Jitter and Basic Requirements of the Reaction Wheel Assembly in the Attitude Control System

Lulu Liu

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1 Brief Introduction

Photometric precision is a major concern in this space mission. A pointing accuracy of 2 arcseconds is required to consistently keep the largest portion of the point spread function of any star over the same CCD pixel (∼20 arcseconds across) orbit after orbit. There are many sources of error and instability that may limit our pointing ability. This document is a starting and fully theoretical look at the jitter on the spacecraft caused by dynamic and static imbalances in the reaction wheels. It does not take into account any natural frequencies in the system– a future, major consideration.

2 The Model

Some significant assumptions were made in creating our simplistic jitter model. First, that the spacecraft is a perfectly rigid body and has no normal vibrational modes. This assumption was necessary as we have not yet made decisions on the final structure and composition of the spacecraft though we know its approximate dimensions and mass distribution. Next, we assumed a reaction wheel assembly of three orthogonal, concentric, identical wheels, spatially aligned with the center of mass location along the Z-axis, see Figure 1.

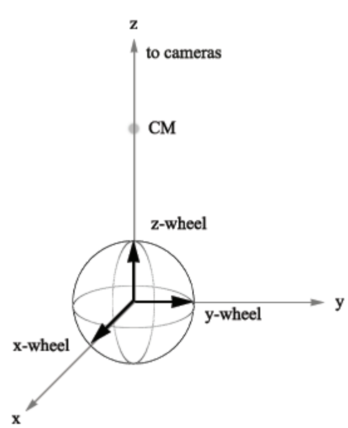


Figure 1: RWA Simplified Model. Each reaction wheel is named for its normal axis.

The static imbalance of a wheel, S, is in dimensions of $[mass] \cdot [length]$, and its disturbance on the system is modeled as a central force: $|f| = S\omega^2$ where ω is the angular velocity of the wheel. The dynamic imbalance of a wheel, D, is in dimensions of $[mass] \cdot [length]^2$, and it imposes a torque on the system of magnitude $|\tau| = D\omega^2$.

Their time-varying effects are then calculated for the pixel in the farthest corner of the camera-array's field of view, 45° off the Z-axis in the Y-direction and 27° off-axis in the X-direction.

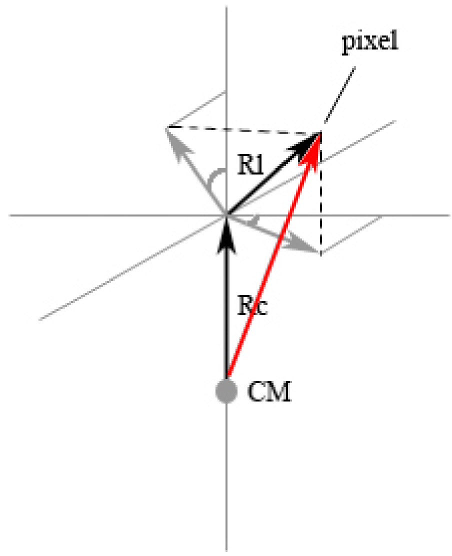


Figure 2: Location of the point of interest with respect to Center of Mass.

3 Calculations

3.1 Static Imbalance

Static imbalances are radial asymmetries in mass distribution. Each spinning wheel with a static imbalance will impose a periodic force on the spacecraft. The magnitude of the force vector is constant however its direction changes with time. In the case of the Z-wheel (wheel whose normal vector is aligned with the Z-axis), the force on the spacecraft breaks down as such:

$$\vec{f}_z = S_z\omega_z^2 \cos(\omega_z t + \varphi_z)\hat{x} + S_z\omega_z^2 \sin(\omega_z t + \varphi_z)\hat{y} \tag{1}$$

These forces are applied at the site of the reaction wheel assembly. As a result, any force not acting through the center of mass (only F_z in this case) also acts as a torque on the spacecraft.

