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SCSR1013 DIGITAL LOGIC

MODULE 2a: NUMBER SYSTEMS

FACULTY OF COMPUTING

Module 2, Part 1: Content

- Decimal Number
- Binary Number
 - Binary-to-Decimal Conversion
 - Decimal-to-Binary Conversion
- Hexadecimal Numbers
- Octal Number
- Binary Coded Decimal (BCD)
- Digital Codes and Parity
- Digital System Application

Numbering system

- A number system is the set of symbols used to express quantities as the basis for counting, determining order, comparing amounts, performing calculations, and representing value.
- Examples include the Arabic, Babylonian, Chinese, Egyptian, Greek, Mayan, and Roman number systems.
- Any positive integer B ($B > 1$) can be chosen as the base or radix of a numbering system.
- If base is B , then B digits ($0, 1, 2, \dots, B - 1$) are used.

Table 2.1: Example of Numbering System

<u>Base/Radix</u>	<u>Name</u>	<u>Numerals</u>
2	Binary	0, 1
3	Trinary	0, 1, 2
4	Quaternary	0, 1, 2, 3
5	Quinary	0, 1, 2, 3, 4
8	Octal	0, 1, 2, 3, 4, 5, 6, 7
10	Decimal	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
16	Hexadecimal	0, 1, 2, 3, 4, 5, 6, 7, 8, 9 A, B, C, D, E, F

Positional Numbering Systems

- All numbering system is known as a positional number system, because the value of the number depends on the position of the digits.
- A number written in positional notation can be expanded in power series:

$$\begin{aligned}N &= (c_3c_2c_1c_0 \bullet c_{-1}c_{-2}c_{-3})_B \\&= (c_3xB^3) + (c_2xB^2) + (c_1xB^1) + (c_0xB^0) \bullet (c_{-1}xB^{-1}) + (c_{-2}xB^{-2}) + (c_{-3}xB^{-3})\end{aligned}$$

where c_i is the coefficient of B^i and $0 \leq c_i \leq B-1$.

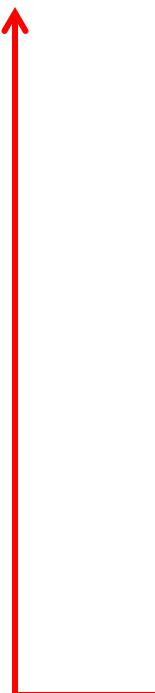
Extra

$$N = (c_3 c_2 c_1 c_0 \cdot c_{-1} c_{-2} c_{-3})_B =$$

$$c_3 \times B^3 + c_2 \times B^2 + c_1 \times B^1 + c_0 \times B^0 + c_{-1} \times B^{-1} + c_{-2} \times B^{-2} + c_{-3} \times B^{-3}$$

Example:

$$N = 4839.72_{10} \rightarrow (4_{\color{red}3} 8_{\color{red}2} 3_{\color{red}1} 9_{\color{red}0} \cdot 7_{\color{red}-1} 2_{\color{blue}-2})_{10}$$



$$\rightarrow (4 \times 10^3) + (8 \times 10^2) + (3 \times 10^1) + \\ (9 \times 10^0) + (7 \times 10^{-1}) + (2 \times 10^{-2})$$

$$\rightarrow (4 \times 1000) + (8 \times 100) + (3 \times 10) + \\ (9 \times 1) + (7 \times 0.1) + (2 \times 0.01)$$

$$\rightarrow (4000) + (800) + (30) + (9) + (0.7) + \\ (0.02)$$

$$\rightarrow 4839.72$$

Terms:

Base b number: $N = a_{q-1}b^{q-1} + \dots + a_0b^0 + \dots + a_{-p}b^{-p}$

$b > 1$, $0 \leq a_i \leq b-1$

Integer part: $a_{q-1}a_{q-2}\dots a_0$

Fractional part: $a_{-1}a_{-2}\dots a_{-p}$

Most significant digit: a_{q-1}

Least significant digit: a_{-p}

Extra

Example:

Most significant bit (MSB)

$N = 4 \underline{8} \underline{3} \underline{9} . \underline{7} 2$

Integer part

Least significant bit (LSB)

10

Base number

Fraction part

Decimal number

<u>Base/Radix</u>	<u>Name</u>	<u>Numerals</u>
10	Decimal	0, 1, 2, 3, 4, 5, 6, 7, 8, 9

... 10^5 10^4 10^3 10^2 10^1 10^0 . 10^{-1} 10^{-2} 10^{-3} 10^{-4} 10^{-5} ...

