Table of Contents

PROBLEM 1

```
clc;
clear;
L=pi;
N=16;
h=L/N;
k=1:N+1;

%exact
kh=(k-1).*h;
figure(1)
plot(kh,kh,LineWidth=1)
hold on;

%4th order central
kdashh= (8.*sin(kh)-sin(2.*kh))./6;
plot(kh,kdashh,LineWidth=1)
hold on;

%Pade
alpha=4;
beta=1;
kdashp=(6.*beta.*sin(kh))./(alpha+2.*cos(kh));
plot(kh,kdashp,LineWidth=1)
grid on;
xlabel('kh')
ylabel("k'h")
title('Modified Wave Number')
legend('Exact kh','4th Order Central','Pade')
saveas(figure(1),'problem1a','jpg')
```

PROBLEM 3A

```
clc;
```

```
clear;
x = (0:0.001:2.*pi);
func=x;
subplot(2,1,1)
plot(x,func,'linewidth',1)
hold on

f=pi;
for k=1:64
      ak=(1/(pi*(k.^2)))*((2.*pi*k*sin(2.*pi*k))+cos(2.*pi*k)-1);
      bk=(1/(pi*(k.^2)))*(sin(2.*pi*k)-(2.*pi*k*cos(2.*pi*k)));
      f= f+ak.*cos(k*x)+bk.*sin(k*x);
end
plot(x,f,'linewidth',1)
xlabel('X')
ylabel('f(x)')
title('N=64')
legend('f(x)','Fourier Rep with N=64')
grid on


x = (0:0.001:2.*pi);
func=x;
subplot(2,1,2)
plot(x,func,'linewidth',1)
hold on

f=pi;
for k=1:128
      ak=(1/(pi*(k.^2)))*((2.*pi*k*sin(2.*pi*k))+cos(2.*pi*k)-1);
      bk=(1/(pi*(k.^2)))*(sin(2.*pi*k)-(2.*pi*k*cos(2.*pi*k)));
      f= f+ak.*cos(k*x)+bk.*sin(k*x);
end
plot(x,f,'linewidth',1)
xlabel('X')
ylabel('f(x)')
title('N=128')
legend('f(x)','Fourier Rep with N=128')
grid on
```

PROBLEM 3B

```
clc;
clear;
x = (0:0.01:2.*pi);
func= zeros(size(x));
func(x<315)=x;
func(316:629) = 2.*pi-x(316:629);
subplot(2,1,1)
plot(x,func,'linewidth',1)
hold on;

f=pi/2;
```

```
for k=1:64
      ak=(1/(pi*(k.^2)))*((2.*cos(pi.*k))-cos(2.*pi*k)-1);
      bk=(1/(pi*(k.^2)))*(2.*sin(pi.*k)-(sin(2.*pi*k)));
      f= f+ak.*cos(k*x)+bk.*sin(k*x);
end
plot(x,f,'linewidth',1)
xlabel('X')
ylabel('f(x)')
title('N=64')
legend('f(x)','Fourier Rep with N=64')
grid on

x = (0:0.01:2.*pi);
func= zeros(size(x));
func(x<33)=x;
func(316:629) = 2.*pi-x(316:629);
subplot(2,1,2)
plot(x,func,'linewidth',1)
hold on;

f=pi/2;
for k=1:128
      ak=(1/(pi*(k.^2)))*((2.*cos(pi.*k))-cos(2.*pi*k)-1);
      bk=(1/(pi*(k.^2)))*(2.*sin(pi.*k)-(sin(2.*pi*k)));
      f= f+ak.*cos(k*x)+bk.*sin(k*x);
end
plot(x,f,'linewidth',1)
xlabel('X')
ylabel('f(x)')
title('N=128')
legend('f(x)','Fourier Rep with N=128')
grid on
```

PROBLEM 4A

```
clc;
clear;

%exact differentiation

L=2.*pi;
N=8;                    %varied for each problem
func=zeros(1,N);
 for j =1:N+1
     x(j)=(L/N).*(j-1);
     func(j)=2.*(cos(2.*x(j)))-24.*(sin(6.*x(j)));
 end

 subplot(3,1,1)
 plot(x,func,Marker="o",LineWidth=1)
 grid on
 hold on
```

```
%differentiation using Fourier Method

k=-N/2:1:(N/2-1);
f_k=zeros(1,N);
f_kdash=zeros(1,N);
f=zeros(1,N);
f_dash=zeros(1,N);
x=zeros(1,N);
 for j = 1:N
    x(j)=(L/N).*(j-1);
    f(j)= sin(2.*x(j))+4.*cos(6.*x(j));
 end
 f_k=fft(f)/N;
 f_k=fftshift(f_k);

 for j =1:N
     f_kdash(j)=f_k(j).*(1i.*(2.*pi./L).*k(j));
 end
%remove the shiftnow
f_kdash=ifftshift(f_kdash);

%calculating inverse fft
 f_dash=ifft(f_kdash.*N);
 %f_dash=(fftshift(f_dash));

 plot(x,f_dash,Marker="*",LineWidth=1)

 %error
 for i=1:N
    error_fourier(i)=func(i)-f_dash(i);
 end
 xlabel('x')
 ylabel("f'(x)")
 title('Exact Vs Fourier for N = 8')
 plot(x,error_fourier,Marker='+',LineWidth=1)
 legend('Exact Derivative','Fourier Transform Derivative','Error')



%N=16

%exact differentiation
clc;clear;
L=2.*pi;
N=16;                    %varied for each problem
func=zeros(1,N);
 for j =1:N+1
     x(j)=(L/N).*(j-1);
     func(j)=2.*(cos(2.*x(j)))-24.*(sin(6.*x(j)));
 end

 subplot(3,1,2)
```

```
