UNIVERSITI TEKNOLOGI MALAYSIA
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SCSR1013 DIGITAL LOGIC

MODULE 2a: NUMBER SYSTEMS

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FACULTY OF COMPUTING

Module 2, Part 1: Content

- Decimal Number
- Binary Number
  - Binary-to-Decimal Conversion
  - Decimal-to-Binary Conversion
- Hexadecimal Numbers
- Octal Number
- Binary Coded Decimal (BCD)
- Digital Codes and Parity
- Digital System Application

Numbering system

- A number system is the set of symbols used to express quantities as the basis for counting, determining order, comparing amounts, performing calculations, and representing value.
- Examples include the Arabic, Babylonian, Chinese, Egyptian, Greek, Mayan, and Roman number systems.
- Any positive integer B ($B > 1$) can be chosen as the base or radix of a numbering system.
- If base is B, then B digits (0, 1, 2, ..., $B - 1$) are used.

Table 2.1: Example of Numbering System

Base/Radix	Name	Numerals
2	Binary	0, 1
3	Trinary	0, 1, 2
4	Quaternary	0, 1, 2, 3
5	Quinary	0, 1, 2, 3, 4
8	Octal	0, 1, 2, 3, 4, 5, 6, 7
10	Decimal	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
16	Hexadecimal	0, 1, 2, 3, 4, 5, 6, 7, 8, 9 A, B, C, D, E, F

Positional Numbering Systems

- All numbering system is known as a positional number system, because the value of the number depends on the position of the digits.
- A number written in positional notation can be expanded in power series:

$$N = (c_3 c_2 c_1 c_0 \bullet c_{-1} c_{-2} c_{-3})_B$$
$$= (c_3 x B^3) + (c_2 x B^2) + (c_1 x B^1) + (c_0 x B^0) \bullet (c_{-1} x B^{-1}) + (c_{-2} x B^{-2}) + (c_{-3} x B^{-3})$$

where c_i is the coefficient of B^i and $0 \le c_i \le B$-1.

$$N = (c_3c_2c_1c_0 \,.\, c_{-1}c_{-2}c_{-3})_B =$$

$$c_3 \times B^3 + c_2 \times B^2 + c_1 \times B^1 + c_0 \times B^0 + c_{-1} \times B^{-1} + c_{-2} \times B^{-2} + c_{-3} \times B^{-3}$$

Extra

Example:

$N = 4839.72_{10}$

- $(4_3 8_2 3_1 9_0 \,.\, 7_{-1} 2_{-2})_{10}$
- $(4 \times 10^3) + (8 \times 10^2) + (3 \times 10^1) + (9 \times 10^0) + (7 \times 10^{-1}) + (2 \times 10^{-2})$
- $(4 \times 1000) + (8 \times 100) + (3 \times 10) + (9 \times 1) + (7 \times 0.1) + (2 \times 0.01)$
- $(4000) + (800) + (30) + (9) + (0.7) + (0.02)$
- 4839.72

Terms:

Base b number: $N = a_{q-1}b^{q-1} + \ldots + a_0b^0 + \ldots + a_{-p}b^{-p}$

$b > 1, \quad 0 <= a_i <= b-1$

Integer part: $a_{q-1}a_{q-2} \ldots a_0$

Fractional part: $a_{-1}a_{-2} \ldots a_{-p}$

Most significant digit: a_{q-1}

Least significant digit: a_{-p}

Example:

Most significant bit (MSB)

Least significant bit (LSB)

$$N = \underbrace{4\,8\,3\,9}_{\text{Integer part}}\,.\,\underbrace{7\,2}_{\text{Fraction part}}\ {}_{10}$$

Base number

Decimal number

Base/Radix	Name	Numerals
10	Decimal	0, 1, 2, 3, 4, 5, 6, 7, 8, 9

$$\ldots\ 10^5\ 10^4\ 10^3\ 10^2\ 10^1\ 10^0.\ 10^{-1} 10^{-2}\ 10^{-3}\ 10^{-4}\ 10^{-5} \ldots$$

Example:

Express decimal 47 as a sum of the values of each digit.

$$47_{10} = (4 \times 10^1) + (7 \times 10^0) = 40 + 7$$

$$= 47$$

10000	1000	100	10	1	0.1	0.01	Decimal values
10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	positional values

