Linear Regression:

12/14

→ Linear relationship

→ $\boxed{y = mx + b} + \varepsilon$ → error

→ Try to find the best-fit line

Gradient descent

$$\boxed{y = 4x^2 + 4x + 5}$$

Ask: Find the value of x, where y is minimal?

$x = 0$; $x = 1$; $x = -1$

$$\boxed{\frac{d}{dx} x^n = n x^{n-1}}$$

(1) $$\frac{dx^5}{dx} = 5x^4$$

$$\frac{dx}{dx} = 1x^{1-1} = 1 \cdot x\, x^0$$

$$= 1 \times 1 = 1$$

(2) $$\frac{dx^2}{dx} = 2x^{2-1} = 2x$$

$$\frac{d(5)}{dx} = 0$$

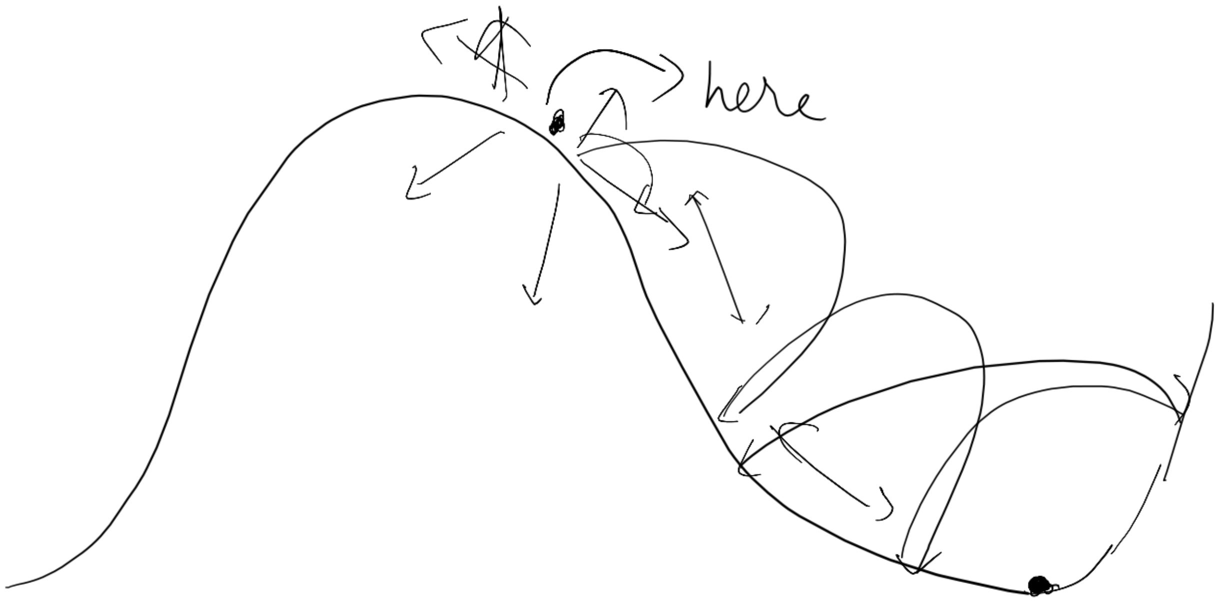
$$\Downarrow$$

$$\frac{d(5)}{dx} = \frac{d(5)x^0}{dx} = 0 \times 5 x^{0-1} = 0$$

