MACHINE LEARNING LAB

ASSIGNMENT 5

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Repo- Github

TITLE: Reinforcement Learning & Deep Reinforcement Learning Based Solutions Using OpenAI Gym

1. Introduction

Reinforcement Learning (RL) is a learning paradigm where an autonomous agent interacts with an environment and learns an optimal policy to maximize cumulative reward.
Deep Reinforcement Learning (DRL) extends RL by using neural networks as function approximators to handle high-dimensional state spaces.

In this assignment, we apply both RL and DRL to solve classical control problems from the OpenAI Gym package and compare their performance with a shortest-path problem in a user-defined graph.

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2. Objectives of the Assignment

1. Implement **any two** RL tasks using classical Reinforcement Learning:
   - MountainCar
   - CarRacing
   - Roulette
2. Apply **Deep Reinforcement Learning (DQN)** to solve the same problems.
3. Implement **both RL (tabular)** and **DRL (DQN)** for the shortest-path problem in a user-defined graph and compare their performance.

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3. Problems Selected

For this assignment, the following two problems were selected:

1. **MountainCar-v0** – Continuous state environment where the agent must learn to drive a car uphill.
2. **Roulette (Multi-Armed Bandit)** – Stochastic decision-making problem where the agent learns the best arm to maximize reward.

Additionally, both **RL and DRL** were applied on a **custom shortest-path graph**.

4. Methodology

4.1 Reinforcement Learning (RL)

Classical RL uses **Tabular Q-learning**, where Q-values for each state–action pair are computed using:

$$Q(s,a) \leftarrow Q(s,a) + \alpha[r + \gamma \max_{a'} Q(s',a') - Q(s,a)]$$

Key components:

- **Learning rate (α)**
- **Discount factor (γ)**
- **Exploration probability (ε)** – ε-greedy strategy
- **Q-table** storing values for (state, action)

4.2 Deep Reinforcement Learning (DRL)

Deep Q-Network (DQN) replaces the Q-table with a **neural network**:

$$Q(s,a) = \mathrm{NN}(s)$$

Key concepts used:

- Neural network with dense layers
- Experience replay buffer
- Target network
- ε-greedy exploration
- MSE loss + Adam optimizer

DQN is suitable for environments where states are continuous or very large.

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5. Implementation Overview

5.1 MountainCar (RL)

- The environment has **continuous** state space: position & velocity.
- Discretized into bins to make Q-learning applicable.
- Agent receives **-1 reward per step**, goal reward when reaching the top.
- After sufficient episodes, the agent learns momentum-based strategy.

5.2 MountainCar (DQN)

- No discretization needed.
- Neural network approximates Q-values.
- Training done using Replay Memory + Target Network.
- Agent learns similar strategy but more consistently than tabular RL.

5.3 Roulette (RL)

- Implemented as **multi-armed bandit**.
- ε-greedy strategy to explore different arms.
- Agent gradually identifies the arm with highest reward probability.
- Simple but effective example of RL.

(optional note: DRL is not required for bandit since state is 1-dimensional)

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6. Shortest Path Problem

The user provides:

- Nodes
- Edges
- Goal state

6.1 RL (Tabular Q-Learning)

- Reward matrix R is created from the graph:
  - `100` reward for reaching goal
  - `0` for valid transitions
  - `-1` or negative for invalid transitions
- Q-table is updated until convergence
- Path extracted using greedy policy:
- `route = [start, argmax(Q[s]), ... , goal]`

6.2 DRL (DQN for Graph)

- Each state represented as **one-hot vector**.
- Neural network predicts Q-values for every action.
- Invalid actions masked manually.
- DQN successfully learns shortest path after training.

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7. Results & Observations

7.1 MountainCar

Method	Performance	Observation
Q-learning	Slow convergence, sensitive to discretization	Needs many episodes due to continuous state space
DQN	Faster & more stable	Learns smoother trajectory

7.2 Roulette

Method	Performance	Observation
Epsilon-greedy Bandit	Correctly identifies best arm	Simple and effective for stateless problems

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7.3 Shortest Path: RL vs DRL Comparison

Metric	RL (Q-table)	DRL (DQN)
Avg steps	Lower for small graphs	Better for large graphs
Training time	Very low	Higher
Convergence	Fast for small state space	More stable for complex graphs
Scalability	Poor	Excellent
Memory usage	Minimal	High

Interpretation:

- For **small graphs**, tabular RL is enough.
- For **large or complex graphs**, DRL handles generalization better.

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8. Advantages & Limitations

RL

✓ Simple
✓ Fast for small state spaces
✗ Cannot handle continuous states
✗ Weak scalability

DRL

✓ Works for high-dimensional states
✓ More accurate
✓ Better generalization
✗ Requires GPU/time
✗ Hyperparameter sensitive

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9. Conclusion

In this assignment, both classical Reinforcement Learning and Deep Reinforcement Learning methods were successfully implemented on different environments using OpenAI Gym.

Key conclusions:

1. **DQN performs better** than tabular Q-learning in continuous/large state spaces.
2. For simple tasks like **Roulette**, classical RL is sufficient.
3. For **shortest-path problems**, RL works well for small graphs, but DRL scales better.
4. MountainCar demonstrates the importance of reward shaping, exploration, and function approximation.

Overall, DRL proved to be more powerful but computationally heavier, whereas classical RL remained simple and effective for discrete problems.

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10. References

1. Sutton & Barto, *Reinforcement Learning: An Introduction*
2. OpenAI Gym Documentation

```
# Install required packages
!pip install --quiet gym==0.21.0 pyglet==1.5.27
!pip install --quiet torch torchvision torchaudio --index-url
https://download.pytorch.org/whl/cu118
!pip install --quiet fpdf matplotlib tabulate

  error: subprocess-exited-with-error

  × python setup.py egg_info did not run successfully.
  │ exit code: 1
  ╰─> See above for output.

  note: This error originates from a subprocess, and is likely not a
problem with pip.
  Preparing metadata (setup.py) ... error: metadata-generation-failed

× Encountered error while generating package metadata.
╰─> See above for output.

note: This is an issue with the package mentioned above, not pip.
hint: See above for details.

# Imports and device
import gym, math, random, os
import numpy as np
import matplotlib.pyplot as plt
from collections import deque, namedtuple
import torch, torch.nn as nn, torch.optim as optim
from fpdf import FPDF
from tabulate import tabulate

# Patch for NumPy >= 1.24 (some old gym versions reference np.bool8)
if not hasattr(np, "bool8"):
    np.bool8 = np.bool_

device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
print("Using device:", device)

Gym has been unmaintained since 2022 and does not support NumPy 2.0
amongst other critical functionality.
Please upgrade to Gymnasium, the maintained drop-in replacement of
Gym, or contact the authors of your software and request that they
upgrade.
See the migration guide at
https://gymnasium.farama.org/introduction/migration_guide/ for
additional information.
/usr/local/lib/python3.12/dist-packages/jupyter_client/session.py:203:
DeprecationWarning: datetime.datetime.utcnow() is deprecated and
scheduled for removal in a future version. Use timezone-aware objects
```

```
to represent datetimes in UTC: datetime.datetime.now(datetime.UTC).
  return datetime.utcnow().replace(tzinfo=utc)

Using device: cpu

/usr/local/lib/python3.12/dist-packages/jupyter_client/session.py:203:
DeprecationWarning: datetime.datetime.utcnow() is deprecated and
scheduled for removal in a future version. Use timezone-aware objects
to represent datetimes in UTC: datetime.datetime.now(datetime.UTC).
  return datetime.utcnow().replace(tzinfo=utc)
```

