COURSE INTRODUCTION

SECI 1013 DISCRETE STRUCTURE

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Overview

- Discrete structure deals with mathematical structures that are fundamentally separate and countable.

- Application in various fields, including computer science, cryptography, operations research, and artificial intelligence. It enables us to solve complex problems efficiently and make informed decisions

Topics

** Refer to course information for weekly scheduled topic

Assessment

Assessment	Percentage
Quiz 1 (W3)	5
Quiz 2 (W6)	5
Quiz 3 (W11)	5
Quiz 4 (W13)	5
Test (W9)	20
Assignment 1 (W2 &W5)	5
Assignment 2 (W8)	5
Assignment 3 (W11 & 12)	5
Assignment 4 (W14)	5
Final Exam	40

References

i.   *Discrete Structure Teaching Module*, Department of Computer Science, UTM

ii.   Johnsonbaugh, R. *Discrete Mathematics*, 8th ed. Pearson Prentice Hall, 2017.

iii.   Malik, D.S. & Sen, M.K. *Discrete Mathematical: Theory and Applications*. Cengage Learning, 2012.

iv.   Kenneth H. R., *Discrete Mathematical And Its Application"*, 7th ed. Mc Graw Hill, 2012.

How you will master Discrete Sturuce

- *I hear and I forget, I see and I remember, I do than I understand*

Chapter 1

SET THEORY
[Part 1: Set & Subset]

Introduction

Why are we studying sets

- The concept of set is basic to all of mathematics and mathematical applications.
- Serves as a basis of description of higher concept and mathematical reasoning
- Set is fundamental in many areas of Computer Science.

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What is SET ?

- A set is a **well-defined collection of distinct objects.**

- These objects are called **members** or **elements** of the set.

- Well-defined means that we can tell for certain whether an object is a member of the collection or not.

- If a set is finite and not too large, we can describe it by listing the elements in it.

Example

- *A* is a set of all positive integers less than 10,
  A={1, 2, 3, 4, 5, 6, 7, 8, 9}

- *B* is a set of first 5 positive odd integers,
  B={1, 3, 5, 7, 9}

- *C* is a set of vowels, *C*={*a, e, i, o, u*}

How we Define Sets?

This can be done by:

- Listing ALL elements of the set within braces.

- Listing enough elements to show the pattern then an ellipsis.

- Use set builder notation to define "rules" for determining membership in the set

Example

1. Listing ALL elements. $A = \{1, 2, 3, 4\}$ explicitly

2. Demonstrating a pattern. $\mathbb{N} = \{1, 2, 3, \dots\}$ implicitly

3. Using set builder notation. $P = \{x \mid x \in \mathbb{R} \text{ and } x \notin \mathbb{C}\}$
implicitly

A set is determined by its elements and not by any particular order in which the element might be listed.

Example, $A=\{1, 2, 3, 4\}$,

A might just as well be specified as

$\{2, 3, 4, 1\}$ or $\{4, 1, 3, 2\}$

Sets

The elements making up a set are assumed to be **distinct**, we may have duplicates in our list, only one occurrence of each element is in the set.

Example

$\{a, b, c, a, c\}$ ⟶ $\{a, b, c\}$

$\{1, 3, 3, 5, 1\}$ ⟶ $\{1, 3, 5\}$

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Sets

- Use uppercase letters *A, B, C …* to denote sets, lowercase denote the elements of set.

- The symbol $\in$ stands for 'belongs to'

- The symbol $\notin$ stands for 'does not belong to'

Example

$$X=\{\ a,\ b,\ c,\ d,\ e\ \},\qquad b \in X\ \text{ and } m \notin X$$

$$A=\{\{1\},\ \{2\},\ 3,\ 4\},\quad \{2\} \in A\ \text{ and }\ 1 \notin A$$

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Sets

- If a set is a large finite set or an infinite set, we can describe it by **listing a property necessary for memberships**

- Let *S* be a set, the notation,

$$A = \{x \mid x \in S, P(x)\} \text{ or } A = \{x \in S \mid P(x)\}$$

means that *A* is the set of all elements *x* of *S* such that *x* satisfies the property *P*.

Example

- Let $A=\{1, 2, 3, 4, 5, 6\}$, we can also write A as,

  $A=\{x \mid x \in Z,\ 0< x <7\}$ if Z denotes the set of integers.

