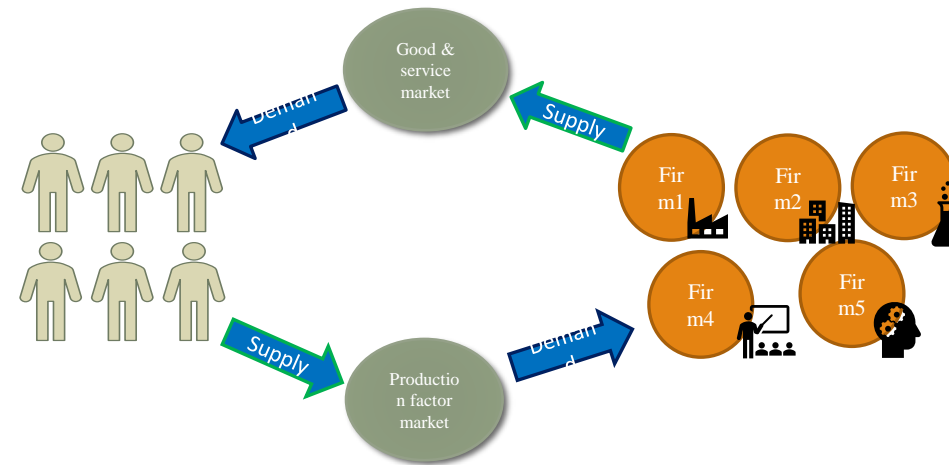


Lecture 3

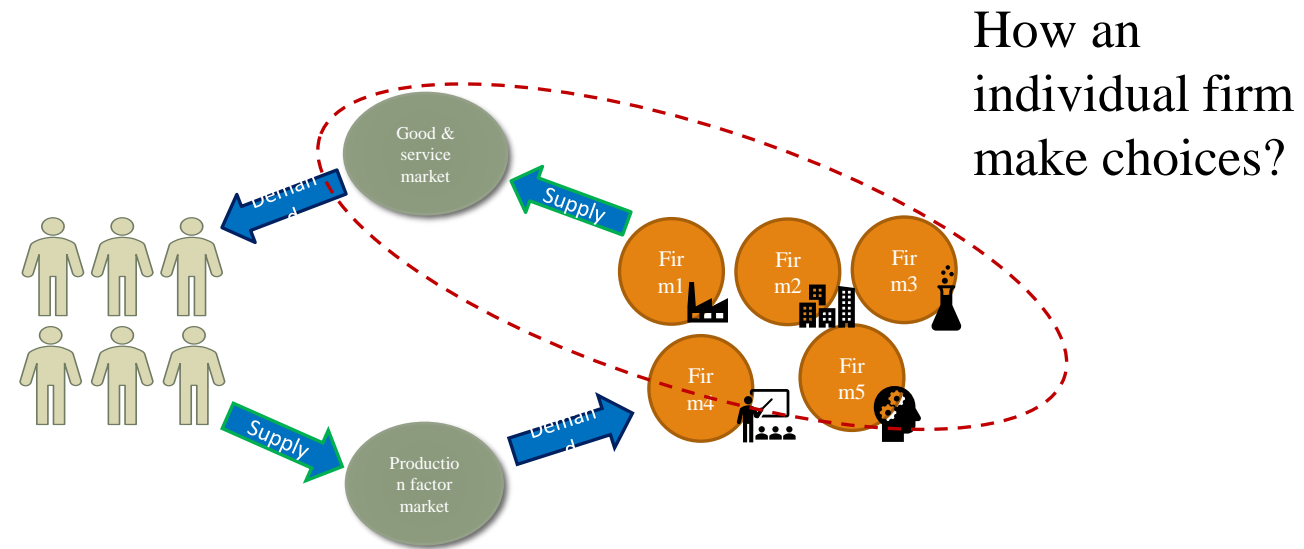
: Producer problem and supply curve

Jengei Hong

Market economy system



Production sector and supply curve



****Keywords****

- What is the producer's optimal decision?
- Production cost
 - Fixed cost and variable cost
 - Average cost and marginal cost
- Production function
 - The law of diminishing marginal production
- Total revenue, average revenue, and marginal revenue

Producer's optimization problem

- Business sector or firms combines inputs of labor, capital, land, and raw or finished component materials to produce outputs.
- This activity of production includes not only manufacturing but also any process or service that creates value, including transportation, distribution, wholesale and retail sales.
- The objective of business : **profit**

Producer's optimization problem

- Profit = Revenue – cost
 - Revenue = Price \times Quantity
 - Cost = the amount that the firm pays to make and sell their products is called total cost.
- The cost structure of firms can be broken down into some common underlying patterns. In the short run, we can classify the costs into fixed costs and variable costs.
 - Fixed costs are the costs that cannot be changed in the short run.
 - Variable costs are the costs that can be changed.

The structure of production cost

- **Fixed costs** are expenditures that do not change regardless of the level of production, at least not in the short term.
 - For example, the cost of building factory or buying heavy equipment, research and development costs, advertising expenses.
- The level of fixed costs varies according to the feature of the products or industry.
- For instance, manufacturing mobile phone or computer chips requires an expensive factory. But, a food-truck or a painting artist can get by with relatively less fixed costs.

The structure of production cost

- **Variable costs** are incurred in the act of producing.
 - For example, wages and cost to buy intermediate goods are variable costs.
- The more you produce, the greater the variable cost.
- **Total cost** = fixed cost + variable cost.

The nature of production

- Two assumptions:
 1. Positivity (with respect to input)
 2. Law of diminishing marginal production
- **Marginal product of any input** : the additional quantity of output obtained from one additional unit of that input.

Example : Pepe's chicken house

- Assume that the fixed costs of operating the chicken house, including the space and equipment, are 100.
- The variable costs are the costs of hiring worker, which in our example is 50 per deliver.
- Variable cost : price of input \times quantity of input.
- When one unit of the input (worker) is added, the cost would be increased as much as the price of the input (50 for the wage).



Example : cost structure of the chicken house

Labor	Quantity produced
1	10
2	18
3	24
4	28
5	30
6	31
7	31.5

The law of diminishing marginal production



Example : revenue of the chicken house

Let's assume the price of 1 chicken = 30!

Labor	Quantity produced	Total revenue	Additional rev (from 1 more worker)
1	10	300	300
2	18	540	240
3	24	720	180
4	28	840	120
5	30	900	60
6	31	930	30
7	31.5	945	15

Example : cost structure of the chicken house

Labor	Quantity produced	Fixed cost	Variable cost	Total cost	Additional cost (1 worker)
1	10	100	50	150	50
2	18	100	100	200	50
3	24	100	150	250	50
4	28	100	200	300	50
5	30	100	250	350	50
6	31	100	300	400	50
7	31.5	100	350	450	50

What is the optimal choice?

Labor	Quantity produced	Additional cost (1 worker)	Additional rev (1 worker)
1	10	50	300
2	18	50	240
3	24	50	180
4	28	50	120
5	30	50	60
6	31	50	30
7	31.5	50	15

What is the optimal choice?

Labor	Quantity produced	Additional cost (1 worker)	Additional rev (1 worker)
1	10	50	300
2	18	50	240
3	24	50	180
4	28	50	120
5	30	50	60
6	31	50	30
7	31.5	50	15

MR = MC

The production function

- We've discussed about the production cost, by using a production table.
- Now, we can use the production function instead of the production table.
- And then, from the production function, we can derive of each cost functions.

The production function

- As the utility function, the form of the production function is usually assumed.
- In our case, I assume a production function with the pattern of diminishing marginal product.
- Because this form is one of the basic form of production function which economists usually uses.

Preparations

- Functions and graphs
- The concept of differentiation
- Formulas for differentiation

Function

- Function : a special type of relation between X and Y. Projection every element of X to exactly one element of Y.
- Denotation : $y = F(x)$
- Examples
 1. $y = x + 10$
 2. $y = 2x^\alpha$
- Illustrating a function is helpful to sketch the characteristic of the function. (note that α determines the feature of y in 2)
 - ✓ Suppose that α is $0 < \alpha < 1$ and illustrate 2.
 - ✓ Suppose that α is $\alpha > 1$ and illustrate 2.
- Usually, horizontal (vertical) axis indicates input (output) variable.

