GARCH

July 31, 2025

1 GARCH Volatility Modeling

1.0.1 Analysis of S&P 500 Volatility Dynamics

This notebook demonstrates advanced volatility modeling techniques using GARCH models applied to S&P 500 data. We progress from basic univariate GARCH to a multivariate model and rolling forecast. Lastly, we'll compile for risk management applications.

Key Analyses:

1. Univariate GARCH modeling and diagnostics
2. Dynamic Conditional Correlation (DCC-GARCH) for multi-asset portfolios
3. Rolling Window Forecasts
4. Risk management applications (VaR, portfolio optimization)

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2 S&P Summary Statistics and Exploratory Data Analysis

2.0.1 S&P 500 Summary Analysis

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S&P 500 Returns Summary Statistics:
Mean: 0.0628%
Std Dev: 1.3229%
Skewness: -0.5443
Kurtosis: 11.5024
Min: -10.9424%
Max: 9.0603%
```

```
'The S&P 500 daily returns exhibit typical characteristics of financial time
series data, with an average daily return of 0.0628% (approximately 16%
annualized). The standard deviation of 1.32% indicates moderate daily
volatility, while the negative skewness of -0.54 reveals a slight tendency
toward larger negative returns than positive ones-a common feature in equity
markets known as the "leverage effect." Most notably, the excess kurtosis of
11.50 demonstrates significant fat tails, indicating that extreme returns (both
positive and negative) occur much more frequently than would be expected under a
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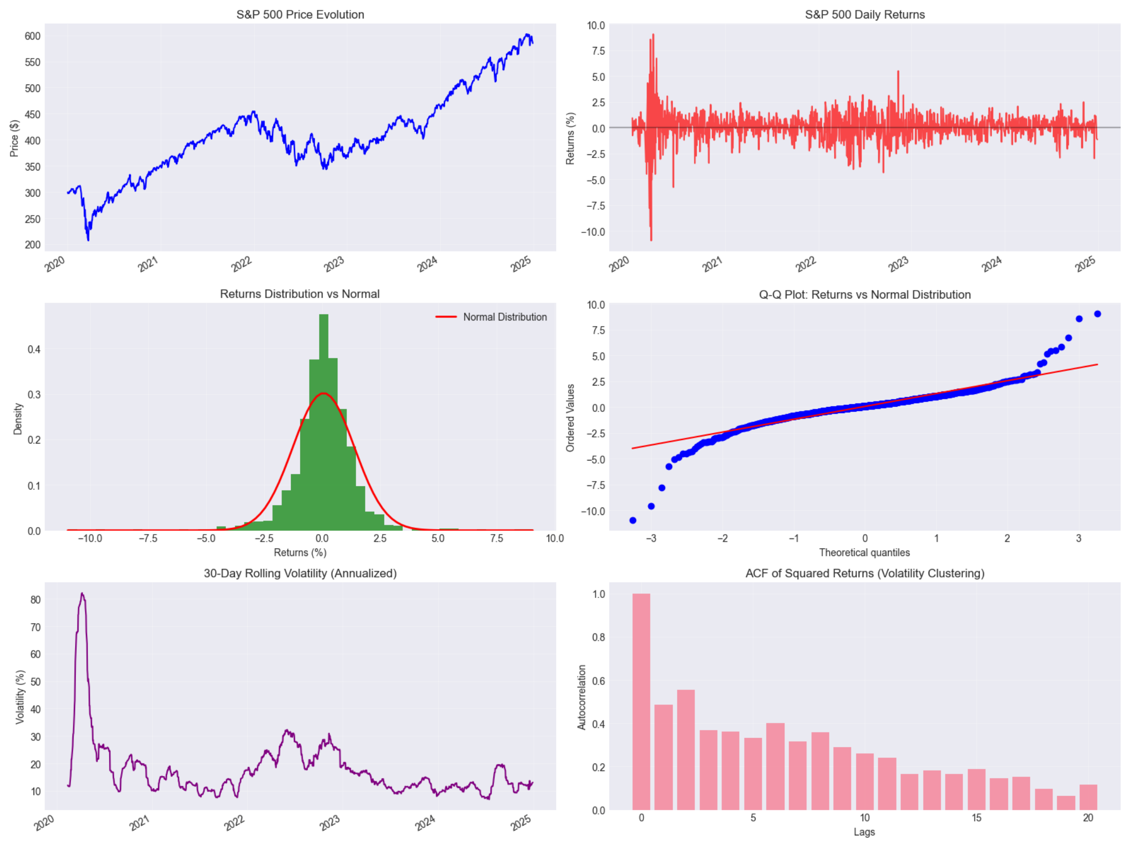
normal distribution. This leptokurtic behavior, combined with the observed range from -10.94% to +9.06%, confirms the presence of volatility clustering and suggests that traditional risk models assuming normal distributions may underestimate tail risks in portfolio management and risk assessment applications.'

