Geometric Modeling

2015

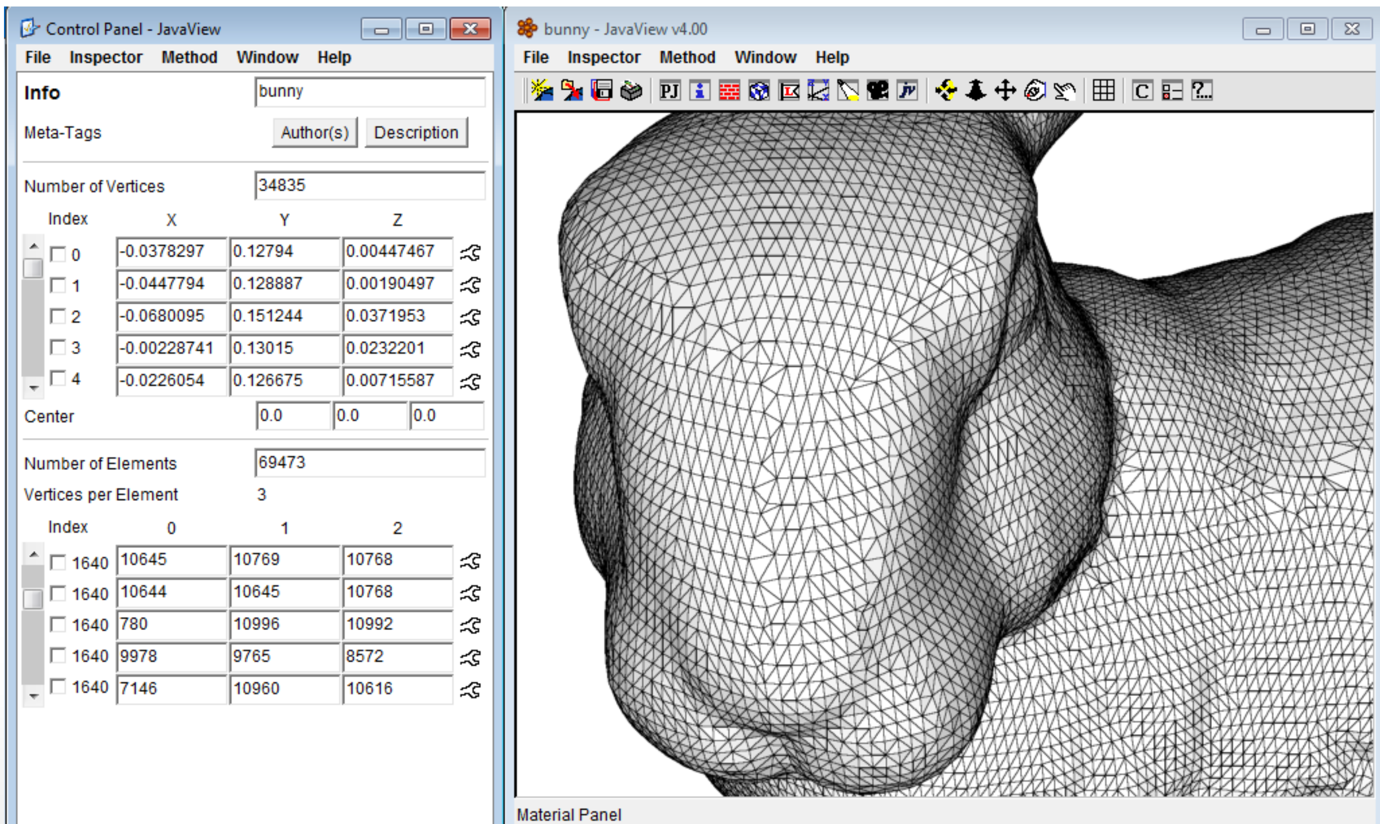
Surface Meshes

TU Delft

Triangle Meshes

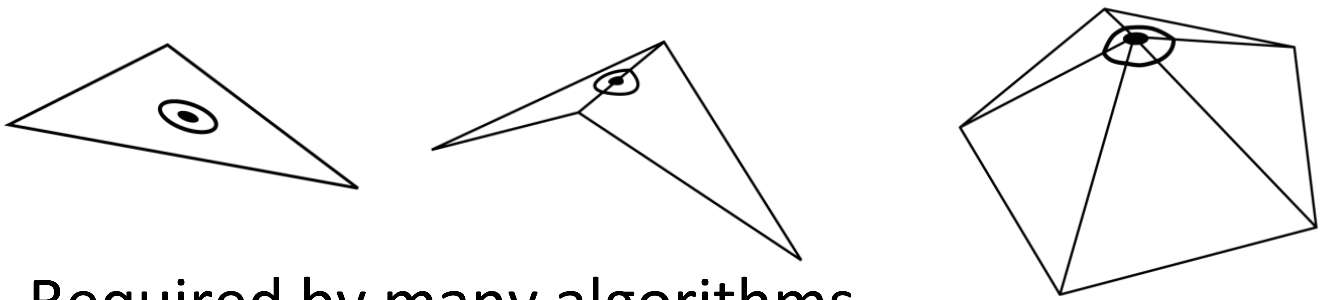
Representation of a triangle mesh in $\mathbb{R}^3$

- Vertices: a finite list $\{v_1, \ldots, v_n\}$ of points in $\mathbb{R}^3$
- Faces: a list of triples, e.g. $\{\{2,34,7\}, \ldots, \{14,7,5\}\}$



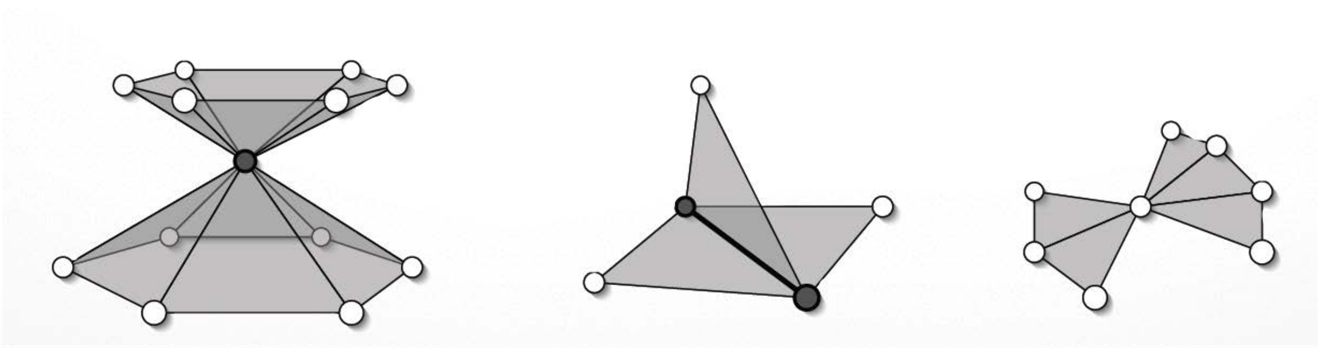
Surfaces

Surface: All points are locally disks (except boundaries)

