

Отчет о проверке на заимствования №1



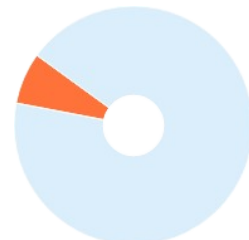
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Проверяющий: ApiCorp
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[08]	0,99%	0%	pustotin_d_a_dobavlenie-vneshney-inf...	09 Июн 2023	Модуль поиска "ВШЭ"	0	1	Источник исключен. Причина: Маленький процент пересечения.
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[16]	0,47%	0%	АННОТАЦИЯ К: Актуальные проблем... http://vo.garant.ru	05 Сен 2022	СПС ГАРАНТ: аналитика	0	1	Источник исключен. Причина: Маленький процент пересечения.
[17]	0,47%	0%	An examination of the effectiveness of s... https://doi.org	21 Июл 2003	Издательство Wiley	0	1	Источник исключен. Причина: Маленький процент пересечения.
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[21]	0,35%	0%	http://www.ru.ac.bd/stat/wp-content/u... http://ru.ac.bd	26 Фев 2022	Интернет Плюс*	0	1	Источник исключен. Причина: Маленький процент пересечения.
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[23]	0,29%	0%	https://www.diva-portal.org/smash/get... https://diva-portal.org	27 Фев 2023	Интернет Плюс*	0	1	Источник исключен. Причина: Маленький процент пересечения.

NATIONAL RESEARCH UNIVERSITY
HIGHER SCHOOL OF ECONOMICS

Faculty of Computer Science
Bachelor's Programme "Applied Mathematics and Informatics"

Research Project Report on the Topic:
Black Scholes Merton Model Drawbacks

Fulfilled by:

Student of БПМИ212 AMI FCS
Mark Litvinov

(signature)

(date)

Assessed by:

Lukyanchenko Peter Pavlovich
Faculty of Computer Science, HSE University

(signature)

(date)

Moscow 2023

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2 Abstract

The purpose of the course work was to study BS model drawbacks and find out solutions suitable for improving the basic model quality. Also compare the Black-Scholes model and its enhanced version, which incorporates SABR volatility, with the test on real market data from Indian exchange.

First, derivation of the Black-Scholes was described along with its solution. This served as a basis for finding out model drawbacks. Turned out that the constant volatility assumption is the strongest shortcoming, so I tried to overcome it with more advanced volatility calculation with SABR model and this led to creation of improved version of basic Black-Scholes. Thus, two models were tested on real data: BS and BS with SABR volatility.

For the calibration step, I used real market data from an Indian exchange. The "Differential Evolution" algorithm was employed for model calibration, executed in Python. This optimization algorithm allowed me to fine-tune the models to better fit the market data.

Subsequently, I conducted a direct comparison of the original Black-Scholes model and its SABR-enhanced version. This enabled me to numerically assess the improve gained by adding SABR implied volatility.

3 Definitions

European option - A derivative instrument that gives the owner the right, but not the obligation, to sell or buy an asset (called underlying) at a stated point of time in future with a stated price.

Volatility - A statistical measure of the dispersion of returns for a given asset or market index, often expressed as a percentage. Higher volatility indicates greater price uncertainty, representing risk or opportunity.

Black-Scholes model (BS) - A mathematical model used for calculating the theoretical value of European-style options. Three outstanding economists created the model, two of whom received the Nobel Prize for it at the time. They were Black, Scholes and Merton.

SABR model - A way to calculate volatility using some market information with usage of stochastic parameters. We will apply it to BS to make the model more precise. It incorporates four parameters α, β, ν, ρ to allow for more flexible volatility modeling.

4 Black-Scholes Equation ⁷

We assume that S (asset price) follows a geometric Brownian motion, which could be written as the following SDE:

$$dS = \mu S dt + \sigma S dz$$

where:

- dS - the component of the price change ¹
- μ - the return expected or drift
- σ - the constant volatility of an underlying asset
- dz - Wiener process or Brownian motion component

In a context where risk is neutralized, the anticipated yield of the asset aligns with the risk-free interest rate, denoted as r . Consequently, the formula undergoes a modification to reflect this:

$$dS = rS dt + \sigma S dz$$

Establish a fresh portfolio value V comprised of a single option and $-\Delta$ shares of the underlying asset. The value for Δ is selected to ensure that the portfolio is devoid of risk and appreciates at the rate equivalent to the risk-free rate. Hence, the value of this portfolio becomes:

$$V = \Delta S - C$$

where C refers to price of option call.

