NAME First name : 2024/2025

05.05.2025 **EDS Exam** Maths Grade 11

Conditions d'évaluation

Calculator : allowed. Duration : 2h00

- The paper contains **5 exercises**.
  You may complete them in any order you like.
  Make sure you have the complete paper before starting.
- The paper is worth 40 points.
- **Remember to write your name on the paper and return it with your answer sheet.**
- Number the pages of your answer sheet (1/n, 2/n, ..., n/n)
- Any element of answer will be taken into account in the grading.

Exercice 1 Complete study of a function *(6 points)*

We consider the function f defined on $\mathbb{R}$ by $f(x) = x^3 + 3x^2 + 3x - 63$. Let $\mathcal{C}$ be its graph in an orthonormal coordinate system.

1. Show that $f'(x) = 3(x+1)^2$.
2. Deduce the sign of $f'(x)$ on $\mathbb{R}$.
3. Establish the variation table of the function f on $\mathbb{R}$.
4. Justify that the tangent to the curve $\mathcal{C}$ at the point with abscissa -1 is the line $\mathcal{D}$ with equation $y = -64$.

BONUS Question :
This question should be answered at the end of your exam IF YOU HAVE TIME. It can earn you up to +1 point.

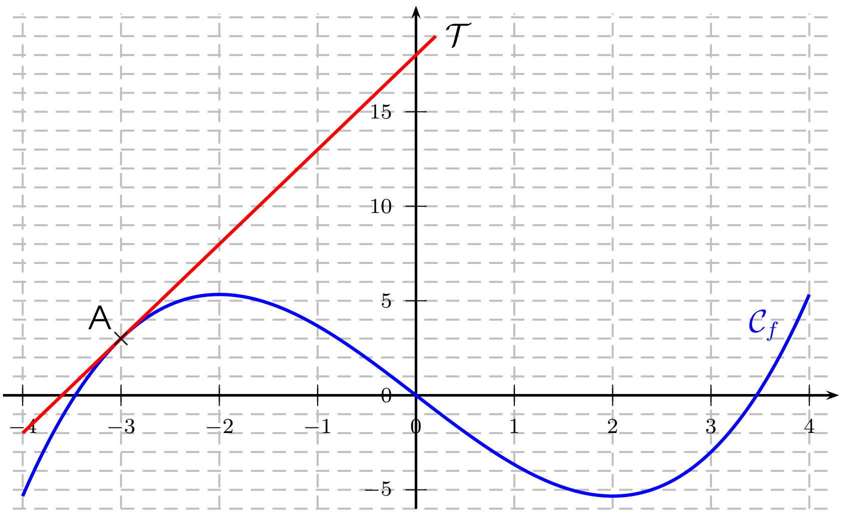
Determine at which points of the curve $\mathcal{C}$ the tangent to the curve is parallel to the line with equation $y = 3x - 100$.

Exercice 2 **Multiple choice questionnaire** *(10 points)*

This exercise is a multiple-choice questionnaire with ten independent questions.
For each question, only one of the proposed answers is correct.
For each question, indicate the number and write the letter corresponding to the chosen answer.
No justification is required, but rough work may help you determine your answer.
Each correct answer is worth 1 point. An incorrect or missing answer neither earns nor deducts any point.

Question 1

Given below is the graph $\mathcal{C}_f$ of a function f. This curve has a tangent $\mathcal{T}$ at the point $A(-3\ ;\ 3)$.

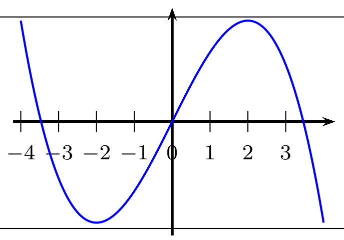
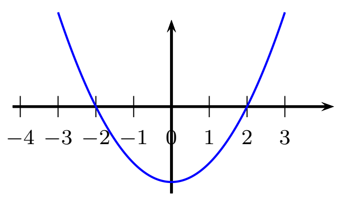
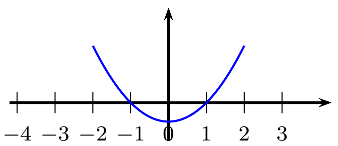
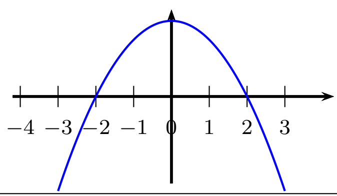