Example: Express 1024.68_{10} as a sum of values of each digit

1	0	2	4 .	6	8	number
10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	positional values

$$
\begin{aligned}
1024.68_{10} &= (1 \times 10^3) + (0 \times 10^2) + (2 \times 10^1) + (4 \times 10^0) + (6 \times 10^{-1}) + (8 \times 10^{-2}) \\
&= (1 \times 1000) + (0 \times 100) + (2 \times 10) + (4 \times 1) + (6 \times 0.1) + (8 \times 0.01) \\
&= (1000) + (0) + (20) + (4) + (0.6) + (0.08)
\end{aligned}
$$

Extra

Exercise 2a.1:

Express 567.23_{10} as a sum of values of each digit.

10000	1000	100	10	1	0.1	0.01	Decimal values
10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	positional values

Solution:

$= (5 \times 10^2) + (6 \times 10^1) + (7 \times 10^0) + (2 \times 10^{-1}) + (3 \times 10^{-2})$

$= (5 \times 100) + (6 \times 10) + (7 \times 1) + (2 \times 0.1) + (3 \times 0.01)$

$= 500 + 60 + 7 + 0.2 + 0.03$

Base/Radix	Name	Numerals
2	Binary	0, 1

2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	positional values
10000_2	1000_2	100_2	10_2	1_2	0.1_2	0.01_2	binary weight values
16	8	4	2	1	0.5	0.25	decimal values

Example:

$$
\begin{aligned}
10011.01_2 &= (1 \times 2^4) + (0 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) + (0 \times 2^{-1}) + (1 \times 2^{-2}) \\
&= (1 \times 16) + (0 \times 8) + (0 \times 4) + (1 \times 2) + (1 \times 1) + (0 \times 0.5) + (1 \times 0.25) \\
&= 16 + 2 + 1 + 0.25
\end{aligned}
$$

Exercise 2a.2:

Express 110100.011_2 as a sum of values of each digit.

Solution:

$= (1 \times 2^5) + (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (0 \times 2^0) + (0 \times 2^{-1}) + (1 \times 2^{-2}) + (1 \times 2^{-3})$

$= (1 \times 32) + (1 \times 16) + (0 \times 8) + (1 \times 4) + (0 \times 2) + (0 \times 1) + (0 \times 0.5) + (1 \times 0.25) + (1 \times 0.125)$

$= (32) + (16) + (4) + (0.25) + (0.125)$

Octal number

Base/Radix	Name	Numerals
8	Octal	0, 1, 2, 3, 4, 5, 6, 7

8^3	8^2	8^1	8^0	8^{-1}	8^{-2}	positional values
1000_8	100_8	10_8	1_8	0.1_8	0.01_8	octal weighted values
512	64	8	1	0.125	0.0625	decimal values

Example:

$$3706.01_8 = (3 \times 8^3) + (7 \times 8^2) + (0 \times 8^1) + (6 \times 8^0) + (0 \times 8^{-1}) + (1 \times 8^{-2})$$

$$= (3 \times 512) + (7 \times 64) + (0 \times 8) + (6 \times 8) + (0 \times 0.125) + (1 \times 0.0625)$$

Is there any errors ?

Base/Radix	Name	Numerals
8	Octal	0, 1, 2, 3, 4, 5, 6, 7

8^3	8^2	8^1	8^0	8^{-1}	8^{-2}	positional values
1000_8	100_8	10_8	1_8	0.1_8	0.01_8	octal weighted values
512	64	8	1	0.125	0.015625	decimal values

Example:

$3706.01_8 = (3 \times 8^3) + (7 \times 8^2) + (0 \times 8^1) + (6 \times 8^0) + (0 \times 8^{-1}) + (1 \times 8^{-2})$

$= (3 \times 512) + (7 \times 64) + (0 \times 8) + (6 \times 1) + (0 \times 0.125) + (1 \times 0.015625)$

(Correction in module)

Exercise 2a.3:

(No digit 8 in octal number system)

Express 568.23_8 as a sum of values of each digit.

Is there any errors ?

Exercise 2a.3:

Express 567.23_8 as a sum of values of each digit.