Below is the set of all forces and torques on the body due to any static imbalance as a function of the wheel angular velocities.

$$\begin{aligned}
\vec{F}_s(\omega_x, \omega_y, \omega_z) &= (S_y\omega_y^2 \sin(\omega_y t + \varphi_y) + S_z\omega_z^2 \cos(\omega_z t + \varphi_z))\hat{x} \\
&\quad + (S_z\omega_z^2 \sin(\omega_z t + \varphi_z) + S_x\omega_x^2 \cos(\omega_x t + \varphi_x))\hat{y} \\
&\quad + (S_x\omega_x^2 \sin(\omega_x t + \varphi_x) + S_y\omega_y^2 \cos(\omega_y t + \varphi_y))\hat{z} \qquad (2) \\
\vec{\tau}_s(\omega_x, \omega_y, \omega_z) &= \vec{R}_w \times \vec{F}(\omega_x, \omega_y, \omega_z) \qquad (3)
\end{aligned}$$

3.2 Dynamic Imbalance

Dynamic imbalances are asymmetries in mass distribution across the thickness of the wheel. Dynamically imbalanced wheels in motion want to "straighten out" its spin along its center of mass plane, a torque exerted on the wheel counteracts this tendency. The spacecraft experiences the equal and opposite.

Combining the effects of all three wheels, the net torque on the spacecraft due to dynamic imbalances D_x, D_y, D_z is

$$\begin{aligned}\vec{\tau}_d(\omega_x,\omega_y,\omega_z) = {} & (D_z\omega_z^2\sin(\omega_z t+\phi'_z) - D_y\omega_y^2\cos(\omega_y t+\phi'_y))\hat{x} \\ & +(D_x\omega_x^2\sin(\omega_x t+\phi'_x) - D_z\omega_z^2\cos(\omega_z t+\phi'_z))\hat{y} \\ & +(D_y\omega_y^2\sin(\omega_y t+\phi'_y) - D_x\omega_x^2\cos(\omega_x t+\phi'_x))\hat{z}\end{aligned} \tag{4}$$

Since these are pure torques, the net force on the spacecraft is zero.

3.3 Constants and Parameters

Taking as our parameters our preliminary spacecraft design and a set of identical Ithaco Type A reaction wheels, the following inputs

Spacecraft Moments of Inertia	$I_{xx} = 110$	$kg \cdot m^2$
	$I_{yy} = 110$	
	$I_{zz} = 42$	
Mass of Spacecraft	$M = 100$	kg
Distance between Wheels and CM	$R_w = 0.35$	m
Distance between CM and Cameras	$R_c = 0.65$	m
Length of each Lens	$R_l = 0.179$	m
Wheel Dynamic Imbalance	$D_x = D_y = D_z = 1 \times 10^-6$	$kg \cdot m^2$
Wheel Static Imbalance	$S_x = S_y = S_z = 5 \times 10^-6$	$kg \cdot m$

4 Analysis and Results

As we can assume a great distance to the stars we are observing, linear displacements can be neglected in our discussion of pointing accuracy. Instead, we focus on the magnitude of periodic angular displacements.

Figure 3 shows the expected jitter in terms of angular and linear displacements as a function of time for the case of two wheels (Y and Z) running at the same speed (3000 RPM) while the third is still. Figure Figure 3 is a similar graph for three distinct wheel speeds. A 3-D plot over all speeds in a two-wheel coupled case is displayed in Figure 5. In all cases, relative phase lags were set to zero.

Our model shows a maximum angular error less than 10^{-7} radians = **0.2 arcseconds**. As optimistic as this number appears to be, it is achieved without consideration of material properties and structural frequency response. It, however, does provide an adequate baseline to be modified by newer design considerations, such as damping, structural stiffness, and results of dynamic modeling.