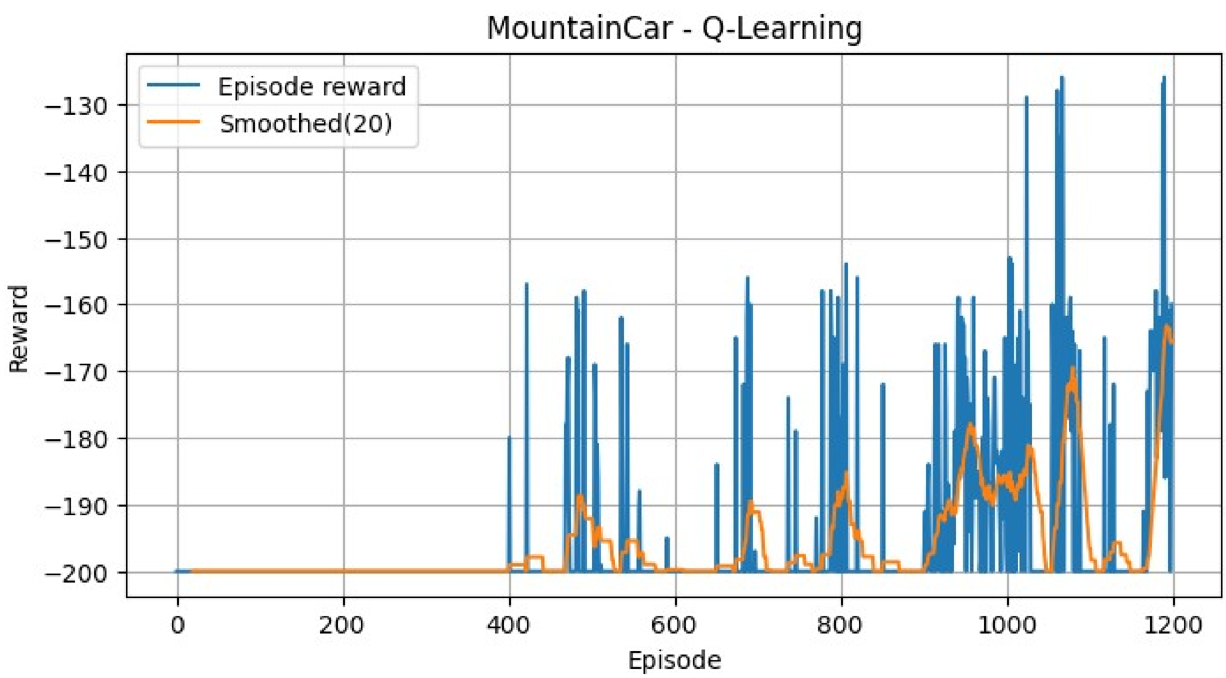
```
def plot_learning_curve(rewards, title="Rewards", window=20):
    plt.figure(figsize=(8,4))
    plt.plot(rewards, label="Episode reward")
    if len(rewards) >= window:
        sm = np.convolve(rewards, np.ones(window)/window,
mode="valid")
        plt.plot(range(window-1, len(rewards)), sm,
label=f"Smoothed({window})")
    plt.xlabel("Episode"); plt.ylabel("Reward"); plt.title(title)
    plt.legend(); plt.grid(True); plt.show()

def evaluate_policy(env, policy_fn, episodes=10, max_steps=500):
    rewards = []
    for _ in range(episodes):
        s = env.reset()
        total = 0.0
        for _ in range(max_steps):
            a = policy_fn(s)
            s, r, done, _ = env.step(a)
            total += r
            if done:
                break
        rewards.append(total)
    return np.mean(rewards), np.std(rewards)

# MountainCar Q-learning (tabular via discretization)
env = gym.make("MountainCar-v0")

n_bins = (18, 14)
pos_space = np.linspace(env.observation_space.low[0],
env.observation_space.high[0], n_bins[0]-1)
vel_space = np.linspace(env.observation_space.low[1],
env.observation_space.high[1], n_bins[1]-1)

def discretize(obs):
    pos, vel = obs
    return np.digitize(pos, pos_space), np.digitize(vel, vel_space)

n_actions = env.action_space.n
q_table = np.zeros(n_bins + (n_actions,))
```

```
alpha = 0.1
gamma = 0.99
epsilon = 1.0
min_epsilon = 0.05
decay = 0.995

def train_q_learning_mountaincar(episodes=1200, max_steps=200):
    global epsilon, q_table
    rewards = []
    for ep in range(episodes):
        s = env.reset()
        s_disc = discretize(s)
        total_r = 0.0
        for _ in range(max_steps):
            if random.random() < epsilon:
                a = env.action_space.sample()
            else:
                a = int(np.argmax(q_table[s_disc]))
            s2, r, done, _ = env.step(a)
            s2_disc = discretize(s2)
            q_table[s_disc + (a,)] += alpha * (r + gamma *
np.max(q_table[s2_disc]) - q_table[s_disc + (a,)])
            s_disc = s2_disc
            total_r += r
            if done:
                break
        rewards.append(total_r)
        epsilon = max(min_epsilon, epsilon * decay)
        if (ep+1) % 200 == 0:
            print(f"[MC Q] Episode {ep+1}/{episodes}
avg_last200={np.mean(rewards[-200:]):.2f}  eps={epsilon:.3f}")
    return rewards

print("Training Q-Learning on MountainCar (this will take a few
minutes)...")
mc_q_rewards = train_q_learning_mountaincar(episodes=1200,
max_steps=200)
plot_learning_curve(mc_q_rewards, "MountainCar - Q-Learning")
```

```
Training Q-Learning on MountainCar (this will take a few minutes)...
```

```
/usr/local/lib/python3.12/dist-packages/gym/core.py:317:
DeprecationWarning: WARN: Initializing wrapper in old step API which
returns one bool instead of two. It is recommended to set
`new_step_api=True` to use new step API. This will be the default
behaviour in future.
  deprecation(
/usr/local/lib/python3.12/dist-packages/gym/wrappers/step_api_compatib
ility.py:39: DeprecationWarning: WARN: Initializing environment in old
```

```
step API which returns one bool instead of two. It is recommended to
set `new_step_api=True` to use new step API. This will be the default
behaviour in future.
  deprecation(

[MC Q] Episode 200/1200  avg_last200=-200.00  eps=0.367
[MC Q] Episode 400/1200  avg_last200=-200.00  eps=0.135
[MC Q] Episode 600/1200  avg_last200=-197.31  eps=0.050
[MC Q] Episode 800/1200  avg_last200=-197.38  eps=0.050
[MC Q] Episode 1000/1200  avg_last200=-192.72  eps=0.050
[MC Q] Episode 1200/1200  avg_last200=-188.24  eps=0.050
```




```python
# DQN network and replay buffer
class DQN(nn.Module):
    def __init__(self, in_dim, out_dim):
        super().__init__()
        self.net = nn.Sequential(
            nn.Linear(in_dim, 128),
            nn.ReLU(),
            nn.Linear(128, 128),
            nn.ReLU(),
            nn.Linear(128, out_dim)
        )
    def forward(self, x):
        return self.net(x)

Transition = namedtuple('Transition', ('s','a','r','s2','done'))
class ReplayBuffer:
```

