

# GARCH Volatility Analysis - S&P 500

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## GARCH Volatility Modeling

### Analysis of S&P 500 Volatility Dynamics

This notebook demonstrates advanced volatility modeling techniques using GARCH models applied to S&P 500 data. We progress from basic univariate GARCH to a multivariate model and rolling forecast. Lastly, we'll compile for risk management applications.

Key Analyses:

1. Univariate GARCH modeling and diagnostics
2. Dynamic Conditional Correlation (DCC-GARCH) for multi-asset portfolios
3. Rolling Window Forecasts
4. Risk management applications (VaR, portfolio optimization)

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Daily S&P 500 returns were loaded for the period from 2020-01-01 to 2024-12-31.  
Additional tickers were loaded for the same period: TLT, GLD, and VXX. These will be used in the multivariate analysis.

## 1. S&P Summary Statistics and Exploratory Data Analysis

### 1.1 S&P 500 Summary Statistics and Analysis

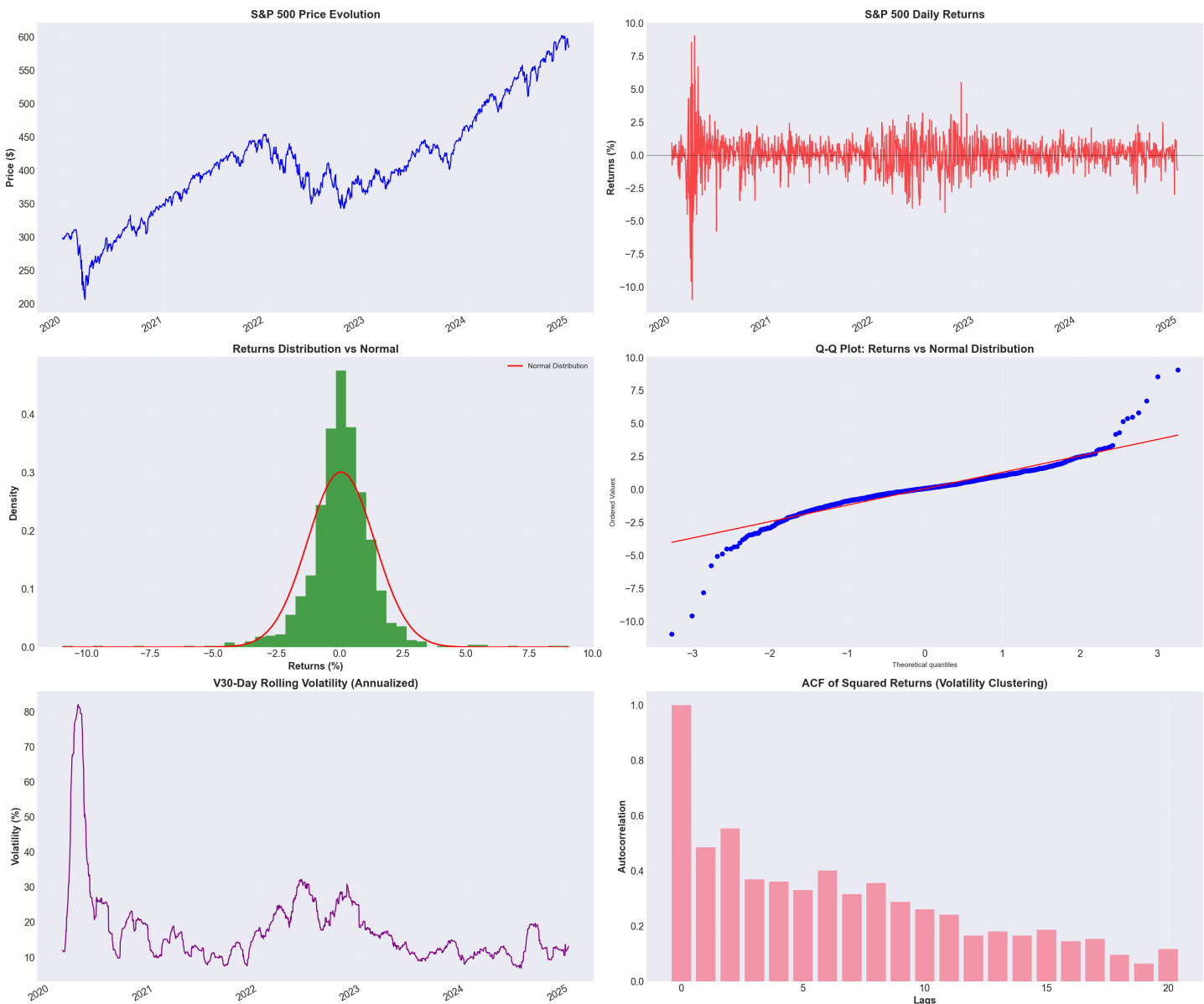
S&P 500 summary statistics results are the following:

Mean: 0.0628%  
Std Dev: 1.3229%  
Skewness: -0.5443  
Kurtosis: 11.5024  
Min: -10.9424%  
Max: 9.0603%

S&P 500 analysis of summary statistics:

The S&P 500 daily returns resemble the characteristics of typical financial time series data, with an average daily return of 0.0628%, or approximately 16% annualized. The standard deviation of 1.32% represents moderate daily volatility, while the negative skewness of -0.54 reveals a slight tendency toward larger negative returns than positive returns. The excess kurtosis of 11.50 demonstrates significant fat tails as the baseline kurtosis is 3.0, resulting in an excess of 8.50. This high kurtosis, combined with negative skewness (-0.54), indicates the S&P 500 has frequent small gains, occasional large losses, and reflects extreme events occurring more frequently than normal distribution theory would predict. This is evidenced by the wide range between the observed minimum and maximum from -10.94% to +9.06%. Finally, this confirms the presence of volatility clustering and suggests that traditional risk models assuming normal distributions may underestimate tail risks for this period of the portfolio.