Example:

Express decimal 47 as a sum of the values of each digit.

$$\begin{aligned}47_{10} &= (4 \times 10^1) + (7 \times 10^0) = 40 + 7 \\&= 47\end{aligned}$$

10000	1000	100	10	1	0.1	0.01	Decimal values
10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	positional values

Example: Express 1024.68_{10} as a sum of values of each digit

1	0	2	4 .	6	8	number
10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	positional values

$$\begin{aligned}
 1024.68_{10} &= (1 \times 10^3) + (0 \times 10^2) + (2 \times 10^1) + (4 \times 10^0) + (6 \times 10^{-1}) + (8 \times 10^{-2}) \\
 &= (1 \times 1000) + (0 \times 100) + (2 \times 10) + (4 \times 1) + (6 \times 0.1) + (8 \times 0.01) \\
 &= (1000) + (0) + (20) + (4) + (0.6) + (0.08)
 \end{aligned}$$



Extra

Exercise 2a.1:

Express 567.23_{10} as a sum of values of each digit.

10000	1000	100	10	1	0.1	0.01	Decimal values
10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	positional values

Solution:

$$= (5 \times 10^2) + (6 \times 10^1) + (7 \times 10^0) + (2 \times 10^{-1}) + (3 \times 10^{-2})$$

$$= (5 \times 100) + (6 \times 10) + (7 \times 1) + (2 \times 0.1) + (3 \times 0.01)$$

$$= 500 + 60 + 7 + 0.2 + 0.03$$

Binary number

<u>Base/Radix</u>	<u>Name</u>	<u>Numerals</u>
-------------------	-------------	-----------------

2	Binary	0, 1
---	--------	------

2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	positional values
10000_2	1000_2	100_2	10_2	1_2	0.1_2	0.01_2	binary weight values
16	8	4	2	1	0.5	0.25	decimal values

Example:

$$\begin{aligned}10011.01_2 &= (1 \times 2^4) + (0 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) + (0 \times 2^{-1}) + (1 \times 2^{-2}) \\&= (1 \times 16) + (0 \times 8) + (0 \times 4) + (1 \times 2) + (1 \times 1) + (0 \times 0.5) + (1 \times 0.25) \\&= 16 + 2 + 1 + 0.25\end{aligned}$$

Exercise 2a.2:

Express 110100.011_2 as a sum of values of each digit.

Solution:

$$\begin{aligned} &= (1 \times 2^5) + (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + \\ &\quad (0 \times 2^0) + (0 \times 2^{-1}) + (1 \times 2^{-2}) + (1 \times 2^{-3}) \\ &= (1 \times 32) + (1 \times 16) + (0 \times 8) + (1 \times 4) + (0 \times 2) + \\ &\quad (0 \times 1) + (0 \times 0.5) + (1 \times 0.25) + (1 \times 0.125) \\ &= (32) + (16) + (4) + (0.25) + (0.125) \end{aligned}$$

Octal number

<u>Base/Radix</u>	<u>Name</u>	<u>Numerals</u>
8	Octal	0, 1, 2, 3, 4, 5, 6, 7

8^3	8^2	8^1	8^0	8^{-1}	8^{-2}	positional values
1000_8	100_8	10_8	1_8	0.1_8	0.01_8	octal weighted values
512	64	8	1	0.125	0.0625	decimal values

Example:

$$\begin{aligned}3706.01_8 &= (3 \times 8^3) + (7 \times 8^2) + (0 \times 8^1) + (6 \times 8^0) + (0 \times 8^{-1}) + (1 \times 8^{-2}) \\&= (3 \times 512) + (7 \times 64) + (0 \times 8) + (6 \times 1) + (0 \times 0.125) + (1 \times 0.0625)\end{aligned}$$

Is there any errors ?

