Chapter 1

Part 3

Fundamental and Elements of Logic

Why Are We Studying Logic?

Some of the reasons:

- Logic is the foundation for computer operation
- Logical conditions are common in programs

  Example:

  Selection: if (score <= max) { ... }

  Iteration: while (i<limit && list[i]!=sentinel) ...
- All manner of structures in computing have properties that need to be proven (and proofs that need to be understood).

  Example: Trees, Graphs, Recursive Algorithms, . . .
- Programs can be proven correct.
- Computational linguistics must represent and reason about human language, and language represents thought (and thus also logic).

PROPOSITION

A **statement** or a **proposition**, is a declarative sentence that is **either TRUE or FALSE, but not both.**

Example:

- 4 is less than 3.
- 7 is an even integer.
- Washington, DC, is the capital of United State.

Example

i) Why do we study mathematics?
ii) Study logic.
iii) What is your name?
iv) Quiet, please.

The above sentences are not propositions. Why ?

(i) & (iii) : is question, not a statement.
(ii) & (iv) : is a command.

Example

i) The temperature on the surface of the planet Venus is 800 F.
ii) The sun will come out tomorrow.

Propositions? Why?

- Is a statement since it is either true or false, but not both.
- However, we do not know at this time to determine whether it is true or false.

CONJUNCTIONS

- Compound propositions formed in English with the word "**and**"
- Formed in logic with the caret symbol **("∧")**
- **True** only when **both participating propositions are true**.

TRUTH TABLE: This tables aid in the evaluation of **compound propositions**.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

True (T), False (F)

Example

p : 2 is an even integer
q : 3 is an odd number

propositions

p ∧ q

symbols

: 2 is an even integer and 3 is an odd number

statements

p : today is Monday
q : it is hot

p ∧ q: today is Monday and it is hot

Example

Proposition

p : 2 divides 4

q : 2 divides 6

Symbol & Statement

$p \land q$: 2 divides 4 and 2 divides 6.

or,

$p \land q$: 2 divides both 4 and 6.

Example

Proposition

p : 5 is an integer

q : 5 is not an odd integer

Symbol & Statement

$p \wedge q$: 5 is an integer and 5 is not an odd integer.

or,

$p \wedge q$: 5 is an integer but 5 is not an odd integer.

DISJUNCTION

- Compound propositions formed in English with the word "**or**",
- Formed in logic with the caret symbol (" V ")
- **True** when **one or both** participating propositions are true.

The truth table for $p \vee q$

p	q	p Vq
T	T	T
T	F	T
F	T	T
F	F	F

Example

$\boldsymbol{p}$: 2 is an integer
$\boldsymbol{q}$: 3 is greater than 5

$\boldsymbol{p} \vee \boldsymbol{q}$: 2 is an integer or 3 is greater than 5

$\boldsymbol{p}$: 1+1=3
$\boldsymbol{q}$: A decade is 10 years

$\boldsymbol{p} \vee \boldsymbol{q}$ **:** 1+1=3 or a decade is 10 years

Example

p : 3 is an even integer
q : 3 is an odd integer

p $\vee$ **q**
3 is an even integer or 3 is an odd integer
or
3 is an even integer or an odd integer

NEGATION

Negating a proposition simply flips its value. Symbols representing negation include:

$\neg x$, $\bar{x}$, $\sim x$, x' (NOT)

Let *p* be a proposition.
The negation of *p, written* ¬ *p* is the statement obtained by negating statement *p.*

The truth table of ¬ *p*

p	¬*p*
T	F
F	T

Example

p : 2 is positive

$\neg$ ***p :*** 2 is not positive.

p : 4 is less than 3

$\neg$ ***p :*** 4 is not less than 3.

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Exercise

p: It will rain tomorrow

q: it will snow tomorrow

Give the negation of the following statement and write the symbol.

It will rain tomorrow or it will snow tomorrow.

Exercise

In each of the following, form the conjunction and the disjunction of ***p*** and ***q*** by writing the symbol and the statements.

i) p: I will drive my car
q: I will be late

ii) p : NUM > 10
q : NUM $\leq$ 15

Exercise

Suppose x is a particular real number. Let ***p***, ***q*** and ***r*** symbolize "$0 < x$", "$x < 3$" and "$x = 3$", respectively. Write the following inequalities symbolically:

a) $x \leq 3$

b) $0 < x < 3$

c) $0 < x \leq 3$

Exercise

State either TRUE or FALSE if ***p*** and ***r*** are TRUE and ***q*** is FALSE.

a) $\sim p \wedge (q \vee r)$

a) $(r \wedge \sim q) \vee (p \vee r)$

CONDITIONAL PROPOSITIONS

Let ***p*** *and* ***q*** be propositions.