 plot(x,func,Marker="o",LineWidth=1)
 grid on
 hold on


%differentiation using Fourier Method

k=-N/2:1:(N/2-1);
f_k=zeros(1,N);
f_kdash=zeros(1,N);
f=zeros(1,N);
f_dash=zeros(1,N);
x=zeros(1,N);
 for j = 1:N
    x(j)=(L/N).*(j-1);
    f(j)= sin(2.*x(j))+4.*cos(6.*x(j));
 end
 f_k=fft(f)/N;
 f_k=fftshift(f_k);

 for j =1:N
     f_kdash(j)=f_k(j).*(1i.*(2.*pi./L).*k(j));
 end
%remove the shiftnow
f_kdash=ifftshift(f_kdash);

%calculating inverse fft
 f_dash=ifft(f_kdash.*N);
% f_dash=(fftshift(f_dash));

 plot(x,f_dash,Marker="*",LineWidth=1)

 %error
 for i=1:N
    error_fourier(i)=func(i)-f_dash(i);
 end
 xlabel('x')
 ylabel("f'(x)")
 title('Exact Vs Fourier for N = 16')
 plot(x,error_fourier,Marker='+',LineWidth=1)
 legend('Exact Derivative','Fourier Transform Derivative','Error')


%N=32

%exact differentiation
clc;clear;
L=2.*pi;
N=128;                    %varied for each problem
func=zeros(1,N);
 for j =1:N+1
     x(j)=(L/N).*(j-1);
     func(j)=2.*(cos(2.*x(j)))-24.*(sin(6.*x(j)));
 end
```

```
 subplot(3,1,3)
 plot(x,func,Marker="o",LineWidth=1)
 grid on
 hold on


%differentiation using Fourier Method

k=-N/2:1:(N/2-1);
f_k=zeros(1,N);
f_kdash=zeros(1,N);
f=zeros(1,N);
f_dash=zeros(1,N);
x=zeros(1,N);
 for j = 1:N
    x(j)=(L/N).*(j-1);
    f(j)= sin(2.*x(j))+4.*cos(6.*x(j));
 end
 f_k=fft(f)/N;
 f_k=fftshift(f_k);

 for j =1:N
     f_kdash(j)=f_k(j).*(1i.*(2.*pi./L).*k(j));
 end
%remove the shiftnow
f_kdash=ifftshift(f_kdash);

%calculating inverse fft
 f_dash=ifft(f_kdash.*N);
% f_dash=(fftshift(f_dash));

 plot(x,f_dash,Marker="*",LineWidth=1)

 %error
 for i=1:N
    error_fourier(i)=func(i)-f_dash(i);
 end
 xlabel('x')
 ylabel("f'(x)")
 title('Exact Vs Fourier for N = 32')
 plot(x,error_fourier,Marker='+',LineWidth=1)
 legend('Exact Derivative','Fourier Transform Derivative','Error')
```

PROBLEM 4A

```
clc;
clear;

%exact differentiation

L=2.*pi;
N=8;                    %varied for each problem
```

```
func=zeros(1,N);
 for j =1:N+1
     x(j)=(L/N).*(j-1);
     func(j)=2.*(cos(2.*x(j)))-24.*(sin(6.*x(j)));
 end

 subplot(3,1,1)
 plot(x,func,Marker="o",LineWidth=1)
 grid on
 hold on

 %Differentiation using finite element
  f=zeros(1,N+1)
 df2=zeros(1,N+1)
 x=zeros(1,N+1);
 for j = 1:N+1
    x(j)=(L/N).*(j-1);
    f(j)= sin(2.*x(j))+4.*cos(6.*x(j));
 end

 df2(1)=(f(2)-f(N))./(2.*(L./N));
 df2(N+1)=df2(1);
 for j = 2:N
    df2(j)= (f(j+1)-f(j-1))/(2.*(L./N));
 end
 plot(x,df2,Marker="*",LineWidth=1)


 %error
 for i=1:N+1
    error_finite(i)=func(i)-df2(i);
 end
 xlabel('x')
 ylabel("f'(x)")
 title('Exact Vs Finite Element for N = 8')
 plot(x,error_finite,Marker='+',LineWidth=1)
 legend('Exact Derivative','Finite Element Derivative','Error')



%N=16

%exact differentiation
clc;clear;
L=2.*pi;
N=16;                    %varied for each problem
func=zeros(1,N);
 for j =1:N+1
     x(j)=(L/N).*(j-1);
     func(j)=2.*(cos(2.*x(j)))-24.*(sin(6.*x(j)));
 end

 subplot(3,1,2)
```

```
plot(x,func,Marker="o",LineWidth=1)
grid on
hold on

%Differentiation using finite element
f=zeros(1,N+1)
df2=zeros(1,N+1)
x=zeros(1,N+1);
for j = 1:N+1
   x(j)=(L/N).*(j-1);
   f(j)= sin(2.*x(j))+4.*cos(6.*x(j));
end

df2(1)=(f(2)-f(N))./(2.*(L./N));
df2(N+1)=df2(1);
for j = 2:N
   df2(j)= (f(j+1)-f(j-1))/(2.*(L./N));
end
plot(x,df2,Marker="*",LineWidth=1)


%error
for i=1:N+1
   error_finite(i)=func(i)-df2(i);
end
xlabel('x')
ylabel("f'(x)")
title('Exact Vs Finite Element for N = 16')
plot(x,error_finite,Marker='+',LineWidth=1)