1.2 Exploratory Data Analysis and Visualizations



S&P 500 price evolution shows an increase of nearly 200% from \$300 to around \$600. While demonstrating strong long-term growth, the data reveals two distinct periods of significant volatility clustering: early-to-mid

2020 during the COVID market crash, and throughout most of 2022 amid aggressive interest rate increases. This clustering behavior is clearly visible in the daily returns plot, where periods of high volatility (large price swings) are followed by continued high volatility, and calm periods persist for extended timeframes. The returns distribution and Q-Q plot confirm substantial deviations from normality, particularly in the tails, indicating the presence of extreme market events that occur more frequently than a normal distribution would predict. The 30-day rolling volatility and autocorrelation function (ACF) of squared returns provide deeper insight into this volatility clustering phenomenon. The rolling volatility shows sustained high-volatility periods reaching 80% annualized during COVID and elevated levels throughout 2022, while the ACF demonstrates strong persistence in volatility shocks—when markets become volatile, they tend to remain volatile for weeks rather than quickly reverting to calm conditions. As evidence, the ACF shows volatility remaining elevated even at 20-day lags. This persistent volatility clustering validates the use of GARCH modeling to capture these time-varying risk dynamics that traditional models assuming constant volatility would miss.

1.3 Statistical Tests

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STATISTICAL TESTS
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Jarque-Bera Test for Normality:
Statistic: 6930.6674
P-value: 0.000000
Result: Reject normality

Ljung-Box Test for ARCH Effects (Volatility Clustering):
P-value (lag 10): 0.000000
Result: Significant ARCH effects detected

The Jarque-Bera and Ljung-Box tests results appear to be typical and expected for financial data with high volatility clustering. The Jarque-Bera tests rejects normality, which is appropriate given the data’s slight skew of -0.54 and fat tails, as indicated in the Q-Q plot. The Ljung-Box test strongly rejects the null hypothesis meaning periods of high volatility tend to be followed by more high volatility, and periods of low volatility tend to be followed by more low volatility. This clustering behavior is visually confirmed in the ACF plot of squared returns, which shows significant autocorrelation persisting for 15-20 days, indicating that volatility shocks have lasting effects rather than quickly reverting to average levels.

3. Univariate GARCH Model

Model Comparison (sorted by AIC):

Model	AIC	BIC	Log-Likelihood	Parameters
GARCH(1,1)-t	3614.103878	3639.786294	-1802.051939	5
GJR-GARCH(1,1)	3637.198188	3662.880604	-1813.599094	5
GARCH(1,1)	3658.670231	3679.216164	-1825.335115	4
GARCH(2,2)	3660.993700	3691.812599	-1824.496850	6
EGARCH(1,1)	3669.637129	3690.183062	-1830.818564	4

Best model: GARCH(1,1)-t

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MODEL DIAGNOSTICS: GARCH(1,1)-t

Constant Mean - GARCH Model Results			
Dep. Variable:	SPY	R-squared:	0.000
Mean Model:	Constant Mean	Adj. R-squared:	0.000
Vol Model:	GARCH	Log-Likelihood:	-1802.05
Distribution:	Standardized Student's t	AIC:	3614.10
Method:	Maximum Likelihood	BIC:	3639.79
		No. Observations:	1257
Date:	Sat, Aug 02 2025	Df Residuals:	1256
Time:	21:20:24	Df Model:	1
Mean Model			

	coef	std err	t	P> t	95.0% Conf. Int.
mu	0.1233	2.314e-02	5.331	9.783e-08	[7.799e-02, 0.169]