The concept of differentiation

- Check the responsiveness of changes in y with respect to changes in x.
- The meaning of slope of a function.
- Differentiation and derivative function
- Denotations : $\frac{dy}{dx}$, $\frac{dF(x)}{dx}$, $F'(x)$

Formulas for differentiation

- Derivative of a constant : $\frac{dA}{dx} = 0$
- Power rule : $\frac{dAx^\alpha}{dx} = \alpha Ax^{\alpha-1}$
- Sum rule : $\frac{d(F(x)+Z(x))}{dx} = \frac{dF(x)}{dx} + \frac{dZ(x)}{dx}$

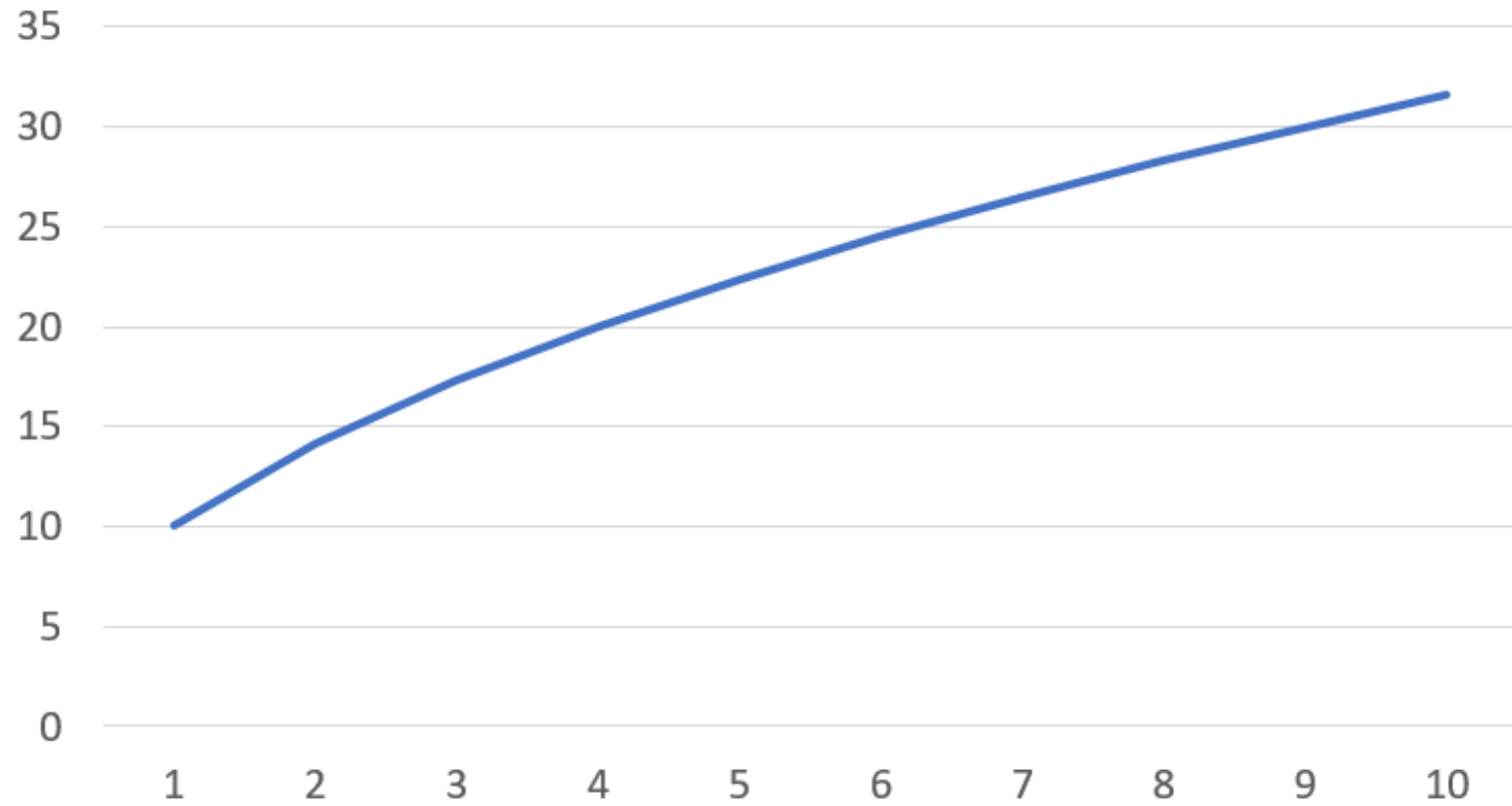
Example 2 : production function

- Let's assume the form of the production function of Pepe's chicken house as,

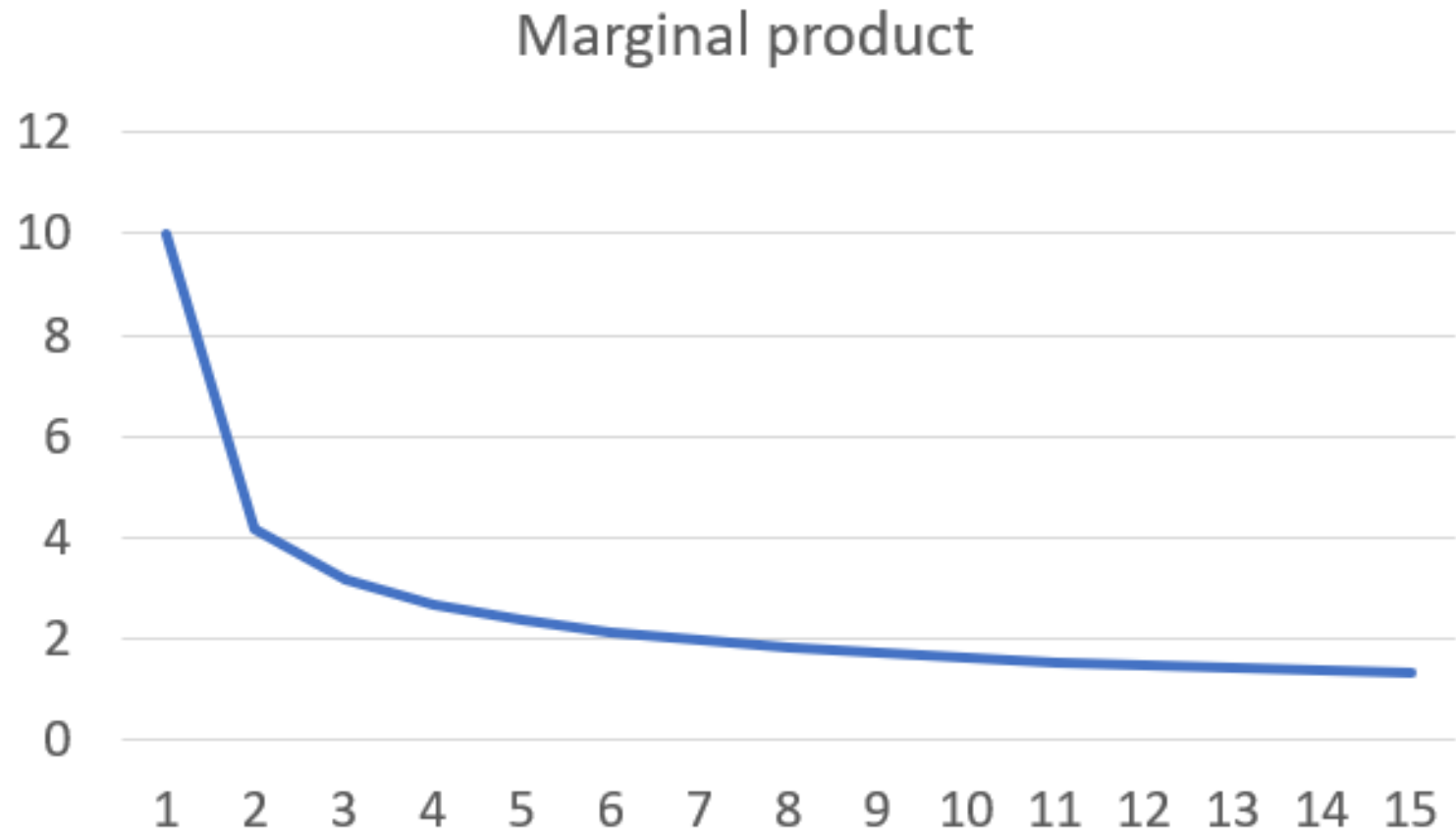
$$Q = 10X^{\frac{1}{2}}$$

where Q is the output, X is input (number of hiring).