legend('Exact Derivative','Finite Element Derivative','Error')



%N=32

%exact differentiation
clc;clear;
L=2.*pi;
N=32;                    %varied for each problem
func=zeros(1,N);
 for j =1:N+1
     x(j)=(L/N).*(j-1);
     func(j)=2.*(cos(2.*x(j)))-24.*(sin(6.*x(j)));
 end

 subplot(3,1,3)
 plot(x,func,Marker="o",LineWidth=1)
 grid on
 hold on

  %Differentiation using finite element
  f=zeros(1,N+1)
 df2=zeros(1,N+1)
```

```
x=zeros(1,N+1);
for j = 1:N+1
   x(j)=(L/N).*(j-1);
   f(j)= sin(2.*x(j))+4.*cos(6.*x(j));
end

df2(1)=(f(2)-f(N))./(2.*(L./N));
df2(N+1)=df2(1);
for j = 2:N
   df2(j)= (f(j+1)-f(j-1))/(2.*(L./N));
end
plot(x,df2,Marker="*",LineWidth=1)


%error
for i=1:N+1
   error_finite(i)=func(i)-df2(i);
end

xlabel('x')
ylabel("f'(x)")
title('Exact Vs Finite Element for N = 32')
plot(x,error_finite,Marker='+',LineWidth=1)
legend('Exact Derivative','Finite Element Derivative','Error')
```

PROBLEM 4A

```
clc;
clear;
L=2.*pi;
N=32;                          %varied for each value of N
a=1;
b=-1;
c=-2;

%calculating f(n)
f_k=zeros(1,N);
k=-(N/2):1:((N/2)-1);
u_n=zeros(1,N);
for m = 1:N
 for j = 1:N
    x(j)=(L/N).*(j-1);

 f_k(m)=f_k(m)+((2+6.*sin(6.*x(j))+4.*cos(10.*x(j))).*exp(-1i.*2.*pi.*k(m).*x(j)/
L));

 end
 f_k(m)=(1/N).*f_k(m);
end

%calculating u_n
for m =2:N
```

```
    u_n(m)=f_k(m)./((-a.*((2.*pi.*(m-((N/2)+1))./L).^2))+(b.*(1i).*(2.*pi.*(m-
((N/2)+1))./L))+c);
end
u_n(1)=f_k(1)./((-a.*((2.*pi.*1./L).^2))+c);


%calculating u(j)
 u_j=zeros(1,N);
 for j= 1:N
     for m= 1:N
         x(j)=(L/N).*(j-1);
         u_j(j)=u_j(j)+(u_n(m).*exp(1i.*2.*pi.*k(m).*x(j)/L));
     end
 end
```

PROBLEM 4B

```
%exact solution
x=-pi/2:0.01:5.*pi/3;                    %varied for each BC
u_x=(9*cos(6*x))/370 - (51*cos(10*x))/1313 - (57*sin(6*x))/370 -
 (5*sin(10*x))/1313 - 1;
figure(1)
plot(x,u_x,LineWidth=1);
xlabel('x')
ylabel('u(x)')
grid on;


L=2.*pi;
N=8;                          %varied for each value of N
a=1;
b=-1;
c=-2;

%calculating f(n)
f_k=zeros(1,N);
k=-(N/2):1:((N/2)-1);
u_n=zeros(1,N);
x=zeros(1,N);
for m = 1:N
 for j = 1:N
    x(j)=(L/N).*(j-(1+(pi/3)));          %varied for each BC

 f_k(m)=f_k(m)+((2+6.*sin(6.*x(j))+4.*cos(10.*x(j))).*exp(-1i.*2.*pi.*k(m).*x(j)/
L));

 end
 f_k(m)=(1/N).*f_k(m);
end

%calculating u_n
for m =2:N
```

```
    u_n(m)=f_k(m)./((-a.*((2.*pi.*(m-((N/2)+1))./L).^2))+(b.*(1i).*(2.*pi.*(m-
((N/2)+1))./L))+c);
end
u_n(1)=f_k(1)./((-a.*((2.*pi.*1./L).^2))+c);


%calculating u(j)
 u_j=zeros(1,N);
 u_x=zeros(1,N);
 for j= 1:N
     for m= 1:N
         x(j)=(L/N).*(j-(1+(pi/3)));
         u_j(j)=u_j(j)+(u_n(m).*exp(1i.*2.*pi.*k(m).*x(j)/L));
         u_x(j)=(9.*cos(6.*x(j)))/370 - (51.*cos(10.*x(j)))/1313 -
 (57.*sin(6*x(j)))/370 - (5.*sin(10.*x(j)))/1313 - 1;
     end
 end
 figure(2)
 subplot(4,1,1)
 plot(x,u_x,LineWidth=1,Marker='o');
 grid on;
 hold on;
 plot(x,u_j,LineWidth=1,Marker='*');
 xlabel('x')
 ylabel('u(x)')
 title('N=8')
 legend('Exact Derivative','Discrete Fourier Method')

 %error
 %Ltwo error
 Ltwoall=0;
 for i= 1:N
     Ltwoall=Ltwoall+(abs(u_j(i)-u_x(i)).^2);
 end
 Ltwo1=(Ltwoall.*(1/N)).^(1/2);

L=2.*pi;
N=16;                          %varied for each value of N
a=1;
b=-1;
c=-2;

%calculating f(n)
f_k=zeros(1,N);
k=-(N/2):1:((N/2)-1);
u_n=zeros(1,N);
for m = 1:N
 for j = 1:N
    x(j)=(L/N).*(j-(1+(pi/3)));

 f_k(m)=f_k(m)+((2+6.*sin(6.*x(j))+4.*cos(10.*x(j))).*exp(-1i.*2.*pi.*k(m).*x(j)/
L));

 end
```

```
 f_k(m)=(1/N).*f_k(m);
end

%calculating u_n
