Доклад про воду и камни

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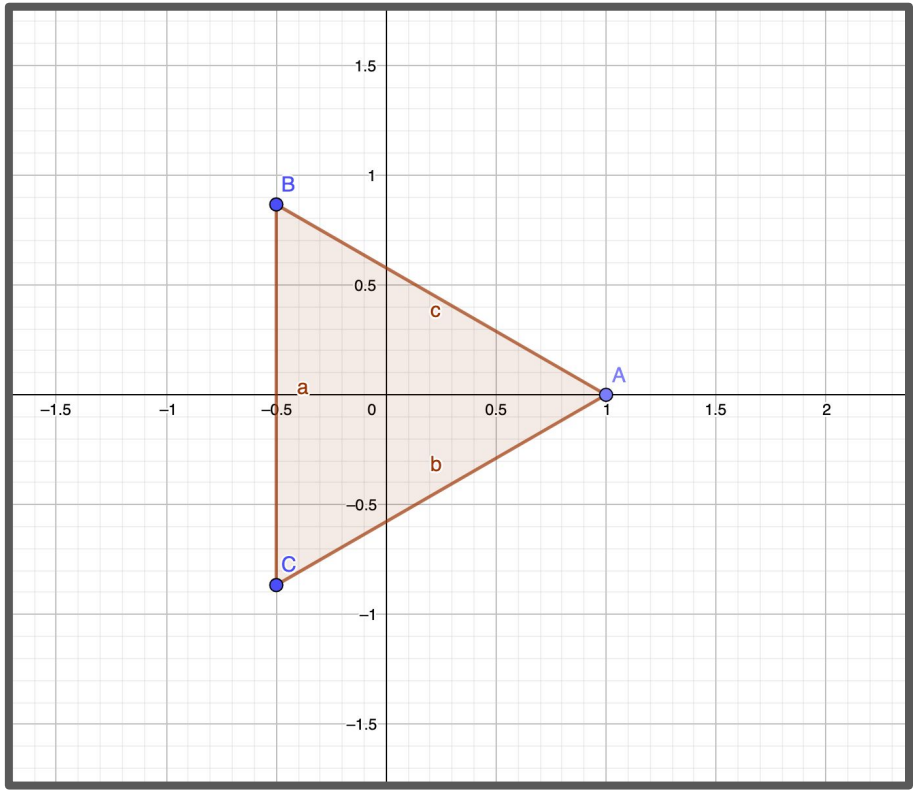
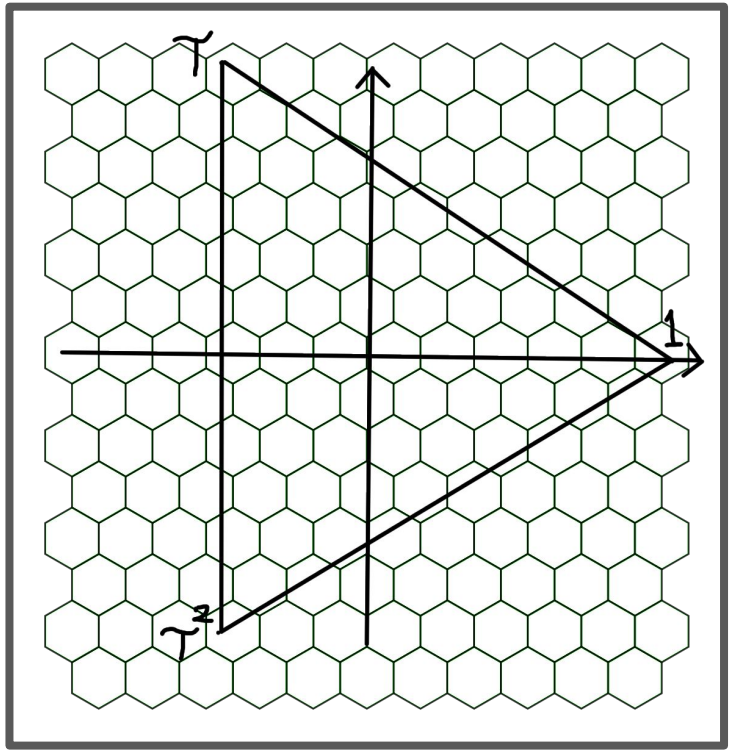
Вводные определения