Utilize Ito's lemma to ascertain a small variation dV :

$$dV = \Delta dS + dC + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} dt$$

To neutralize risk, equate dV with $rV dt$:

$$dV = rVdt$$

Incorporating the formulas for dV and V results in:

$$\Delta dS + dC + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} dt = r(\Delta S - C)dt$$

To nullify dS , let Δ be:

$$\Delta = \frac{\partial C}{\partial S}$$

This is commonly referred to as the hedge coefficient or "delta."

Upon reducing the equation, the resultant form is the Black-Scholes Partial Differential Equation (PDE):

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS \frac{\partial C}{\partial S} - rC = 0$$

To derive the Black-Scholes formula for European call and put options⁷, we need to resolve the corresponding Partial Differential Equation (PDE) using suitable boundary conditions.

$$C = S_0 N(d_1) - X e^{-rT} N(d_2)$$

$$P = X e^{-rT} N(-d_2) - S_0 N(-d_1)$$

where:

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

where:

- S_0 - spot price
- X - option's strike
- T - time to expiration in years
- r - risk-free rate
- σ - constant volatility parameter
- N - normal distribution function

See the Attachments section for an example of call option pricing with BS.

5 Black-Scholes Shortcomings

The Black-Scholes-Merton (BSM) model operates on a set of premises that may not necessarily reflect actual market conditions. To better align this model with real-world scenarios, it's crucial to address these assumptions, ranking them from most to least significant. The following considerations should be taken into account for such modifications:

1 Constant Volatility and Rates:

This is the most critical assumption. The base model may not be fully relevant to actual financial markets due to its oversimplified volatility outlook. There's both short-term and long-term volatility to consider. In the face of variable volatility, we will try to solve it with SABR volatility.

2 No Dividend Payments:

Modifying the model to account for dividends is quite uncomplicated. By reducing the existing asset price by its dividend's present value, the adjustment can be smoothly integrated into the model. This is done by incorporating a continuous dividend yield q into the formula:

$$C = S_0 e^{-qT} N(d_1) - X e^{-rT} N(d_2)$$

3 No Taxes and Transaction Costs:

Taking into account taxes and transaction costs could be complicated and often requires numerical solutions. However, for large trading entities that act as liquidity providers, this may be less of a concern and is therefore not a primary area of study.

4 Market Efficiency:

If markets are not perfectly efficient and there's predictive capacity, this can be incorporated. For example, machine learning algorithms could forecast price or volatility trends. This aspect remains ambiguous in terms of adjustment.

5 European-Style Options:

To address American options, which can be exercised early, models like the Cox-Ross-Rubinstein tree model can be employed. However, this is considered a less conventional and less pressing issue for most researchers.

6 Risk-Free Borrowing and Lending:

If borrowing and lending are associated with some risk, then the cost of borrowing and the yield on lending could be factored in, though they often have a lesser impact compared to changes in volatility.

In summary, the importance of each assumption varies, and the BSM model can be adjusted in various ways to more closely reflect the complexities of real-world markets.

6 Problem with volatility

BS model is highly regarded as a cornerstone for pricing financial derivatives, but it assumes that market volatility remains stable. While this assumption allows for more tractable equations, it often fails to accurately capture the fluctuating nature of financial markets. In practice simple Black-Scholes model has a lot of mispricing cases which leads to loss of capital and more sophisticated approaches.

Addressing this shortcoming, SABR model allows us to react to some changes in volatility depending on different combinations of underlying price, strike and time to expiration. The main goal of the model is to calculate implied volatility for a stated set of parameters, which could be used in several ways.

7 SABR Volatility

The stochastic alpha, beta, rho (SABR) model is extensively employed in practice for more advanced options pricing. It also allows to capture the volatility smile, phenomena observed when implied volatility varies with strike prices. This feature allows financial professionals to better understand and manage the risks associated with options portfolios, making the SABR model a valuable tool for pricing, risk assessment, and portfolio optimization in the ever-evolving world of finance.

Talking about math side of the model it describes the evolution of an asset price F and its volatility σ using stochastic differential equations. The model's SDEs are:

$$dF_t = \sigma_t F_t^\beta dW_{1t},$$

$$d\sigma_t = \nu \sigma_t dW_{2t}$$

Here, α is the initial volatility, β governs the asset behavior, ν is the vol-vol, ρ - correlation between dW_{1t} and dW_{2t} - Brownian motions, defined as $dW_{1t}dW_{2t} = \rho dt$. These four parameters α, β, ν, ρ determine how the asset and its volatility evolve over time.

