t-J interladder conductivity for 4 holes on 10 rungs

current operator

There are 4 relevant hopping terms, $t_{\perp 2}, t_{\perp 3}, t_{\perp 5}, t_{\perp 7}$. The current operator for $t_{\perp 2}$ term can be written as

$$j_2 \propto \mathrm{i} t_{\perp 2} \sum_i \left[-c_{ui}^{1\dagger} \left(c_{\ell i}^2 + c_{\ell i+1}^2 \right) + c_{\ell i}^{2\dagger} \left(c_{ui}^1 + c_{ui-1}^1 \right) \right] \tag{1}$$

The two ladders are labeled $1, 2$, u, ℓ denotes the upper and lower legs, respectively. i is the unit cell index. Transforming into momentum space operators

$$\sum_i c_{ui}^{1\dagger} \left(c_{\ell i}^2 + c_{\ell i+1}^2 \right) = \sum_{ikq} \frac{1}{N} c_{uk}^{1\dagger} \left(c_{\ell q}^2 + c_{\ell q}^2 \mathrm{e}^{-\mathrm{i} q a} \right) \mathrm{e}^{-\mathrm{i}(k-q) r_i} = \sum_k c_{uk}^{1\dagger} c_{\ell k}^2 \left(1 + \mathrm{e}^{-\mathrm{i} k a} \right) \tag{2}$$

The mirror symmetry gives us

$$c_{uk} = \frac{1}{\sqrt{2}} (c_{0k} + c_{\pi k}), \tag{3}$$

$$c_{\ell k} = \frac{1}{\sqrt{2}} (c_{0k} - c_{\pi k}). \tag{4}$$

$$j_2 \propto \mathrm{i} t_{\perp 2} \sum_k \left[-\left(c_{0k}^{1\dagger} + c_{\pi k}^{1\dagger} \right) \left(c_{0k}^2 - c_{\pi k}^2 \right) \frac{1 + \mathrm{e}^{-\mathrm{i} k a}}{2} + \left(c_{0k}^{2\dagger} - c_{\pi k}^{2\dagger} \right) \left(c_{0k}^1 + c_{\pi k}^1 \right) \frac{1 + \mathrm{e}^{\mathrm{i} k a}}{2} \right] \tag{5}$$

After expanding the brackets,

$$j_2 \propto \mathrm{i} t_{\perp 2} \sum_k \left[\left(c_{0k}^{2\dagger} c_{0k}^1 + c_{0k}^{2\dagger} c_{\pi k}^1 - c_{\pi k}^{2\dagger} c_{0k}^1 - c_{\pi k}^{2\dagger} c_{\pi k}^1 \right) \frac{1 + \mathrm{e}^{\mathrm{i} k a}}{2} - \mathrm{H.c.} \right]. \tag{6}$$

The current operator for $t_{\perp 5}$ is

$$j_5 \propto \mathrm{i} t_{\perp 5} \sum_i \left[-c_{ui}^{1\dagger} \left(c_{\ell i-1}^2 + c_{\ell i+2}^2 \right) + c_{\ell i}^{2\dagger} \left(c_{ui+1}^1 + c_{ui-2}^1 \right) \right], \tag{7}$$

and differs from j_2 only in the exponential factors,

$$j_5 \propto \mathrm{i} t_{\perp 5} \sum_k \left[-\left(c_{0k}^{1\dagger} + c_{\pi k}^{1\dagger} \right) \left(c_{0k}^2 - c_{\pi k}^2 \right) \frac{\mathrm{e}^{2\mathrm{i} k a} + \mathrm{e}^{-\mathrm{i} k a}}{2} + \left(c_{0k}^{2\dagger} - c_{\pi k}^{2\dagger} \right) \left(c_{0k}^1 + c_{\pi k}^1 \right) \frac{\mathrm{e}^{-2\mathrm{i} k a} + \mathrm{e}^{\mathrm{i} k a}}{2} \right], \tag{8}$$

$$j_5 \propto \mathrm{i} t_{\perp 5} \sum_k \left[\left(c_{0k}^{2\dagger} c_{0k}^1 + c_{0k}^{2\dagger} c_{\pi k}^1 - c_{\pi k}^{2\dagger} c_{0k}^1 - c_{\pi k}^{2\dagger} c_{\pi k}^1 \right) \frac{\mathrm{e}^{-2\mathrm{i} k a} + \mathrm{e}^{\mathrm{i} k a}}{2} - \mathrm{H.c.} \right]. \tag{9}$$

The current operator for $t_{\perp 3}$ shares the exponential factors with $t_{\perp 2}$, but the k_y transformation gives different combination of terms,

$$j_3 \propto \mathrm{i} t_{\perp 3} \sum_i \left[-c_{\ell i}^{1\dagger} \left(c_{\ell i}^2 + c_{\ell i+1}^2 \right) + c_{\ell i}^{2\dagger} \left(c_{\ell i}^1 + c_{\ell i-1}^1 \right) \right], \tag{10}$$

$$j_3 \propto \mathrm{i} t_{\perp 3} \sum_k \left[-\left(c_{0k}^{1\dagger} - c_{\pi k}^{1\dagger} \right) \left(c_{0k}^2 - c_{\pi k}^2 \right) \frac{1 + \mathrm{e}^{-\mathrm{i} k a}}{2} + \left(c_{0k}^{2\dagger} - c_{\pi k}^{2\dagger} \right) \left(c_{0k}^1 - c_{\pi k}^1 \right) \frac{1 + \mathrm{e}^{\mathrm{i} k a}}{2} \right], \tag{11}$$

$$j_3 \propto \mathrm{i} t_{\perp 3} \sum_k \left[\left(c_{0k}^{2\dagger} c_{0k}^1 - c_{0k}^{2\dagger} c_{\pi k}^1 - c_{\pi k}^{2\dagger} c_{0k}^1 + c_{\pi k}^{2\dagger} c_{\pi k}^1 \right) \frac{1 + \mathrm{e}^{\mathrm{i} k a}}{2} - \mathrm{H.c.} \right]. \tag{12}$$

