

- MIN CHEOL SEO, *Proof, Computer and Mathematical Agent*.  
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Since Appel's and Haken's proof of the Four-Colour Theorem, machine-assisted proofs has become an integral practice in mathematics, to the point that computer has become an indispensable partner. Philosophers have approached this transformation of computer from a mere calculator to human-machine coalition in epistemic term—i.e., what warrants *trust* in machine output? do we have a genuine mathematical knowledge from machine-assisted proofs?—but only a handful of philosophers engaged the fine-grained structure of mathematical practice itself. Hamami and Morris do so in drawing on Bratman's theory of temporally extended planning, suggest that understanding proofs tantamount to recognising and reconstructing underlying *plan* of mathematicians. Extending Hamami's and Morris' framework, I recast proof construction as a hierarchy of nested intentions in which computers occupy well-defined but partial roles. Transposing Bratman's planning to mathematics, I shall give a hierarchical structure of intentions in mathematical proofs as:

- L<sub>0</sub> Long-term intention:** prove or refute a theorem  $T$ .
- L<sub>1</sub> Strategic commitments:** select overall method (induction, variational bound, etc.)
- L<sub>2</sub> Tactical sub-intentions:** state lemmas, fix parameterisations, choose normal forms.
- L<sub>3</sub> Micro-steps:** verify an inequality, explore a search tree, run a proof-assistant tactic.

From this hierarchical structure, I map contemporary practice onto it yielding six recurring roles: (i) exhaustive enumeration— $L_3$ ; (ii) rigorous numerics— $L_3$ ; (iii) SAT/SMT certification— $L_3$ — $L_2$ ; (iv) automated theorem proving— $L_2$ ; (v) interactive proof assistance— $L_3$ ; (vi) heuristic optimisation— $L_2$ . In this, each role associated with *sub-intentions* whose outputs are diffused upward level, which can be reflected through a series of case studies. Finally, I conclude with some remarks on a new approach to the conception of mathematical agents, through an agency-centered methodology for mathematical practice.