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==================================================
STATISTICAL TESTS
==================================================
Jarque-Bera Test for Normality:
  Statistic: 6930.6603
  P-value: 0.000000
  Result: Reject normality

Ljung-Box Test for ARCH Effects (Volatility Clustering):
  P-value (lag 10): 0.000000
  Result: Significant ARCH effects detected
```

3. Univariate GARCH Model

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SECTION 3: UNIVARIATE GARCH MODELING
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Model Comparison (sorted by AIC):
         Model         AIC         BIC  Log-Likelihood  Parameters
  GARCH(1,1)-t 3614.104916 3639.787332    -1802.052458           5
GJR-GARCH(1,1) 3637.199779 3662.882195    -1813.599889           5
    GARCH(1,1) 3658.671270 3679.217203    -1825.335635           4
    GARCH(2,2) 3660.994868 3691.813767    -1824.497434           6
   EGARCH(1,1) 3669.637863 3690.183795    -1830.818931           4

Best model: GARCH(1,1)-t


============================================================
MODEL DIAGNOSTICS: GARCH(1,1)-t
============================================================
                        Constant Mean - GARCH Model Results
================================================================================
====
Dep. Variable:                          SPY   R-squared:
0.000
Mean Model:                   Constant Mean   Adj. R-squared:
0.000
Vol Model:                            GARCH   Log-Likelihood:
-1802.05
Distribution:      Standardized Student's t   AIC:
3614.10
Method:                  Maximum Likelihood   BIC:
3639.79
                                              No. Observations:
1257
Date:                      Thu, Jul 31 2025   Df Residuals:
1256
Time:                              09:06:09   Df Model:
1
                                Mean Model
==========================================================================
                 coef    std err          t      P>|t|    95.0% Conf. Int.
--------------------------------------------------------------------------
mu             0.1233  2.314e-02      5.331  9.785e-08 [7.799e-02,  0.169]
                              Volatility Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
omega          0.0380  1.277e-02      2.978  2.900e-03 [1.300e-02,6.306e-02]
```
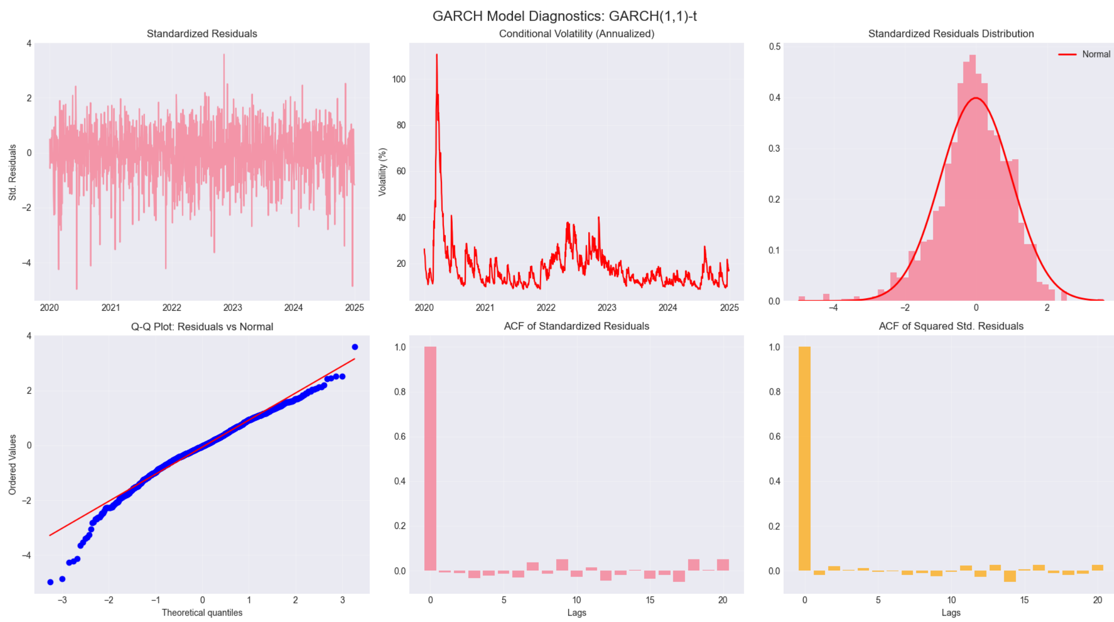

```
alpha[1]        0.1560  2.872e-02      5.431  5.614e-08   [9.969e-02,  0.212]
beta[1]         0.8247  2.894e-02     28.495 1.360e-178     [  0.768,  0.881]
                              Distribution
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
nu              7.1748      1.408      5.095  3.491e-07 [  4.415,  9.935]
========================================================================

Covariance estimator: robust
```




```
Residual Diagnostics:
Jarque-Bera (normality): 208.4697 (p=0.000000)
Ljung-Box on squared residuals (ARCH): 0.989380
```