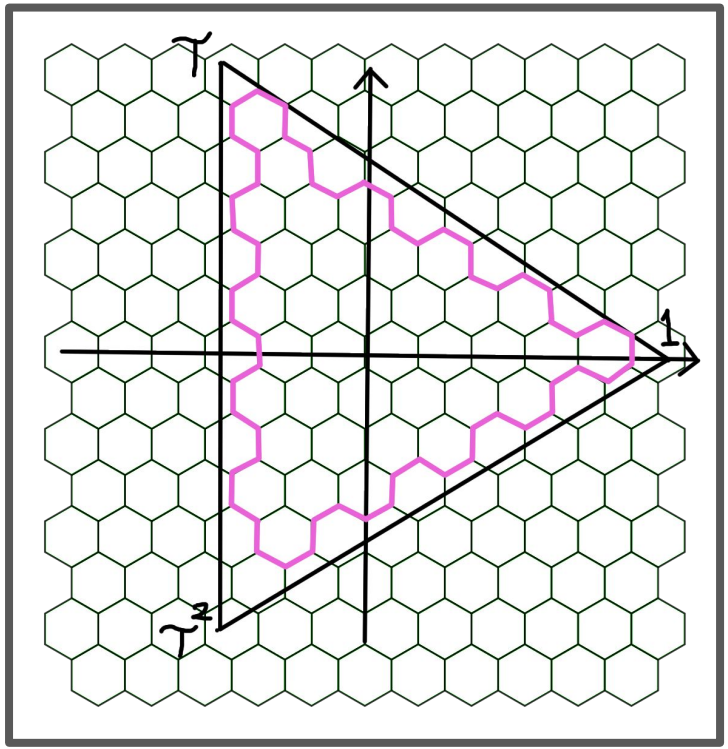
$$\tau = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$$