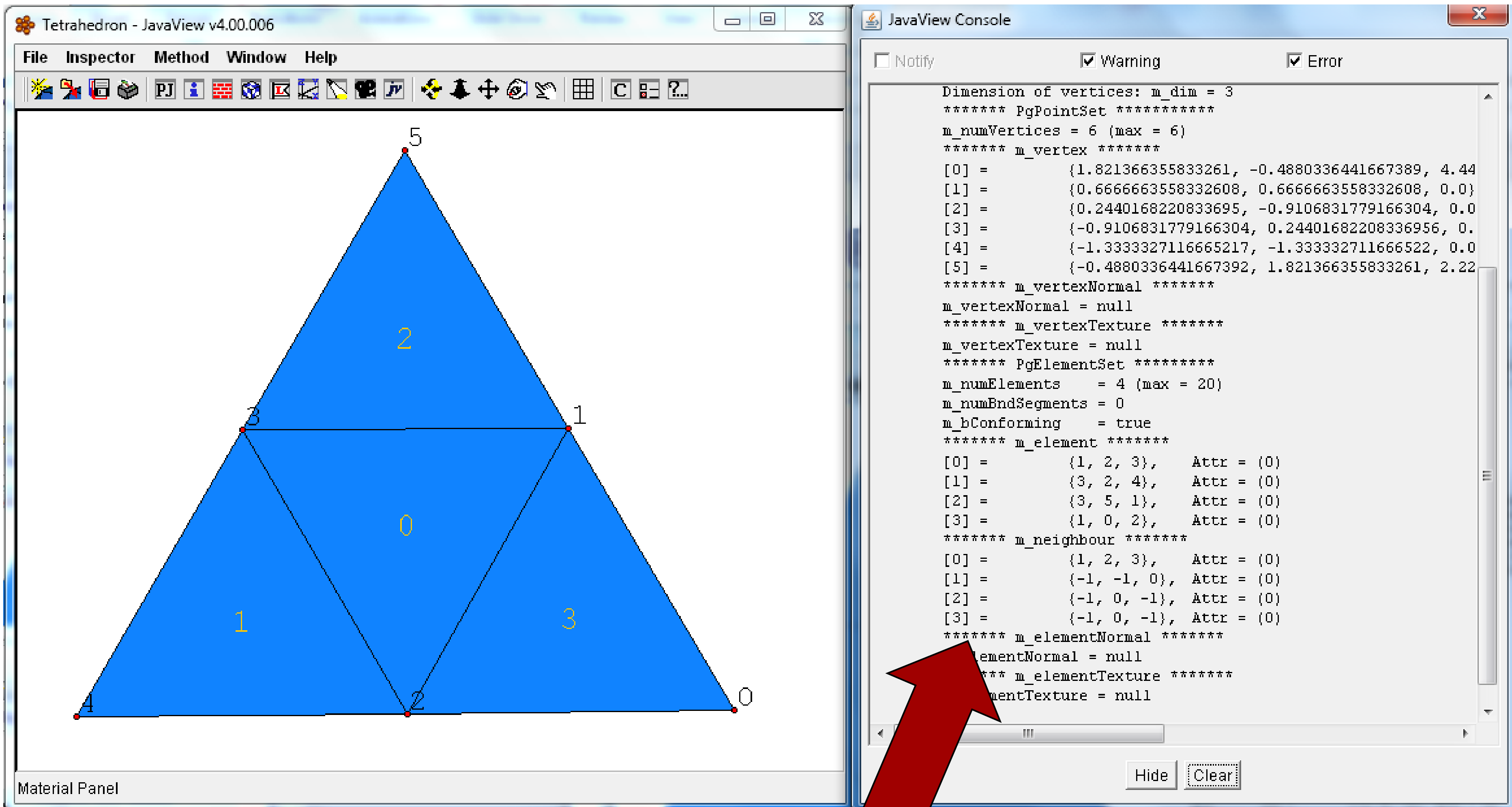


- Required by many algorithms

Examples: Non-manifold



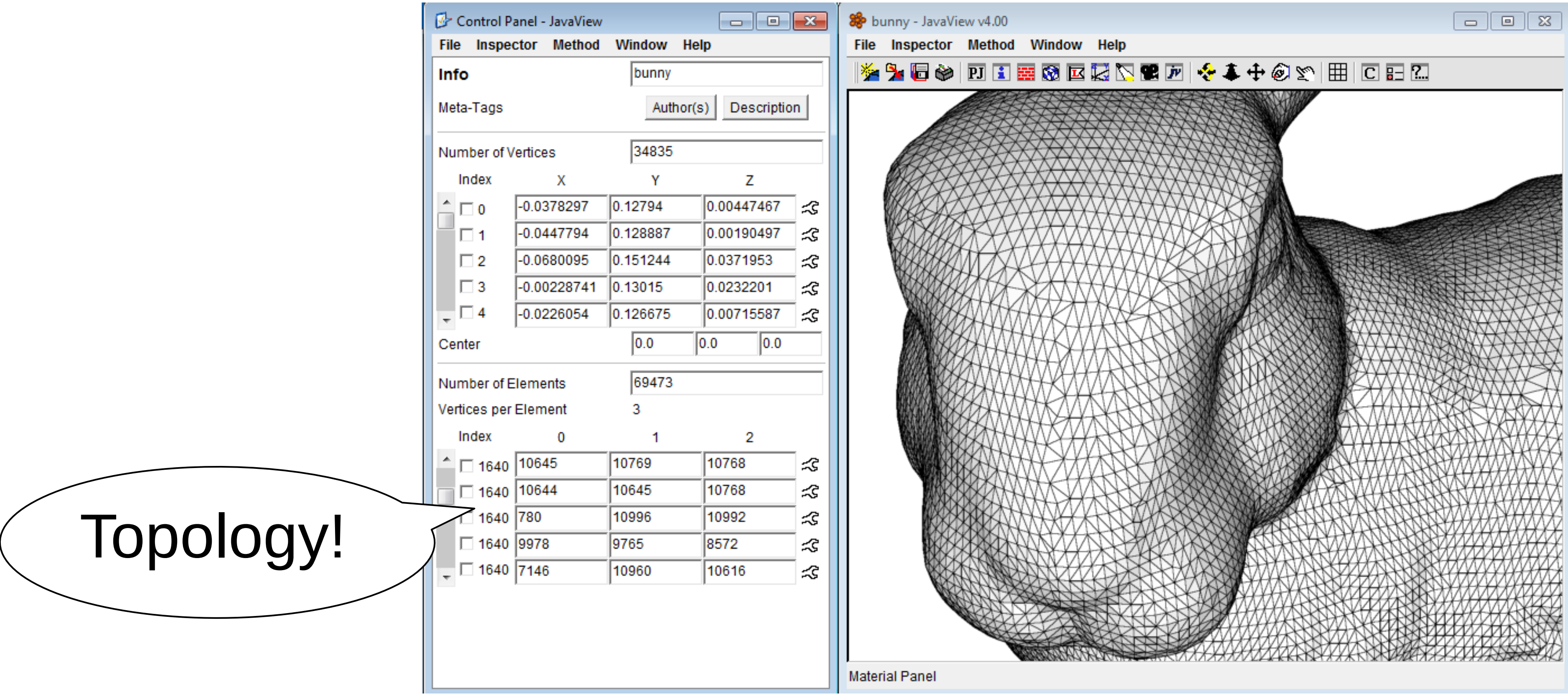
Local Neighborhoods



Index of the triangle opposite to the vertex (with the same position in the array) is stored and -1 for boundaries

Topology

Topology

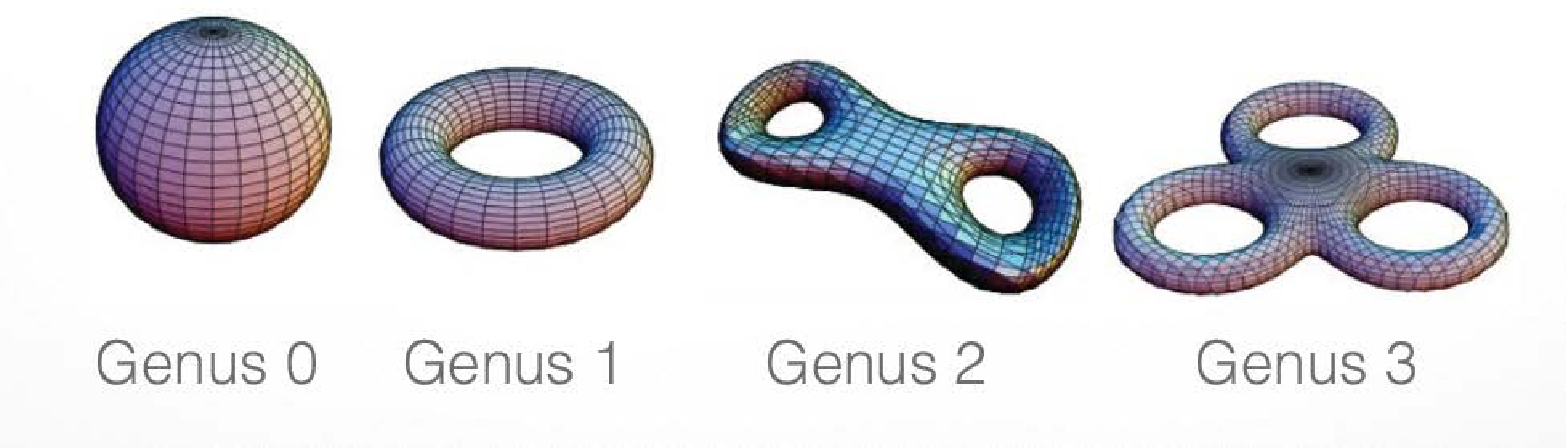


The face array contains information about the surface that is independent of the choice of vertex positions