8 SABR Output Volatility

Before delving into the complex formula for SABR (Stochastic Alpha, Beta, Rho) volatility, it's important to set up the notation:

T – time to expiration (years),

s – strike,

f – forward price,

α – 'volatility-like' parameter,

β – characterizes the asset's behavior,

ν – volatility of the underlying volatility,

ρ – value ranged from -1 to 1: correlation of the asset price and its volatility

In the notation made, the implied volatility formula is the following:

$$\sigma_{impl} = \frac{\alpha}{D} \times mult \times d,$$

Where:

$$\begin{aligned} D &= \sqrt{A} \left(1 + \frac{C}{24} + \frac{C^2}{1920} \right), \\ d &= 1 + T \left(\frac{(1 - \beta)^2 \alpha^2}{24A} + \frac{\rho \beta \nu \alpha}{4\sqrt{A}} + \frac{(2 - 3\rho^2) \nu^2}{24} \right), \\ mult &= \frac{z}{\log \left(\frac{\sqrt{B} + z - \rho}{1 - \rho} \right)}, \\ B &= 1 - 2\rho z + z^2, \\ z &= \sqrt{A} (\nu / \alpha) \log \frac{f}{s}, \\ A &= (f \cdot s)^{1 - \beta}, \\ C &= (1 - \beta)^2 \times \log^2 \frac{f}{s} \end{aligned}$$

Find example of volatility surface with SABR model, improved model pricing results and price surface difference in Attachments section.

9 Two Models

In the realm of financial modeling for option pricing, the Black-Scholes (BS) model stands as a seminal framework that calculates the theoretical value of European-style options based on a set of parameters: spot, strike, time to expiration, rate, volatility (constant).

While effective under certain conditions, its limitation lies in its assumption of constant volatility. An advanced modification to this model incorporates volatility values generated from the Stochastic Alpha, Beta, Rho (SABR) model for each corresponding set of parameters. Instead of assuming a constant volatility, the SABR model produces a dynamic volatility parameter to capture volatility smile. This hybrid approach results in a more accurate and market-adaptive option pricing mechanism.

Two ways were thought of to add SABR volatility into the basic BS. The first way was to plug the SABR volatility into the existing Black-Scholes formula. The second way was to change the Black-Scholes equation itself by adding the SABR volatility and then solving the new equation. I chose the first way to quickly fix the main problem of the original model, which is that it assumes volatility stays the same. Adding dynamic, or changing, volatility from the SABR model made the Black-Scholes model much better. This improvement was clear when the updated model was

tested using real market data.

10 Implementation

The "Implementation" section details the work done in Python for option pricing. Two models were developed: the classic Black-Scholes (BS) and its improvement that integrates SABR volatility, based on existing literature. Both models were tested using real market data from an Indian exchange.

The models were calibrated and evaluated using the RMSE (Root Mean Square Error) metric with help of Differential evolution algorithm - an optimization algorithm that starts with a random population of candidate solutions and evolves them over time. Through steps of mutation, crossover, and selection, it aims to find the solution that optimizes a given function, often used for real-valued, multi-dimensional problems. The classic BS model yielded an RMSE of 62 on the test data, while its improved version had a significantly better RMSE of 45. In the context of RMSE, this improvement is quite strong.

To make the comparison more visual, I created 3D and 2D graphs comparing the results from both models. These graphs clearly show that the improved model often provides a more accurate price estimation, although neither is perfect, especially for options with a far-off expiry date. This is likely because the available data may contain some insider factors that are very hard to account for on top of the current model assumptions. Some graphs could be found in Attachments sections, all the graphs gained are stored in **pngs/compare_test** folder in the project repository.

To see the whole solution find ipython notebook in repository.

11 Conclusion

In this study, I delved into the intricacies of the Black-Scholes (BS) model for option pricing, focusing particularly on its limitations, such as the assumption of constant volatility. This rigid assumption often makes the model less adaptable to real-world market conditions, leading to discrepancies between the predicted and actual option prices.

To address this shortcoming, I explored the Stochastic Alpha, Beta, Rho (SABR) model, known for its capability to capture dynamic volatility. I implemented both the classic BS and an improved BS+SABR model, evaluating their performance using real market data from an Indian exchange. The results demonstrated a significant improvement in the Root Mean Square Error

metric, confirming the worth of challenging the constant volatility assumption in the BS model.

