TP2 Report: Plug Play Algorithms

Imane Si Salah

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1 Find optimal λ for TV denoiser

1. QUESTION 2: Provide the optimal value of λ and the RMSEs / PSNRs that you obtain for TV and the different PyTorch denoisers. Which denoiser provides the best performance?

   The optimal value of lambda, is obtained after only one step is $\lambda =$ **0.1079** it resulted in **PSNRs = 0.90** and **RMSE=0.0630 (24.0 dB)**

   and we can see the resulting image as:

Figure 1: TV denoising for $\sigma = 40$, PSNRs = 0.90 and RMSE=0.0630 (24.0 dB)

   we then tested other denoisers and for different σ values, the results are displayed in Table 6

2. QUESTION 3: Is there a big difference between DnCNN and RealSN_DnCNN? Explain why or why not.

   We see that the RealSN_DnCNN has exactly the same performance. this can be explained by the fact that we applied just a Gaussian noise to the test image. the performance might be different if we tested on images with real noise which more complex than the gaussian noise.

	metrics	$\sigma = 40$	$\sigma = 15$	$\sigma = 5$
TV Denoiser	optimal λ	0.1079	6.0248e-03	4.6261e-04
	PSNR	0.90	0.19	0.04
	RMSE	20.0630 (24.0 dB)	0.0508 (25.9 dB)	0.0192 (34.4 dB)
DnCNN	PSNR	0.94	0.89	0.83
	RMSE	0.0450 (26.9 dB)	0.0236 (32.5 dB)	0.0120 (38.4 dB)
RealSN_DnCNN	PSNR	0.94	0.89	0.83
	RMSE	0.0450 (26.9 dB)	0.0236 (32.5 dB)	0.0120 (38.4 dB)
BM3D	PSNR	0.95	0.87	0.72
	RMSE	0.0417 (27.6 dB)	0.0233 (32.7 dB)	0.0124 (38.2 dB)

Table 1: comparison of TV denoiser to other Pytorch denoisers

2 Plug Play ADMM / DRS for Gaussian denosing

2.1 $s = \sigma$

3. we use the PnP DRS algorithm to denoise an image with noise variance $s^2 = \sigma^2$ however this is not realistic in practice

2.2 $s \neq \sigma$

Now use the provided PnP DRS algorithm to denoise an image with noise variance $s^2 \neq \sigma^2$.

In this case the regularization parameter γ needs to be adapted to find the optimal value.

4. QUESTION 5: Take $s = 30$ and use the same strategy as for TV denoising to search for the optimal γ for $\sigma = 5, 15, 40$. Make a table with the optimal values of γ for each σ and the corresponding PSNRs.

σ	γ	PSNR
40	1.33	22.96
15	0.5	22.37
5	0.17	14.77

Table 2: ADMM / DRS results with Real_DnCNN denoiser

- What is the maximum value of γ that ensures convergence?

  from the results, the largest γ that resulted in convergence was 1.33 for$\sigma = 40$.

- Is convergence guaranteed in all cases? Which ones?

  in the results the algorithm has converged for each case tested (different values of σ)

- Which value of σ provides the best results? Can you explain why?
  he best PSNR was achieved with $\sigma = 40$, probably higher noise levels are more visible and easier for the algorithm to remove.

5. QUESTION 6: Compare the results obtained with RealSN_DnCNN and with DnCNN, TV and BM3D. Which one provides the best performance? Can you explain why?

   the best performance was acheived with the BM3D denoiser, it may be due to the fact that it can effectively remove gaussian noise while keeping the details in the image since it is based on collaborative filtering followed by the DnCNN which ca surprisingly denoise images with different noise levels than the ones it was trained with when used as the regularization

	σ	γ	PSNR
DnCNN Denoiser	40	1.33	22.94
	15	0.5	22.62
	5	0.17	14.73
BM3D Denoiser	40	1.33	23.69
	15	0.5	22.17
	5	0.17	13.68
TV Denoiser	40	1.33	14.39
	15	0.5	15.88
	5	0.17	18.00

Table 3: ADMM / DRS results with DnCNN denoiser

6. QUESTION 7: Test different initialisations, including 0 and random images. How important is the initialization to obtain a good result? Is this consistent with the convergence results seen in the Lecture ?

   The results are displayed as follows:

   We see that the initialization does not have a big impact in the resulting PSNR values for a single simga. this is consistent with what we have seen in lecture since the optimizer converges to the global minimum of our objective function independently of the initialization.

3 PnP ADMM algorithm

7. QUESTION 8:

	σ	initialization type	γ	PSNR
DnCNN Denoiser	40	zeros	1.33	21.53
		random	1.33	21.53
		noisy	1.33	21.53
	15	zeros	0.5	15.28
		random	0.5	15.28
		noisy	0.5	15.28
	5	zeros	0.16	13.39
		random	0.16	13.39
		noisy	0.16	13.39
BM3D Denoiser	40	zeros	1.33	23.23
		random	1.33	23.23
		noisy	1.33	23.23
	15	zeros	0.5	14.89
		random	0.5	14.89
		noisy	0.5	14.89
	5	zeros	0.16	13.21
		random	0.16	13.21
		noisy	0.16	13.21
TV Denoiser	40	zeros	1.33	14.03
		random	1.33	14.03
		noisy	1.33	14.03
	15	zeros	0.5	15.88
		random	0.5	15.88
		noisy	0.5	15.88
	5	zeros	0.16	18.00
		random	0.16	18.00
		noisy	0.16	18.00

Table 4: ADMM / DRS results with various denoisers and initializations

By tracking the expansion parameter, we see that the maximal factor around the theoretical gamma is 0.94 for both admm and drs which implies the convergence of both, corresponding to $\gamma = 0.584$

b) A renaming of variables with a reordering of the three updates allows to show the equivalence of ADMM and DRS. Prove this equivalence.