The simplified equation of this tangent is :

a. $y = \frac{1}{5}x - 3,7$	**b.** $y = \frac{1}{5}x + 18$	**c.** $y = 5x + 18$	**d.** $y = 5x - 3,7.$

Question 2

We take the function f from the previous question. The graph of its derivative function is :

 **a.**	 **b.**
 **c.**	 **d.**

Question 3

The expression $\cos(x + \pi) + \sin(x + \frac{\pi}{2})$ is equal to :

a. $-2\cos(x)$	**b.** 0	**c.** $\cos(x) + \sin(x)$	**d.** $2\cos(x).$

Question 4
We consider the function polynôme du second degré f defined on $\mathbb{R}$ by

$$f(x) = -2x^2 + 4x + 6$$

This function is strictly positive on the interval :

a. $]-\infty;-1[\cup]3;+\infty[$	**b.** $]-1;3[$
c. $]-\infty;-3[\cup]1;+\infty[$	**d.** $]-3;1[$.

Question 5

Let (u_n) be the arithmetic sequence with first term $u_0 = 2$ and common difference 0.9. We have :

a. $u_{50} = 47$	**b.** $u_{50} = 100,9$	**c.** $u_{50} = -47$	**d.** $u_{50} = -100,9$.

Question 6

Let (v_n) be the geometric sequence with first term $v_0 = 2$ and ratio 0.9. The sum of the first 37 terms of the sequence (v_n) is :

a. $2 \times \dfrac{1-0,9^{38}}{1-0,9}$	**b.** $2 \times \frac{1-0,9^{37}}{1-0,9}$	**c.** $0,9 \times \dfrac{1-2^{38}}{1-2}$	**d.** $0,9 \times \dfrac{1-2^{37}}{1-2}$.

Question 7

A Python program that returns the sum of the integers from 1 to 100 is :

a.
def Somme() :
s=0
While s<100 :
    s=s+1
return (s)
	b.
	def Somme() :
	s=0
	While s<100 :
	    s=2*s+1
	return (s)
		c.
		def Somme() :
		s=0
		for k in range 101 :
		    s=s+k
		return(s)
			d.
			def Somme() :
			s=0
			for k in range . 100 :
			    s=s+k
			return(s)

Question 8

Given that $x \in \left[-\dfrac{\pi}{2} \ ; \ 0\right]$ and $\cos x = 0.8$, then :

a. $\sin x = 0,6$	**b.** $\sin x = -0,6$	**c.** $\sin x = -0,2$	**d.** $\sin x = 0,2$.

Question 9

The number $\frac{13\pi}{4}$ corresponds to the same point on the unit circle as the real number :

a. $-\dfrac{14\pi}{4}$	**b.** $-\dfrac{3\pi}{4}$	**c.** $\dfrac{7\pi}{4}$	**d.** $\dfrac{19\pi}{4}$.

Question 10
In an orthonormal coordinate system, we consider the points A$(3\ ;-1)$, B$(4;2)$, and C$(1;\ 1)$.
The dot product $\overrightarrow{AB} \cdot \overrightarrow{AC}$ is equal to :

a. -4	**b.** 2	**c.** 4	**d.** 8

Exercice 3 **Production sites** *(10 points)*

A company that manufactures needles has two production sites : site A and site B.
Site A produces three-quarters of the needles, and site B the remaining quarter.
Some needles may be defective. A quality control study revealed that :

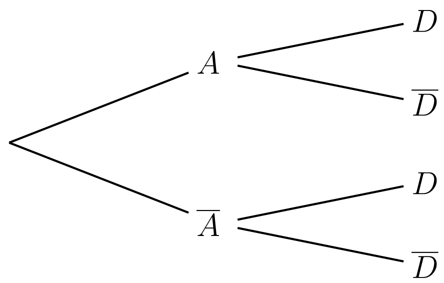
- 2% of the needles from site A are defective ;
- 4% of the needles from site B are defective.

The needles from both sites are mixed and sold together in batches.
We randomly select a needle from a batch and consider the following events :

- A : the needle comes from site A ;
- B : the needle comes from site B ;
- D : the needle is defective.