In conclusion, the incorporation of dynamic volatility from the SABR model proved to be an effective heuristic for enhancing the BS model's performance. This advancement not only improves the model's market adaptability but also provides a promising avenue for future research to make even more accurate and reliable option pricing models.

12 Attachments

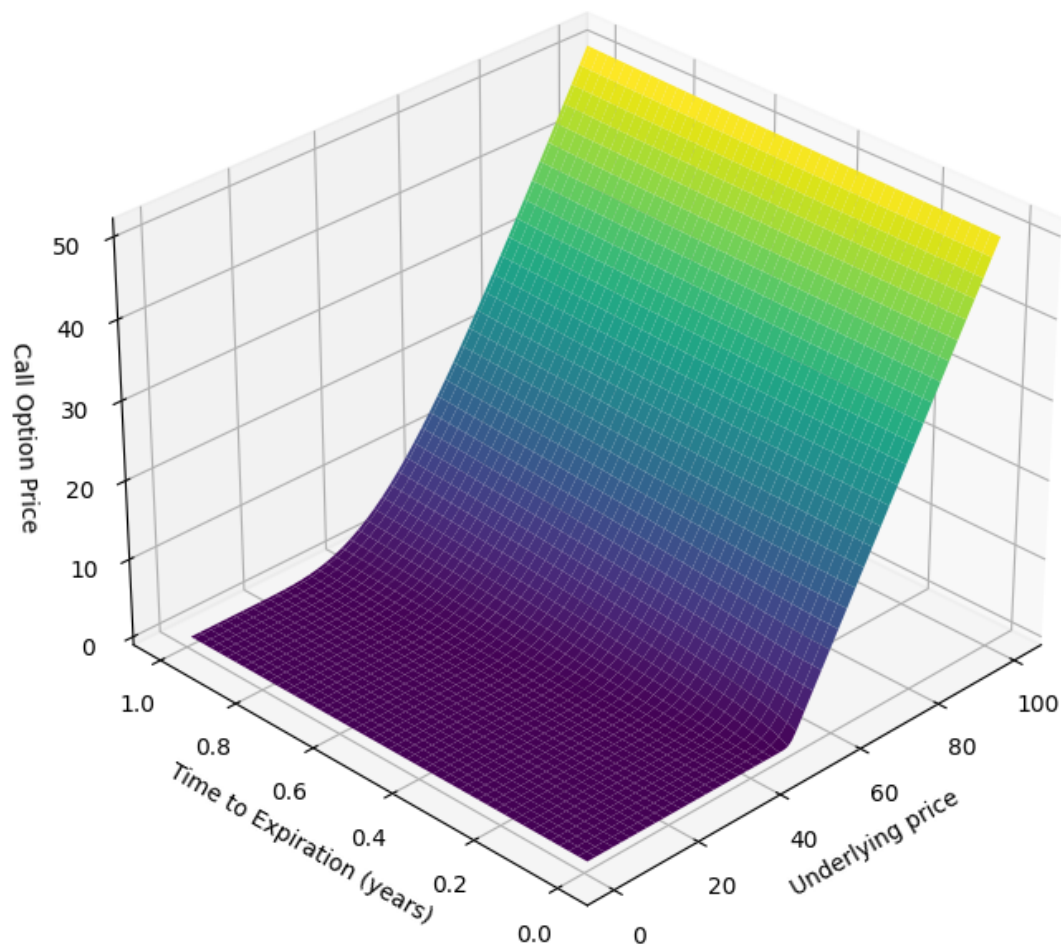


Figure 12.1: BS model pricing call option with parameters:
strike = 50, rate = 2%, volatility = 0.34