Example 2 : production function



Example 2 : marginal production



Producer's optimization problem

- Remind that, Profit = Revenue – cost
- Let's illustrate cost & revenue.
 - Production cost & marginal cost of production
 - Revenue & marginal revenue

Arrangement for the production cost function

- Production cost (with respect to quantity produced) : cost paid to produce that much of quantity
- Total cost = Fixed cost + Variable cost(=input price * input)
- Production cost = Fixed cost + input price * input required to produce that much of products

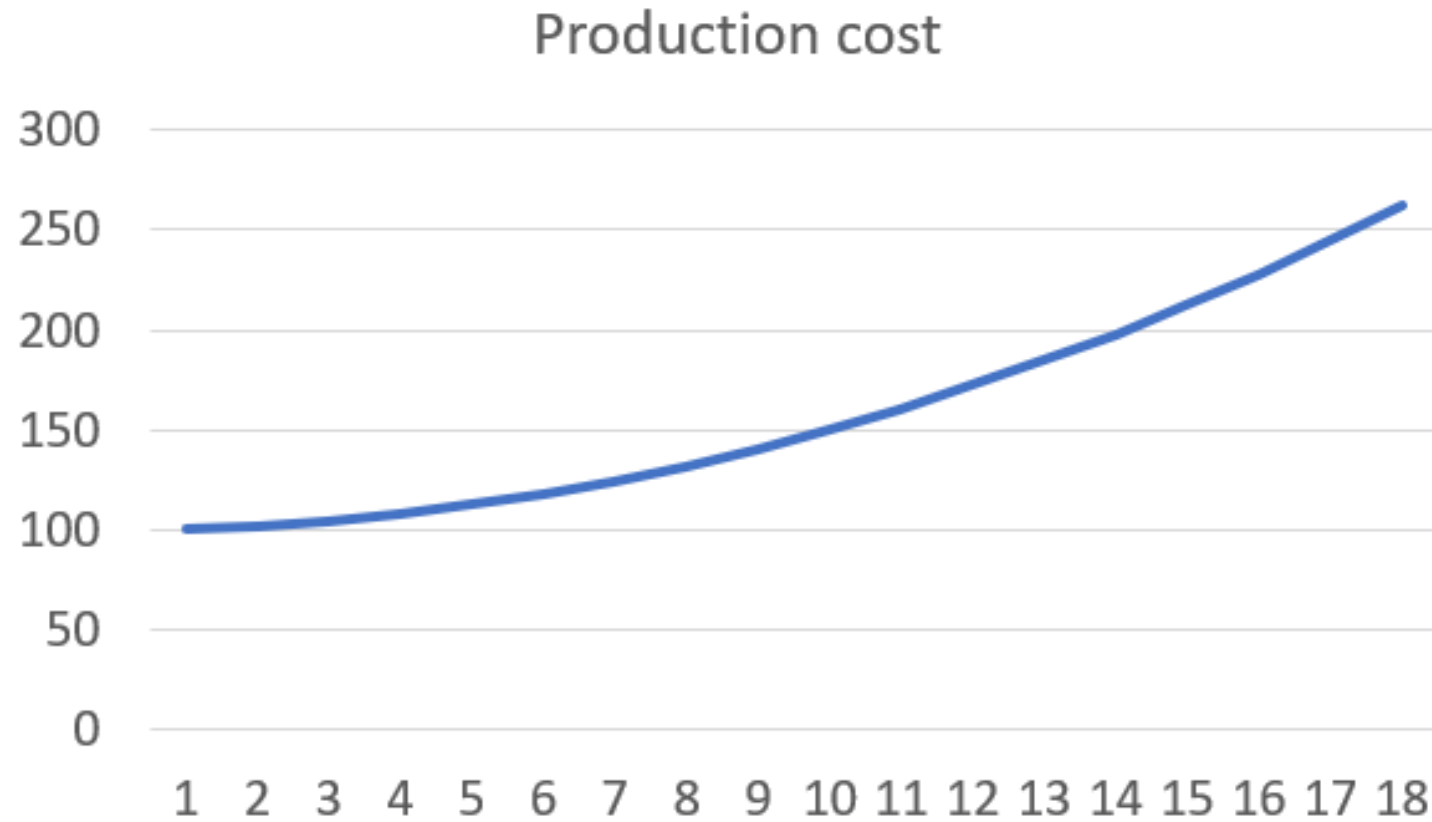
Example 2 : Production cost function

- In our example,

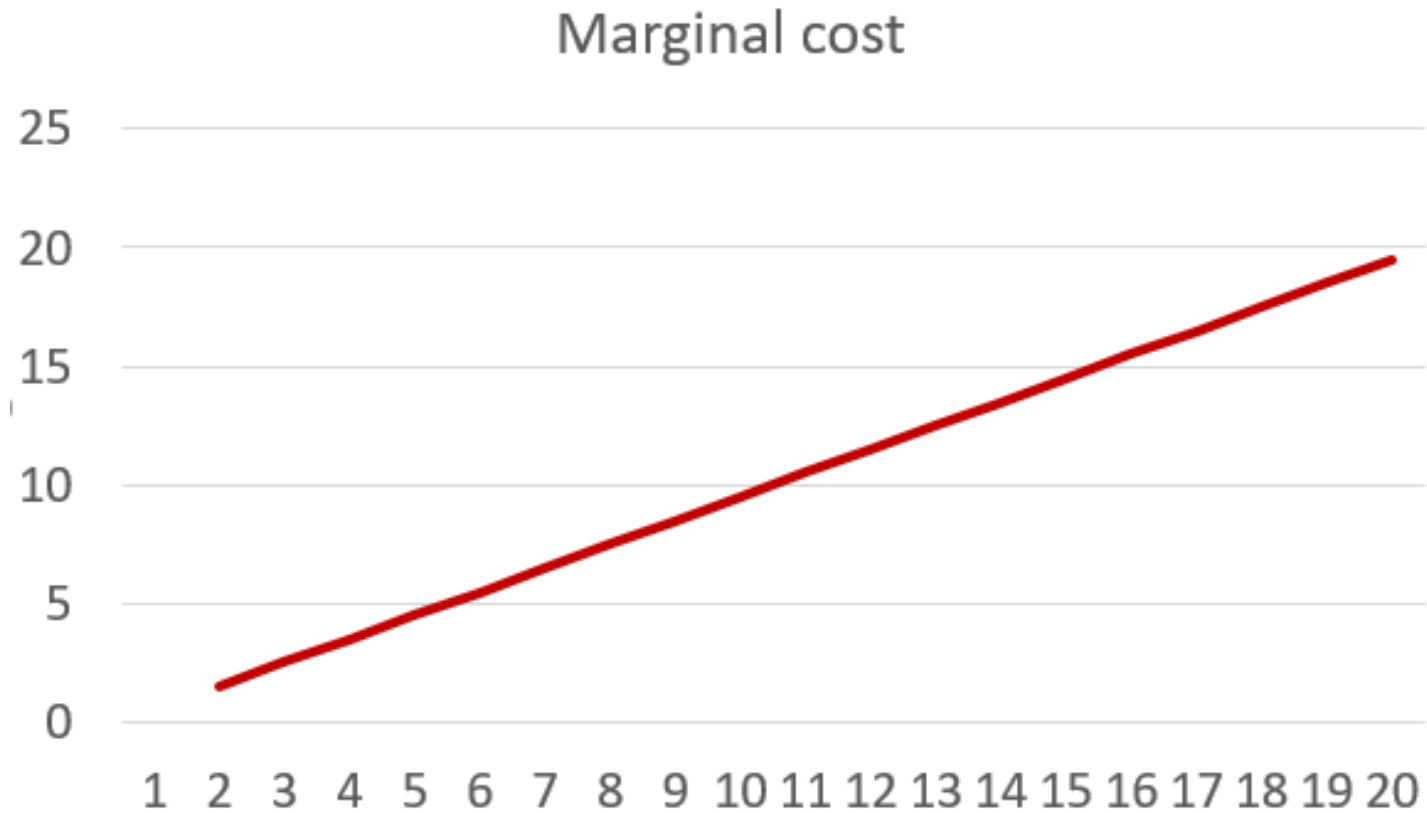
$$Q = 10X^{\frac{1}{2}}$$
$$\Rightarrow$$
$$X = \frac{Q^2}{100}$$

- Production cost (to produce Q) = $100 + 50 * \frac{Q^2}{100}$
- Marginal cost = Q (derivative function of the production cost with respect to Q)