n-замощение

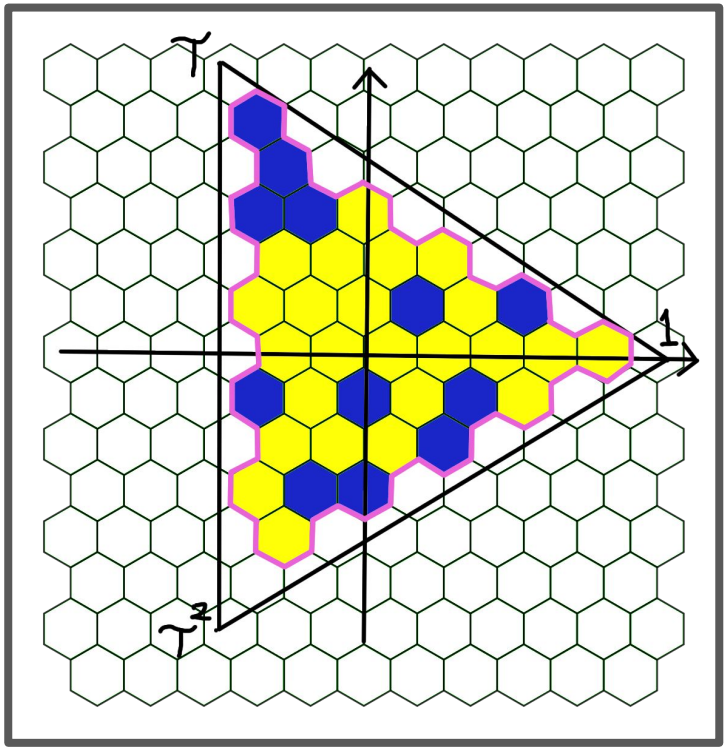




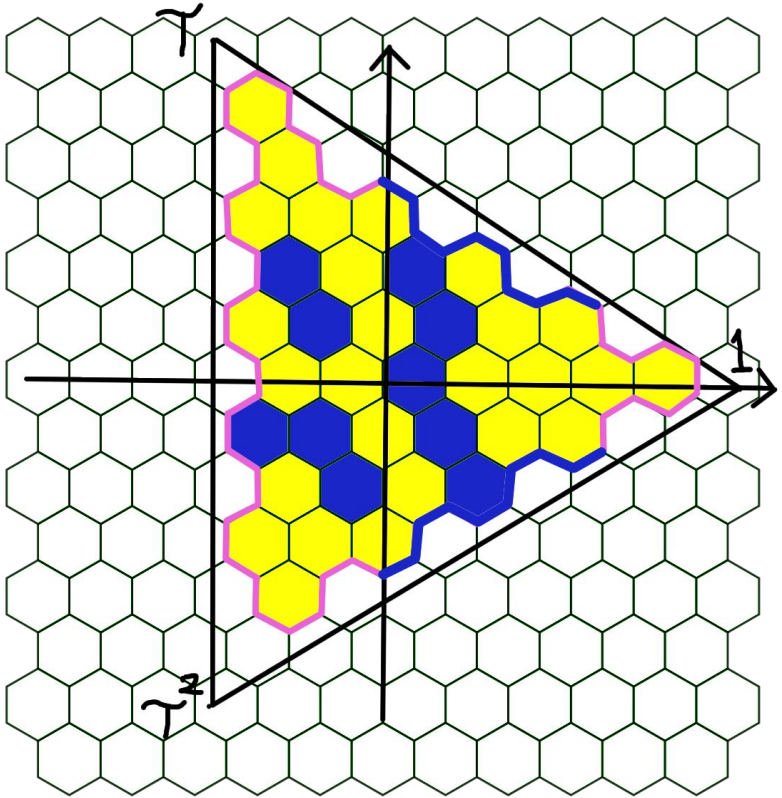
M_n



Раскраски шестиугольников из M_n

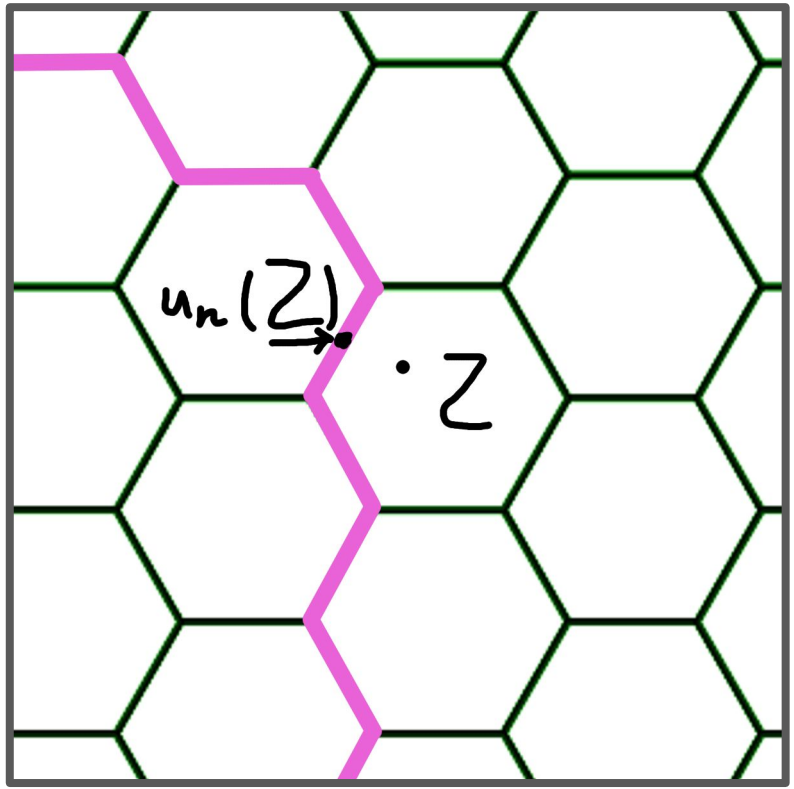