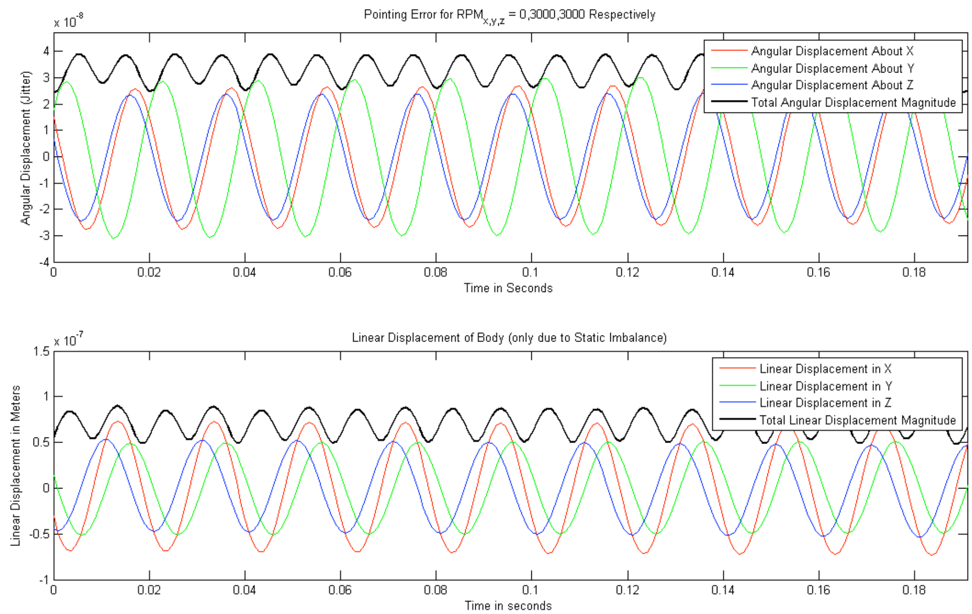


Figure 3: Jitter profile for the case of zero phase lags, X-wheel: 0 RPM, Y-wheel: 3000 RPM, Z-wheel: 3000 RPM

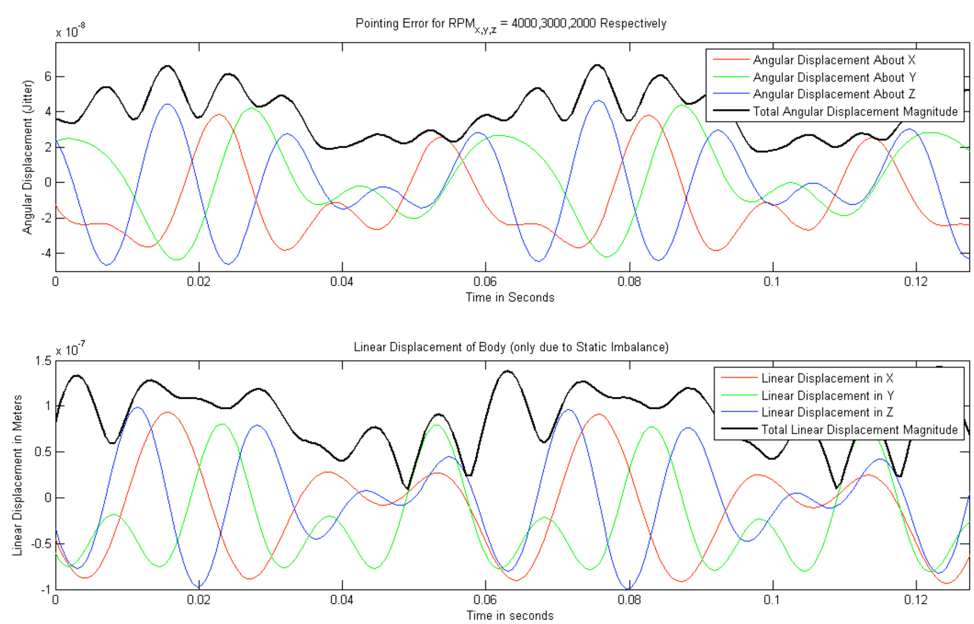


Figure 4: Jitter profile for the case of zero phase lags, X-wheel: 4000 RPM, Y-wheel: 3000 RPM, Z-wheel: 2000 RPM

