Work Sheet 4 **September 05, 2019**

Course Name: Introduction to Linear and Non Linear Programming Lab
Course Number: CS4101
Name of Instructor: Pritam Rooj and Tapan Naskar
Total Marks: 10

1. Using Quadratic Fit Search and 1D-Line Search Method find the extrema of the following functions.
   (a) Minima: $f(x) = \dfrac{-x^2}{x^3 + \cos(x)}$, Domain $[0,\ 5]$.
   (b) Minima: $f(x) = \left(x^4 - x^3 - x^2\right)\exp(-x^2)$, Domain $[0,\ 5]$.
   (c) Minima: $f(x) = \dfrac{\sin(x)}{\sqrt{x}}$, Domain $[0,\ 5]$.
   (d) Minima: $f(x) = \dfrac{\cos(x)}{\sqrt{x}}$, Domain $[0,\ 5]$.
   (e) Minima: $f(x) = (x+1)(x-3)^2(x+7)^{-2}$, Domain $[0,\ 5]$.
   (f) Minima: $f(x) = (x-2)(x-3)^2(x-5)^3$, Domain $[0,\ 5]$.
   (g) Minima: $f(x) = 1 - \exp\left(-(x-3)^2\right)$, Domain $[0,\ 5]$.
   (h) Minima: $f(x) = J_1(x) - \exp\left(-(x-3)\right)$, Domain $[0,\ 5]$.
   where, $J_p(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,(n+p)!}\left(\frac{x}{2}\right)^{2n+p}$.
   (i) Minima: $f(x) = \dfrac{1}{2}x^2 + 2\cos(x)$, Domain $[0,\ 5]$.
   (j) Minima: $f(x) = x^{100}$, Domain $[0,\ 5]$.
2. Compute the above problems using Newton Raphson method and compare your result.
3. In Quadratic Fit Search the extrema is given by

$$x = \frac{1}{2}\,\frac{y_a\left(b^2 - c^2\right) + y_b\left(c^2 - a^2\right) + y_c\left(a^2 - b^2\right)}{y_a(b-c) + y_b(c-q) + y_c(a-b)}$$

   where $a,\ b,\ c$ are the ordinates of the points on the curve whose extrema is to be computed, such that $a < b < c$. $y_a,\ ,y_b,\ y_c$ are the abscissa of the points on the curve such that $y_a = f(a),\ y_b = f(b),\ y_c = f(c)$.

(a) Show that this can also be reduced to

$$x = b - \frac{1}{2}\frac{(b-a)^2(y_b - y_c) - (b-c)^2(y_b - y_a)}{(b-a)(y_b - y_c) - (b-c)(y_b - y_a)}$$

(b) Write a program to find the minima of $f(x) = (x-1)(x-3)^2$, with $a = 1,\ b = 2, c = 4$.

(c) What are the difference in results if any between two formulas and why?

(d) What are the other problems?

(e) How to modify the algorithm (algorithm 3.4 of Mykel, J Kochenderfer) to eliminate these errors.

(f) Write a code which do not have such problems.

4. Prerequisite for applications to higher dimensions line search.

   (a) Write a program which compute the transpose of a matrix.

   (b) Write a program which multiply two matrices.

   (c) Write a program which compute the determinant of a given matrix.

   (d) Write a program which take a matrix as input and print inverse of that matrix as output.

   You may call library function of your programming language for the above task.

5. Find the minima of the following function in domain $-10 \leq x_1,\ x_2 \leq 10$,using Gradient Descent Algorithm.

   (a) $f(x_1, x_2) = (x_1 - x_2)^2 + (x_1 + x_2)^2$.

   (b) $f(x_1, x_2) = (1 - x_1)^2 + 100\left(x_2 - x_1^2\right)^2$.

   (c) $f(x_1, x_2) = (1.5 - x_1 + x_1 x_2)^2 + \left(2.25 - x_1 + x_1 x_2^2\right)^2 + \left(2.625 - x_1 + x_1 x_2^3\right)^2$.

   (d) $f(x_1, x_2) = \sin^2 3\pi x_1 + (x_1 - 1)^2\left(1 + \sin^2 3\pi x_2\right) + (x_2 - 1)^2\left(1 + \sin^2 3\pi x_2\right)$.

   (e) $f(x_1, x_2) = -\cos x_1 \cos x_2 \exp\left(-(x_1 - \pi)^2 - (x_2 - \pi)^2\right)$.