4. Multivariate GARCH Model

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SECTION 4: MULTIVARIATE GARCH MODELING
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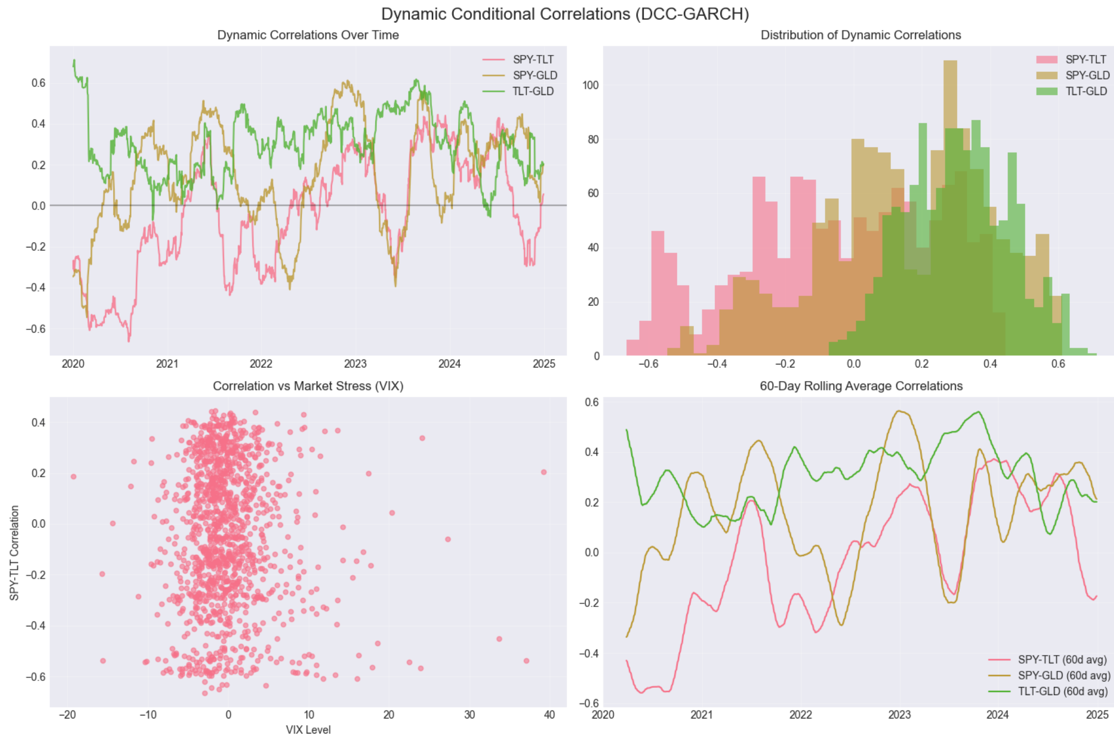


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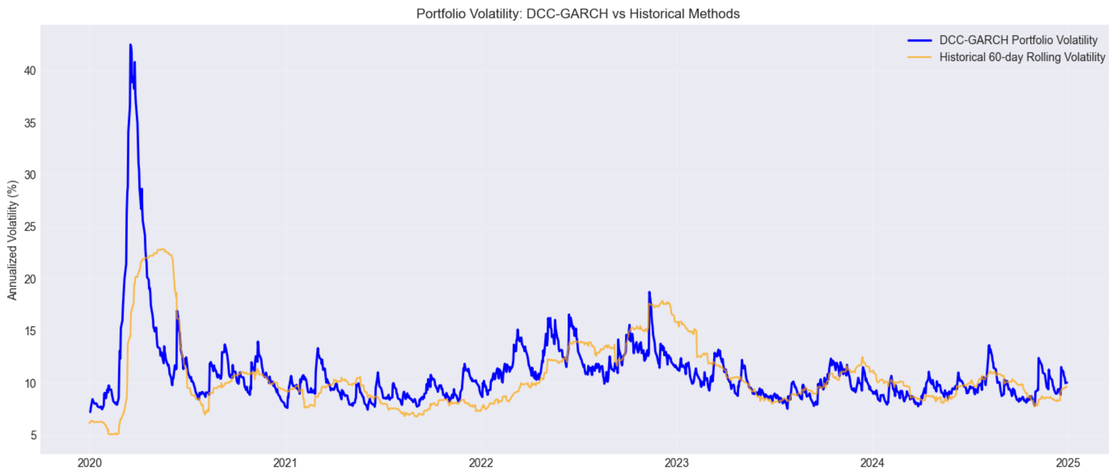
Correlation During High VIX Periods (VIX > 2.4):
  SPY_TLT: Crisis=-0.080, Normal=-0.034, Difference=-0.046
  SPY_GLD: Crisis=0.140, Normal=0.155, Difference=-0.015
  TLT_GLD: Crisis=0.303, Normal=0.304, Difference=-0.001


