Imports

```
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np
```

Data Processing

```
# Import Data
df = pd.read_csv('car_launch_data_5.txt', header=1, delim_whitespace=True)


# Clean data
# Recenter y data; Translate Y data, shift lowest y to 0
m = min(df['y'])
df['y'] = df['y'].apply(lambda x: x-m)

# Make t start at 0
# t_offset = min(df['x'])
# df['t'] = df['t'].apply(lambda x: x-t_offset)

print(df)
```

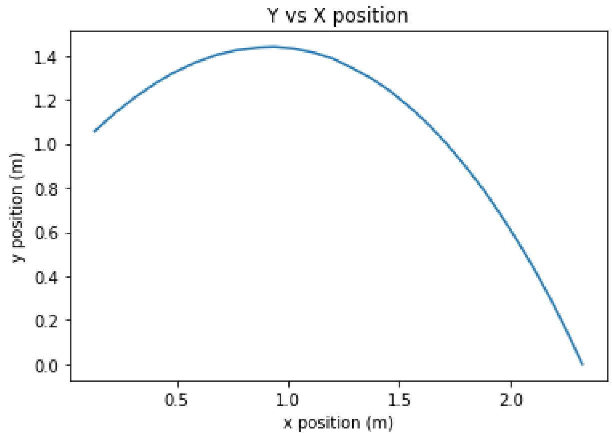
	t	x	y
0	0.000	0.132	1.057
1	0.033	0.223	1.140
2	0.067	0.317	1.214
3	0.100	0.411	1.278
4	0.133	0.480	1.319
5	0.167	0.585	1.369
6	0.200	0.678	1.403
7	0.233	0.770	1.425
8	0.267	0.872	1.437
9	0.300	0.938	1.440
10	0.333	1.027	1.432
11	0.367	1.115	1.414
12	0.400	1.201	1.387
13	0.433	1.290	1.343
14	0.467	1.376	1.296
15	0.500	1.461	1.238
16	0.533	1.564	1.152
17	0.567	1.634	1.085
18	0.600	1.715	0.998
19	0.633	1.803	0.891
20	0.667	1.881	0.789
21	0.700	1.957	0.677
22	0.733	2.033	0.557
23	0.767	2.108	0.430
24	0.800	2.181	0.294
25	0.833	2.253	0.150
26	0.867	2.323	0.000

Initial Plots

```
# Initial visualization of the data
plt.plot(df['x'], df['y'])
plt.xlabel("x position (m)")
```

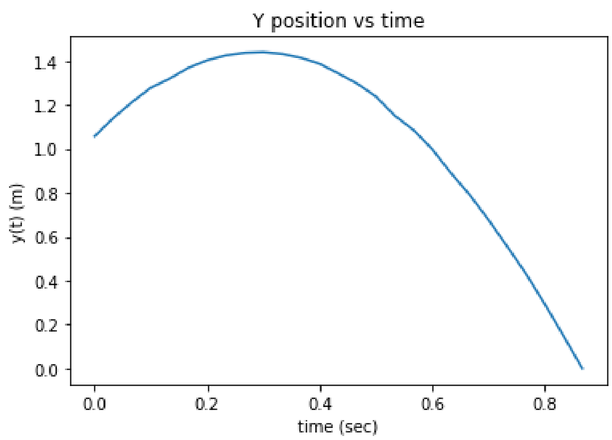
```
plt.ylabel("y position (m)")
```

```
Text(0.5, 1.0, 'Y vs X position')
```



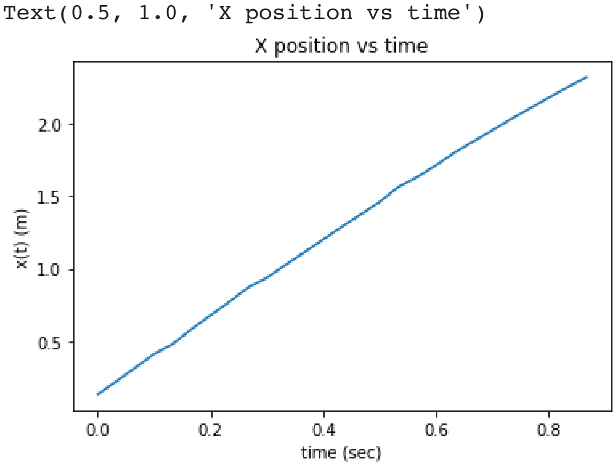

```
plt.plot(df['t'], df['y'])
plt.xlabel("time (sec)")
plt.ylabel("y(t) (m)")
plt.title("Y position vs time")
```

```
Text(0.5, 1.0, 'Y position vs time')
```




```
plt.plot(df['t'], df['x'])
plt.xlabel("time (sec)")
plt.ylabel("x(t) (m)")
plt.title("X position vs time")
```

```
Text(0.5, 1.0, 'X position vs time')
```