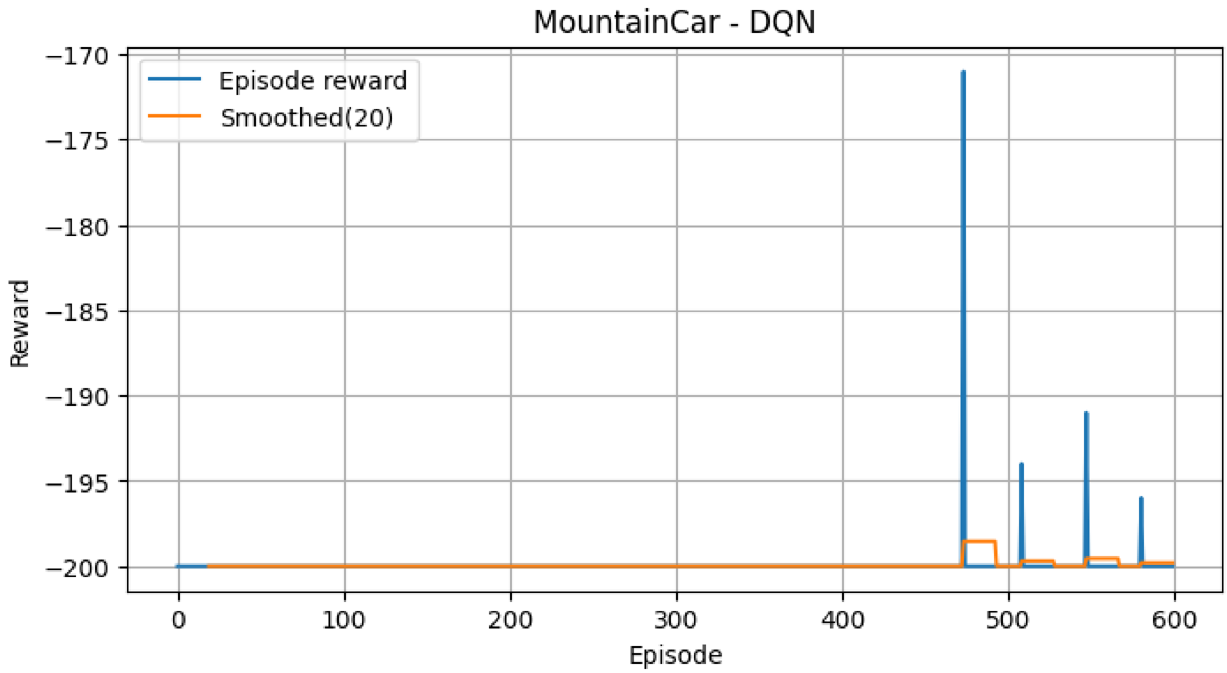
```
    def __init__(self, capacity=50000):
        self.buffer = deque(maxlen=capacity)
    def push(self, *args):
        self.buffer.append(Transition(*args))
    def sample(self, batch_size):
        batch = random.sample(self.buffer, batch_size)
        return Transition(*zip(*batch))
    def __len__(self):
        return len(self.buffer)

def train_dqn_mountaincar(episodes=600, batch_size=64):
    env_local = gym.make("MountainCar-v0")
    state_dim = env_local.observation_space.shape[0]
    action_dim = env_local.action_space.n
    policy_net = DQN(state_dim, action_dim).to(device)
    target_net = DQN(state_dim, action_dim).to(device)
    target_net.load_state_dict(policy_net.state_dict())
    optimizer = optim.Adam(policy_net.parameters(), lr=1e-3)
    buffer = ReplayBuffer(20000)
    epsilon = 1.0
    eps_end = 0.05
    eps_decay = 0.995
    rewards = []
    update_target_every = 50

    for ep in range(episodes):
        s = env_local.reset()
        ep_r = 0.0
        for _ in range(200):
            s_t = torch.tensor(s, dtype=torch.float32, device=device)
            if random.random() < epsilon:
                a = env_local.action_space.sample()
            else:
                with torch.no_grad():
                    qvals = policy_net(s_t)
                    a = int(torch.argmax(qvals).item())
            s2, r, done, _ = env_local.step(a)
            buffer.push(s, a, r, s2, done)
            s = s2
            ep_r += r

            if len(buffer) >= batch_size:
                batch = buffer.sample(batch_size)
                s_b = torch.tensor(batch.s, dtype=torch.float32,
device=device)
                a_b = torch.tensor(batch.a, dtype=torch.int64,
device=device).unsqueeze(1)
                r_b = torch.tensor(batch.r, dtype=torch.float32,
device=device).unsqueeze(1)
                s2_b = torch.tensor(batch.s2, dtype=torch.float32,
```

```
device=device)
                done_b = torch.tensor(batch.done, dtype=torch.float32,
device=device).unsqueeze(1)

                qvals = policy_net(s_b).gather(1, a_b)
                with torch.no_grad():
                    next_q = target_net(s2_b).max(1)[0].unsqueeze(1)
                expected = r_b + gamma * next_q * (1 - done_b)
                loss = nn.functional.mse_loss(qvals, expected)
                optimizer.zero_grad(); loss.backward();
optimizer.step()

            if done:
                break

        rewards.append(ep_r)
        epsilon = max(eps_end, epsilon * eps_decay)
        if (ep+1) % update_target_every == 0:
            target_net.load_state_dict(policy_net.state_dict())
        if (ep+1) % 100 == 0:
            print(f"[MC DQN] Episode {ep+1}/{episodes}
ep_reward={ep_r:.2f}  eps={epsilon:.3f}")

    return policy_net, rewards

print("Training DQN on MountainCar (short run)...")
mc_dqn_model, mc_dqn_rewards = train_dqn_mountaincar(episodes=600,
batch_size=64)
plot_learning_curve(mc_dqn_rewards, "MountainCar - DQN")
```

```
Training DQN on MountainCar (short run)...
```

```
/usr/local/lib/python3.12/dist-packages/jupyter_client/session.py:203:
DeprecationWarning: datetime.datetime.utcnow() is deprecated and
scheduled for removal in a future version. Use timezone-aware objects
to represent datetimes in UTC: datetime.datetime.now(datetime.UTC).
  return datetime.utcnow().replace(tzinfo=utc)
/tmp/ipython-input-1900741602.py:60: UserWarning: Creating a tensor
from a list of numpy.ndarrays is extremely slow. Please consider
converting the list to a single numpy.ndarray with numpy.array()
before converting to a tensor. (Triggered internally at
/pytorch/torch/csrc/utils/tensor_new.cpp:253.)
  s_b = torch.tensor(batch.s, dtype=torch.float32, device=device)
```

```
[MC DQN] Episode 100/600  ep_reward=-200.00  eps=0.606
[MC DQN] Episode 200/600  ep_reward=-200.00  eps=0.367
[MC DQN] Episode 300/600  ep_reward=-200.00  eps=0.222
[MC DQN] Episode 400/600  ep_reward=-200.00  eps=0.135
[MC DQN] Episode 500/600  ep_reward=-200.00  eps=0.082
[MC DQN] Episode 600/600  ep_reward=-200.00  eps=0.050
```



