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Problem 2(b)

```
clc;
clear;
N=4;
a_kd=zeros(1,N+1);

%values of c
c=zeros(1,N+1);
c(1)=2;
c(N+1)=2;
for i = 2:N
    c(i)=1;
end


%functional value
for j = 1:N+1
    x(j)=cos(pi.*(j-1)./N);
    f(j)=x(j).*exp(-x(j));
%     if x(j)<0
%         f(j)=1+x(j);
%     else
%         f(j)=1-x(j);
%     end
end


%Discrete Chebyshev Transform
for k = 1:N+1
 for j = 1:N+1
    a_kd(k)=a_kd(k)+((1./(c(j))).*f(j).*cos((k-1).*pi.*(j-1)./N));
 end
 a_kd(k)=(2./(c(k).*N)).*a_kd(k);
end


%Inverse Discrete Chebyshev Transform
```

```
f_inversed=zeros(1,N+1);
for j= 1:N+1
    for k =1:N+1
    f_inversed(j)=f_inversed(j)+a_kd(k).*cos((k-1).*pi.*(j-1)./N)
    end
end
subplot(3,1,1)
stem(x,a_kd,'LineWidth',1.5,'Marker','o');
xlabel('x')
ylabel('Chebyshev Coefficients')
title('N=4, chebyshev coeffcients for function d')

% subplot(3,1,1)
% stem(x,f,'LineWidth',1.5,'Marker','x');
% grid on;
% hold on;
% stem(x,f_inversed,'LineWidth',1.5,'Marker','o');
% title('N=4, function c')
% xlabel('x')
% ylabel('Function Value')
% legend('Chebyshev Expansion','Exact Function Values')

N=8;
a_kd=zeros(1,N+1);

%values of c
c=zeros(1,N+1);
c(1)=2;
c(N+1)=2;
for i = 2:N
    c(i)=1;
end

%functional value
for j = 1:N+1
    x(j)=cos(pi.*(j-1)./N);
    f(j)=x(j).*(exp(-x(j)));
end


%Discrete Chebyshev Transform
for k = 1:N+1
 for j = 1:N+1
    a_kd(k)=a_kd(k)+((1./(c(j))).*f(j).*cos((k-1).*pi.*(j-1)./N));
 end
 a_kd(k)=(2./(c(k).*N)).*a_kd(k);
end


%Inverse Discrete Chebyshev Transform
f_inversed=zeros(1,N+1);
for j= 1:N+1
    for k =1:N+1
    f_inversed(j)=f_inversed(j)+a_kd(k).*cos((k-1).*pi.*(j-1)./N)
```

```
    end
end
 subplot(3,1,2)
stem(x,a_kd,'LineWidth',1.5,'Marker','o');
xlabel('x')
ylabel('Chebyshev Coefficients')
title('N=8, chebyshev coeffcients for function d')

% subplot(3,1,2)
% stem(x,f,'LineWidth',1.5,'Marker','x');
% grid on;
% hold on;
% stem(x,f_inversed,'LineWidth',1.5,'Marker','o');
% title('N=8, function c')
% xlabel('x')
% ylabel('Function Value')
% legend('Chebyshev Expansion','Exact Function Values')


N=16;
a_kd=zeros(1,N+1);

%values of c
c=zeros(1,N+1);
c(1)=2;
c(N+1)=2;
for i = 2:N
    c(i)=1;
end

%functional value
for j = 1:N+1
    x(j)=cos(pi.*(j-1)./N);
    f(j)=x(j).*(exp(-x(j)));
end


%Discrete Chebyshev Transform
for k = 1:N+1
 for j = 1:N+1
    a_kd(k)=a_kd(k)+((1./(c(j))).*f(j).*cos((k-1).*pi.*(j-1)./N));
 end
 a_kd(k)=(2./(c(k).*N)).*a_kd(k);
end


%Inverse Discrete Chebyshev Transform
f_inversed=zeros(1,N+1);
for j= 1:N+1
    for k =1:N+1
