



UTM
UNIVERSITI TEKNOLOGI MALAYSIA

Chapter 1

SET THEORY

[Part 2: Operation on Set]



Topics

- ☐ Union (\cup)
- ☐ Intersection (\cap)
- ☐ Disjoint
- ☐ Difference
- ☐ Symmetric Difference
- ☐ Compliment
- ☐ Properties of Set

Consider the universal set $U = \{a, o, u, c, d, l, m, s\}$.

The following subsets are considered:

$A = \{c, a, s, o\}$ and $B = \{m, a, l, o\}$.

- Calculate $A \cup B, A \cap B, A - B, B - A, A', B', A' \cap B$
- Represent the sets A, B and U in a Venn diagram.

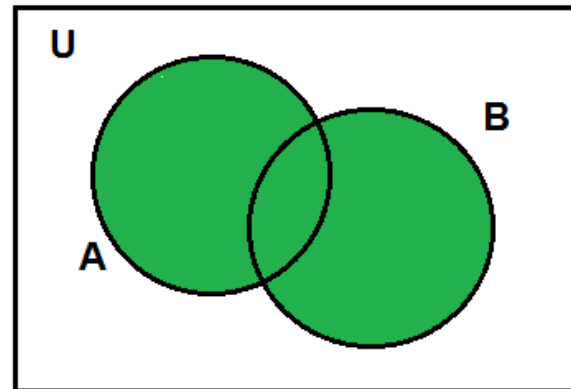
Union

- The **union** of two sets A and B , denoted by $A \cup B$, is defined to be the set

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

- The union consists of all elements belonging to either A or B (or both)

Venn diagram of $A \cup B$



Example

$A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6\}$ and $C = \{8, 9\}$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

$$A \cup C = \{1, 2, 3, 4, 5, 8, 9\}$$

$$B \cup C = \{2, 4, 6, 8, 9\}$$

$$A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 8, 9\}$$

Union

If A and B are finite sets, the **cardinality** of $A \cup B$,

$$|A \cup B| = |A| + |B| - |A \cap B|$$

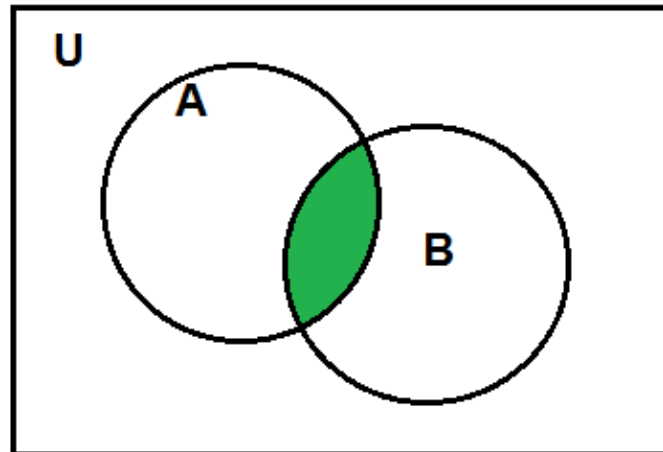
Intersection

- The **intersection** of two sets A and B , denoted by $A \cap B$, is defined to be the set

$$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$$

- The **intersection** consists of all elements belonging to both A and B .

Venn diagram of $A \cap B$



Example

$A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6, 8, 10\}$ and $C = \{1, 2, 8, 10\}$