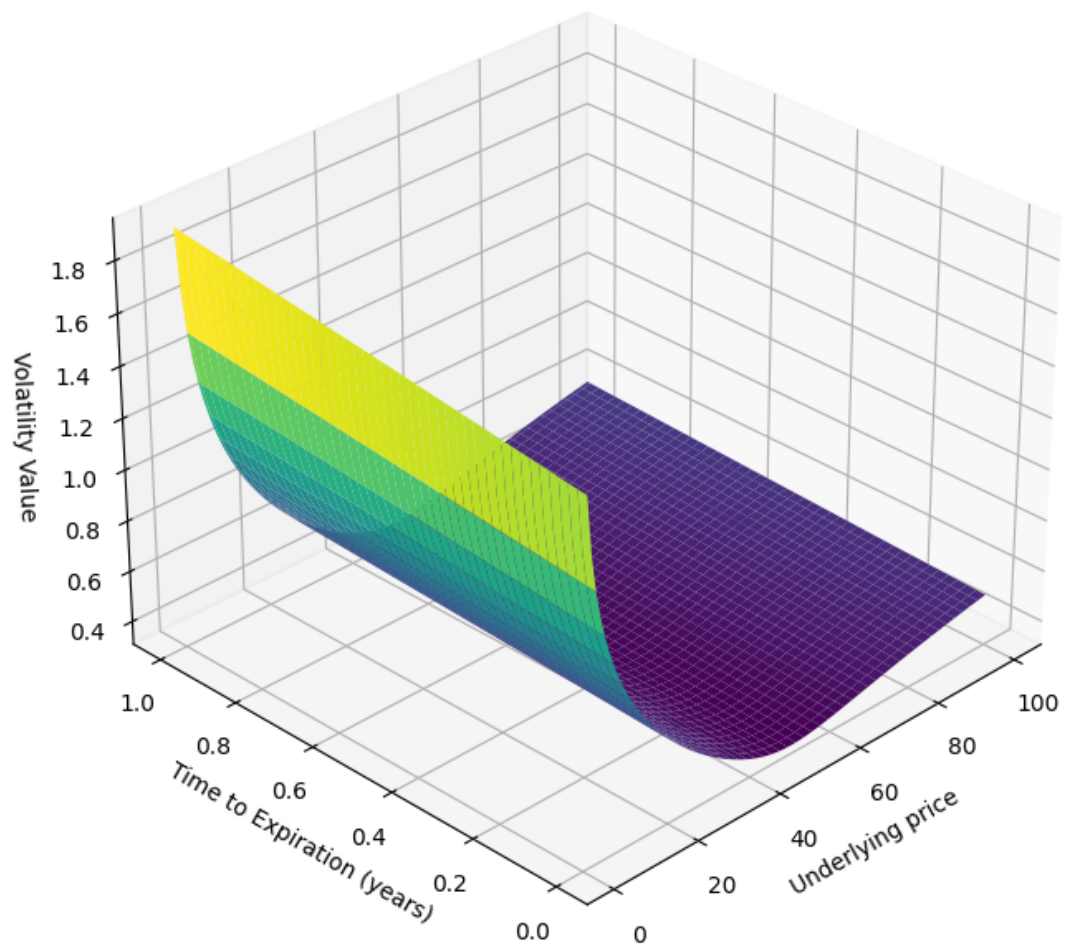


Figure 12.2: SABR volatility surface with parameters:
strike = 50, rate = 2%, $\alpha = 1.63$, $\beta = 0.6$, $\rho = -0.09$, $\nu = 1.3$