Portfolio Optimization Using Dynamic Correlations:
Average correlations:
  SPY-TLT: -0.042
  SPY-GLD: 0.145
  TLT-GLD: 0.290
```




```
Average Portfolio Volatility (DCC): 10.95%
Average Portfolio Volatility (Historical): 10.66%

Portfolio Analysis:
Number of observations: 1257
Date range: 2020-01-02 to 2024-12-30

Volatility Statistics:
DCC Portfolio Volatility:
  Mean: 10.95%
  Std:  3.85%
  Min:  7.22%
  Max:  42.49%
```

```
2020-01-02     7.220731
2020-01-03     7.663375
2020-01-06     8.460043
2020-01-07     8.354543
2020-01-08     8.164549
                 …
2024-12-23    11.113840
2024-12-24    10.753672
2024-12-26    10.468055
2024-12-27     9.999121
2024-12-30    10.050365
Length: 1257, dtype: float64
```

5. Rolling Window Forecast

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SECTION 5: ROLLING WINDOW FORECASTING
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Simple Rolling GARCH Forecast
Window size: 252 days
Forecast horizon: 30 days
Forecasting every 10 days
----------------------------------------
2020-12-31: Forecast=10.67%, Actual=15.09%
2021-01-15: Forecast=13.01%, Actual=17.39%
2021-02-01: Forecast=28.24%, Actual=16.34%
2021-02-16: Forecast=13.32%, Actual=16.68%
2021-03-02: Forecast=21.91%, Actual=13.54%
2021-03-16: Forecast=16.42%, Actual=11.46%
2021-03-30: Forecast=15.29%, Actual=10.60%
2021-04-14: Forecast=11.95%, Actual=13.41%
2021-04-28: Forecast=13.32%, Actual=12.06%
2021-05-12: Forecast=14.89%, Actual=12.21%
2021-05-26: Forecast=14.32%, Actual=8.23%
2021-06-10: Forecast=11.00%, Actual=10.90%
2021-06-24: Forecast=13.20%, Actual=10.09%
2021-07-09: Forecast=13.08%, Actual=10.19%
2021-07-23: Forecast=13.36%, Actual=8.03%
2021-08-06: Forecast=11.74%, Actual=8.26%
2021-08-20: Forecast=12.84%, Actual=12.58%
2021-09-03: Forecast=9.26%, Actual=14.16%
2021-09-20: Forecast=13.33%, Actual=13.82%
2021-10-04: Forecast=17.04%, Actual=9.61%
2021-10-18: Forecast=14.34%, Actual=9.80%
2021-11-01: Forecast=10.30%, Actual=14.79%
2021-11-15: Forecast=10.77%, Actual=17.51%
2021-11-30: Forecast=18.89%, Actual=17.09%
2021-12-14: Forecast=14.92%, Actual=15.91%
2021-12-29: Forecast=13.50%, Actual=18.03%
2022-01-12: Forecast=12.52%, Actual=21.14%
2022-01-27: Forecast=15.32%, Actual=24.53%
2022-02-10: Forecast=17.78%, Actual=24.58%
2022-02-25: Forecast=21.92%, Actual=22.05%
2022-03-11: Forecast=25.97%, Actual=20.21%
2022-03-25: Forecast=20.42%, Actual=25.73%
2022-04-08: Forecast=15.73%, Actual=29.99%
2022-04-25: Forecast=26.82%, Actual=31.37%
2022-05-09: Forecast=36.52%, Actual=30.86%
2022-05-23: Forecast=32.65%, Actual=27.91%
2022-06-07: Forecast=24.69%, Actual=26.97%
2022-06-22: Forecast=34.96%, Actual=20.27%
2022-07-07: Forecast=21.78%, Actual=18.09%
2022-07-21: Forecast=22.92%, Actual=20.20%
```