$$A \cap B = \{2, 4, 6\}$$

$$A \cap C = \{1, 2\}$$

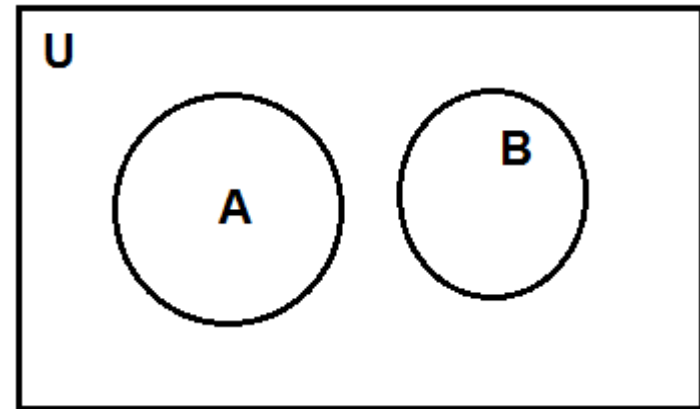
$$C \cap B = \{2, 8, 10\}$$

$$A \cap B \cap C = \{2\}$$

Disjoint

Two sets A and B are said to be **disjoint** if, $A \cap B = \emptyset$

Venn diagram, $A \cap B = \emptyset$



Example

$$A = \{1, 3, 5, 7, 9, 11\}, B = \{2, 4, 6, 8, 10\}$$

$$A \cap B = \emptyset$$

Difference

The set,

$$A-B = \{x \mid x \in A \text{ and } x \notin B\}$$

is called the **difference**.

The difference $A-B$ consists of all elements in A that are not in B .

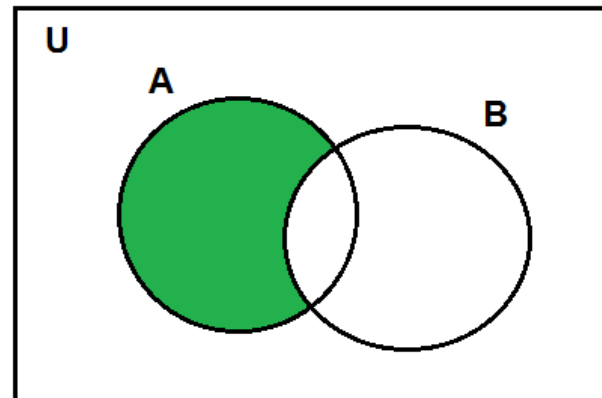
Venn diagram of $A-B$

Example

$$A = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

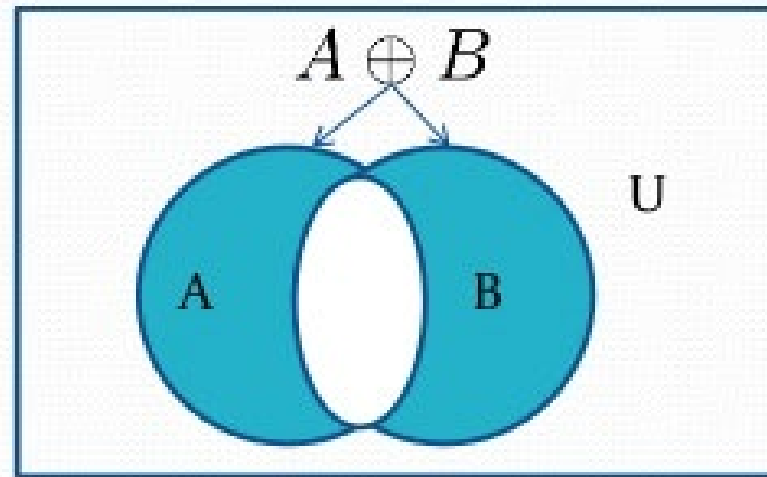
$$B = \{2, 4, 6, 8\}$$

$$A-B = \{1, 3, 5, 7\}$$



Symmetric Difference

The symmetric difference of set A and set B , denoted by $A \oplus B$ is the set $(A - B) \cup (B - A)$