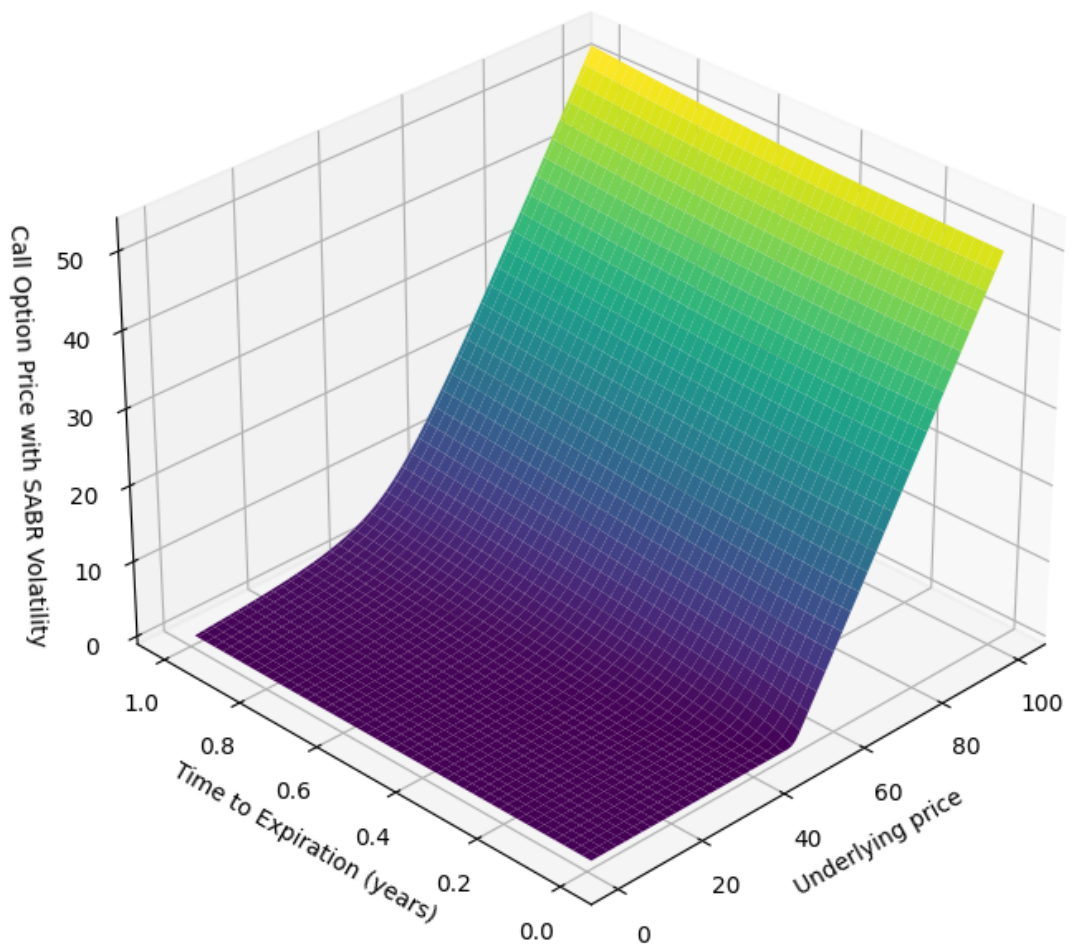


Figure 12.3: BS+SABR model price prediction with parameters:
strike = 50, rate = 2%, $\alpha = 1.63$, $\beta = 0.6$, $\rho = -0.09$, $\nu = 1.3$