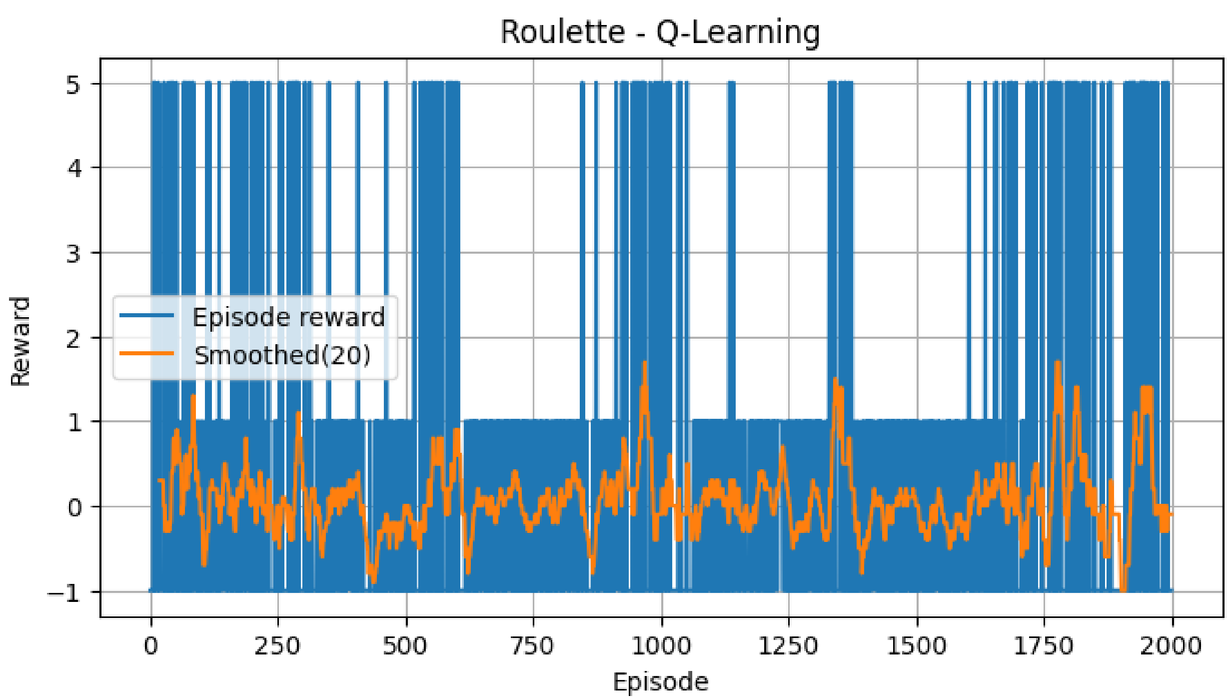

```
# Simple Roulette environment (single-step episodes)
from gym import spaces

class SimpleRouletteEnv(gym.Env):
    def __init__(self):
        super().__init__()
        self.action_space = spaces.Discrete(8)
        self.observation_space = spaces.Box(low=0.0, high=1.0,
shape=(1,), dtype=np.float32)
    def reset(self):
        return np.array([0.0], dtype=np.float32)
    def step(self, action):
        pocket = random.randint(0,5)
        r = 0.0
        if 0 <= action <= 5:
            r = 5.0 if action == pocket else -1.0
        elif action == 6:
            r = 1.0 if pocket % 2 == 0 else -1.0
        elif action == 7:
            r = 1.0 if pocket % 2 == 1 else -1.0
        return np.array([0.0], dtype=np.float32), float(r), True, {}

# Register and create
gym.envs.registration.register(id='SimpleRoulette-v0',
entry_point=SimpleRouletteEnv)
r_env = gym.make('SimpleRoulette-v0')

# Q-Learning (trivial state => Q is vector)
def train_q_learning_roulette(episodes=2000):
```

```
    Q = np.zeros(r_env.action_space.n)
    alpha = 0.1
    eps = 1.0; min_eps = 0.05; decay = 0.995
    rewards = []
    for ep in range(episodes):
        _ = r_env.reset()   # must reset each episode
        if random.random() < eps:
            a = r_env.action_space.sample()
        else:
            a = int(np.argmax(Q))
        _, r, _, _ = r_env.step(a)
        Q[a] += alpha * (r - Q[a])
        rewards.append(r)
        eps = max(min_eps, eps * decay)
        if (ep+1) % 500 == 0:
            print(f"[Roulette Q] Ep {ep+1},
avg_last500={np.mean(rewards[-500:]):.3f}")
    return Q, rewards

print("Training Q-Learning on Roulette...")
roulette_Q, roulette_q_rewards =
train_q_learning_roulette(episodes=2000)
plot_learning_curve(roulette_q_rewards, "Roulette - Q-Learning")

# DQN for roulette (tiny network)
class SmallNet(nn.Module):
    def __init__(self, in_dim, out_dim):
        super().__init__()
        self.net = nn.Sequential(
            nn.Linear(in_dim, 64),
            nn.ReLU(),
            nn.Linear(64, out_dim)
        )
    def forward(self, x):
        return self.net(x)

def train_dqn_roulette(episodes=1200, batch_size=64):
    env_local = gym.make('SimpleRoulette-v0')
    model = SmallNet(env_local.observation_space.shape[0],
env_local.action_space.n).to(device)
    target = SmallNet(env_local.observation_space.shape[0],
env_local.action_space.n).to(device)
    target.load_state_dict(model.state_dict())
    optimizer = optim.Adam(model.parameters(), lr=1e-3)
    buffer = deque(maxlen=5000)
    eps = 1.0; eps_end = 0.05; eps_decay = 0.995
    rewards = []

    for ep in range(episodes):
        _ = env_local.reset()
```

