

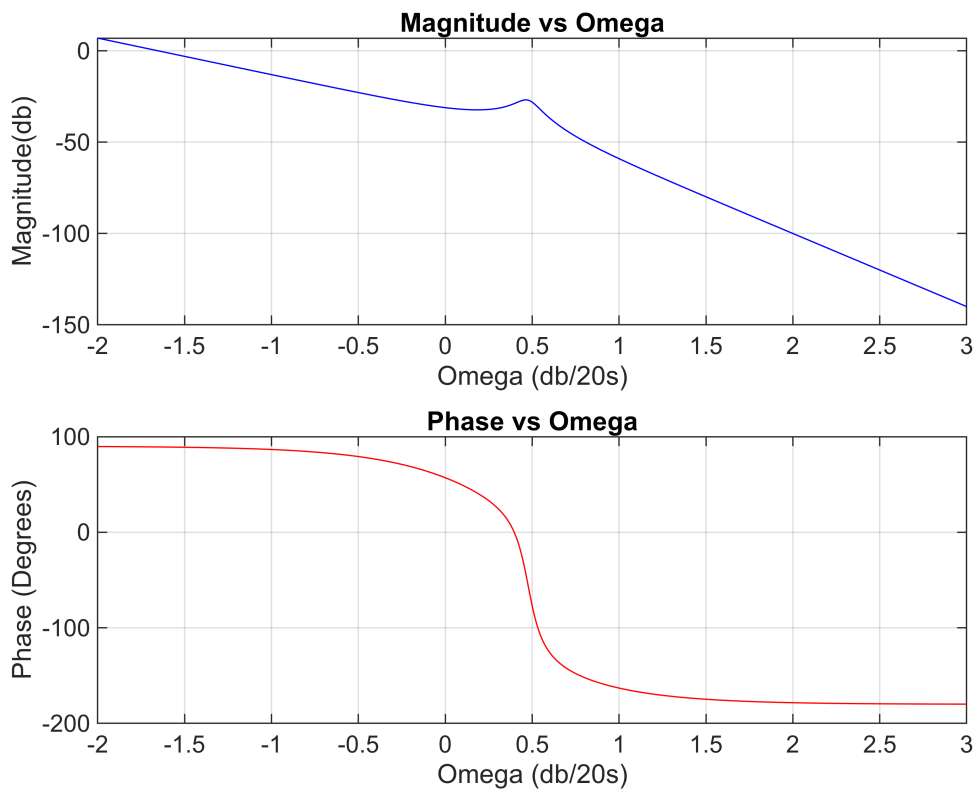
1. Bode extraction

1.1. Load the data

```
load('Data.mat');  
omega = log10(Data.omega);  
magnitude = 20.*log10(Data.magnitude);  
phase = Data.phase ;
```

1.2. Plot

```
figure;  
subplot(2,1,1);  
plot(omega, magnitude, 'b'); % Plot Magnitude vs Omega  
xlabel('Omega (db/20s)');  
ylabel('Magnitude(db)');  
title('Magnitude vs Omega');  
grid on;  
subplot(2,1,2);  
plot(omega, phase, 'r'); % Plot Phase vs Omega  
xlabel('Omega (db/20s)');  
ylabel('Phase (Degrees)');  
title('Phase vs Omega');  
grid on;
```



2. System identification

2.1. System type

As the slope in the Magnitude part is -20dB/dec, we assume that we have a pole in the Origin so obviously the answer to step function should be "0" and the system is type 1.