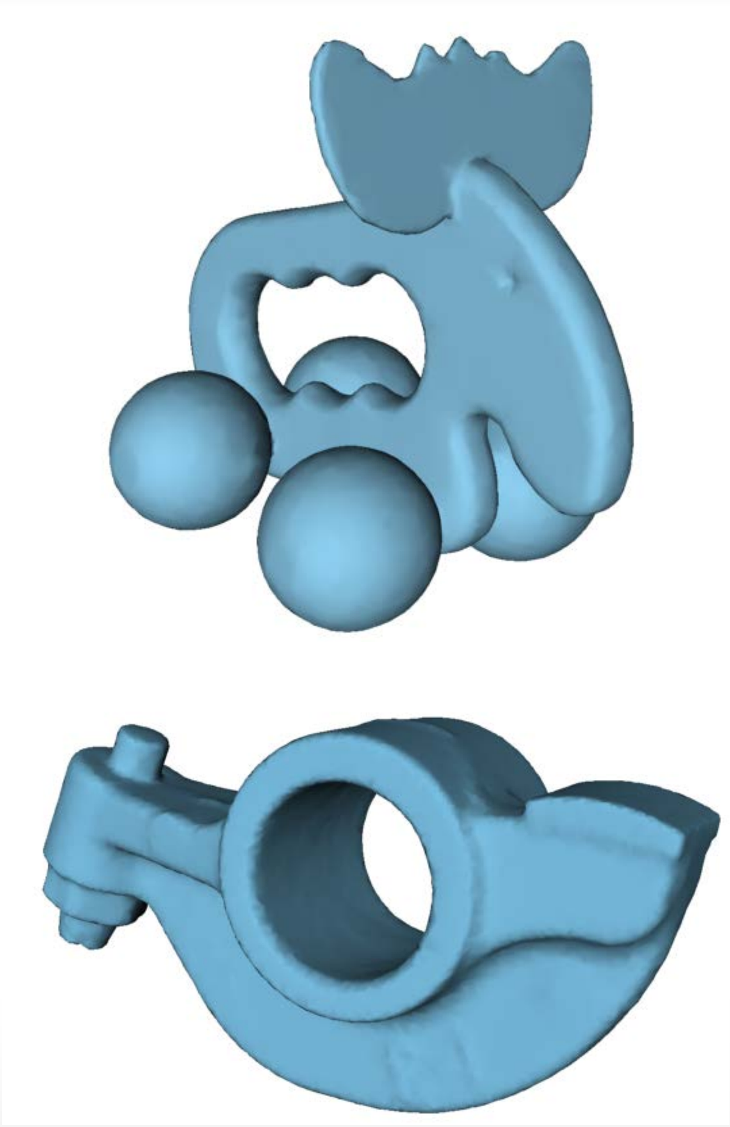
Topological Classification

Genus **(compact, orientable surfaces)**

- Half the maximal number of closed paths that do not disconnect the surface
- Intuition: Number of holes (or handles)



What is the Genus of...



