Black Scholes Model Drawbacks

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Problem Description

The Black-Scholes Model is a mathematical framework for pricing options, contracts that give the right to buy or sell an asset on stock exchange. However, its assumptions can limit its accuracy in real-world applications.

Underestimating its assumptions can lead to mispricing of options, potentially resulting in substantial losses.

The task was to analyse Black-Scholes models with its drawbacks, find out ways to solve them.

Actuality of the Problem

Option contracts contribute to economic stability. This makes pricing options crucial as it helps investors manage risk and capitalize on market volatility.

The Black-Scholes Model is the default mathematical model for pricing options due to its simplicity and analytical tractability. Understanding the drawbacks of the Black-Scholes Model is essential to improve its accuracy.

Black-Scholes Formula

The basic solution was invented by Black, Scholes and Merton, which provides an exact solution to the Black-Scholes partial differential equation:

$$C = S_0 N(d1) - X e^{-rT} N(d2)$$

Constant Volatility Problem and SABR

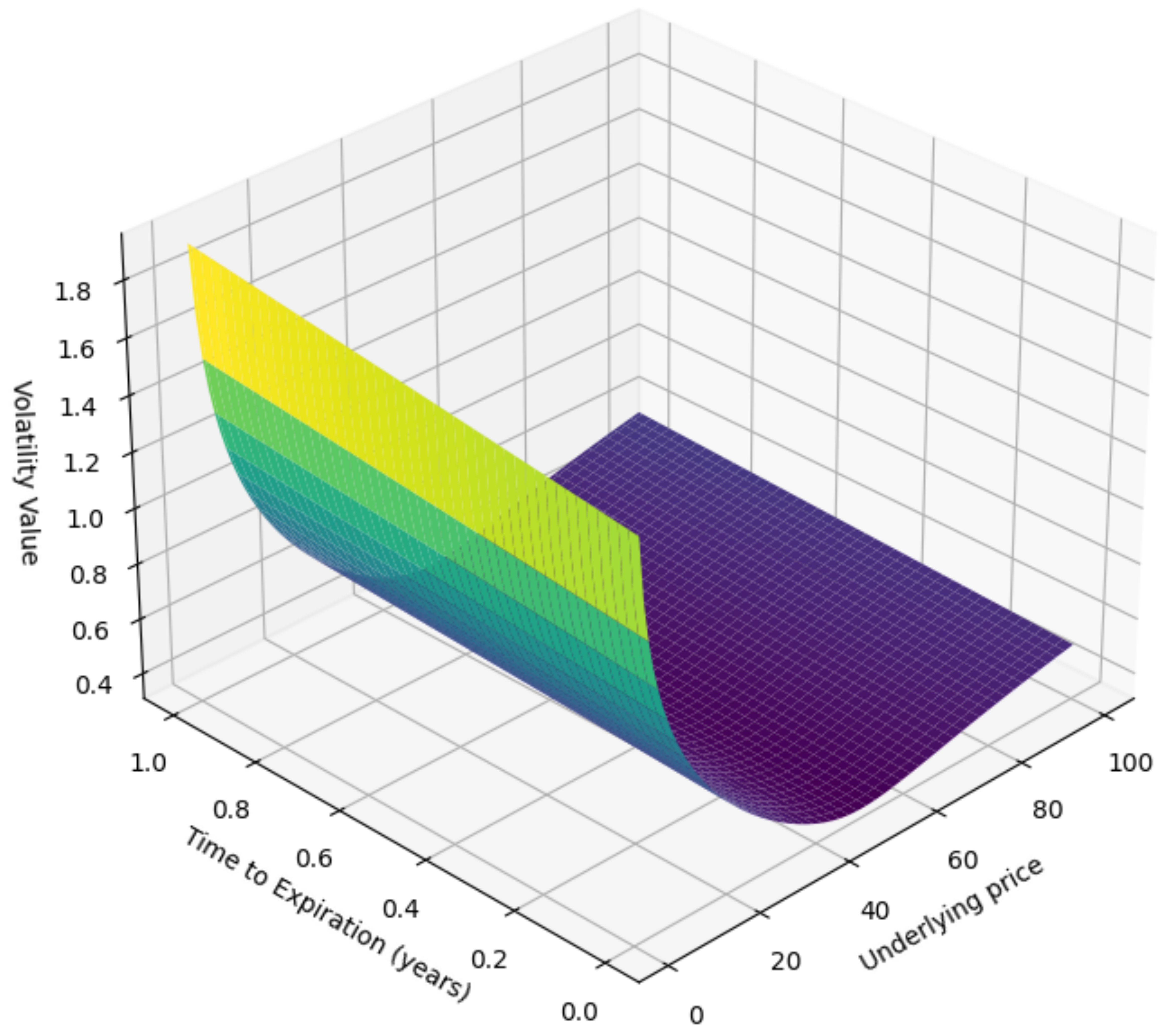
Constant volatility in the Black-Scholes Model is crucial because volatility is a key factor in determining the price of options. This leads to significant mispricing, hence SABR model was used.