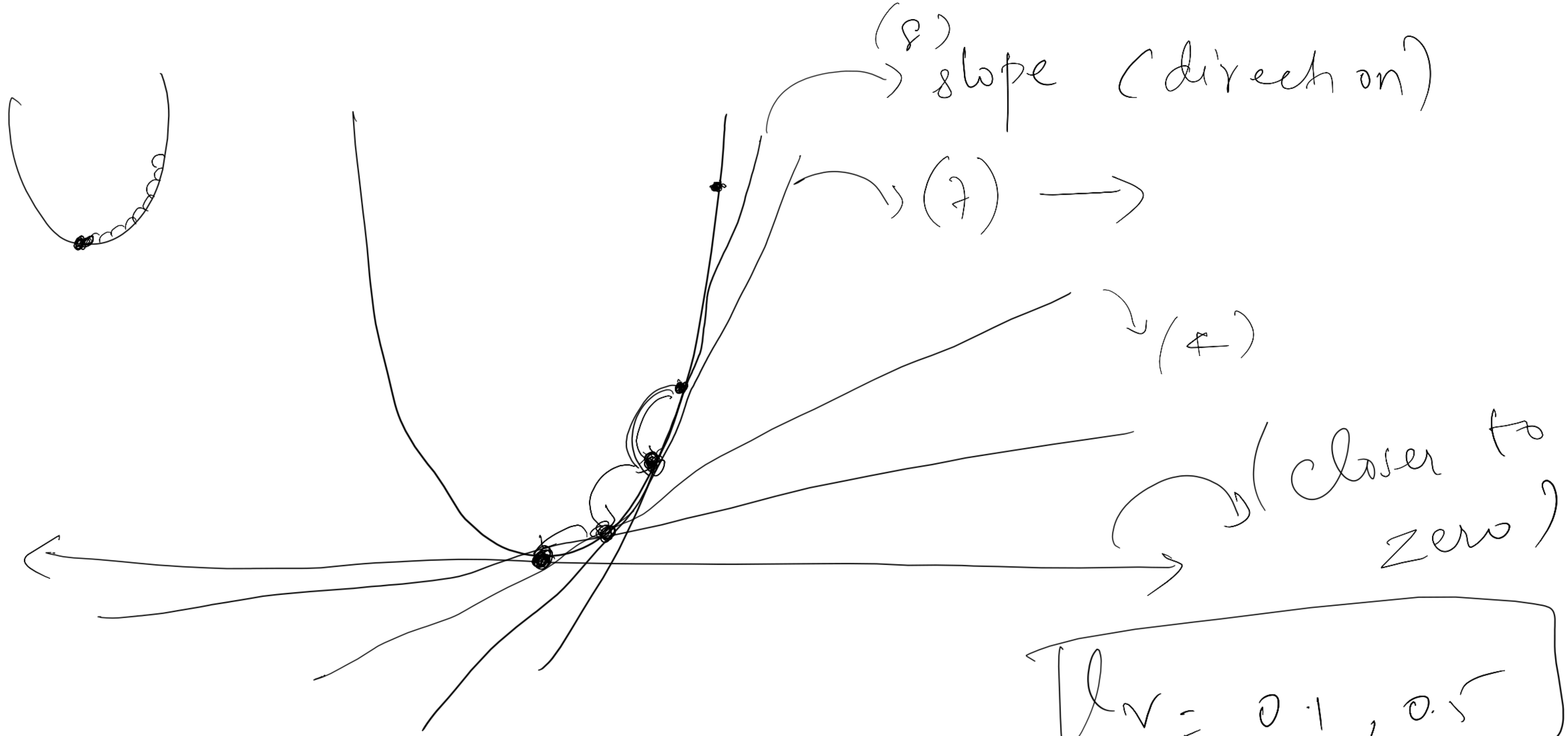
$$y = 4x^2 + 4x + 5$$

$$\frac{dy}{dx} = 8x + 4 + 0$$

$$= 8x + 4$$



1. Direction (differentiation)
2. Momentum (Step size) => we set



$$\boxed{lr = 0.1,\ 0.5}$$

$$y = 4x^2 + 4x + 5 \Rightarrow \frac{dy}{dx} = 8x + 4$$

Steps :-

- ✓ → Differentiate the equation
- ✓ → Set a random number
- ✓ → Substitute the number in the differential eqn
- ✓ → Update the learning rate $\boxed{\pm 0.01}$
- → With the new point, find the differentiation
- → Repeat ~~the~~ differentiation, until it come closer to zero