Протекание между ломаными

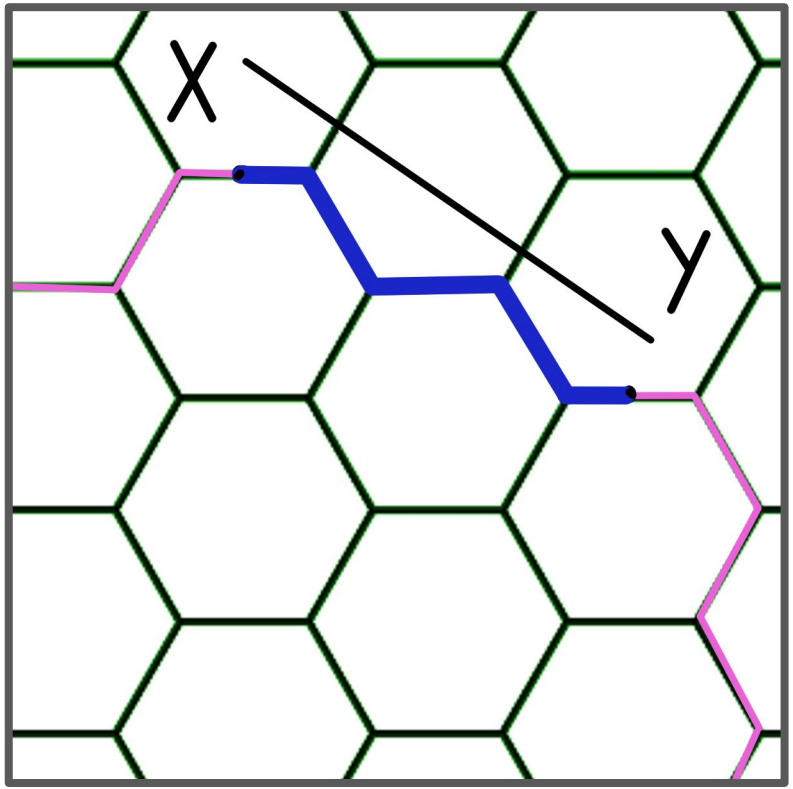


Ближайшая середина стороны M_n

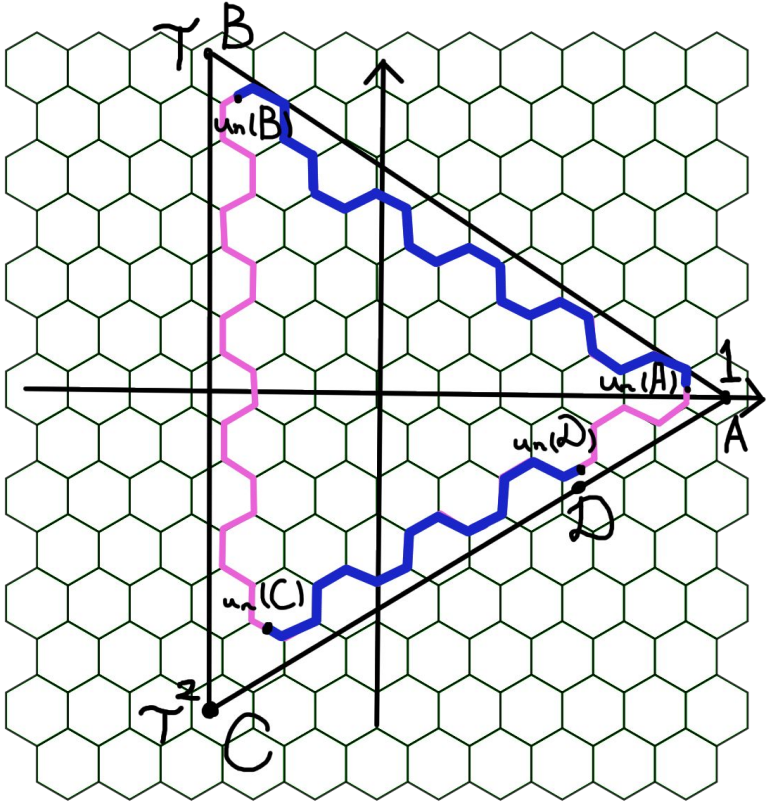
$$Z \to u_n(Z)$$