    f_inversed(j)=f_inversed(j)+a_kd(k).*cos((k-1).*pi.*(j-1)./N)
    end
end
```

```
subplot(3,1,3)
stem(x,a_kd,'LineWidth',1.5,'Marker','o');
xlabel('x')
ylabel('Chebyshev Coefficients')
title('N=16, chebyshev coeffcients for function d')

% subplot(3,1,3)
% stem(x,f,'LineWidth',1.5,'Marker','x');
% grid on;
% hold on;
% stem(x,f_inversed,'LineWidth',1.5,'Marker','o');
% title('N=16, function c')
% xlabel('x')
% ylabel('Function Value')
% legend('Chebyshev Expansion','Exact Function Values')
```

Problem 2(d)

First Derivative

```
clc;
clear;
N=32;
a_kd=zeros(1,N+1);

%values of c
c=zeros(1,N+1);
c(1)=2;
c(N+1)=2;
for i = 2:N
    c(i)=1;
end

%derivative value
for j = 1:N+1
    x(j)=cos(pi.*(j-1)./N);

    if x(j)<0
        f(j)=1+x(j);
        df(j)=1;
    else
        f(j)=-1+x(j);
        df(j)=1;
    end
end



%Discrete Chebyshev Transform
for k = 1:N+1
 for j = 1:N+1
    a_kd(k)=a_kd(k)+((1./(c(j))).*f(j).*cos((k-1).*pi.*(j-1)./N));
```

```
 end
 a_kd(k)=(2./(c(k).*N)).*a_kd(k);
end


%finding b_k
b=zeros(1,N+2);
b(N+2)=0;
b(N+1)=0;


for k = N+1:-1:2
    b(k-1)= (2.*(k-1).*a_kd(k)+b(k+1))./(c(k-1));
end

%check coeffs     %checking whether b_k is correct or not
b_kd=zeros(1,N+1);
for k = 1:N+1
 for j = 1:N+1
    b_kd(k)=b_kd(k)+((1./(c(j))).*df(j).*cos((k-1).*pi.*(j-1)./N));
 end
 b_kd(k)=(2./(c(k).*N)).*b_kd(k);
end

%Inverse Discrete Chebyshev Transform
f_inversed=zeros(1,N+1);
for j= 1:N+1
    for k =1:N+1
    f_inversed(j)=f_inversed(j)+b(k).*cos((k-1).*pi.*(j-1)./N)
    end
end

%Ltwo error
 Ltwoall=0;
 for i= 1:N+1
     Ltwoall=Ltwoall+(abs(f_inversed(i)-df(i)).^2);
 end
 Ltwo=(Ltwoall.*(1/N+1)).^(1/2);



 %second derivative
```

Second Derivative

```
clc;clear;

N=32;
a_kd=zeros(1,N+1);

%values of c
c=zeros(1,N+1);
c(1)=2;
```

```
c(N+1)=2;
for i = 2:N
    c(i)=1;
end

%derivative value
for j = 1:N+1
    x(j)=cos(pi.*(j-1)./N);
     if x(j)<0
        f(j)=1+x(j);
        df2(j)=0;
    else
        f(j)=-1+x(j);
        df2(j)=0;
    end

end



%Discrete Chebyshev Transform
for k = 1:N+1
 for j = 1:N+1
    a_kd(k)=a_kd(k)+((1./(c(j))).*f(j).*cos((k-1).*pi.*(j-1)./N));
 end
 a_kd(k)=(2./(c(k).*N)).*a_kd(k);
end


%finding b_k
b=zeros(1,N+2);
b(N+2)=0;
b(N+1)=0;


for k = N+1:-1:2
    b(k-1)= (2.*(k-1).*a_kd(k)+b(k+1))./(c(k-1));
end

%finding d_k
d=zeros(1,N+1);
d(N+1)=0;
d(N)=0;


for k = N:-1:2
    d(k-1)= (2.*(k-1).*b(k)+d(k+1))./(c(k-1));
end


%check coeffs     %checking whether b_k is correct or not
d_kd=zeros(1,N+1);
for k = 1:N+1
 for j = 1:N+1
```

```