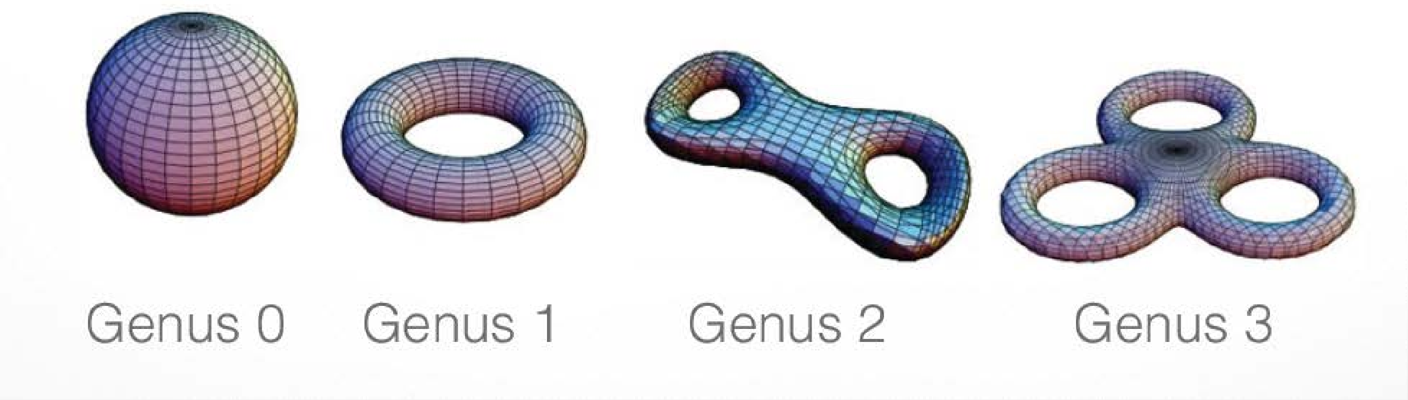
Euler Characteristic

For a triangle (or polygonal) mesh (without boundary)
Euler's formula

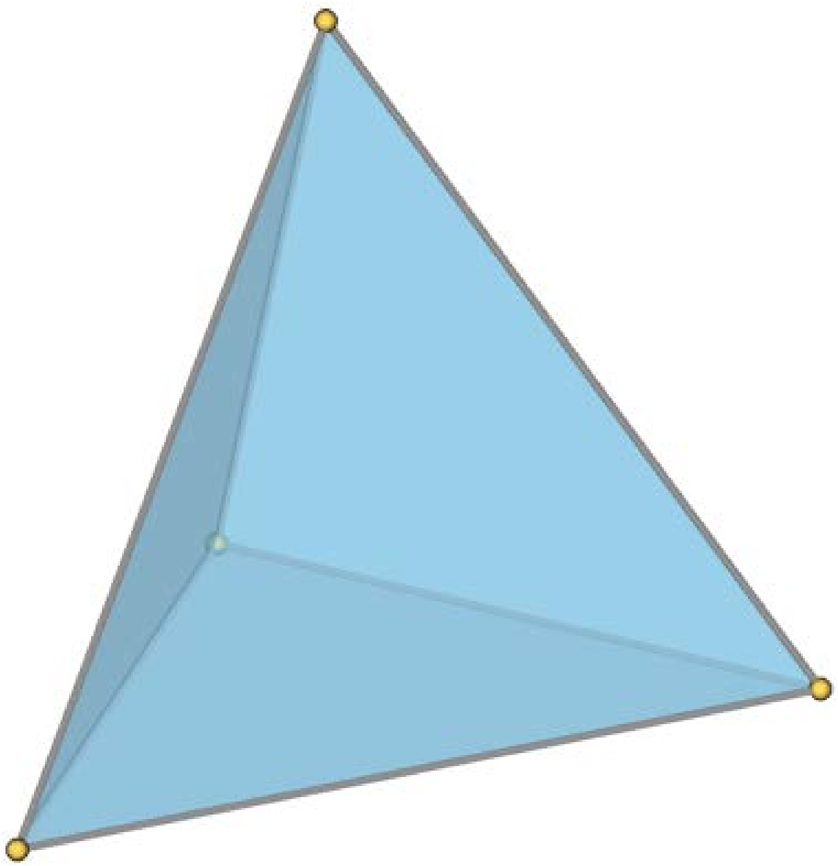
$$V - E + F = 2(1 - g)$$

relates the number of vertices V, edges E, faces F, and the **genus** g

The term $2(1 - g)$ is called the **Euler characteristic** X



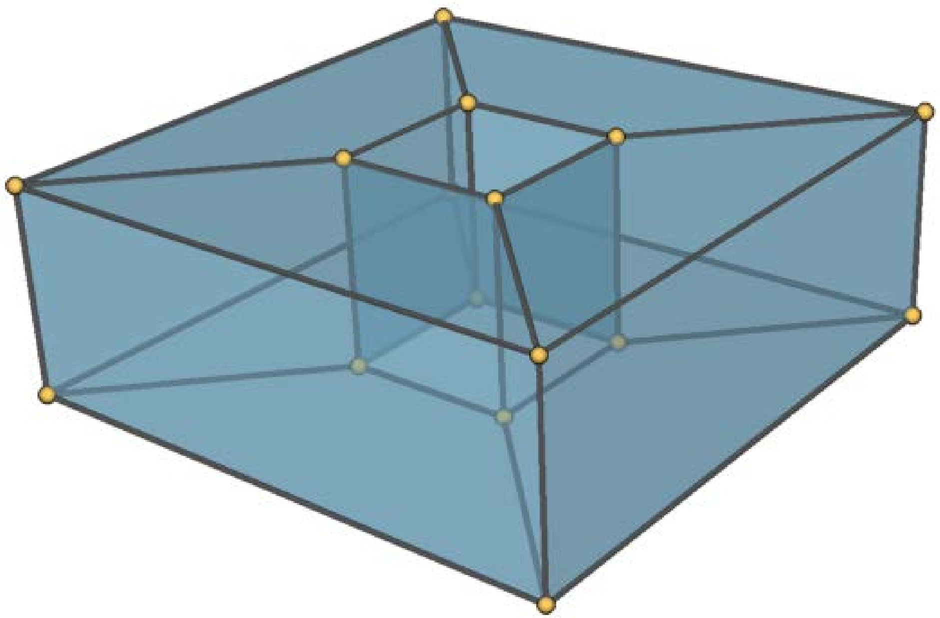
Example: Genus 0



$$V - E + F = 2(1 - g)$$