for m =2:N
    u_n(m)=f_k(m)./((-a.*((2.*pi.*(m-((N/2)+1))./L).^2))+(b.*(1i).*(2.*pi.*(m-
((N/2)+1))./L))+c);
end
u_n(1)=f_k(1)./((-a.*((2.*pi.*1./L).^2))+c);


%calculating u(j)
 u_j=zeros(1,N);
 u_x=zeros(1,N);
 for j= 1:N
     for m= 1:N
         x(j)=(L/N).*(j-(1+(pi/3)));
         u_j(j)=u_j(j)+(u_n(m).*exp(1i.*2.*pi.*k(m).*x(j)/L));
         u_x(j)=(9.*cos(6.*x(j)))/370 - (51.*cos(10.*x(j)))/1313 -
 (57.*sin(6*x(j)))/370 - (5.*sin(10.*x(j)))/1313 - 1;
     end
 end
 subplot(4,1,2)
 plot(x,u_x,LineWidth=1,Marker='o')
 hold on;
 grid on;
 plot(x,u_j,LineWidth=1,Marker='*')
 xlabel('x')
 ylabel('u(x)')
 title('N=16')
 legend('Exact Derivative','Discrete Fourier Method')

 %error
 %Ltwo error
 Ltwoall=0;
 for i= 1:N
     Ltwoall=Ltwoall+(abs(u_j(i)-u_x(i)).^2);
 end
 Ltwo2=(Ltwoall.*(1/N)).^(1/2);

 L=2.*pi;
N=32;                          %varied for each value of N
a=1;
b=-1;
c=-2;

%calculating f(n)
f_k=zeros(1,N);
k=-(N/2):1:((N/2)-1);
u_n=zeros(1,N);
for m = 1:N
 for j = 1:N
    x(j)=(L/N).*(j-(1+(pi/3)));
```

```
 f_k(m)=f_k(m)+((2+6.*sin(6.*x(j))+4.*cos(10.*x(j))).*exp(-1i.*2.*pi.*k(m).*x(j)/
L));

 end
 f_k(m)=(1/N).*f_k(m);
end

%calculating u_n
for m =2:N
    u_n(m)=f_k(m)./((-a.*((2.*pi.*(m-((N/2)+1))./L).^2))+(b.*(1i).*(2.*pi.*(m-
((N/2)+1))./L))+c);
end
u_n(1)=f_k(1)./((-a.*((2.*pi.*1./L).^2))+c);


%calculating u(j)
 u_j=zeros(1,N);
 u_x=zeros(1,N);
 for j= 1:N
     for m= 1:N
         x(j)=(L/N).*(j-(1+(pi/3)));
         u_j(j)=u_j(j)+(u_n(m).*exp(1i.*2.*pi.*k(m).*x(j)/L));
         u_x(j)=(9.*cos(6.*x(j)))/370 - (51.*cos(10.*x(j)))/1313 -
 (57.*sin(6*x(j)))/370 - (5.*sin(10.*x(j)))/1313 - 1;
     end
 end
 subplot(4,1,3)
 plot(x,u_x,LineWidth=1,Marker='o')
 hold on;
 grid on;
 plot(x,u_j,LineWidth=1,Marker='*')
 xlabel('x')
 ylabel('u(x)')
 title('N=32')
 legend('Exact Derivative','Discrete Fourier Method')

 %error
 %Ltwo error
 Ltwoall=0;
 for i= 1:N
     Ltwoall=Ltwoall+(abs(u_j(i)-u_x(i)).^2);
 end
 Ltwo3=(Ltwoall.*(1/N)).^(1/2);

 L=2.*pi;
N=64;                        %varied for each value of N
a=1;
b=-1;
c=-2;

%calculating f(n)
f_k=zeros(1,N);
k=-(N/2):1:((N/2)-1);
```

```
u_n=zeros(1,N);
for m = 1:N
 for j = 1:N
    x(j)=(L/N).*(j-(1+(pi/3)));

 f_k(m)=f_k(m)+((2+6.*sin(6.*x(j))+4.*cos(10.*x(j))).*exp(-1i.*2.*pi.*k(m).*x(j)/
L));

 end
 f_k(m)=(1/N).*f_k(m);
end

%calculating u_n
for m =2:N
    u_n(m)=f_k(m)./((-a.*((2.*pi.*(m-((N/2)+1))./L).^2))+(b.*(1i).*(2.*pi.*(m-
((N/2)+1))./L))+c);
end
u_n(1)=f_k(1)./((-a.*((2.*pi.*1./L).^2))+c);


%calculating u(j)
 u_j=zeros(1,N);
 u_x=zeros(1,N);
 for j= 1:N
     for m= 1:N
         x(j)=(L/N).*(j-(1+(pi/3)));
         u_j(j)=u_j(j)+(u_n(m).*exp(1i.*2.*pi.*k(m).*x(j)/L));
         u_x(j)=(9.*cos(6.*x(j)))/370 - (51.*cos(10.*x(j)))/1313 -
 (57.*sin(6*x(j)))/370 - (5.*sin(10.*x(j)))/1313 - 1;
     end
 end
 subplot(4,1,4)
 plot(x,u_x,LineWidth=1,Marker='o')
 hold on;
 grid on;
 plot(x,u_j,LineWidth=1,Marker='*')
 xlabel('x')
 ylabel('u(x)')
 title('N=64')
 legend('Exact Derivative','Discrete Fourier Method')

 %error
 %Ltwo error
 Ltwoall=0;
 for i= 1:N
     Ltwoall=Ltwoall+(abs(u_j(i)-u_x(i)).^2);
 end
 Ltwo4=(Ltwoall.*(1/N)).^(1/2);
```

PROBLEM 4B

```
clc;
clear;
```

```
%exact differentiation

L=2.*pi;
N=8;                    %varied for each problem
func=zeros(1,N);
 for j =1:N+1
     x(j)=(L/N).*(j-1);
     func(j)=3+2.*x(j);
 end

 subplot(3,1,1)
 plot(x,func,Marker="o",LineWidth=1)
 grid on
 hold on

 %Differentiation using finite element