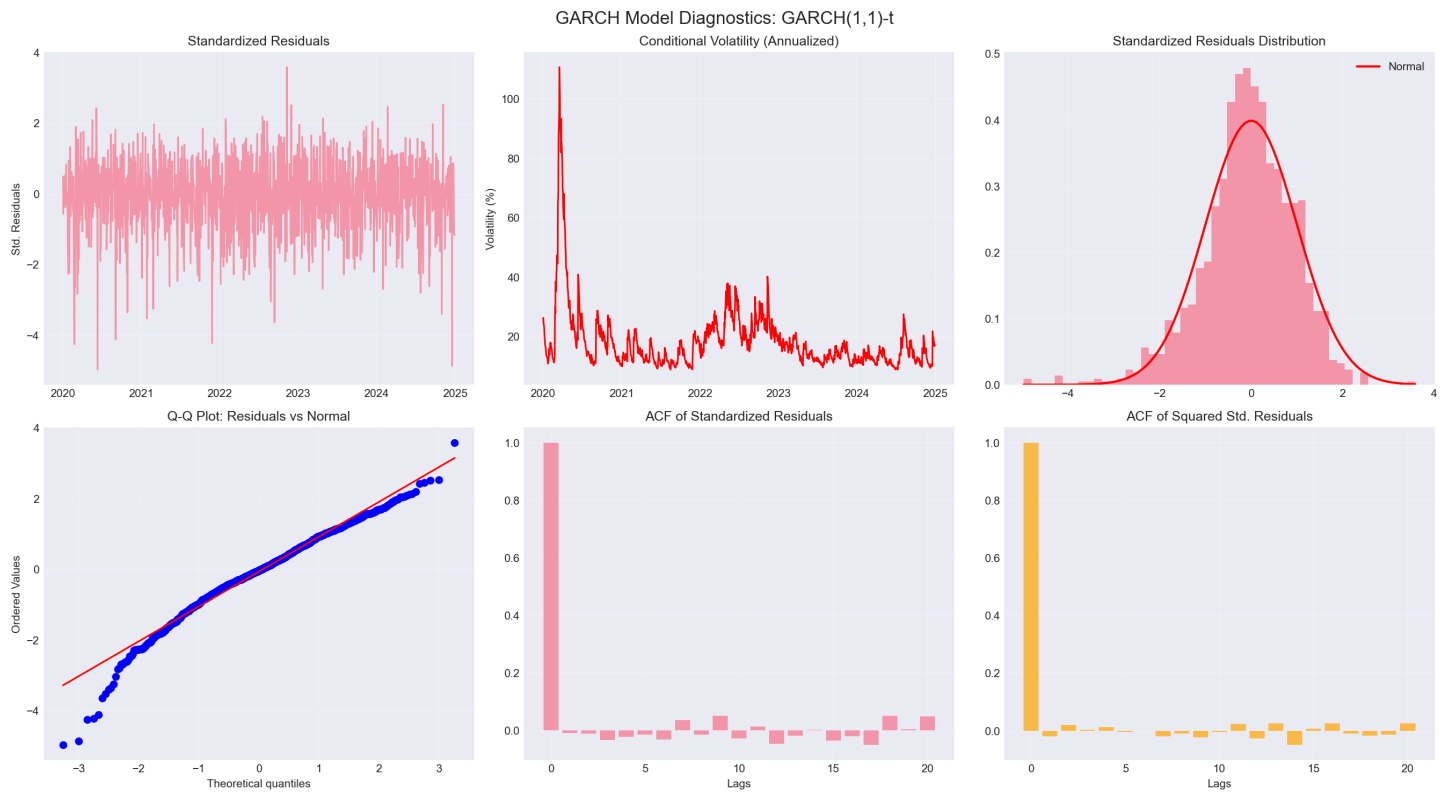
Volatility Model

	coef	std err	t	P> t	95.0% Conf. Int.
omega	0.0380	1.277e-02	2.978	2.900e-03	[1.300e-02, 6.306e-02]
alpha[1]	0.1560	2.872e-02	5.431	5.613e-08	[9.969e-02, 0.212]
beta[1]	0.8247	2.894e-02	28.495	1.349e-178	[ 0.768, 0.881]

Distribution

	coef	std err	t	P> t	95.0% Conf. Int.
nu	7.1747	1.408	5.095	3.488e-07	[ 4.415, 9.935]

Covariance estimator: robust



Residual Diagnostics:

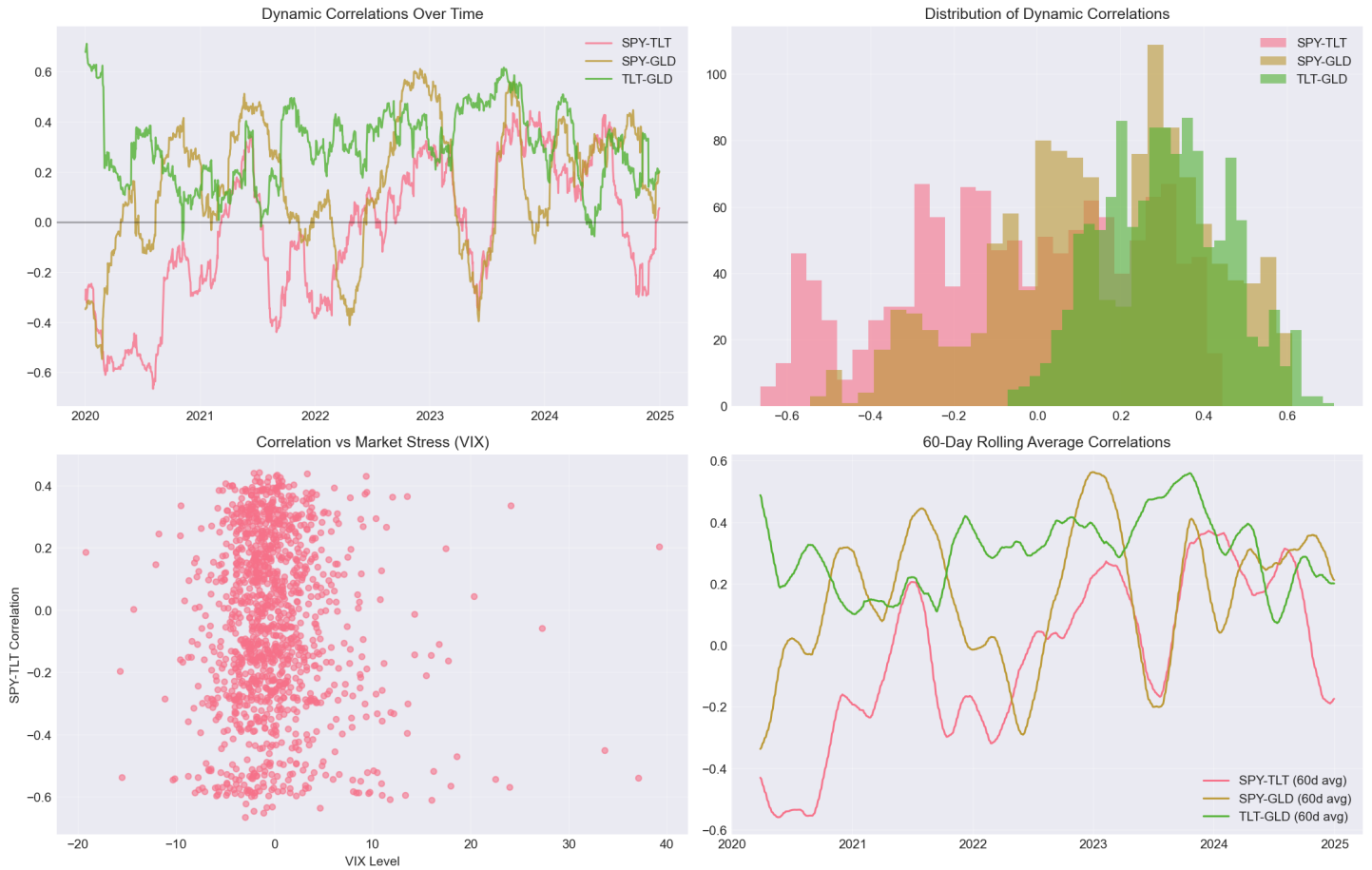
Jarque-Bera (normality): 208.4680 (p=0.000000)