8^3	8^2	8^1	8^0	8^{-1}	8^{-2}	positional values
1000_8	100_8	10_8	1_8	0.1_8	0.01_8	octal weighted values
512	64	8	1	0.125	0.015625	decimal values

Solution:

$= (5 \times 8^2) + (6 \times 8^1) + (7 \times 8^0) + (2 \times 8^{-1}) + (3 \times 8^{-2})$

$= (5 \times 64) + (6 \times 8) + (7 \times 1) + (2 \times 0.125) + (3 \times 0.015625)$

$= 320 + 48 + 7 + 0.25 + 0.046875$

Hexadecimal number

Base/Radix	Name	Numerals
16	Hexadecimal	0, 1, 2, 3, 4, 5, 6, 7, 8, 9 A, B, C, D, E, F

Base 16

- 16 possible symbols
- 0-9 and A-F
- $(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \boxed{A, B, C, D, E, F)_{16}}$
  $(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \boxed{10, 11, 12, 13, 14, 15)_{10}}$

Representation of decimal value into hexadecimal value

16^3	16^2	16^1	16^0	16^{-1}	positional values
1000_{16}	100_{16}	10_{16}	1_{16}	0.1_{16}	hexadecimal weighted values
4096	256	16	1	0.0625	decimal values

Example:

$$
\begin{aligned}
A21C.D_{16} &= (A \times 16^3) + (2 \times 16^2) + (1 \times 16^1) + (C \times 16^0) + (D \times 16^{-1}) \\
&= (A \times 4096) + (2 \times 256) + (1 \times 16) + (C \times 1) + (D \times 0.0625) \\
&= (10 \times 4096) + (2 \times 256) + (1 \times 16) + (12 \times 1) + (13 \times 0.0625)
\end{aligned}
$$

Exercise 2a.4:

Express 567.23_{16} as a sum of values of each digit.

16^3	16^2	16^1	16^0	16^{-1}	positional values
1000_{16}	100_{16}	10_{16}	1_{16}	0.1_{16}	hexadecimal weighted values
4096	256	16	1	0.0625	decimal values

Solution:

$= (5 \times 16^2) + (6 \times 16^1) + (7 \times 16^0) + (2 \times 16^{-1}) + (3 \times 16^{-2})$

$= (5 \times 256) + (6 \times 16) + (7 \times 1) + (2 \times 0.0625) + (3 \times 0.00390625)$

$= 1280 + 96 + 7 + 0.125 + 0.1171875$

Exercise 2a.4b:

Express $5A7.2F_{16}$ as a sum of values of each digit.

16^3	16^2	16^1	16^0	16^{-1}	positional values
1000_{16}	100_{16}	10_{16}	1_{16}	0.1_{16}	hexadecimal weighted values
4096	256	16	1	0.0625	decimal values

Solution:

$= (5 \times 16^2) + (A \times 16^1) + (7 \times 16^0) + (2 \times 16^{-1}) + (F \times 16^{-2})$

$= (5 \times 256) + (10 \times 16) + (7 \times 1) + (2 \times 0.0625) + (15 \times 0.00390625)$

$= 1280 + 160 + 7 + 0.125 + 0.05859375$

Convert From Any Base To Decimal

- The summation of the equation is the value in decimal.

Note:
- All examples in previous slide are converting into decimal number without the total.
- Calculate the value in decimal to those example.

Example 3: $2132.413_5 = \_\_\_\_\_\_\_\_\_\__{10}$

$(2 \times 5^3) + (1 \times 5^2) + (3 \times 5^1) + (2 \times 5^0) + (4 \times 5^{-1}) + (1 \times 5^{-2}) + (3 \times 5^{-3})$

$= (2 \times 125) + (1 \times 25) + (3 \times 5) + (2 \times 1) + (4 \times 0.2) + (1 \times 0.04) + (3 \times 0.008)$

$= 250 + 25 + 15 + 2 + 0.8 + 0.04 + 0.024 = 290.864_{10}$

Extra

Calculate the value in decimal to all previous exercise.