$$4 - 6 + 4 = 2(1 - 0)$$

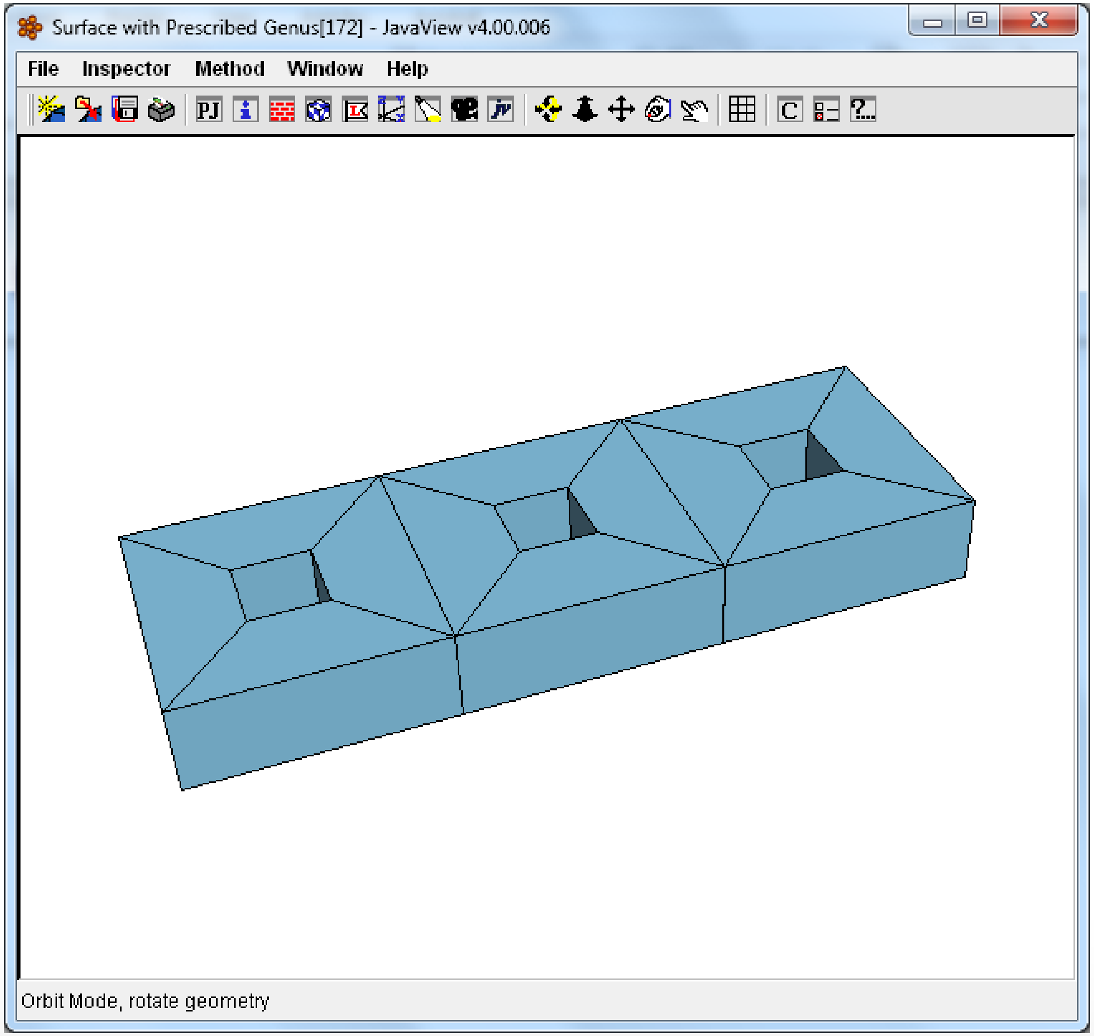
Example: Genus 1



$$V - E + F = 2(1 - g)$$

$$16 - 32 + 16 = 2(1 - 1)$$

Genus Builder



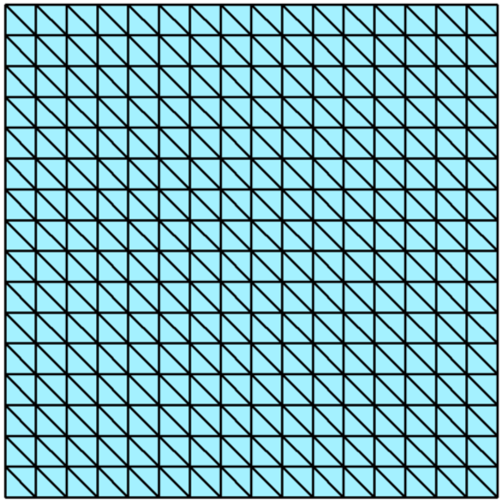
Average Valences

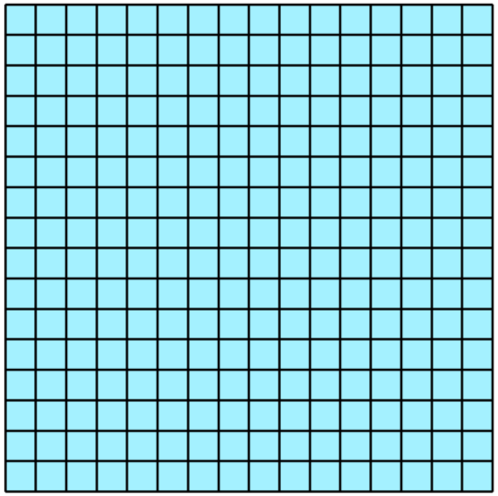
Valence of a vertex

- Number of adjacent edges

Average valence (for large number of vertices)

- Triangle meshes: 6
- Quad meshes: 4





Average Valences

Why?