Ljung-Box on squared residuals (ARCH): 0.989379

#### 4. Multivariate GARCH Model

#### SECTION 4: MULTIVARIATE GARCH MODELING

## Dynamic Conditional Correlations (DCC-GARCH)



### Correlation During High VIX Periods (VIX > 2.4):

SPY\_TLT: Crisis=-0.080, Normal=-0.034, Difference=-0.046

SPY\_GLD: Crisis=0.140, Normal=0.155, Difference=-0.015

TLT\_GLD: Crisis=0.303, Normal=0.304, Difference=-0.001

### Portfolio Optimization Using Dynamic Correlations:

#### Average correlations:

SPY-TLT: -0.042

SPY-GLD: 0.145

TLT-GLD: 0.290

Portfolio Volatility: DCC-GARCH vs Historical Methods



Average Portfolio Volatility (DCC): 10.95%  
Average Portfolio Volatility (Historical): 10.66%

Portfolio Analysis:  
Number of observations: 1257  
Date range: 2020-01-02 to 2024-12-30

Volatility Statistics:  
DCC Portfolio Volatility:  
Mean: 10.95%  
Std: 3.85%  
Min: 7.22%  
Max: 42.49%

```
2020-01-02    7.220807
2020-01-03    7.663453
2020-01-06    8.460070
2020-01-07    8.354567
2020-01-08    8.164562
...
2024-12-23   11.113895
2024-12-24   10.753728
2024-12-26   10.468114
2024-12-27    9.999184
2024-12-30   10.050422
Length: 1257, dtype: float64
```

5. Rolling Window Forecast

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SECTION 5: ROLLING WINDOW FORECASTING

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# Simple Rolling GARCH Forecast

Window size: 252 days

Forecast horizon: 30 days

Forecasting every 10 days

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2020-12-31:	Forecast=10.67%,	Actual=15.09%
2021-01-15:	Forecast=13.01%,	Actual=17.39%
2021-02-01:	Forecast=28.24%,	Actual=16.34%
2021-02-16:	Forecast=13.32%,	Actual=16.68%
2021-03-02:	Forecast=21.91%,	Actual=13.54%
2021-03-16:	Forecast=16.42%,	Actual=11.46%
2021-03-30:	Forecast=15.29%,	Actual=10.60%
2021-04-14:	Forecast=11.95%,	Actual=13.41%
2021-04-28:	Forecast=13.32%,	Actual=12.06%
2021-05-12:	Forecast=14.89%,	Actual=12.21%
2021-05-26:	Forecast=14.32%,	Actual=8.23%
2021-06-10:	Forecast=11.00%,	Actual=10.90%
2021-06-24:	Forecast=13.20%,	Actual=10.09%
2021-07-09:	Forecast=13.08%,	Actual=10.19%
2021-07-23:	Forecast=13.36%,	Actual=8.03%
2021-08-06:	Forecast=11.74%,	Actual=8.26%
2021-08-20:	Forecast=12.84%,	Actual=12.58%
2021-09-03:	Forecast=9.26%,	Actual=14.16%
2021-09-20:	Forecast=13.33%,	Actual=13.82%
2021-10-04:	Forecast=17.04%,	Actual=9.61%
2021-10-18:	Forecast=14.34%,	Actual=9.80%
2021-11-01:	Forecast=10.30%,	Actual=14.79%
2021-11-15:	Forecast=10.77%,	Actual=17.51%
2021-11-30:	Forecast=18.89%,	Actual=17.09%
2021-12-14:	Forecast=14.92%,	Actual=15.91%
2021-12-29:	Forecast=13.50%,	Actual=18.03%
2022-01-12:	Forecast=12.52%,	Actual=21.14%
2022-01-27:	Forecast=15.32%,	Actual=24.53%
2022-02-10:	Forecast=17.78%,	Actual=24.58%
2022-02-25:	Forecast=21.92%,	Actual=22.05%
2022-03-11:	Forecast=25.97%,	Actual=20.21%
2022-03-25:	Forecast=20.42%,	Actual=25.73%
2022-04-08:	Forecast=15.73%,	Actual=29.99%
2022-04-25:	Forecast=26.82%,	Actual=31.37%
2022-05-09:	Forecast=36.52%,	Actual=30.86%
2022-05-23:	Forecast=32.65%,	Actual=27.91%
2022-06-07:	Forecast=24.69%,	Actual=26.97%
2022-06-22:	Forecast=34.96%,	Actual=20.27%
2022-07-07:	Forecast=21.78%,	Actual=18.09%
2022-07-21:	Forecast=22.92%,	Actual=20.20%
2022-08-04:	Forecast=21.51%,	Actual=22.46%
2022-08-18:	Forecast=17.10%,	Actual=23.09%
2022-09-01:	Forecast=22.53%,	Actual=26.63%
2022-09-16:	Forecast=27.45%,	Actual=26.26%
2022-09-30:	Forecast=24.80%,	Actual=30.90%
2022-10-14:	Forecast=27.17%,	Actual=26.20%
2022-10-28:	Forecast=25.90%,	Actual=25.63%
2022-11-11:	Forecast=27.70%,	Actual=18.70%