2.2. System order

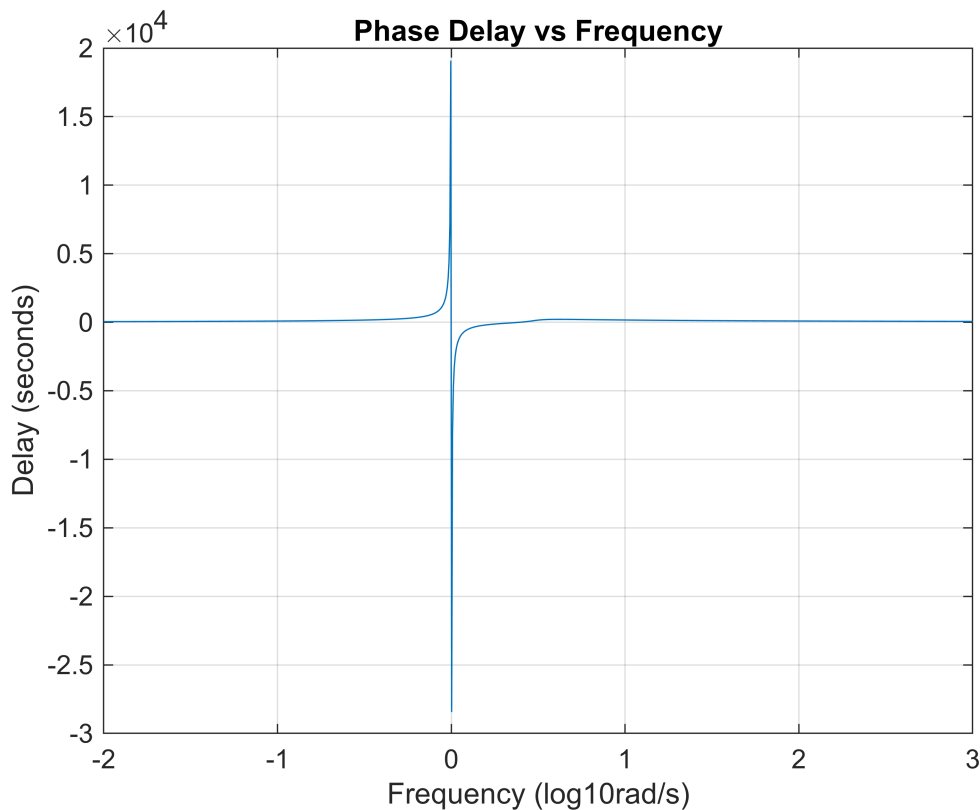
As explained in section 2.4 we have 3 poles so the system is from the third order.

2.3. System delay

To estimate the delay τ , you can calculate the slope of the phase curve at different frequencies.

```
delay = -phase ./ omega; % Phase delay at each frequency (in seconds)
```

```
figure;  
plot(omega, delay) % Plot delay vs frequency (log scale for omega)  
xlabel('Frequency (log10rad/s)');  
ylabel('Delay (seconds)');  
grid on;  
title('Phase Delay vs Frequency');
```