    d_kd(k)=d_kd(k)+((1./(c(j))).*df2(j).*cos((k-1).*pi.*(j-1)./N));
 end
 d_kd(k)=(2./(c(k).*N)).*d_kd(k);
end

%Inverse Discrete Chebyshev Transform
f2_inversed=zeros(1,N+1);
for j= 1:N+1
    for k =1:N+1
    f2_inversed(j)=f2_inversed(j)+d(k).*cos((k-1).*pi.*(j-1)./N);
    end
end

%Ltwo error
 Ltwoall=0;
 for i= 1:N+1
     Ltwoall=Ltwoall+(abs(f2_inversed(i)-df2(i)).^2);
 end
 Ltwo=(Ltwoall.*(1/N+1)).^(1/2);

subplot(4,1,1)
stem(x,df2,'LineWidth',1.5,'Marker','x');
grid on;
hold on;
stem(x,f2_inversed,'LineWidth',1.5,'Marker','o');
title('N=4, function d')
xlabel('x')
ylabel('Second Derivative')
legend('Exact Second Derivative','Chebyshev Expansion')


N=8;
a_kd=zeros(1,N+1);

%values of c
c=zeros(1,N+1);
c(1)=2;
c(N+1)=2;
for i = 2:N
    c(i)=1;
end

%derivative value
for j = 1:N+1
    x(j)=cos(pi.*(j-1)./N);
    if x(j)<0
        f(j)=1+x(j);
        df2(j)=0;
    else
        f(j)=-1+x(j);
        df2(j)=0;
    end
end
```

```
%Discrete Chebyshev Transform
for k = 1:N+1
 for j = 1:N+1
    a_kd(k)=a_kd(k)+((1./(c(j))).*f(j).*cos((k-1).*pi.*(j-1)./N));
 end
 a_kd(k)=(2./(c(k).*N)).*a_kd(k);
end


%finding b_k
b=zeros(1,N+2);
b(N+2)=0;
b(N+1)=0;


for k = N+1:-1:2
    b(k-1)= (2.*(k-1).*a_kd(k)+b(k+1))./(c(k-1));
end

%finding d_k
d=zeros(1,N+1);
d(N+1)=0;
d(N)=0;


for k = N:-1:2
    d(k-1)= (2.*(k-1).*b(k)+d(k+1))./(c(k-1));
end


%check coeffs     %checking whether b_k is correct or not
d_kd=zeros(1,N+1);
for k = 1:N+1
 for j = 1:N+1
    d_kd(k)=d_kd(k)+((1./(c(j))).*df2(j).*cos((k-1).*pi.*(j-1)./N));
 end
 d_kd(k)=(2./(c(k).*N)).*d_kd(k);
end

%Inverse Discrete Chebyshev Transform
f2_inversed=zeros(1,N+1);
for j= 1:N+1
    for k =1:N+1
    f2_inversed(j)=f2_inversed(j)+d(k).*cos((k-1).*pi.*(j-1)./N);
    end
end

subplot(4,1,2)
stem(x,df2,'LineWidth',1.5,'Marker','x');
grid on;
hold on;
stem(x,f2_inversed,'LineWidth',1.5,'Marker','o');
```

```
title('N=8, function d')
xlabel('x')
ylabel('Second Derivative')
legend('Exact Second Derivative','Chebyshev Expansion')


N=16;
a_kd=zeros(1,N+1);

%values of c
c=zeros(1,N+1);
c(1)=2;
c(N+1)=2;
for i = 2:N
    c(i)=1;
end

%derivative value
for j = 1:N+1
    x(j)=cos(pi.*(j-1)./N);
    if x(j)<0
        f(j)=1+x(j);
        df2(j)=0;
    else
        f(j)=-1+x(j);
        df2(j)=0;
    end
end



%Discrete Chebyshev Transform
for k = 1:N+1
 for j = 1:N+1
    a_kd(k)=a_kd(k)+((1./(c(j))).*f(j).*cos((k-1).*pi.*(j-1)./N));
 end
 a_kd(k)=(2./(c(k).*N)).*a_kd(k);
end


%finding b_k
b=zeros(1,N+2);
b(N+2)=0;
b(N+1)=0;


for k = N+1:-1:2
    b(k-1)= (2.*(k-1).*a_kd(k)+b(k+1))./(c(k-1));