<u>Base/Radix</u>	<u>Name</u>	<u>Numerals</u>
8	Octal	0, 1, 2, 3, 4, 5, 6, 7

8^3	8^2	8^1	8^0	8^{-1}	8^{-2}	positional values
1000_8	100_8	10_8	1_8	0.1_8	0.01_8	octal weighted values
512	64	8	1	0.125	0.015625	decimal values

Example:

$$\begin{aligned}
 3706.01_8 &= (3 \times 8^3) + (7 \times 8^2) + (0 \times 8^1) + (6 \times 8^0) + (0 \times 8^{-1}) + (1 \times 8^{-2}) \\
 &= (3 \times 512) + (7 \times 64) + (0 \times 8) + (6 \times 1) + (0 \times 0.125) + (1 \times 0.015625) \\
 &\quad \text{(Correction in module)}
 \end{aligned}$$

Exercise 2a.3:

(No digit 8 in octal number system)

Express 568.23_8 as a sum of values of each digit.

Is there any errors ?

Exercise 2a.3:

Express 567.23_8 as a sum of values of each digit.

8^3	8^2	8^1	8^0	8^{-1}	8^{-2}	positional values
1000_8	100_8	10_8	1_8	0.1_8	0.01_8	octal weighted values
512	64	8	1	0.125	0.015625	decimal values

Solution:

$$= (5 \times 8^2) + (6 \times 8^1) + (7 \times 8^0) + (2 \times 8^{-1}) + (3 \times 8^{-2})$$

$$= (5 \times 64) + (6 \times 8) + (7 \times 1) + (2 \times 0.125) + (3 \times 0.015625)$$

$$= 320 + 48 + 7 + 0.25 + 0.046875$$

Hexadecimal number

<u>Base/Radix</u>	<u>Name</u>	<u>Numerals</u>
16	Hexadecimal	0, 1, 2, 3, 4, 5, 6, 7, 8, 9 A, B, C, D, E, F

Base 16

- 16 possible symbols
- 0-9 and A-F
- $(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F)_{16}$
- $(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)_{10}$

Representation of decimal value into hexadecimal value

16^3	16^2	16^1	16^0	16^{-1}	positional values
1000_{16}	100_{16}	10_{16}	1_{16}	0.1_{16}	hexadecimal weighted values
4096	256	16	1	0.0625	decimal values

Example:

$$\begin{aligned}
 A21C.D_{16} &= (A \times 16^3) + (2 \times 16^2) + (1 \times 16^1) + (C \times 16^0) + (D \times 16^{-1}) \\
 &= (A \times 4096) + (2 \times 256) + (1 \times 16) + (C \times 1) + (D \times 0.0625) \\
 &= (10 \times 4096) + (2 \times 256) + (1 \times 16) + (12 \times 1) + (13 \times 0.0625)
 \end{aligned}$$

Exercise 2a.4:

Express 567.23_{16} as a sum of values of each digit.

16^3	16^2	16^1	16^0	16^{-1}	positional values
1000_{16}	100_{16}	10_{16}	1_{16}	0.1_{16}	hexadecimal weighted values
4096	256	16	1	0.0625	decimal values

Solution:

$$= (5 \times 16^2) + (6 \times 16^1) + (7 \times 16^0) + (2 \times 16^{-1}) + (3 \times 16^{-2})$$

$$= (5 \times 256) + (6 \times 16) + (7 \times 1) + (2 \times 0.0625) + (3 \times 0.00390625)$$

$$= 1280 + 96 + 7 + 0.125 + 0.1171875$$

Exercise 2a.4b:

Express $5A7.2F_{16}$ as a sum of values of each digit.