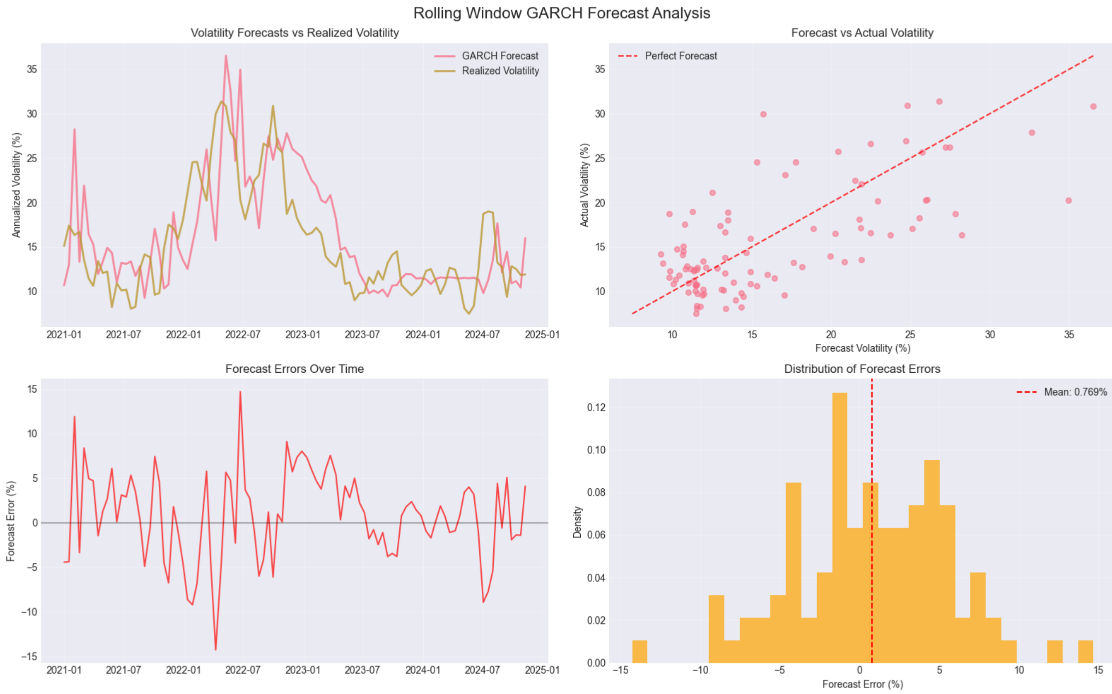
```
2022-08-04: Forecast=21.51%, Actual=22.46%
2022-08-18: Forecast=17.10%, Actual=23.09%
2022-09-01: Forecast=22.50%, Actual=26.63%
2022-09-16: Forecast=27.45%, Actual=26.26%
2022-09-30: Forecast=24.80%, Actual=30.90%
2022-10-14: Forecast=27.18%, Actual=26.20%
2022-10-28: Forecast=25.73%, Actual=25.63%
2022-11-11: Forecast=27.80%, Actual=18.70%
2022-11-28: Forecast=26.02%, Actual=20.31%
2022-12-12: Forecast=25.56%, Actual=18.24%
2022-12-27: Forecast=25.12%, Actual=17.10%
2023-01-11: Forecast=23.71%, Actual=16.37%
2023-01-26: Forecast=22.49%, Actual=16.57%
2023-02-09: Forecast=21.87%, Actual=17.16%
2023-02-24: Forecast=20.24%, Actual=16.47%
2023-03-10: Forecast=19.97%, Actual=13.92%
2023-03-24: Forecast=20.84%, Actual=13.29%
2023-04-10: Forecast=18.15%, Actual=12.77%
2023-04-24: Forecast=14.65%, Actual=14.33%
2023-05-08: Forecast=14.93%, Actual=10.82%
2023-05-22: Forecast=13.83%, Actual=11.02%
2023-06-06: Forecast=13.98%, Actual=8.98%
2023-06-21: Forecast=11.99%, Actual=9.75%
2023-07-06: Forecast=10.99%, Actual=9.86%
2023-07-20: Forecast=9.78%, Actual=11.58%
2023-08-03: Forecast=10.08%, Actual=10.87%
2023-08-17: Forecast=9.83%, Actual=12.28%
2023-08-31: Forecast=10.19%, Actual=11.31%
2023-09-15: Forecast=9.40%, Actual=13.19%
2023-09-29: Forecast=10.65%, Actual=14.10%
2023-10-13: Forecast=10.70%, Actual=14.49%
2023-10-27: Forecast=11.47%, Actual=10.71%
2023-11-10: Forecast=11.95%, Actual=10.18%
2023-11-27: Forecast=11.90%, Actual=9.55%
2023-12-11: Forecast=11.44%, Actual=10.01%
2023-12-26: Forecast=11.49%, Actual=10.71%
2024-01-10: Forecast=11.35%, Actual=12.29%
2024-01-25: Forecast=10.81%, Actual=12.50%
2024-02-08: Forecast=11.37%, Actual=11.28%
2024-02-23: Forecast=11.57%, Actual=9.69%
2024-03-08: Forecast=11.51%, Actual=10.79%
2024-03-22: Forecast=11.57%, Actual=12.66%
