Optimization on Power Splitting Ratio Design for K-tier HCNs with Opportunistic Energy Harvesting

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***Abstract*—Future small cells are expected to be energy-efficient and utilize green technologies. To this end, a promising solution is to employ automatic energy harvesting techniques, such as power splitting (PS) which harvests energy from ambient radio frequency (RF) signals in modern communication systems. In this paper, optimization on PS ratio design is investigated to maximize the average harvested energy in the context of general large-scale K-tier heterogeneous cellular networks (HCNs). Specifically, coverage probabilities and average energy harvesting expressions are derived with the stochastic geometry treatment to elucidate the performance of future green networks. Then optimal fixed PS ratios for each tier are obtained under coverage performance constraints. Moreover, with receivers' position information effortlessly provided in future networks, a dynamic location-based PS ratio design (DLPS) is proposed to further enhance the energy harvesting performance. Simulation results are given to demonstrate that the average harvested energy is effectively increased by more than 30% when coverage probability requirement is greater than 0.7 by our proposed DLPS compared with the optimal fixed PS ratio while maintaining the coverage performance. Furthermore, rather than drawing the conclusions about the merits of our PS strategy, this work is to provide a tractable analytical framework for addressing the energy harvesting issues in such HCNs.**

I. Introduction

Energy harvesting is of paramount importance towards a fifth generation (5G) world with energy-efficient and green infrastructures in heterogeneous cellular networks (HCNs) [1, 2]. Since ambient radio-frequency (RF) signals have been an applicable new source for energy harvesting, simultaneous wireless information and power transfer (SWIPT) has become one of feasible solutions for energy harvesting by opportunistically harvesting energy from surrounding RF signals [3], and consequently leads to efficient utilization of available energy, prolongs system lifetime and reduces the operation cost [4]. However, there are two key challenges in the realization process of SWIPT in the context of large-scale networks.

First, to enable communication networks with RF energy harvesting capabilities, an accurate and effective theoretical analysis is indispensable. The energy source of RF signals has some characteristics including inexhaustible but unreliable, sufficient but unstable. Moreover, cells are getting smaller, denser, more random and chaotic in future ultra dense networks (UDNs) and base stations (BSs) may even support "drop and play" deployments. As a result, a more applicable way is to model the BS locations as random and drawn from a spatial stochastic process, such as the Poisson point process (PPP) [5–7]. The stochastic geometry theory [8] has advantages of a more accurate and tractable analysis on such irregular and opportunistic cellular networks.

Second, how to set energy harvesting configurations is a challenging issue. One of the practical RF mechanisms for SWIPT introduced by [9] is named power splitting (PS), where one part of the received signal is used for information decoding, while the other part is used for RF energy harvesting. To the best of our knowledge, lots of contributions [9–11] have been made to cope with the problems under PS treatment, but most of existing work focus on a point-to-point multiple-input multiple-output (MIMO) channel in a fixed or single-tier network with specific users, neglecting inter-cell interference or simplifying the network models. For instance, the authors derive the optimal transmission strategy characterized by the boundary of a so-called rate-energy region in a point-to-point network [9]; the work in [10] investigates rate-energy performance trade-off for dynamic PS with a point-to-point flat-fading channel; [11] considers a large-scale network with random number of transmitter-receiver pairs in a single tier, but it is not a general K-tier cellular network model, etc. Besides, what is the performance of the whole networks when adopting PS scheme and how PS helps and are also worth investigating.

This paper analyzes the performance of a general K-tier cellular network with PS scheme and deriving a desired PS ratio design. The main contributions of this paper are as follows: 1) coverage probability and average energy harvesting expressions with PS scheme for a general K-tier cellular networks are obtained under stochastic geometry framework, which is efficiently computed numerically; 2) an optimal fixed PS ratio for each tier is given under coverage performance constraints; 3) with receivers' position information effortlessly provided in future networks, a dynamic location-based PS ratio design (DLPS) is proposed to further enhance energy harvesting performance; and 4) theoretical and simulated results are presented, which strongly corroborates the accuracy of our analysis. Nevertheless, rather than presenting the conclusions about the merits of our PS strategy, this paper is to provide a

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tractable analytical framework as a guideline for addressing the energy harvesting problems in large-scale cellular networks.