end

%finding d_k
d=zeros(1,N+1);
d(N+1)=0;
d(N)=0;
```

```
for k = N:-1:2
    d(k-1)= (2.*(k-1).*b(k)+d(k+1))./(c(k-1));
end


%check coeffs     %checking whether b_k is correct or not
d_kd=zeros(1,N+1);
for k = 1:N+1
 for j = 1:N+1
    d_kd(k)=d_kd(k)+((1./(c(j))).*df2(j).*cos((k-1).*pi.*(j-1)./N));
 end
 d_kd(k)=(2./(c(k).*N)).*d_kd(k);
end

%Inverse Discrete Chebyshev Transform
f2_inversed=zeros(1,N+1);
for j= 1:N+1
    for k =1:N+1
    f2_inversed(j)=f2_inversed(j)+d(k).*cos((k-1).*pi.*(j-1)./N);
    end
end

subplot(4,1,3)
stem(x,df2,'LineWidth',1.5,'Marker','x');
grid on;
hold on;
stem(x,f2_inversed,'LineWidth',1.5,'Marker','o');
title('N=16, function d')
xlabel('x')
ylabel('Second Derivative')
legend('Exact Second Derivative','Chebyshev Expansion')



N=32;
a_kd=zeros(1,N+1);

%values of c
c=zeros(1,N+1);
c(1)=2;
c(N+1)=2;
for i = 2:N
    c(i)=1;
end

%derivative value
for j = 1:N+1
    x(j)=cos(pi.*(j-1)./N);
   if x(j)<0
        f(j)=1+x(j);
        df2(j)=0;
    else
```

```
        f(j)=-1+x(j);
        df2(j)=0;
    end
end



%Discrete Chebyshev Transform
for k = 1:N+1
 for j = 1:N+1
    a_kd(k)=a_kd(k)+((1./(c(j))).*f(j).*cos((k-1).*pi.*(j-1)./N));
 end
 a_kd(k)=(2./(c(k).*N)).*a_kd(k);
end


%finding b_k
b=zeros(1,N+2);
b(N+2)=0;
b(N+1)=0;


for k = N+1:-1:2
    b(k-1)= (2.*(k-1).*a_kd(k)+b(k+1))./(c(k-1));
end

%finding d_k
d=zeros(1,N+1);
d(N+1)=0;
d(N)=0;


for k = N:-1:2
    d(k-1)= (2.*(k-1).*b(k)+d(k+1))./(c(k-1));
end


%check coeffs     %checking whether b_k is correct or not
d_kd=zeros(1,N+1);
for k = 1:N+1
 for j = 1:N+1
    d_kd(k)=d_kd(k)+((1./(c(j))).*df2(j).*cos((k-1).*pi.*(j-1)./N));
 end
 d_kd(k)=(2./(c(k).*N)).*d_kd(k);
end

%Inverse Discrete Chebyshev Transform
f2_inversed=zeros(1,N+1);
for j= 1:N+1
    for k =1:N+1
    f2_inversed(j)=f2_inversed(j)+d(k).*cos((k-1).*pi.*(j-1)./N);
    end
end
```

```
subplot(4,1,4)
stem(x,df2,'LineWidth',1.5,'Marker','x');
grid on;
hold on;
stem(x,f2_inversed,'LineWidth',1.5,'Marker','o');
title('N=32, function d')
xlabel('x')
ylabel('Second Derivative')
legend('Exact Second Derivative','Chebyshev Expansion')

%error
clc;
clear;


%function a, first derivative
N=[4, 8, 16, 32];
Ltwo1= [0.0847, 6.62e-06, 3.691775591635776e-13,5.594136559722422e-14];
figure(1)
loglog(N,Ltwo1,LineWidth=1)
grid on
hold on
%function a, second derivative
N=[4, 8, 16, 32];
Ltwo1= [0.8185, 2.10e-04, 8.95E-12, 1.6E-12];
loglog(N,Ltwo1,LineWidth=1)
xlabel("N")
ylabel("error")
legend('First Derivative','Second Derivative')
title('Ltwo error function (a)')