$y = 4x^2 + 4x + 5$

$$\frac{dy}{dx} = 8x + 4 + 0$$

$\boxed{\text{Derivative} = \text{Slope}}$

$\boxed{\text{random\_number} = 0}$

$$\frac{dy}{dx} = 8(0) + 4 = 4 \quad (\text{where } x = 0)$$

slope / derivative

$\boxed{\text{current\_x} = 0}$

$\boxed{\text{l\_r} = 0.01}$

Update $\Rightarrow$ $\boxed{\text{current\_x} - \text{der}(x) * \text{learning\_rate}}$

$= 0 - (4 * 0.01)$

$\text{curr\_x} = 0 - (0.04) = -0.04$

iter2: $\frac{dy}{dx} = 8x + 4$

$= 8(-0.04) + 4$

$= -0.32 + 4 = 3.68$ → Slope / derivative

$\text{curr\_x} = -0.04 - (3.68 * 0.01)$

$\boxed{\text{curr\_x} = -0.07}$

$$\frac{dy}{dx} = 8x + 4$$

$$= 8(-0.07) + 4$$

$$= 3.44$$ → slope / derivative

iter 3

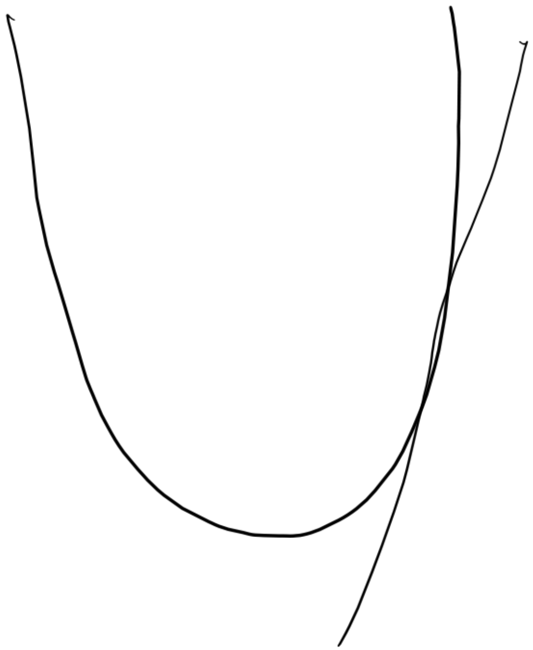
$$\text{cur\_x} = -0.07 - (3.44 * 0.01)$$ → lr

$$\text{cur\_x} = \boxed{-0.10}$$

cur_x

$\Rightarrow$ $0 \rightarrow -0.04 \rightarrow -0.07 \rightarrow -0.1$

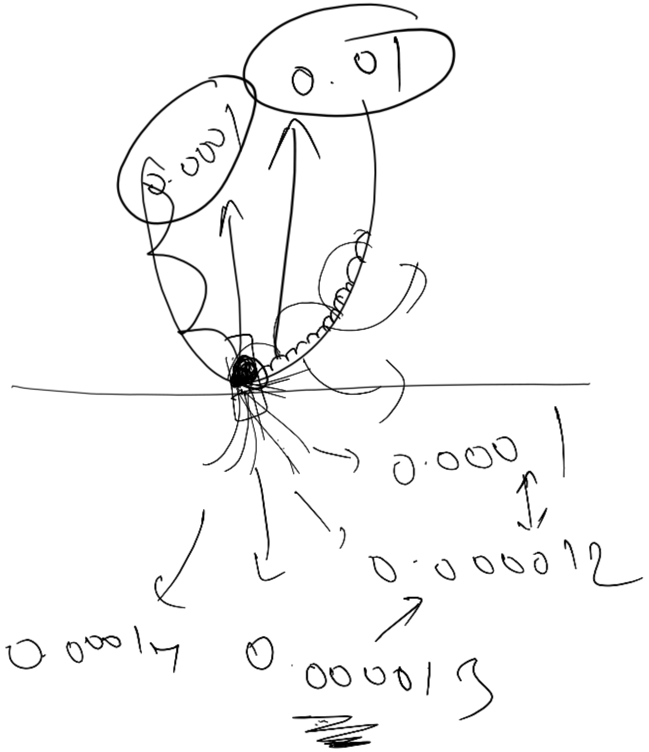
Slope: $4 \rightarrow 3.68 \rightarrow 3.44$