2024-04-08: Forecast=11.52%, Actual=12.43%
2024-04-22: Forecast=11.45%, Actual=10.71%
2024-05-06: Forecast=11.52%, Actual=8.09%
2024-05-20: Forecast=11.47%, Actual=7.47%
2024-06-04: Forecast=11.53%, Actual=8.35%
2024-06-18: Forecast=11.34%, Actual=12.44%
```

```
2024-07-03: Forecast=9.80%, Actual=18.72%
2024-07-18: Forecast=11.28%, Actual=18.98%
2024-08-01: Forecast=13.49%, Actual=18.87%
2024-08-15: Forecast=17.65%, Actual=13.23%
2024-08-29: Forecast=12.13%, Actual=12.72%
2024-09-13: Forecast=14.46%, Actual=9.38%
2024-09-27: Forecast=10.91%, Actual=12.82%
2024-10-11: Forecast=11.14%, Actual=12.52%
2024-10-25: Forecast=10.43%, Actual=11.83%
2024-11-08: Forecast=15.99%, Actual=11.90%

  Generated 98 forecasts


Evaluating Forecast Accuracy…
Forecast Accuracy Metrics:
  Mean Absolute Error (MAE): 3.918%
  Root Mean Square Error (RMSE): 4.922%
  Mean Absolute Percentage Error (MAPE): 26.43%
  Correlation: 0.6867
  Direction Accuracy: 100.00%
  Number of Observations: 98

 {'MAE': 3.9177580497776483,
  'RMSE': 4.922394046111131,
  'MAPE': 26.42842547270497,
  'Correlation': 0.6866832348974241,
  'Direction_Accuracy': 100.0,
  'N_Observations': 98}
```




```
           Volatility_Forecast  Actual_Volatility
2020-12-31           10.666828          15.094922
2021-01-15           13.009151          17.387831
2021-02-01           28.242275          16.336778
2021-02-16           13.316422          16.676188
2021-03-02           21.909158          13.538013
...                        ...                ...
2024-09-13           14.455022           9.375735
2024-09-27           10.905954          12.820220
2024-10-11           11.137445          12.515732
2024-10-25           10.425922          11.831633
2024-11-08           15.988153          11.895724

[98 rows x 2 columns]
```