$$dF_t = \sigma_t F_t^{\beta} dW_{1t},$$

$$d\sigma_t = \nu \sigma_t dW_{2t}$$

$$dW_{1t} dW_{2t} = \rho dt$$

Implied Volatility from SABR as Solution



Two Models Implementation

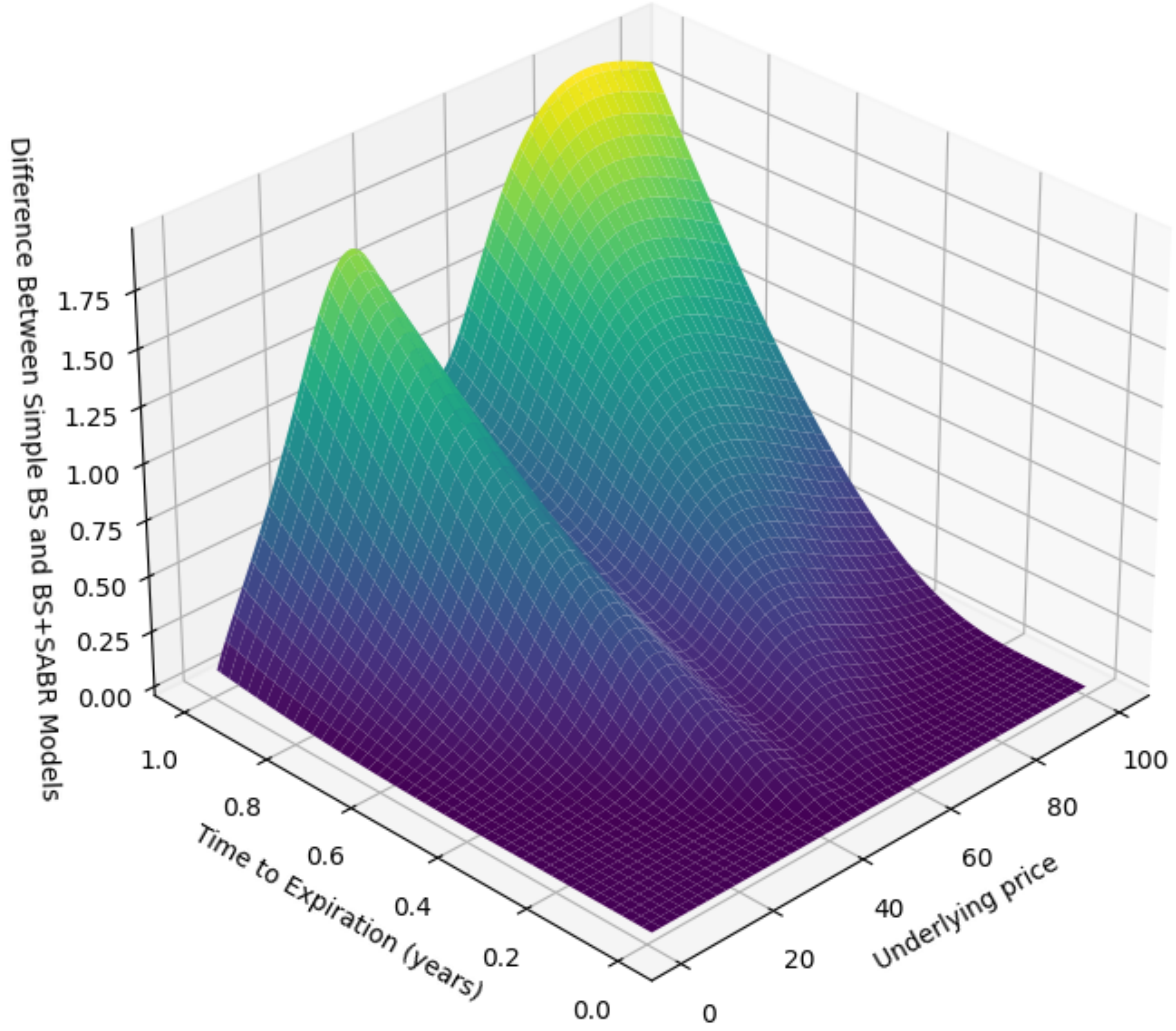
1. Black-Scholes model:

$$C_{BS} = S_0 N(d_1(\sigma)) - Xe^{-rT} N(d_2(\sigma))$$

2. Black-Scholes model with SABR implied volatility:

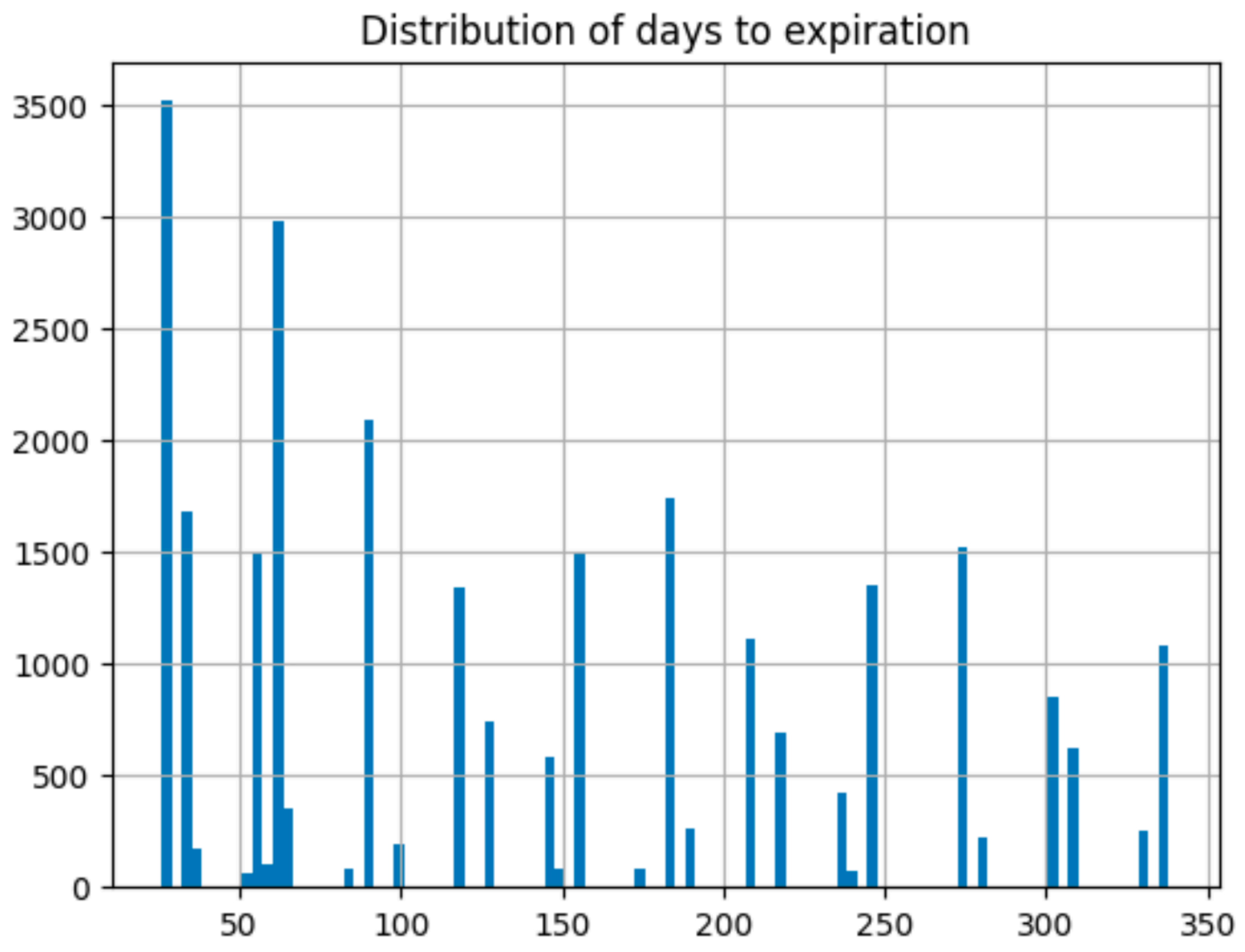
$$C_{SABR} = S_0 N(d_1(\sigma_{impl})) - Xe^{-rT} N(d_2(\sigma_{impl}))$$