16^3	16^2	16^1	16^0	16^{-1}	positional values
1000_{16}	100_{16}	10_{16}	1_{16}	0.1_{16}	hexadecimal weighted values
4096	256	16	1	0.0625	decimal values

Solution:

$$= (5 \times 16^2) + (\text{A} \times 16^1) + (7 \times 16^0) + (2 \times 16^{-1}) + (\text{F} \times 16^{-2})$$

$$= (5 \times 256) + (\text{10} \times 16) + (7 \times 1) + (2 \times 0.0625) + (\text{15} \times 0.00390625)$$

$$= 1280 + 160 + 7 + 0.125 + 0.05859375$$

Convert From Any Base To Decimal

- The summation of the equation is the value in decimal.

Note:

- All examples in previous slide are converting into decimal number without the total.
- Calculate the value in decimal to those example.

Example 3: $2132.413_5 = \underline{\hspace{2cm}}_{10}$

$$\begin{aligned}(2 \times 5^3) + (1 \times 5^2) + (3 \times 5^1) + (2 \times 5^0) + (4 \times 5^{-1}) + (1 \times 5^{-2}) + (3 \times 5^{-3}) \\= (2 \times 125) + (1 \times 25) + (3 \times 5) + (2 \times 1) + (4 \times 0.2) + (1 \times 0.04) + (3 \times 0.008) \\= 250 + 25 + 15 + 2 + 0.8 + 0.04 + 0.024 = 290.864_{10}\end{aligned}$$



Extra

Calculate the value in decimal to all previous exercise.

Exercise 2a.2:

$$\begin{aligned}110100.011_2 &= (32) + (16) + (4) + (0.25) + (0.125) \\&= 52.375_{10}\end{aligned}$$

Exercise 2a.3:

$$\begin{aligned}567.23_8 &= (320) + (48) + (7) + (0.25) + (0.046875) \\&= 375.296875_{10}\end{aligned}$$

Exercise 2a.4:

$$\begin{aligned}567.23_{16} &= (1280) + (96) + (7) + (0.125) + (0.1171875) \\&= 1383.24219_{10}\end{aligned}$$

Exercise 2a.4b:

$$\begin{aligned}5A7.2F_{16} &= (1280) + (160) + (7) + (0.125) + (0.05859375) \\&= 1447.18359_{10}\end{aligned}$$

Exercise 2a.5:

Simple Deduction: Binary Number

- Fill in the blank spaces.

(a)			(b)		(c)	
Binary	Decimal	2^x	Binary	Decimal	Binary	Decimal
10	2	2^1	1	1	101	5
100	4	2^2	11	3	1010	10
1000	8	2^3	111	7	10001	17
10000	16	2^4	1111	15	11010	26
100000	32	2^5	11111	31	100011	35

(a)			(b)		(c)	
Binary	Decimal	2^x	Binary	Decimal	Binary	Decimal
10	2	2^1	1	1	101	5
100	4	2^2	11	3	1010	10
1000	8	2^3	111	7	10001	17
10000	16	2^4	1111	15	11010	26
100000	32	2^5	11111	31	100011	35

Solution:

- ii. Based on the answers that you have calculated in the table, select the correct answer below that best described the deduced observation for (a), (b) and (c).

- (c) An even number will have a zero as the last bit while an odd number will have a one as the last bit.
- (a) The power of two is equivalent to the number of zeroes in the binary representation number.
- (b) A binary number that is equal to $2^x - 1$, will consist of all ones.