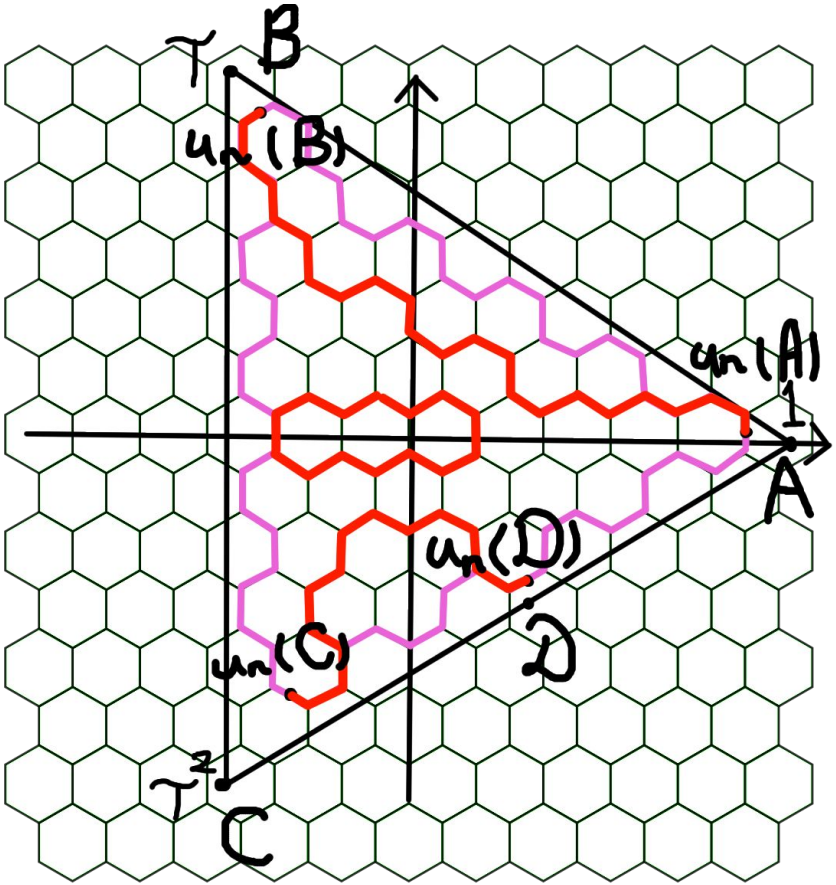
$$XY \to u_n(X)u_n(Y)$$



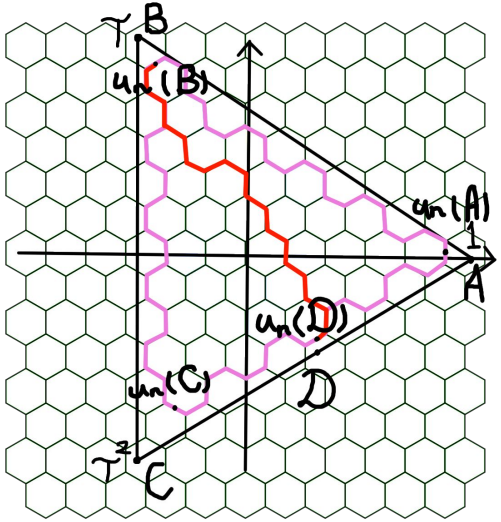
$$\lim_{n\to\infty} u_n(D)u_n(A) \leftrightarrow u_n(C)u_n(B) = \frac{CD}{AB}$$