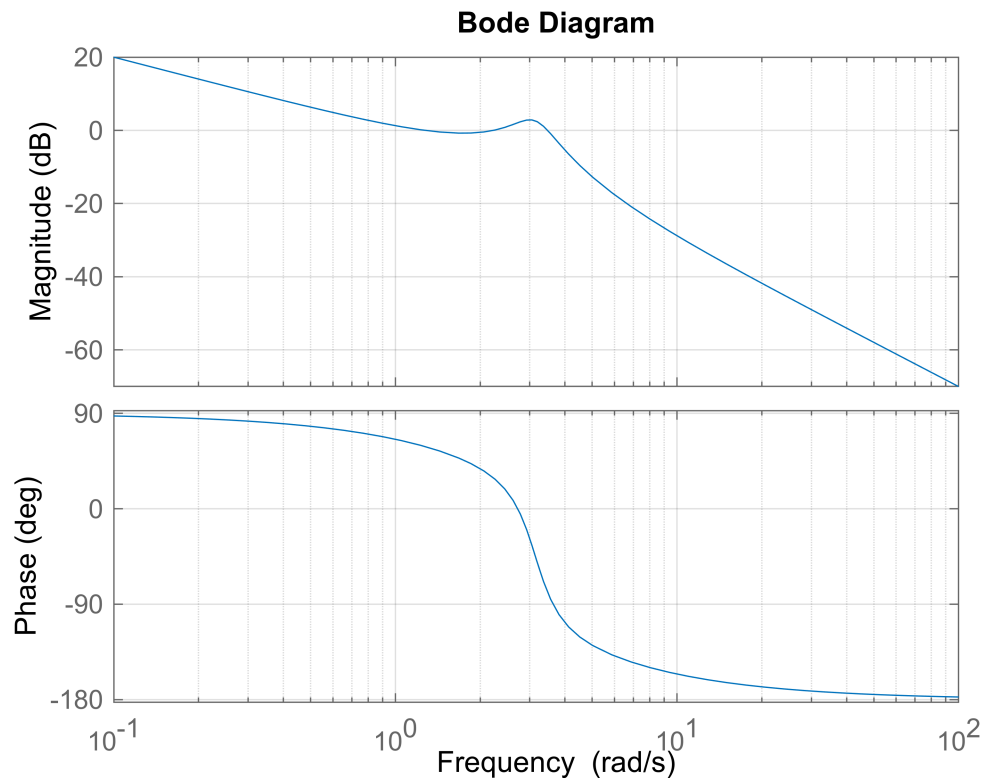
2.4. Minimum phase

As it can be seen in the Bode diagram, we start the magnitude with a -20dB/dec slope and a phase of 90 which shows that we have a "-s" and a second order func pole with positive middle coefficient and a zero at the same frequency, So it has a minimum phase zero.

3. Transfer function

The estimated transfer function is gonna be like the following

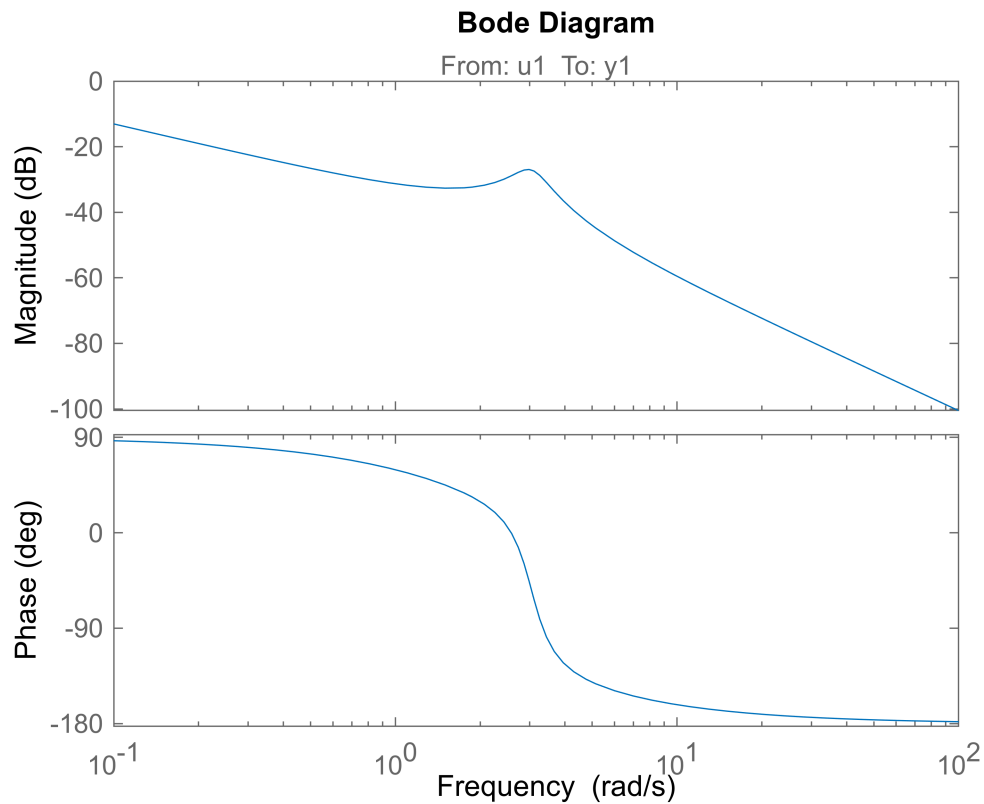
```
num = [1/pi -1];  
den = conv([1/(pi^2) 1/(3*pi) 1], [1 0]);  
sys = tf(num,den);  
bode(sys);  
grid on;
```



which actually looks the same as the upper expresion.