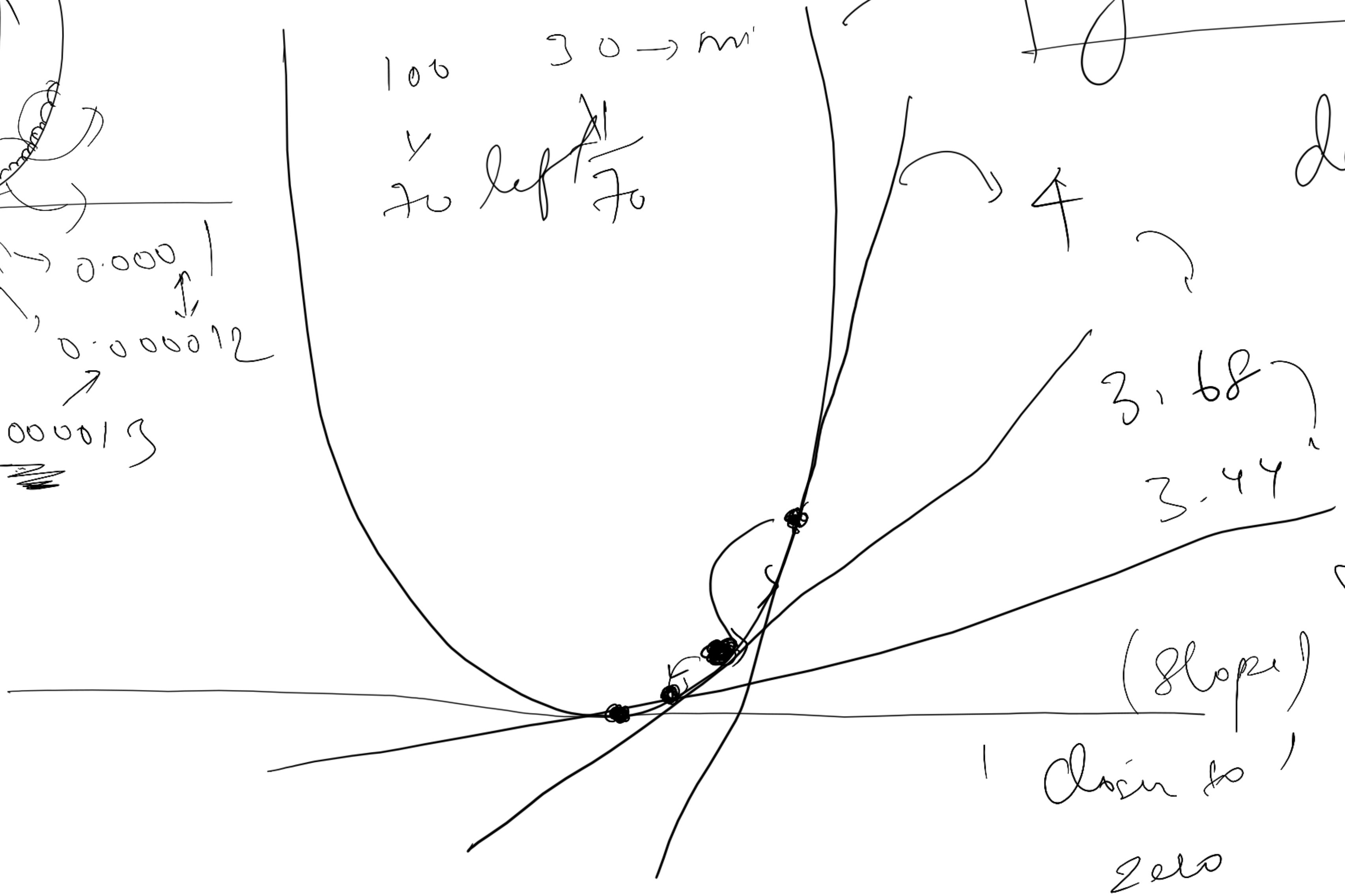


Derivative

$\downarrow$

tangent





$\boxed{y \longrightarrow y \text{ is minimal?}}$

derivative

⇓

tangent

⇓

'slope'

(or)

derivative

Let's get back at

7:55 IST

Error : ? $y - \hat{y}$

MSE

$$\frac{\sum (y - \hat{y})^2}{n}$$

$$MSE = \frac{1}{n} \sum \left(y - (mx + b)\right)^2$$

Find the value of m & b where MSE is minimum?