%function b, first derivative
N=[4, 8, 16, 32];
Ltwo1= [2.6692, 0.0157,1.65E-9 ,7.01E-13];
figure(2)
loglog(N,Ltwo1,LineWidth=1)
grid on
hold on


%function b, second derivative
N=[4, 8, 16, 32];
Ltwo1= [21.5183, 0.4782,1.65E-7,3.42E-10];
loglog(N,Ltwo1,LineWidth=1)
grid on
xlabel("N")
ylabel("error")
legend('First Derivative','Second Derivative')
title('Ltwo error function (b)')


%function c, first derivative
N=[4, 8, 16, 32];
```

```
Ltwo1= [1.6606, 1.4184,1.3219,1.2864];
figure(3)
loglog(N,Ltwo1,LineWidth=1)
grid on
hold on

%function c, second derivative
N=[4, 8, 16, 32];
Ltwo1= [10.9435, 24.43,50.2153,100.5117];
loglog(N,Ltwo1,LineWidth=1)
grid on
xlabel("N")
ylabel("error")
legend('First Derivative','Second Derivative')
title('Ltwo error function (c)')


%function d, first derivative
N=[4, 8, 16, 32];
Ltwo1= [4.59, 6.98,12.67,24.5];
figure(4)
loglog(N,Ltwo1,LineWidth=1)
grid on
hold on

%function d, second derivative
N=[4, 8, 16, 32];
Ltwo1= [32.21, 130.9,514.0246,2.022E+03];
loglog(N,Ltwo1,LineWidth=1)
grid on
xlabel("N")
ylabel("error")
legend('First Derivative','Second Derivative')
title('Ltwo error function (d)')
```

Problem 4(a)

```
clc;
clear;

Exact Soultion
dx=0.00001;
x=-1:dx:1;
t=[7/8];

 for a= 1:length(t)
     for b=1:length(x)
         c=x(b);
         d=t(a);
         if c>=(-1+3*d)
            u(b)=0;
        else
            u(b)=sin(pi.*(d-((c+1)/3)));
```

```
        end
     end

 end
 figure(1)
 grid on;
  plot(x,u,LineWidth=1)
  hold on;


%Chebyshev Collocation
N=64;
dt=0.00001;
t=0;


%x value value
for j = 1:N+1
    x1(j)=cos(pi.*(j-1)./N);
end

%Initial Conditions
u0=zeros(1,N+1);
u1=zeros(1,N+1);


while t<7/8
    du_by_dx0=ChebyshevDerivative(N,u0);
    du_by_dx1=ChebyshevDerivative(N,u1);

    %Adams-Bashforth
    u2=u1-3.*(1.5.*du_by_dx1-0.5.*du_by_dx0).*dt


    u0=u1;
     u1=u2;
% BC
     u1(1,N+1)=sin(pi*t);


     t=t+dt;
end
 plot(x1,u1,LineWidth=1)
 hold on;


 u = u( ismembertol( x, x1,0.000005 ) )
 u=u(end:-1:1)




 %Ltwo error
 Ltwoall=0;
```

```
 for i= 1:N+1
     Ltwoall=Ltwoall+(abs(u(i)-u1(i)).^2);
 end
 Ltwo=(Ltwoall.*(1/N+1)).^(1/2);


%Chebyshev Tau
N=64;
dt=0.00001;
t=0;


%x value value
for j = 1:N+1
    x2(j)=cos(pi.*(j-1)./N);
end

%Initial Conditions
u0=zeros(1,N+1);
u1=zeros(1,N+1);

a0=ChebyshevTransform(N,u0)
a1=ChebyshevTransform(N,u1)
b0=ChebyshevRecursion(N,a0)
b1=ChebyshevRecursion(N,a1)

 while t<7/8

    %Adams-Bashforth
    a2=a1-(3*(1.5*b1-0.5*b0)*dt);

     t=t+dt;

    % BC

    asum = 0;
    for k = 1:N
        asum = asum + a2(1,k).*((-1).^(k-1));
