# **Appendix 1: Data Description**

The data used in this paper come from a variety of sources.

* GDP per capita

GDP per capita is measured in 2010 US dollars at constant prices, published by the *World Bank national accounts data*, and *OECD National Accounts data files*, available at:

<https://data.worldbank.org/indicator/NY.GDP.PCAP.KD?view=chart>

* Human capital return

Human capital return is calibrated by the return to another year of schooling based on the Annex Table 1 in Montenegro & Patrinos (2014) published by the *World Bank Education Global Practice Group*, available at: <http://econ.worldbank.org>

* Personal income tax allowance

The personal income tax allowance is calculated by the sum of the threshold of annual personal income tax plus annual personal deductions, transferred by the average foreign exchange rate against the US dollars in 2016. Sources of the tax information in each country are listed below.

Canada: *Government of Canada*, available at:

<https://www.canada.ca/en/revenue-agency/news/newsroom/fact-sheets/fact-sheets-2015/2016-indexation-adjustment-personal-income-tax-benefit-amounts.html>

China: *Ministry of Finance of the People’s Republic of China*, available at:

<http://szs.mof.gov.cn/shuizhijianjie/200806/t20080630_54460.htm>

France, Germany, Italy, US: *Taxing Wages 2017 published by* *OECD*, available at:

<https://www.oecd-ilibrary.org/taxation/taxing-wages-2017_tax_wages-2017-en>

Japan: *Ministry of Finance Japan*, available at:

<https://www.mof.go.jp/english/tax_policy/tax_system/income/index.html>

UK: *GOV.UK*, available at:

<https://www.gov.uk/government/publications/rates-and-allowances-income-tax/income-tax-rates-and-allowances-current-and-past>

* Poverty line

The poverty line is computed following OECD methodology, i.e., the half of the median disposable household income. Data for G7 countries are collected in the OECD Database, available at: <https://data.oecd.org/hha/household-disposable-income.htm>

Data for China are from National Bureau of Statistics of China, available at:

<http://data.stats.gov.cn/easyquery.htm?cn=C01>

* Marginal consumption propensity

It is estimated from the consumption equation, which is equal to the ratio of (consumption per capita – poverty line) over disposable income per capita. The data on household consumption per capita and disposable income per capita are collected from the OECD Database.

# **Appendix 2: Transition Matrices**

Table 10 Social Mobility Transition Matrices of Thought Experiments

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Tax Model** | **Bottom** | **Lower** | **Middle** | **Upper** | **Top** |
|  | **20%** | **20%** | **20%** | **20%** | **20%** |
| Bottom 20% | 46.5% | 21.5% | 18.0% | 9.0% | 5.0% |
| Lower 20% | 24.5% | 26.0% | 25.5% | 14.0% | 10.0% |
| Middle 20% | 16.0% | 21.0% | 23.5% | 22.5% | 17.0% |
| Upper 20% | 9.5% | 19.5% | 19.5% | 27.5% | 24.0% |
| Top 20% | 3.5% | 12.0% | 13.5% | 27.0% | 44.0% |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Growth Model** | **Bottom** | **Lower** | **Middle** | **Upper** | **Top** |
|  | **20%** | **20%** | **20%** | **20%** | **20%** |
| Bottom 20% | 21.5% | 21.0% | 17.5% | 21.0% | 19.0% |
| Lower 20% | 21.5% | 16.5% | 16.5% | 26.0% | 19.5% |
| Middle 20% | 19.5% | 20.0% | 26.0% | 15.5% | 19.0% |
| Upper 20% | 19.0% | 17.5% | 18.5% | 20.5% | 24.5% |
| Top 20% | 18.5% | 25.0% | 21.5% | 17.0% | 18.0% |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Human Capital Model** | **Bottom** | **Lower** | **Middle** | **Upper** | **Top** |
|  | **20%** | **20%** | **20%** | **20%** | **20%** |
| Bottom 20% | 35.5% | 31.5% | 22.0% | 10.0% | 1.0% |
| Lower 20% | 30.5% | 36.5% | 19.5% | 12.0% | 1.5% |
| Middle 20% | 20.5% | 20.0% | 28.5% | 26.5% | 4.5% |
| Upper 20% | 13.0% | 11.5% | 27.5% | 28.0% | 20.0% |
| Top 20% | 0.5% | 0.5% | 2.5% | 23.5% | 73.0% |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Physical Capital Model** | **Bottom** | **Lower** | **Middle** | **Upper** | **Top** |
|  | **20%** | **20%** | **20%** | **20%** | **20%** |
| Bottom 20% | 82.5% | 11.5% | 0.5% | 5.0% | 0.5% |
| Lower 20% | 0.0% | 60.5% | 8.0% | 20.0% | 11.5% |
| Middle 20% | 9.5% | 1.0% | 54.5% | 19.5% | 15.5% |
| Upper 20% | 5.5% | 22.5% | 26.5% | 34.5% | 11.0% |
| Top 20% | 2.5% | 4.5% | 10.5% | 21.0% | 61.5% |

