Problem 2: a and b are alternated in the same code

```
clc;
clear;

%Analytical Solution of 2a
N=2048;
Lx=2.*pi;
Ly=2.*pi;
Q=zeros(N,N);

 for j = 1:N
    for p=1:N
      x(j)=(Lx/N).*(j-1);
      y(p)=(Ly/N).*(p-1);
      Q(j,p)= -2.*(x(j)-(x(j).^2))-2.*(y(p)-(y(p).^2));

  %Q(j,p)=((1./4).*sin(2.*x(j)).*sin(4.*y(p)))+((1./2).*sin(12.*x(j)).*sin(y(p)));
    end
 end

%in x-direction
 for p=1:N
     Q_x=fft(Q);
 end
 Q_x=Q_x/N;

 for i =1:N
    Q_x_shifted(1:N,i)=fftshift(Q_x(1:N,i));
 end
%in y-direction
 for m=1:N
         Q_y(m,:)=fft(Q_x_shifted(m,:));
 end
 Q_xy=Q_y/N;

 for i =1:N
   Q_xy_shifted(i,1:N)=fftshift(Q_xy(i,1:N));
 end

 %phi_cap(m,n)

 for  m=1:N
     for n=1:N
         k(m)=(2.*pi.*(m-(N/2+1)))./Lx;
         l(n)=(2.*pi.*(n-(N/2+1)))./Ly;
         phi_cap(m,n)=-Q_xy_shifted(m,n)./((k(m).^2)+(l(n).^2));
     end
 end
```

```
 phi_cap(N/2+1,N/2+1)=0;


 for i =1:N
     phi_cap(i,1:N)=ifftshift(phi_cap(i,1:N));
 end


 %inverse 2D FFT for phi(j,p)

 %2D inverse FFT

 %inverse in y dir:
for m=1:N
          phi_y(m,1:N)=(ifft(phi_cap(m,1:N).*N));
end


for i= 1:N
      phi_y(1:N,i)=fftshift(phi_y(1:N,i));
  end

 %in x-direction
 for p=1:N
     phi_x_e=(ifft(phi_y.*N));
 end
 phi_x_e=real(phi_x_e);
contourf(phi_x_e)
% title('Analytical Solution with large N for 2(a)')



%Analytical Solution for 2(b)
clc;
clear;

N=1024;
Lx=2.*pi;
Ly=2.*pi;
Q=zeros(N,N);

for j = 1:N
    for p=1:N
      x(j)=(Lx/N).*(j-1);
      y(p)=(Ly/N).*(p-1);

 phii(j,p)=((-1./16).*sin(2.*x(j)).*sin(4.*y(p)))+((-1./2).*sin(12.*x(j)).*sin(y(p)));
    end
end

contourf(phii)
title('Analytical Solution for 2(b)')
```

```
N=8;
Lx=2.*pi;
Ly=2.*pi;
Q=zeros(N,N);
Q_x=zeros(N,N);
Q_x_shifted=zeros(N,N);
Q_y=zeros(N,N);
Q_xy=zeros(N,N);
Q_xy_shifted=zeros(N,N);
phi_cap=zeros(N,N);
phi_y=zeros(N,N);
phi_x=zeros(N,N);

 for j = 1:N
    for p=1:N
      x(j)=(Lx/N).*(j-1);
      y(p)=(Ly/N).*(p-1);
      %Q(j,p)= -2.*(x(j)-(x(j).^2))-2.*(y(p)-(y(p).^2));

 Q(j,p)=((1./4).*sin(2.*x(j)).*sin(4.*y(p)))+((1./2).*sin(12.*x(j)).*sin(y(p)));
    end
 end

%in x-direction
 for p=1:N
     Q_x=fft(Q);
 end
 Q_x=Q_x/N;

 for i =1:N
    Q_x_shifted(1:N,i)=fftshift(Q_x(1:N,i));
 end
%in y-direction
 for m=1:N
         Q_y(m,:)=fft(Q_x_shifted(m,:));
 end
 Q_xy=Q_y/N;

 for i =1:N
   Q_xy_shifted(i,1:N)=fftshift(Q_xy(i,1:N));
 end

% phi_cap(m,n)

 for  m=1:N
     for n=1:N
         k(m)=(2.*pi.*(m-(N/2+1)))./Lx;