6. RISK MANAGEMENT APPLICATIONS

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SECTION 5: RISK MANAGEMENT APPLICATIONS
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Calculating Dynamic VaR (1-day horizon)
Confidence levels: [95, 99]%
Distribution: t
```

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--------------------------------------------------
Volatility forecast: 1.117%
95% VaR: -2.171%
99% VaR: -3.511%
--------------------------------------------------

Portfolio VaR (Portfolio Value: $1,000,000)
--------------------------------------------------
95% VaR: -2.171% = $21,708
99% VaR: -3.511% = $35,109
--------------------------------------------------


VaR Summary Table:
============================================================
Confidence Level VaR (%) Volatility Forecast (%) VaR ($)
             95% -2.171%                  1.117% $21,708
             99% -3.511%                  1.117% $35,109
============================================================

Interpretation:
• 5% chance of daily loss exceeding 2.17% ($21,708)
• 1% chance of daily loss exceeding 3.51% ($35,109)
STRESS TESTING SCENARIOS
============================================================
Portfolio Value: $1,000,000
============================================================

STRESS TEST RESULTS
--------------------------------------------------------------------------------

BASE CASE:
  Description: Normal market conditions (1-day GARCH forecast)
  Annual Volatility: 17.7%
  95% Daily VaR: -2.17% ($21,708)
  99% Daily VaR: -3.51% ($35,109)
  Expected Annual Loss: 7.1% ($70,937)

MODERATE STRESS:
  Description: 1.5x current market volatility
  Annual Volatility: 19.6%
  95% Daily VaR: -38.02% ($380,175)
  99% Daily VaR: -61.49% ($614,850)
  Expected Annual Loss: 7.8% ($78,258)

CRISIS 2008:
  Description: 2008-style financial crisis (45% volatility)
  Annual Volatility: 45.0%
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  95% Daily VaR: -87.44% ($874,431)
  99% Daily VaR: -141.42% ($1,414,201)
  Expected Annual Loss: 27.0% ($270,000)

EXTREME COVID:
  Description: COVID-style market shock (60% volatility)
  Annual Volatility: 60.0%
  95% Daily VaR: -116.59% ($1,165,908)
  99% Daily VaR: -188.56% ($1,885,601)
  Expected Annual Loss: 48.0% ($480,000)

================================================================================

 {'Base_Case': {'description': 'Normal market conditions (1-day GARCH forecast)',
   'volatility': 17.734332405627537,
   'var_95': -2.1708392667411975,
   'var_99': -3.51085693954857,
   'expected_annual_loss': 7.093732962251015,
   'var_95_dollar': 21708.392667411976,
   'var_99_dollar': 35108.5693954857,
   'annual_loss_dollar': 70937.32962251015},
  'Moderate_Stress': {'description': '1.5x current market volatility',
   'volatility': 19.564595846833342,
   'var_95': -38.01753684342106,
   'var_99': -61.48503719100118,
   'expected_annual_loss': 7.8258383387333375,
   'var_95_dollar': 380175.3684342106,
   'var_99_dollar': 614850.3719100117,
   'annual_loss_dollar': 78258.38338733338},
  'Crisis_2008': {'description': '2008-style financial crisis (45% volatility)',
   'volatility': 45.0,
   'var_95': -87.44311261767517,
   'var_99': -141.42007814809435,
   'expected_annual_loss': 27.0,
   'var_95_dollar': 874431.1261767517,
   'var_99_dollar': 1414200.7814809433,
   'annual_loss_dollar': 270000.0},
  'Extreme_COVID': {'description': 'COVID-style market shock (60% volatility)',
   'volatility': 60.0,
   'var_95': -116.5908168235669,
   'var_99': -188.56010419745914,
   'expected_annual_loss': 48.0,
   'var_95_dollar': 1165908.168235669,
   'var_99_dollar': 1885601.0419745916,
   'annual_loss_dollar': 480000.0}}

Creating GARCH Risk Management Dashboard…
<arch.univariate.base.ARCHModelForecast object at 0x000001B51F622310>
```