Difference on grid



Market data

	expiration	ts	strike	price	spot	dt	dt_days
0	2017-12-15	2017-10-18	2140.0	237.35	2560.5	58 days	58
1	2017-12-15	2017-10-18	2150.0	241.60	2560.5	58 days	58
2	2017-12-15	2017-10-18	2160.0	254.30	2560.5	58 days	58
3	2017-12-15	2017-10-18	2165.0	247.25	2560.5	58 days	58
4	2017-12-15	2017-10-18	2170.0	246.60	2560.5	58 days	58
…	…	…	…	…	…	…	
27213	2012-06-15	2011-09-16	1290.0	206.75	1206.5	273 days	273
27214	2012-06-15	2011-09-16	1295.0	194.15	1206.5	273 days	273
27215	2012-06-15	2011-09-16	1300.0	202.80	1206.5	273 days	273
27216	2012-06-15	2011-09-16	1305.0	190.25	1206.5	273 days	273
27217	2012-06-15	2011-09-16	1310.0	198.95	1206.5	273 days	273



Differential Evolution usage

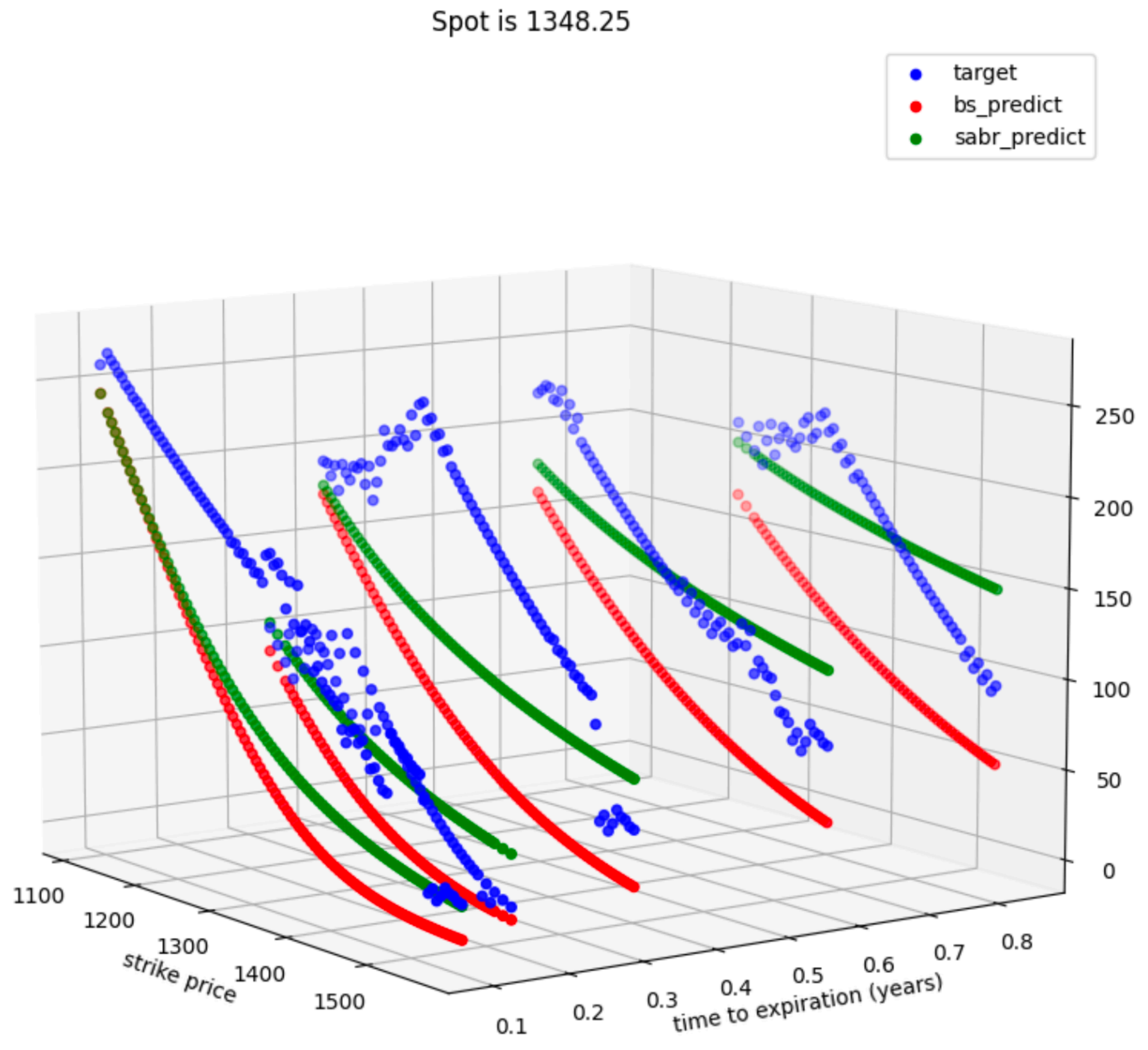
```
differential_evolution step 1: f(x)= 76.1874
differential_evolution step 2: f(x)= 72.967
differential_evolution step 3: f(x)= 72.967
differential_evolution step 4: f(x)= 72.5026
differential_evolution step 5: f(x)= 70.7206
differential_evolution step 6: f(x)= 70.7206
differential_evolution step 7: f(x)= 70.6201
```