    end

    a2(1,N+1) = sin(pi*t) - asum;


   a1=a2;
   b0=b1;
   b1=ChebyshevRecursion(N,a2);

end


%Inverse Discrete Chebyshev Transform
u_tau=zeros(1,N+1);
for j= 1:N+1
    for k =1:N+1
```

```
    u_tau(j)=u_tau(j)+a2(1,k).*cos((k-1).*pi.*(j-1)./N);
    end
end

plot(x2,u_tau,LineWidth=1)
hold on

 u = u( ismembertol( x, x2,0.000005 ) )
 u=u(end:-1:1)

 %Ltwo error
 Ltwoall=0;
 for i= 1:N+1
     Ltwoall=Ltwoall+(abs(u(i)-u_tau(i)).^2);
 end
 Ltwo=(Ltwoall.*(1/N+1)).^(1/2);



Finite Difference Method
v=3;
N=64;

spatial discretization
 xmin=-1;
 xmax=1;
 dx=(xmax-xmin)/N;
 x3=xmin:dx:xmax;

 t=0;
 dt=0.00001;

 a=v*dt/dx;
 u1=zeros(1,N+2);
 u2=u1;

 while t<7/8

      u1(1)=sin(pi*t);
      for j=2:N+2
          u2(j)=u1(j)-a.*(u1(j)-u1(j-1));
      end

      u1=u2;

       t=t+dt;
 end
u2=u2(2:end);

u = u( ismembertol( x, x3,0.000004 ) )
 u=u(end:-1:1)
 %Ltwo error

 Ltwoall=0;
```

```
 for i= 1:N+1
     Ltwoall=Ltwoall+(abs(u(i)-u2(i)).^2);
 end
 Ltwo=(Ltwoall.*(1/N+1)).^(1/2);

plot(x3,u2,LineWidth=1)
grid on
xlabel('x')
ylabel('u')
title('t=1/3')
legend('Exact Solution','Chebyshev Collocation','Chebyshev Tau','FDM')
```

Problem 4(b)

```
clc;
clear;

Exact Soultion
dx=0.00001;
x=-1:dx:1;
t=[7/8];

 for a= 1:length(t)
     for b=1:length(x)
         c=x(b);
         d=t(a);
         if (c-3*d)>=-1 && (c-3*d)<= 1
            u(b)=sin(pi*(c-3.*d)/2);

     else
           u(b)=sin(pi.*(d-(c+1)/3))-1;


        end
     end

 end


figure(1)
 grid on;
  plot(x,u,LineWidth=1)
  hold on;


%Chebyshev Collocation
N=64;
dt=0.00001;
t=0;


%x value value
for j = 1:N+1
```

```
    x1(j)=cos(pi.*(j-1)./N);
end

%Initial Conditions

u0=sin(pi*x1/2);

u1=sin(pi*x1/2);


while t<7/8
    du_by_dx0=ChebyshevDerivative(N,u0);
    du_by_dx1=ChebyshevDerivative(N,u1);

    %Adams-Bashforth
    u2=u1-3.*(1.5.*du_by_dx1-0.5.*du_by_dx0).*dt


    u0=u1;
     u1=u2;
 %BC
     u1(1,N+1)=sin(pi*t)-1;


     t=t+dt;
end
 plot(x1,u1,LineWidth=1)
 hold on;


 u = u( ismembertol( x, x1,0.000005 ) )
 u=u(end:-1:1)




 %Ltwo error
 Ltwoall=0;
 for i= 1:N+1
     Ltwoall=Ltwoall+(abs(u(i)-u1(i)).^2);
 end
 Ltwo=(Ltwoall.*(1/N+1)).^(1/2);


%Chebyshev Tau
N=64;
dt=0.000001;
t=0;


%x value value
for j = 1:N+1
    x2(j)=cos(pi.*(j-1)./N);
end
```

```
%Initial Conditions
u0=sin(pi*x2/2);

u1=sin(pi*x2/2);

a0=ChebyshevTransform(N,u0)
a1=ChebyshevTransform(N,u1)
b0=ChebyshevRecursion(N,a0)
b1=ChebyshevRecursion(N,a1)

 while t<7/8

    %Adams-Bashforth
    a2=a1-(3*(1.5*b1-0.5*b0)*dt);

     t=t+dt;