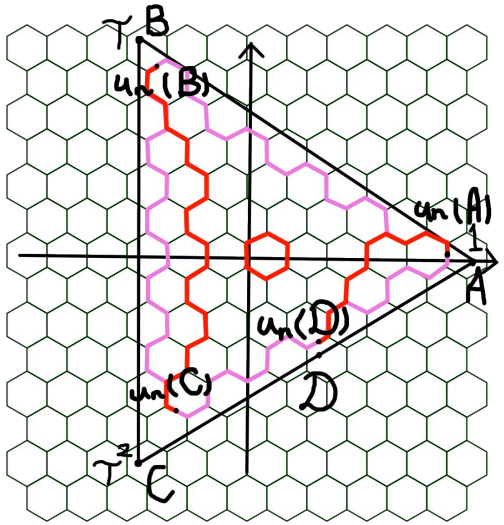
Пути из красных отрезков


T
B
$u_n(B)$
$u_n(A)$
1
A
$u_n(D)$
D
$u_n(C)$
T^2
C

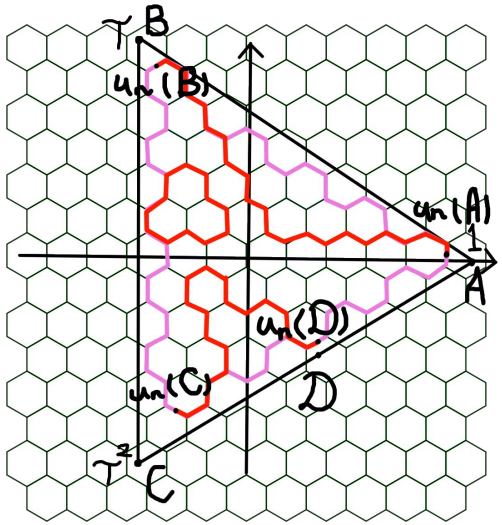
3 типа



$$P(u_n(D) \leftrightarrow u_n(B))$$

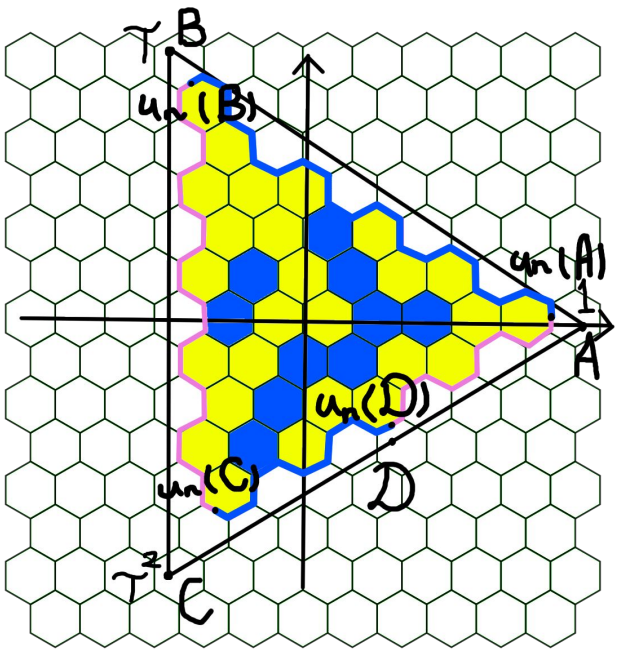


$$P(u_n(D) \leftrightarrow u_n(C))$$

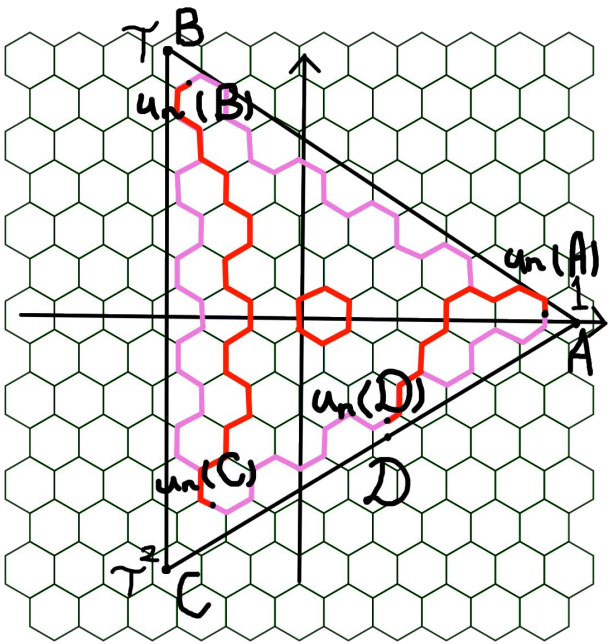


$$P(u_n(D) \leftrightarrow u_n(A))$$

Предложение 1



$$u_n(C)u_n(D) \leftrightarrow u_n(A)u_n(B)$$



$$P(u_n(D) \leftrightarrow u_n(A))$$

Наблюдаемая f

Рассмотрим функцию, определенную на множестве середин ребер M_n

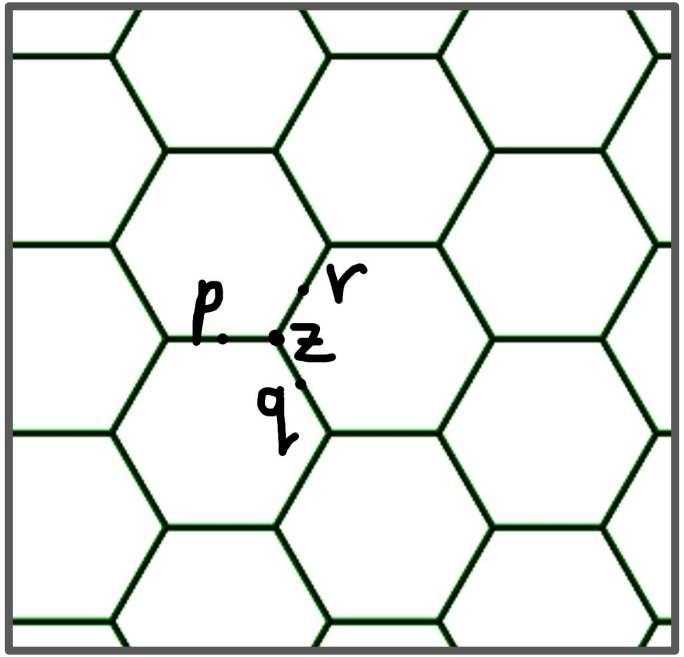
$$f_n(u_n(D)) \stackrel{def}{=} P(u_n(D) \leftrightarrow u_n(A))$$