- For triangles: $3F = 2E$ (every traingle has 3 edges and every edge is in 2 triangles)
- Euler Formula

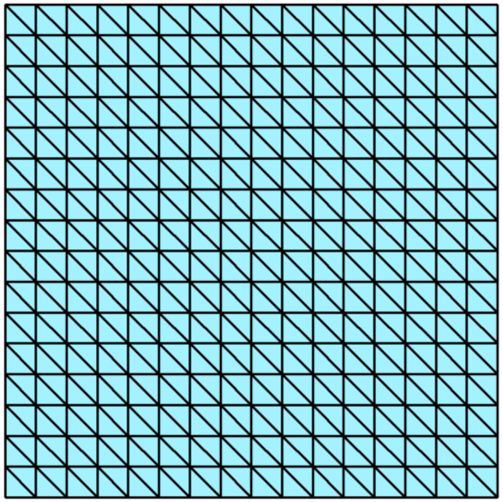
$$V + F - E \ = \ V + 2E/3 - E \ = \ 2 - 2g$$

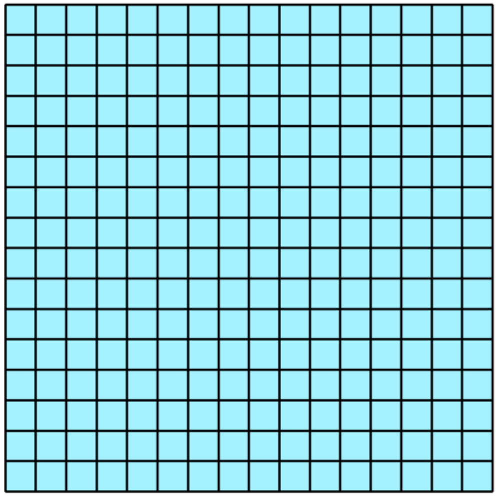
- Solving for E

$$E \ = \ 3(V - 2 + 2g)$$

- For large V and small g, we get

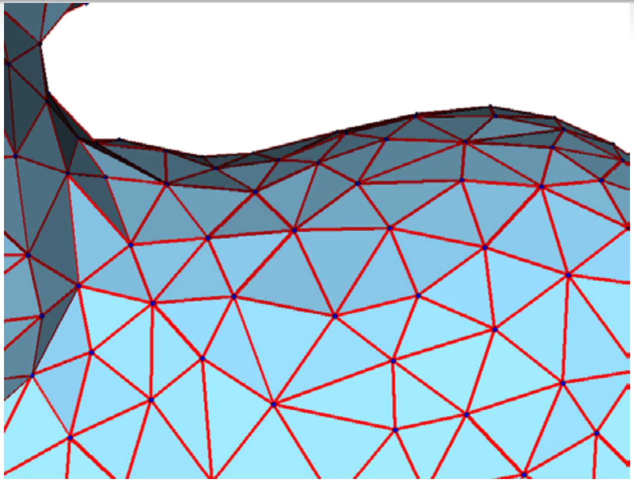
$$\frac{2E}{V} = \frac{6(V - 2 + 2g)}{V} \approx 6$$