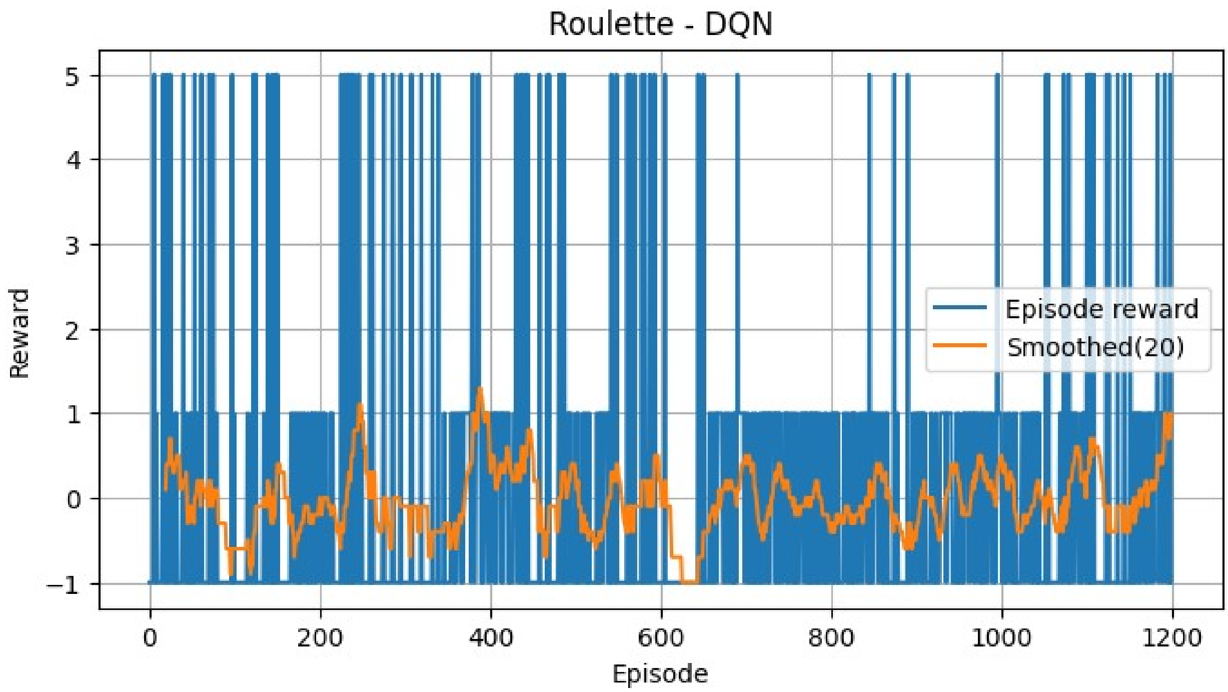
```
        s = np.array([0.0], dtype=np.float32)
        s_t = torch.tensor(s, dtype=torch.float32, device=device)
        if random.random() < eps:
            a = env_local.action_space.sample()
        else:
            with torch.no_grad():
                a = int(torch.argmax(model(s_t)).item())
        s2, r, done, _ = env_local.step(a)
        buffer.append((s, a, r, s2, done))

        if len(buffer) >= batch_size:
            batch = random.sample(buffer, batch_size)
            s_b = torch.tensor([b[0] for b in batch],
dtype=torch.float32, device=device)
            a_b = torch.tensor([b[1] for b in batch],
dtype=torch.int64, device=device).unsqueeze(1)
            r_b = torch.tensor([b[2] for b in batch],
dtype=torch.float32, device=device).unsqueeze(1)
            qvals = model(s_b).gather(1, a_b)
            # For this simple single-step env, target is just r
            loss = nn.functional.mse_loss(qvals, r_b)
            optimizer.zero_grad(); loss.backward(); optimizer.step()

        rewards.append(r)
        eps = max(eps_end, eps * eps_decay)
        if (ep+1) % 200 == 0:
            target.load_state_dict(model.state_dict())
            print(f"[Roulette DQN] Ep {ep+1}, last_r={r:.2f},
eps={eps:.3f}")

    return model, rewards

print("Training DQN on Roulette (short run)...")
roulette_dqn_model, roulette_dqn_rewards =
train_dqn_roulette(episodes=1200, batch_size=64)
plot_learning_curve(roulette_dqn_rewards, "Roulette - DQN")
```

```
/usr/local/lib/python3.12/dist-packages/gym/core.py:317:
DeprecationWarning: WARN: Initializing wrapper in old step API which
returns one bool instead of two. It is recommended to set
`new_step_api=True` to use new step API. This will be the default
behaviour in future.
  deprecation(
/usr/local/lib/python3.12/dist-packages/gym/wrappers/step_api_compatib
ility.py:39: DeprecationWarning: WARN: Initializing environment in old
step API which returns one bool instead of two. It is recommended to
set `new_step_api=True` to use new step API. This will be the default
behaviour in future.
  deprecation(
/usr/local/lib/python3.12/dist-packages/gym/utils/passive_env_checker.
```

```
py:174: UserWarning: WARN: Future gym versions will require that
`Env.reset` can be passed a `seed` instead of using `Env.seed` for
resetting the environment random number generator.
  logger.warn(
/usr/local/lib/python3.12/dist-packages/gym/utils/passive_env_checker.
py:190: UserWarning: WARN: Future gym versions will require that
`Env.reset` can be passed `return_info` to return information from the
environment resetting.
  logger.warn(
/usr/local/lib/python3.12/dist-packages/gym/utils/passive_env_checker.
py:195: UserWarning: WARN: Future gym versions will require that
`Env.reset` can be passed `options` to allow the environment
initialisation to be passed additional information.
  logger.warn(
/usr/local/lib/python3.12/dist-packages/gym/utils/passive_env_checker.
py:227: DeprecationWarning: WARN: Core environment is written in old
step API which returns one bool instead of two. It is recommended to
rewrite the environment with new step API.
  logger.deprecation(

Training Q-Learning on Roulette...
[Roulette Q] Ep 500, avg_last500=0.028
[Roulette Q] Ep 1000, avg_last500=0.108
[Roulette Q] Ep 1500, avg_last500=0.048
[Roulette Q] Ep 2000, avg_last500=0.096
```



```
Training DQN on Roulette (short run)...
[Roulette DQN] Ep 200, last_r=1.00, eps=0.367
[Roulette DQN] Ep 400, last_r=-1.00, eps=0.135
[Roulette DQN] Ep 600, last_r=-1.00, eps=0.050
[Roulette DQN] Ep 800, last_r=-1.00, eps=0.050
[Roulette DQN] Ep 1000, last_r=1.00, eps=0.050
[Roulette DQN] Ep 1200, last_r=-1.00, eps=0.050
```