         l(n)=(2.*pi.*(n-(N/2+1)))./Ly;
         phi_cap(m,n)=-Q_xy_shifted(m,n)./((k(m).^2)+(l(n).^2));
     end
 end
```

```
 phi_cap(N/2+1,N/2+1)=0;


 for i =1:N
     phi_cap(i,1:N)=ifftshift(phi_cap(i,1:N));
 end


 %inverse 2D FFT for phi(j,p)

% 2D inverse FFT

 %inverse in y dir:
for m=1:N
          phi_y(m,1:N)=(ifft(phi_cap(m,1:N).*N));
end


for i= 1:N
      phi_y(1:N,i)=fftshift(phi_y(1:N,i));
  end

% in x-direction
 for p=1:N
     phi_x=(ifft(phi_y.*N));
 end
 phi_x=real(phi_x);
figure(2)
%subplot(2,1,1)
contourf(phi_x)
title('Numerical Solution at N=8 for 2(a)')



%error
 %Ltwo error
 Ltwoall=0;
 for j= 1:N
     for p=1:N
     Ltwoall=Ltwoall+(abs(phi_x(j,p)-phi_x_e(j*2048/N,p*2048/N)).^2);
     end
 end
 Ltwo1=(Ltwoall.*(1/(N.*N))).^(1/2);


clc;clear;
N=16;
Lx=2.*pi;
Ly=2.*pi;
Q=zeros(N,N);

 for j = 1:N
    for p=1:N
      x(j)=(Lx/N).*(j-1);
```

```
     y(p)=(Ly/N).*(p-1);
    % Q(j,p)= -2.*(x(j)-(x(j).^2))-2.*(y(p)-(y(p).^2));

 Q(j,p)=((1./4).*sin(2.*x(j)).*sin(4.*y(p)))+((1./2).*sin(12.*x(j)).*sin(y(p)));
    end
 end

%in x-direction
 for p=1:N
     Q_x=fft(Q);
 end
 Q_x=Q_x/N;

 for i =1:N
    Q_x_shifted(1:N,i)=fftshift(Q_x(1:N,i));
 end
%in y-direction
 for m=1:N
         Q_y(m,:)=fft(Q_x_shifted(m,:));
 end
 Q_xy=Q_y/N;

 for i =1:N
   Q_xy_shifted(i,1:N)=fftshift(Q_xy(i,1:N));
 end

 %phi_cap(m,n)

 for  m=1:N
     for n=1:N
         k(m)=(2.*pi.*(m-(N/2+1)))./Lx;
         l(n)=(2.*pi.*(n-(N/2+1)))./Ly;
         phi_cap(m,n)=-Q_xy_shifted(m,n)./((k(m).^2)+(l(n).^2));
     end
 end


 phi_cap(N/2+1,N/2+1)=0;


 for i =1:N
     phi_cap(i,1:N)=ifftshift(phi_cap(i,1:N));
 end


 %inverse 2D FFT for phi(j,p)

 %2D inverse FFT

 %inverse in y dir:
for m=1:N
          phi_y(m,1:N)=(ifft(phi_cap(m,1:N).*N));
end
```

```
for i= 1:N
      phi_y(1:N,i)=fftshift(phi_y(1:N,i));
  end

 %in x-direction
 for p=1:N
     phi_x=(ifft(phi_y.*N));
 end

subplot(2,1,2)
contourf(phi_x)
title('Numerical Solution at N=16 for 2(b)')



clc;clear;
N=32;
Lx=2.*pi;
Ly=2.*pi;
Q=zeros(N,N);

 for j = 1:N
    for p=1:N
      x(j)=(Lx/N).*(j-1);
      y(p)=(Ly/N).*(p-1);
      %Q(j,p)= -2.*(x(j)-(x(j).^2))-2.*(y(p)-(y(p).^2));

 Q(j,p)=((1./4).*sin(2.*x(j)).*sin(4.*y(p)))+((1./2).*sin(12.*x(j)).*sin(y(p)));
    end
 end

%in x-direction
 for p=1:N
     Q_x=fft(Q);
 end
 Q_x=Q_x/N;

 for i =1:N
    Q_x_shifted(1:N,i)=fftshift(Q_x(1:N,i));
 end
%in y-direction
 for m=1:N
         Q_y(m,:)=fft(Q_x_shifted(m,:));
 end