2022-11-28: Forecast=26.05%, Actual=20.31%  
2022-12-12: Forecast=25.56%, Actual=18.24%  
2022-12-27: Forecast=25.12%, Actual=17.10%  
2023-01-11: Forecast=23.71%, Actual=16.37%  
2023-01-26: Forecast=22.49%, Actual=16.57%  
2023-02-09: Forecast=21.87%, Actual=17.16%  
2023-02-24: Forecast=20.24%, Actual=16.47%  
2023-03-10: Forecast=19.97%, Actual=13.92%  
2023-03-24: Forecast=20.84%, Actual=13.29%  
2023-04-10: Forecast=18.15%, Actual=12.77%  
2023-04-24: Forecast=14.65%, Actual=14.33%  
2023-05-08: Forecast=14.93%, Actual=10.82%  
2023-05-22: Forecast=13.83%, Actual=11.02%  
2023-06-06: Forecast=13.98%, Actual=8.98%  
2023-06-21: Forecast=11.99%, Actual=9.75%  
2023-07-06: Forecast=10.99%, Actual=9.86%  
2023-07-20: Forecast=9.78%, Actual=11.58%  
2023-08-03: Forecast=10.08%, Actual=10.87%  
2023-08-17: Forecast=9.83%, Actual=12.28%  
2023-08-31: Forecast=10.20%, Actual=11.31%  
2023-09-15: Forecast=9.41%, Actual=13.19%  
2023-09-29: Forecast=10.67%, Actual=14.10%  
2023-10-13: Forecast=10.68%, Actual=14.49%  
2023-10-27: Forecast=11.48%, Actual=10.71%  
2023-11-10: Forecast=11.91%, Actual=10.18%  
2023-11-27: Forecast=11.90%, Actual=9.55%  
2023-12-11: Forecast=11.45%, Actual=10.01%  
2023-12-26: Forecast=11.49%, Actual=10.71%  
2024-01-10: Forecast=11.33%, Actual=12.29%  
2024-01-25: Forecast=10.80%, Actual=12.50%  
2024-02-08: Forecast=11.37%, Actual=11.28%  
2024-02-23: Forecast=11.64%, Actual=9.69%  
2024-03-08: Forecast=11.52%, Actual=10.79%  
2024-03-22: Forecast=11.55%, Actual=12.66%  
2024-04-08: Forecast=11.51%, Actual=12.43%  
2024-04-22: Forecast=11.45%, Actual=10.71%  
2024-05-06: Forecast=11.71%, Actual=8.09%  
2024-05-20: Forecast=11.45%, Actual=7.47%  
2024-06-04: Forecast=11.54%, Actual=8.35%  
2024-06-18: Forecast=11.27%, Actual=12.44%  
2024-07-03: Forecast=9.79%, Actual=18.72%  
2024-07-18: Forecast=11.28%, Actual=18.98%  
2024-08-01: Forecast=13.49%, Actual=18.87%  
2024-08-15: Forecast=17.65%, Actual=13.23%  
2024-08-29: Forecast=12.13%, Actual=12.72%  
2024-09-13: Forecast=14.45%, Actual=9.38%  
2024-09-27: Forecast=10.91%, Actual=12.82%  
2024-10-11: Forecast=11.14%, Actual=12.52%  
2024-10-25: Forecast=10.43%, Actual=11.83%  
2024-11-08: Forecast=15.99%, Actual=11.90%