3.1. Optional first

```
systemIdentification;  
bode(tf1);
```



tf1

```
tf1 =
  From input "u1" to output "y1":
    0.09401 s - 0.2059
  -----
  s^3 + 0.861 s^2 + 9.266 s - 8.868e-07
```

Name: tf1
Continuous-time identified transfer function.

Parameterization:
Number of poles: 3 Number of zeros: 1
Number of free coefficients: 5
Use "tfdata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using TFEST on frequency response data "mydata".
Fit to estimation data: 99.82%
FPE: 7.3e-07, MSE: 7.196e-07

Model Properties

4. Ruth hurwitz

```
syms s;

% Define the characteristic equation
char_eq = (1/(pi^2)) * s^3 + (1/(3*pi)) * s^2 + 1 * s;
```

```
% Get the coefficients as a numeric vector
coeffs = double(sym2poly(char_eq)); % Ensure numeric values

% Compute the Routh array
routh_table = routh_hurwitz(coeffs);

% Display Routh-Hurwitz Table in Live Script
disp('Routh-Hurwitz Table:');
```

Routh-Hurwitz Table:

```
disp(routh_table);
```

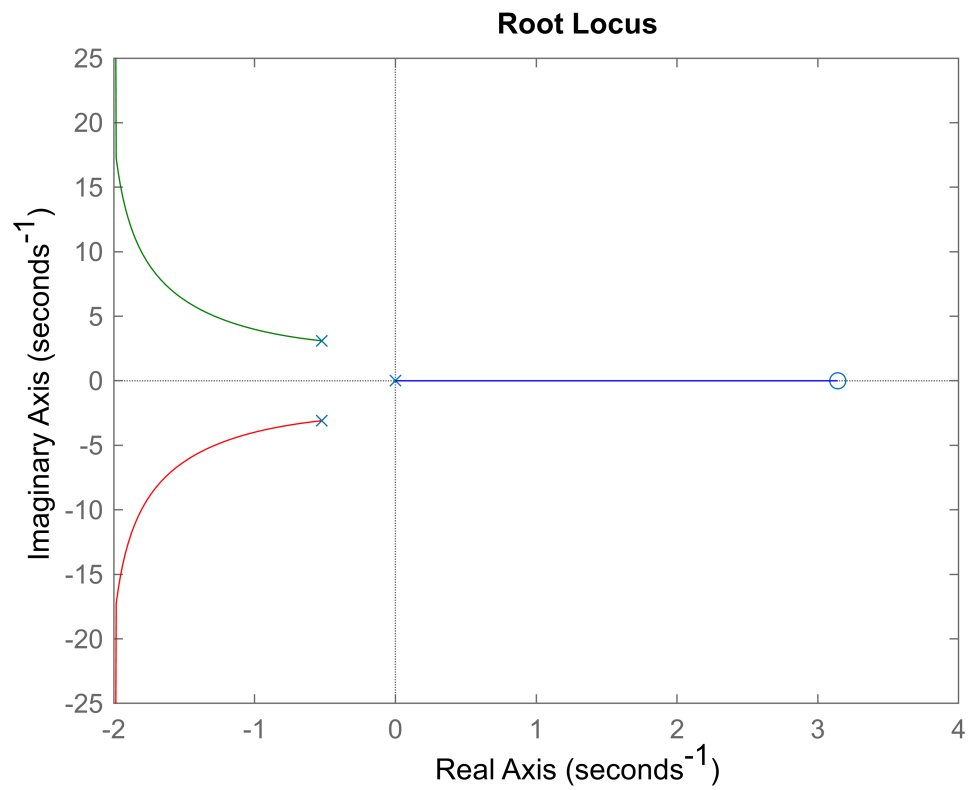
```
0.1013    1.0000
0.1061         0
1.0000         0
0           0
```

I tried to do the computing completely on matlab but I couldn't figure it out, the computing goes on by writing the characteristic equation considering K and being close loop. It converts to $\frac{s^3}{\pi^2} + \frac{s^2}{3\pi} + (1+k)s - k\pi$ which translates to the Routh's table's first column to be $\frac{1}{\pi^2} \quad \frac{1}{3\pi} \quad (1+4k) \quad -k\pi$ which means that k is to be $-\frac{1}{4} < k < 0$