- Let $B=\{x \mid x \in Z,\ x>0\}$, $B=\{1, 2, 3, 4, \ldots\}$

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The set of natural numbers: $\mathbb{N} = \{1, 2, 3, \ldots\}$

The set of integers: $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

The set of positive integers: $\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$

The set of Rational Numbers (fractions): $\frac{1}{2}, \frac{2}{3}, \frac{5}{7}, etc \in \mathbb{Q}$

More formally: $\mathbb{Q} = \left\{ \frac{a}{b} \,\middle|\, a, b \in \mathbb{R}, b \neq 0 \right\}$

The set of Irrational Numbers: $\sqrt{2}, \pi,$ or e are irrational

The Real numbers $= \mathbb{R} =$ the union of the rational numbers with the irrational numbers

The standard form of notation for this is called "set builder notation".

For instance, $\{x \mid x$ is an odd positive integer$\}$ represents the set $\{1, 3, 5, 7, 9, \dots\}$

$\{x \mid x$ is an odd positive integer$\}$ is read as

"the set consisting of all x such that x is an odd positive integer".

The vertical bar, " $\mid$ ", stands for "such that"

Other "short-hand" notation used in working with sets

"$\forall$" stands for "for every"

"$\cup$" stands for "union"

"$\subseteq$" stands for "is a subset of"

"$\not\subset$" stands for "is a not a (proper) subset of"

"$\in$" stands for "is an element of"

"$\times$" stands for "cartesian cross product"

"$\exists$" stands for "there exists"

"$\cap$" stands for "intersection"

"$\subset$" stands for "is a (proper) subset of"

"$\emptyset$" stands for the "empty set"

"$\notin$" stands for "is not an element of"

"$=$" stands for "is equal to"

Subset

If every element of *A* is an element of *B*, we say that *A* is a subset of *B* and write $A \subseteq B$.

$A=B$, if $A \subseteq B$ and $B \subseteq A$

The empty set ($\varnothing$) is a subset of every set.

Example $A=\{1, 2, 3\}$

Subset of *A*,

$\varnothing$, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$, $\{1, 2, 3\}$

Note: **A** is a subset of **A**

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Answer true or false

a) $\{x\} \subseteq \{x\}$

b) $\{x\} \in \{x\}$

c) $\{x\} \in \{x,\{x\}\}$

d) $\{x\} \subseteq \{x,\{x\}\}$

e) $\{\{x\}\} \subseteq \{x,\{x\}\}$

f) $x \in \{x,\{x\}\}$

Proper Subset

If $A \subseteq B$ and B contains an element that is not in A, then we say "A is a **proper subset** of B": $A \subset B$ or $B \supset A$.

Formally: $A \subseteq B$ means $\forall x [x \in A \rightarrow x \in B]$.

For all sets: $A \subseteq A$.

Note: If **A** is a subset of **B** and **A** does not equal B, we say that **A** is a proper subset of **B** ($A \subseteq B$ and $A \neq B$ ($B \not\subseteq A$))

Example

- Let, $A=\{1, 2, 3\}$

  Proper subset of A,
  $\varnothing$, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$

- Let, $B=\{1, 2, 3, 4, 5, 6\}$

  A is proper subset of B.

Example

A={a,b,c,d,e,f,g,h}
B={b,d,e}
C={a,b,c,d,e}
D={r,s,d,e}

Proper subset of A ??

Empty Sets

The **empty set** $\varnothing$ or {} but not {$\varnothing$}
is the set without elements.

Note:

- Empty set has no elements
- Empty set is a subset of any set
- There is exactly one empty set
- Properties of empty set:

  $A \cup \varnothing = A, A \cap \varnothing = \varnothing$

  $A \cap A' = \varnothing, A \cup A' = U$

  $U' = \varnothing, \varnothing' = U$

Example

$$\varnothing = \{x \mid x \text{ is a real number and } x^2 = -3\}$$
$$\varnothing = \{x \mid x \text{ is positive integer and } x^3 < 0\}$$

Equal Sets

The sets A and B are **equal** (A=B) if and only if each element of A is an element of B and vice versa.

Formally: A=B means $\forall x \, [x \in A \leftrightarrow x \in B]$.