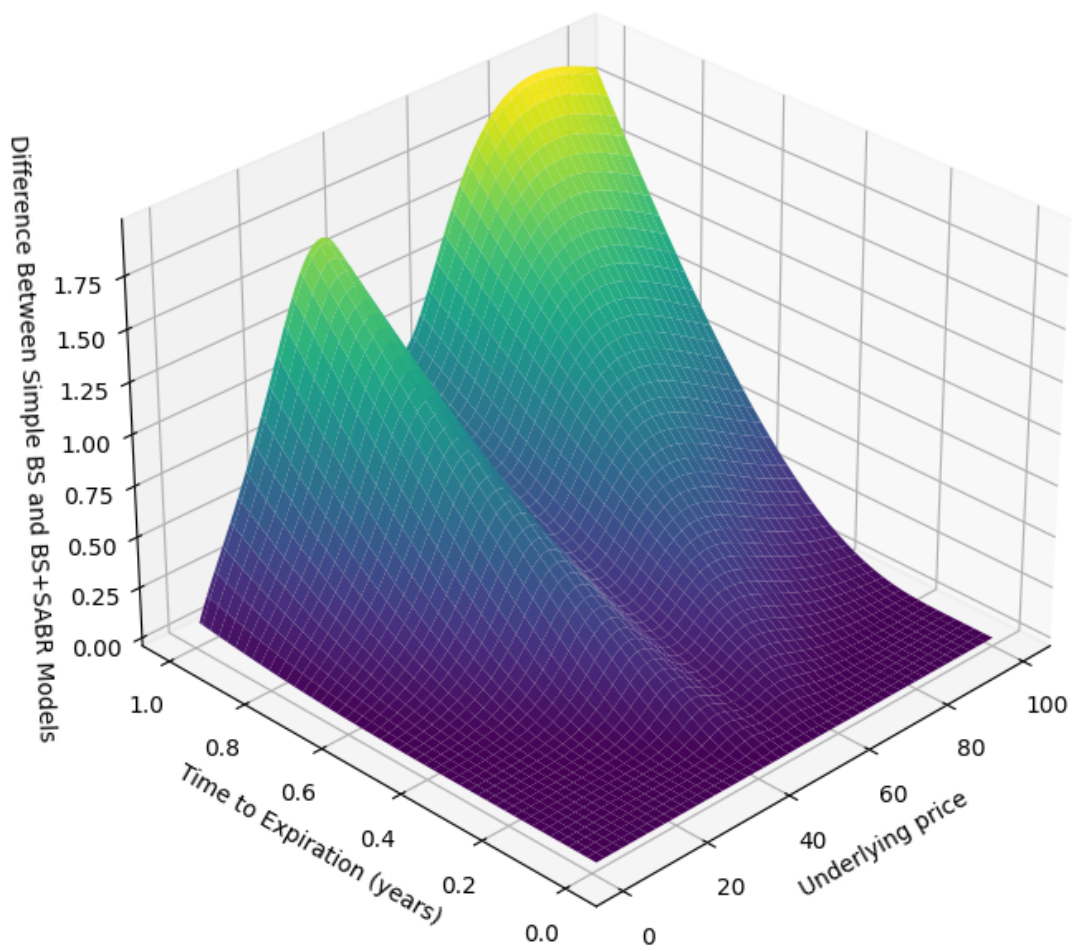


Figure 12.4: Difference of two models

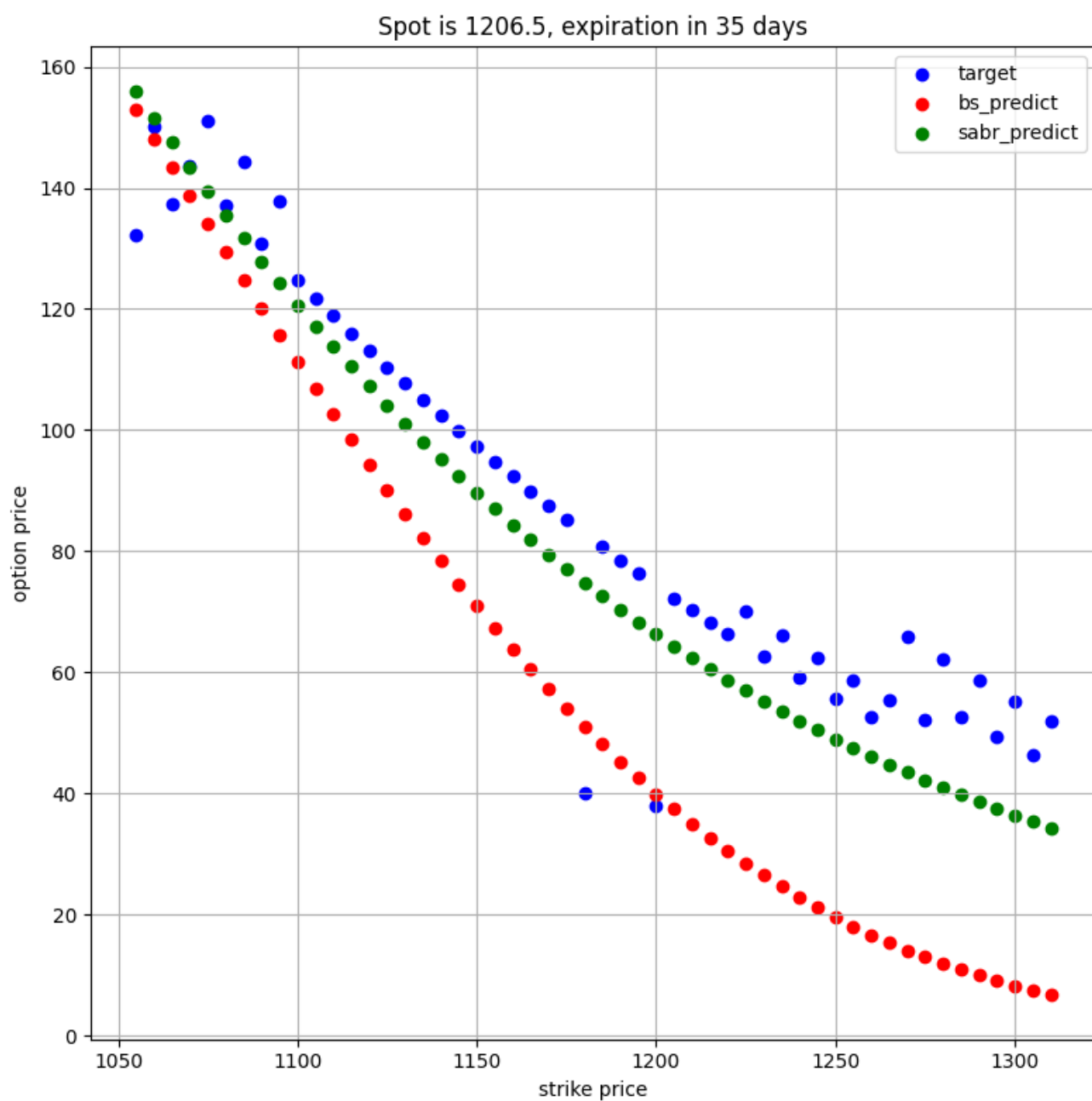