"if ***p***, *then* ***q***"

is a statement called a **conditional proposition**, written as

$$p \rightarrow q$$

CONDITIONAL PROPOSITIONS

The truth table of $p \rightarrow q$

=> Cause and effect relationship

FALSE if
***p* = True**
and *q*
=false

p	q	p→ q
T	T	T
T	F	F
F	T	T
F	F	T

TRUE if
both
true or
***p*=false**
for any
value of
q

Example

p : today is Sunday

q : I will go for a walk

***p* → *q* :** If today is Sunday, then I will go for a walk.

p : I get a bonus

q : I will buy a new car

p → ***q***: If I get a bonus, then I will buy a new car

Example

p : x/2 is an integer.

q : x is an even integer.

$p \rightarrow q$: if x/2 is an integer, then x is an even integer.

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BICONDITIONAL

Let ***p*** *and* ***q*** *be propositions.*

“***p*** *if and only if* ***q***”

is a statement called a **biconditional proposition**, written as

$$p \leftrightarrow q$$

BICONDITIONAL

The truth table of $p \leftrightarrow q$:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Example

***p** :* my program will compile
***q** :* it has no syntax error.

$p \leftrightarrow q$: My program will compile if and only if it has no syntax error.

p : x is divisible by 3
q : x is divisible by 9

$p \leftrightarrow q$: x is divisible by 3 if and only if x is divisible by 9.

LOGICAL EQUIVALENCE

- The compound propositions ***Q*** and ***R*** are made up of the propositions $p_1, \ldots, p_n$.

- ***Q*** and ***R*** are logically equivalent and write,

  $Q \equiv R$

  provided that given any truth values of $p_1, \ldots, p_n$, either ***Q*** and ***R*** are **both true** or ***Q*** and ***R*** are **both false**.

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Example

$\boldsymbol{Q} = p \rightarrow q \qquad \boldsymbol{R} = \neg q \rightarrow \neg p$

Show that, $\boldsymbol{Q} \equiv \boldsymbol{R}$

The truth table shows that, $Q \equiv R$

p	q	$p \rightarrow q$	$\neg q \rightarrow \neg p$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Example

Show that, $\neg(p \rightarrow q) \equiv p \wedge \neg q$

The truth table shows that, $\neg(p \rightarrow q) \equiv p \wedge \neg q$

p	q	$\neg(p \rightarrow q)$	$p \wedge \neg q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	F	F

PRECEDENCE OF LOGICAL CONNECTIVES

	Precedence of logical connectives is as follows:		
not	¬	↑	Highest
and	$\wedge$		
or	$\vee$		
If…then	$\rightarrow$		
If and only if	$\leftrightarrow$		Lowest

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Example

Construct the truth table for, **A** = $\neg(p \vee q) \rightarrow (q \wedge p)$

Solution

p	q	$(p \vee q)$	$\neg(p \vee q)$	$(q \wedge p)$	A
T	T	T	F	T	T
T	F	T	F	F	T
F	T	T	F	F	T
F	F	F	T	F	F

Exercise

Construct the truth table for each of the following statements:

i) $\neg p \land q$

ii) $\neg(p \lor q) \rightarrow q$

iii) $\neg(\neg p \land q) \lor q$

iv) $(p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$

LOGIC & SET THEORY

Logic and set theory go very well togather. The previous definitions can be made very succinct:

$$x \notin A \text{ if and only if } \neg(x \in A)$$
$$A \subseteq B \text{ if and only if } (x \in A \rightarrow x \in B) \text{ is True}$$
$$x \in (A \cap B) \text{ if and only if } (x \in A \wedge x \in B)$$
$$x \in (A \cup B) \text{ if and only if } (x \in A \vee x \in B)$$
$$x \in A - B \text{ if and only if } (x \in A \wedge x \notin B)$$
$$x \in A \Delta B \text{ if and only if } (x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin A)$$
$$x \in A' \text{ if and only if } \neg(x \in A)$$
$$X \in P(A) \text{ if and only if } X \subseteq A$$