Conversion of Decimal to Other Number Bases

- Apply method of successive division.
 - Divide the decimal number by the base of the converted value and get the quotient and remainder.
 - Successively divide the quotients and keep the remainder until the quotient is 0. The answer is the string of remainders (read from bottom to up)

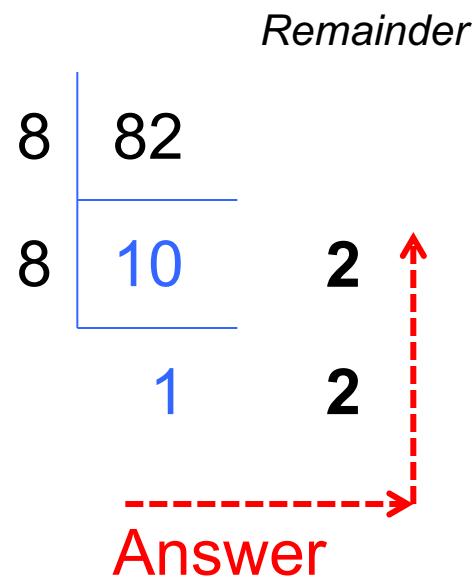
Successive Division:

Example 1: $82_{10} = \underline{\hspace{2cm}} \text{ } 8$

$82/8 =$

$10/8 =$

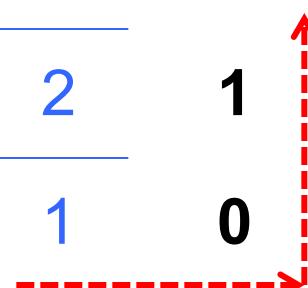
$1/8 =$



Successive Division:

		<i>Remainder</i>
2	42	
2	21	0
2	10	1
2	5	0
2	2	1
1	1	0

Answer



$$\begin{array}{r} \text{Example 2: } 42_{10} = \underline{\quad 1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad} \quad 2 \\ 42/2 = \\ 21/2 = \\ 10/2 = \\ 5/2 = \\ 2/2 = \\ 1/2 = \end{array}$$

Hex, Octal, Binary and Decimal Numbering System

Hexadecimal	Octal	Binary	Decimal
0	0	0000	0
1	1	0001	1
2	2	0010	2
3	3	0011	3
4	4	0100	4
5	5	0101	5
6	6	0110	6
7	7	0111	7
8	10	1000	8
9	11	1001	9
A	12	1010	10
B	13	1011	11
C	14	1100	12
D	15	1101	13
E	16	1110	14
F	17	1111	15

$(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F)_{16}$
 $(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)_{10}$