Example 2 : production cost



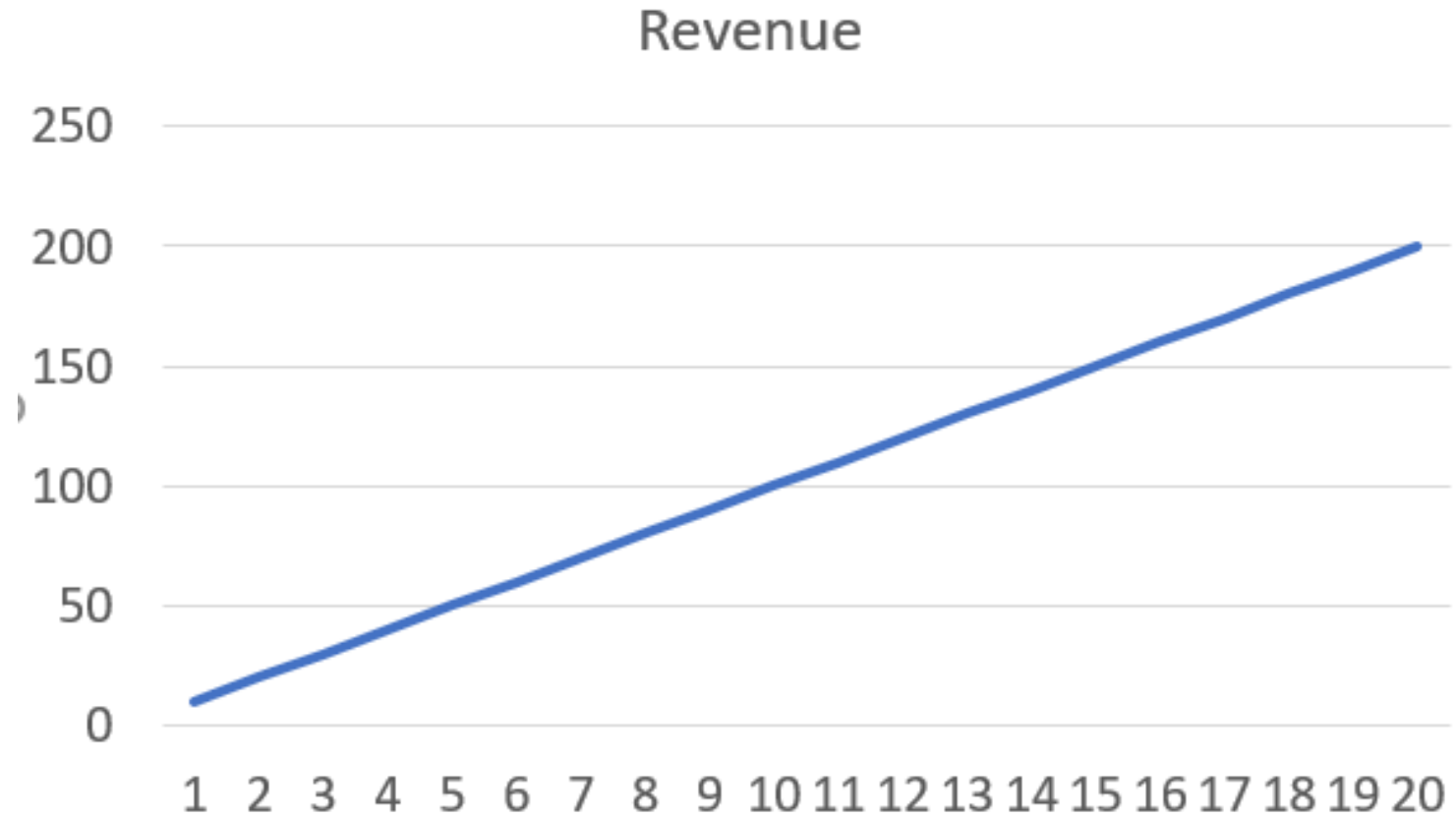
Example 2 : marginal cost of production



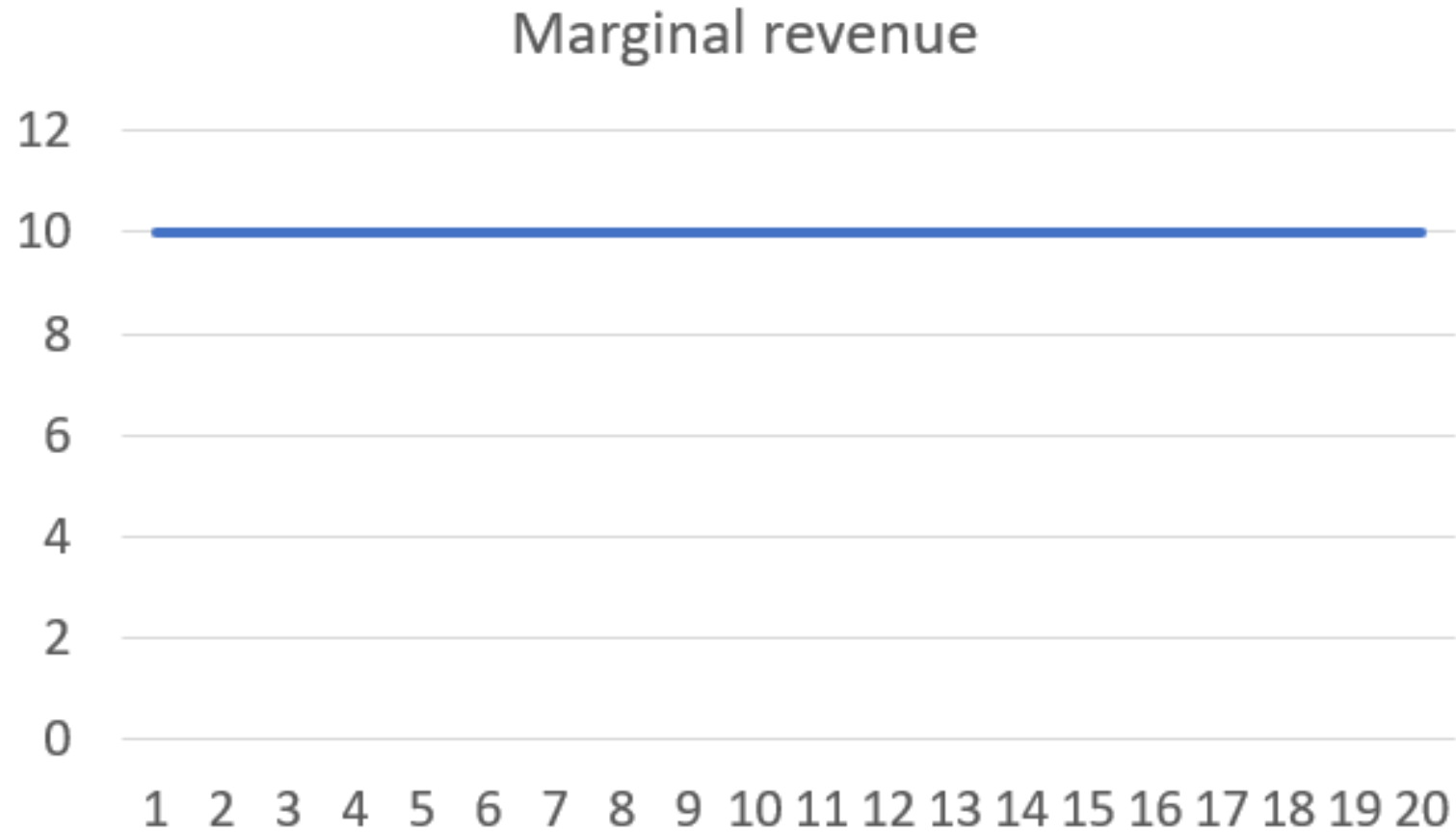
Revenue and marginal revenue

- Revenue = Price \times Quantity
- Marginal revenue : additional revenue from producing & selling 1 more unit of product = price
- To solve it, we have to assume the nature of price
- For simplicity, let's suppose the price is fixed at 10.