Simulation Helpers

```
# Constants
g   = 9.8               # gravity (m/s^2)

t0  = 0                 # initial Time (s)
dt  = .033              # time resolution

tf  = 0.867             # total time to simulate
nsteps = int(tf/dt) + 1 # number of time steps
```

```
# Calculate inital conditions based on two given (early) coordinate points
def initial_conditions(coord1, coord2):
    t    = coord2[0] - coord1[0]
    x    = coord2[1] - coord1[1]
    y    = coord2[2] - coord1[2]
    return [x/t, y/t, coord1[1], coord1[2]]


```

Euler Method

```
# Initial Conditions

# Getting initial velocity components
i_coord = initial_conditions(df.loc[0], df.loc[1])

print("X vel:", i_coord[0], "m/s")
print("Y vel:", i_coord[1], "m/s")
print("X pos:", i_coord[2], "m")
print("Y pos:", i_coord[3], "m")

y0  = i_coord[3]        # initial y (m)
x0  = i_coord[2]        # initial x (m)

vy0 = i_coord[1]        # iniital y velocity (m/s)
vx0 = i_coord[0]        # initial x velocity (m/s)
```

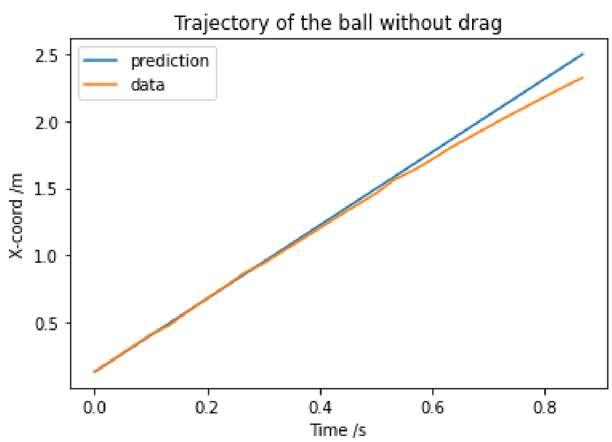
```
X vel: 2.7575757575757573 m/s
Y vel: 2.515151515151514 m/s
X pos: 0.132 m
Y pos: 1.057 m
```

```
# Euler method setting up - time evolution
t = np.linspace(t0, tf, nsteps)

vx = np.zeros([nsteps])
vy = np.zeros([nsteps])
y  = np.zeros([nsteps])
x  = np.zeros([nsteps])

y[0]  = y0
x[0]  = x0
vy[0] = vy0
vx[0] = vx0

## time evolving
```

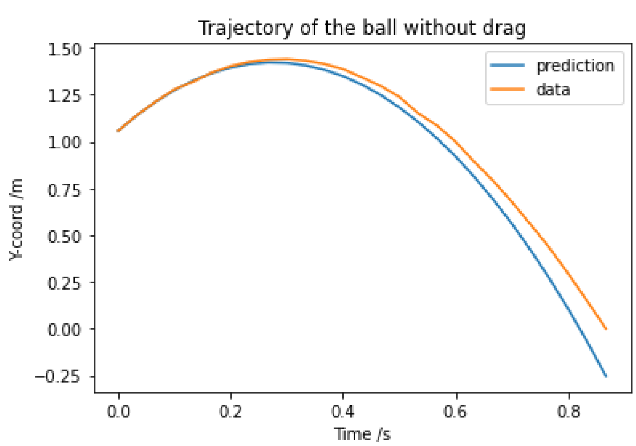
```
for i in range(0, nsteps-1):
    y[i+1]  = y[i] + dt * vy[i]
    vy[i+1] = vy[i] - dt * g

    x[i+1]  = x[i] + dt * vx[i]
    vx[i+1] = vx[i]
```