Example 6: $2047_{10} = \underline{\quad\quad\quad}_{16}$ **7 F F**

$$2047/16 = 127$$

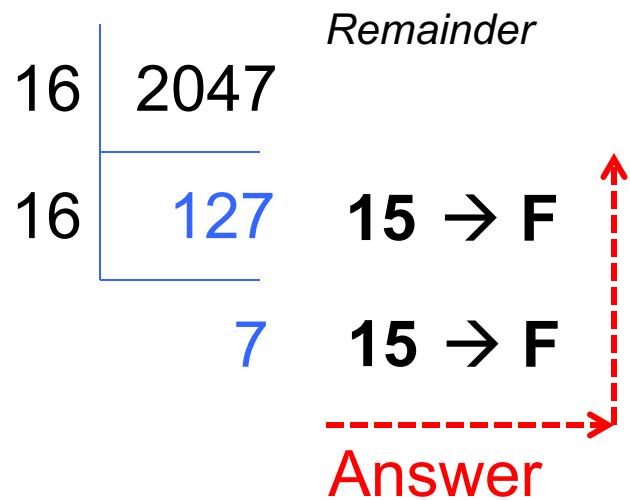
remainder 15 = F ↑

$$127/16 = 7$$

remainder 15 = F ↑

$$7/16 = 0$$

remainder 7

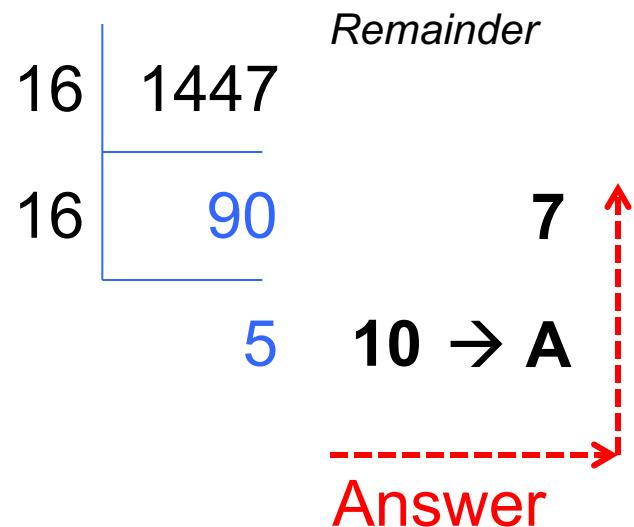


Extra

Exercise 2a.6:

$$1447_{10} = \underline{\quad 5 A 7 \quad}_{16}$$

Successive Division:



(inverted from exercise 4b)

Conversion of Fractions to Other Numbering System

- Repetitive multiplication
 - Step 1:
Multiply the fraction number by base of the required numbering system
 - Step 2:
Separate the whole (part of the answer) and the fraction.
 - Step 3:
Repeat (1) with the new fraction from (2)
 - Stop when the answer of the multiplication = 0
 - Or until reaching the desired fractional point

Example 1: $0.3125_{10} = \underline{\quad.0\ 1\ 0\ 1\quad}_2$

Answer:

$$0.3125 \times 2 = 0.\boxed{625} \rightarrow 0$$

$$0.625 \times 2 = 1.\boxed{25} \rightarrow 1$$

$$0.25 \times 2 = 0.\boxed{5} \rightarrow 0$$

$$0.5 \times 2 = 1.\boxed{0} \rightarrow 1$$

Example 4: $0.798_{10} = \underline{\quad . \text{C C 4 9}}_{16}$

Answer:

$$0.798 \times 16 = 12.768 \rightarrow 12 = \text{C}$$

$$0.768 \times 16 = 12.288 \rightarrow 12 = \text{C}$$

$$0.288 \times 16 = 4.608 \rightarrow 4$$

$$0.608 \times 16 = 9.728 \rightarrow 9$$

Or until reaching the desired fractional point

Whole and Fraction Conversion

- Given a number $(c_3c_2c_1c_0.c_{-1}c_{-2}c_{-3})_B$
- To convert the number to the base x:
 - Successive division for $c_3c_2c_1c_0$ by x
 - Successive multiplication for $c_{-1}c_{-2}c_{-3}$ by x

Exercise 2a.7:

Extra

$$1447.18359_{10} = \underline{5 A 7 . 2 E F}_{16}$$

$$1447 + 0.18359$$

Successive Division:
(Whole part)

$$\begin{array}{r}
 & \text{Remainder} \\
 16 | & 1447 \\
 & \hline
 16 | & 90 \\
 & \hline
 & 5 \quad 10 \rightarrow A
 \end{array}$$

A vertical dashed red arrow points from the digit 7 up to the digit 7 in the sequence 5 A 7. A horizontal dashed red arrow points from the digit 10 to the digit A.

Successive Multiplication:
(Fraction part)

$$\begin{aligned}
 0.18359 \times 16 &= 2.93744 \rightarrow 2 \\
 0.93744 \times 16 &= 14.99904 \rightarrow E \\
 0.99904 \times 16 &= 15.98464 \rightarrow F
 \end{aligned}$$

A vertical dashed red arrow points from the digit 2 down to the digit 2 in the sequence 5 A 7. A vertical dashed red arrow points from the digit E down to the digit F.