  f=zeros(1,N+1)
 df2=zeros(1,N+1)
 x=zeros(1,N+1);
 for j = 1:N+1
    x(j)=(L/N).*(j-1);
    f(j)= 3.*x(j)+(x(j).^2);
 end

  df2(1)=(f(2)-f(N))./(2.*(L./N));
  df2(N+1)=df2(1);
 for j = 2:N
    df2(j)= (f(j+1)-f(j-1))/(2.*(L./N));
 end
 plot(x,df2,Marker="*",LineWidth=1)


 %error
 for i=1:N+1
    error_finite(i)=func(i)-df2(i);
 end
 xlabel('x')
 ylabel("f'(x)")
 title('Exact Vs Finite Element for N = 8')
 plot(x,error_finite,Marker='+',LineWidth=1)
 legend('Exact Derivative','Finite Element Derivative','Error')



%N=16

%exact differentiation
clc;clear;
L=2.*pi;
N=16;                    %varied for each problem
func=zeros(1,N);
 for j =1:N+1
```

```
    x(j)=(L/N).*(j-1);
    func(j)=3+2.*x(j);
end

subplot(3,1,2)
plot(x,func,Marker="o",LineWidth=1)
grid on
hold on

%Differentiation using finite element
f=zeros(1,N+1)
df2=zeros(1,N+1)
x=zeros(1,N+1);
for j = 1:N+1
   x(j)=(L/N).*(j-1);
   f(j)= 3.*x(j)+(x(j).^2);
end

df2(1)=(f(2)-f(N))./(2.*(L./N));
df2(N+1)=df2(1);
for j = 2:N
   df2(j)= (f(j+1)-f(j-1))/(2.*(L./N));
end
plot(x,df2,Marker="*",LineWidth=1)


%error
for i=1:N+1
   error_finite(i)=func(i)-df2(i);
end
xlabel('x')
ylabel("f'(x)")
title('Exact Vs Finite Element for N = 16')
plot(x,error_finite,Marker='+',LineWidth=1)
legend('Exact Derivative','Finite Element Derivative','Error')



%N=32

%exact differentiation
clc;clear;
L=2.*pi;
N=32;                    %varied for each problem
func=zeros(1,N);
 for j =1:N+1
     x(j)=(L/N).*(j-1);
     func(j)=3+2.*x(j);
 end

 subplot(3,1,3)
 plot(x,func,Marker="o",LineWidth=1)
 grid on
```

```
 hold on

  %Differentiation using finite element
  f=zeros(1,N+1)
 df2=zeros(1,N+1)
 x=zeros(1,N+1);
 for j = 1:N+1
    x(j)=(L/N).*(j-1);
    f(j)=3.*x(j)+(x(j).^2);
 end

 df2(1)=(f(2)-f(N))./(2.*(L./N));
 df2(N+1)=df2(1);
 for j = 2:N
    df2(j)= (f(j+1)-f(j-1))/(2.*(L./N));
 end
 plot(x,df2,Marker="*",LineWidth=1)


 %error
 for i=1:N+1
    error_finite(i)=func(i)-df2(i);
 end

 xlabel('x')
 ylabel("f'(x)")
 title('Exact Vs Finite Element for N = 32')
 plot(x,error_finite,Marker='+',LineWidth=1)
 legend('Exact Derivative','Finite Element Derivative','Error')
```

PROBLEM 5A

```
 clc;
clear;
L=2.*pi;
N=32;                          %varied for each value of N
a=1;
b=-1;
c=-2;

%calculating f(n)
f_k=zeros(1,N);
k=-(N/2):1:((N/2)-1);
u_n=zeros(1,N);
for m = 1:N
 for j = 1:N
    x(j)=(L/N).*(j-1);

 f_k(m)=f_k(m)+((2+6.*sin(6.*x(j))+4.*cos(10.*x(j))).*exp(-1i.*2.*pi.*k(m).*x(j)/
L));

 end
 f_k(m)=(1/N).*f_k(m);
```

```
end

%calculating u_n
for m =2:N
    u_n(m)=f_k(m)./((-a.*((2.*pi.*(m-((N/2)+1))./L).^2))+(b.*(1i).*(2.*pi.*(m-
((N/2)+1))./L))+c);
end
u_n(1)=f_k(1)./((-a.*((2.*pi.*1./L).^2))+c);


%calculating u(j)
 u_j=zeros(1,N);
 for j= 1:N
     for m= 1:N
         x(j)=(L/N).*(j-1);
         u_j(j)=u_j(j)+(u_n(m).*exp(1i.*2.*pi.*k(m).*x(j)/L));
     end
 end
```

PROBLEM 5B

```
clc;clear;

%exact solution
x=-pi/2:0.01:5.*pi/3;                    %varied for each BC
u_x=(9*cos(6*x))/370 - (51*cos(10*x))/1313 - (57*sin(6*x))/370 -
 (5*sin(10*x))/1313 - 1;
figure(1)
plot(x,u_x,LineWidth=1);
xlabel('x')
ylabel('u(x)')
grid on;


L=2.*pi;
N=8;                          %varied for each value of N
a=1;
b=-1;
c=-2;

%calculating f(n)
f_k=zeros(1,N);
k=-(N/2):1:((N/2)-1);
u_n=zeros(1,N);
x=zeros(1,N);
for m = 1:N
 for j = 1:N
    x(j)=(L/N).*(j-(1+(pi/3)));          %varied for each BC

 f_k(m)=f_k(m)+((2+6.*sin(6.*x(j))+4.*cos(10.*x(j))).*exp(-1i.*2.*pi.*k(m).*x(j)/
L));