Example 2 : revenue



Example 2 : marginal revenue

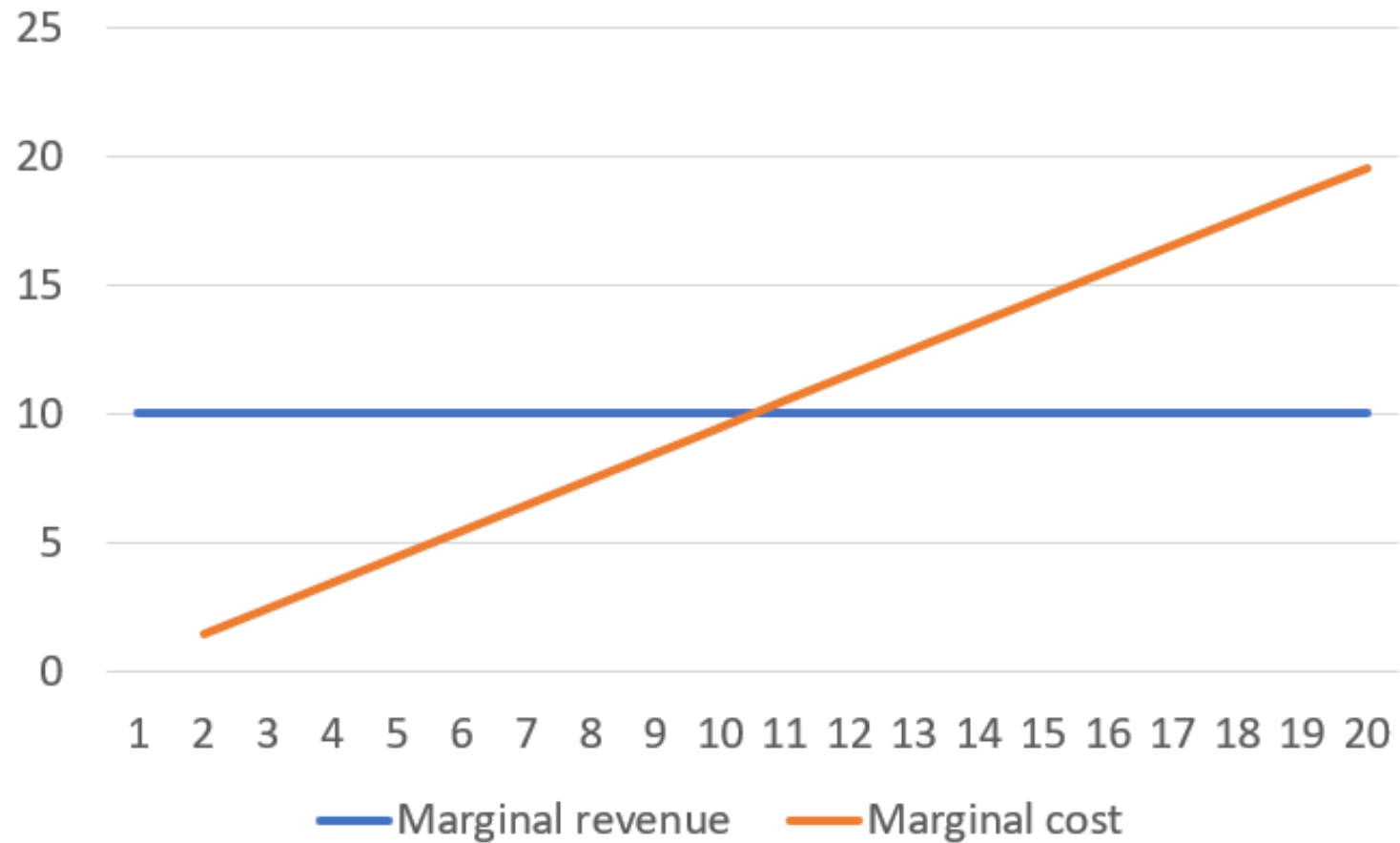


Producer's optimization problem

- The rule of profit maximization

$$\textit{Marginal revenue} = \textit{Marginal cost}$$

Example 2 : optimization



Producer's optimization problem

- In the example,

$$\text{Marginal revenue} = 10$$

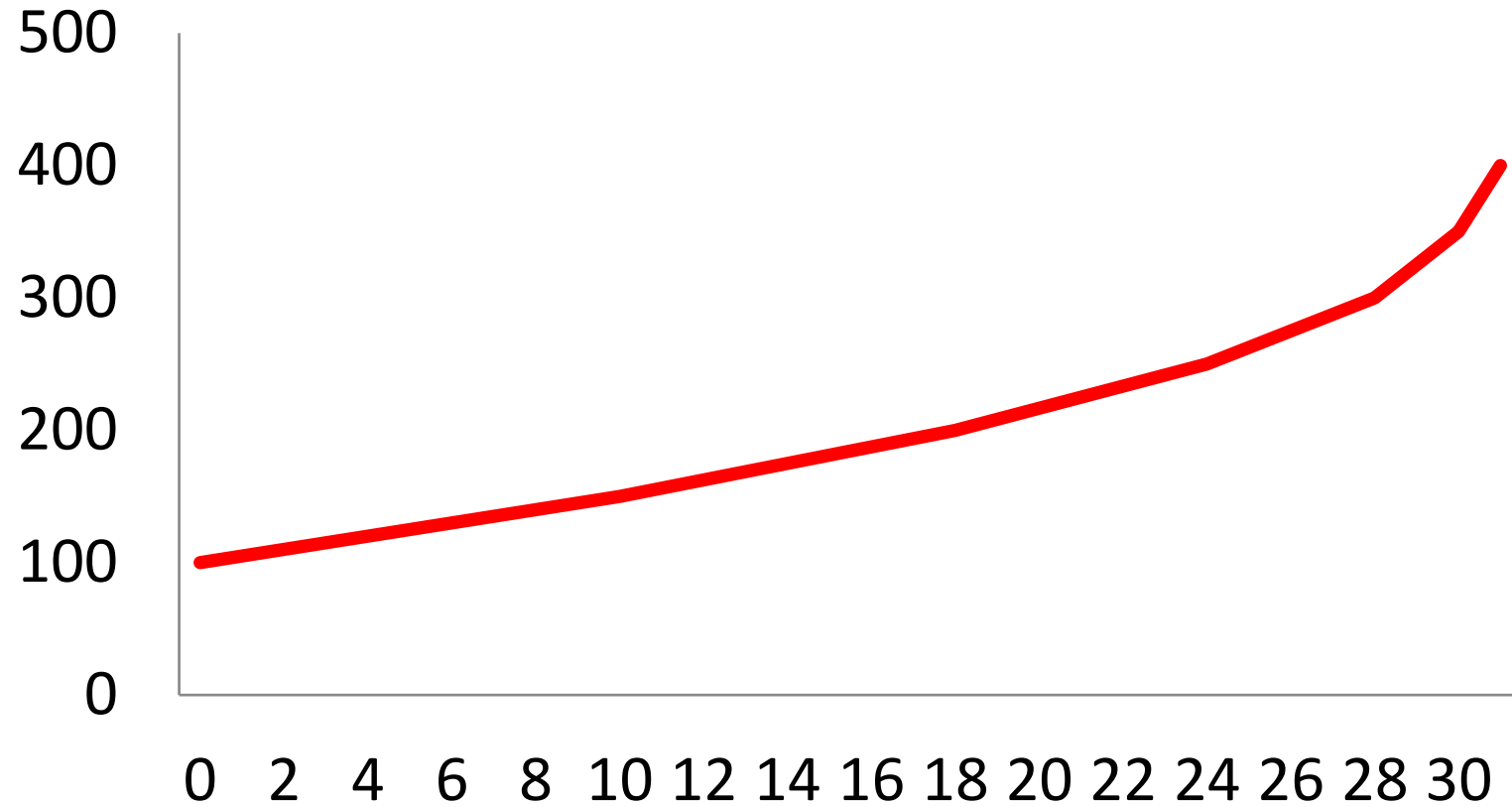
$$\text{Marginal cost} = Q$$

- Then, $MR=MC$ implies that $Q = 10$.
- Hence, the optimal amount of production is 10.

The relationship between output and cost

- The relationship 'quantity produced' and total cost.
- Note that the production process have diminishing marginal products.
- So, the amount of change in output to the additional one unit of input is decreasing.
- Actually, It is same with the statement that the amount of additional input needed to produce additional one unit of output is increasing.
- It implies that the amount of cost to produce one more unit of output is increasing.

The relationship between output and cost



The structure of production cost

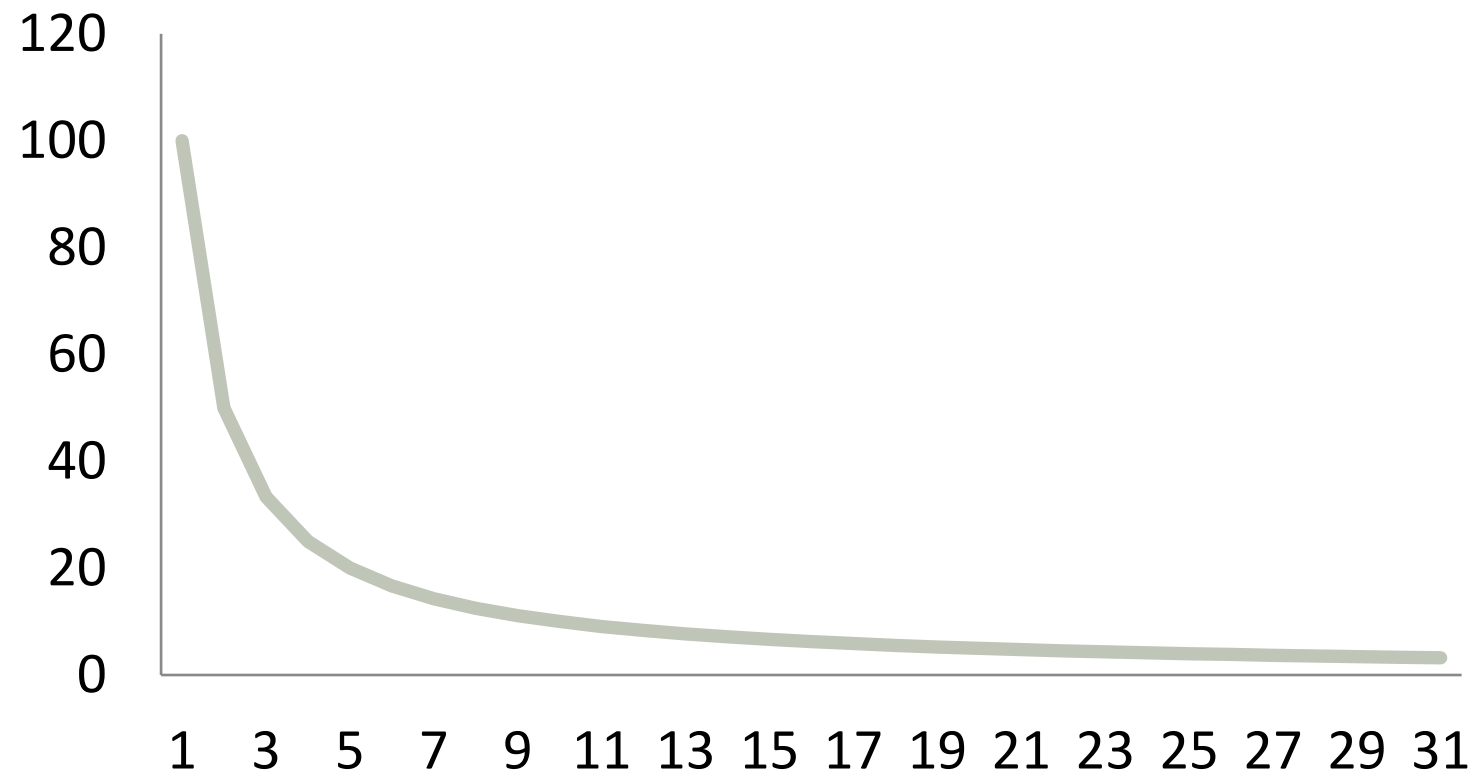
- 4 concepts of cost.
 - Average total cost
 - Average fixed cost
 - Average variable cost
 - Marginal cost