(up to 3 fractional points)

(inverted from exercise 4b)

Binary to Octal & Hex Conversion

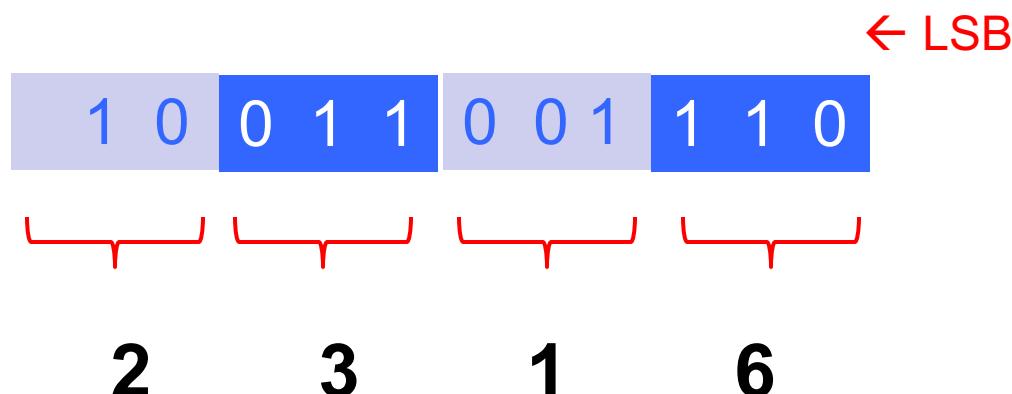
- Convert from binary to octal by grouping bits in three starting with the LSB.
 - Each group is then converted to the octal equivalent.
- Convert from binary to hex by grouping bits in four starting with the LSB.
 - Each group is then converted to the hex equivalent.
- Leading zeros can be added to the left of the MSB to fill out the last group.

Binary₂ → Octal₈

Example 2: $10011001110_2 = \underline{\hspace{2cm}2\ 3\ 1\ 6}$ ₈

Grouping bits in **3** starting with the LSB.

$$2^n = 8$$
$$n = 3$$

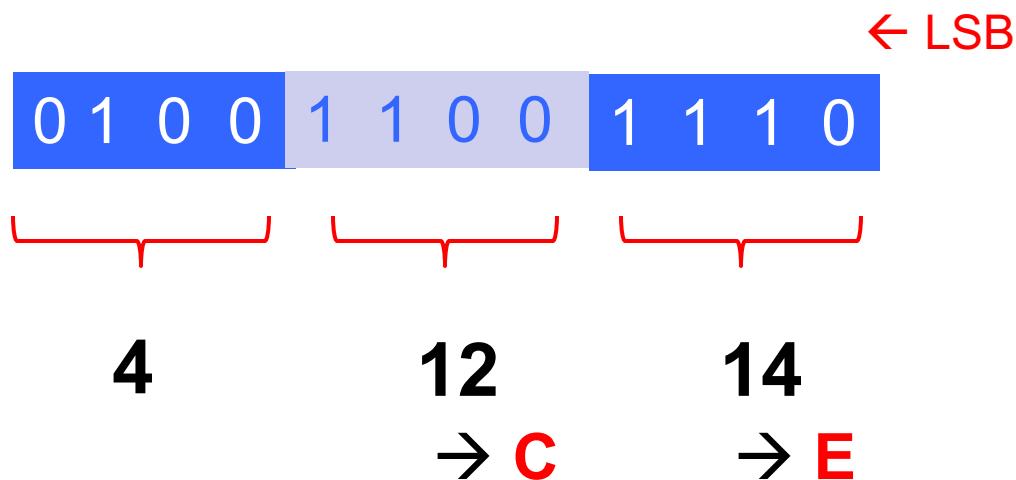


Binary₂ → Hexadecimal₁₆

Example 2: $10011001110_2 = \underline{\hspace{2cm}}^{4\text{ C E}}$ $_{16}$

$$2^n = 16 \\ n = 4$$

Grouping bits in 4 starting with the LSB.



Binary Fraction to Octal & Hex Conversion

- For fractional binary number, the grouping of bits start from the radix point :
 - Octal: 3-bit group
 - Hexadecimal: 4-bit group
 - Add the necessary number of 0's

- For **whole** and **fraction** binary, the process is divided into two steps:
 - Step 1: Group the whole (integer part) portion starting from the radix point and moving to the left. Add the necessary number of 0's to the left.
 - Step 2: Group the fractional portion starting from the radix point and moving to the right. Add the necessary number of 0's to the right.