Table 11 Social Mobility Matrices of Hybrid Models (G7+C)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Canada** | **Bottom** | **Lower** | **Middle** | **Upper** | **Top** |
|  | **20%** | **20%** | **20%** | **20%** | **20%** |
| Bottom 20% | 69.5% | 26.5% | 4.0% | 0.0% | 0.0% |
| Lower 20% | 51.5% | 34.0% | 14.5% | 0.0% | 0.0% |
| Middle 20% | 9.0% | 9.5% | 75.0% | 6.5% | 0.0% |
| Upper 20% | 0.0% | 0.0% | 6.5% | 89.0% | 4.5% |
| Top 20% | 0.0% | 0.0% | 0.0% | 4.5% | 95.5% |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **China** | **Bottom** | **Lower** | **Middle** | **Upper** | **Top** |
|  | **20%** | **20%** | **20%** | **20%** | **20%** |
| Bottom 20% | 34.5% | 44.5% | 18.5% | 2.5% | 0.0% |
| Lower 20% | 36.0% | 36.5% | 24.0% | 3.5% | 0.0% |
| Middle 20% | 29.0% | 29.5% | 29.0% | 12.5% | 0.0% |
| Upper 20% | 0.5% | 3.0% | 15.0% | 73.0% | 8.5% |
| Top 20% | 0.0% | 0.0% | 0.0% | 8.5% | 91.5% |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **France** | **Bottom** | **Lower** | **Middle** | **Upper** | **Top** |
|  | **20%** | **20%** | **20%** | **20%** | **20%** |
| Bottom 20% | 66.0% | 32.5% | 1.5% | 0.0% | 0.0% |
| Lower 20% | 46.5% | 44.0% | 9.5% | 0.0% | 0.0% |
| Middle 20% | 1.5% | 9.5% | 82.0% | 7.0% | 0.0% |
| Upper 20% | 0.0% | 0.0% | 7.0% | 89.0% | 4.0% |
| Top 20% | 0.0% | 0.0% | 0.0% | 4.0% | 96.0% |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Germany** | **Bottom** | **Lower** | **Middle** | **Upper** | **Top** |
|  | **20%** | **20%** | **20%** | **20%** | **20%** |
| Bottom 20% | 68.5% | 21.5% | 10.0% | 0.0% | 0.0% |
| Lower 20% | 59.5% | 25.5% | 15.0% | 0.0% | 0.0% |
| Middle 20% | 14.0% | 11.0% | 69.5% | 5.5% | 0.0% |
| Upper 20% | 0.0% | 0.0% | 5.5% | 90.0% | 4.5% |
| Top 20% | 0.0% | 0.0% | 0.0% | 4.5% | 95.5% |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Italy** | **Bottom** | **Lower** | **Middle** | **Upper** | **Top** |
|  | **20%** | **20%** | **20%** | **20%** | **20%** |
| Bottom 20% | 69.0% | 30.5% | 0.5% | 0.0% | 0.0% |
| Lower 20% | 36.5% | 53.5% | 10.0% | 0.0% | 0.0% |
| Middle 20% | 0.0% | 10.5% | 83.0% | 6.5% | 0.0% |
| Upper 20% | 0.0% | 0.0% | 6.5% | 90.0% | 3.5% |
| Top 20% | 0.0% | 0.0% | 0.0% | 3.5% | 96.5% |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Japan** | **Bottom** | **Lower** | **Middle** | **Upper** | **Top** |
|  | **20%** | **20%** | **20%** | **20%** | **20%** |
| Bottom 20% | 75.5% | 24.5% | 0.0% | 0.0% | 0.0% |
| Lower 20% | 24.5% | 60.0% | 15.5% | 0.0% | 0.0% |
| Middle 20% | 0.0% | 15.5% | 74.5% | 10.0% | 0.0% |
| Upper 20% | 0.0% | 0.0% | 10.0% | 83.0% | 7.0% |
| Top 20% | 0.0% | 0.0% | 0.0% | 7.0% | 93.0% |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **UK** | **Bottom** | **Lower** | **Middle** | **Upper** | **Top** |
|  | **20%** | **20%** | **20%** | **20%** | **20%** |
| Bottom 20% | 56.5% | 39.0% | 4.5% | 0.0% | 0.0% |
| Lower 20% | 44.0% | 41.0% | 15.0% | 0.0% | 0.0% |
| Middle 20% | 9.5% | 10.0% | 73.0% | 7.5% | 0.0% |
| Upper 20% | 0.0% | 0.0% | 7.5% | 87.5% | 5.0% |
| Top 20% | 0.0% | 0.0% | 0.0% | 5.0% | 95.0% |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **US** | **Bottom** | **Lower** | **Middle** | **Upper** | **Top** |
|  | **20%** | **20%** | **20%** | **20%** | **20%** |
| Bottom 20% | 39.0% | 45.5% | 12.0% | 3.5% | 0.0% |
| Lower 20% | 31.5% | 44.5% | 21.0% | 3.0% | 0.0% |
| Middle 20% | 30.0% | 42.5% | 20.0% | 7.5% | 0.0% |
| Upper 20% | 2.0% | 4.0% | 8.0% | 83.5% | 2.5% |
| Top 20% | 0.0% | 0.0% | 0.0% | 2.5% | 97.5% |