$$+ \tau\, P(u_n(D) \leftrightarrow u_n(B))$$

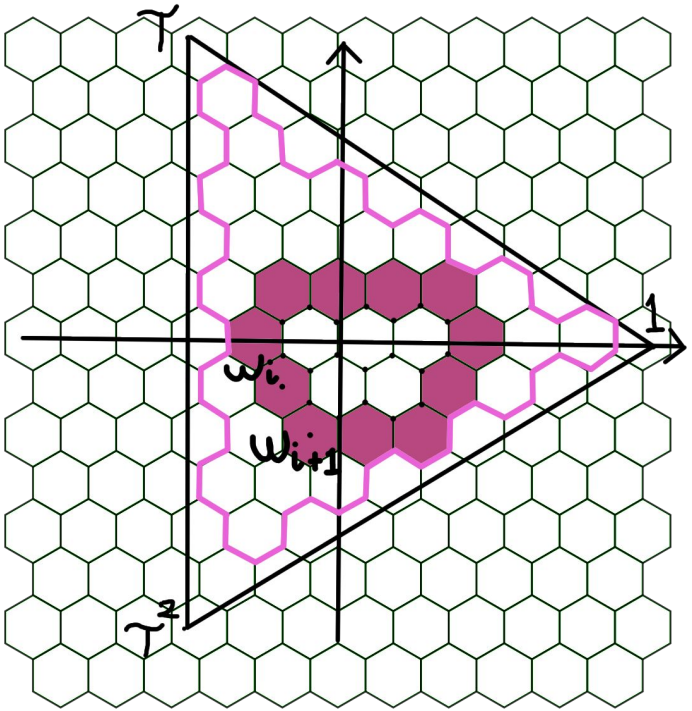
$$+ \tau^2\, P(u_n(D) \leftrightarrow u_n(C))$$

Тождество треугольника

$$(p-z)f_n(p)+(q-z)f_n(q)+(r-z)f_n(r)=0.$$



$$\sum_{j=1}^{k} f\left(\frac{w_{j+1}+w_j}{2}\right)(w_{j+1}-w_j)=0.$$



Ключевые утверждения

Теорема 1. Существует функция $F(Z)$, заданная на всем треугольнике ABC, включая границу, и принимающая комплексные значения, такая что:

$$\lim_{n\to\infty} \max_{Z\in ABC} |F(Z) - f_n(u_n(Z))| = 0.$$

Непрерывность

$$\lim_{n\to\infty} \max_{Z\in ABC} |F(Z) - F(u_n(Z))| = 0.$$

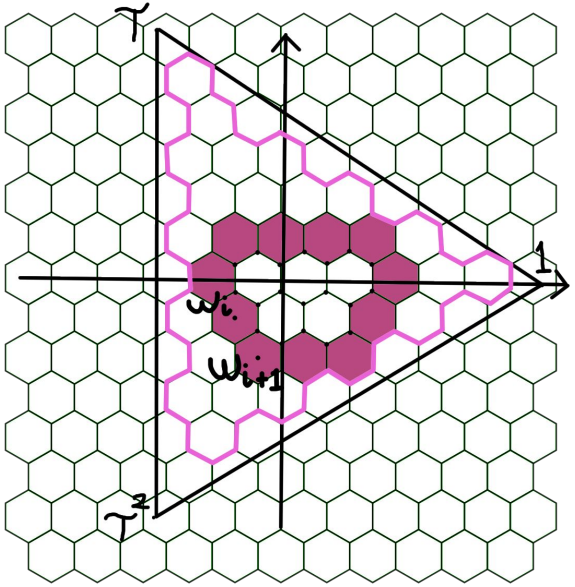
Доказательство:

$$|F(Z) - F(u_n(Z))| \leq |F(Z) - f_n(u_n(Z))| +$$

$$+ |F(u_n(Z)) - f_n(u_n(Z))| \leq \frac{\varepsilon_0}{2} + \frac{\varepsilon_0}{2} = \varepsilon_0.$$

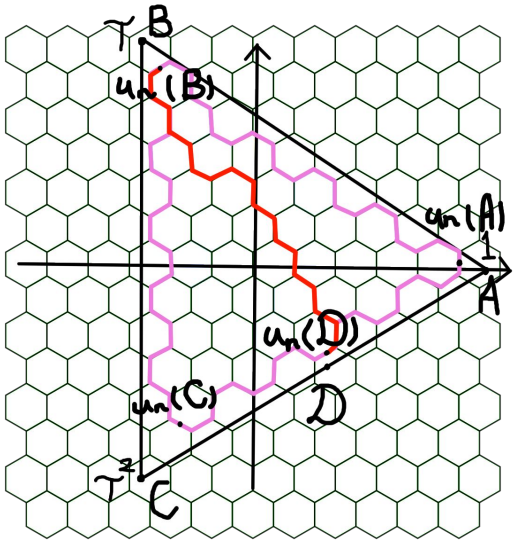
Аналитичность в некотором виде

$$\lim_{n\to\infty}\sum_{j=1}^{l_n} F\left(\frac{w_{j+1,n}+w_{j,n}}{2}\right)(w_{j+1,n}-w_{j,n})=0$$