The rest of this paper is organized as follows. In Section II, we first introduce the system models, and then give coverage probability and average energy harvesting definitions. In Section III, coverage probability and average energy harvesting expressions with PS scheme are derived, an optimal fixed PS ratio is also presented. Further a DLPS scheme is proposed in Section III. Section IV describe theoretical and simulation results. Finally, Section V concludes the paper.

II. System Model

A. Assumptions

We consider a large-scale celluar wireless network model consisting of K tiers of BSs that are distinguished by their spatial densities and transmit powers, such as those of macro-cells, femtocells or picocells. The BSs in the k-th tier arrange according to some homogeneous PPP Φ_k of intensity λ_k respectively in the Euclidean plane, and the BSs distributions of each tier are independent and each BS operate at a fixed transmit power of P_k.

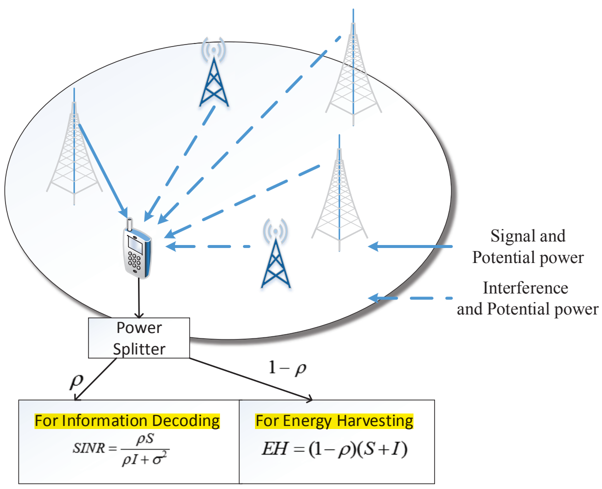

Fig. 1. Example of downlink HCNs with two tiers of BSs. Users are equipped with PS supporting devices and PS scheme is adopted for energy harvesting. S and I denote the received signal power and inter-cell interference power respectively.

In this model, each BS has a continuous power supply while the users are equiped with energy harvesting devices so as to have RF energy harvesting capabilities from the received electromagnetic radiation. Based on the PS technique, as shown in Fig. 1, the hybrid receiver can split the received signal into two parts, namely ρ portion of the received energy is used for signal processing and information decoding and the the remaining $1-\rho$ portion is converted to DC voltage for energy storage, where $\rho \in [0, 1]$, and ρ_k is used when receiver is connected to the k-th tier.

As for power loss propagation model, we denote α as the path loss exponent for large-scale fading. Rayleigh fading with unit average power models the random channel fast fading, which is denoted by $h \sim \exp(1)$, namely the channel random variable h follows an exponential distribution with mean 1. Consider a typical random user located at the origin of this plane. Hence, the received signal power at a typical random user from the serving BS at point x_k (belonging to k-th tier) is $P_k h_{x_k} \|x_k\|^{-\alpha}$. And the thermal noise is additive and has constant power σ_A^2 but no specific distribution is assumed. The signal process noise during the baseband conversion phase [9] is modelled as Gaussian and has zero mean and variance σ_P^2. The associated Signal to Interference Plus Noise Ratio (SINR) is given by:

$$SINR(x_k) = \frac{\rho_k P_k h_{x_k} \|x_k\|^{-\alpha}}{\rho_k (I + \sigma_A^2) + \sigma_P^2}, \tag{1}$$

where $I = \sum_{j=1}^{K} \sum_{y \in \Phi_j \setminus x_k} P_j h_y \|y\|^{-\alpha}$ denotes inter-cell interference. Note that σ_A^2 can be ignored in interference-limited networks for the typical networks are interference limited and hence thermal noise has a very limited effect on SINR distribution, however, σ_P^2 is essential because it may become the dominant noise when ρ is small enough.