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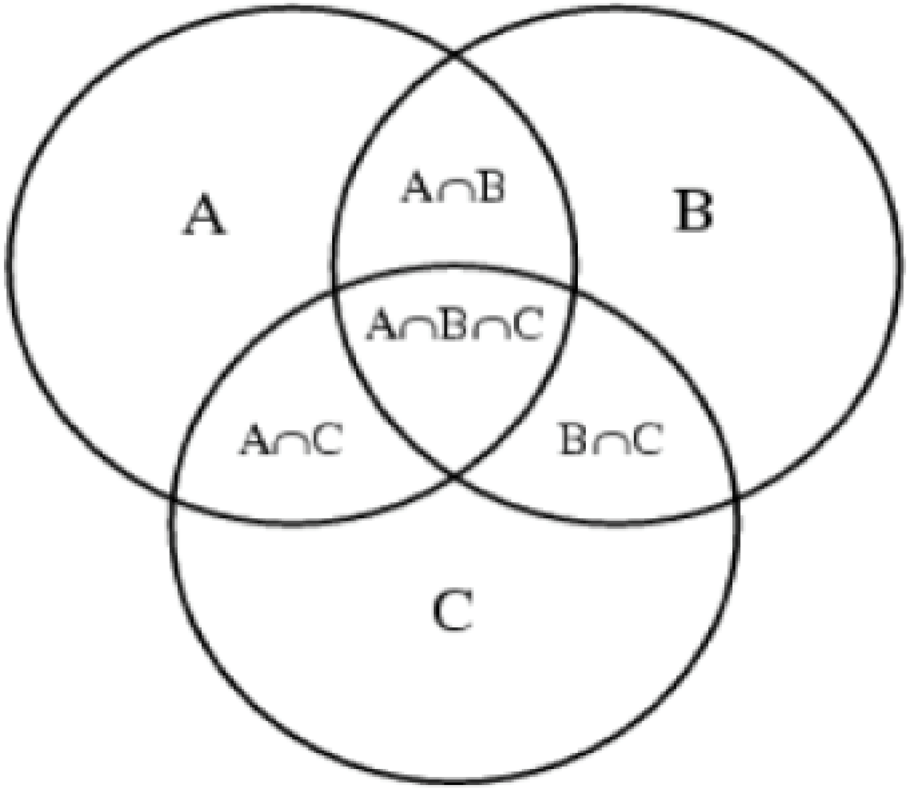
Venn Diagrams

Venn Diagrams are used to depict the various unions, subsets, complements, intersections etc. of sets.



Logic and Sets
are closely related

Tautology	Set Operation Identity
$p \vee q \leftrightarrow q \vee p$	$A \cup B = B \cup A$
$p \wedge q \leftrightarrow q \wedge p$	$A \cap B = B \cap A$
$p \vee (q \vee r) \leftrightarrow (p \vee q) \vee r$	$A \cup (B \cup C) = (A \cup B) \cup C$
$p \wedge (q \wedge r) \leftrightarrow (p \wedge q) \wedge r$	$A \cap (B \cap C) = (A \cap B) \cap C$
$p \vee (q \wedge r) \leftrightarrow (p \vee q) \wedge (p \vee r)$	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$p \wedge \neg q \leftrightarrow p \wedge \neg (p \wedge q)$	$A - B = A - (A \cap B)$
$p \wedge \neg (q \vee r) \leftrightarrow (p \wedge \neg q) \wedge (p \wedge \neg r)$	$A - (B \cap C) = (A - B) \cup (A - C)$
$p \wedge \neg (q \wedge r) \leftrightarrow (p \wedge \neg q) \vee (p \wedge \neg r)$	$A - (B \cup C) = (A - B) \cap (A - C)$
$p \wedge (q \wedge \neg r) \leftrightarrow (p \wedge q) \wedge \neg (p \wedge \neg r)$	$A \cap (B - C) = (A \cap B) - (A \cap C)$
$p \vee (q \wedge \neg r) \leftrightarrow (p \vee q) \wedge \neg (r \wedge \neg p)$	$A \cup (B - C) = (A \cup B) - (C - A)$
$p \wedge \neg \vee (q \wedge \neg r) \leftrightarrow (p \wedge \neg q) \vee (p \wedge r)$	$A - (B - C) = (A - B) \cup (A \cap C)$

The above identities serve as the basis for an "algebra of sets".