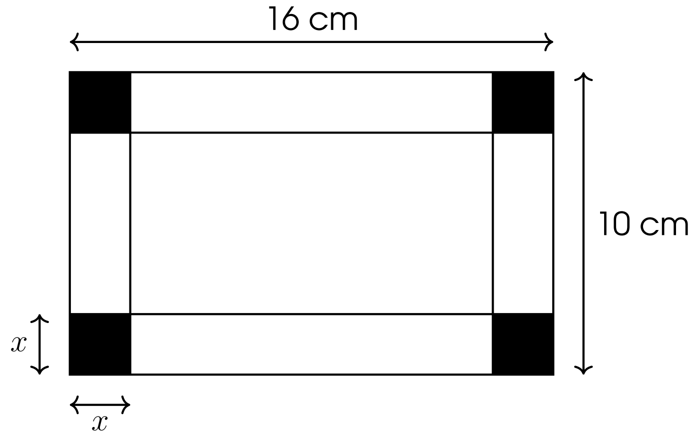
The opposite event of D is denoted $\overline{D}$.

1. Based on the information given, state the value of the probability of event A, denoted $P(A)$.
2. Copy and complete the probability tree below, indicating the probabilities on the branches.
3. What is the probability that the needle is defective and comes from site A ?
4. Show that $P(D) = 0.025$.
5. After inspection, the selected needle turns out to be defective.
   What is the probability that it was produced at site A ?



Exercice 4 Net of a box *(7 points)*

We want to create, using the net below, a rectangular box **without a lid**. All lengths are given in cm.



1. What are the possible values of x *(no justification needed)*?
2. Verify that the volume V of this box can be expressed as a function of x by :

$$\mathcal{V}(x) = 4x^3 - 52x^2 + 160x.$$

3. (a) Verify that :

$$\mathcal{V}'(x) = 12x^2 - 104x + 160$$

   (b) Construct the sign chart of $\mathcal{V}'(x)$ and then the variation table of the function $\mathcal{V}$ over the interval $[0\ ;\ 5]$, knowing that $\mathcal{V}(0) = \mathcal{V}(5) = 0$.

   (c) What is the maximum volume of the box? For which height?

Exercice 5 Dietary plan *(7 points)*

For health reasons, Mrs. Lefèvre decides each day between two lunch options : a vegetarian meal (denoted V) or a meat-based meal (denoted M).
If she eats a vegetarian meal one day, she will choose a vegetarian meal again the next day with a probability of 0.7.
If she eats a meat-based meal one day, she will choose a vegetarian meal the next day with a probability of 0.5.

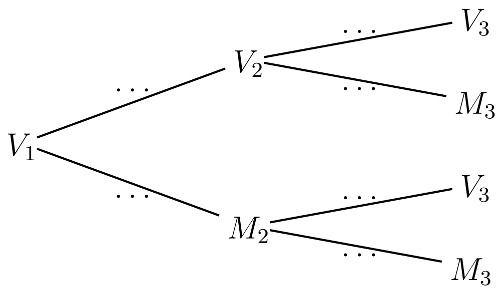
For every non-zero natural number n, we define :

- V_n : the event Mrs. Lefèvre eats a vegetarian meal on the n-th day ;
- M_n : the event Mrs. Lefèvre eats a meat-based meal on the n-th day ;
- v_n : the probability that Mrs. Lefèvre eats a vegetarian meal on the n-th day. In other words, $\mathbb{P}(V_n) = v_n$.

On the first day, she starts with a vegetarian meal, so $v_1 = 1$.

Part A Study of the first days

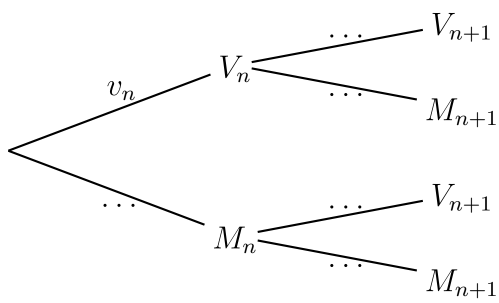
1. Copy and complete the weighted tree diagram below representing the situation for the 2nd and 3rd days :



2. Calculate v_3.
3. On the 3rd day, Mrs. Lefèvre eats a vegetarian meal. What is the probability that she ate a meat-based meal the day before ?

Part B Long-term study

1. Copy and complete the weighted tree diagram below representing the situation between the n-th and the $(n+1)$-th day :



2. Show that, for all $n \geq 1$, $v_{n+1} = 0,2v_n + 0,5$