We consider a maximum long-term received power (MLRP) connectivity model: A mobile user is associated with the strongest BS in terms of long-term averaged received power at the user.

B. Definitions

Here we give the definitions of the main metrics, i.e., coverage probability and average energy harvesting. We first define the probability of coverage which is the key measure of the downlink HCNs.

In the context of this paper, we define coverage probability as the probability that a user's instantaneous SINR is greater than a predetermined threshold value θ_k for each tier, mathematically the coverage probability of tier k is:

$$C_k = \mathbb{P}(SINR(x_k) > \theta_k), \tag{2}$$

then A_k denotes the probability of a typical user is associated with k-th tier and using the law of total probability, the coverage probability is given as:

$$C = \sum_{k=1}^{K} C_k A_k. \tag{3}$$

Next we define average energy harvesting. Since $1-\rho$ portion of the received energy is used for energy storage, the average harvested energy at the typical receiver, whose serving BS belongs to tier k (noted as event S_k), is expressed as

$$EH_k = \eta \mathbb{E}\left[(1-\rho_k) \sum_{j=1}^{K} \sum_{x_j \in \Phi_j} P_j h_{x_j} \|x_j\|^{-\alpha} \middle| S_k \right], \tag{4}$$

where $\eta \in (0, 1]$ denotes the conversion efficiency from RF signal to DC voltage. And the average harvested energy of the whole network is:

$$EH_k = \sum_{k=1}^{K} EH_k A_k. \tag{5}$$

III. Energy Harvesting Performance in K-tier HCNs with PS Scheme under Coverage Performance Constraints

This section first presents coverage probability expressions with the PS scheme. In the case when all the tiers have predetermined SINR thresholds θ_k, coverage probability can be given precisely. Since the interference power easily dominates thermal noise, we ignore σ_A^2 for its negligible effect on SINR distribution as well as for tractable analysis. With this understanding, we now derive the probability of coverage for a randomly located mobile user.

A randomly located user is possibly covered by any tier and BSs in different tiers are distributed independently.

Proposition 1. *In a K-tier network, the coverage probability $C(\{\rho_k\})$ with PS scheme is given by*

$$C(\{\rho_k\}) = \sum_{k=1}^{K} 2\pi\lambda_k \int_0^\infty r \exp\left(-\pi r^2 \sum_{j=1}^{K} W_j\right) \exp\left(\frac{-\theta_k \sigma_P^2 r^\alpha}{P_k \rho_k}\right) dr. \tag{6}$$

where

$$W_j = (\theta_{k2}F_1[1, 0.5; 1.5; -\theta_k] + 1)\lambda_j \left(\frac{P_j}{P_k}\right)^{2/\alpha},$$

with ${}_2F_1[.]$ denotes the Gauss hypergeometric function. ■

Proof: The proof is provided in Appendix.

The results of the coverage probability expression we obtain is efficiently computed numerically, which shows the PS ratios ρ_k do have significant effects on the coverage probability,

Then we presents the average energy harvesting expression. In this part, we modify the pathloss degradation model to avoid singularity as well as to ensure the accuracy of our pathloss model for short distances. For a randomly located user at the origin point, the received signal power from BS x is given by

$$P_{x,k} h_{x,k} = P_k \left[\max(\|x\|, r_0)\right]^{-\alpha} h_{x,k}, \tag{7}$$

where the parameter $r_0 > 1$ refers to the minimum possible pathloss degradation. Besides, this modification does not affect the analysis of coverage probability for a network that is not too dense and λ_k is small enough compared to $1/r_0^2$ [12]. Then the average energy harvesting expression is derived by Proposition 2.