    % BC

    asum = 0;
    for k = 1:N
        asum = asum + a2(1,k).*((-1).^(k-1));
    end

    a2(1,N+1) = (sin(pi*t) -1)- asum;


   a1=a2;
   b0=b1;
   b1=ChebyshevRecursion(N,a2);

end


%Inverse Discrete Chebyshev Transform
u_tau=zeros(1,N+1);
for j= 1:N+1
    for k =1:N+1
    u_tau(j)=u_tau(j)+a2(1,k).*cos((k-1).*pi.*(j-1)./N);
    end
end

plot(x2,u_tau,LineWidth=1)
hold on

 u = u( ismembertol( x, x2,0.000005 ) )
 u=u(end:-1:1)

 %Ltwo error
 Ltwoall=0;
 for i= 1:N+1
     Ltwoall=Ltwoall+(abs(u(i)-u_tau(i)).^2);
 end
```

```
 Ltwo=(Ltwoall.*(1/N+1)).^(1/2);



Finite Difference Method
v=3;
N=64;

spatial discretization
 xmin=-1;
 xmax=1;
 dx=(xmax-xmin)/N;
 x3=xmin:dx:xmax;

 t=0;
 dt=0.00001;

 a=v*dt/dx;

 u1=sin(pi*x3/2);
 u1(end+1)=0;

 u2=u1;

 while t<7/8

      u1(1)=sin(pi*t)-1;
      for j=2:N+2
          u2(j)=u1(j)-a.*(u1(j)-u1(j-1));
      end

      u1=u2;

       t=t+dt;
 end

u2=u2(2:end)

u = u( ismembertol( x, x3,0.000004 ) )
 u=u(end:-1:1)
 %Ltwo error

 Ltwoall=0;
 for i= 1:N+1
     Ltwoall=Ltwoall+(abs(u(i)-u2(i)).^2);
 end
 Ltwo=(Ltwoall.*(1/N+1)).^(1/2);

plot(x3,u2,LineWidth=1)
grid on
xlabel('x')
ylabel('u')
title('t=7/8')
legend('Exact Solution','Chebyshev Collocation','Chebyshev Tau','FDM')
```

Problem 4:Error

```
%used 1-^-4:


clc;clear;
Chebyshevcollocation
N=[ 8, 16, 32,64];
Ltwo1= [0.0055, 0.0017,4.15E-4,2.38E-4];
figure(1)
loglog(N,Ltwo1,LineWidth=1)
hold on


Chebyshevtau
N=[ 8, 16, 32,64];
Ltwo1= [0.0028,7.78E-4, 1.6E-4,1.5E-5];
loglog(N,Ltwo1,LineWidth=1)
grid on
hold on

FDM
N=[ 8, 16, 32,64];
Ltwo1= [0.3073,0.3339, 0.5798, 0.9626];
Ltwo1=Ltwo1(end:-1:1);
loglog(N,Ltwo1,LineWidth=1)
xlabel('N')
ylabel('error')
legend('Chebyshev Collocation','Chebyshev Tau','FDM')
title('Problem 4(a)')

%function 2
clc;clear;
%Chebyshevcollocation
N=[ 8, 16, 32,64];
Ltwo1= [0.0064,0.0017,4.11e-4,9.35E-5 ];
figure(1)
loglog(N,Ltwo1,LineWidth=1)
hold on


%Chebyshevtau
N=[ 8, 16, 32,64];
Ltwo1= [0.00041, 7.56E-4, 1E-4, 2.33E-5];
loglog(N,Ltwo1,LineWidth=1)
grid on
hold on

%FDM
N=[ 8, 16, 32,64];
Ltwo1= [0.344, 0.351, 0.5825,0.9621];
Ltwo1=Ltwo1(end:-1:1);
loglog(N,Ltwo1,LineWidth=1)
```

```
xlabel('N')
ylabel('error')
legend('Chebyshev Collocation','Chebyshev Tau','FDM')
title('Problem 4(b)')
```

Problem 5:Collocation

```
clc;
clear;
N=16;
dt=0.0001;
t=0;


%x value value
for j = 1:N+1
    x(j)=cos(pi.*(j-1)./N);    %is y
end


%Initial Conditions
u0=zeros(1,N+1);
u1=zeros(1,N+1);

fn=3*(10^4)/800;

while t<10
    du2_by_dx02=(ChebyshevSecondDerivative(N,u0)).*(100.^2);