Prediction Plots

```
plt.plot(t, x, label="prediction")
plt.plot(df['t'], df['x'], label="data")
plt.plot()
plt.title('Trajectory of the ball without drag')
plt.xlabel('Time /s')
plt.ylabel('X-coord /m')
plt.legend()
plt.show()
```



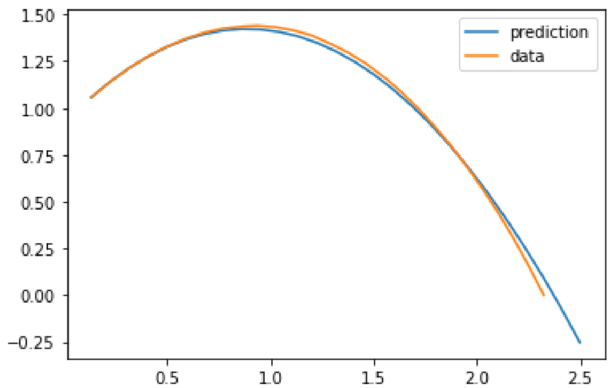

```
plt.plot(t, y, label="prediction")
plt.plot(df['t'], df['y'], label = "data")
plt.title('Trajectory of the ball without drag')
plt.xlabel('Time /s')
plt.ylabel('Y-coord /m')
plt.legend()
plt.show()
```




```
plt.plot(x,y, label="prediction")
plt.plot(df['x'], df['y'], label="data")
```

```
plt.legend()
```

```
<matplotlib.legend.Legend at 0x7f608ced79a0>
```



Error

```
def err(actual, expected):
    if not len(actual) == len(expected):
        return "number of data points do not match"
    else:
        return [round(abs(actual[i] - expected[i]), 5) if actual[i] > 0 and expected[i] > 0 else 0 for
```

```
x_data = df['x'].values
x_pred = x

y_data = df['y'].values
y_pred = y

print(len(x_data), len(x_pred))
print(x_data[-1], x_pred[-1])

print(len(y_data), len(y_pred))
print(y_data[-1], y_pred[-1])
```

```
27 27
2.323 2.4980000000000007
27 27
0.0 -0.2534650000000009
```

```
x_err = err(x_data, x_pred)
y_err = err(y_data, y_pred)
print(y_err)
```

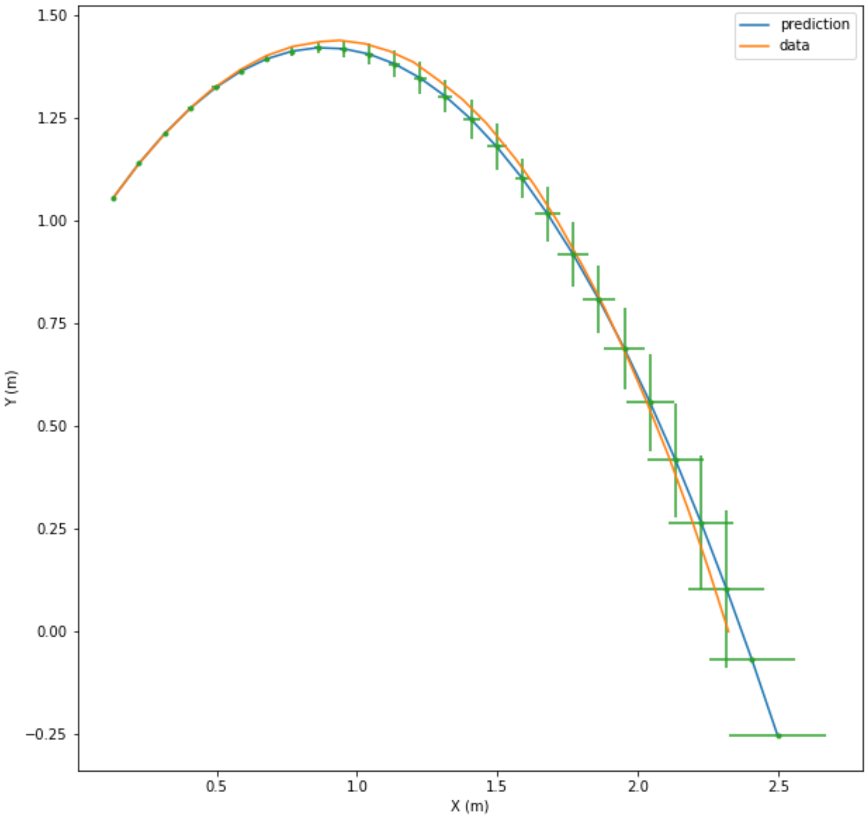
```
[0.0, 0.0, 0.00167, 0.00402, 0.00597, 0.00372, 0.00808, 0.01112, 0.01482, 0.0202, 0.02525, 0.03097, 0.
```