Proposition 2. *In a K-tier network, the average harvested energy power with PS scheme is given by*

$$\begin{aligned} EH &= \eta \sum_{k=1}^{K} \mathbb{E}\left[(1-\rho_k) \sum_{j=1}^{K} \sum_{x\in\Phi_j} P_{x,j} h_{x,j} \middle| S_k\right] A_k \\ &= \eta \sum_{k=1}^{K} (1-\rho_k) A_k \sum_{j=1}^{K} \lambda_j \int_{\mathbb{R}^2} P_j [\max(\|x\|, r_0)]^{-\alpha} dx \\ &= \eta \sum_{k=1}^{K} (1-\rho_k) A_k \sum_{j=1}^{K} 2\pi\lambda_j P_j \left(\int_0^{r_0} x r_0^{-\alpha} dx + \int_{r_0}^{\infty} x^{1-\alpha} dx\right) \\ &= \frac{\pi\eta\alpha r_0^{2-\alpha}}{\alpha-2} \frac{\sum_{j=1}^{K} \lambda_j P_j}{\sum_{j=1}^{K} \lambda_j P_j^{2/\alpha}} \sum_{k=1}^{K} (1-\rho_k)\lambda_k P_k^{2/\alpha}, \end{aligned} \tag{8}$$

where the expression of A_k is given in Appendix.

In order to pursue a unified study on coverage performance and energy harvesting performance, we formulate a theoretical problem which jointly determines the optimal PS ratios $\{\rho_k\}$ to maximize the average harvested energy while guaranteeing the coverage probability is under constraint, i.e., it is higher than a given expected value ϵ as follows (noted as **P1**):

$$\begin{aligned} \textbf{P1} \qquad & \max_{\{\rho_k\}} \ EH \\ & \text{s. t.} \ \ C(\{\rho_k\}) \geq \epsilon, \\ & \qquad 0 \leq \rho_k \leq 1. \end{aligned} \tag{9}$$

To solve this problem, we can rewrite $C(\{\rho_k\})$ to $\sum_{k=1}^{K} C'_k(\rho_k)$, EH to $\sum_{k=1}^{K} EH'_k$, , and each term of the summation only concludes the single variable ρ_k. If this problem has feasible solution, ϵ can be expressed as $\sum_{k=1}^{K} \epsilon_k$, where $0 \leq \epsilon_k \leq C'_k(1)$. It is noticeable that $C_k(\rho_k)$ is a monotone increasing function of ρ_k from Proposition 1 and EH_k is a monotone decreasing function of ρ_k from Proposition 2. These trends are also consistent with our intuitive feelings. For a given ϵ_k, if $C'_k(\rho_k)$ is constrained by $C'_k(\rho_k) \geq \epsilon_k$, the optimal ρ_k (denoted as $\rho_k^\star$) is derived at the boundary conditions, i.e.,

$$C'_k(\rho_k^\star) = \epsilon_k \tag{10}$$

or $\rho_k^\star = C'^{-1}_k(\epsilon_k)$ because of monotonicity, which can be easily solved by bisection method [13] for some certain ϵ_k. The global optimal solution $\{\rho_k^\star\}$ is obtained when every $C'_k(\rho_k)$ reaches ϵ_k. Introducing relaxed variables to further transform the constraints into standard linear ones, the optimization problem has the format below

$$\begin{aligned} \min_{\{\epsilon_k\}} \quad & -EH = -\frac{\pi\eta\alpha r_0^{2-\alpha}}{\alpha-2} \frac{\sum_{j=1}^{K} \lambda_j P_j}{\sum_{j=1}^{K} \lambda_j P_j^{2/\alpha}} \sum_{k=1}^{K} (1 - C'^{-1}_k(\epsilon_k))\lambda_k P_k^{2/\alpha} \\ & \text{s. t.} \ \sum_{k=1}^{K} \epsilon_k = \epsilon, \\ & \qquad \epsilon_k + \epsilon'_k = C'_k(1), \\ & \qquad \epsilon_k, \epsilon'_k \geq 0, \end{aligned} \tag{11}$$

where $\epsilon'_k, k \in \{1, 2, ..., K\}$ denotes the relaxed variables. Since the optimization problem has linear constraints, we can use some search algorithms, such as Zoutendijks methods [14], to obtain the optimal solution easily. Follow the feasible direction, this linear programming can be rapidly implemented by computer. Consequently the optimal solution $\{\rho_k^\star\}$ in **P1** is derived.