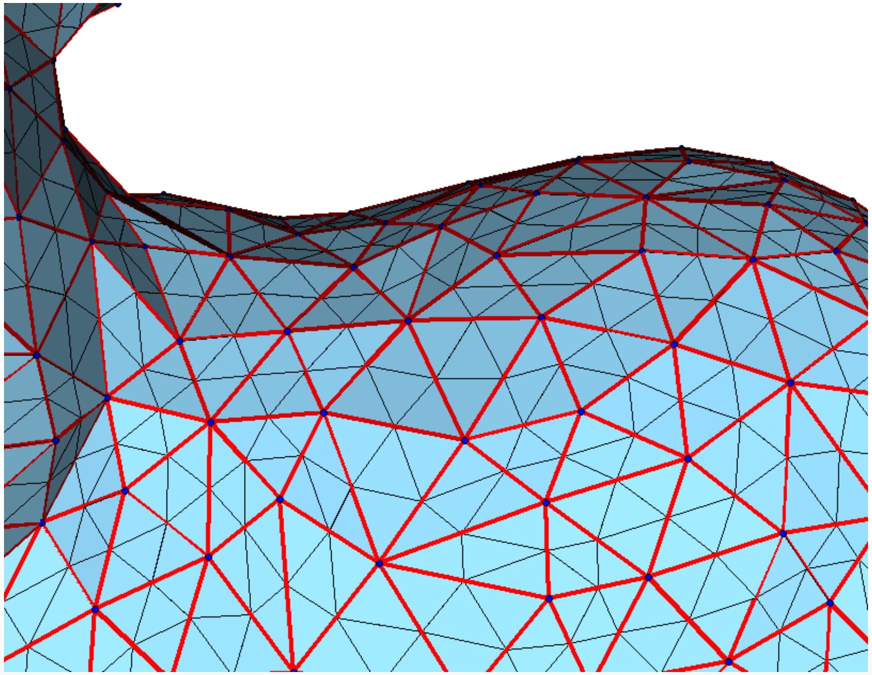


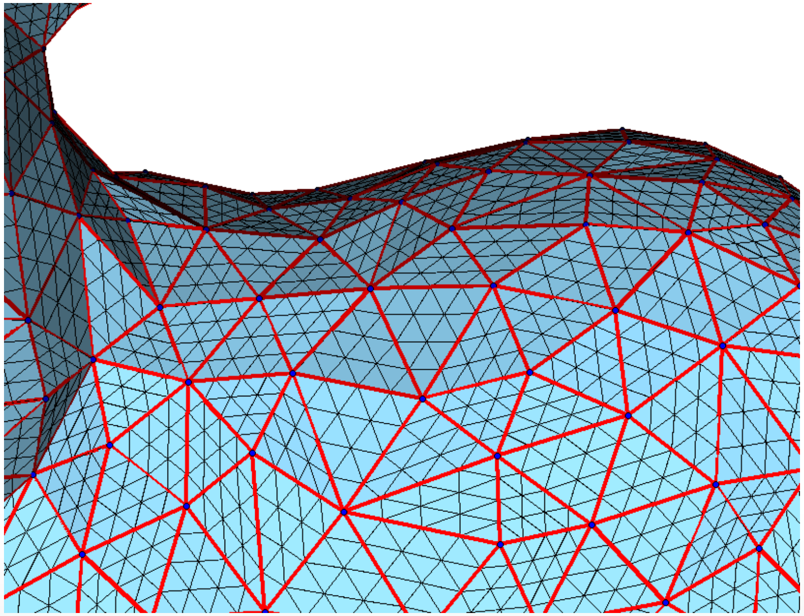


Example: One-to-Four Refinement

All new vertices have valence 6



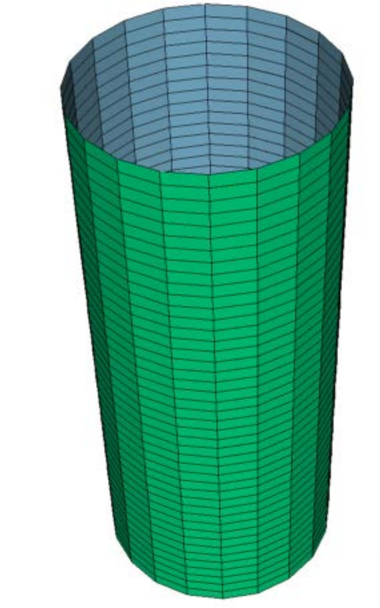


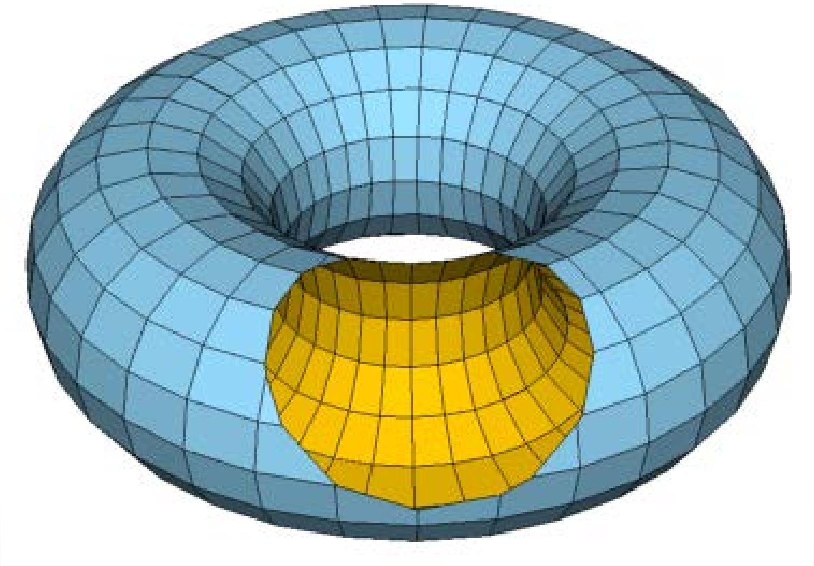


Orientable Surfaces

Orientable

- Surface has a consistent normal field
- “Can color the inside and outside with different colors”

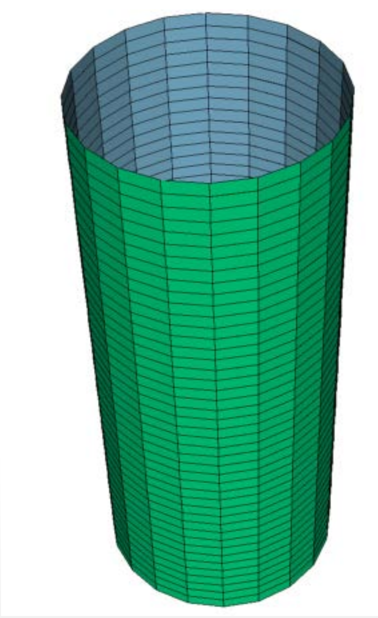


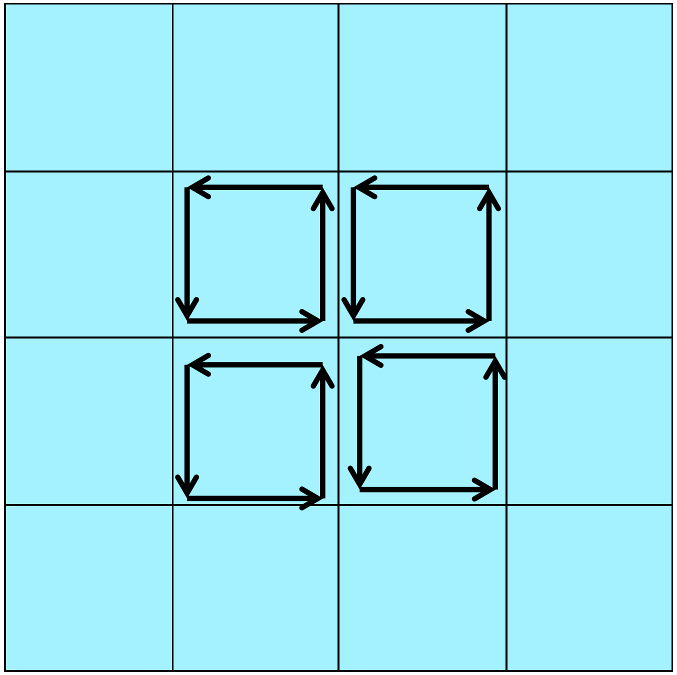


Orientable Surfaces

Orientable

- Can be decided from the face array (vertex positions not needed)
  - Can all edges can be oriented consistently?