Exercise 2a.2:

$110100.011_2 = (32) + (16) + (4) + (0.25) + (0.125)$

$= 52.375_{10}$

Exercise 2a.3:

$567.23_8 = (320) + (48) + (7) + (0.25) + (0.046875)$

$= 375.296875_{10}$

Exercise 2a.4:

$567.23_{16} = (1280) + (96) + (7) + (0.125) + (0.1171875)$

$= 1383.24219_{10}$

Exercise 2a.4b:

$5A7.2F_{16} = (1280) + (160) + (7) + (0.125) + (0.05859375)$

$= 1447.18359_{10}$

Exercise 2a.5:

Simple Deduction: Binary Number

- Fill in the blank spaces.

(a)			(b)		(c)	
Binary	Decimal	2^x	Binary	Decimal	Binary	Decimal
10	2	2^1	1	1	101	5
100	4	2^2	11	3	1010	10
1000	8	2^3	111	7	10001	17
10000	16	2^4	1111	15	11010	26
100000	32	2^5	11111	31	100011	35

(a)			(b)		(c)	
Binary	Decimal	2^x	Binary	Decimal	Binary	Decimal
10	2	2^1	1	1	101	5
100	4	2^2	11	3	1010	10
1000	8	2^3	111	7	10001	17
10000	16	2^4	1111	15	11010	26
100000	32	2^5	11111	31	100011	35

Solution:

ii. Based on the answers that you have calculated in the table, select the correct answer below that best described the deduced observation for (a), (b) and (c).

(c) An even number will have a zero as the last bit while an odd number will have a one as the last bit.

(a) The power of two is equivalent to the number of zeroes in the binary representation number.

(b) A binary number that is equal to $2^x - 1$, will consist of all ones.

Conversion of Decimal to Other Number Bases

- Apply method of successive division.
  - Divide the decimal number by the base of the converted value and get the quotient and remainder.
  - Successively divide the quotients and keep the remainder until the quotient is 0. The answer is the string of remainders (read from bottom to up)

Successive Division:

Example 1: 82_{10} = $\underline{\quad 1\ 2\ 2 \quad}_{8}$

82/8 =

10/8 =

1/8 =

		Remainder
8	82	
8	10	**2**
	1	**2**

Answer

Successive Division:

Divisor	Quotient	*Remainder*
2	42	
2	21	**0**
2	10	**1**
2	5	**0**
2	2	**1**
	1	**0**

Answer

Example 2: 42_{10} = **1 0 1 0 1 0** $_2$

42/2 =

21/2 =

10/2 =

5/2 =

2/2 =

1/2 =

Hex, Octal, Binary and Decimal Numbering System

Hexadecimal	Octal	Binary	Decimal
0	0	0000	0
1	1	0001	1
2	2	0010	2
3	3	0011	3
4	4	0100	4
5	5	0101	5
6	6	0110	6
7	7	0111	7
8	10	1000	8
9	11	1001	9
A	12	1010	10
B	13	1011	11
C	14	1100	12
D	15	1101	13
E	16	1110	14
F	17	1111	15

$(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F)_{16}$

$(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)_{10}$

Example 6: 2047_{10} = __7 F F__ $_{16}$

2047/16	=	127	remainder 15 = F
127/16	=	7	remainder 15 = F
7/16	=	0	remainder 7

		Remainder
16	2047	
16	127	**15 □ F**
	7	**15 □ F**

Answer

Exercise 2a.6:

$1447_{10} = \underline{\quad 5\,A\,7 \quad}_{16}$

Successive Division:

		Remainder
16	1447	
16	90	**7**
	5	**10 □ A**

Answer

(inverted from exercise 4b)

Conversion of Fractions to Other Numbering System

- Repetitive multiplication
  - Step 1:
    Multiply the fraction number by base of the required numbering system
  - Step 2:
    Separate the whole (part of the answer) and the fraction.
  - Step 3:
    Repeat (1) with the new fraction from (2)
  - Stop when the answer of the multiplication = 0
  - Or until reaching the desired fractional point

Example 1: 0.3125_{10} = $\underline{\ .\ 0\ 1\ 0\ 1\ }_2$

Answer:

$0.3125 \times 2 = 0.625 \rightarrow 0$

$0.625 \times 2 = 1.25 \rightarrow 1$

$0.25 \times 2 = 0.5 \rightarrow 0$

$0.5 \times 2 = 1.0 \rightarrow 1$

Example 4: 0.798_{10} = . C C 4 9 $_{16}$

Answer:

$0.798 \times 16 = 12.768 \rightarrow 12 = C$

$0.768 \times 16 = 12.288 \rightarrow 12 = C$

$0.288 \times 16 = 4.608 \rightarrow 4$

$0.608 \times 16 = 9.728 \rightarrow 9$

Or until reaching the desired fractional point

Whole and Fraction Conversion

- Given a number $(c_3c_2c_1c_0.c_{-1}c_{-2}c_{-3})_B$
- To convert the number to the base x:

- Successive division for $c_3c_2c_1c_0$ by x

- Successive multiplication for $c_{-1}c_{-2}c_{-3}$ by x

Exercise 2a.7:

Extra

$$1447.18359_{10} = \underline{5A7.2EF}_{16}$$

1447 + 0.18359

Successive Division:
(Whole part)

		Remainder
16	1447	
16	90	7
	5	10 □ A

Successive Multiplication:
(Fraction part)

0.18359 x 16 = 2.93744 □ 2

0.93744 x 16 = 14.99904 □ E

0.99904 x 16 = 15.98464 □ F

(up to 3 fractional points)

(inverted from exercise 4b)

Binary to Octal & Hex Conversion

- Convert from binary to octal by grouping bits in three starting with the LSB.
  - Each group is then converted to the octal equivalent.
- Convert from binary to hex by grouping bits in four starting with the LSB.
  - Each group is then converted to the hex equivalent.
- Leading zeros can be added to the left of the MSB to fill out the last group.

Example 2: 10011001110_2 = **2 3 1 6** $_8$

$2^n = 8$

$n = 3$

Grouping bits in 3 starting with the LSB.

□ LSB

1 0	0 1 1	0 0 1	1 1 0
2	**3**	**1**	**6**

Example 2: 10011001110_2 = **4 C E** $_{16}$

$2^n = 16$

$n = 4$

Grouping bits in 4 starting with the LSB.

□ LSB

0 1 0 0	1 1 0 0	1 1 1 0
4	**12** □ **C**	**14** □ **E**

Binary Fraction to Octal & Hex Conversion

- For fractional binary number, the grouping of bits start from the radix point :
  - Octal: 3-bit group
  - Hexadecimal: 4-bit group
  - Add the necessary number of 0's

- For whole and fraction binary, the process is divided into two steps:
  - Step 1: Group the whole (integer part) portion starting from the radix point and moving to the left. Add the necessary number of 0's to the left.
  - Step 2: Group the fractional portion starting from the radix point and moving to the right. Add the necessary number of 0's to the right.

Whole fraction (Binary$_2$ □ Octal$_8$)

Recall:
$2^n = 8$
$n = 3$

Example 3: 10001101.1101001_2 = 2 1 5 . 6 4 4 $_8$

Part 1: Group of 3 bits starting from the radix point moving to the left.

010 001 101

2 1 5

Part 2: Group of 3 bits starting from the radix point moving to the right.

110 100 1**00**

6 4 4

Whole fraction (Binary$_2$ □ Hexadecimal$_{16}$)

Recall:
$2^n = 16$
$n = 4$

8 D . D 2

Example 3: 10001101.1101001_2 = ______________ $_{16}$

Part 1: Group of 4 bits starting from the radix point moving to the left.

1000	1101
8	13 = D

Part 2: Group of 4 bits starting from the radix point moving to the right.

1101	001**0**
13 = D	2

Octal & Hex to Binary Conversion

- Convert every digit into groups of binary bits:
  - Octal: 3 bits
  - Hexadecimal: 4 bits
- When converting octal to hexadecimal and vice-verse, it is advisable to uses binary representative as an intermediate conversion.