IV. Dynamic Power Splitting Ratio Design

In Section III, we obtain the optimal ρ_k for each tier to maximize the average harvested energy under coverage performance constraints. However, fixed PS ratios for all users seem not to be preferable intuitively since it is easy to reach the SINR requirement when a user is close to its serving BS and more received energy power can be stored for future use and prolonging system lifetime. Thus we propose a DLPS scheme to further enhance energy harvesting performance.

We firstly give our proposed expressions of ρ_k as follows:

$$\rho_k(r) = 1 - \exp\left[-\left(\frac{1}{1-\rho_k^\star} - 1\right)\pi \sum_{j=1}^{K} \lambda_j \left(\frac{P_j}{P_k}\right)^{2/\alpha} r^2\right], \tag{12}$$

where r denotes the distance between the user and its serving BS, and $\rho_k^\star$ can be obtained in Section III.

This design is inspired by

$$\begin{cases} \mathbb{E}[\rho_k] = \int_0^\infty \rho_k(r) f_{R_k}(r)dr = \rho_k^\star, & \text{(13a)} \\ 0 < \rho_k(r) < 1, & \text{(13b)} \\ \rho_k(r_1) < \rho_k(r_2)\ for\ \forall r_1 < r_2 \in (0,\infty), & \text{(13c)} \end{cases}$$

where $f_{R_k}(r)$ is the probability density function of the distance between the user and its serving BS (belonging to the k-th tier), and its expression is given in Appendix.

Equation (13a) maintains the expectation of $\rho_k(r)$ to fixed optimal $\rho_k^\star$, equation (13b) guarantees $\rho_k(r)$ is feasible and equation (13c) ensures the desired monotonicity of r. Constrained by these conditions, we propose a satisfied expression of $\rho_k(r)$, namely equation (12), which effectively achieve better energy harvesting performance while the coverage performance keeps very closed to the optimal solution for cases of fixed PS ratios. The accuracy of this conclusion will also be validated in this paper via simulations in Section V.

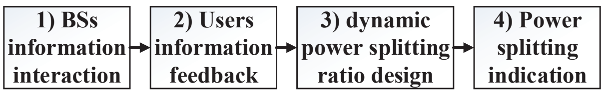

Fig. 2. Flow chart of the pratical application of opportunistically energy harvesting with DLPS scheme.

As Fig. 2 shows, we draw a flow chart to describe a practical application of opportunistically energy harvesting with our proposed DLPS scheme. This consists of the following steps:

1) *Step 1, BSs information interaction*: Each BS periodically get informed from surrounding BSs about the active BS density of each tier λ_k.(Note that some BSs may be operate in sleep mode or may support "drop and play".)
2) *Step 2, User information feedback*: Users should feed back information which mainly includes position estimation and channel status information (CSI), etc.
3) *Step 3, Dynamic power splitting ratio design*: Calculate the optimal fixed ρ_k firstly as shown in Section III and then regard ρ_k as a parameter to further enhance PS ratios by proposed DLPS scheme according to (12).
4) *Step 4, Power splitting indication*: BSs indicate receivers how to configure power splitters according to the results from step 3.