2 variable $\begin{cases} \text{'m'} \\ \text{'b'} \end{cases}$

$\downarrow$

Partial derivative

$$\frac{\partial}{\partial m} \quad , \quad \frac{\partial}{\partial b}$$

$$y = 6x^2 + 2x + 7z + 10$$

$$\frac{\partial y}{\partial x} = \boxed{12x + 2} + \boxed{0 + 0}$$

Constant

$$\frac{\partial y}{\partial z} = \boxed{0 + 0} + 7 + 0 = 7$$

Constant

Derivative

$$MSE = \frac{1}{n} \sum (y - (mx + b))^2$$

$$\boxed{\text{Partial derivative}}$$

w.r.to -> m

$$\frac{\partial(mse)}{\partial m} = ?$$

$$\frac{\partial(mse)}{\partial b} = ?$$

$$MSE \rightarrow \frac{1}{n} \sum \left(y - (mx + b)\right)^2$$

$$\boxed{(a-b)^2 = a^2 + b^2 - 2ab}$$

$$= \frac{1}{n} \sum \left[y^2 + (mx + b)^2 - 2y(mx + b) \right]$$

$$= \frac{1}{n} \sum \left[y^2 + m^2x^2 + b^2 + 2mxb - 2ymx - 2yb \right]$$

$$\frac{\partial E}{\partial m} = \frac{1}{n} \sum \left[0 + 2mx^2 + 0 + 2xb - 2yx - 0 \right]$$

$$=$$

$$= \frac{1}{n} \sum \left[2mx^2 + 2xb - 2yx \right]$$

$$= \frac{2}{n} \sum \left[mx^2 + xb - yx \right]$$

$$= \frac{2}{n} \sum x \left[mx + b - y \right]$$

$$\frac{\partial}{\partial m} = \frac{2}{n} \sum \left[-x \left(-mx - b \right] \right.$$

$$\frac{\partial}{\partial m} = \frac{2}{n} \sum x \left[(mx + b) - y \right]$$

$$\frac{\partial}{\partial m} = \frac{2}{n} \sum \left[-x \left(y - (mx + b) \right) \right]$$

$$= -\frac{2}{n} \sum \left[x \left(y - mx + b \right) \right]$$

"Direction with respect to slope 'm'"

$$MSE = \frac{1}{n} \sum \left(y^2 + (mx+b)^2 - 2y(mx+b) \right]$$

$$= \frac{1}{n} \sum \left(y^2 + m^2 x^2 + b^2 + 2mxb - 2ymx - 2yb \right]$$

$$\frac{\partial E}{\partial b} = \frac{1}{n} \sum \left(0 + 0 + 2b + 2mx - 0 - 2y \right)$$