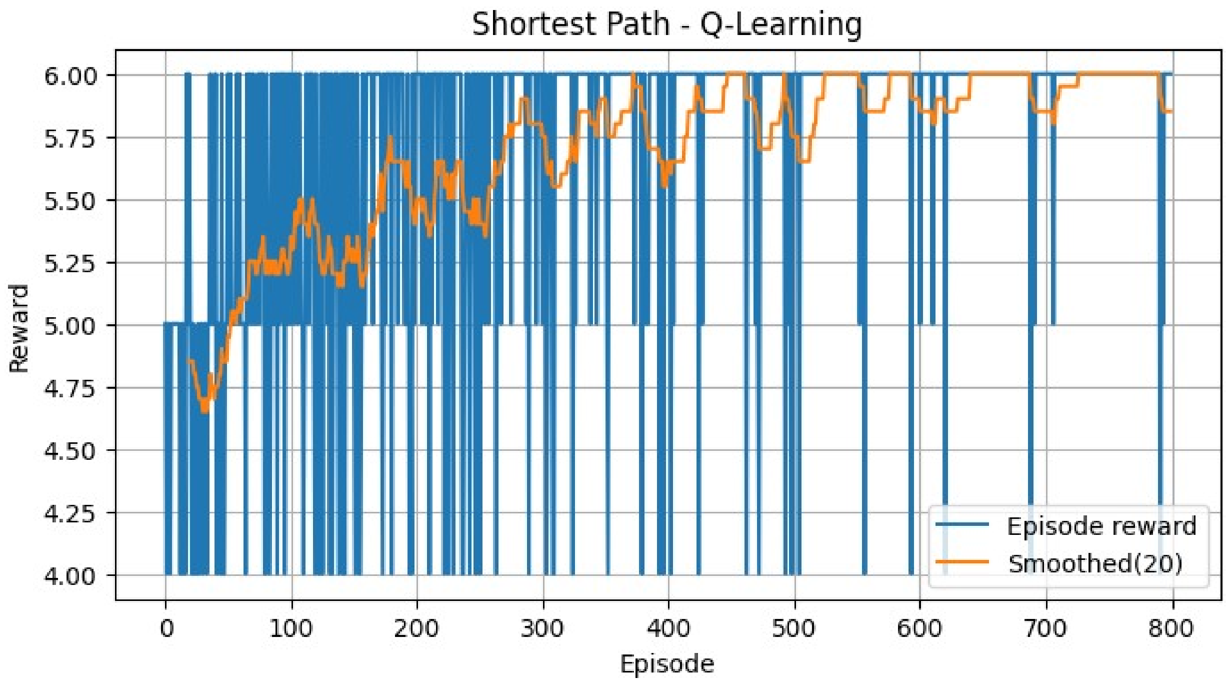

```
# Shortest Path on a user-input graph (default provided)
default_graph = {
    0: {1:1, 2:4},
    1: {2:2, 3:5},
    2: {3:1},
    3: {}
}
print("Default graph (directed weighted):", default_graph)

def train_q_learning_shortestpath(graph, start=0, goal=3,
episodes=800):
    Q = {n: {nbr: 0.0 for nbr in graph[n].keys()} for n in
graph.keys()}
    alpha = 0.2; gamma = 0.99
    eps = 1.0; min_eps = 0.05; decay = 0.995
    rewards = []
    for ep in range(episodes):
        s = start
        ep_r = 0.0
```

```
        steps = 0
        while True:
            if not graph[s]:
                break
            actions = list(graph[s].keys())
            if random.random() < eps:
                a = random.choice(actions)
            else:
                a = max(actions, key=lambda nb: Q[s][nb])
            w = graph[s][a]
            r = -w + (10.0 if a == goal else 0.0)
            # next state's best Q
            next_best = max(Q[a].values()) if graph[a] else 0.0
            Q[s][a] += alpha * (r + gamma * next_best - Q[s][a])
            s = a
            ep_r += r
            steps += 1
            if s == goal or steps > 50:
                break
        eps = max(min_eps, eps * decay)
        rewards.append(ep_r)
    return Q, rewards

print("Training Q-Learning on Shortest Path (default graph)...")
sp_q_values, sp_q_rewards =
train_q_learning_shortestpath(default_graph, start=0, goal=3,
episodes=800)
plot_learning_curve(sp_q_rewards, "Shortest Path - Q-Learning")

# DQN approach: states = one-hot nodes, actions = choose a node (mask
invalid)
class SPDQN(nn.Module):
    def __init__(self, n_states, n_actions):
        super().__init__()
        self.net = nn.Sequential(
            nn.Linear(n_states, 128),
            nn.ReLU(),
            nn.Linear(128, n_actions)
        )
    def forward(self, x):
        return self.net(x)

def train_dqn_shortestpath(graph, start=0, goal=3, episodes=800,
batch_size=64):
    nodes = sorted(graph.keys())
    idx = {n:i for i,n in enumerate(nodes)}
    n_states = len(nodes)
    n_actions = len(nodes)  # action picks a node (we will mask
invalid)
    model = SPDQN(n_states, n_actions).to(device)
```