TABLE I
NOTATIONS AND PARAMETERS

Deployment 1		Deployment 2	
Symbol	Default Value	Symbol	Default Value
λ_1	$10^{-5}m^{-2}$	λ_1	$1.5\times10^{-4}m^{-2}$
λ_2	$4\times10^{-5}m^{-2}$	λ_2	$3\times10^{-4}m^{-2}$
P_1	$41dBm$	P_1	$32dBm$
P_2	$33dBm$	P_2	$20dBm$
θ_1	$-8dB$	θ_1	$-6dB$
θ_2	$-8dB$	θ_2	$-6dB$
α	4	α	4
σ_A^2	$-174dBm/Hz$	σ_A^2	$-174dBm/Hz$
σ_P^2	$10^{-7}W$	σ_P^2	$10^{-7}W$

V. NUMERICAL RESULTS AND DISCUSSIONS

In the following, we present comparison of the Monte Carlo simulations and the analytical results to evaluate the coverage and energy harvesting performance with PS scheme. We use the default values in Table I unless otherwise stated and simulate a two-tier network in a 10km × 10km square area with the reference receiver at the origin. We sampled 10000 realizations and the simulated probability is computed as (realizations with $SINR \geq$ threshold)/10000 and the average energy harvesting is obtained by averaging these 10000 results using equation (4). This simulation also presents two different kinds of deployments denoted as $D1$ and $D2$.

Fig. 3 and 4 depict coverage probability and average energy harvesting performance in a two-tier HCN. In Fig. 3, PS ratios are variable but identical in each tier. Fig. 4 presents a heat map to show how variable ρ_1 and ρ_2 effects average harvested energy and the cases of coverage probabilities below requirement are printed by dark blue. First of all, results show that our theoretical results closely match the corresponding simulation result. With the MLPR connectivity model, both figures show that PS ratios play an important role in the process of energy harvesting. In detail, we note that coverage probability is monotone increasing with the increasing of ρ while the average harvested energy is monotone decreasing with the the increasing of ρ, and the changes of coverage probability gradually flattened while average harvested energy variation keeps linear. The maximum value is derived exactly when coverage probability meets the requirement. These results confirm we could obtain the optimal ρ_k to maximize average harvested energy under a certain coverage performance constraint if this problem has feasible solutions.

Fig. 5 illustrates the coverage probability performance under our proposed DLPS scheme. The black line is a reference standing for the required coverage probability. With fixed PS scheme, the coverage probability exactly stabilizes at the required value and this is because this mechanism tries to satisfy the lowest requirement. By comparing with DLPS scheme, the results reveal that DLPS remains coverage performance since this design reasonably utilize the property of the distance distribution between a user and its serving BS as well as a user

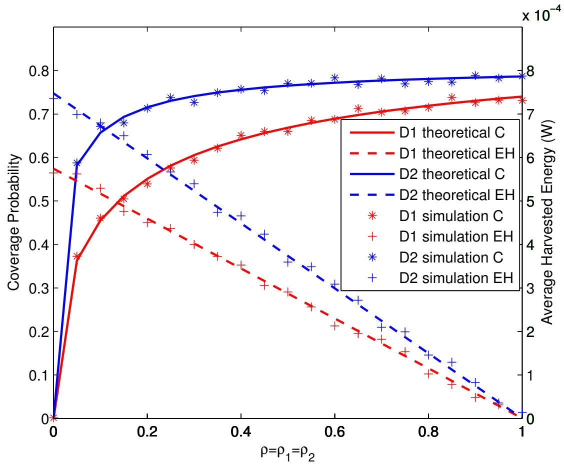

Fig. 3. Coverage probabilities and average harvested energy in two tier HCNs for varying PS ratio $\rho = \rho_1 = \rho_2$, where C stands for coverage probability and EH stands for average harvested energy.

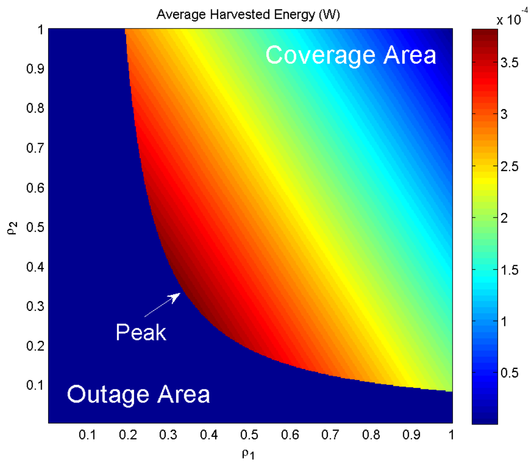

Fig. 4. Average harvested energy in two tier HCNs (D1) for varying PS ratio ρ_1 and ρ_2. Coverage probability requirement $\epsilon = 0.62$. The outage area (dark blue) refers to the cases of coverage probabilities below ϵ.