Example

$A=\{a, b, c\}$, $B=\{b, c, a\}$,　　　$A=B$

$C=\{1, 2, 3, 4\}$
$D=\{x \mid x$ is a positive integer and $2x < 10\}$,
$C=D$

Determine whether each pair of sets is equal

a) $\{1, 2, 2, 3\}$, $\{1, 3, 2\}$
b) $\{ x \mid x^2 + x = 2 \}$, $\{1, -2\}$
c) $\{x \mid x$ is a real number and $0 < x \leq 2\}$, $\{1, 2\}$

Equivalent Sets

Two sets, A and B, are equivalent if there exists a one-to-one correspondence between them.

When we say sets "have the same size", we mean that they are equivalent.

Example

Set A= {A, B, C, D, E} and **Set** B={1, 2, 3, 4, 5}

Note:
- An equivalent set is simply a set with an **equal number of elements**.
- The sets do not have to have the same exact elements, just the same number of elements.

Finite Sets

A set A is finite

if it is empty

or

if there is a natural number n such that set A is equivalent to $\{1, 2, 3, \ldots n\}$.

Example

$A = \{1, 2, 3, 4\}$

$B = \{x \mid x \text{ is an integer, } 1 \leq x \leq 4\}$

Note:

There exists a nonnegative integer n such that *A* has *n* elements (*A* is called a finite set with *n* elements)

Infinite Sets

- An infinite set is a set whose **elements can not be counted**.
- An infinite set is one that has **no last element**

Are all infinite sets equivalent?

An infinite set is a set that can be placed into a **one-to-one correspondence** with a proper subset of itself.

Example

Infinite sets

$Z = \{x \mid x$ is an integer$\}$
or $Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

$S = \{x \mid x$ is a real number and $1 \leq x \leq 4\}$
$D = \{x \mid x$ is an integer, $x > 0\}$

Finite Sets

$C = \{5, 6, 7, 8, 9, 10\}$

$B = \{x \mid x$ is an integer, $10 < x < 20\}$

Universal Set

- Sometimes we are dealing with sets all of which are subsets of a set *U*.

- This set *U* is called a universal set or a universe.

- The set *U* must be explicitly given or inferred from the context

Typically we consider a set A
a part of a **universal set $\mathcal{U}$**,
which consists of all possible elements.
To be entirely correct we should say

$$\forall x \in \mathcal{U} \, [x \in A \leftrightarrow x \in B]$$

instead of

$$\forall x \, [x \in A \leftrightarrow x \in B] \text{ for } A=B.$$

Note that $\{ x \mid 0<x<5\}$ is can be ambiguous.
Compare $\{ x \mid 0<x<5, x \in N \}$ with $\{ x \mid 0<x<5, x \in Q \}$

Example

- The sets *A*={1,2,3}, *B*={2,4,6,8} and *C*={5,7}

- One may choose *U*={1,2,3,4,5,6,7,8} as a universal set.

- Any superset of *U* can also be considered a universal set for these sets *A*, *B*, and *C*.

  For example, *U*={*x* | *x* is a positive integer}

Cardinality of Set

- Let *S* be a finite set with *n* distinct elements, where $n \geq 0$.

- Then we write $|S|=n$ and say that the cardinality (or the number of elements) of *S* is *n*.

Example

$A = \{1, 2, 3\}, \quad |A|=3$

$B = \{a,b,c,d,e,f,g\}, \quad |B|=7$

If *M* is finite, determine the |*M*|

a) *M* = {1, 2, 3, 4}
b) *M* = {4, 4, 4}
c) *M* = {}
d) *M* = {ø,{ø}, {ø, {ø}}}

Power Set

- The set of all subsets of a set A, denoted $P(A)$, is called the power set of A.

$$P(A) = \{X \mid X \subseteq A\}$$

If $|A| = n$, then $|P(A)| = 2^n$

Example $A = \{1,2,3\}$

The power set of A,

$P(A) = \{\varnothing, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

Notice that $|A| = 3$, and $|P(A)| = 2^3 = 8$

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List the member of $P(\{a, b, c, d\})$.
Which are proper subset of $\{a, b, c, d\}$?

1. Let $U = \{1, 2, 3, 4, 5, 6, 7, \dots.\}$, Let $S = \{x|x \text{ is less than } 10\}$

   a) How many subsets does S have?

   b) How many proper subsets does S have?

2. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}, X = \{2, 4, 6, 8, 10\}$,

   W = {x|x is an odd number} , $Y = \{3\}$

   state whether the following TRUE or FALSE

    1. $X' = W$

    2. $Y \subseteq W$

    3. $Y \subset W$

    4. $X \subseteq U$

    5. $\{ \} \subseteq X$

Thank You

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