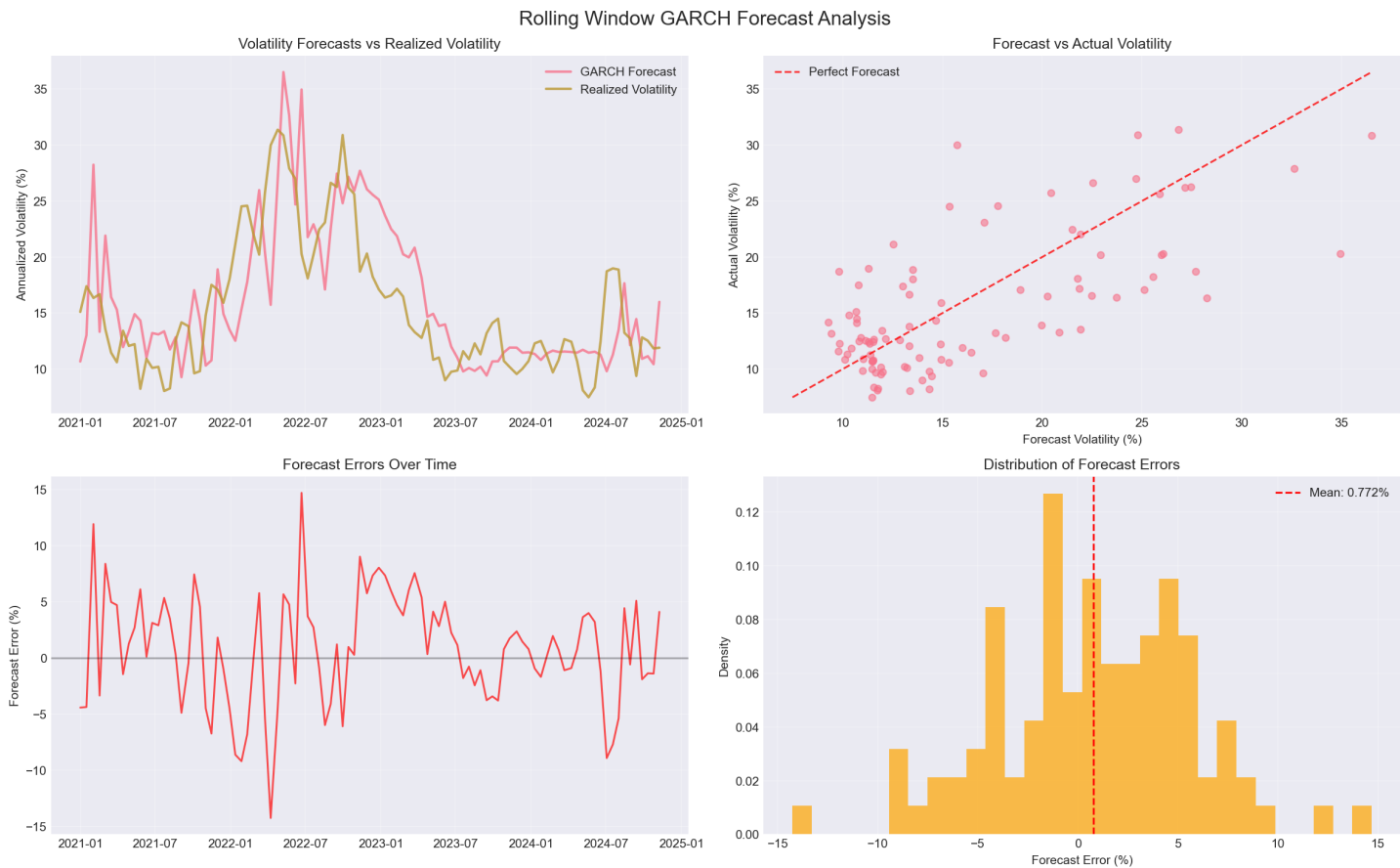
Generated 98 forecasts

Evaluating Forecast Accuracy...

Forecast Accuracy Metrics:

Mean Absolute Error (MAE): 3.922%  
Root Mean Square Error (RMSE): 4.923%  
Mean Absolute Percentage Error (MAPE): 26.47%  
Correlation: 0.6867  
Direction Accuracy: 100.00%  
Number of Observations: 98

```
{'MAE': 3.9217526100812976,  
 'RMSE': 4.922504859308641,  
 'MAPE': 26.46604513906678,  
 'Correlation': 0.6867310442618569,  
 'Direction_Accuracy': 100.0,  
 'N_Observations': 98}
```



	Volatility_Forecast	Actual_Volatility
2020-12-31	10.666763	15.094853
2021-01-15	13.009069	17.387812
2021-02-01	28.242080	16.336801
2021-02-16	13.316347	16.676184
2021-03-02	21.909222	13.537983
...	...	...
2024-09-13	14.454979	9.375789
2024-09-27	10.905995	12.820242

	Volatility_Forecast	Actual_Volatility
2024-10-11	11.137358	12.515752
2024-10-25	10.425896	11.831648
2024-11-08	15.988180	11.895717

6. RISK MANAGEMENT APPLICATIONS

SECTION 5: RISK MANAGEMENT APPLICATIONS

Calculating Dynamic VaR (1-day horizon)  
Confidence levels: [95, 99]%  
Distribution: t

Volatility forecast: 1.117%  
95% VaR: -2.171%  
99% VaR: -3.511%

Portfolio VaR (Portfolio Value: \$1,000,000)

95% VaR: -2.171% = \$21,708  
99% VaR: -3.511% = \$35,109

VaR Summary Table:

Confidence Level	VaR (%)	Volatility Forecast (%)	VaR (\$)
95%	-2.171%	1.117%	\$21,708
99%	-3.511%	1.117%	\$35,109

Interpretation:

- 5% chance of daily loss exceeding 2.17% (\$21,708)
- 1% chance of daily loss exceeding 3.51% (\$35,109)

STRESS TESTING SCENARIOS

Portfolio Value: \$1,000,000

STRESS TEST RESULTS

BASE CASE:

Description: Normal market conditions (1-day GARCH forecast)  
Annual Volatility: 17.7%

95% Daily VaR: -2.17% (\$21,708)  
99% Daily VaR: -3.51% (\$35,109)  
Expected Annual Loss: 7.1% (\$70,937)

MODERATE STRESS:

Description: 1.5x current market volatility  
Annual Volatility: 19.6%  
95% Daily VaR: -38.02% (\$380,175)  
99% Daily VaR: -61.48% (\$614,850)  
Expected Annual Loss: 7.8% (\$78,258)

CRISIS 2008:

Description: 2008-style financial crisis (45% volatility)  
Annual Volatility: 45.0%  
95% Daily VaR: -87.44% (\$874,431)  
99% Daily VaR: -141.42% (\$1,414,201)  
Expected Annual Loss: 27.0% (\$270,000)

EXTREME COVID:

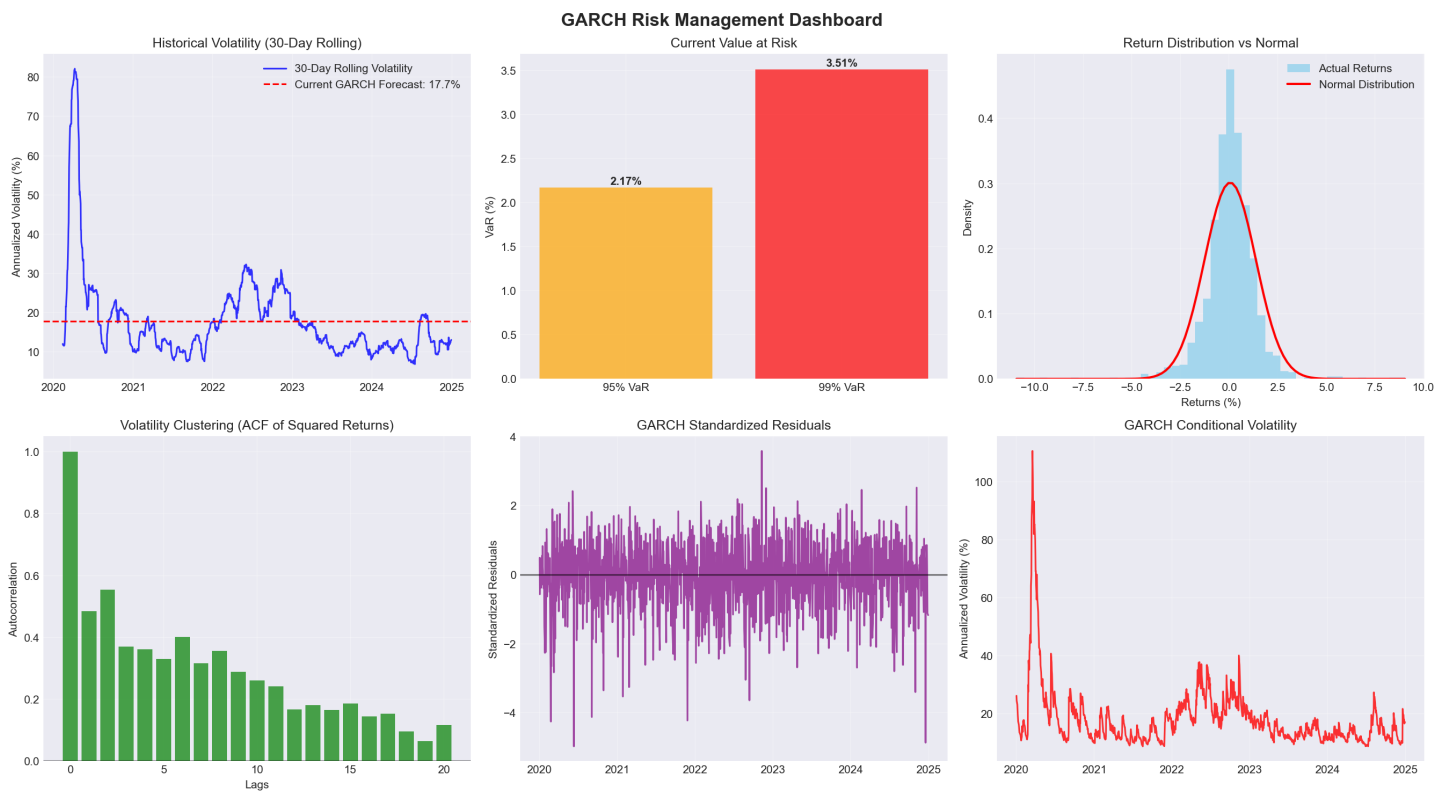
Description: COVID-style market shock (60% volatility)  
Annual Volatility: 60.0%  
95% Daily VaR: -116.59% (\$1,165,908)  
99% Daily VaR: -188.56% (\$1,885,601)  
Expected Annual Loss: 48.0% (\$480,000)

```
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{'Base_Case': {'description': 'Normal market conditions (1-day GARCH forecast)',
  'volatility': 17.734337349931923,
  'var_95': -2.17083987196782,
  'var_99': -3.510857918370043,
  'expected_annual_loss': 7.093734939972769,
  'var_95_dollar': 21708.398719678204,
  'var_99_dollar': 35108.57918370043,
  'annual_loss_dollar': 70937.3493997277},
'Moderate_Stress': {'description': '1.5x current market volatility',
  'volatility': 19.56458373277353,
  'var_95': -38.01751330361893,
  'var_99': -61.48499912052819,
  'expected_annual_loss': 7.825833493109413,
  'var_95_dollar': 380175.13303618925,
  'var_99_dollar': 614849.9912052818,
  'annual_loss_dollar': 78258.33493109413},
'Crisis_2008': {'description': '2008-style financial crisis (45% volatility)',
  'volatility': 45.0,
  'var_95': -87.44311261767517,
  'var_99': -141.42007814809435,
  'expected_annual_loss': 27.0,
  'var_95_dollar': 874431.1261767517,
  'var_99_dollar': 1414200.7814809433,
  'annual_loss_dollar': 270000.0},
'Extreme_COVID': {'description': 'COVID-style market shock (60% volatility)',
  'volatility': 60.0,
```

```
'var_95': -116.5908168235669,  
'var_99': -188.56010419745914,  
'expected_annual_loss': 48.0,  
'var_95_dollar': 1165908.168235669,  
'var_99_dollar': 1885601.0419745916,  
'annual_loss_dollar': 480000.0}}
```