 Q_xy=Q_y/N;

 for i =1:N
   Q_xy_shifted(i,1:N)=fftshift(Q_xy(i,1:N));
 end

 %phi_cap(m,n)

 for  m=1:N
```

```
    for n=1:N
        k(m)=(2.*pi.*(m-(N/2+1)))./Lx;
        l(n)=(2.*pi.*(n-(N/2+1)))./Ly;
        phi_cap(m,n)=-Q_xy_shifted(m,n)./((k(m).^2)+(l(n).^2));
    end
end


phi_cap(N/2+1,N/2+1)=0;


for i =1:N
    phi_cap(i,1:N)=ifftshift(phi_cap(i,1:N));
end


%inverse 2D FFT for phi(j,p)

%2D inverse FFT

%inverse in y dir:
for m=1:N
         phi_y(m,1:N)=(ifft(phi_cap(m,1:N).*N));
end


for i= 1:N
      phi_y(1:N,i)=fftshift(phi_y(1:N,i));
  end

%in x-direction
for p=1:N
    phi_x=(ifft(phi_y.*N));
end
figure(2)
subplot(2,1,1)
contourf(phi_x)
title('Numerical Solution at N=32 for 2(b)')


clc;clear;
N=64;
Lx=2.*pi;
Ly=2.*pi;
Q=zeros(N,N);

for j = 1:N
   for p=1:N
     x(j)=(Lx/N).*(j-1);
     y(p)=(Ly/N).*(p-1);
     %Q(j,p)= -2.*(x(j)-(x(j).^2))-2.*(y(p)-(y(p).^2));

Q(j,p)=((1./4).*sin(2.*x(j)).*sin(4.*y(p)))+((1./2).*sin(12.*x(j)).*sin(y(p)));
   end
```

```
 end

%in x-direction
 for p=1:N
     Q_x=fft(Q);
 end
 Q_x=Q_x/N;

 for i =1:N
    Q_x_shifted(1:N,i)=fftshift(Q_x(1:N,i));
 end
%in y-direction
 for m=1:N
         Q_y(m,:)=fft(Q_x_shifted(m,:));
 end
 Q_xy=Q_y/N;

 for i =1:N
   Q_xy_shifted(i,1:N)=fftshift(Q_xy(i,1:N));
 end

 %phi_cap(m,n)

 for  m=1:N
     for n=1:N
         k(m)=(2.*pi.*(m-(N/2+1)))./Lx;
         l(n)=(2.*pi.*(n-(N/2+1)))./Ly;
         phi_cap(m,n)=-Q_xy_shifted(m,n)./((k(m).^2)+(l(n).^2));
     end
 end


 phi_cap(N/2+1,N/2+1)=0;


 for i =1:N
     phi_cap(i,1:N)=ifftshift(phi_cap(i,1:N));
 end


 %inverse 2D FFT for phi(j,p)

 %2D inverse FFT

 %inverse in y dir:
for m=1:N
          phi_y(m,1:N)=(ifft(phi_cap(m,1:N).*N));
end


for i= 1:N
      phi_y(1:N,i)=fftshift(phi_y(1:N,i));
  end
```

```
 %in x-direction
 for p=1:N
     phi_x=(ifft(phi_y.*N));
 end
 phi_x=real(phi_x);
subplot(2,1,2)
contourf(phi_x)
title('Numerical Solution at N=64 for 2(b)')
```

Problem 2: Error Calculations

```
%2a error
clc;
clear;

N=[8, 16, 32, 64];
Ltwo1= [0.0040, 0.0020, 9.85E-4, 4.77E-4 ];
figure(1)
loglog(N,Ltwo1,LineWidth=1)
grid on
xlabel("N")
ylabel("error")
legend('Ltwo error')
title('Ltwo error')

%2b error
clc;
clear;

N=[8, 16, 32, 64, 128, 256];
Ltwo1= [1.407E-4, 0.0173, 0.0060, 0.0032, 0.0016, 7.47E-4];
figure(1)
loglog(N,Ltwo1,LineWidth=1)
grid on
xlabel("N")
ylabel("error")
legend('Ltwo error')
title('Ltwo error, 2b')
```

Published with MATLAB® R2022b