The structure of production cost

- Average fixed cost(AFC) is the fixed cost is divided by quantity of output.

$$AFC = \text{fixed cost} / \text{output}$$

The structure of production cost : AFC

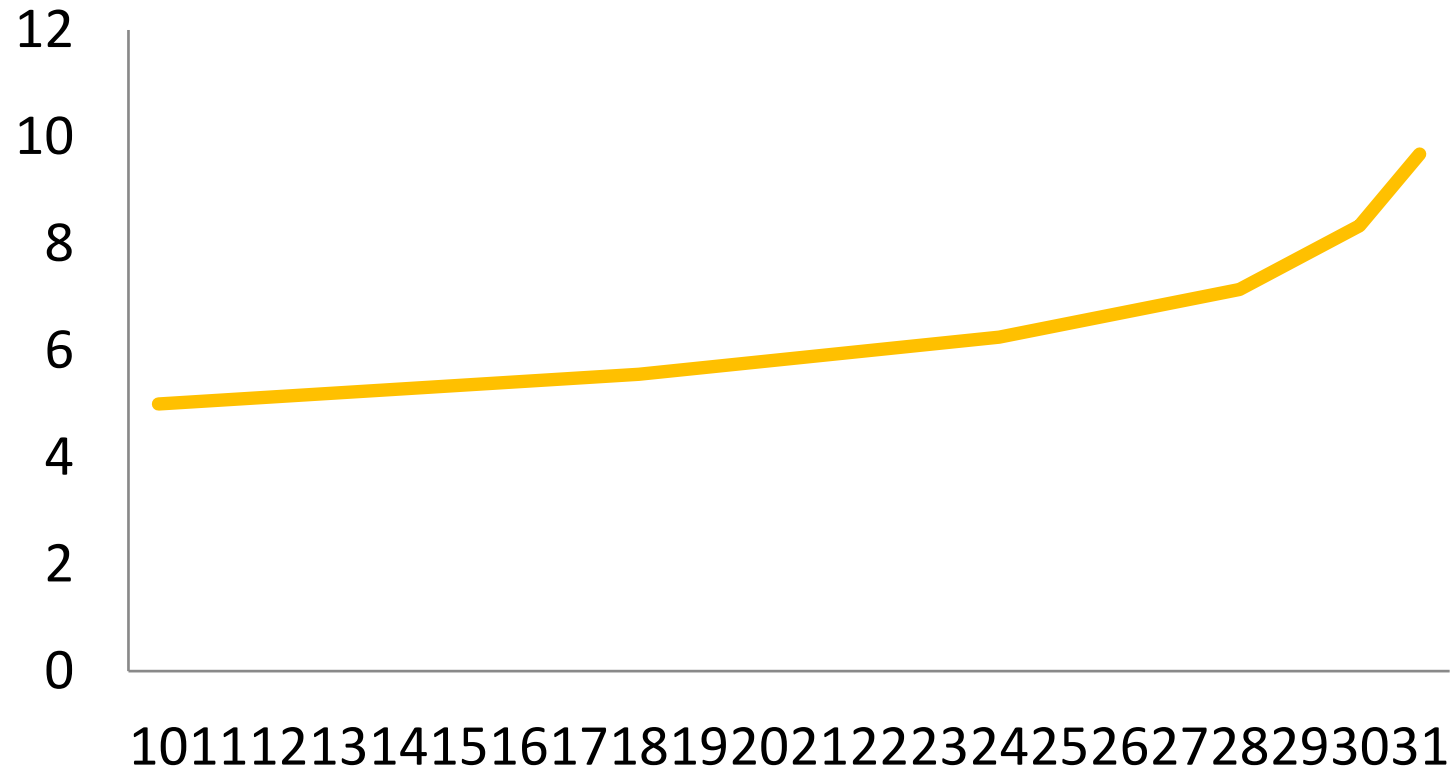


The structure of production cost

- Average variable cost is the variable cost divided by quantity of output.

$$AVC = \text{total variable cost} / \text{output}$$

The structure of production cost : AVC



The structure of production cost

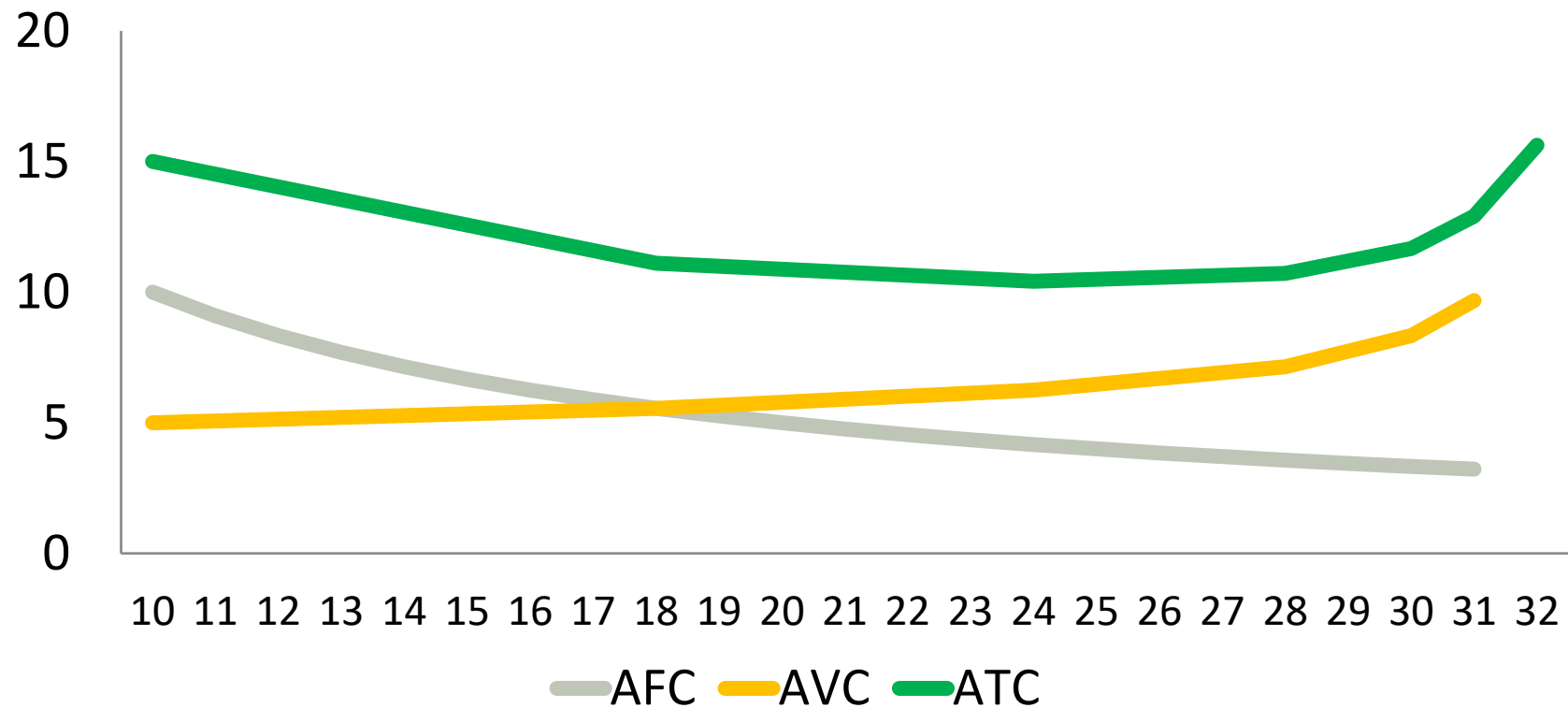
- Average total cost is total cost divided by the quantity of output.