Error Plots

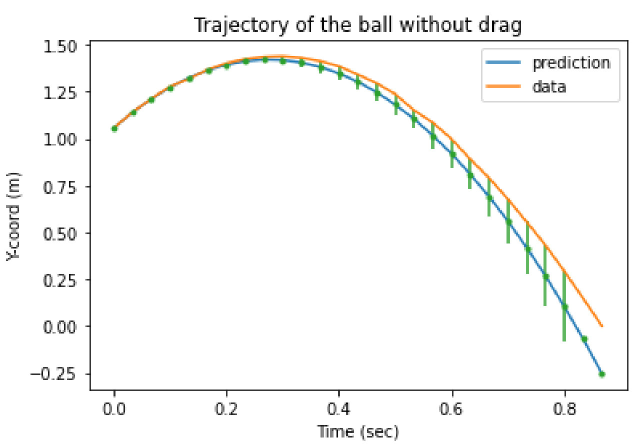
```
fig = plt.figure(figsize=(10, 10))
plt.plot(x,y, label="prediction")
plt.plot(df['x'], df['y'], label="data")
plt.errorbar(x, y,
             xerr = x_err,
             yerr = y_err,
             fmt='.')
plt.xlabel('X (m)')
```

```
plt.ylabel('Y (m)')

plt.legend()
```

```
<matplotlib.legend.Legend at 0x7f608ce46d00>
```

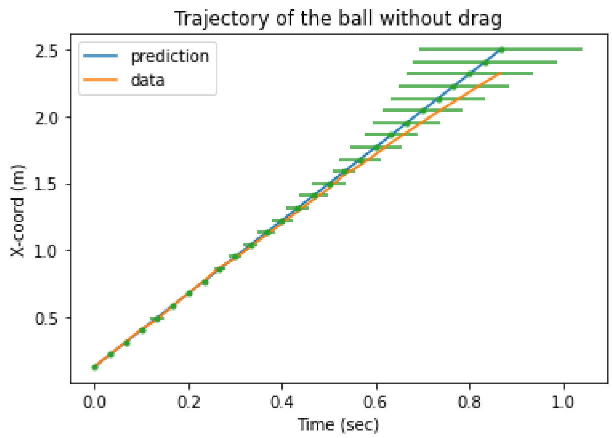



```
plt.plot(t, y, label="prediction")
plt.plot(df['t'], df['y'], label = "data")
plt.title('Trajectory of the ball without drag')
plt.xlabel('Time (sec)')
plt.ylabel('Y-coord (m)')
plt.legend()
plt.errorbar(t, y, yerr=y_err, fmt='.')
plt.show()
```




```
plt.plot(t, x, label="prediction")
plt.plot(df['t'], df['x'], label="data")
```

```
plt.plot()
plt.title('Trajectory of the ball without drag')
plt.xlabel('Time (sec)')
plt.ylabel('X-coord (m)')
plt.errorbar(t, x, xerr=x_err, fmt='.')
plt.legend()
plt.show()
```



Bashforth-Adams

The bashforth-adams algorithm is apparently an improvement on the Euler algorithm so we will make an attempt at comparing them here

For a given position component, we have:

$$y_{n+1} = y_n + \frac{3}{2}\delta t f\left(t_n, y_n\right) - \frac{1}{2}\delta t f\left(t_{n-1}, y_{n-1}\right)$$