Whole fraction (Binary₂ → Octal₈)

Example 3: $10001101.1101001_2 = \underline{\hspace{2cm}2\ 1\ 5\ .\ 6\ 4\ 4}$ 8

Recall:

$$2^n = 8$$

$$n = 3$$

Part 1: Group of 3 bits starting from the radix point moving to the left.

010 001 101
2 1 5

Part 2: Group of 3 bits starting from the radix point moving to the right.

110 100 100
6 4 4

Whole fraction (Binary₂ → Hexadecimal₁₆)

Example 3: $10001101.1101001_2 = \underline{\hspace{2cm}}_{16}$

Recall:

$$2^n = 16$$

$$n = 4$$

Part 1: Group of 4 bits starting from the radix point moving to the left.

1000	1101
8	13 = D

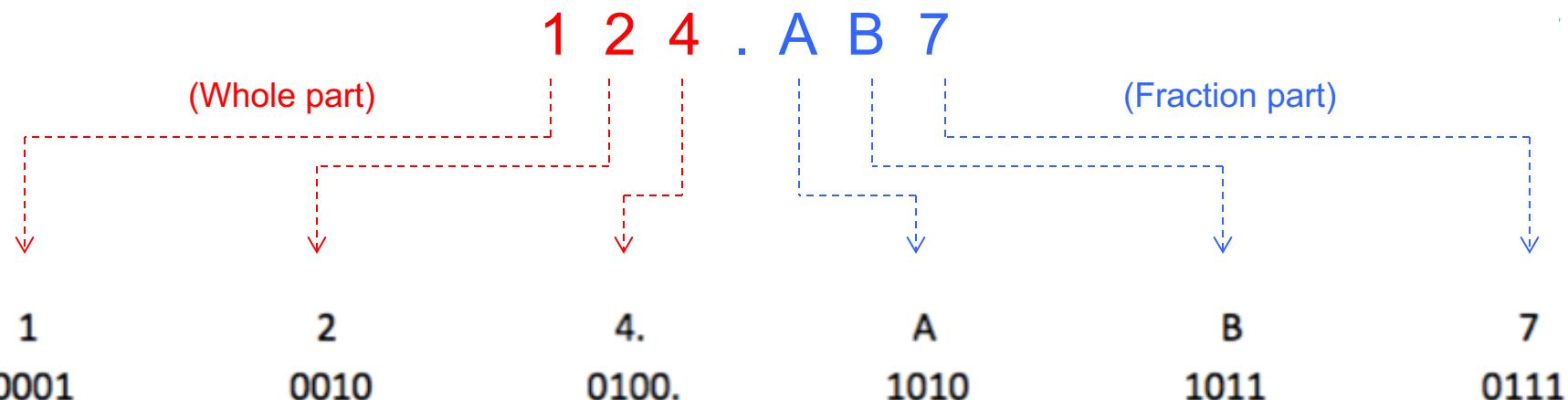
Part 2: Group of 4 bits starting from the radix point moving to the right.

1101	0010
13 = D	2

Octal & Hex to Binary Conversion

- Convert every digit into groups of binary bits:
 - Octal: 3 bits
 - Hexadecimal: 4 bits
- When converting octal to hexadecimal and vice-versa, it is advisable to use binary representative as an intermediate conversion.

Example 1: $124.AB7_{16} = \underline{\hspace{1cm}}_2$



$$124.AB7_{16} = 000100100100.101010110111_2$$

Example 2: $623.53_8 = \underline{\hspace{1cm}}_2$

6	2	3.	5	3
110	010	011.	101	011

$$623.53_8 = 110010011.101011_2$$



Extra

Exercise 2a.8:

$$AF90.7B_{16} = \underline{101011110010000.01111011}_2$$

Solution:

(Convert each digit into group of 4 bits)

A	F	9	0	.	7	B
1010	1111	1001	0000		0111	1011

Extra

Summary of Number Systems Conversion