The current operator for $t_{\perp 7}$ shares the exponential factors with $t_{\perp 5}$ and the k_y combination of terms with $t_{\perp 3}$,

$$j_7 \propto \mathrm{i} t_{\perp 7} \sum_i \left[-c_{\ell i}^{1\dagger} \left(c_{\ell i-1}^2 + c_{\ell i+2}^2 \right) + c_{\ell i}^{2\dagger} \left(c_{\ell i+1}^1 + c_{\ell i-2}^1 \right) \right], \tag{13}$$

$$j_7 \propto \mathrm{i} t_{\perp 7} \sum_k \left[-\left(c_{0k}^{1\dagger} - c_{\pi k}^{1\dagger} \right) \left(c_{0k}^2 - c_{\pi k}^2 \right) \frac{\mathrm{e}^{2\mathrm{i} k a} + \mathrm{e}^{-\mathrm{i} k a}}{2} + \left(c_{0k}^{2\dagger} - c_{\pi k}^{2\dagger} \right) \left(c_{0k}^1 - c_{\pi k}^1 \right) \frac{\mathrm{e}^{-2\mathrm{i} k a} + \mathrm{e}^{\mathrm{i} k a}}{2} \right], \tag{14}$$

$$j_7 \propto \mathrm{i} t_{\perp 7} \sum_k \left[\left(c_{0k}^{2\dagger} c_{0k}^1 - c_{0k}^{2\dagger} c_{\pi k}^1 - c_{\pi k}^{2\dagger} c_{0k}^1 + c_{\pi k}^{2\dagger} c_{\pi k}^1 \right) \frac{\mathrm{e}^{-2\mathrm{i} k a} + \mathrm{e}^{\mathrm{i} k a}}{2} - \mathrm{H.c.} \right]. \tag{15}$$

The combination of these amplitudes yields different factors for the various hopping terms

$$c_{0k}^{2\dagger} c_{0k}^{1} : \mathrm{i}\left[(t_2 + t_3)\,\frac{1+\mathrm{e}^{\mathrm{i}ka}}{2} + (t_5 + t_7)\,\frac{\mathrm{e}^{-2\mathrm{i}ka} + \mathrm{e}^{\mathrm{i}ka}}{2}\right] \tag{16}$$

$$c_{0k}^{2\dagger} c_{\pi k}^{1} : \mathrm{i}\left[(t_2 - t_3)\,\frac{1+\mathrm{e}^{\mathrm{i}ka}}{2} + (t_5 - t_7)\,\frac{\mathrm{e}^{-2\mathrm{i}ka} + \mathrm{e}^{\mathrm{i}ka}}{2}\right] \tag{17}$$

$$c_{\pi k}^{2\dagger} c_{0k}^{1} : \mathrm{i}\left[(t_2 - t_3)\,\frac{1+\mathrm{e}^{\mathrm{i}ka}}{2} - (t_5 + t_7)\,\frac{\mathrm{e}^{-2\mathrm{i}ka} + \mathrm{e}^{\mathrm{i}ka}}{2}\right] \tag{18}$$

$$c_{\pi k}^{2\dagger} c_{\pi k}^{1} : \mathrm{i}\left[(t_2 + t_3)\,\frac{1+\mathrm{e}^{\mathrm{i}ka}}{2} - (t_5 - t_7)\,\frac{\mathrm{e}^{-2\mathrm{i}ka} + \mathrm{e}^{\mathrm{i}ka}}{2}\right] \tag{19}$$

conductivity

$$\mathrm{Re}\sigma(\omega) \propto \sum_M \frac{\left|\langle M\,|j|\,GS\rangle\right|^2}{E_M - E_{GS}}\left[\delta(\omega - (E_M - E_{GS})) + \delta(\omega + (E_M - E_{GS}))\right] \tag{20}$$

the uppercase states are eigenstates of two ladder system.

$$A_q(\omega) = A_q^+(\omega) + A_q^-(\omega) \tag{21}$$

$$A_q^+(\omega) = \sum_m \left|\langle m\,|c_q^\dagger|\,gs\rangle\right|^2 \delta(\omega - (E_m - E_{gs})) \tag{22}$$

the lowercase states are eigenstates of a single ladder system. The $\omega > 0$ part of the real conductivity is given by convolution of spectral functions, e.g.

$$\mathrm{Re}\Delta\sigma(\omega > 0) \propto |f(k,q)|^2 \int \mathrm{d}\omega' A_k^+(\omega') A_q^-(\omega - \omega') = \frac{|f(k,q)|^2}{\omega} \sum_{mn} \left|\left\langle m\left|c_k^\dagger\right| gs\right\rangle\right|^2 \left|\langle n\,|c_q|\,gs\rangle\right|^2 \delta(\omega - \Delta_{mn}), \tag{23}$$

where $f(k,q)$ is the geometric factor and $\Delta_{mn} = E_m + E_n - 2E_{gs}$, gives contribution from a single term in the current operator j if the initial state of the two ladders is identical and the two ladders are not correlated. All cross-terms in $\left|\langle M\,|j|\,GS\rangle\right|^2$ are zero, therefore the contributions from the four current operators differ only in the geometrical factor, the value of $t_\perp$ and the along the rung direction length of the hopping.