Граничное условие

$$F(D) \in \begin{cases} [1; \tau^2] & при\ D \in [A,\ C], \\ [\tau^2; \tau] & при\ D \in [C,\ B], \\ [\tau; 1] & при\ D \in [B,\ A]. \end{cases}$$



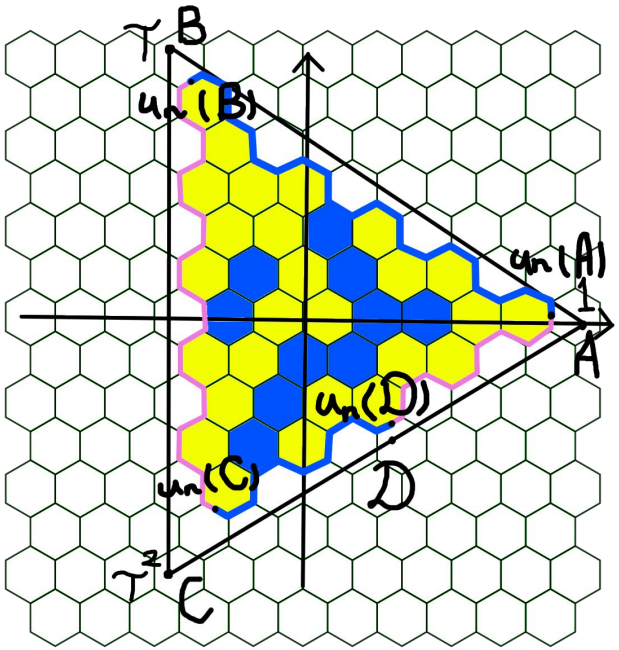
Теорема 2. Существует не более одной функции $F(Z)$, заданной на всем треугольнике ABC, включая границу, принимающей комплексные значения и удовлетворяющей трем предыдущим условиям.

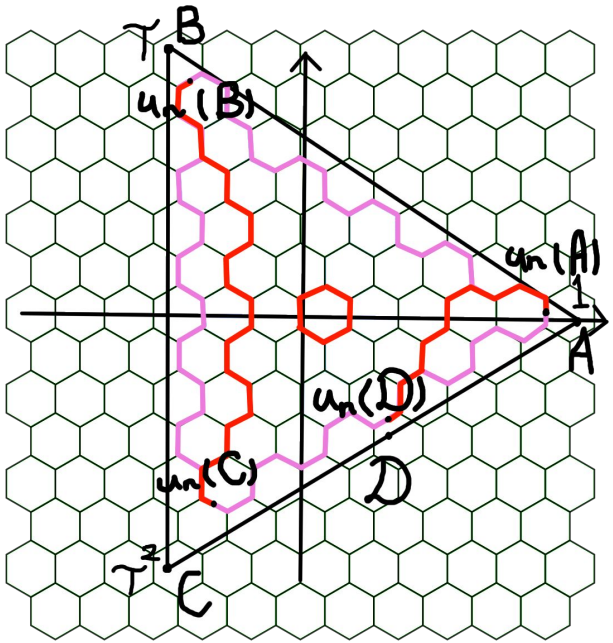
Лемма 1. Тождественная функция $F(Z) = Z$ удовлетворяет предыдущим условиям.

Доказательство теоремы

Равенство пределов

$$\lim_{n\to\infty} u_n(D)u_n(A) \leftrightarrow u_n(C)u_n(B) \quad = \quad \lim_{n\to\infty} P(u_n(D) \leftrightarrow u_n(A))$$

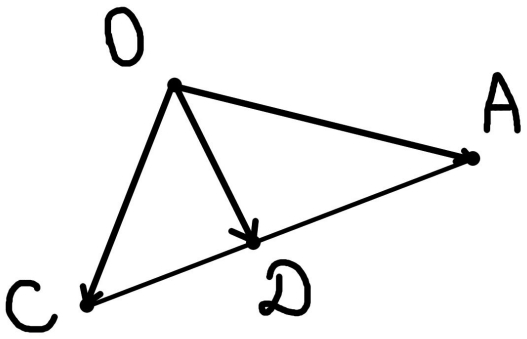




$$D = F(D) = \lim_{n\to\infty} f_n(u_n(D)) =$$

$$= \lim_{n\to\infty} \left(P(u_n(D) \leftrightarrow u_n(A)) + \tau^2 P(u_n(D) \leftrightarrow u_n(C))\right)$$

$$D = 1 * \frac{DC}{AC} + \tau^2 * \frac{AD}{AC}$$



$$\Rightarrow \lim_{n\to\infty} P(u_n(D) \leftrightarrow u_n(A)) = \frac{DC}{AC} \qquad \square$$