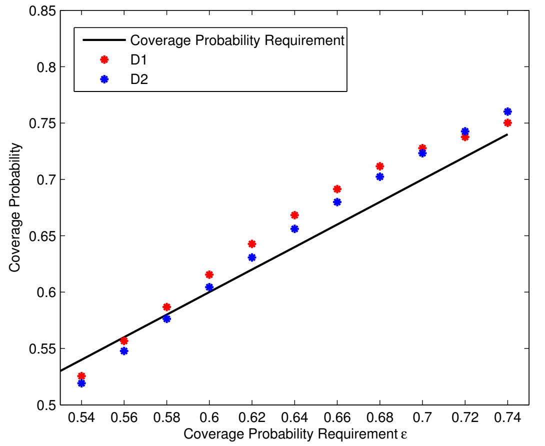

Fig. 5. Coverage probabilities of our proposed DLPS scheme. The black line is a reference standing for the coverage probability requirement.

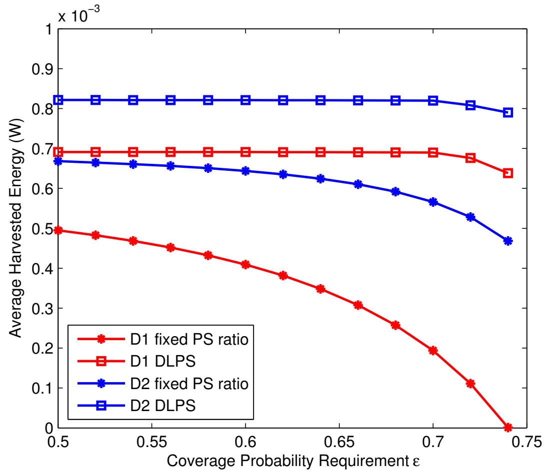

Fig. 6. Average harvested energy with optimal fixed PS ratio scheme and our proposed DLPS scheme.

could reach higher SINR when getting closer to its serving BS.

Fig. 6 elucidates how DLPS effects average harvested energy. We compare the performance of energy harvesting for the cases of DLPS scheme and fixed PS ratio scheme. Results show that the average harvested energy steadily goes down with the increase of coverage probability requiremnet. However, the average harvested energy with DLPS scheme is markedly greater than the fixed PS ratio scheme. This result is a consequence of DLPS scheme collecting more energy when a user stands at a advantageous lacation. We also observe that the gap between these two schemes it getting greater with the increasing SINR threshold values. This indicates when HCN operates with higher coverage performance requirement, both schemes present lower average harvested energy but the merit of setting a more appropriate PS ratio is gradually highlighted.

VI. Conclusion

In this work, a tractable analytical framework for PS scheme in a general K-tier heterogeneous cellular networks has been developed. Leveraging the tools from stochastic geometry, we have analyzed the coverage performance and energy harvesting performance. Firstly optimal fixed PS ratios for each tier are obtained to maximize the average harvested energy under coverage performance constraints and secondly a DLPS scheme is proposed to further enhance the energy harvesting performance. Thirdly, the simulation results validate the derived expressions, which are efficiently computed numerically. The results also demonstrate the proposed DLPS scheme significantly improves the average harvested energy while maintaining the coverage performance. Further more, our work can provide guidelines for PS configurations in future multi-tier networks to enhance energy harvesting performance.