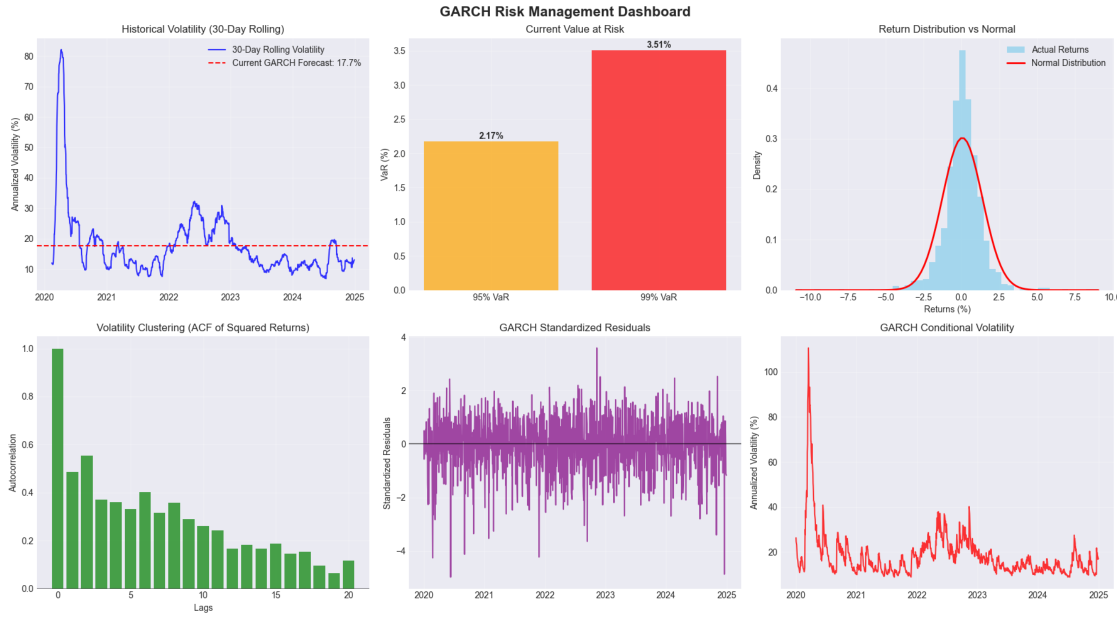


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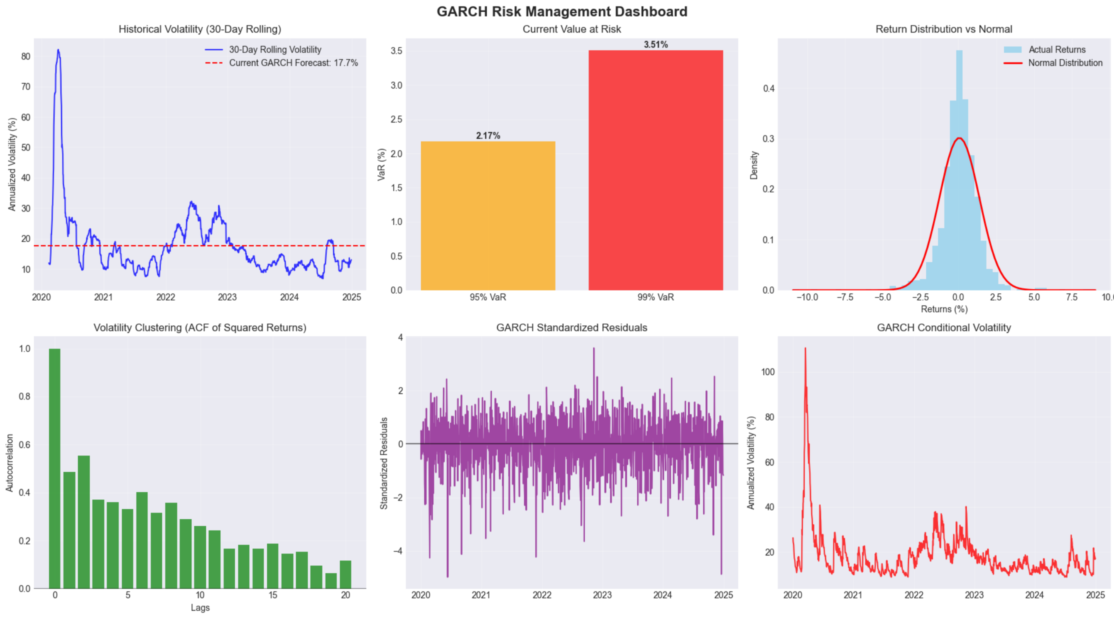
============================================================
RISK DASHBOARD SUMMARY
============================================================
Current Volatility Forecast: 17.73%
30-Day Realized Volatility: 13.04%
1-Year Realized Volatility: 12.57%

95% VaR (1-day): 2.17%
99% VaR (1-day): 3.51%

Risk Assessment:
  MODERATE RISK: Current volatility elevated
============================================================
```

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==================================================
PORTFOLIO RISK SUMMARY
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Individual Asset Volatilities:
  SPY: 21.00% (Weight: 25.0%)
  TLT: 17.97% (Weight: 25.0%)
  GLD: 15.53% (Weight: 25.0%)
  VXX: 75.92% (Weight: 25.0%)

Portfolio Volatility: 17.27%

Correlation Matrix:
       SPY    TLT    GLD    VXX
SPY  1.000 -0.150  0.158 -0.710
TLT -0.150  1.000  0.267  0.129
GLD  0.158  0.267  1.000 -0.095
VXX -0.710  0.129 -0.095  1.000

Diversification Analysis:
  Weighted Average Volatility: 32.60%
  Portfolio Volatility: 17.27%
  Diversification Benefit: 15.33%
==================================================
```

```
{'portfolio_vol': 17.270001792850035,
 'individual_vols': SPY    21.000075
```

```
TLT    17.969708
GLD    15.525967
VXX    75.923741
dtype: float64,
'diversification_benefit': 15.33487094512623,
'correlation_matrix':            SPY       TLT       GLD       VXX
SPY  1.000000 -0.149611  0.158440 -0.710000
TLT -0.149611  1.000000  0.266832  0.129034
GLD  0.158440  0.266832  1.000000 -0.095294
VXX -0.710000  0.129034 -0.095294  1.000000}
```