Venn Diagram

Example

$$U = \{0,1,2,3,4,5,6,7,8,9,10\}$$

$$A = \{1,2,3,4,5\}; B = \{4,5,6,7,8\}$$

$$A \oplus B = (A - B) \cup (B - A) = \{1,2,3,6,7,8\}$$



$$A - B = \{1,2,3\}$$



$$B - A = \{6,7,8\}$$

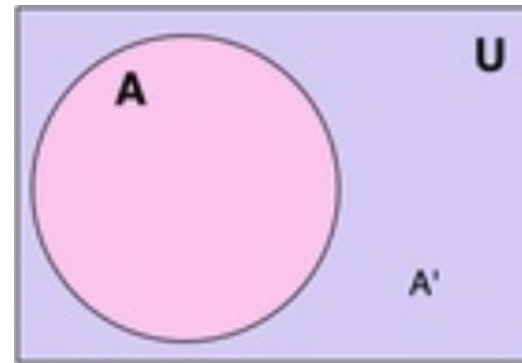
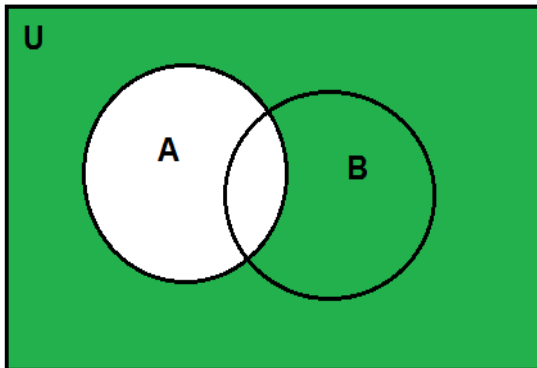
Complement

The complement of a set A with respect to a universal set U , denoted by A' is defined to be

$$A' = \{x \in U \mid x \notin A\}$$

$$A' = U - A$$

Venn diagram of A'



Example

Let U be a universal set,

$$U = \{ 1, 2, 3, 4, 5, 6, 7 \}$$

$$A = \{ 2, 4, 6 \}$$

$$A' = U - A = \{ 1, 3, 5, 7 \}$$

Exercise

Let,

$$U = \{ a, b, c, d, e, f, g, h, i, j, k, l, m \}$$

$$A = \{ a, c, f, m \}$$

$$B = \{ b, c, g, h, m \}$$

Find:

$$A \cup B, A \cap B, |A \cup B|, A - B \text{ dan } A'.$$

Exercise

Let the universe be the set $U=\{1, 2, 3, 4, \dots, 10\}$.

Let $A=\{1, 4, 7, 10\}$, $B=\{1, 2, 3, 4, 5\}$ and $C=\{2, 4, 6, 8\}$.

List the elements of each set:

- a) U'
- b) $B' \cap (C - A)$
- c) $B - A$
- d) $(A \cup B) \cap (C - B)$

Set Identities (Properties of Set)

- **Commutative laws**

$$A \cap B = B \cap A, \quad A \cup B = B \cup A$$

- **Associative laws**

$$A \cap (B \cap C) = (A \cap B) \cap C$$

$$A \cup (B \cup C) = (A \cup B) \cup C$$

- **Distributive laws**

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Set Identities (Properties of Set)

▪ Absorption laws

$$A \cup (A \cap B) = A$$

$$A \cap (A \cup B) = A$$

▪ Idempotent laws

$$A \cap A = A, \quad A \cup A = A$$

▪ De Morgan's laws

$$(A \cap B)' = A' \cup B'$$

$$(A \cup B)' = A' \cap B'$$

Set Identities (Properties of Set)

- **Complement laws**

$$A \cap A' = \emptyset \qquad A \cup A' = U$$

- **Double complement laws**

$$(A')' = A$$

- **Complement of U and \emptyset**

$$\emptyset' = U \qquad U' = \emptyset$$

Set Identities (Properties of Set)