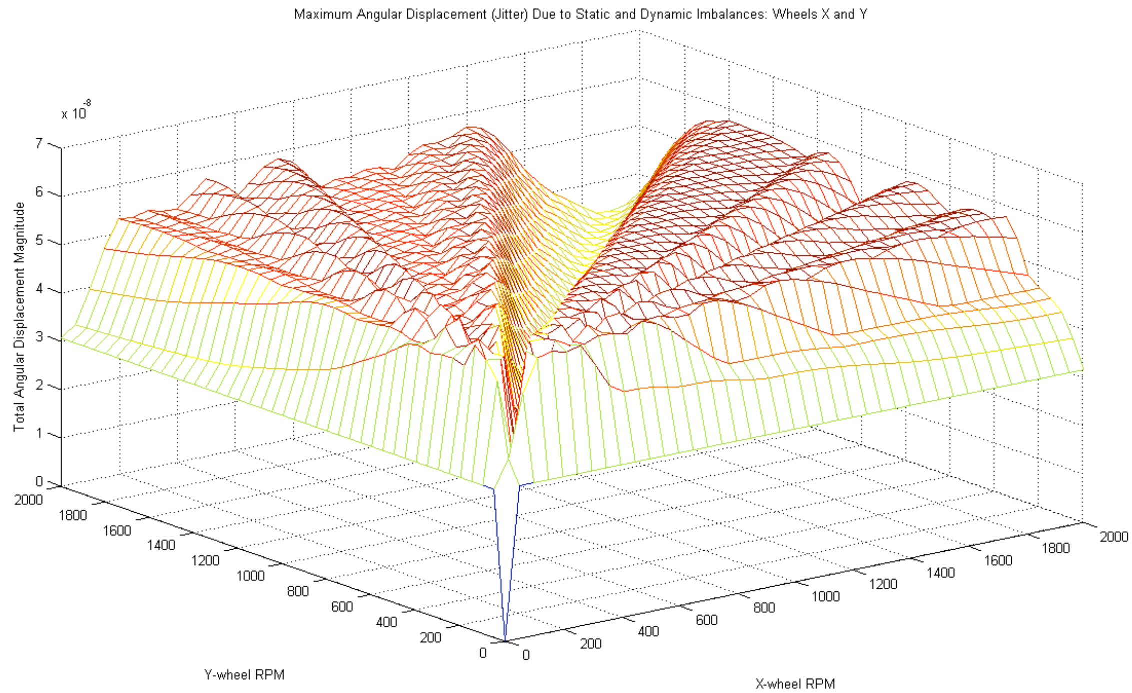


Figure 5: Mesh plot: Magnitude of Maximum Angular Displacement vs. X-wheel speed, Y-wheel speed.

5 Calculation of Minimum Wheel Requirements

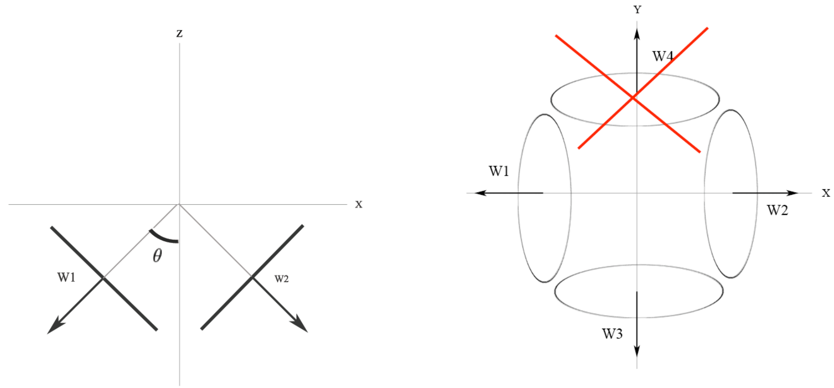
For this analysis we abandon the three-wheel model of the reaction wheel assembly and introduce a more often used four-wheel coupled setup. The main benefit of coupling on each axis is redundancy. In case of malfunction, we preserve all three degrees of rotational freedom.

This study is necessary for the determination of minimum wheel requirements that will eventually result in the choice of a reaction wheel model that best suits our mission needs. From the geometry of the system and the mission plan, we will calculate the minimum required momentum capacity and the minimum required torque. The most demanding mid-orbit maneuver is the 18° horizontal slew during orbit-night which needs to be completed along one axis within one minute- we will use this maneuver as a base to determine our minimum requirements.

$18°/60s = 0.3°/s$ for a constant angular velocity over one minute. A much more realistic model, a constant angular acceleration first half and deceleration second half, would naturally result in a peak angular velocity requirement twice as high at $0.6°/s$. Double this for a margin of 100

$$|\vec{L}_{slew}| = 2.31\ kgm^2/s$$
$$|\vec{\tau}_{slew}| = 0.077\ kgm^2/s^2$$

The normal vectors of each wheel are kept at an angle of θ from the negative Z-axis. As seen from above the wheels are at an angle of 90° with respect to each other and together form the four sides of a square. The first arrangment aligns the XY-plane projections of the wheels' normal vectors with the X- and Y- axes. See Figure 6 (a). Since the incentive for coupling was the case of one-wheel failure, it makes sense to conduct the analysis as if one wheel cannot function.



