \right)$$

$$= \sum_{i=1}^{n-1} \left(\frac{n(n-1)}{2} - i(n-i) \right)$$

$$= \sum \left(\frac{n^2}{2} - \frac{n}{2} - ni + i^2 \right)$$

$$= \frac{n^2}{2} \sum 1 - \frac{n}{2} \sum 1 - n \sum i + \sum i^2$$

$$= \frac{n^2(n-1)}{2} - \frac{n(n-1)}{2} - n\left(\frac{n(n-1)}{2} \right) + \frac{n(n-1)(2n-1)}{2}$$

$$= \frac{n^3}{2} - \frac{n^2}{2} - \frac{n^2}{2} + \frac{n}{2} - \frac{n^3}{2} + \frac{n^2}{2} + 2n^3 - 3n^2 + n$$

$$= 2n^3 - \frac{7}{2}n^2 + \frac{3}{2}n$$

Strat 1 : T(n) Total

$1+1+1+1+n \quad + n(n+1)/2 - 1 + 2n^3 - 3n^2 + n$
$+ 12n^3 - 21n^2 + 9n + 10n^3 - 15n^2 + 5n$
$+ 3(4n^3 - 6n^2 + 2n) + 2n^3 - 3n^2 + n + 1$

$= 4 + n + n^2/2 + n/2 + 2n^3 - 3n^2 + n + 12n^3 - 21n^2 + 9n$
$+ 10n^3 - 15n^2 + 5n + 12n^3 - 18n^2 + 6n + 2n^3 - 3n^2 + n$

$= 4 + \frac{47}{2}n + n^2/2 + 2n^3 - 3n^2 + 12n^3 - 21n^2 + 10n^3 - 15n^2$
$+ 12n^3 - 18n^2 + 2n^3 - 3n^2$

$= 4 + \frac{47}{2}n - \frac{119}{2}n^2 + 38n^3$

$= \frac{1}{2}(76n^3 - 119n^2 + 47n + 8)$