Figure 12.5: Comparing two models on a test part of real market data

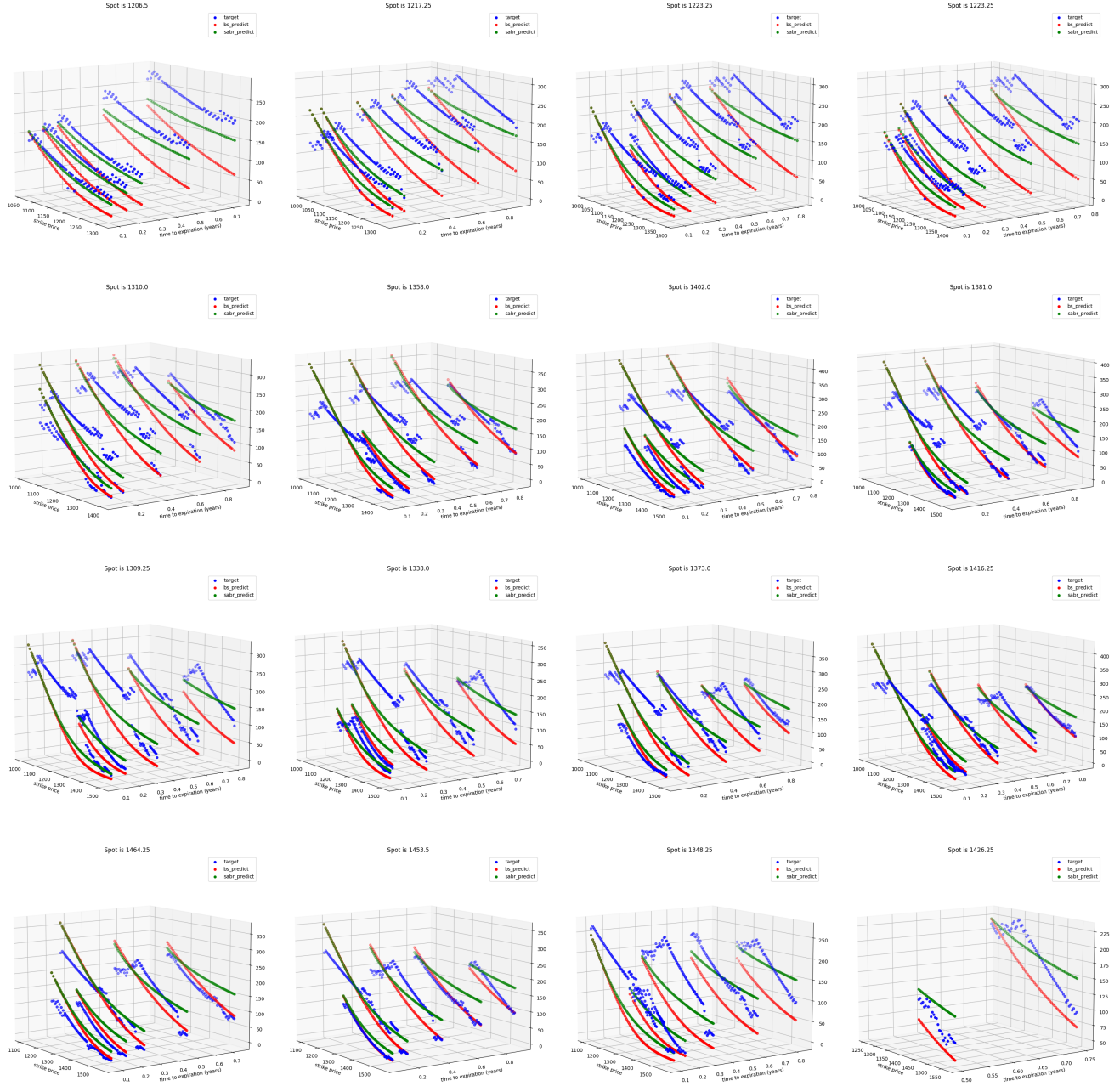


Figure 12.6: Summary of comparison