$$= \frac{1}{n} \sum \left(2b + 2mx - 2y \right)$$

$$\frac{\partial}{\partial b} = \frac{2}{n} \sum (b + mx - y)$$

$$\partial/\partial b = -\frac{2}{n} \sum (y - (mx + b))$$

↳ "Direction with respect to 'b' intercept

$$y = 4x^2 + 4x + 5$$

obj

↳ find 'y' is minimal

$$\frac{dy}{dx} = 8x + 4$$

Differentiation → 1 var

$\boxed{y \text{ is minimal}}$

$$MSE = \frac{\Sigma(y - (mx+b))^2}{n}$$

MSE ⇒ minimal

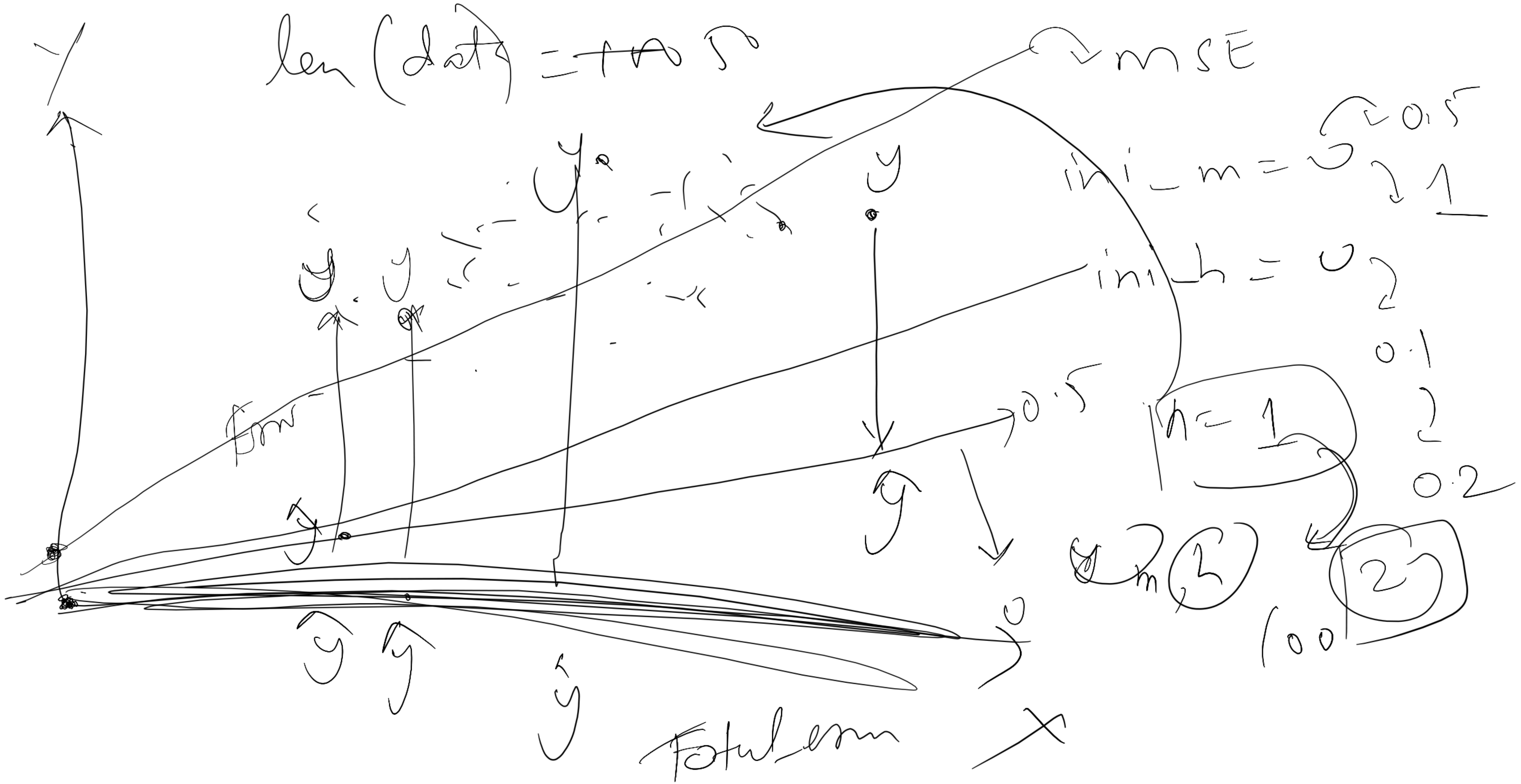
2 var → m & b

Partial derivative

↳ m

↳ b

MSE is minimal



Y
len(data)
MSE
init_m = 5
0.5
1
init_b = 0
0.1
0.2
0.5
b = 1
2
100
X