ATC

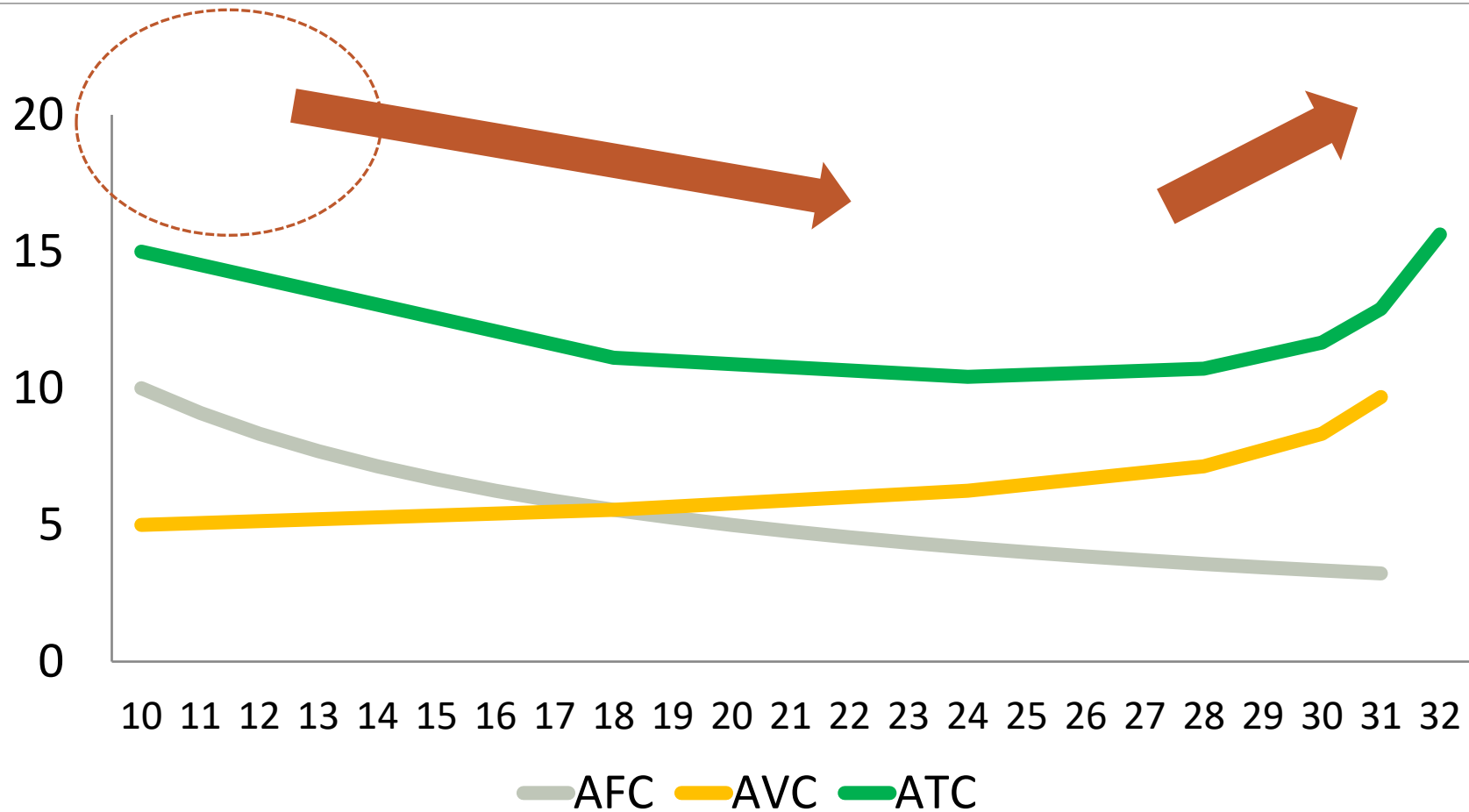
= total cost / output

= (fixed cost + variable cost) / output

The structure of production cost : ATC



The structure of production cost



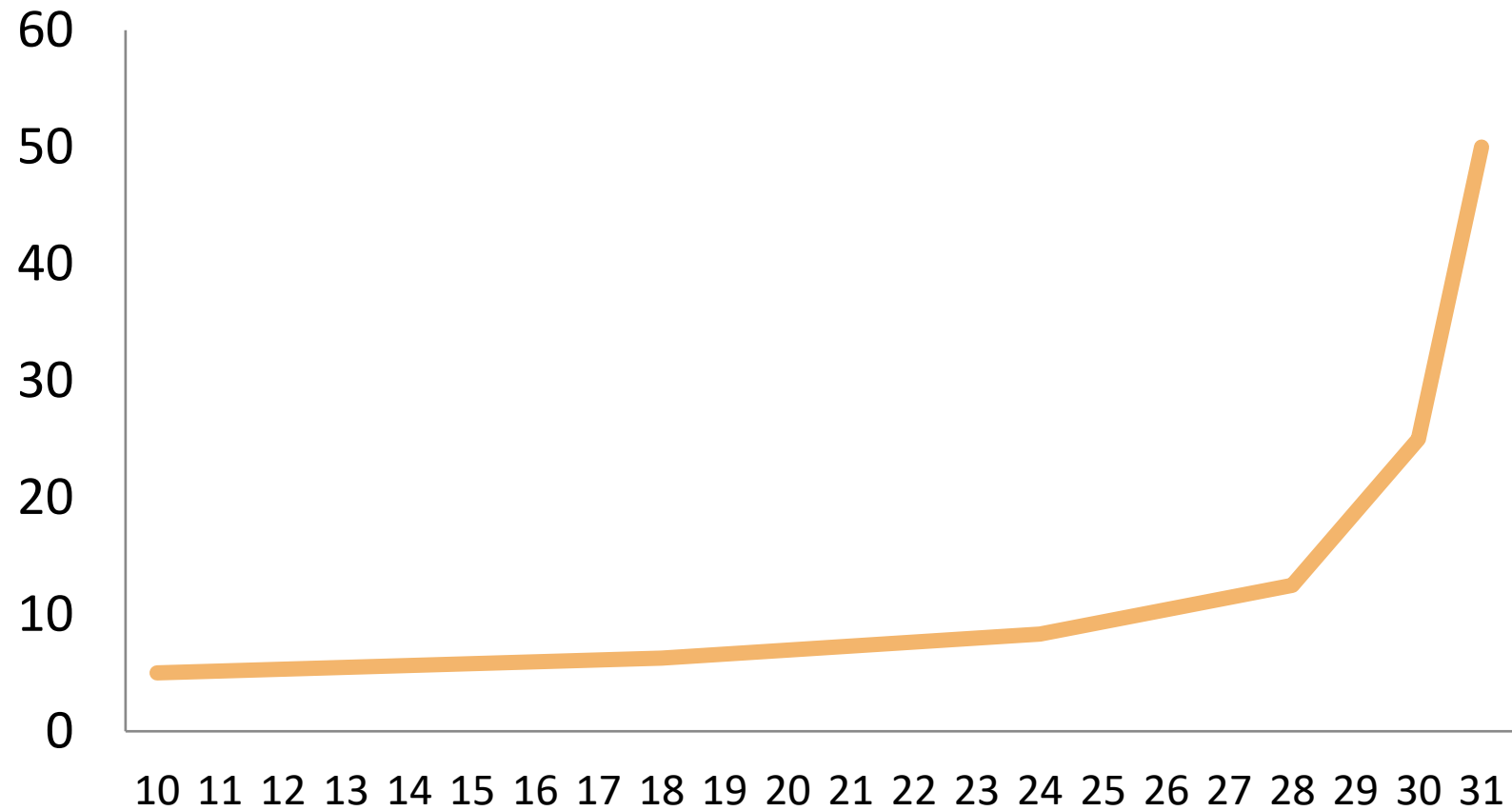
The structure of production cost

- Marginal cost(MC) is the additional cost of producing one more unit of output.