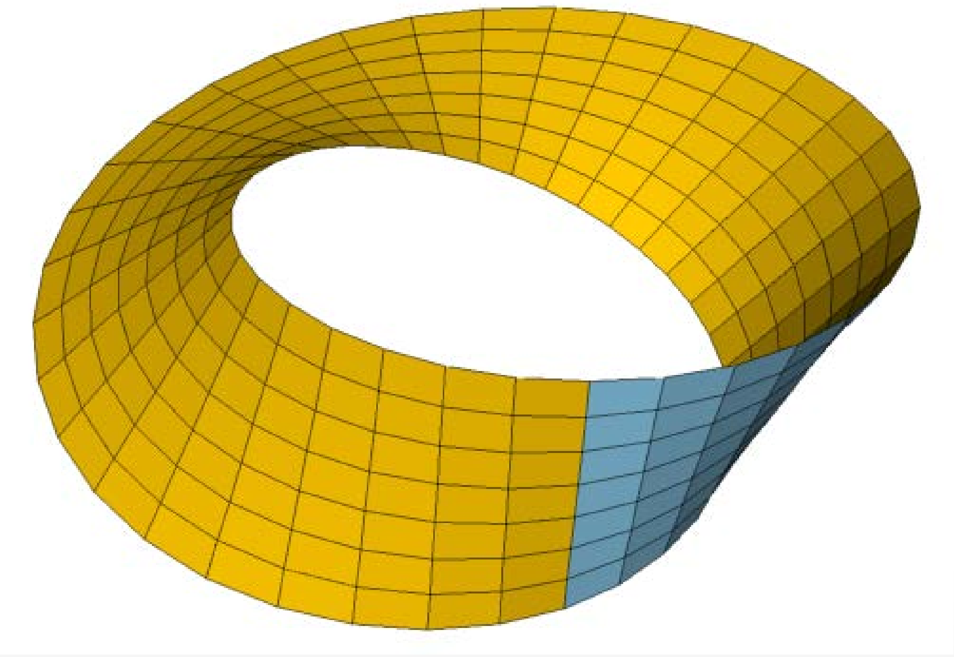


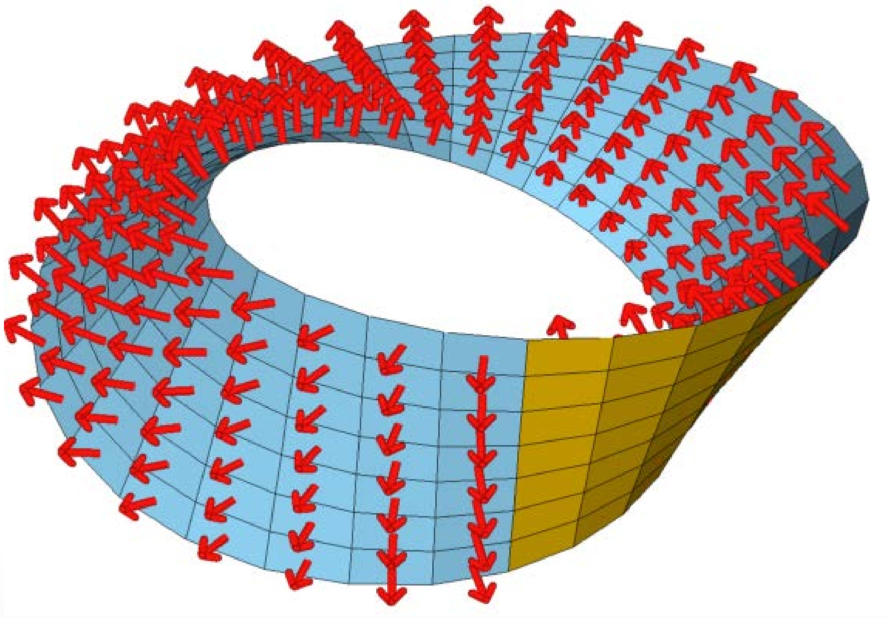


Non-Orientable Surfaces

Möbius Strip

- Dangerous, algorithms may crash
- Example: How do you compute vertex normals?

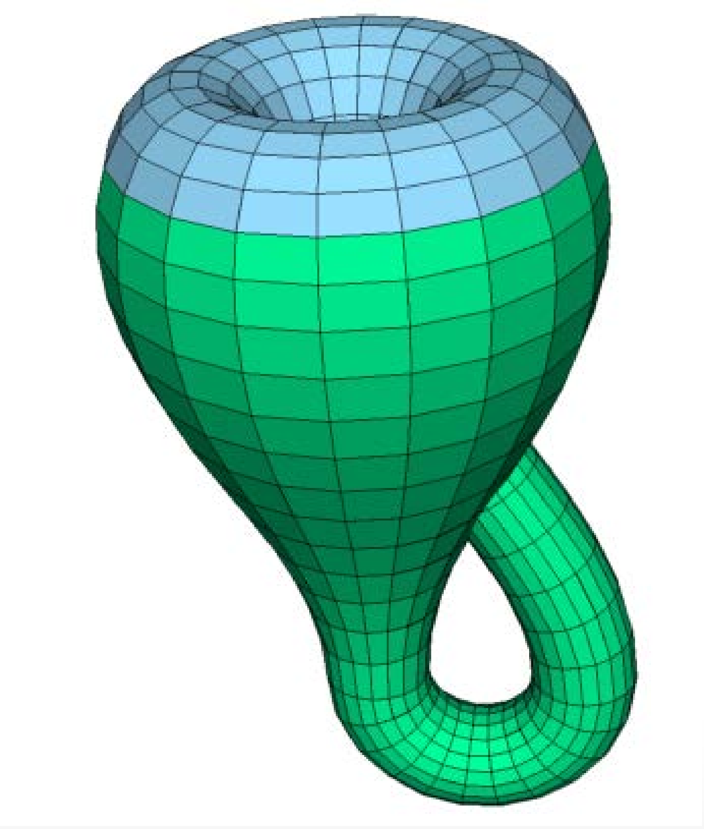




Non-Orientable Surfaces

Klein Bottle (Klein's surface)

- Euler characteristic: $X = 0$



Costa-Hoffman-Meeks Surface

Orientable?