To show the equivalence between DRS and ADMM steps, we just do the following changes:

Rename y in ADMM to v in DRS.

Reverse the order of x update and y update in ADMM.

8. QUESTION 9: Write the potential and the proximal operator for this degradation (it has a closed form), and solve the inverse problem using PnP-ADMM as before.

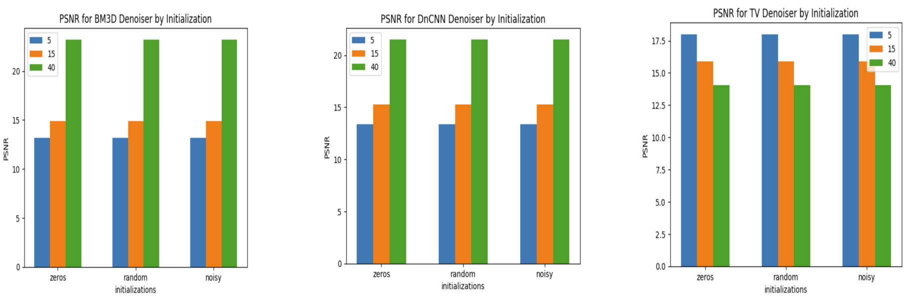

Figure 2: PSNR for different initialization and different denoisers

DRS Steps	**ADMM Steps**
$v = prox_F(x+u)$	$x = prox_G(y-u)$
$x = prox_G(2v-x-u)$	$y = prox_F(x+u)$
$u = u+x-v$	$u = u+x-y$

Table 5: Equivalence of DRS and ADMM algorithm steps.

The derivation of the proximal operator of the data fidelity term can be derived similarly to the case of the gaussian denoising as follows: we set

$$F(x) = \begin{cases} -\frac{\|y_i - x_i\|^2}{2\sigma^2} & \text{if } m_i = 1, \text{(i pixel position)} \\ 0 & \text{otherwise.} \end{cases}$$

$$\begin{aligned} Prox_{\alpha F}(x) &= \arg\min_v \frac{\|y_i - x\|^2}{2} + \alpha F(v) \\ &= \begin{cases} -\frac{x + y\alpha/\sigma^2}{1+\alpha/\sigma^2} & \text{if } m_i = 1, (ipixelposition) \\ x & \text{otherwise.} \end{cases} \end{aligned}$$

PnP-ADMM was applied for this missing pixels problem using this prox formulation. Starting from an image with 90% missing pixels ($psnr = -9.52$), we first searched for an optimal empirical gamma value using the same function written above, we obtained that the best empirical gamma was $\gamma = 1.84$ then computing the optimal gamma ensuring convergence as was done with the DRS optimizer around this empirical gamma, we get $\gamma = 0.21$ for a $psnr = 16.17dB$, we see that we could retrieve some features of the image, but it is still overly smoothed. this can be due to the fact that the found gamma was too large so the regularization term was too strong. furthermore this can be due to a flaw in the updating strategy that is used.

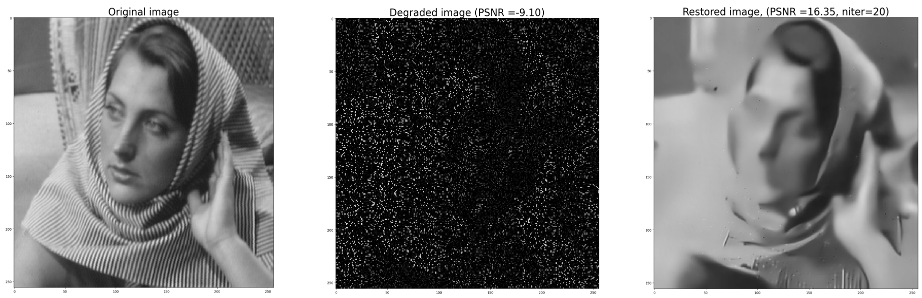

Figure 3: Original image, image with 90% missing pixels, retreived image

4 PnP ISTA (optional)

9. QUESTION 13 (optional): Complete the code below to compute the pnp ista algorithm find the condition on γ to garantee the convergence of PnP ISTA find the optimal γ for $\sigma = 40, 15, 5$ does it satisfy the convergence condition ? For which σ do you obtain the best reconstruction ?

σ	γ	PSNR
40	1.33	22.94
15	0.5	22.13
5	0.17	14.89

Table 6: ISTA results with Real_DnCNN denoiser

Figure 4: Original image, recovered images using ISTA for $\sigma = 40, 15, 5$ respectively

We see that the best result is obtained for $\sigma = 40$ with $psnr = 22.94dB$