5. Routh locus analysis

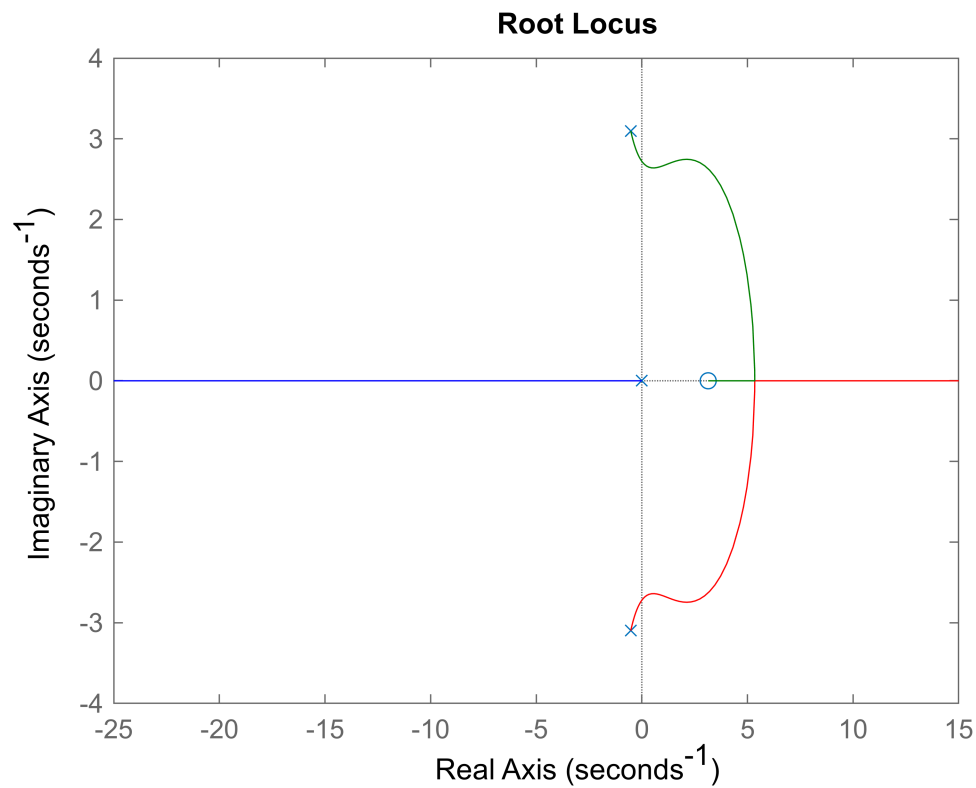
5.1. Plotting

```
rlocus(sys);
```



As seen, we have a complete branch in the ORHP which means that we can't manage stabilizing the system using only the k coefficient.

```
rlocus(-sys);
```