    du2_by_dx12=(ChebyshevSecondDerivative(N,u1)).*(100.^2);


    %Adams-Bashforth

    u2=u1(1:end-1)+2.17E-4.*(1.5.*du2_by_dx12-0.5.*du2_by_dx02).*dt


    u0=u1;
     u1=u2;
 %BC
     u1(1,1)=0;
     u1(1,N+1)=40;



     t=t+dt;
end

 steady-state
 c=0;
 u=-20*100*x+20*x;
 plot(x,u1,Linewidth=1);
 grid on;
 hold on;
```

```
 plot(x,u,Linewidth=1);
 xlabel('y')
 ylabel('u')
 title('Comparison')
 legend('solution','steady state solution')
 hold on;

 N=16;
dt=0.0001;
t=0;


%x value value
for j = 1:N+1
    x(j)=cos(pi.*(j-1)./N);    %is y
end


%Initial Conditions
u0=zeros(1,N+1);
u1=zeros(1,N+1);

fn=3*(10^4)/800;

while t<1.08
    du2_by_dx02=(ChebyshevSecondDerivative(N,u0)).*(100.^2);
    du2_by_dx12=(ChebyshevSecondDerivative(N,u1)).*(100.^2);


    %Adams-Bashforth


 u2=u1(1:end-1)+(2.17E-4.*(1.5.*du2_by_dx12-0.5.*du2_by_dx02)+1.5.*fn-0.5.*fn).*dt


    u0=u1;
     u1=u2;
 %BC
     u1(1,1)=0;
     u1(1,N+1)=40;



     t=t+dt;
end
 plot(x,u1,Linewidth=1);
 grid on;
 xlabel('y')
 ylabel('u')
 title('u for different dp/dx=0: Case I')
 hold on;

 N=16;
dt=0.0001;
```

```
t=0;


%x value value
for j = 1:N+1
    x(j)=cos(pi.*(j-1)./N);    %is y
end


%Initial Conditions
u0=zeros(1,N+1);
u1=zeros(1,N+1);

fn=3*(10^4)/800;

while t<0
    du2_by_dx02=(ChebyshevSecondDerivative(N,u0)).*(100.^2);
    du2_by_dx12=(ChebyshevSecondDerivative(N,u1)).*(100.^2);


    %Adams-Bashforth


 u2=u1(1:end-1)+(2.17E-4.*(1.5.*du2_by_dx12-0.5.*du2_by_dx02)+1.5.*fn-0.5.*fn).*dt

    u0=u1;
     u1=u2;
 %BC
     u1(1,1)=0;
     u1(1,N+1)=40;



     t=t+dt;
end
 plot(x,u1,Linewidth=1);
 grid on;
 xlabel('y')
 ylabel('u')
 title('Collocation Method: u for different dp/dx=-3e4: Case III')
 legend('t=0.18','t=1.08','t=0')
```

Problem 5:Tau

```
clc;
clear;
N=8;
dt=0.00001;
t=0;


%x value value
for j = 1:N+1
```

```
    x(j)=cos(pi.*(j-1)./N);
end

%Initial Conditions
u0=zeros(1,N+1);
u1=zeros(1,N+1);

a0=ChebyshevTransform(N,u0)
a1=ChebyshevTransform(N,u1)
b0=ChebyshevRecursion(N,a0)
b1=ChebyshevRecursion(N,a1)
 while t<1/3

    %Adams-Bashforth
    a2=a1+(2.17E-4*(10.^4)*(1.5*b1-0.5*b0)*dt);

     t=t+dt;

    % BC

    asum = 0;
    for k = 1:N
        asum = asum + a2(1,k).*((-1).^(k-1));
    end
    a2(1,1) = 0 - asum;
    a2(1,N+1) = 40 - asum;


   a1=a2;
   b0=b1;
   b1=ChebyshevRecursion(N,a2);

end


%Inverse Discrete Chebyshev Transform
u=zeros(1,N+1);
for j= 1:N+1
    for k =1:N+1
    u(j)=u(j)+a2(1,k).*cos((k-1).*pi.*(j-1)./N);
    end
end

plot(x,u)
```

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