 end
```

```
 f_k(m)=(1/N).*f_k(m);
end

%calculating u_n
for m =2:N
    u_n(m)=f_k(m)./((-a.*((2.*pi.*(m-((N/2)+1))./L).^2))+(b.*(1i).*(2.*pi.*(m-
((N/2)+1))./L))+c);
end
u_n(1)=f_k(1)./((-a.*((2.*pi.*1./L).^2))+c);


%calculating u(j)
 u_j=zeros(1,N);
 u_x=zeros(1,N);
 for j= 1:N
     for m= 1:N
         x(j)=(L/N).*(j-(1+(pi/3)));
         u_j(j)=u_j(j)+(u_n(m).*exp(1i.*2.*pi.*k(m).*x(j)/L));
         u_x(j)=(9.*cos(6.*x(j)))/370 - (51.*cos(10.*x(j)))/1313 -
 (57.*sin(6*x(j)))/370 - (5.*sin(10.*x(j)))/1313 - 1;
     end
 end
 figure(2)
 subplot(4,1,1)
 plot(x,u_x,LineWidth=1,Marker='o');
 grid on;
 hold on;
 plot(x,u_j,LineWidth=1,Marker='*');
 xlabel('x')
 ylabel('u(x)')
 title('N=8')
 legend('Exact Derivative','Discrete Fourier Method')

 %error
 %Ltwo error
 Ltwoall=0;
 for i= 1:N
     Ltwoall=Ltwoall+(abs(u_j(i)-u_x(i)).^2);
 end
 Ltwo1=(Ltwoall.*(1/N)).^(1/2);

L=2.*pi;
N=16;                          %varied for each value of N
a=1;
b=-1;
c=-2;

%calculating f(n)
f_k=zeros(1,N);
k=-(N/2):1:((N/2)-1);
u_n=zeros(1,N);
for m = 1:N
 for j = 1:N
    x(j)=(L/N).*(j-(1+(pi/3)));
```

```
 f_k(m)=f_k(m)+((2+6.*sin(6.*x(j))+4.*cos(10.*x(j))).*exp(-1i.*2.*pi.*k(m).*x(j)/
L));

 end
 f_k(m)=(1/N).*f_k(m);
end

%calculating u_n
for m =2:N
    u_n(m)=f_k(m)./((-a.*((2.*pi.*(m-((N/2)+1))./L).^2))+(b.*(1i).*(2.*pi.*(m-
((N/2)+1))./L))+c);
end
u_n(1)=f_k(1)./((-a.*((2.*pi.*1./L).^2))+c);


%calculating u(j)
 u_j=zeros(1,N);
 u_x=zeros(1,N);
 for j= 1:N
     for m= 1:N
         x(j)=(L/N).*(j-(1+(pi/3)));
         u_j(j)=u_j(j)+(u_n(m).*exp(1i.*2.*pi.*k(m).*x(j)/L));
         u_x(j)=(9.*cos(6.*x(j)))/370 - (51.*cos(10.*x(j)))/1313 -
 (57.*sin(6*x(j)))/370 - (5.*sin(10.*x(j)))/1313 - 1;
     end
 end
 subplot(4,1,2)
 plot(x,u_x,LineWidth=1,Marker='o')
 hold on;
 grid on;
 plot(x,u_j,LineWidth=1,Marker='*')
 xlabel('x')
 ylabel('u(x)')
 title('N=16')
 legend('Exact Derivative','Discrete Fourier Method')

 %error
 %Ltwo error
 Ltwoall=0;
 for i= 1:N
     Ltwoall=Ltwoall+(abs(u_j(i)-u_x(i)).^2);
 end
 Ltwo2=(Ltwoall.*(1/N)).^(1/2);

 L=2.*pi;
N=32;                        %varied for each value of N
a=1;
b=-1;
c=-2;

%calculating f(n)
f_k=zeros(1,N);
k=-(N/2):1:((N/2)-1);
```

```
u_n=zeros(1,N);
for m = 1:N
 for j = 1:N
    x(j)=(L/N).*(j-(1+(pi/3)));

 f_k(m)=f_k(m)+((2+6.*sin(6.*x(j))+4.*cos(10.*x(j))).*exp(-1i.*2.*pi.*k(m).*x(j)/
L));

 end
 f_k(m)=(1/N).*f_k(m);
end

%calculating u_n
for m =2:N
    u_n(m)=f_k(m)./((-a.*((2.*pi.*(m-((N/2)+1))./L).^2))+(b.*(1i).*(2.*pi.*(m-
((N/2)+1))./L))+c);
end
u_n(1)=f_k(1)./((-a.*((2.*pi.*1./L).^2))+c);


%calculating u(j)
 u_j=zeros(1,N);
 u_x=zeros(1,N);
 for j= 1:N
     for m= 1:N
         x(j)=(L/N).*(j-(1+(pi/3)));
         u_j(j)=u_j(j)+(u_n(m).*exp(1i.*2.*pi.*k(m).*x(j)/L));
         u_x(j)=(9.*cos(6.*x(j)))/370 - (51.*cos(10.*x(j)))/1313 -
 (57.*sin(6*x(j)))/370 - (5.*sin(10.*x(j)))/1313 - 1;
     end
 end
 subplot(4,1,3)
 plot(x,u_x,LineWidth=1,Marker='o')
 hold on;
 grid on;
 plot(x,u_j,LineWidth=1,Marker='*')
 xlabel('x')
 ylabel('u(x)')
 title('N=32')
 legend('Exact Derivative','Discrete Fourier Method')

 %error
 %Ltwo error
 Ltwoall=0;
 for i= 1:N
     Ltwoall=Ltwoall+(abs(u_j(i)-u_x(i)).^2);
 end
 Ltwo3=(Ltwoall.*(1/N)).^(1/2);

 L=2.*pi;
N=64;                          %varied for each value of N
a=1;
b=-1;
c=-2;
```

```
%calculating f(n)
f_k=zeros(1,N);
k=-(N/2):1:((N/2)-1);
u_n=zeros(1,N);
for m = 1:N
 for j = 1:N
    x(j)=(L/N).*(j-(1+(pi/3)));