# **Appendix 3: Proof of a diminishing thickness index as drops**

The complementary CDF or tail distribution (defined as 1 minus the CDF) which follows a power law can be written as:

If the entire distribution follows the same power law, then for any , we have:

Now assume that, for all values including , the distribution follows an exponential law. The cutting-off point is where the two distributions meet. The complementary CDF is therefore:

for

The PDFs of the two distributions can be obtained by differentiating CDF, so:

Power law:

Exponential law:

We now prove the following theorem.

[Theorem] The estimated thickness index diminishes as more observations from exponential distribution are added in the sample.

We break down the proof into two steps. First, the difference between the two distributions is shown. Second, the effect of the difference on the estimated is derived.

[Step 1]

The PDF of the exponential part is lower than the power-law counterpart for . This can be shown by resorting to two special points: and .

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Both PDFs are continuous and monotonic, so for . We use the following figure to show the intuition of the proof of step 1. The overall PDF includes the exponential-law part when and the power-law part when .

density

A

C

B

[Step 2]

According to the property of power-law distribution, we have the condition below:

Without losing generality. let’s pick and use the fact that is just the integration of the PDF for . The equation above can be expressed with the help of the illustrations in the figure:

If observations below come from an exponential distribution and we force the use of the condition above to estimate the thickness , then we have:

Simple analysis leads to the conclusion that .

In fact, this proof is not limited in exponential distribution. Any thinner distribution than the richest tail will have the same effect. That is why the rolling window estimation of has a downward trend. A reverse trend is also possible if the distribution is thicker. The intuition behind this theorem is that the thickness of the tail for a combined distribution is boosted if the lower end is thinner.