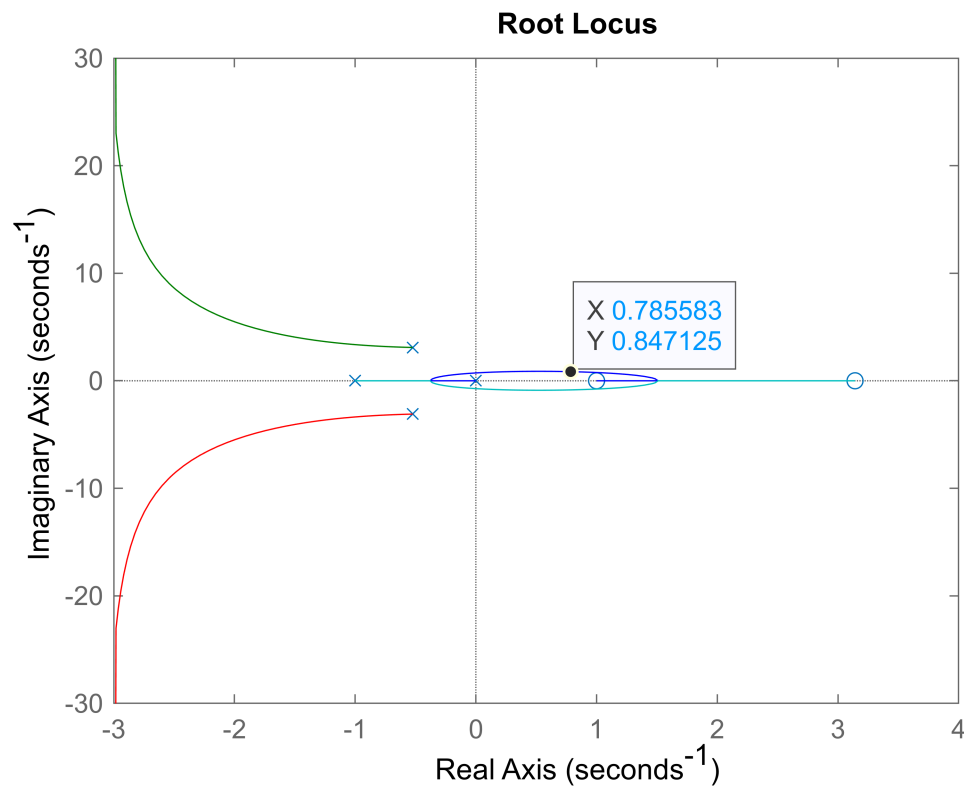


Working with the negative K_s , we have a really small gap in the branches to work with which actually matches what we saw in ruth's method.

5.2. Configure a PI or PD controller

We can add another minimum phase zero and another pole in the PLHP so that it leads to appearing a new branch moving from the OLHP to ORHP so that we find the chance to consider areas for K to bring all the poles in the OLHP so it shapes a PID.

```
num = conv([1/pi -1] , [1 -1]);
den = conv([1/(pi^2) 1/(3*pi) 1], [1 1 0]);
sys_rc = tf(num,den);
rlocus(sys_rc);
```



As you see now we have a little area in positive Ks to bring all the poles in the OLHP and make the system stable.

6. Control overshoot

Our closed loop transfer function will be $\frac{k\pi^2}{s^2 + \frac{\pi}{3}s + (k+1)\pi^2}$ which is a shape of $\frac{k\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$ and as

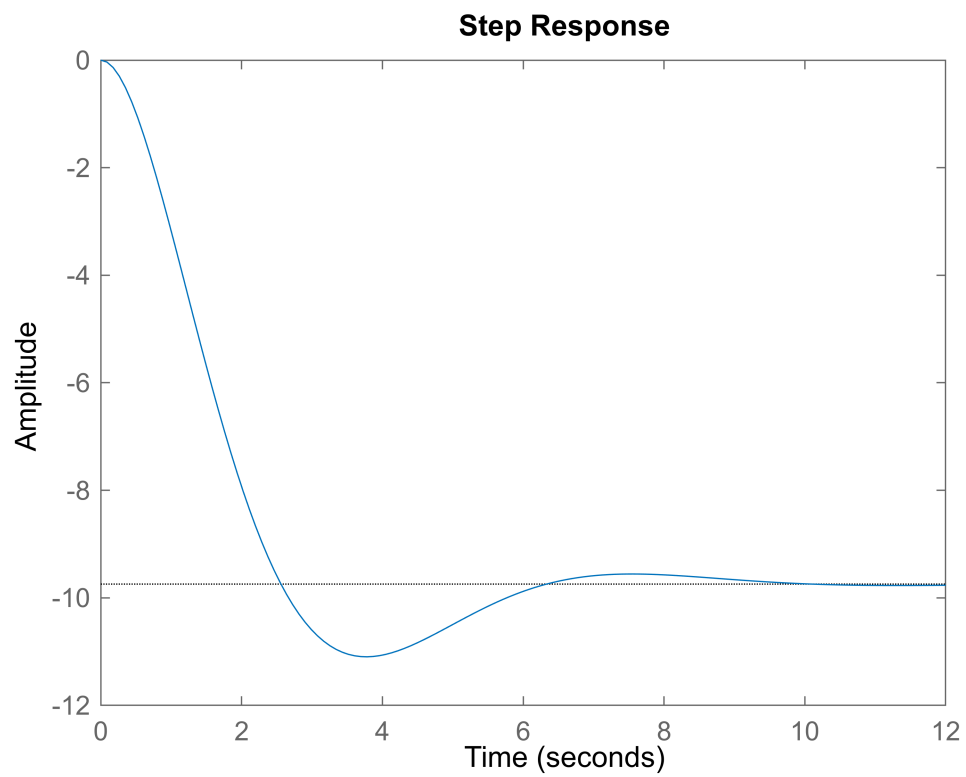
$M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$ it concludes that $3.96 < \xi < 5.8$ and settling time with a $\delta = 2\%$ is equal to $\frac{4}{\xi\omega_n}$ so we conclude