- **Properties of universal set**

$$A \cup U = U \quad A \cap U = A$$

- **Set difference laws**

$$A - B = A \cap B'$$

- **Identity laws**

$$A \cup \emptyset = A \quad A \cap U = A$$

- **Properties of empty set**

$$A \cup \emptyset = A \quad A \cap \emptyset = \emptyset$$

Example

- Let A , B and C denote the subsets of a set S and let C' denote a complement of C in S .
- If $A \cap C = B \cap C$ and $A \cap C' = B \cap C'$, then prove that $A = B$

$$\begin{aligned} A &= A \cap S \\ &= A \cap (C \cup C') \\ &= (A \cap C) \cup (A \cap C') && \text{(distributive)} \\ &= (B \cap C) \cup (B \cap C') && \text{(the given conditions)} \\ &= B \cap (C \cup C') && \text{(distributive)} \\ &= B \cap S \\ &= B \end{aligned}$$

Example

By referring to the properties of set operations, show that:

$$A - (A \cap B) = A - B$$

set difference

$$A - B = A \cap B'$$

$$\begin{aligned} A - (A \cap B) &= A \cap (A \cap B)' \\ &= A \cap (A' \cup B') \\ &= (A \cap A') \cup (A \cap B') \\ &= \emptyset \cup (A \cap B') \\ &= (A \cap B') \cup \emptyset \\ &= A \cap B' \\ &= A - B \end{aligned}$$

[set difference laws]

[De Morgan's laws]

[distributive laws]

[complement laws]

[commutative]

[Identity laws]

Exercise

1) Let A , B and C be sets. Show that

$$(A \cup (B \cap C))' = A' \cap (B' \cup C')$$

2) Let A , B and C be sets such that

$$A \cap B = A \cap C \text{ and } A \cup B = A \cup C$$

Prove that $B = C$

$$B = B \cap (B \cup A)$$

$$= (B \cap B) \cup (B \cap A)$$

$$= B \cup (B \cap A)$$

$$= B \cup (C \cap A)$$

$$= (B \cup C) \cap (B \cup A)$$

$$= (B \cup C) \cap (C \cup A)$$

$$= C \cup (B \cap A)$$

$$= C \cup (C \cap A)$$

$$= C$$

(absorption law)

(distributive law)

(Idempotent law)

(given $A \cap B = A \cap C$)

(given $A \cup B = A \cup C$)

(distributive law)

(given $A \cap B = A \cap C$)

(absorption law)

The ***union*** of a collection of sets is the set that contains those elements that are members of **at least one set** in the collection.

Notation:

$$\bigcup_{i=1}^n A_i = A_1 \cup A_2 \cup \dots \cup A_n = \{x \in U \mid x \in A_i \text{ for at least one } i = 0, 1, 2, \dots, n\}$$

$$\bigcup_{i=1}^{\infty} A_i = A_1 \cup A_2 \cup \dots \cup A_n = \{x \in U \mid x \in A_i \text{ for at least one nonnegative integer } i\}$$

The ***intersection*** of a collection of sets is the set that contains those elements that are members of **all the sets** in the collection.

Notation:

$$\bigcap_{i=1}^n A_i = A_1 \cap A_2 \cap \dots \cap A_n = \{x \in U \mid x \in A_i \text{ for all } i = 0, 1, 2, \dots, n\}$$

$$\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap \dots \cap A_n = \{x \in U \mid x \in A_i \text{ for all nonnegative integer } i\}$$

Example

For $i = 1, 2, \dots$, let $A_i = \{i, i+1, i+2, \dots\}$. Then,

$$\bigcup_{i=1}^n A_i = \bigcup_{i=1}^n \{i, i+1, i+2, \dots\} = \{1, 2, 3, \dots\},$$

and

$$\bigcap_{i=1}^n A_i = \bigcap_{i=1}^n \{i, i+1, i+2, \dots\} = \{n, n+1, n+2, \dots\} = A_n$$

Cartesian Product

- Let A and B be sets. An **ordered pair** of elements $a \in A$ dan $b \in B$ written (a, b) is a listing of the elements a and b in a specific order.
- The ordered pair (a, b) specifies that a is the first element and b is the second element.
- An ordered pair (a, b) is considered distinct from ordered pair (b, a) , unless $a=b$.