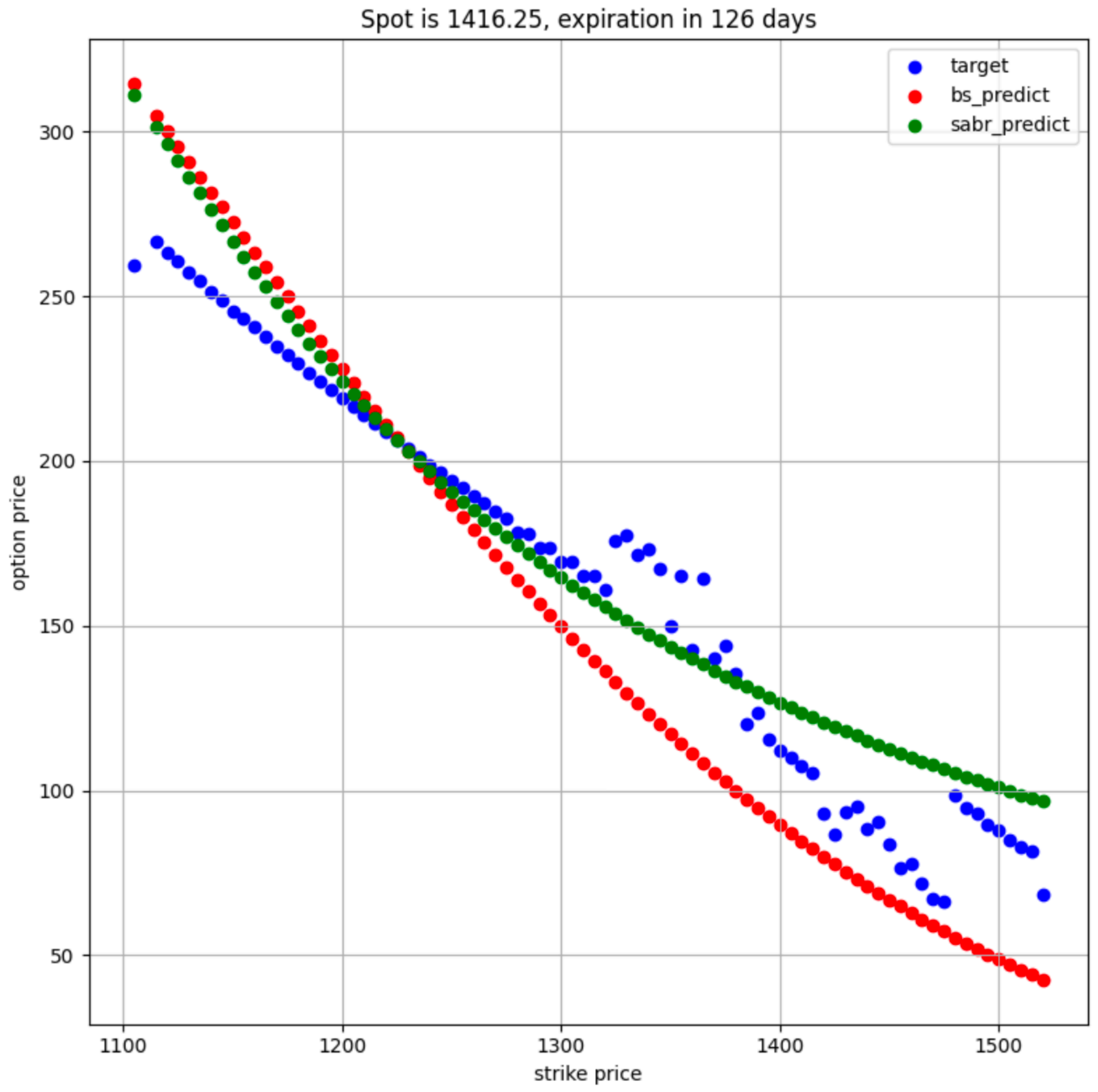
```
differential_evolution step 209: f(x)= 65.2141
differential_evolution step 210: f(x)= 65.2141
differential_evolution step 211: f(x)= 65.2141
differential_evolution step 212: f(x)= 65.2141
Polishing solution with 'L-BFGS-B'

Best Solution: [-5.99982849  2.06388218 -0.67433803  0.30102893  0.25200498]
Best RMSE Reached: 65.21412561155364
```

Evaluating Performance

```
bs_rmse_score = np.sqrt(mean_squared_error(X_test['target'],␣
 ↪X_test['bs_predict']))
sabr_rmse_score = np.sqrt(mean_squared_error(X_test['target'],␣
 ↪X_test['sabr_predict']))

bs_rmse_score, sabr_rmse_score
```

```
(62.93773615689657, 45.647942369951814)
```





Conclusion

Black-Scholes constant volatility assumption was studied, SABR model was implemented to fight this limitation, two models were implemented in practice and tested on real marketdata. Results were compared, the hypothesis was confirmed: heuristics with volatility do increase model quality significantly.

QnA