Answer to the question no - 02

For implementation I,

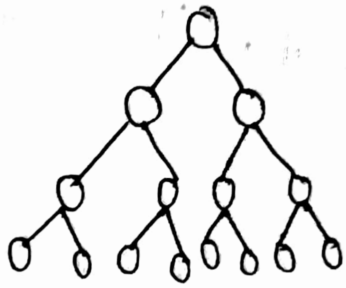
In this case, the function is called two times in each ~~set~~ step with the summation of previous two numbers.

In step - 0, 2^0

In step - 1, 2^1

In step - 2, 2^2

In step - 3, 2^3



$\vdots$

In ~~set~~ step - n, 2^n

$\therefore$ Time complexity for implementation I, $O(2^n)$.

For implementation II,

In this case, for $n < 0$, $O(1)$

for $n \leq 2$, $O(1)$

for n-th step, $O(n)$

$\therefore$ Time complexity for implementation II,

$O(1) + O(1) + O(n)$

$= O(n)$

Here,

$O(n^2) > O(n)$

So, the second implimementation is better because the time complexity of first implimentation is higher than the second.

Ans to the question no - 04

In this case, there are a nested loop which includes three loops with the range of n.

$\therefore$ The time complexity $= n \times n \times n$

$= n^3$

$= O(n^3)$ Ans.

Answer to the question no - 05

(i) $T(n) = T(n/2) + n - 1, \quad T(1) = 0$

According to masters theorem,

$$T(n) = aT\left(\frac{n}{b}\right) + cn^k, \text{ where } T(1) = c$$

$\therefore a = 1$

$\therefore b = 2$

$\therefore c = 1$

$\therefore k = 1$

$\therefore b^k = 2^1 = 2$

Here,

$b^k > a$

$\therefore$ Time complexity $= O(n^k)$

$= O(n^1)$

$= O(n)$

Ans.

(2) $T(n) = T(n-1) + n-1 \quad , \quad T(1) = 0$

According to masters theorem,

$$T(n) = aT(n-b) + f(n)$$

Here,

$a > 0 \quad , \quad b > 0$

$f(n) = O(n^k)$, where $k \geq 0$

$\therefore a = 1$

$\therefore b = 1$

$\therefore k = 1$

$\therefore$ Time complexity $= O(n^{k+1})$

$= O(n^{1+1})$

$= O(n^2)$

Ans.

(3) $T(n) = T\left(\frac{n}{3}\right) + 2T\left(\frac{n}{3}\right) + n$

Here,

$$T(n) = 3T\left(\frac{n}{3}\right) + n$$

According to masters theorem,

$$T(n) = aT\left(\frac{n}{b}\right) + cn^k$$

$\therefore a = 3$

$\therefore b = 3$

$\therefore c = 1$

$\therefore k = 1$

$\therefore b^k = 3^1 = 3$

Here,

$b^k = a$

$\therefore$ Time complexity $= O\left(n^k \log n\right)$

$= O\left(n^1 \log n\right)$

$= O\left(n \log n\right)$

Ans.

④ $T(n) = 2T\left(\frac{n}{2}\right) + n^2$

According to masters theorem,

$$T(n) = aT\left(\frac{n}{b}\right) + cn^k$$

$\therefore a = 2$

$\therefore b = 2$

$\therefore c = 1$

$\therefore k = 2$

$\therefore b^k = 2^2 = 4$

Here,

$b^k > a$

$\therefore$ Time complexity $= O(n^k)$

$= O(n^2)$

Ans.