 f_k(m)=f_k(m)+((2+6.*sin(6.*x(j))+4.*cos(10.*x(j))).*exp(-1i.*2.*pi.*k(m).*x(j)/
L));

 end
 f_k(m)=(1/N).*f_k(m);
end

%calculating u_n
for m =2:N
    u_n(m)=f_k(m)./((-a.*((2.*pi.*(m-((N/2)+1))./L).^2))+(b.*(1i).*(2.*pi.*(m-
((N/2)+1))./L))+c);
end
u_n(1)=f_k(1)./((-a.*((2.*pi.*1./L).^2))+c);


%calculating u(j)
 u_j=zeros(1,N);
 u_x=zeros(1,N);
 for j= 1:N
     for m= 1:N
         x(j)=(L/N).*(j-(1+(pi/3)));
         u_j(j)=u_j(j)+(u_n(m).*exp(1i.*2.*pi.*k(m).*x(j)/L));
         u_x(j)=(9.*cos(6.*x(j)))/370 - (51.*cos(10.*x(j)))/1313 -
 (57.*sin(6*x(j)))/370 - (5.*sin(10.*x(j)))/1313 - 1;
     end
 end
 subplot(4,1,4)
 plot(x,u_x,LineWidth=1,Marker='o')
 hold on;
 grid on;
 plot(x,u_j,LineWidth=1,Marker='*')
 xlabel('x')
 ylabel('u(x)')
 title('N=64')
 legend('Exact Derivative','Discrete Fourier Method')

 %error
 %Ltwo error
 Ltwoall=0;
 for i= 1:N
     Ltwoall=Ltwoall+(abs(u_j(i)-u_x(i)).^2);
 end
 Ltwo4=(Ltwoall.*(1/N)).^(1/2);
```

PROBLEM 5 ERROR

```
clc;
clear;

%Plotting error norms for function (i)
N=[8, 16, 32, 64];
Ltwo1= [0.737, 0.0473, 9.1883e-16, 5.5937e-16];
Ltwo2 = [0.5172,0.0473,2.1519e-15, 2.7272e-15];
figure(1)
loglog(N,Ltwo1,LineWidth=1)
grid on
hold on
loglog(N,Ltwo2,LineWidth=1)
xlabel("N")
ylabel("error")
legend('Ltwo, function w/ BC i','Ltwo, function w/ BC ii')
title('Ltwo error')
```

PROBLEM 6

```
clc;clear;



L=2.*pi;
N=1024;                         %varied for each value of N
a=3;
b=2;
c=1;

%calculating f(n)
f_k=zeros(1,N);
k=-(N/2):1:((N/2)-1);
u_n=zeros(1,N);
x=zeros(1,N);
for m = 1:N
 for j = 1:N
    x(j)=(L/N).*(j-1);          %varied for each BC
    f_k(m)=f_k(m)+(cos((x(j).^2)./(2.*pi)));

 end
 f_k(m)=(1/N).*f_k(m);
end

%calculating u_n
for m =2:N
    u_n(m)=f_k(m)./((-a.*((2.*pi.*(m-((N/2)+1))./L).^2))+(b.*(1i).*(2.*pi.*(m-
((N/2)+1))./L))+c);
end
u_n(1)=f_k(1)./((-a.*((2.*pi.*1./L).^2))+c);
```

```
%calculating u(j)
 u_j_e=zeros(1,N);
 u_x=zeros(1,N);
 for j= 1:N
     for m= 1:N
         x(j)=(L/N).*(j-1);
         u_j_e(j)=u_j_e(j)+(u_n(m).*exp(1i.*2.*pi.*k(m).*x(j)/L));
     end
 end

subplot(3,1,1)
 grid on;
 hold on;
 plot(x,u_j_e,LineWidth=1,Marker='o');
 xlabel('x')
 ylabel('u(x)')
 title('N=16')
```

```
L=2.*pi;
N=1024;                          %varied for each value of N
a=3;
b=2;
c=1;

%calculating f(n)
f_k=zeros(1,N);
k=-(N/2):1:((N/2)-1);
u_n=zeros(1,N);
x=zeros(1,N);
for m = 1:N
 for j = 1:N
    x(j)=(L/N).*(j-1);           %varied for each BC
    f_k(m)=f_k(m)+(cos((x(j).^2)./(2.*pi)));

 end
 f_k(m)=(1/N).*f_k(m);
end

%calculating u_n
for m =2:N
    u_n(m)=f_k(m)./((-a.*((2.*pi.*(m-((N/2)+1))./L).^2))+(b.*(1i).*(2.*pi.*(m-
((N/2)+1))./L))+c);
end
u_n(1)=f_k(1)./((-a.*((2.*pi.*1./L).^2))+c);


%calculating u(j)
 u_j_16=zeros(1,N);
```

```
 u_x=zeros(1,N);
 for j= 1:N
     for m= 1:N
         x(j)=(L/N).*(j-1);
         u_j_16(j)=u_j_16(j)+(u_n(m).*exp(1i.*2.*pi.*k(m).*x(j)/L));
     end
 end

 grid on;
 plot(x,u_j_16,LineWidth=1,Marker='*');
 legend('Exact Derivative','Discrete Fourier Method')
%error
 %Ltwo error
 Ltwoall=0;
 for i= 1:N
     Ltwoall=Ltwoall+(abs(u_j_16(i)-u_j_e(i)).^2);
 end
 Ltwo1=(Ltwoall.*(1/N)).^(1/2);



 L=2.*pi;
N=1024;                         %varied for each value of N
a=3;
b=2;
c=1;

%calculating f(n)
f_k=zeros(1,N);
k=-(N/2):1:((N/2)-1);
u_n=zeros(1,N);
x=zeros(1,N);
for m = 1:N
 for j = 1:N
    x(j)=(L/N).*(j-1);          %varied for each BC
    f_k(m)=f_k(m)+(cos((x(j).^2)./(2.*pi)));

 end
 f_k(m)=(1/N).*f_k(m);
end

%calculating u_n
for m =2:N