(a) XZ-plane view. (b) XY-plane view.

Figure 6: On-axis coupled assembly.

Only one wheel controls rotations about the Y-axis, the extent of which is a function of θ only.

$$|\vec{L}_{req}| = |\vec{L}_{slew}| / \sin\theta$$
$$|\vec{\tau}_{req}| = |\vec{\tau}_{slew}| / \sin\theta$$

A quick calculation reveals,

$\theta = 45°$:

$$|\vec{L}_{req}| = 3.27\ kgm^2/s$$
$$|\vec{\tau}_{req}| = 0.109\ kgm^2/s^2$$

$\theta = 55°$:

$$|\vec{L}_{req}| = 2.82\ kgm^2/s$$
$$|\vec{\tau}_{req}| = 0.094\ kgm^2/s^2$$

The off-axis assembly, pictured below in Figure 7, obeys this system of equations:

$$\begin{pmatrix} \frac{\sin\theta}{\sqrt{2}} & -\frac{\sin\theta}{\sqrt{2}} & -\frac{\sin\theta}{\sqrt{2}} \\ \frac{\sin\theta}{\sqrt{2}} & -\frac{\sin\theta}{\sqrt{2}} & \frac{\sin\theta}{\sqrt{2}} \\ \cos\theta & \cos\theta & \cos\theta \end{pmatrix} \begin{pmatrix} L_{1,req} \\ L_{2,req} \\ L_{3,req} \end{pmatrix} = \begin{pmatrix} L_{x,slew} \\ L_{y,slew} \\ L_{z,slew} \end{pmatrix} \tag{5}$$

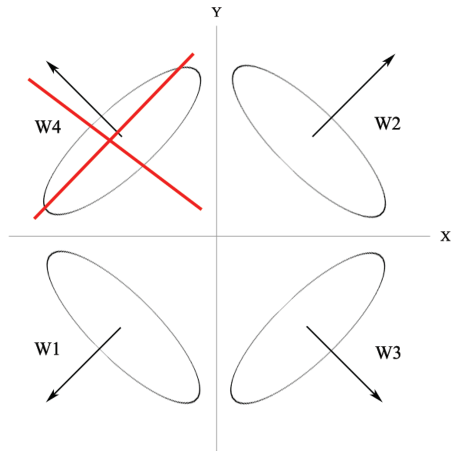


Figure 7: Off-axis coupled assembly.

The calculated required momentum and torque capacity follow,

$\theta = 45°$:

$$\begin{pmatrix} L_{1,req} \\ L_{2,req} \\ L_{3,req} \end{pmatrix} = \begin{pmatrix} 2.3 \\ 0 \\ -2.3 \end{pmatrix} \; kgm^2/s, \quad \begin{pmatrix} \tau_{1,req} \\ \tau_{2,req} \\ \tau_{3,req} \end{pmatrix} = \begin{pmatrix} 0.077 \\ 0 \\ -0.077 \end{pmatrix} \; kgm^2/s^2$$

$\theta = 55°$:

$$\begin{pmatrix} L_{1,req} \\ L_{2,req} \\ L_{3,req} \end{pmatrix} = \begin{pmatrix} 1.99 \\ 0 \\ -1.99 \end{pmatrix} \; kgm^2/s, \quad \begin{pmatrix} \tau_{1,req} \\ \tau_{2,req} \\ \tau_{3,req} \end{pmatrix} = \begin{pmatrix} 0.066 \\ 0 \\ -0.066 \end{pmatrix} \; kgm^2/s^2$$

From the brief calculations, it is evident that a θ of 55° enabled more coupling in the X- and Y- axes and reduced the load on a single wheel in the event of a one-wheel malfunction. In addition, off-setting the wheels from the body rotation axes also had a positive (similar) effect on the load distribution during rotation about each of these axes.

According to these results, even the relatively lightweight and balanced Ithaco A-Wheels can more than provide the maneuverability needed for this mission.