Work Sheet 1 **August. 08. 2019**
Course Name: Introduction to Linear and Non Linear Programming Lab
Course Number: CS4101
Name of Instructor: Pritam Rooj and Tapan Naskar
Total Marks: 10

First Order Derivative:

Forward Difference Formula:

$$f'(x) = \frac{f(x+h) - f(x)}{h} \tag{1}$$

Backward Difference Formula:

$$f'(x) = \frac{f(x) - f(x-h)}{h} \tag{2}$$

Central Difference Formula:

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} \tag{3}$$

Second Order Derivative

Forward Difference Formula:

$$f''(x) = \frac{f(x+2h) - 2f(x+h) + f(x)}{h^2} \tag{4}$$

Backward Difference Formula:

$$f''(x) = \frac{f(x) - 2f(x-h) + f(x-2h)}{h^2} \tag{5}$$

Central Difference Formula:

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} \tag{6}$$

1. Find the numerical derivative $f'(x)$ of the given functions $f(x)$ below, using the given formulas. Then compare the result with actual calculated derivative using calculus.

   (a) Find $f(x) = x - \ln(x)$

x	h	$f(x)$	$f'(x)$ Computed	$f'(x)$ Calculated	Error
0.09	0.001				
0.09	0.0001				
0.09	0.00001				
0.09					$\sim 10^{-10}$
0.09					$\sim 10^{-15}$

   (b) Change the value of x and see if magnitude of error is same or different for the given values of h. Make similar tables. What changes, if any in percentage error, defined as: Percentage Error=$\frac{f'\text{Computed} - f'\text{Calculated}}{f'\text{Calculated}}$.

(c) Is magnitude of error depends on h and x? What about percentage error?

(d) Keeping x fixed at a certain value, plot h vs percentage error. Repeat this with different values of x.

(e) Write your numerical code such way that it can be reuse later, just by changing the function $f(x)$, variable x and step size h.

2. Repeat all the parts of Question(1) with the following functions:
   (a) $f(x) = \cos(x^2), \quad 0 \leq x \leq \pi$
   (b) $f(x) = \sin(x^2), \quad 0 \leq x \leq \pi$
   (c) $f(x) = x^2 - \dfrac{\cos(x)}{x^2}, \quad 0 < x \leq \pi$
   (d) $f(x) = x\exp(x^2)$
   (e) $f(x) = \tan(x)$

3. Among all the difference formula, which one gives minimum error and why?

4. Plot the given functions:
   (a) $f(x) = 2 - \dfrac{1}{x}$
   (b) $f(x) = x + \ln(x)$
   (c) $f(x) = \dfrac{2 - x^2}{5 + x^2}$
   (d) $f(x) = \dfrac{2 - x^2}{5 + x^2}$
   (e) $f(x) = x^3 + 5x^2 - 10x - 2$

5. Using the functions given in Question (4) find the roots using the given formulas.
   (a) Bi-Section Method.
      i. If $f(x_1) < 0$ and $f(x_2) > 0$, then root lies between x_1 and x_2. Estimate x_1 and x_2 from the plot.
      ii. $x_1(old) = x_1, \quad x_2(old) = x_2$.
      iii. Redefine $x_m = \dfrac{x_1(old) + x_2(old)}{2}$.
      iv. Test the sign of the function at x_m, find where the function crosses x-axis.
      v. Repeat the process.
      vi. Write an algorithm and implement in your program.
   (b) Newton's Method.
      i. Select x close to root from the plot. Take any small value of h.
      ii. $f(x+h) \approx f(x) + h\dfrac{\mathrm{df(x)}}{\mathrm{dx}}$.
      iii. Assume $f(x+h) \approx 0$.

iv. $h = -\dfrac{f(x)}{f'(x)}$.

v. $x_{\text{new}} = x_{\text{old}} + h$.

vi. Repeat.

vii. Write an algorithm and implement in your program.

6. Using the code you have developed for earlier questions on *First Order Derivatives*, find the extrema of functions given in above questions.

7. Using numerical scheme for second order derivatives, find the maxima, minima and inflection points of the above functions.

Write the code in a way so that you can use it later in this course. Here is a scheme C++.

```
double f(double x)
{
.......
val=your function
return (val);
}

main()
{
.......
.......
cout<<f(x)<<endl;
...
return 0;
}
```

- Write a computer program which will find the maxima, minima and inflection points of any function with-in a given range. This program should be general enough, so that you only need to change the function and range.
- In this lab session, we will give you few more functions on the spot and you need to find the maxima, minima and inflection points.
- Make a file in PDF containing:
  - Task with dates and Work sheet number.
  - Codes.
  - Results and plots using given functions.