    u_n(m)=f_k(m)./((-a.*((2.*pi.*(m-((N/2)+1))./L).^2))+(b.*(1i).*(2.*pi.*(m-
((N/2)+1))./L))+c);
end
u_n(1)=f_k(1)./((-a.*((2.*pi.*1./L).^2))+c);


%calculating u(j)
 u_j_e=zeros(1,N);
 u_x=zeros(1,N);
 for j= 1:N
     for m= 1:N
```

```
        x(j)=(L/N).*(j-1);
        u_j_e(j)=u_j_e(j)+(u_n(m).*exp(1i.*2.*pi.*k(m).*x(j)/L));
    end
 end

subplot(3,1,2)
 grid on;
 hold on;
 plot(x,u_j_e,LineWidth=1,Marker='o');
 xlabel('x')
 ylabel('u(x)')
 title('N=32')

L=2.*pi;
N=32;                        %varied for each value of N
a=3;
b=2;
c=1;

%calculating f(n)
f_k=zeros(1,N);
k=-(N/2):1:((N/2)-1);
u_n=zeros(1,N);
x=zeros(1,N);
for m = 1:N
 for j = 1:N
    x(j)=(L/N).*(j-1);          %varied for each BC
    f_k(m)=f_k(m)+(cos((x(j).^2)./(2.*pi)));

 end
 f_k(m)=(1/N).*f_k(m);
end

%calculating u_n
for m =2:N
    u_n(m)=f_k(m)./((-a.*((2.*pi.*(m-((N/2)+1))./L).^2))+(b.*(1i).*(2.*pi.*(m-
((N/2)+1))./L))+c);
end
u_n(1)=f_k(1)./((-a.*((2.*pi.*1./L).^2))+c);


%calculating u(j)
 u_j_32=zeros(1,N);
 u_x=zeros(1,N);
 for j= 1:N
     for m= 1:N
         x(j)=(L/N).*(j-1);
         u_j_32(j)=u_j_32(j)+(u_n(m).*exp(1i.*2.*pi.*k(m).*x(j)/L));
     end
 end

 grid on;
 plot(x,u_j_32,LineWidth=1,Marker='*');
 legend('Exact Derivative','Discrete Fourier Method')
```

```
%error
 %Ltwo error
 Ltwoall=0;
 for i= 1:N
     Ltwoall=Ltwoall+(abs(u_j_32(i)-u_j_e(i)).^2);
 end
 Ltwo2=(Ltwoall.*(1/N)).^(1/2);


  L=2.*pi;
N=1024;                         %varied for each value of N
a=3;
b=2;
c=1;

%calculating f(n)
f_k=zeros(1,N);
k=-(N/2):1:((N/2)-1);
u_n=zeros(1,N);
x=zeros(1,N);
for m = 1:N
 for j = 1:N
    x(j)=(L/N).*(j-1);          %varied for each BC
    f_k(m)=f_k(m)+(cos((x(j).^2)./(2.*pi)));

 end
 f_k(m)=(1/N).*f_k(m);
end

%calculating u_n
for m =2:N
    u_n(m)=f_k(m)./((-a.*((2.*pi.*(m-((N/2)+1))./L).^2))+(b.*(1i).*(2.*pi.*(m-
((N/2)+1))./L))+c);
end
u_n(1)=f_k(1)./((-a.*((2.*pi.*1./L).^2))+c);


%calculating u(j)
 u_j_e=zeros(1,N);
 u_x=zeros(1,N);
 for j= 1:N
     for m= 1:N
         x(j)=(L/N).*(j-1);
         u_j_e(j)=u_j_e(j)+(u_n(m).*exp(1i.*2.*pi.*k(m).*x(j)/L));
     end
 end

subplot(3,1,3)
 grid on;
 hold on;
 plot(x,u_j_e,LineWidth=1,Marker='o');
 xlabel('x')
 ylabel('u(x)')
 title('N=64')
```

```
L=2.*pi;
N=64;                          %varied for each value of N
a=3;
b=2;
c=1;

%calculating f(n)
f_k=zeros(1,N);
k=-(N/2):1:((N/2)-1);
u_n=zeros(1,N);
x=zeros(1,N);
for m = 1:N
 for j = 1:N
    x(j)=(L/N).*(j-1);          %varied for each BC
    f_k(m)=f_k(m)+(cos((x(j).^2)./(2.*pi)));

 end
 f_k(m)=(1/N).*f_k(m);
end

%calculating u_n
for m =2:N
    u_n(m)=f_k(m)./((-a.*((2.*pi.*(m-((N/2)+1))./L).^2))+(b.*(1i).*(2.*pi.*(m-
((N/2)+1))./L))+c);
end
u_n(1)=f_k(1)./((-a.*((2.*pi.*1./L).^2))+c);


%calculating u(j)
 u_j_64=zeros(1,N);
 u_x=zeros(1,N);
 for j= 1:N
     for m= 1:N
         x(j)=(L/N).*(j-1);
         u_j_64(j)=u_j_64(j)+(u_n(m).*exp(1i.*2.*pi.*k(m).*x(j)/L));
     end
 end

 grid on;
 plot(x,u_j_64,LineWidth=1,Marker='*');
 legend('Exact Derivative','Discrete Fourier Method')
%error
 %Ltwo error
 Ltwoall=0;
 for i= 1:N
     Ltwoall=Ltwoall+(abs(u_j_64(i)-u_j_e(i)).^2);
 end
 Ltwo3=(Ltwoall.*(1/N)).^(1/2);
```

PROBLEM 6 ERROR

```
 clc;
clear;

N=[8, 16, 32, 64, 128, 256, 512, 1024];
Ltwo1= [0.2508, 0.2436, 0.2310, 0.2118, 0.1940, 0.1977, 0.1770, 0];
figure(1)
loglog(N,Ltwo1,LineWidth=1)
grid on
xlabel("N")
ylabel("error")
legend('Ltwo')
title('Ltwo error')
```

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