that $\xi > \frac{4}{10\pi\sqrt{k+1}}$ and we can consider $-\frac{63}{64} > k > -\frac{3339}{3364}$ and we can find a k between these using an old rule :)))

$$k = -\frac{63 + 3339}{64 + 3364}$$

```
num = -9.432;
den = [1 pi/3 0.968];
sys_c = tf(num,den);
step(sys_c);
```

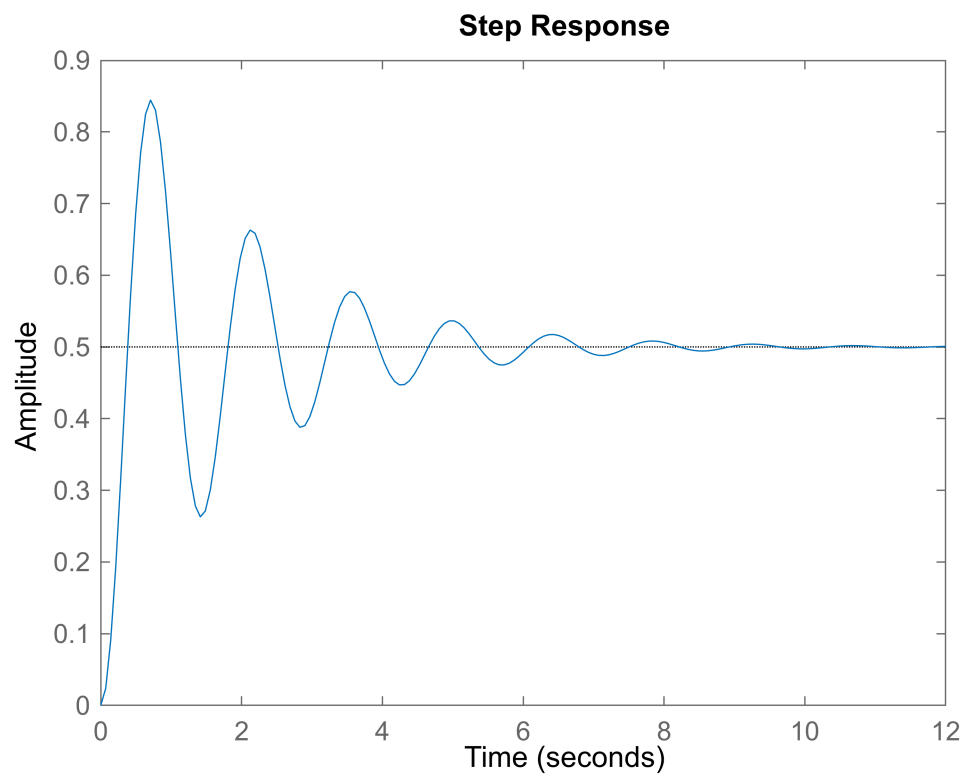
% a little help from instincts :)



```
stepinfo(sys_c)
```

```
ans = struct with fields:
    RiseTime: 1.7318
    TransientTime: 5.8792
    SettlingTime: 5.8792
    SettlingMin: -11.0961
    SettlingMax: -8.9610
    Overshoot: 13.8785
    Undershoot: 0
    Peak: 11.0961
    PeakTime: 3.7819
```

```
num = pi^2;
den = [1 pi/3 2*pi^2];
sys_uc = tf(num,den);
step(sys_uc);
```



```
stepinfo(sys_uc)
```

```
ans = struct with fields:  
    RiseTime: 0.2575  
    TransientTime: 7.2524  
    SettlingTime: 7.2524  
    SettlingMin: 0.2630  
    SettlingMax: 0.8443  
    Overshoot: 68.8609  
    Undershoot: 0  
    Peak: 0.8443  
    PeakTime: 0.7071
```