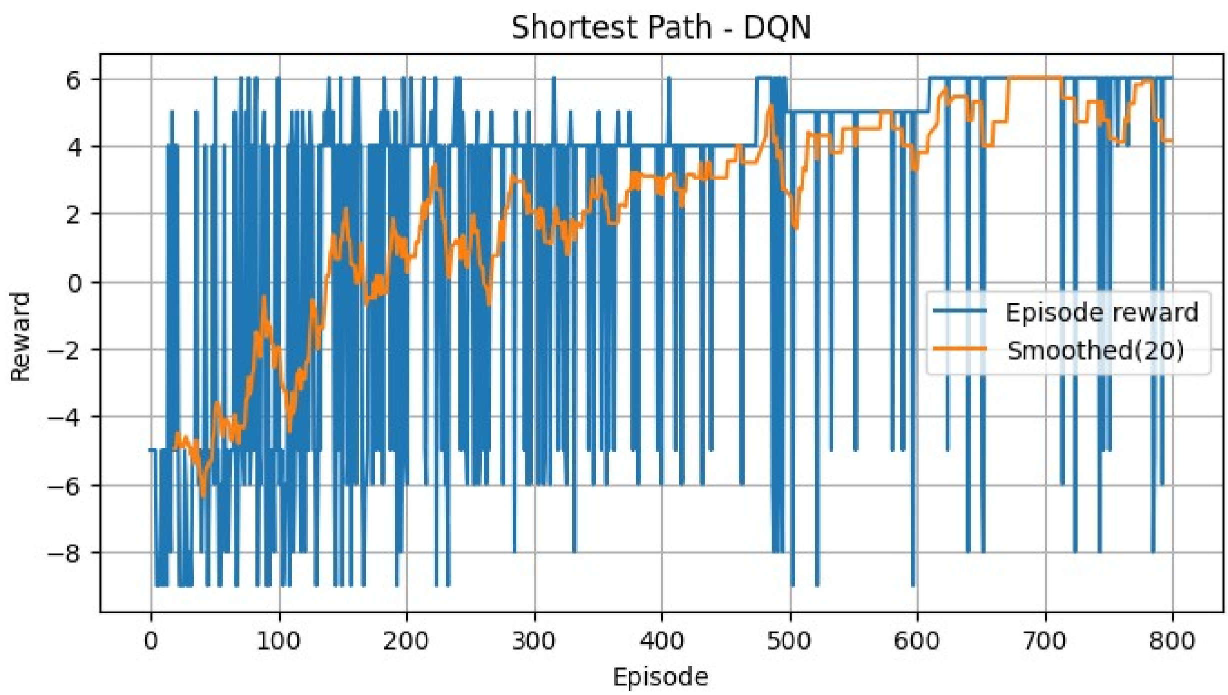
```
    target = SPDQN(n_states, n_actions).to(device)
    target.load_state_dict(model.state_dict())
    optimizer = optim.Adam(model.parameters(), lr=1e-3)
    buffer = deque(maxlen=20000)
    eps = 1.0; min_eps = 0.05; decay = 0.995
    rewards = []

    for ep in range(episodes):
        s = start
        ep_r = 0.0
        steps = 0
        while True:
            s_vec = np.zeros(n_states, dtype=np.float32);
s_vec[idx[s]] = 1.0
            if random.random() < eps:
                a_idx = random.randrange(n_actions)
            else:
                with torch.no_grad():
                    qvals = model(torch.tensor(s_vec,
dtype=torch.float32, device=device))
                    q_np = qvals.cpu().numpy()
                    # mask invalid moves
                    mask = np.array([ -1e9 if nodes[i] not in graph[s]
else 0.0 for i in range(n_actions)], dtype=np.float32)
                    a_idx = int(np.argmax(q_np + mask))
            a_node = nodes[a_idx]
            if a_node not in graph[s]:
                # invalid action: penalize and end episode step
                r = -5.0
                buffer.append((s_vec, a_idx, r, s_vec, True))
                ep_r += r
                break
            w = graph[s][a_node]
            r = -w + (10.0 if a_node == goal else 0.0)
            s2_vec = np.zeros(n_states, dtype=np.float32);
s2_vec[idx[a_node]] = 1.0
            done = (a_node == goal)
            buffer.append((s_vec, a_idx, r, s2_vec, done))
            ep_r += r
            s = a_node
            steps += 1
            if done or steps > 50:
                break

            # Train from buffer
            if len(buffer) >= batch_size:
                batch = random.sample(buffer, batch_size)
                s_b = torch.tensor([b[0] for b in batch],
dtype=torch.float32, device=device)
```

```
                a_b = torch.tensor([b[1] for b in batch],
dtype=torch.int64, device=device).unsqueeze(1)
                r_b = torch.tensor([b[2] for b in batch],
dtype=torch.float32, device=device).unsqueeze(1)
                s2_b = torch.tensor([b[3] for b in batch],
dtype=torch.float32, device=device)
                done_b = torch.tensor([b[4] for b in batch],
dtype=torch.float32, device=device).unsqueeze(1)

                qvals = model(s_b).gather(1, a_b)
                with torch.no_grad():
                    next_q = target(s2_b)
                    # For each s2 in batch, mask invalid actions
before taking max:
                    next_q_np = next_q.cpu().numpy()
                    max_next = []
                    for i in range(next_q_np.shape[0]):
                        s2_node_idx =
int(np.argmax(s2_b[i].cpu().numpy()))
                        s2_node = nodes[s2_node_idx]
                        if not graph[s2_node]:
                            max_next.append(0.0)
                        else:
                            legal_idxs = [idx[nn] for nn in
graph[s2_node].keys()]
                            max_next.append(float(next_q[i,
legal_idxs].max().cpu().numpy()))
                    max_next = torch.tensor(max_next,
dtype=torch.float32, device=device).unsqueeze(1)
                expected = r_b + 0.99 * max_next * (1 - done_b)
                loss = nn.functional.mse_loss(qvals, expected)
                optimizer.zero_grad(); loss.backward();
optimizer.step()

        rewards.append(ep_r)
        eps = max(min_eps, eps * decay)
        if (ep+1) % 200 == 0:
            target.load_state_dict(model.state_dict())
            print(f"[SP DQN] Episode {ep+1}/{episodes}
ep_r={ep_r:.2f} eps={eps:.3f}")

    return model, rewards

print("Training DQN on Shortest Path (short run)...")
sp_dqn_model, sp_dqn_rewards = train_dqn_shortestpath(default_graph,
start=0, goal=3, episodes=800, batch_size=64)
plot_learning_curve(sp_dqn_rewards, "Shortest Path - DQN")
```

```
Default graph (directed weighted): {0: {1: 1, 2: 4}, 1: {2: 2, 3: 5},
2: {3: 1}, 3: {}}
Training Q-Learning on Shortest Path (default graph)...
```




```
Training DQN on Shortest Path (short run)...
[SP DQN] Episode 200/800  ep_r=4.00 eps=0.367
[SP DQN] Episode 400/800  ep_r=4.00 eps=0.135
[SP DQN] Episode 600/800  ep_r=5.00 eps=0.050
[SP DQN] Episode 800/800  ep_r=6.00 eps=0.050
```




```
# --- Evaluation policies ---
mc_q_policy = lambda s: int(np.argmax(q_table[discretize(s)]))
mc_dqn_policy = lambda s:
int(torch.argmax(mc_dqn_model(torch.tensor(s, dtype=torch.float32,
device=device))).item())

# MountainCar eval
mc_q_mean, mc_q_std = evaluate_policy(env, mc_q_policy, episodes=10)
mc_dqn_mean, mc_dqn_std = evaluate_policy(gym.make("MountainCar-v0"),
mc_dqn_policy, episodes=10)

# Roulette eval
roulette_q_policy = lambda s: int(np.argmax(roulette_Q))
def roulette_dqn_policy(s):
    s_t = torch.tensor(s, dtype=torch.float32, device=device)
    with torch.no_grad():
        return int(torch.argmax(roulette_dqn_model(s_t)).item())

rq_q_mean, _ = evaluate_policy(r_env, roulette_q_policy, episodes=200)
rq_dqn_mean, _ = evaluate_policy(gym.make("SimpleRoulette-v0"),
roulette_dqn_policy, episodes=200)

# Shortest path eval helpers
def eval_sp_q(graph, Q, start=0, goal=3, trials=50):
    rewards = []; lengths = []
    for _ in range(trials):
        s = start; total = 0.0; l = 0
        while s != goal and l < 50:
```

```
            if not graph[s]:
                break
            a = max(graph[s].keys(), key=lambda nb: Q[s].get(nb, -
1e9))
            total += -graph[s][a] + (10.0 if a == goal else 0.0)
            s = a; l += 1
        rewards.append(total); lengths.append(l)
    return np.mean(rewards), np.mean(lengths)

def eval_sp_dqn(graph, model, start=0, goal=3, trials=50):
    nodes = sorted(graph.keys()); idx = {n:i for i,n in
enumerate(nodes)}
    rewards = []; lengths = []
    for _ in range(trials):
        s = start; total = 0.0; l = 0
        while s != goal and l < 50:
            sv = np.zeros(len(nodes), dtype=np.float32); sv[idx[s]] =
1.0
            with torch.no_grad():
                q = model(torch.tensor(sv, dtype=torch.float32,
device=device)).cpu().numpy()
            mask = np.array([ -1e9 if nodes[i] not in graph[s] else
0.0 for i in range(len(nodes)) ], dtype=np.float32)
            a_idx = int(np.argmax(q + mask))
            a_node = nodes[a_idx]
            if a_node not in graph[s]:
                total += -5.0; break
            total += -graph[s][a_node] + (10.0 if a_node == goal else
0.0)
            s = a_node; l += 1
        rewards.append(total); lengths.append(l)
    return np.mean(rewards), np.mean(lengths)

sp_q_mean, sp_q_len = eval_sp_q(default_graph, sp_q_values, start=0,
goal=3)
sp_dqn_mean, sp_dqn_len = eval_sp_dqn(default_graph, sp_dqn_model,
start=0, goal=3)

# --- Comparison table ---
table = [
    ["Environment", "Method", "Avg Reward", "Std/Notes", "Avg Path
Len"],
    ["MountainCar", "Q-Learning", f"{mc_q_mean:.2f}",
f"{mc_q_std:.2f}", "-"],
    ["MountainCar", "DQN", f"{mc_dqn_mean:.2f}", f"{mc_dqn_std:.2f}",
"-"],
    ["Roulette", "Q-Learning", f"{rq_q_mean:.2f}", "-", "-"],
    ["Roulette", "DQN", f"{rq_dqn_mean:.2f}", "-", "-"],
    ["ShortestPath", "Q-Learning", f"{sp_q_mean:.2f}", "-",
f"{sp_q_len:.2f}"],
```

```
    ["ShortestPath", "DQN", f"{sp_dqn_mean:.2f}", "-",
f"{sp_dqn_len:.2f}"]
]
print(tabulate(table, headers="firstrow", tablefmt="github"))

# --- Save outputs and create PDF report ---
os.makedirs("outputs", exist_ok=True)
# save plots
plt.figure(); plt.plot(mc_q_rewards); plt.title("MC Q-Learning");
plt.savefig("outputs/mc_q_rewards.png"); plt.close()
plt.figure(); plt.plot(mc_dqn_rewards); plt.title("MC DQN");
plt.savefig("outputs/mc_dqn_rewards.png"); plt.close()
plt.figure(); plt.plot(roulette_q_rewards); plt.title("Roulette Q");
plt.savefig("outputs/r_q_rewards.png"); plt.close()
plt.figure(); plt.plot(roulette_dqn_rewards); plt.title("Roulette
DQN"); plt.savefig("outputs/r_dqn_rewards.png"); plt.close()
plt.figure(); plt.plot(sp_q_rewards); plt.title("SP Q");
plt.savefig("outputs/sp_q_rewards.png"); plt.close()
plt.figure(); plt.plot(sp_dqn_rewards); plt.title("SP DQN");
plt.savefig("outputs/sp_dqn_rewards.png"); plt.close()

# save DQN models (optional)
torch.save(mc_dqn_model.state_dict(), "outputs/mc_dqn.pt")
torch.save(roulette_dqn_model.state_dict(), "outputs/roulette_dqn.pt")
torch.save(sp_dqn_model.state_dict(), "outputs/sp_dqn.pt")

| Environment   | Method     |   Avg Reward | Std/Notes    | Avg Path
Len   |
|---------------|------------|--------------|-------------|-----------
-----|
| MountainCar   | Q-Learning |      -167.7  | 16.37        | -
|
| MountainCar   | DQN        |      -200    | 0.00         | -
|
| Roulette      | Q-Learning |         0.2  | -            | -
|
| Roulette      | DQN        |        -0.25 | -            | -
|
| ShortestPath  | Q-Learning |         6    | -            | 3.00
|
| ShortestPath  | DQN        |         6    | -            | 3.00
|

import gym
import matplotlib.pyplot as plt
import numpy as np

env = gym.make("MountainCar-v0", render_mode="rgb_array")

frames = []
```

```python
state, _ = env.reset()
done = False
for _ in range(200):
    action = np.argmax(q_table[state])  # Use your learned Q-table
    state, reward, terminated, truncated, _ = env.step(action)
    frames.append(env.render())
    if terminated or truncated:
        break

env.close()

# Save as GIF (works everywhere)
import imageio
imageio.mimsave("mountaincar_agent.gif", frames, fps=30)
print("� Saved 'mountaincar_agent.gif' successfully!")
```

```
/usr/local/lib/python3.12/dist-packages/pygame/pkgdata.py:25:
DeprecationWarning: pkg_resources is deprecated as an API. See
https://setuptools.pypa.io/en/latest/pkg_resources.html
  from pkg_resources import resource_stream, resource_exists
/usr/local/lib/python3.12/dist-packages/jupyter_client/session.py:203:
DeprecationWarning: datetime.datetime.utcnow() is deprecated and
scheduled for removal in a future version. Use timezone-aware objects
to represent datetimes in UTC: datetime.datetime.now(datetime.UTC).
  return datetime.utcnow().replace(tzinfo=utc)
/usr/local/lib/python3.12/dist-packages/pkg_resources/__init__.py:3154
: DeprecationWarning: Deprecated call to
`pkg_resources.declare_namespace('google')`.
Implementing implicit namespace packages (as specified in PEP 420) is
preferred to `pkg_resources.declare_namespace`. See
https://setuptools.pypa.io/en/latest/references/keywords.html#keyword-
namespace-packages
  declare_namespace(pkg)
/usr/local/lib/python3.12/dist-packages/pkg_resources/__init__.py:3154
: DeprecationWarning: Deprecated call to
`pkg_resources.declare_namespace('google.cloud')`.
Implementing implicit namespace packages (as specified in PEP 420) is
preferred to `pkg_resources.declare_namespace`. See
https://setuptools.pypa.io/en/latest/references/keywords.html#keyword-
namespace-packages
  declare_namespace(pkg)
/usr/local/lib/python3.12/dist-packages/pkg_resources/__init__.py:3154
: DeprecationWarning: Deprecated call to
`pkg_resources.declare_namespace('sphinxcontrib')`.
Implementing implicit namespace packages (as specified in PEP 420) is
preferred to `pkg_resources.declare_namespace`. See
https://setuptools.pypa.io/en/latest/references/keywords.html#keyword-
namespace-packages
  declare_namespace(pkg)
```

```
---------------------------------------------------------------------------
IndexError                                Traceback (most recent call last)
/tmp/ipython-input-2346433252.py in <cell line: 0>()
      9 done = False
     10 for _ in range(200):
---> 11     action = np.argmax(q_table[state])  # Use your learned Q-table
     12     state, reward, terminated, truncated, _ = env.step(action)
     13     frames.append(env.render())

IndexError: only integers, slices (`:`), ellipsis (`...`), numpy.newaxis (`None`) and integer or boolean arrays are valid indices
```