Acknowledgment

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Appendix

Proof of Proposition 1

Consider a typical user at the origin associated with the k-th tier. Denote R_k as the distance between the user and its serving BS. The probability density function $f_{R_k}(x)$ of the distance R_k and the probability that the user is associated with the k-th tier are respectively given by [15]:

$$f_{R_k}(r) = \frac{2\pi\lambda_k}{A_k} r \exp\left\{-\pi \sum_{j=1}^{K} \lambda_j \left(\frac{P_j}{P_k}\right)^{2/\alpha} r^2\right\}, \tag{14}$$

$$A_k = \frac{\lambda_k P_k^{2/\alpha}}{\sum_{j=1}^{K} \lambda_j P_j^{2/\alpha}}. \tag{15}$$

Conditioning on the distance R_k,

$$\begin{aligned} C &= \sum_{k=1}^{K} A_k \mathbb{E}_{R_k} \mathbb{P}(SINR > \theta_k)|R_k) \\ &= \sum_{k=1}^{K} A_k \int_0^\infty \mathbb{P}(SINR > \theta_k)|r) f_{R_k}(r) dr. \end{aligned} \tag{16}$$

Focus on $\mathbb{P}(SINR > \theta_k)|r)$ which we denote by P_k,

$$\begin{aligned} P_k &= \mathbb{P}\left(\frac{P_k h_{x_k} r^{-\alpha}}{\sum_{j=1}^{K} \sum_{y\in\Phi_j \backslash x_k} P_j h_y \|y\|^{-\alpha} + \sigma_P^2/\rho_k} \geq \theta_k \middle| r\right) \\ &\overset{(a)}{=} \mathbb{E}\left[\exp\left(-\theta_k P_k^{-1} r^\alpha \left(\sum_{j=1}^{K} \sum_{y\in\Phi_j \backslash x_k} P_j h_y \|y\|^{-\alpha} + \frac{\sigma_P^2}{\rho_k}\right)\right) \middle| r\right] \\ &= \mathbb{E}\left[\prod_{j=1}^{K} \prod_{y\in\Phi_j \backslash x_k} \exp\left(-\theta_k P_k^{-1} r^\alpha P_j h_y \|y\|^{-\alpha}\right) \middle| r\right] \exp\left(\frac{-\theta_k \sigma_P^2 r^\alpha}{P_k \rho_k}\right), \end{aligned} \tag{17}$$

where (a) follows from the distribution of h_{x_k}. And the first multiplier of P_k results from the inter-cell interference, we denote by $\mathcal{I}_k$ and obtain

$$\begin{aligned} \mathcal{I}_k &= \mathbb{E}\left[\prod_{j=1}^{K} \prod_{y\in\Phi_j \backslash x_k} \exp\left(-\theta_k P_k^{-1} r^\alpha P_j h_y \|y\|^{-\alpha}\right) \middle| r\right] \\ &\overset{(b)}{=} \mathbb{E}\left[\prod_{j=1}^{K} \prod_{y\in\Phi_j \backslash x_k} \frac{1}{1 + \theta_k P_k^{-1} r^\alpha P_j \|y\|^{-\alpha}} \middle| r\right] \\ &\overset{(c)}{=} \prod_{j=1}^{K} \exp\left(-\pi\lambda_j \int_r^\infty \frac{y}{1 + \theta_k P_k^{-1} r^\alpha P_j y^{-\alpha}} dy\right) \\ &\overset{(d)}{=} \exp\left(-\pi r^2 \theta_{k2} F_1[1, 0.5; 1.5; -\theta_k] \sum_{j=1}^{K} \lambda_j \left(\frac{P_j}{P_k}\right)^{2/\alpha}\right), \end{aligned} \tag{18}$$

where (b) follows from the distribution of h_y since the fades are independent and exponentially distributed with unit mean, (c) follows from the probability generating functional (PGFL) of PPP [16]. (d) results from the use of some properties of Gauss hypergeometric function and some algebraic manipulations.

Un-conditioning on R_k, substitute (14), (17) and (18) into (16), we have **Proposition 1**. ■

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