7.Controller design

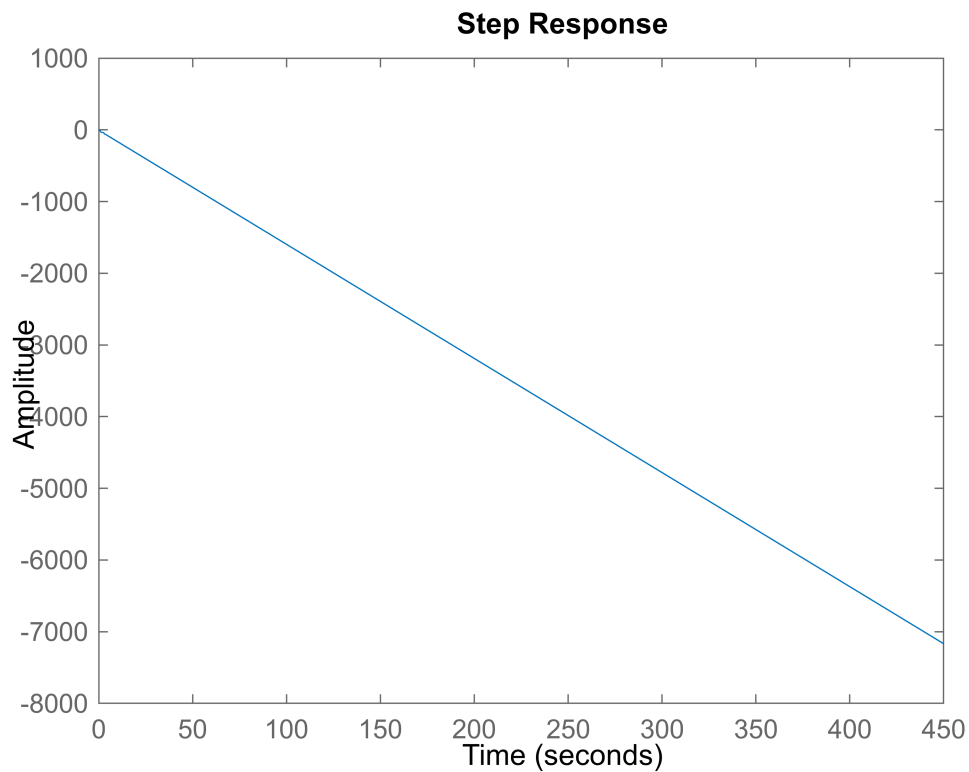
7.1.Maximum of 2% for e_{sr}

$\frac{1}{s}S(s) = \frac{1}{s^2(1+L(s))}$ so $y(s) = \frac{1}{s^3(1+L(s))}$ and the $e_{sr} = \lim_{s \rightarrow 0} \frac{1}{s \left(1 + \frac{s-\pi}{s \left(\frac{s^2}{\pi^2} + \frac{s}{3\pi} + 1 \right)} \right)} = -\frac{1}{\pi} = \frac{1}{k_v} < \frac{1}{50}$ so we need a

$\frac{50}{\pi}$ gain to reach the wanted result.

Also we add a zero at origin.

```
K = (50/pi) -1;  
T = (1/25)*(((K/0.05)^2)-1)^0.5;  
num = conv([T 50/pi] , [1/pi -1 0]);  
den = conv([1/(pi^2) 1/(3*pi) 1] , [1 0 0]); %having ramp response is equal to adding  
sys_fc = tf(num,den); %a pole at origin then computing the step  
step(sys_fc); %response.
```



Thanks for your attention!

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