Logic and Sets
are closely related

Tautology

$p \wedge p \leftrightarrow p$

$p \vee p \leftrightarrow p$

$p \wedge \neg(q \wedge \neg q) \leftrightarrow p$

$p \vee \neg(q \wedge \neg q) \leftrightarrow p$

Set Operation Identity

$A \cap A = A$

$A \cup A = A$

$A - \varnothing = A$

$A \cup \varnothing = A$

Contradiction

$p \wedge \neg p$

$p \wedge (q \wedge \neg q)$

$p \wedge \neg p$

Set Operation Identity

$A - A = \varnothing$

$A \cap \varnothing = \varnothing$

$A - A = \varnothing$

The above identities serve as the basis for an "algebra of sets".

Theorem for Logic

Let ***p***, ***q*** and ***r*** be propositions.

Idempotent laws:

$p \wedge p \equiv p$

$p \vee p \equiv p$

Truth table

p	$p \wedge p$	$p \vee p$
T	T	T
F	F	F

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Theorem for Logic

Double negation law:

$$\neg \neg p \equiv p$$

Commutative laws:

$$p \wedge q \equiv q \wedge p$$
$$p \vee q \equiv q \vee p$$

Associative laws:

$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$
$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$

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Theorem for Logic

Distributive laws:

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

p	q	r	$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

PROVE

Theorem for Logic

Absorption laws:

$$p \wedge (p \vee q) \equiv p$$
$$p \vee (p \wedge q) \equiv p$$

p	q	$p \wedge (p \vee q)$	$p \vee (p \wedge q)$
T	T	T	T
T	F	T	T
F	T	F	F
F	F	F	F

PROVE

Theorem for Logic

De Morgan's laws:

$$\neg(p \land q) \equiv (\neg p) \lor (\neg q)$$
$$\neg(p \lor q) \equiv (\neg p) \land (\neg q)$$

The truth table for $\neg(p \lor q) \equiv (\neg p) \land (\neg q)$

p	q	$\neg(p \lor q)$	$\neg p \land \neg q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

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Exercise

Given,

$\boldsymbol{R} = p \wedge (\neg\, q \vee r)$

$\boldsymbol{Q} = p \vee (q \wedge \neg\, r)$

State whether or not $\boldsymbol{R} \equiv \boldsymbol{Q}$.

Exercise

Propositional functions ***p***, ***q*** and ***r*** are defined as follows:

- ***p*** is "$n = 7$"
- ***q*** is "$a > 5$"
- ***r*** is "$x = 0$"

Write the following expressions in terms of ***p***, ***q*** and ***r***, and show that each pair of expressions is **logically equivalent**. State carefully which of the above laws are used at each stage.

(a) $((n = 7) \vee (a > 5))\ (x = 0)$
$((n = 7)\ (x = 0)) \vee ((a > 5)\ (x = 0))$

(b) $\neg((n = 7)\ (a \leq 5))$
$(n \neq 7) \vee (a > 5)$

(c) $(n = 7) \vee (\neg((a \leq 5)\ (x = 0)))$
$((n = 7) \vee (a > 5)) \vee (x \neq 0)$

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Exercise

Propositions ***p***, ***q***, ***r*** and ***s*** are defined as follows:
p is "I shall finish my Coursework Assignment"
q is "I shall work for forty hours this week"
r is "I shall pass Maths"
s is "I like Maths"

Write each sentence in symbols:
(a) I shall not finish my Coursework Assignment.
(b) I don't like Maths, but I shall finish my Coursework Assignment.
(c) If I finish my Coursework Assignment, I shall pass Maths.
(d) I shall pass Maths only if I work for forty hours this week and finish my Coursework Assignment.

Write each expression as a sensible (if untrue!) English sentence:
(e) $q \vee p$
(f) $\neg p \rightarrow \neg r$

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Exercise

For each pair of expressions, construct truth tables to see if the two compound propositions are logically equivalent:

(a) $p \vee (q \wedge \neg p)$

$p \vee q$

(b) $(\neg p \wedge q) \vee (p \wedge \neg q)$

$(\neg p \wedge \neg q) \vee (p \wedge q)$

Thank You

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