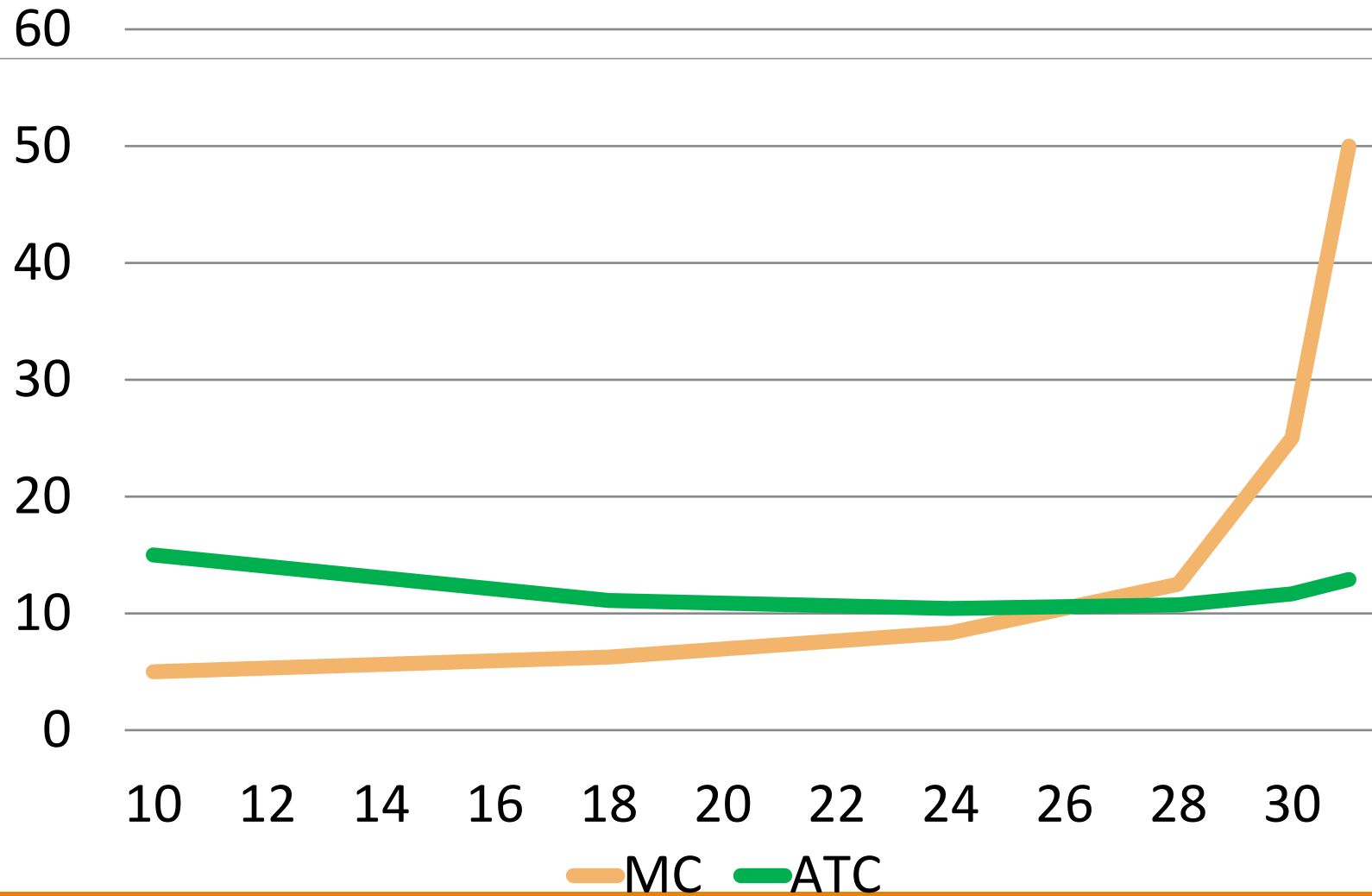
$$MC = \text{change in total cost} / \text{change in output}$$

For example, as quantity produced increases from 10 to 18 chickens, total costs rise by 50 (Thus, the marginal cost for one chicken will be 50/8).

The structure of production cost : MC



5. The structure of production cost



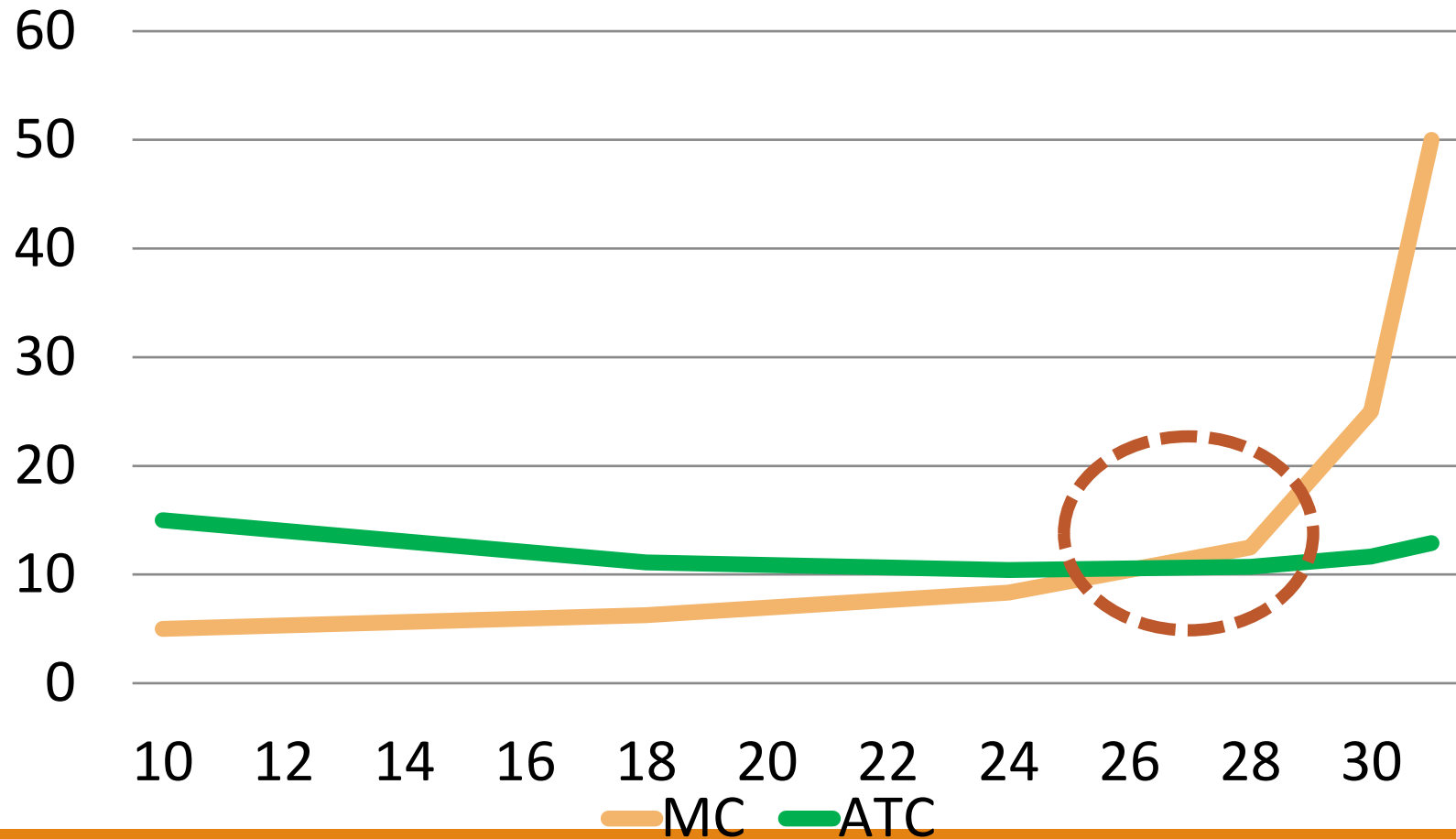
The structure of production cost : the relationship between ATC and MC

- Suppose that a 100-meter runner counting his record.
- And suppose that the average of 10 records was 10 seconds, and the 11th record is 9 seconds.
- How will his average record include the new record change? It will go down a little.
- But, If the 11th record was larger than the former average, such as 11 seconds, average record with the additional record would rise.

The structure of production cost : the relationship between ATC and MC

- The ATC has the similar concept with the average record, and MC is a concept similar to the 11th new record.
- Average total cost is somewhat cumulative cost, while marginal cost is the additional cost.
- Average total cost become larger and smaller if the marginal cost is larger and smaller than the average total cost.

The structure of production cost : the relationship between ATC and MC



Practice

I assume the form of the production function like this.

$$Q = 2 X^{(1/2)}$$

where Q is the output, X is input.

*Suppose that the fixed cost is 100, and the price of each input(variable cost) is 10.

Practice

1. Derive the equation for the total cost function to input.
2. Derive the equation for the total cost function to output.
3. Derive the equation for the average fixed cost.
4. Derive the equation for the average variable cost.
5. Derive the equation for the average total cost.
6. Derive the equation for the marginal cost to output.

Thank you!