Example 1: $124.AB7_{16}$ = ________________ $_2$

1 2 4 . A B 7

(Whole part) (Fraction part)

1	2	4.	A	B	7
0001	0010	0100.	1010	1011	0111

$124.AB7_{16} = 000100100100.101010110111_2$

Example 2: 623.53_8 = ______________ $_2$

6	2	3.	5	3
110	010	011.	101	011

$623.52_8 = 110010011.101011_2$

Extra

Exercise 2a.8:

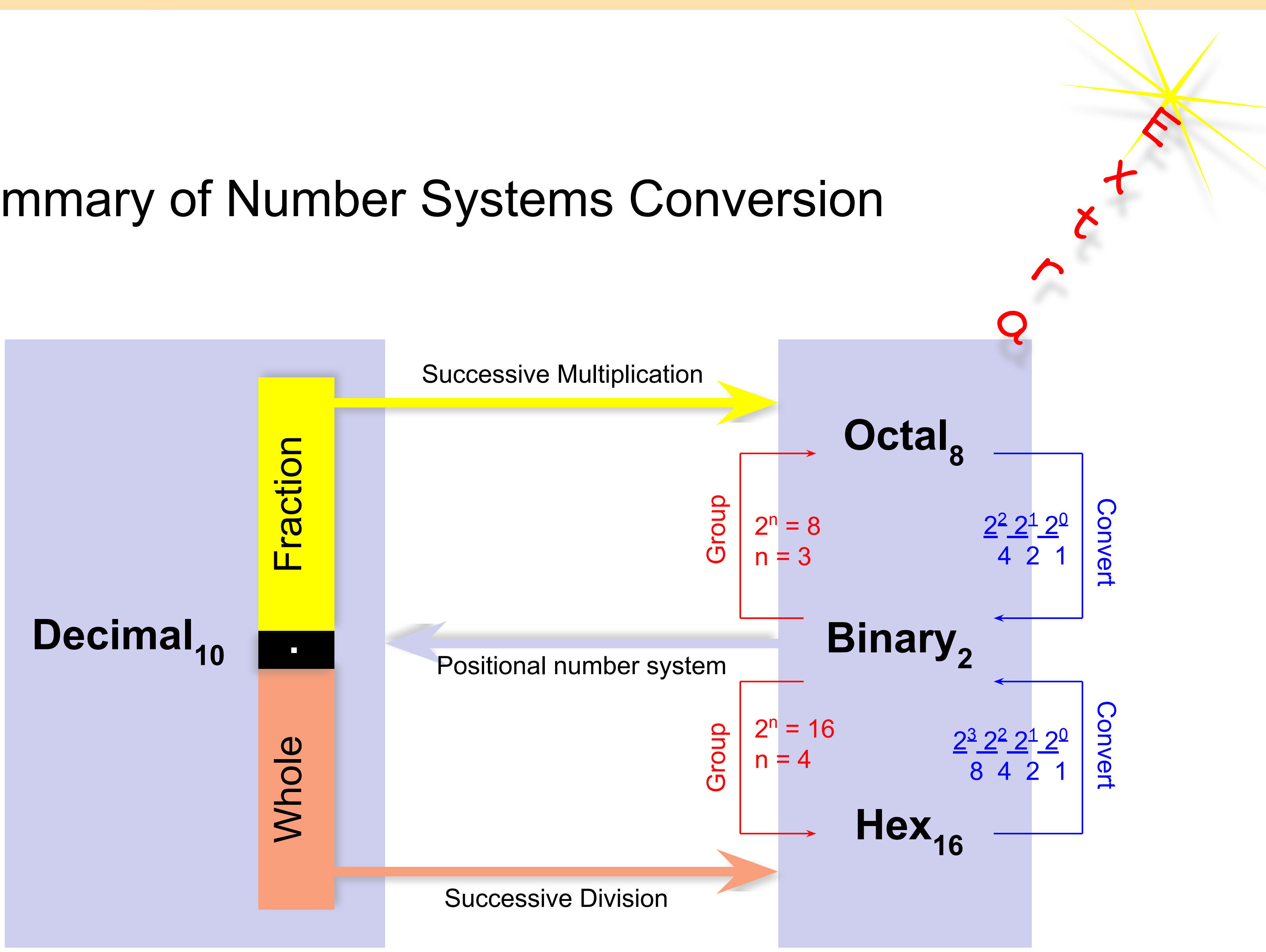
$AF90.7B_{16} = \underline{\mathbf{1010111110010000.01111011}}_2$

Solution:

(Convert each digit into group of 4 bits)

A	F	9	0	.	7	B
1010	1111	1001	0000		0111	1011

Summary of Number Systems Conversion



Summary of Number Systems Conversion