Creating GARCH Risk Management Dashboard...  
<arch.univariate.base.ARCHModelForecast object at 0x000002536A74E210>



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RISK DASHBOARD SUMMARY

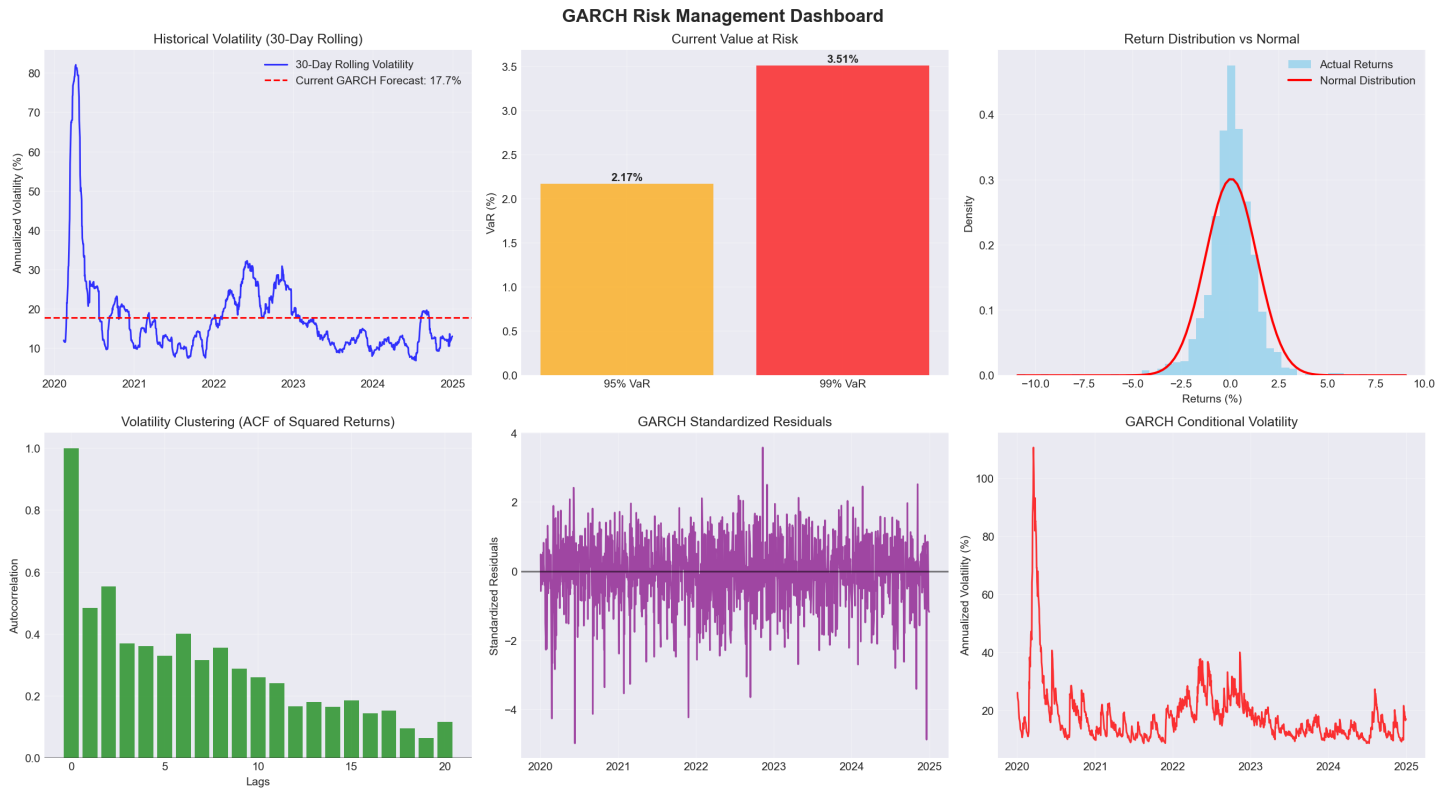
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Current Volatility Forecast: 17.73%  
30-Day Realized Volatility: 13.04%  
1-Year Realized Volatility: 12.57%

95% VaR (1-day): 2.17%  
99% VaR (1-day): 3.51%

Risk Assessment:  
MODERATE RISK: Current volatility elevated

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PORTFOLIO RISK SUMMARY

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Individual Asset Volatilities:

SPY: 21.00% (Weight: 25.0%)  
TLT: 17.97% (Weight: 25.0%)  
GLD: 15.53% (Weight: 25.0%)  
VXX: 75.92% (Weight: 25.0%)

Portfolio Volatility: 17.27%

Correlation Matrix:

	SPY	TLT	GLD	VXX
SPY	1.000	-0.150	0.158	-0.710
TLT	-0.150	1.000	0.267	0.129
GLD	0.158	0.267	1.000	-0.095
VXX	-0.710	0.129	-0.095	1.000

Diversification Analysis:

Weighted Average Volatility: 32.60%  
Portfolio Volatility: 17.27%  
Diversification Benefit: 15.33%

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```
{'portfolio_vol': 17.269997082784307,  
 'individual_vols': SPY    21.000071  
                    TLT    17.969673  
                    GLD    15.525967
```

```
VXX      75.923741
dtype: float64,
'diversification_benefit': 15.334866151843102,
'correlation_matrix':
      SPY      TLT      GLD      VXX
SPY  1.000000 -0.149609  0.158439 -0.710000
TLT -0.149609  1.000000  0.266833  0.129033
GLD  0.158439  0.266833  1.000000 -0.095294
VXX -0.710000  0.129033 -0.095294  1.000000}
```