It seems that most people are defining the function f as the derivative of the position component.

```
# BA Initial Conditions

i_coord0 = initial_conditions(df.loc[0], df.loc[1])
i_coord1 = initial_conditions(df.loc[1], df.loc[2])

y0  = i_coord0[3]         # initial y (m)
x0  = i_coord0[2]         # initial x (m)
vy0 = i_coord0[1]         # iniital y velocity (m/s)
vx0 = i_coord0[0]         # initial x velocity (m/s)

y1  = i_coord1[3]         # initial +1 y (m)
x1  = i_coord1[2]         # initial +1 x (m)
vy1 = i_coord1[1]         # iniital +1 y velocity (m/s)
vx1 = i_coord1[0]         # initial +1 x velocity (m/s)

print(i_coord0,i_coord1)

```

```
[2.7575757575757573, 2.515151515151514, 0.132, 1.057] [2.764705882352941, 2.1764705882352957, 0.223, 1
```

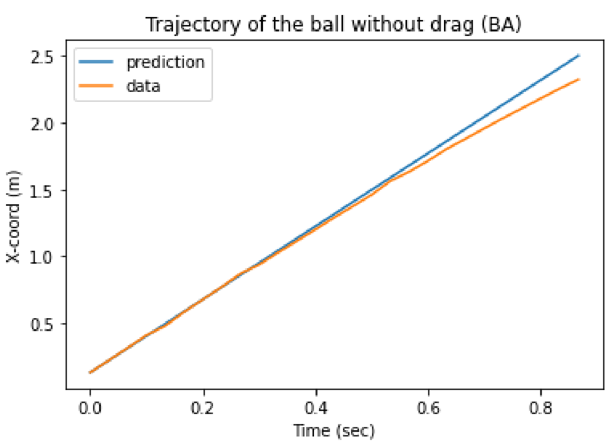
```
# Bashforth-Adams method setting up - time evolution
t = np.linspace(t0, tf, nsteps)

```

```
vy_ba = np.zeros([nsteps])
vx_ba = np.zeros([nsteps])
y_ba  = np.zeros([nsteps])
x_ba  = np.zeros([nsteps])

# Init time 0 and 1
y_ba[0]  = y0
x_ba[0]  = x0
y_ba[1]  = y1
x_ba[1]  = x1
vy_ba[0] = vy0
vx_ba[0] = vx0
vy_ba[1] = vy1
vx_ba[1] = vx1
```

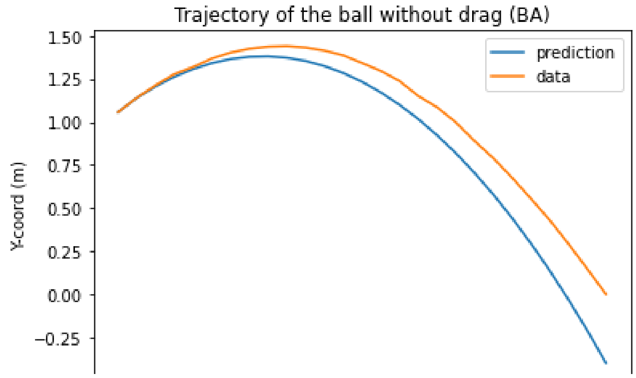
```
# Time evolving
for i in range(0, nsteps-2):
    y_ba[i+2]  = y_ba[i+1] + dt * (3/2*vy_ba[i+1] - 1/2*vy_ba[i])
    vy_ba[i+2] = vy_ba[i+1] - dt * g

    x_ba[i+2]  = x_ba[i+1] + dt * (3/2*vx_ba[i+1] - 1/2*vx_ba[i])
    vx_ba[i+2] = vx_ba[i]

```

Prediction Plots

```
plt.plot(t, x_ba, label="prediction")
plt.plot(df['t'], df['x'], label="data")
plt.plot()
plt.title('Trajectory of the ball without drag (BA)')
plt.xlabel('Time (sec)')
plt.ylabel('X-coord (m)')
plt.legend()
plt.show()
```




```
plt.plot(t, y_ba, label="prediction")
plt.plot(df['t'], df['y'], label = "data")
plt.title('Trajectory of the ball without drag (BA)')
plt.xlabel('Time (sec)')
plt.ylabel('Y-coord (m)')
plt.legend()
plt.show()
```




```
plt.plot(x_ba,y_ba, label="prediction")
plt.plot(df['x'], df['y'], label="data")
plt.xlabel("x (m)")
plt.ylabel("y (m)")
plt.title("Y vs X position")
plt.legend()
```

```
<matplotlib.legend.Legend at 0x7f608cea7640>
```