Example $(1, 2) \neq (2, 1)$

Cartesian Product

- The Cartesian product of two sets A and B , written $A \times B$ is the set,

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

- For any set A ,

$$A \times \emptyset = \emptyset \times A = \emptyset$$

Example

$$A = \{a, b\}, B = \{1, 2\}.$$

$$A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$$

$$B \times A = \{(1, a), (1, b), (2, a), (2, b)\}$$

Cartesian Product

- if $A \neq B$, then $A \times B \neq B \times A$.
- if $|A| = m$ and $|B| = n$, then $|A \times B| = mn$.

Example $A = \{1, 3\}$, $B = \{2, 4, 6\}$.

$$A \times B = \{(1, 2), (1, 4), (1, 6), (3, 2), (3, 4), (3, 6)\}$$

$$B \times A = \{(2, 1), (2, 3), (4, 1), (4, 3), (6, 1), (6, 3)\}$$

$$A \neq B, A \times B \neq B \times A$$

$$|A| = 2, |B| = 3, |A \times B| = 2 \cdot 3 = 6.$$

Cartesian Product

- The Cartesian product of sets A_1, A_2, \dots, A_n is defined to be the set of all n -tuples

(a_1, a_2, \dots, a_n) where $a_i \in A_i$ for $i=1, \dots, n$;

- It is denoted $A_1 \times A_2 \times \dots \times A_n$

$$|A_1 \times A_2 \times \dots \times A_n| = |A_1| \cdot |A_2| \cdot \dots \cdot |A_n|$$

Example

$$A = \{a, b\}, B = \{1, 2\}, C = \{x, y\}$$

$$A \times B \times C = \{(a, 1, x), (a, 1, y), (a, 2, x), (a, 2, y), \\ (b, 1, x), (b, 1, y), (b, 2, x), (b, 2, y)\}$$

$$|A \times B \times C| = 2 \cdot 2 \cdot 2 = 8$$

Exercise

Let $A = \{w, x\}$, $B = \{1, 2\}$ and $C = \{nm, ds, ps\}$.

- 1) Find $|A \times B|$, $|B \times C|$, $|A \times C|$, $|A \times B \times C|$, $|B \times C \times A|$, $|A \times B \times A \times C|$
- 2) Determine the following set,
 - a) $A \times B$, $B \times C$, $A \times C$
 - b) $A \times B \times C$
 - c) $B \times C \times A$
 - d) $A \times B \times A \times C$

Exercise

Let $X = \{1, 2\}$, $Y = \{a\}$ and $Z = \{b, d\}$.

List the elements of each set.

- a) $X \times Y$
- b) $Y \times X$
- c) $X \times Y \times Z$
- d) $X \times Y \times Y$
- e) $X \times X \times X$
- f) $Y \times X \times Y \times Z$



Exercise

1. Let the universal set be the set Z of all integer numbers and let $A = \{x \in Z | 0 < x \leq 2\}$, $B = \{x \in Z | 1 \leq x < 4\}$, and $C = \{x \in Z | 3 \leq x < 9\}$. Find each of the following:
 - a) $A \cup B$
 - b) $A \cap B$
 - c) $(A \cup B) \cap C$
 - d) $(A \cup B) \cap (A \cup C)$
 - e) $|A \cup C|$
 - f) $B - C$
 - g) $A \times (A \cup B)$
 - h) $P(B)$
2. In a classroom of 44 students, there are 20 of them who are taking a Language exam and 18 who are taking a Maths exam. If there are 10 of them who are taking a Language exam but not a Maths one:
 - a) How many of them do not take any exam?
 - b) How many are taking any of the two exams?
 - c) How many are taking both exams?

Thank You